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**DEPARTMENT OF MECHANICAL ENGINEERING**

Topic Fundamental concepts of FEA Unit No. 1

• Historical Notes:-

- > The Finite Element Method (FEM) was first developed in 1943 by Richard Courant (1888-1972), a German mathematician as a method for dividing a continuum in small discrete elements.
- > The theorem first appeared in 1922 as a proof for a Mapping theorem. The first application was done in 1943 as a solution for a torsion problem.
- > The basic ideas of the finite element Method as known today were presented in the papers of Turner, Clough, Martin and Topper and Argyris and Kelsey.
- > The Name finite element was coined by Clough. Reference element application of simple finite elements (pin-jointed bar and triangular plate with in-plane load) for the analysis of aircraft structure and is considered as one of the key contributions in the development of the finite element method.

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Topic : Fundamental Concepts of FEA Unit No. 1

- What is Meant by Finite element analysis ?
  - Finite element analysis (FEA) is the process of predicting an object's behavior based on calculations made with the finite element method (FEM)
- principle of FEA :-
  - Finite element analysis are based on principle that include boundary conditions such as forces and pressure, as well as three governing equations :-
    - Equilibrium equations, which find when the opposing forces OR influences are balanced.
- FEA Application :-
  - Structural Engineering
  - Aerospace Engineering
  - Automobile Engineering
  - Thermal application
  - Metal Forming
  - Soil Mechanics
  - Dynamics
  - Medical and Dental Application

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Topic .....

Unit No. ....

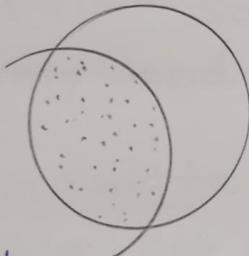
- Degree of Freedom :-
  - It is defined as ability of a Node translate or transmit the load. In the cantilever beam example, you are only concentrate with the displacement and forces.
  - By making one end point fixed, one degree of freedom for displacement is removed from the three possible degree of freedom, so now the model has - 2 degree of freedom the no of D.O.F in a model determines the no. of equations to solve the mathematical model.
- Nodes :-
  - An independent entity in a space called Node.
  - Nodes are similar to the points in geometry and represent the corner as well as mid points of an element
  - The element shape can be changed by moving the nodes in a space.

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Topic FEA ..... Unit No. 1 .....

• Element :-

It is an entity into which the system under study is divided. An element shape is specified by Nodes. The shape (area, Length and Volume) depends on the nodes with which is made.



$$\text{No. of points} = \infty$$

$$\text{dof per point} = 6$$

$$\text{Total equations} = \infty$$



$$\text{No. of Nodes} = 8$$

$$\text{Dof per Node}$$

$$= 6$$

$$\text{Total eqn} = 48$$

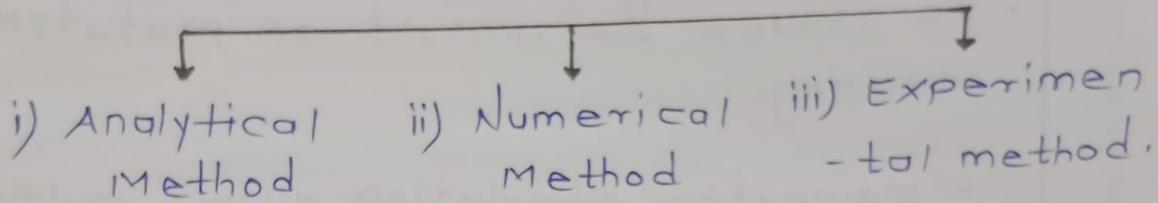
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Topic ....Fundamental concepts of FEA Unit No. 1

- Need of FEA :-
  - To reduce the amount of prototype testing.
    - computer simulation allows multiple "What-if" scenarios to be tested quickly and effectively.
  - To simulate designs that are not suitable for prototype testing
    - Example :- Surgical implants , such as artificial knee.
  - The bottom line :-
    - cost savings
    - time savings ... reduce time to market!
    - create more reliable, better-quality designs.

Topic .....

