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Process scheduling under uncertainty: Review and challenges

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Abstract

Uncertainty is a very important concern in production scheduling since it can cause infeasibilities and production disturbances. Thus scheduling under uncertainty has received a lot of attention in the open literature in recent years from chemical engineering and operations research communities. The purpose of this paper is to review the main methodologies that have been developed to address the problem of uncertainty in production scheduling as well as to identify the main challenges in this area. The uncertainties in process scheduling are first analyzed, and the different mathematical approaches that exist to describe process uncertainties are classified. Based on the different descriptions for the uncertainties, alternative scheduling approaches and relevant optimization models are reviewed and discussed. Further research challenges in the field of process scheduling under uncertainty are identified and some new ideas are discussed.

Keywords: Process uncertainty; Reactive scheduling; Stochastic scheduling; Robust optimization; Sensitivity analysis; Parametric programming

1. Introduction

Scheduling is an important decision-making process in process industries. In an industrial process, each task requires certain amounts of specified resources for a specific time interval called the processing time. The resources include the use of equipment, the utilization of raw material or intermediates, the employment of operators, etc., and tasks involve the chemical or physical transformation of materials, transportation of products or intermediates, cleaning, and maintenance operations, etc. The purpose of scheduling is to optimally allocate limited resources to processing tasks over time. The scheduling objective can take many forms such as minimizing the time required to complete all the tasks (the makespan), minimizing the number of orders completed after their committed due dates, maximizing customer satisfaction by completing orders in a timely fashion, maximizing plant throughput, maximizing profit or minimizing production costs. Scheduling decisions to be determined include the optimal sequence of tasks taking place in each unit, the amount of material being processed at each time in each unit and the processing time of each task in each unit.

There are a great variety of aspects that need to be considered when developing scheduling models. The basis of scheduling is a proper description of the production process. There is a plethora of different approaches that appear in the literature to address the problem of scheduling formulation, a recent review about classification of scheduling problems is given by Méndez, Cerdá, Grossmann, Harjunkoski, and Fahl (2006). One major classification is based on the nature of the production facility to manufacture the required number of products utilizing a limited set of units. If every job consists of the same set of tasks to be performed in the same order and the units are accordingly arranged in production lines, it is classified as a multiproduct plant (also called flow-shop problem). If production orders have different routes (require different sequences of tasks) and some orders may even visit a given unit several times, it is known as multipurpose plant (also called job-shop problem).

A number of alternative ways of formulating the scheduling problem exist in the open literature. One distinguishing characteristic is the time representation, according to which the approaches are classified into two broad categories. Early attempts of formulating the scheduling problem were mainly concentrated on the discrete-time formulation, where the time horizon is divided into a number of intervals of equal duration (Bassett, Pekny, & Reklaitis, 1996; Elkamel, Zentner, Pekny, & Reklaitis, 1997; Kondili, Pantelides, & Sargent, 1993). Recently,

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a large number of publications have focused on developing efficient methods based on a continuous-time representation (Floudas & Lin, 2004). Following the continuous time representation a number of alternatives have appeared in the literature targeting the reduction of computational complexity of the resulting model. Thus there are formulations using global time intervals that coincide with the discrete time representation, global time points that uses a common time grid for all resources, unit-specific time events that utilizes different event points for different units in the production facility, as well as synchronous/asynchronous time slots that use a set of predefined time slots of unknown duration.

Most of the work in the area of scheduling deals with the deterministic optimization model where all the parameters are considered known. Along with the studies in deterministic scheduling, consideration of uncertainties in the scheduling problem has got more attention in recent yeas. In real plants uncertainty is a very important concern that is coupled with the scheduling process since many of the parameters that are associated with scheduling are not known exactly. Parameters like raw material availability, prices, machine reliability, and market requirements vary with respect to time and are often subject to unexpected deviations. Having ways to systematically consider uncertainty is as important as having the model itself. Methodologies for process scheduling under uncertainty aim at producing feasible, robust and optimal schedules. In essence, uncertainty consideration plays the role of validating the use of mathematical models and preserving plant feasibility and viability during operations.

The scope of this paper is to provide an analysis of the sources of uncertainties in process scheduling, present different methods of describing the uncertain parameters and give a detailed literature review on existing approaches that address the problem of uncertainty in scheduling. Following these introductory remarks Section 2 is devoted in the uncertain parameter description. Section 3 discusses the alternative approaches that exist in the literature for scheduling under uncertainty followed by a discussion regarding the remaining challenges and future research directions that are presented in Sections 4 and 5, respectively.

2. Uncertainties in process scheduling

2.1. Motivating example

Uncertainty in process operations can originate from many aspects, such as demand or changes in product orders or order priority, batch or equipment failures, processing time variability, resource changes, recipe variations, or both etc. Based on the nature of the source of uncertainty in a process, a suitable classification has been proposed by Pistikopoulos (1995) as follows: (i) model-inherent uncertainty, such as kinetic constants, physical properties, mass/heat transfer coefficients; (ii) process-inherent uncertainty, such as flow rate and temperature variations, stream quality fluctuations, processing time and equipment availability; (iii) external uncertainty, including feed stream availability, product demands, prices and environmental conditions; (iv) discrete uncertainty, such as equipment availability and other random discrete events, operational personnel absence.

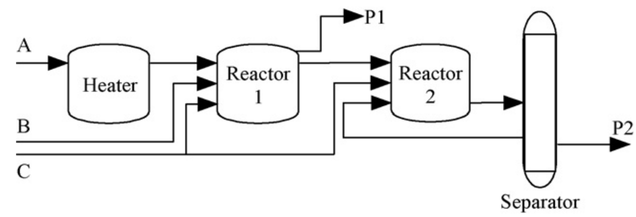

Fig. 1. Flowsheet of the example problem.

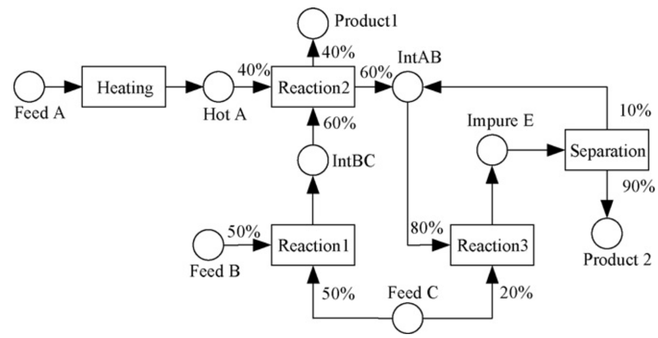

Fig. 2. STN representation of the example problem.

