Theory of Electromagnetic Methods

Lecture Notes

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1 Remarks

These are the notes for the lecture on **Theory of Electromagnetic Methods**.

All materials are stored in the GitHub repo https://github.com/ruboerner/ThEM.

Have fun!

Part I

Introduction

2 Electromagnetic fields

In electromagnetics we deal with the following *fields*:

- electric field $\mathbf{E}$, unit V/m
- magnetic field $\mathbf{H}$, unit A/m
- electric displacement field $\mathbf{D}$, unit As/m^2
- magnetic flux density $\mathbf{B}$, unit Vs/m^2
- electic current density field $\mathbf{J}$, unit A/m^2

All considered fields are functions of *space* $\mathbf{r}$ and *time* t, i.e.,

$$\mathbf{e}(\mathbf{r}, t), \mathbf{h}(\mathbf{r}, t), \mathbf{d}(\mathbf{r}, t), \mathbf{b}(\mathbf{r}, t), \mathbf{j}(\mathbf{r}, t) \tag{2.1}$$

or a function of the *angular frequency* $\omega = 2\pi f$, such that

$$\mathbf{E}(\mathbf{r}, \omega), \mathbf{H}(\mathbf{r}, \omega), \mathbf{D}(\mathbf{r}, \omega), \mathbf{B}(\mathbf{r}, \omega), \mathbf{J}(\mathbf{r}, \omega). \tag{2.2}$$

In the latter case, the time dependency of any field $\mathbf{F}$ is always defined as

$$\mathbf{F}(\mathbf{r}, \omega) = \mathbf{F}_0(\mathbf{r}) e^{i\omega t},$$

and the quantity of interest is $\mathbf{F}_0$.

Convention: Upper case letters: Frequency domain, lower case letters: Time domain.

See Equation **??** for a definition of the Fourier transform.

2.1 Material properties

In electromagnetics we have the following material properties:

- electrical conductivity σ
- dielectrical permittivity ε
- magnetic permeability μ.

In the context of geo-electromagnetics, these parameters are associated with particular rock properties which are studied in *petrophysics*.

2.2 Simplifications

As we know from theroretical physics, the relations between the fields and the associated parameters are very general and allow, e.g., strong frequency-dependency or non-linearities.

In geo-electromagnetics, however, we can allow for a few simplifications.

All rock parameters are supposed to be

- linear with respect to the fields
- stationary, and
- isotropic.

We will see later that anisotropy is a quite general rock property which needs to be considered in the interpretation of geo-electromagnetic field data.

Moreover, we will first study the general properties of the EM induction in a uniform full-space by neglecting any spatial inhomogeneities of the parameters.