## Methods of solve Any Engineering problems



- Advantages of FEA :-
  - can readily handle complex geometry :-
    - = The heart and power of the FEM
  - can handle complex analysis type :-
    - vibration
    - Transients
    - fluids
    - Non-linear
    - Heat transfer
  - can handle complex Loading's :-
    - Node-based loading (point loads).
    - Element-based loading (pressure, thermal, inertial forces)
  - Special material effect are handled :-
    - plasticity
    - creep
    - Temp dependent properties

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Topic ...Fundamental concepts of FEA... Unit No. .... 1

- Disadvantages :-
  - Cost of the hardware and software is high.
  - Need of trained personnel is essential.
  - Accuracy of technology is not 100%.  
(A very good analysis will still be at 90% of actual)
- Future scope :-
  - i) Driven by better hardware.
  - ii) Software enhancement for parallel processing.
  - iii) will enable users to solve application rather than components or assemblies

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Topic .....

Unit No. 1

• General description of the finite element analysis:-

- In the finite element method, the actual continuum or body of matter, such as a solid, liquid or gas is represented as an assemblage of sub-divisions called finite elements.
- These elements are considered to be inter-connected at specified joints called nodes or node points
- The nodes usually lie on the element boundaries, where adjacent element are considered to be connected.
- Since the actual variation of the field variable [E.g :- Displacement, stress, temperature pressure or velocity]

Inside the continuum is not known, we assume that the variation of the field variable inside a finite element can be approximated by a simple function

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Topic ... Fundamental concepts of FEA Unit No. 1

These approximating functions (also called interpolation models) are defined in terms of the values of the field variables at the nodes.

- When field equations (like equilibrium equations) for the whole continuum are written, the new unknowns will be the nodal values of the field variable.
- By solving the field equations, which are generally in the form of matrix equations, the nodal values of the field variable will be known.
- Once these are known, the approximating functions define the field variable throughout the assemblage of elements.
- The solution of a general continuum problem by the finite element method, always follows an orderly step-by-step process. With reference to static structural problems, the step-by-step procedure can be stated as follows.

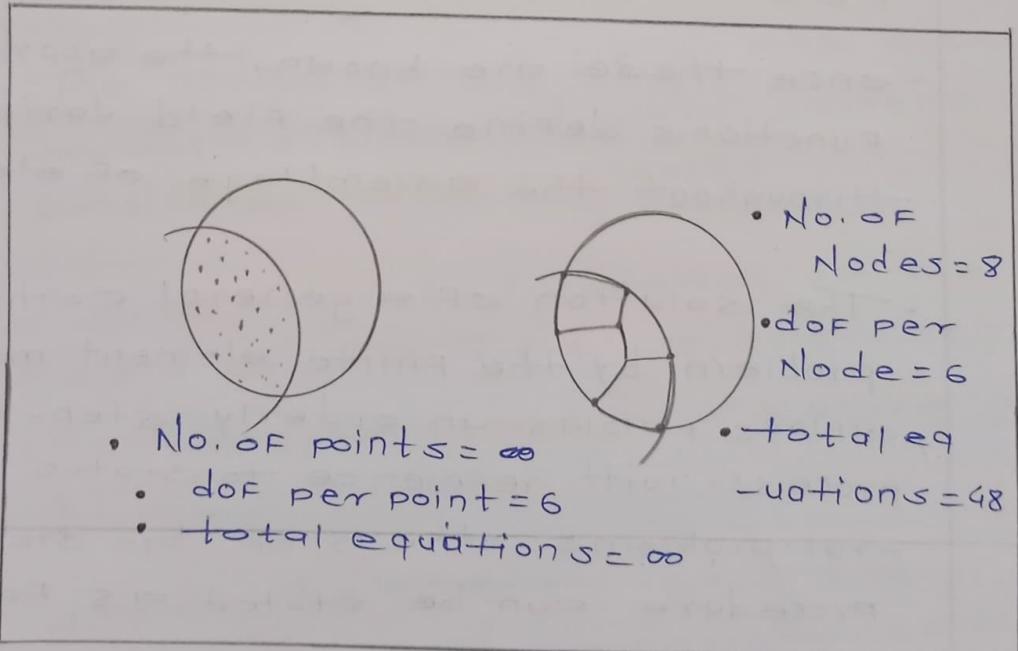
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Topic ..... Unit No. 1

Step 1:- (1) - Discretization of the structure.

- The first step in the finite element method is to divide the structure OR solution region into sub-divisions OR elements,
- Hence, the structure is to be modelled with suitable finite element.
- The Number, type, size, and arrangement of the element are to be decided.



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Topic Fundamental Concepts of FEA Unit No. 1

Step [ii] :- Selection of a proper interpolation or Displacement Model.