Let's consider the following example presented by Kondili et al. (1993) which will be used throughout this paper to illustrate the basic ideas and modeling frameworks. The plant flowsheet and the State Task Network (STN) representation of the problem are shown in Figs. 1 and 2, respectively. The STN representation models the products, intermediates and products as states and that are consumed, produced or both by the processing tasks. All tasks must coincide with the discrete moments of time set by the time domain. The duration and move-out times both have their corresponding constraints. A unified representation Resource Task Network (RTN) was proposed by Pantelides (1994) that employs a uniform treatment for all available resources including equipment units.

Table 1
Data for the example problem

Unit	Capacity	Suitability	Mean processing time ($\tau_{ij}^{\mathrm{mean}}$)
Heater	100	Heating	1.0
Reactor 1	50	Reaction 1,2,3	2.0,2.0,1.0
Reactor 2	80	Reaction 1,2,3	2.0,2.0,1.0
Still	200	Separation	2.0

State	Storage capacity	Initial amount	Price
Feed A	Unlimited	Unlimited	0.0
Feed B	Unlimited	Unlimited	0.0
Feed C	Unlimited	Unlimited	0.0
Hot A	100	0.0	0.0
Int AB	200	0.0	0.0
Int BC	150	0.0	0.0
Impure E	200	0.0	0.0
Product 1	Unlimited	0.0	10.0
Product 2	Unlimited	0.0	10.0

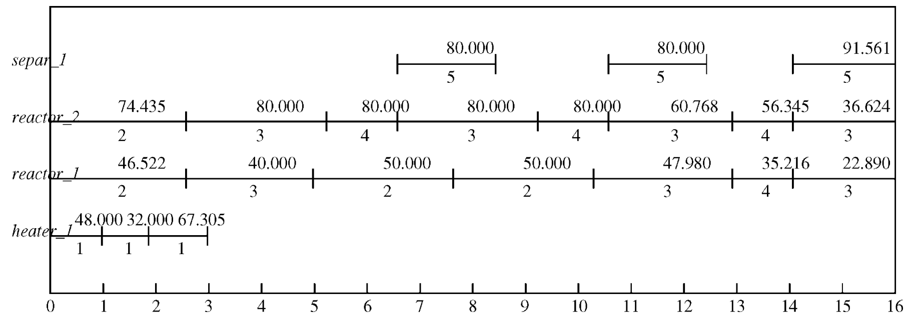

Fig. 3. Optimal schedule of assuming nominal values of the uncertain parameters.

The example problem involves the production of two products using three raw materials, and the data are given in Table 1.

Utilizing the continuous time model of Ierapetritou and Floudas (1998a,b), the mathematical formulation for this problem is as follows:

$$\min H \quad \text{or} \quad \max \sum_{s,n} \text{price}_s\, d_{s,n} \tag{1}$$

$$\text{subject to} \sum_{i \in I_j} wv_{i,j,n} \leq 1 \tag{2}$$

$$st_{s,n} = st_{s,n-1} - d_{s,n} - \sum_{i \in I_j} \rho^{P}_{s,i} \sum_{j \in J_i} b_{i,j,n} + \sum_{i \in I_j} \rho^{c}_{s,i} \sum_{j \in J_i} b_{i,j,n-1} \tag{3}$$

$$st_{s,n} \leq st^{\max}_{s} \tag{4}$$

$$v^{\min}_{i,j} wv_{i,j,n} \leq b_{i,j,n} \leq v^{\max}_{i,j} wv_{i,j,n} \tag{5}$$

$$\sum_{n} d_{s,n} \geq r_s \tag{6}$$

$$Tf_{i,j,n} = Ts_{i,j,n} + \alpha_{i,j} wv_{i,j,n} + \beta_{i,j} b_{i,j,n} \tag{7}$$

$$Ts_{i,j,n+1} \geq Tf_{i,j,n} - U(1 - wv_{i,j,n}) \tag{8}$$

$$Ts_{i,j,n} \geq Tf_{i',j,n} - U(1 - wv_{i',j,n}) \tag{9}$$

$$Ts_{i,j,n} \geq Tf_{i',j',n} - U(1 - wv_{i',j',n}) \tag{10}$$

$$Ts_{i,j,n+1} \geq Ts_{i,j,n} \tag{11}$$

$$Tf_{i,j,n+1} \geq Tf_{i,j,n} \tag{12}$$

$$Ts_{i,j,n} \leq H \tag{13}$$

$$Tf_{i,j,n} \leq H \tag{14}$$

In the above formulation, allocation constraints (2) state that only one of the tasks can be performed in each unit at an event point (n). Constraints (3) represent the material balances for each state (s) expressing that at each event point (n) the amount $st_{s,n}$ is equal to that at event point ($n-1$), adjusted by any amounts produced and consumed between event points ($n-1$) and (n), and delivered to the market at event point (n). The storage and capacity limitations of production units are expressed by constraints (4) and (5). Constraints (6) are written to satisfy the demands of final products. Constraints (7)–(14) represent time limitations due to task duration and sequence requirements in the same or different production units.

Assuming nominal values of all the parameters in the problem, the optimal production for time horizon of 16 hr is 147 for product 1 and 226 for product 2, the objective value (profit) is 3737.1 and the corresponding schedule in the form of a Gantt-chart is shown in Fig. 3 (Wu, 2005).

Uncertainty can appear at different parameters in the process, such as the heat transfer coefficient, stream feed quality, product demand, processing time, etc. To illustrate the effect of uncertainty, let's consider price change, such as that the price of product 1 rises to 11 and the price of product 2 remains the same, the schedule is different from the previous one which is no longer optimal (Fig. 4).

Moreover, let's consider the demand variability. When the demand of product 2 is equal to 50, we can find this optimal schedule that satisfies the demand that requires a makespan of 7 hr. To illustrate the effect of uncertainty let's take an extreme case that the demand of product 2 is increased by 60 percent to a value of 80 units. Using the optimal schedule obtained before for

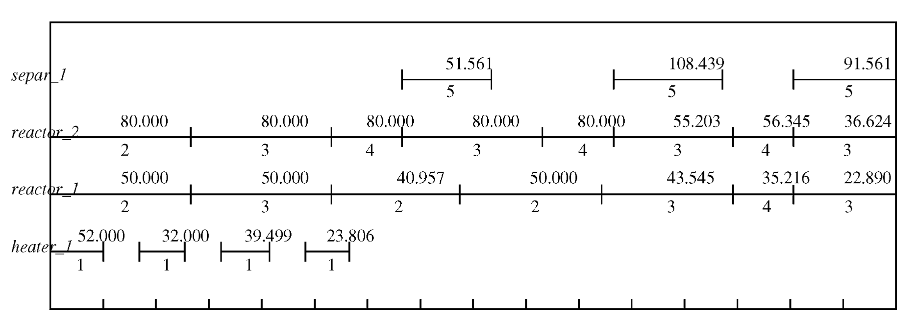

Fig. 4. Optimal schedule considering price variability.

demand equal to 50, the maximum production is 67.5. Therefore, part of the demand cannot be met. So the schedule obtained using deterministic model could lead to infeasible operations or delayed orders.

