§1 Problemset January 18, 2023

Instructions: Solve [36♣]. If you have time, solve [42♣]. Problems with red weights are mandatory.

[2♣] **Problem 1.** The positive integers A, B, $A-B$, and $A+B$ are all prime numbers. The sum of these four primes is

A) even B) divisible by 3 C) divisible by 5 D) divisible by 7 E) prime

[3♣] **Required Problem 2.** In the equation below, A and B are consecutive positive integers, and A, B and $A+B$ represent number bases:

$$132_A + 43_B = 69_{A+B}.$$

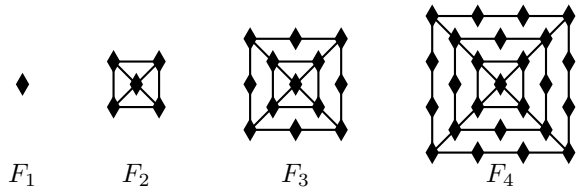
What is $A+B$?

A) 9 B) 11 C) 13 D) 15 E) 17

[2♣] **Problem 3.** If $y+4=(x-2)^2$, $x+4=(y-2)^2$, and $x \neq y$, what is the value of x^2+y^2?

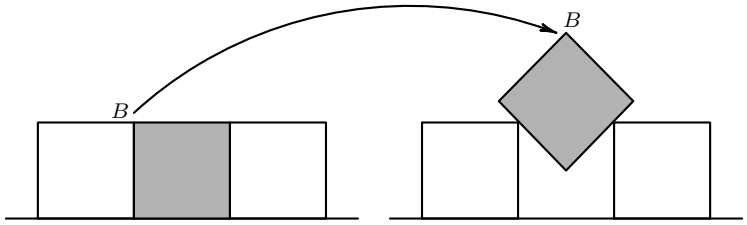
A) 10 B) 15 C) 20 D) 25 E) 30

[5♣] **Problem 4.** The figures F_1, F_2, F_3, and F_4 shown are the first in a sequence of figures. For $n \geq 3$, F_n is constructed from F_{n-1} by surrounding it with a square and placing one more diamond on each side of the new square than F_{n-1} had on each side of its outside square. For example, figure F_3 has 13 diamonds. How many diamonds are there in figure F_{20}?



A) 410 B) 485 C) 585 D) 626 E) 761

[5♣] **Required Problem 5.** Three one-inch squares are palced with their bases on a line. The center square is lifted out and rotated 45°, as shown. Then it is centered and lowered into its original location until it touches both of the adjoining squares. How many inches is the point B from the line on which the bases of the original squares were placed?



A) 1 B) $\sqrt{2}$ C) $\frac{3}{2}$ D) $\sqrt{2}+\frac{1}{2}$ E) 2

[9♣] **Problem 6.** Two unit squares are selected at random without replacement from an $n \times n$ grid of unit squares. Find the least positive integer n such that the probability that the two selected unit square sare horizontally or vertically adjacent is less than $\frac{1}{2015}$.

[3♣] **Problem 7.** A group of 12 pirates agree to divide a treasure chest of gold coins among themselves as follows. The k^{th} pirate to take a share takes $\frac{k}{12}$ of the coins that remian in the chest. The number of coins initially in the chest is the smallest number for which this arrangement will allow each pirate to receive a positive whole number of coins. How many coins does the 12^{th} pirate receive?

A) 720 B) 1296 C) 1728 D) 1925 E) 3850

[2♣] **Problem 8.** The repeating decimals $0.abab\overline{ab}$ and $0.abcabc\overline{abc}$ satisfy

$$0.abab\overline{ab} + 0.abcabc\overline{abc} = \frac{33}{37},$$

where a, b, and c are (not necessarily distinct) digits. Find the three-digit number abc.

[5♣] **Problem 9.** Let $ABCDEF$ be a regular hexagon with side length 1. Denote by X, Y, and Z the midpoints of sides $\overline{AB}$, $\overline{CD}$, and $\overline{EF}$, respectively. What is the area of the convex hexagon whose interior is the intersection of the interiors of $\triangle ACE$ and $\triangle XYZ$?

A) $\frac{3}{8}\sqrt{3}$ B) $\frac{7}{16}\sqrt{3}$ C) $\frac{15}{32}\sqrt{3}$ D) $\frac{1}{2}\sqrt{3}$ E) $\frac{9}{16}\sqrt{3}$

[9♣] **Required Problem 10.** Eddie has a birthday party, in which he invited his 8, very generous, best friends. Each friend brought a certain number of gifts for Eddie. It is observed that for every group of 4 friends, at least 2 of them brought the same number of gifts. Moreover, the person with the most gifts brought 4, while the least brought 1. What is the total number of possible values of the amount of gifts Eddie received in total?

[9♣] **Problem 11.** There are two distinguishable flagpoles, and there are 19 flags, of which 10 are identical blue flags, and 9 are identical green flags. Let N be the number of distinguishable arrangements using all of the flags in which each flagpole has at least one flag and no two green flags on either pole are adjacent. Find the remainder when N is divided by 1000.

Sources: 2002 AMC 12B #11, 2012 AMC 12B #11, 2015 AMC 10A #16, 2009 AMC 12A #11, 2005 AMC 10A #19, 2015 AIME II #5, 2013 AMC 10A #21, 2014 AIME II #4, 2018 AMC 10B #24, 2018 CMC 12A #17, 2008 AIME II #12.