- Since the displacement solution of a complex structure under any specified load conditions can not be predict exactly,
- We assume some suitable solution within an element to approximate the unknown solution.
- The assumed solution must be simple from a computation standpoint, but it should satisfy certain convergence requirement.
- In general, the solution or the interpolation model is taken in the form of a polynomial.

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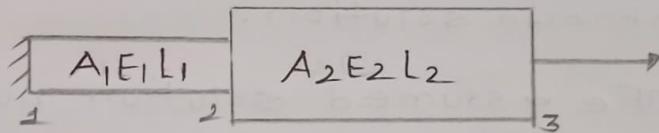
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Unit No. 1

Topic

Step [iii] :- Derivation of element stiffness matrices and load vectors.

- From the assumed displacement model the stiffness Matrix and the load vector of element are to be derived by using either equilibrium conditions or a suitable variational principle.



Step [iv] :- Assemblage of element eqn to obtain the overall equilibrium equation.

- Since the structure is composed of several finite element, the individual element stiffness matrices and load vectors are to be assembled in a suitable manner and the overall equilibrium eqn have to be formulated so.

$$[K]\vec{\Phi} = \vec{P}$$

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Topic ... Fundamental concepts of FEA Unit No. 1

Step [v] :- Solution for the unknown nodal displacements.

- The overall equilibrium equations have to be modified to account for the boundary conditions of the problem.
- After the incorporation of the boundary conditions, the equilibrium equations can be expressed as.

$$[K]\vec{\phi} = \vec{P}$$

Step (vi) :- Computation of element strains and stresses

- From the known nodal displacements, if required the element strains and stresses can be computed by using the necessary equations of solid or structural mechanics.

Topic ..... Unit No. .... 1 .....

- What is consistent units system?

- For the analysis results to be accurate or meaningful, all input data must be specified in consistent unit.
- A system is consistent when the derive units (Force, stress, energy, power) are correctly expressed in terms of the chosen base units : -  
(length, mass, time, temperature)
- Essential and Natural boundary conditions
- The differential equations is subjected to boundary conditions,
- Which is generally two types :-
- i) The essential / Geometric / Dirichlet boundary conditions .
- ii) The Natural / Force boundary condition

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Topic ... Fundamental concepts of FEA Unit No. 1

- The essential boundary conditions are the set of conditions that are sufficient for solving the differential eqn completely.

Ex :-  $u$  and  $\tau$  (primary variables)

- The Natural boundary conditions are the boundary conditions involving higher order derivative terms and are not sufficient for solving the differential equation completely, requiring atleast one essential boundary condition.

Ex :-  $F$  and  $KA \frac{dT}{dx}$  (secondary variables)

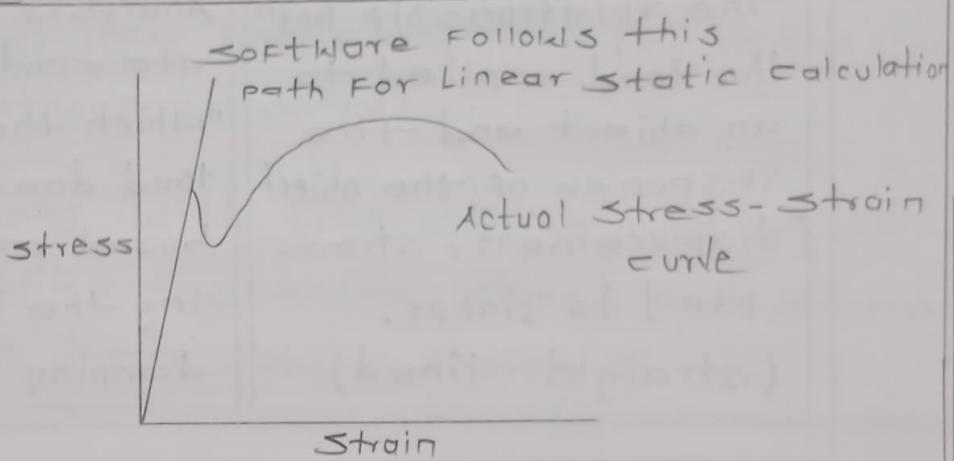
- Symmetric boundary conditions

- S.B.C are often applied to minimize the size of the FEA model. hence simulation time.
- Owing to the symmetrical geometry of the corroded riser pipe being studied the pipe can be effectively simplified and modelled as a quarter symmetry structure.