The solution can also become infeasible when there is uncertainty in the processing times of the tasks. For this same example process, Lin, Janak, and Floudas (2004) showed that if the processing time of each task is increased by simply 0.1 percent of its nominal value, then the nominal schedule will become infeasible and the optimal schedule with the slightly increased processing times will be significantly different from the nominal schedule.

2.2. Uncertainty description

To include the description of uncertain parameters within the optimization model of the scheduling problem, several methods have been used:

(i) *Bounded form.* In many cases, there is not enough information in order to develop an accurate description of the probability distribution that characterize the uncertain parameters, but only error bounds can be obtained. In this case interval mathematics can be used for uncertainty estimation, as this method does not require information about the type of uncertainty in the parameters. Uncertain parameter is described by an interval $\theta \in [\theta_{\min}, \theta_{\max}]$ or $|\tilde{\theta} - \theta| \leq \varepsilon|\theta|$, where $\tilde{\theta}$ is "true" value, θ is the nominal value, and $\varepsilon > \theta$ is a given uncertainty level. This is the typical and readily applicable method to describe the uncertain parameters. The bounds represent the ranges of all possible realizations of the uncertain parameters. The upper and lower bounds can be determined with the analysis of the historical data, customer's orders and market indicators.

(ii) *Probability description.* This is a common approach for the treatment of uncertainties when information about the behavior of uncertainty is available since it requires the use of probabilistic models to describe the uncertain parameters. In the probabilistic approach, uncertainties are characterized by the probabilities associated with events. The probability of an event can be interpreted in terms of the frequency of occurrence of that event. When a larger number of samples or experiments are considered, the probability of an event is defined as the ratio of the number of times the event occurs to the total number of samples or experiments. The probability distribution of the variable X can be uniquely described by its probability distribution function $F(x)$, which is defined by $F(x) = P(X \leq x)$, $x \in R$. If X is a discrete random variable and it attains value $x_1, x_2, \ldots$ with probability $P(X = x_i) = p(x_i)$ the probability distribution function is expressed as $F(x) = P(X \leq x) = \sum_{x_i \leq x} p(x_i)$. If probability distribution function is continuous, then X is a continuous random variable, if furthermore $F(x)$ is absolutely continuous, then probability density function $f(x)$ can be defined from $f(x) = \mathrm{d}F(x)/\mathrm{d}x$. In other words, a probability density function $f(x)$ is any function that determines the probability density in terms of the input variable x as follows: the probability that the random variable in question is in any particular interval $[a, b]$ is the integral of the function $f(x)$ from a to b. The uncertainty is modeled using either discrete probability distributions or the discretization of continuous probability distribution function. Uncertainties such as equipment breakdown, failure of process operations are generally described with discrete parameters.

(iii) *Fuzzy description.* Fuzzy sets allow modeling of uncertainty in cases where historical (probabilistic) data are not readily available. The resulting scheduling models based on fuzzy sets have the advantage that they do not require the use of complicated integration schemes needed for the continuous probabilistic models and they do not need a large number of scenarios as the discrete probabilistic uncertainty representations (Balasubramanian & Grossmann, 2003).

In the classical set theory, the truth value of a statement can be given by the membership function $\mu_A(x)$ in the following way:

$$\mu_A(x) = \begin{cases} 1 & \text{iff } x \in A \\ 0 & \text{iff } x \notin A \end{cases}$$

Fuzzy theory allows for a continuous value of $\mu_A(x)$ between 0 and 1:

$$\mu_A(x) = \begin{cases} 1 & \text{iff } x \in A \\ 0 & \text{iff } x \notin A \\ p; 0 < p < 1 & \text{if } x \text{ partially belongs to } A \end{cases}$$

Instead of probability distributions, these quantities make use of membership functions, based on possibility theory. A fuzzy set is a function that measures the degree of membership to a set. A high value of this membership functions implies a high possibility, while a low value implies a poor possibility. For example, consider a linear constraint $ax \leq \theta$ in terms of decision vector x and assume that the random right-hand-side θ can take values in the range from θ to $\theta + \Delta\theta$, $\Delta\theta \geq 0$, then the linear membership function of this constraint is defined as:

$$\mu_A(x) = \begin{cases} 1 & \text{if } ax \leq \theta \\ 0 & \text{if } \theta + \Delta\theta \leq ax \\ 1 - \dfrac{ax - \theta}{\Delta\theta} & \text{if } \theta \leq ax \leq \theta + \Delta\theta \end{cases}$$

Following the alternative description methods for uncertainty, different scheduling models and optimization approaches have been developed that are reviewed in the next section.

3. Scheduling under uncertainty

The two key elements in scheduling are the schedule generation and the scheduling revisions (Churh & Uzsoy, 1992; Sabuncuoglu & Bayiz, 1998). Scheduling generation acts as a predictive mechanism that determines planned start and completion times of production tasks based on given requirements and constraints prior to the production process. Schedule revision is a reactive part, which monitors execution of schedule and deals with unexpected events. Scheduling approaches can also be divided into offline/online scheduling (Karabuk & Sabuncuoglu, 1997). For offline scheduling, all available jobs are scheduled all at once for the entire planning horizon, whereas online scheduling makes decisions at a time that are needed.

Reactive scheduling is a process to modify the created schedule during the manufacturing process to adapt change (uncertainty) in production environment, such as disruptive events, rush order arrivals, order cancellations or machine breakdowns. For this type of uncertainty there is not enough information prior to realization of the uncertain parameters that will allow a protective action. Preventive scheduling, on the otherhand, can deal with parameter uncertainties, such as processing times, demand of products or prices. For this type of uncertainty, historical data and forecasting techniques can be used to derive information about the behavior of uncertain parameters in future in the form of range of parameters or stochastic distributions as described in the previous section.

Preventive scheduling, although the decisions may be modified as time passes, serves as the basis for planning support activities because commitments are made based on preventive schedule. For preventive scheduling we distinguish the following approaches: stochastic based approaches, robust optimization methods, fuzzy programming methods, sensitivity analysis and parametric programming methods.

3.1. Reactive scheduling

In operational research community, most existing reactive scheduling methods are characterized by least commitment strategies such as real-time dispatching that create partial schedules based on local information. One extension of this dispatching approach is to allow the system to select dispatching rule dynamically as the state of the shop changes. Another extension of these completely reactive approaches are those based on a number of independent, intelligent agents each trying to optimize its own objective function (Aytug, Lawley, McKay, Mohan, & Uzsoy, 2005).

The approaches that exist in the process scheduling literature to address the problem of reactive scheduling deal with the following two types of disturbances: (a) machine breakdown or changes in machine operation that affects the processing times of the tasks in these units; (b) order modification or cancellation that changes the product demands and due dates. The purpose of the proposed approaches is thus to update the current production schedule in order to provide an immediate response to the unexpected event. The original schedule is obtained in a deterministic manner and the reactive scheduling corrections are performed either at or right before the execution of scheduled operations. Reactive scheduling activity is by itself a short-term scheduling problem with some additional characteristics, mainly the possibility that all due dates be not fulfilled.

The reactive scheduling actions are based on various underlying strategies. It relies either on very simple techniques aimed at a quick schedule consistency restoration, or it involves a full scheduling of the tasks that have to be executed after the unexpected event occurs. Such an approach will be referred to as rescheduling and it can use any deterministic performance measure, such as the makespan of the new project. Cott and Macchietto (1989) considered fluctuations of processing times and used a shifting algorithm to modify the starting times of processing steps of a batch by the maximum deviation between the expected and actual processing times of all related processing steps. The proposed approach is easy to implement but is limited in terms of the unexpected events it can address.

A number of the techniques presented in the literature solve the reactive scheduling problem through mathematical programming approaches relying mostly on Mixed Integer Linear Programming (MILP) and the application of heuristic rules.

Rodrigues, Gimeno, Passos, and Campos (1996) considered uncertain processing times and proposed a reactive scheduling technique based on a modified batch-oriented MILP model according to the discrete-time STN formulation proposed by Kondili et al. (1993). A rolling horizon approach is utilized to determine operation starting times with look-ahead characteristics taking into account possible violations of future due dates. Honkomp, Mockus, and Reklaitis (1999) proposed a reactive scheduling framework for processing time variations and equipment breakdown by coupling a deterministic schedule optimizer with a simulator incorporating stochastic events.

Vin and Ierapetritou (2000) considered two types of disturbances in multiproduct batch plants: machine breakdown and rush order arrival. They applied a continuous-time scheduling formulation and reduced the computational effort required for the solution of the resulting MILP problems by fixing binary variables involved in the period before an unexpected event occurs. For the motivation example in Section 2.1, the deterministic schedule was first solved using the data given in Table 1 to maximize the total profit within a fixed time horizon of 8 hr. A machine breakdown is considered for reactor 2 at time $T = 3$ hr, which requires a maintenance time of 1 hr. The approach presented by Vin and Ierapetritou (2000) is applied with the objective of profit maximization. The machine breakdown results in significant profit reduction from 1498.19 units to 896.23 units (40 percent) as expected. The approach proposed can accommodate the requirement to minimize the changes from the original schedule by incorporating a penalty term in the objective function. This however results in further profit reduction to 708.29 units (52 percent).

Méndez and Cerdá (2003) used several different rescheduling operations to perform reactive scheduling in multiproduct, sequential batch plants. They considered start time shifting, local reordering, and unit reallocation of old batches as well as insertion of new batches. In Méndez and Cerdá (2004), they extended their work to include limited discrete renewable resources where only start time shifting, local batch reordering, and resource reallocation of existing batches are allowed. Rescheduling was performed by first reassigning resources to tasks that still need to be processed and then reordering the sequence of processing tasks for each resource item.

Ruiz, Cantón, Mara, Espuna, and Puigjaner (2001) presented a Fault Diagnosis System (FDS) that interacts with a schedule optimizer for multipurpose batch plants to perform reactive scheduling in the event of processing time variability or unit unavailability. The proposed system consists of an artificial neural network structure supplemented with a knowledge-based expert system. The information needed to implement the FDS includes a historical database, a hazard and operability analysis, and a model of the plant. When a deviation from the predicted schedule occurs, the FDS activates the reactive scheduling tools to minimize the effect of this deviation on the remaining schedule.

There are several dispatching rules which are considered as heuristics in reactive scheduling. These rules use certain empirical criteria to prioritize all the batches that are waiting for processing on a unit. Kanakamedala, Reklaitis, and Venkatasubramanian (1994) considered the problem of reactive scheduling in batch processes where there are deviations in processing times and unit availabilities in multipurpose batch plants. They developed a least impact heuristic search approach for schedule modification that allowed time shifting and unit replacement. Huercio, Espuna, and Puigjaner (1995) and Sanmarti, Huercio, Espuña, and Puigjaner (1996) proposed reactive scheduling techniques to deal with variations in task processing times and equipment availability. They used heuristic equipment selection rules for modification of task starting times and reassignment of alternative units. Two rescheduling strategies were employed, shifting of task starting times and reassignment of tasks to alternative units. Their method generates a set of decision trees using alternative unit assignments, each based on a conflict in the real production schedule caused by a deviation in the real schedule from the nominal schedule. Branches of the trees are then pruned according to heuristic equipment selection rules. Janak and Floudas (2006) presented a reactive scheduling framework which provides an immediate response to unexpected events such as equipment breakdown or the addition or modification of orders. The proposed mathematical framework avoided full-scale rescheduling of each production schedule by fixing binary variables for a subset of tasks from the original production schedule. The subset of tasks to fix is determined using a detailed set of rules that reflect the production needs and can be modified for different production facilities. The fixing of tasks results in a reduced computational effort required to solve the resulting MILP problem.

Sanmartí, Espuna, and Puigjaner (1997) presented a different approach for the scheduling of production and maintenance tasks in multipurpose batch plants in the face of equipment failure uncertainty. They computed a reliability index for each unit and for each scheduled task and formulated a nonconvex Mixed Integer Nonlinear Programming (MINLP) model to maximize the overall schedule reliability. Because of the significant difficulty in the rigorous solution of the resulting problem, a heuristic method was developed to find solutions that improve the robustness of an existing schedule. Roslöf, Harjunkoski, Bjorkqvist, Karlsson, and Westerlund (2001) developed an MILP based heuristic algorithm that can be used to improve an existing schedule or to reschedule jobs in the case of changed operational parameters by iteratively releasing a set of jobs in an original schedule and optimally reallocating them.