Topic FEA

Unit No. 1

- ◆ Types of Analysis
- Linear static Analysis :-



- ◆ Linear static analysis is simply an analysis that has 2 main assumption.
  - This analysis is linear, which means several important things that I will discuss right after.
  - This analysis is static, which means that it doesn't depend on time.

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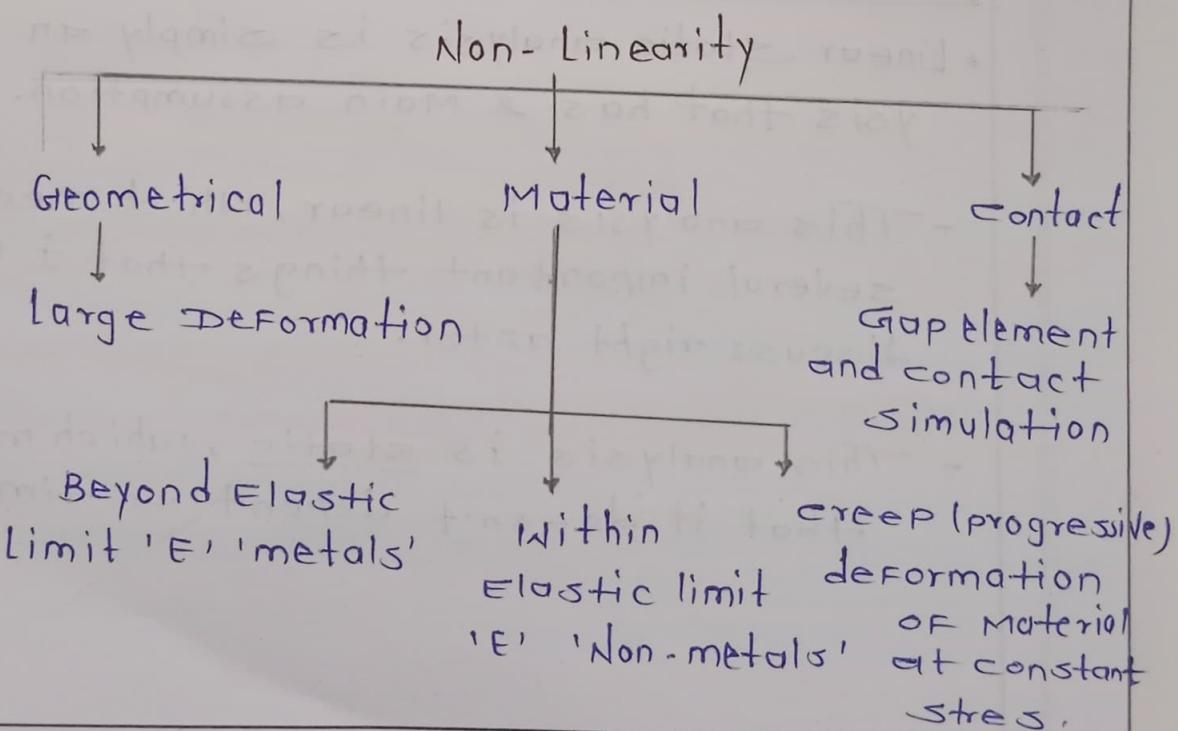
Topic ..... Unit No. .. 1 ..

### "Linear static analysis"

The relationship betn the load applied to an object and the response of the object [displacement, stress etc.] is Linear, (straight - lined)

Analysis of approximate conditions in which the applied load does not vary based on time. ignoring the inertial and damping forces.

#### ♦ Non-Linear Analysis :-



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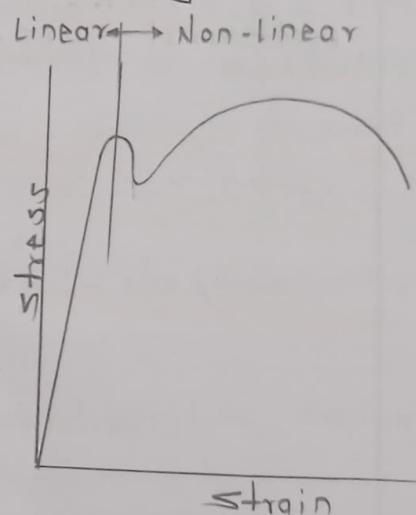
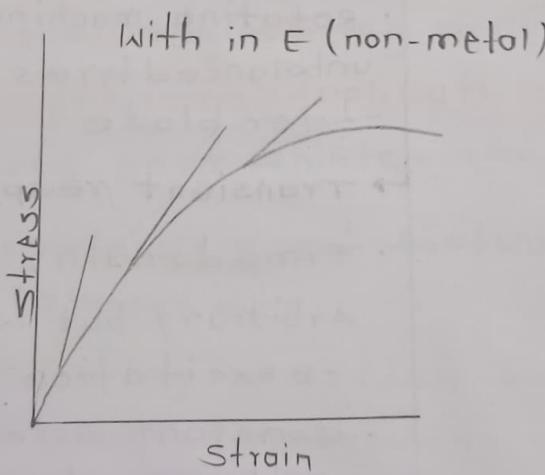
Topic FEA