3.2. Stochastic scheduling

Stochastic scheduling is the most commonly used approach in the literature for preventive scheduling, in which the original deterministic scheduling model is transformed into stochastic model treating the uncertainties as stochastic variables. In this type of approach, either discrete probability distributions or the discretization of continuous probability distribution functions is used. The expectation of a certain performance criterion, such as the expected makespan, is optimized with respect to the scheduling decision variables. Stochastic programming models are divided into the following categories: two-stage or multi-stage stochastic programming; chance constraint programming based approach.

In the two-stage stochastic programming, the first-stage variables are those that have to be decided before the actual realization of the uncertain parameters, a recourse decision can then be made in the second-stage that compensates for any bad effects that might have been experienced as a result of the realization of uncertain parameters. The optimal policy from such a model is a single first-stage policy and a collection of recourse decisions (a decision rule) defining which second-stage action should be taken in response to each random outcome. Considering the motivation example in Section 2.1, in a stochastic scheduling framework, the binary variables can be treated as the first-stage variable, denoting the assignment of the tasks to the units whereas the continuous variables can serve as a recourse in the second-stage problem to adapt to uncertain parameter realization (Vin & Ierapetritou, 2001). Using the same idea, two-stage stochastic programming is also extended to multi-stage approach. Ierapetritou and Pistikopoulos (1996) addressed the scheduling of single-stage and multistage multiproduct continuous plants for a single production line. At each stage they considered uncertainty in product demands. They used Gaussian quadrature integration to evaluate the expected profit and formulated MILP models for the stochastic scheduling problem. Bonfill, Bagajewicz, Espuña, and Puigjaner (2004) used stochastic optimization approach to manage risk in the short-term scheduling of multiproduct batch plants with uncertain demand. The problem is modeled using a two-stage stochastic optimization approach accounting for the maximization of the expected profit. Management of risk is addressed by adding a control measure as a new objective to be considered, thus leading to multi-objective optimization formulations. Bonfill, Espuña, and Puigjaner (2005) addressed the short-term scheduling problem in chemical batch processes with variable processing times, to identify robust schedules able to face the major effects driving the operation of batch processes with uncertain times, i.e., idle and waiting times. The problem is modeled using a two-stage stochastic approach accounting for the minimization of a weighted combination of the expected makespan and the expected waiting times. Balasubramanian and Grossmann (2002) proposed a multiperiod MILP model for scheduling multistage flow-shop plants with uncertain processing times described by discrete or continuous (using discretization schemes) probability distributions. Balasubramanian and Grossmann (2004) presented a multistage stochastic MILP model based on the one formulated by Goel and Grossmann (2004), wherein certain decisions are made irrespective of the realization of the uncertain parameters and some decisions are made upon realization of the uncertainty. They proposed an approximation strategy that consists of solving a series of two-stage stochastic programs within a shrinking-horizon framework to overcome the large computational expense associated with the solution of the multistage stochastic program. Acevedo and Pistikopoulos (1997a) proposed a hybrid parametric/stochastic approach for the solution of stochastic MILP problem, which was applied on the capacity planning problem. The approach is based on a two-stage stochastic programming framework and avoids the repetitive solution of the second-stage subproblems by solving instead multiparametric programs, resulting in a more efficient evaluation of the expected value.

Following the probabilistic or chance-constraint based approach, the focus is on the reliability of the system, i.e., the system's ability to meet feasibility in an uncertain environment. The reliability is expressed as a minimum requirement on the probability of satisfying constraints. Orçun, Altinel, and Hortacsu (1996) proposed a mathematical programming model for optimal scheduling of the operations of a batch processing chemical plant. They considered uncertain processing times in batch processes and employed chance constraints to account for the risk of violation of timing constraints under certain conditions such as uniform distribution functions. Petkov and Maranas (1997) addressed the multiperiod planning and scheduling of multiproduct plants under demand uncertainty. The proposed stochastic model is an extension of the deterministic model introduced by Birewar and Grossmann (1990). The stochastic elements of the model are expressed with equivalent deterministic forms of the chance constraints, eliminating the need for discretization or sampling techniques. The resulting equivalent deterministic optimization models are MINLP problems with convex continuous parts.

Other than the previous two methods in modeling scheduling under uncertainty using stochastic programming ideas, other methods involve simulation based approaches such as the approach proposed by Bassett, Pekny, and Reklaitis (1997) that takes into account processing time fluctuations, equipment reliability/availability, process yields, demands, and manpower changes. They used Monte Carlo sampling to generate random instances of the uncertain parameters, determined a schedule for each instance, and generated a distribution of aggregated properties to infer operating policies. In the reactive scheduling framework of Honkomp et al. (1999), a deterministic schedule optimizer and a simulator incorporating stochastic events were developed. Replicated simulations were used to determine the performance of fixed deterministic schedules in light of uncertainty and for the validation of reactive scheduling techniques. Subramanian, Pekny, and Reklaitis (2000) studied the stochastic optimization problem inherent to the management of an R&D pipeline and developed a computing architecture, Sim-Opt, which combines combinatorial optimization and discrete event system simulation to assess the uncertainty and control the risk present in the R&D pipeline.

3.3. Robust optimization method

Robust scheduling focuses on building the preventive scheduling to minimize the effects of disruptions on the performance measure and tries to ensure that the predictive and realized schedules do not differ drastically, while maintaining a high level of schedule performance.

Mulvey, Vanderbei, and Zenios (1995) developed the concept of Robust Optimization (RO) to handle the trade-off associated with solution and model robustness. A solution to an optimization is considered to be solution robust if it remains close to the optimal for all scenarios, and model robust if it remains feasible for most scenarios. The basic idea of robust optimization is that by reformulating the original problem, or by solving a sequence of problems, we may find a solution which is robust to the uncertainty in the data. One of the earliest papers on robust optimization, by Soyster (1973), considered simple perturbations in the data and aimed to find a reformulation of the original problem such that the resulting solution would be feasible under all possible perturbations. Subsequent, pioneering work by Ben-Tal and Nemirovski (1999), Bertsimas and Sim (2003), and El-Ghaoui, Oustry, and Lebret (1998), and, extended the framework of robust optimization, and included sophisticated solution techniques with non-trivial uncertainty sets describing the data. The major advantages of robust optimization compared to stochastic programming are that no assumptions are needed regarding the underlying probability distribution of the uncertain data and that it provides a way of incorporating different attitudes toward risk. Robust optimization has been applied to several areas in research and practice, such as production planning (Escudero, Kamesan, King, & Wets, 1993), machine scheduling (Daniels & Kouvelis, 1995; Laguna, 1998) and logistics (Escudero, Quintana, & Salmeron, 1999; Yu and Li, 2000).

To improve the schedule flexibility prior to its execution, it is important to measure the performance of a deterministic schedule under changing conditions due to uncertainty. Standard deviation (SD) is one of the most commonly used metric for evaluating the robustness of a schedule. To evaluate the SD, the deterministic model with a fixed sequence of tasks ($wv_{i,j,n}$) is solved for different realizations of uncertain parameters that define the set of scenarios k that results in different makespans H_k. The SD is the defined as:

$$SD_{\text{avg}} = \sqrt{\sum_k \frac{(H_k - H_{\text{avg}})^2}{p_{\text{tot}} - 1}}, \quad H_{\text{avg}} = \frac{\sum_k H_k}{p_{\text{tot}}}$$

where H_{avg} is the average makespan over all the scenarios, and p_{tot} denotes the total number of scenarios. Another similar robustness metric is proposed by Vin and Ierapetritou (2001), where the infeasible scenarios were taken into consideration. In case of infeasibility, the problem is solved to meet the maximum demand possible by incorporating slack variables in the demand constraints. Then the inventory of all raw materials and intermediates at the end of the schedule are used as initial conditions in a new problem with the same schedule to satisfy the unmet demand. The makespan under infeasibility (H_{corr}) is determined as the sum of those two makespans. Then the robustness metric is defined as:

$$SD_{\text{corr}} = \sqrt{\sum_{k} \frac{(H_{\text{act}} - H_{\text{avg}})^2}{p_{\text{tot}} - 1}}$$

where $H_{\text{act}} = H_k$ if scenario k is feasible and $H_{\text{act}} = H_{\text{corr}}$ if scenario k is infeasible.

Lin et al. (2004) proposed a robust optimization method to address the problem of scheduling with uncertain processing times, market demands, or prices. The robust optimization model was derived from its deterministic model considering the worst-case values of the uncertain parameters, so they treated the uncertainty in a bounded form. And as a further research, Janak, Lin, and Floudas (2007) studied the case that uncertainty is described by known probability distribution, where the robust optimization formulation introduces a small number of auxiliary variables and additional constraints into the original MILP problem, generating a deterministic robust counterpart problem which provides the optimal/feasible solution given the (relative) magnitude of the uncertain data, a feasibility tolerance, and a reliability level. The robust optimization approach is then applied to the problem of short-term scheduling under uncertainty. Vin and Ierapetritou (2001) addressed the problem of quantifying the schedule robustness under demand uncertainty, introduced several metrics to evaluate the robustness of a schedule and proposed a multiperiod programming model using extreme points of the demand range as scenarios to improve the schedule performance of batch plants under demand uncertainty. Using flexibility analysis, they observed that the schedules from the multiperiod programming approach were more robust than the deterministic schedules.

Jia and Ierapetritou (2006a,b) recently proposed a multi-objective robust optimization model to deal with the problem of uncertainty in scheduling considering the expected performance (makespan), model robustness and solution robustness. Normal Boundary Intersection (NBI) technique is utilized to solve the multi-objective model and successfully produce Pareto optimal surface that captures the trade-off among different objectives in the face of uncertainty. Considering the example given in Section 2.1, the nominal demand for both products 1 and 2 is 80 and is assumed to exhibit a variability of ±50 percent. The resulting Pareto optimal surface is shown in Fig. 5. Point A is in the area of solutions that favor model robustness. On the otherhand, point B represents a different schedule that favors the expected makespan and solution robustness, at the expense of low model robustness.

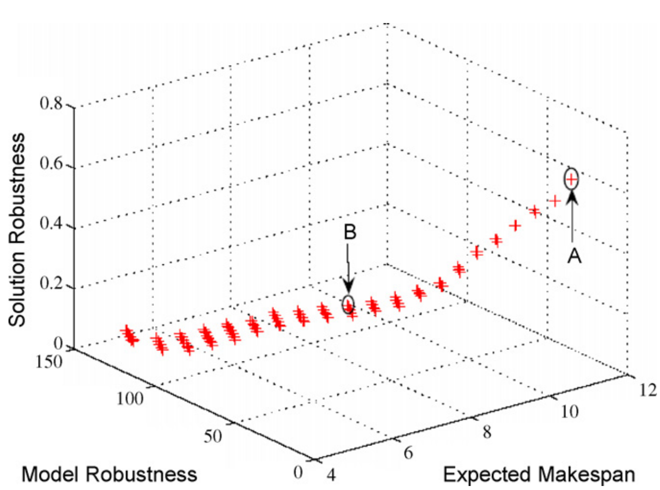

Fig. 5. Pareto set of solutions for example problem.

The schedules obtained by solving this multi-objective optimization problem include robust assignments that can accommodate the demand uncertainty. This robust optimization model is written for general batch plant scheduling, other scheduling problems that have different objectives and constraints can be formulated on this basis.

3.4. Fuzzy programming method

The approaches presented so far rely on the use of probabilistic models that describe the uncertain parameters in terms of probability distributions. However, sometimes such information is not available. For such cases an alternative approach is the use of fuzzy set theory and interval arithmetic to describe the imprecision and uncertainties in process parameters.

Fuzzy programming also addresses optimization problems under uncertainty. Though the uncertainties in process scheduling are generally described through probabilistic models, fuzzy set theory has been applied to scheduling optimization using heuristic search techniques during the past decades. The principal difference between the stochastic and fuzzy optimization approaches is in the way uncertainty is modeled. Here, fuzzy programming considers random parameters as fuzzy numbers and constraints are treated as fuzzy sets. Some constraint violation is allowed and the degree of satisfaction of a constraint is defined as the membership function of the constraint. Objective functions in fuzzy mathematical programming are treated as constraints with the lower and upper bounds of these constraints defining the decision makers' expectations.

Balasubramanian and Grossmann (2003) applied a non-probabilistic approach to the treatment of processing time uncertainty in two scheduling problems: (i) flow-shop plants; (ii) the new product development process. The examples considered show that very good estimates of the uncertain makespan and income can be obtained by using fairly coarse discretizations and that these models can be solved with little computational effort. In these examples the improvement in the estimation of the completion time by using a denser discretization was not significant enough to warrant the order of magnitude increase in computation time required. Wang (2004) developed a robust scheduling methodology based on fuzzy set theory for uncertain product development projects. The imprecise temporal parameters involved in the project were represented by fuzzy sets. The measure of schedule robustness was proposed to guide the Genetic Algorithm (GA) to determine the schedule with the best worst-case performance. The proposed GA approach can obtain the robust schedule with acceptable performance. Petrovic and Duenas (2006) recently used fuzzy programming method to deal with parallel machine scheduling/rescheduling in the presence of uncertain disruptions. A predictive–reactive approach is defined as a two-step process, where the first step consists of adding idle time to the jobs' processing times in order to generate a schedule capable of absorbing the negative effects of uncertain material shortages. The second step is rescheduling where two questions are addressed, namely when to reschedule and which rescheduling method to apply.