Unit No. 1

- Force (stress) vs Displacement (strain)  
curve non linear (polynomial)
- Stiffness [k] is function of displacement [d] (For linear analysis [k] is constant, independent of [d])

Deals with true stress and strain  
(unlike engineering stress and strain  
in linear static Analysis)

- A material based on linearity



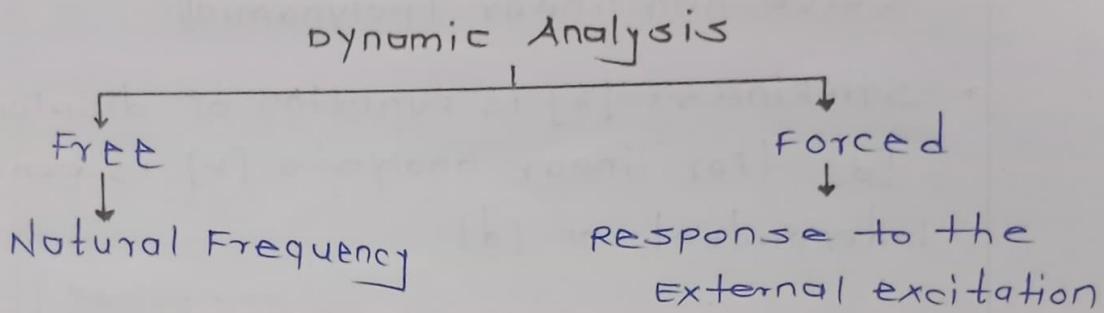
- Stress - strain diagram along with hardening rule for the material is required as input data.

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Topic

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- Dynamic Analysis :-



The frequency with which any object will vibrate if disturbed and allowed to vibrate on its own without any external force

- Frequency Response
- Frequency domain, steady state sinusoidal excitation.
- Limited linear elastic structures.
- Rotating machinery, unbalanced tyres Helicopter blade

→ Transient response

- Time domain,
- Arbitrary but well defined excitation
- Constant acceleration OR Force at a given frequency, road excitation etc.

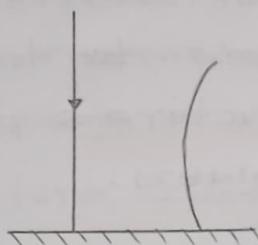
→ Random vibration

Random nature of loading, usually input in the form of PSD (Power spectral density)

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Topic ... FEA ..... Unit No. .... 1

- Linear-Buckling Analysis :-



- Applicable for only compressive load
- slender beams and sheet metal parts
- Bending stiffness <<< Axial stiffness
- Large lateral deformation
- output from software :- critical value of load
- practical application :- commonly used for civil engineering applications, Mechanical Engineering Applications - Vacuum vessel, long gear shifter rod analysis etc.
- commonly used software :- Nastan, Ansys, Abaqus etc.
- Buckling is sudden, uncontrollable failure phenomenon occurring in thin structures subjected to compressive stresses. For the material to fail in buckling, the compressive stresses do not have to reach the ultimate compressive stresses that material is capable of withstanding.