3.5. Sensitivity analysis and parametric programming method

In addition to scheduling problems, most problems in the area of process design and operations are commonly formulated as MILP problems. An alternative way, compared to the approaches presented before, to incorporate uncertainty into these problems is using MILP sensitivity analysis and parametric programming methods. These methods are important as they can offer significant analytical results to problems related to uncertainty.

3.5.1. Sensitivity analysis

Sensitivity analysis (SA) is used to ascertain how a given model output depends upon the input parameters. This is an important method for checking the quality of a given model, as well as a powerful tool for checking the robustness and reliability of any solution. Due to the combinatorial nature of the scheduling problem, sensitivity analysis poses some unique issues.

Not much work has appeared in the literature in sensitivity analysis for the scheduling problem. A recent review in the literature by Hall and Posner (2004) pointed out a number of issues associated with the application of sensitivity analysis (SA) in scheduling problems that involve: the applicability of SA to special classes of scheduling problems; the efficiency of SA approaches when simultaneous parameter changes occur; the selection of a schedule with minimum sensitivity; the computational complexity of answering SA questions for intractable scheduling problems, etc.

Samikoglu, Honkomp, Pekny, and Reklaitis (1998) studied the effect of perturbations on the optimal solution and developed a series of perturbed solutions that span a specified bounded region of the parameter space. This work attempts to reveal parameters most sensitive to perturbations and those that have the greatest impact on the solution. The interaction between the resource constraints and the objective function are explored using single parameter variations. A branch and bound procedure enhanced with logical programming is suggested to reveal these interactions in a more general framework. Penz, Rapine, and Trystram (2001) studied the performance of static scheduling policies in presence of on-line disturbances using sensitivity analysis and showed that in the case of independent tasks, the sensitivity can be guaranteed not to exceed the square root of the magnitude of the perturbation. Guinand, Moukrim, and Sanlaville (2004) considered the problem of sensitivity analysis of statically computed schedules for the problem when the actual communication delays differ from the estimated ones.

Jia and Ierapetritou (2004) handled uncertainty in short-term scheduling based on the idea of inference-based sensitivity analysis for MILP problems and utilization of a branch-and-bound solution methodology. The basic idea of the proposed method is to utilize the information obtained from the sensitivity analysis of the deterministic solution to determine (a) the importance of different parameters and constraints; (b) the range of parameters where the optimal solution remains unchanged. The main steps of the proposed approach are shown in Fig. 6. More specifically, the proposed analysis consists of two parts. In the first part, important information about the effects of different parameters is extracted following the sensitivity analysis step, whereas in the second part, alternative schedules are determined and evaluated for different uncertainty ranges.

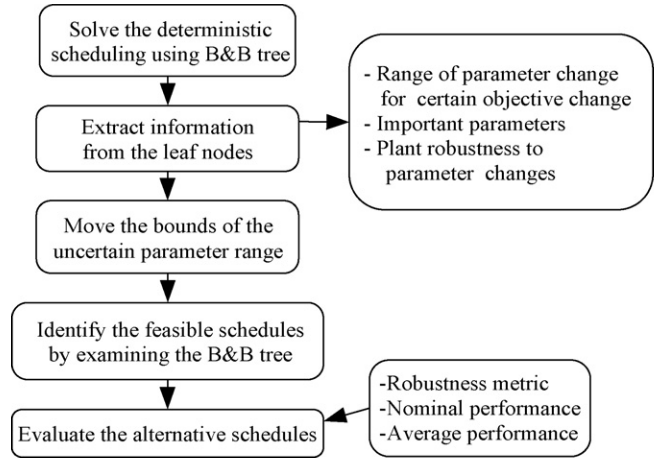

Fig. 6. Flowchart of the sensitivity analysis method (Jia and Ierapetritou, 2004).

Considering the motivating example presented in Section 2.1, after the sensitivity analysis is performed, the following information is obtained: the most critical task of the production line is reaction 2. If the processing capacity of reaction 2 in reactor 1 or reactor 2 is decreased by 11 units, the profit will be reduced by five percent, In contrast, very little or no change is observed in the objective function upon a change in the reaction 1 or reaction 3 processing capacity in both reactors. The objective value is also not sensitive to the change of other parameters; for example, the processing capacity of separation in the separator can drop by up to 120 units without decreasing the profit. Another important modeling issue that can be addressed is the question of constraint redundancy. Here, the importance of storage constraints is investigated, and it is found that these constraints are redundant because they are not active in any of the solution branch-and-bound nodes. More interestingly, the duration constraints are also found to be redundant, which means that the maximum processing capacities are already reached with the current processing times, so that the profit cannot be improved even with zero processing times assuming a fixed number of event points. For the second part of the analysis, the demand of product 2 is considered to be the uncertain parameter, varying within the range of [20, 80], and the objective function is modified to minimize the makespan. A branch-and-bound tree is constructed at the nominal point $r(\text{'}p2\text{'}) = 50$, and the dual information is stored at each node. Applying the inference duality sensitivity analysis, the following expression is obtained regarding the range of demand change following specific objective change (ΔH):$-0.0297\Delta d \leq \Delta H$, which means that, if the

Table 2
Comparison of alternative schedules for the example problem

	Schedule 1	Schedule 2	Schedule 3
H_{nom} (h)	7.00	7.14	7.40
H_{avg} (h)	8.15	7.24	7.40
SD_{corr} (h)	2.63	0.29	0.27

demand is increased by Δd, the new makespan becomes at most $H_{nom} + 0.0297\Delta d$ When r('$p2$') is increased from 50 to 80, schedule 1 (derived by using nominal demand 50 with objective of minimum makespan) becomes infeasible. Then, we solve the LP problem at each leaf node with the demand of 80 and check the leaf nodes with the objective function values below the 7.89 value that is obtained using this inequality in the branch-and-bound tree. The new optimal solution is found to be schedule 2, and schedule 3 is one feasible solution. The schedules are then evaluated with respect to the mean and nominal makespan and standard deviation within the demand range of [20, 80], and the values are shown in Table 2. Compared with schedule 1, schedule 2 and schedule 3 have larger mean makespan and lower standard deviation, which means higher robustness.

3.5.2. Parametric programming

Parametric optimization also serves as an analytic tool in process synthesis under uncertainty mapping the uncertainties in the definition of the synthesis problem to optimal design alternatives. From this point of view, it is the exact mathematical solution of the uncertainty problem.