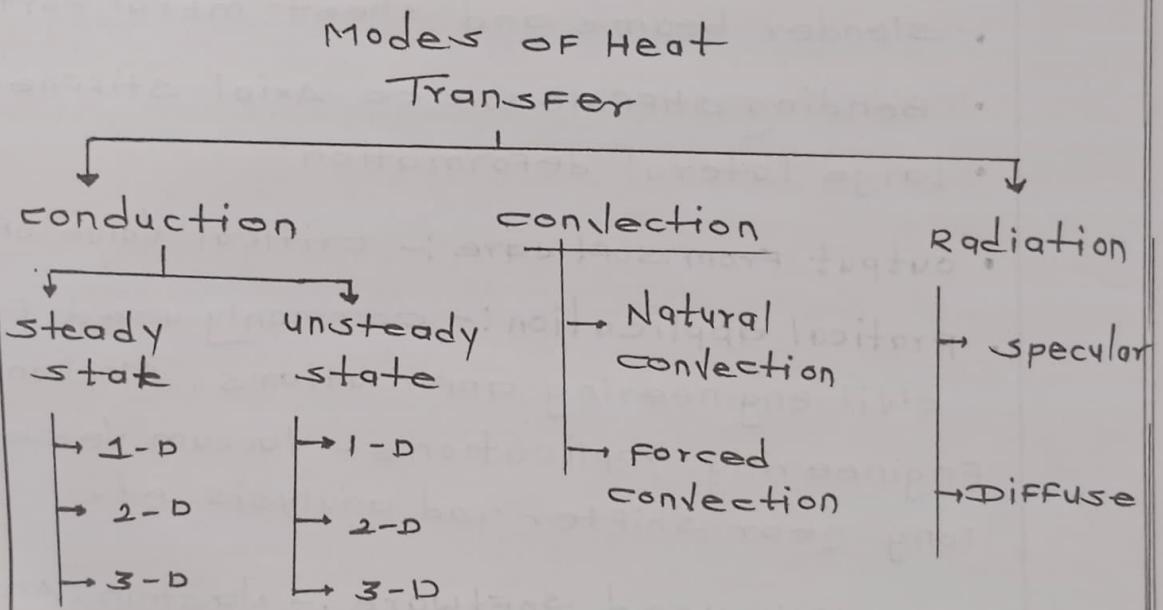
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Topic .....

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This failure phenomenon can occur in different type of structural members like columns, beams, ~~trusses~~, and thin walled structures present in bridges and towers.

• Thermal Analysis :-



Practical Application :- Engine, radiator exhaust system, heat exchanger, power plants, satellite design etc.

commonly used software :-

- Ansys, Nastan, Abaqus I-deas NX etc

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Topic FEA

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- Heat transfer is one of the most common physical phenomena. Thermal analysis can provide useful information for the design of an engineering product.
- > Temperature distributions
- > Heat Flux paths - important information in evaluating insulations.
- > As a boundary condition for the analysis of thermal stresses.
- > Thermal analyses have become increasingly important as designers push for higher performance machines.
- > This means that a good understanding of temperature distribution and thermal loads can often be vital.
- > An incorrect constraint in a system working under elevated temperature can lead to unexpected high thermal stress which may lead to failure.
- > The accuracy of thermal analyses is high dependent on correct boundary conditions being defined.

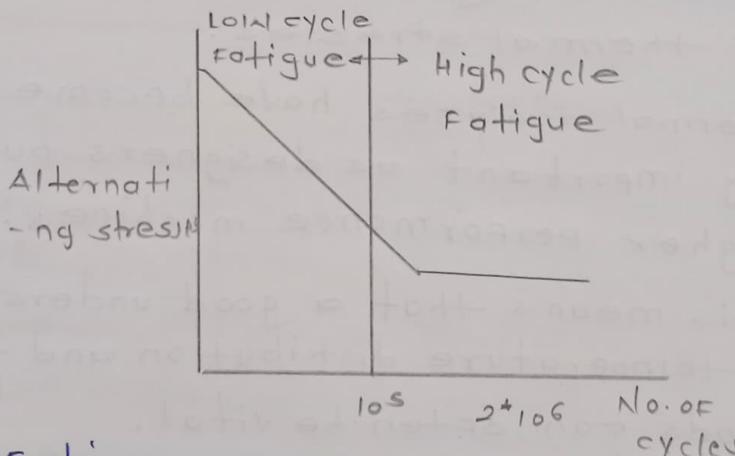
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- Fatigue analysis :-
- Calculations for life of the structure when subjected to repetitive load.
- S-N curve (alternating stress vs cycles) OR  $\epsilon$ -N (alternating strain vs reversals) is the base for fatigue calculations (like  $\sigma$ -E diagram for static analysis)



- A Fatigue analysis is performed to calculate whether a structure will fail after certain number of repeated loading and unloading, so called load cycles rather than after one load cycles as simulated in static analysis.
- Fatigue failure is due to initiation and propagation of a crack somewhere in the component.

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Topic FEA ..... Unit No. 1

fatigue failure analysis helps identify the points of failure and prevent product malfunctions , recalls.