Thompson and Zawack (1985) proposed a zero–one integer programming model for the job-shop scheduling problem with minimum makespan criterion is presented. The algorithm consists of two parts: (a) a branch and bound parametric linear programming code for solving the job-shop problem with fixed completion time; (b) a problem expanding algorithm for finding the optimal completion time. Ryu, Dua, and Pistikopoulos (2004) addressed the problem of bilevel decision-making under uncertainty in the context of enterprise-wide supply chain optimization with one level corresponding to a plant planning problem, while the other to a distribution network problem. The corresponding bilevel programming problem under uncertainty has been solved by parametric optimization techniques. Acevedo and Pistikopoulos (1997b) addressed linear process engineering problems under uncertainty using a branch-and-bound algorithm, base on the solution of multi-parametric linear programs at each node of the tree and the evaluation of the uncertain parameters space for which a node must be considered.

Most recently, Jia and Ierapetritou (2006a,b) proposed a new method of uncertainty analysis on the right hand side (RHS) for MILP problems:

$$\begin{aligned} &\min z = cx \\ \text{subject to}: \quad & Ax \geq \theta + \Delta\theta, \quad x \geq 0, \quad x_j \text{ integer}, \quad j = 1, \ldots, k \end{aligned} \tag{P1}$$

The proposed solution procedure starts with the B&B tree of the MILP problem at the nominal value of the uncertain parameter and requires two iterative steps, Linear Programming (LP)/multi-parametric Linear Programming (mpLP) sensitivity analysis and updating the B&B tree (Fig. 7).

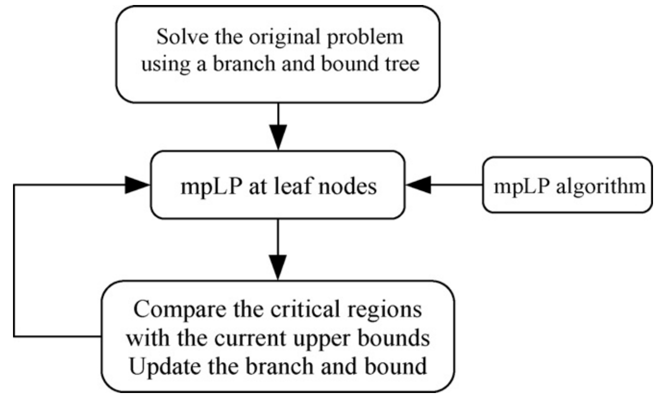

Fig. 7. Flow chart of uncertainty analysis (Jia and Ierapetritou, 2006b).

Based on this framework for uncertainty analysis of scheduling, our current work involves the extension of this approach in two fundamental directions. The first one addresses the issue of infeasibility by providing a description of the feasible region prior to the implementation of the parametric MILP algorithm. They developed a pre-processing step that enables the consideration of wide uncertainty range by exploiting the idea of projection into the constraint set. The feasible region was determined using a projection algorithm, which maps the space into the subspace spanned by uncertain parameters. Thus the complete description of the feasible region was determined before solving the parametric MILP. The second extension is the consideration of uncertainty in the objective function coefficients and problem constraints. For the case of uncertain objective function coefficients, the mpLP procedure is similar to the case of uncertainty in RHS parameters using the basic duality concepts. Considering the uncertain constraint coefficients gives rise to linear bilevel programming problem (BPP). When uncertainties exist in objective function, constraint coefficients and RHS parameters simultaneously, the objective functions in both upper and lower problems of this BPP are nonlinear. Global optimization approaches are then utilized to solve the nonlinear bilevel problems. The results of this work will be a subject of future publication.

4. Challenges

Most scheduling formulations belong to the class of NP-complete problems even when simplifications in comparison to practical problems are introduced. The inclusion of uncertainty in process scheduling problems transform the original deterministic model to stochastic or parametric formulations, which makes the problem more complicated.

In the direction of stochastic scheduling, reducing the computational cost is still a major issue. The use of problem specific heuristics can lead to efficient solution procedures. Such heuristic algorithms incorporate specific knowledge which often leads to good solutions that can be obtained in an acceptable amount of time. However, the use of heuristics has the disadvantage that they cannot usually guarantee convergence and good quality solutions. Research on sensitivity analysis and parametric programming has emerged in the area of scheduling. However, most of the existing methods are dealing with linear models that represent a simplification of the realistic processes. Thus, in order to be able to address nonlinear models, effective sensitivity analysis and parametric programming for Mixed Integer Nonlinear Programming (MINLP) problems are needed. Towards this direction most of the progress done recently is due to the work of Pistikopoulos and co-workers (Dua & Pistikopoulos, 1998, 1999; Hene, Dua, & Pistikopoulos, 2002). Because this is a complete and rigorous way to analyze uncertainty, the main challenge in this direction is the consideration of processing time uncertainties because it shows up in the constraint matrix of the MILP model which causes nonlinear optimal objective function and nonlinear critical region constraints. The computational cost in the case of large number of binary variables is another issue that needs to be addressed.

On the otherhand, considering that a purely preventive approach is too rigid in scheduling, combining reactive scheduling and an approach that accounts for uncertainty should be a promising direction. For reactive scheduling, current capabilities of optimization methods are still very restrictive and mostly focused on sequential batch processes. More general, efficient and systematic rescheduling tools are required for recovering feasibility and efficiency with short reaction time and minimum additional cost. The main effort should be oriented towards avoiding a time-expensive full-scale rescheduling, allowing during the rescheduling process only limited changes to the scheduling decisions already made at the beginning of the time horizon.

To incorporate uncertainty in the process industries, it is worth to investigate a new unified framework for planning and scheduling under uncertainty. Another challenging issue is to be able to handle a large number of uncertain parameters. Most of the existing work addresses just part of the problem due to increasing problem complexity. However, since uncertainties appear in different levels of process operations and in different forms as described in the previous sections, a systematic consideration of all uncertain parameters is of vital importance.

5. Summary

This paper presented a review of the scheduling techniques under uncertainties. A detailed description of process uncertainty was given as the basis to model the scheduling process. According to the different treatment of uncertainty, scheduling methods were divided into two groups: reactive scheduling and preventive scheduling. Reactive scheduling, dynamic scheduling, rescheduling, and online scheduling deal with the problem of modify the original scheduling policy or generate scheduling policy on time when uncertainty occurs. Preventive scheduling generates robust scheduling policy before the uncertainty occurs. Detail classification of preventive scheduling includes: stochastic scheduling, robust optimization method, fuzzy programming method and prosperous sensitivity analysis and parametric programming method. Future work in the field of scheduling under uncertainty requires extended work in the direction of more effective and general method for dealing with the uncertainties in process industry.

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