Chap - 01

01. Fundamental Concept of FEA

FEA: Finite Element Analysis is approximate mathematical method to solve simple as well as complex engg problems.

Advantages of FEA:

1) It can handle complex structures & loading.
2) Saves time.
3) Reduce cost.

Disadvantage:

1) Approximate answer
2) Cost of software & hardware is high.
3) Need skilled workers.

- 26/07/2024.

Types of Analysis:

1) Linear Analysis
2) Non-linear Analysis.
3) Dynamic Analysis
4) Linear Buckling Analysis
5) Thermal Analysis
6) Fatigue Analysis
7) Crush Analysis

- linearity:
  elastic material → linear → metal.

Static: $ ~~Same~~ with time doesn't change.

Stiffness - resistant to change shape.
toughness - to absorbe max load / energy
fatigue -
creep - continuos load causes removal of material.
Strength - capacity to carry load.
hardness - resist deformatn

- Plain strain → considering obj to be 2D for our convinience.

Step- 1) preprocessing
2) Solⁿ
3) Post

Type of forces / Load:-

1. point load
2. Body force
3. Traction

Types of Element:
1) 1D Beam truss
2) 2D △, □
3) 3D tetrahydral.

	No.	Element	D of per node	no. of nodes	used for
1D	1.	Beam Element	1	2	Beam Analysis.
	2.	~~St~~ Truss Element	1	2	Truss & rods.
2D	3.	Triangle element	2	3	plain stress & plain strain conditⁿ.
	4.	Quadilateral	2	4	plain-stres & plain strain conditon.
3D	5.	Tetrahydral	6	4	3D Analysis
	6.	Hexahydral	6	20	3D Anaylis

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Engineering Applicatⁿ of FEA:

1) Nuclear Engg:-
Analysis of nuclear pressure vessels and containment structure, steady steate temp distribution in reactor componens.

2) Bio-Medical Engg:
Stress Analysis of bones, teeth, load bearing capacity of implant and prosthetic sys.

3) Mechanical Design:

4) Electrical Machine and Electromagnetics:
Steady state anyalysis of inductⁿ machines edy curry and core losses in electric machines.

5) Civil Engg Structures - Static Analysis of trusses, frames, shell roofs, shear walls, bridges, concreate structures.

6) Air Craft Structure :-
Static Analysis of Air Craft wings, rockets, space Crafts and miseil structure.

Viva: - which type of element is used?
- Boundry condⁿ :- Spport & fix locatⁿ

7) Heat Conductⁿ:
Steady state temp distributⁿ in solids or liq.

8) Geomechanics:
Analysis of underground openings, soil structure stress Analysis of Dams and machine foundatⁿ.

9) Hydraulic and Water Resources Engg:-
Analysis of potential flow, free surface flows, boundry layer flow, viscous flow Analysis of Hydraulic Structure and Dams.

- General Steps of FEA:-

1) Discreetization of Structure
2) Selectⁿ of Proper Interpolentⁿ or displacement Model
3) Deviation of Element stiffness Matrices and load vector.
4) Assembly of Element eqⁿ (Stiffness matrix) to obtain overall Equilibrillium.
5) solⁿ for unknown Nodal displacement
6)* Computatⁿ of Element Strain & stresses.

- Boundry Conditions :-
Boundry conditⁿ are in which surrounding structure is applied by force and supported by different ~~of~~ types of Constrains.

- Types of Boundry Condi:-

1) Geometric / Essential BC.
2) force or Natural B.C.

1) Geometric:
In geometric B.C. prescribed displacement and slopes are included.

2) Force / Natural BC:
In these BC, prescribed forces and moments in structural mechanic problems are included.

- Consistant Unit System:-
- Most of s/w used consistant unit sys w/o any conversion of unit inside the program
- It is therefore users responsibility to input data and interpriate o/p in appropriate units.

Unit II

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1D Element Analysis.

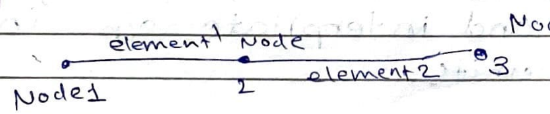
1D - Beam, Truss.
2D - Triangle,

- The shapes, sizes, numbers, configuratⁿ of elements have to be choosen carefully such that; the original body or domain is simulated as closely as possible w/o ↑ computational efforts needed for solⁿ. Mostly the ~~cho~~ ~~type~~ choice of type of elements is dected by geometry of Body and no. of independent co-ordinates neccessary to describe the sys.

- keep the geometry

- Size of Element influences conversions of solⁿ directly and hence has to be choosen with care.
- If size of element is small, final solⁿ is expected to be more accurate But computational time for solⁿ is high.

Locatⁿ of Nodes :→



- If the Body has no abrupt changes in geometry, material properties and external conditⁿ like load & temp, Body can be divided into equal sub-divisions and hence spacing of nodes can be uniform.

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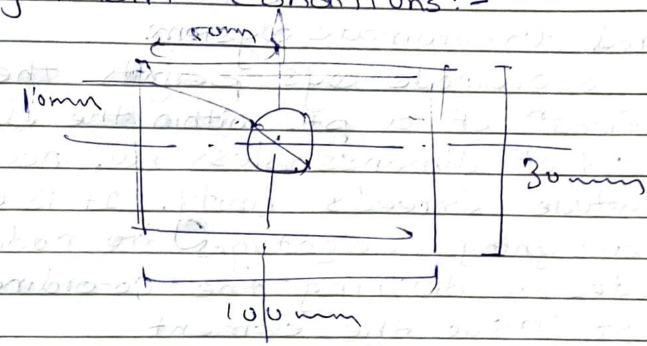
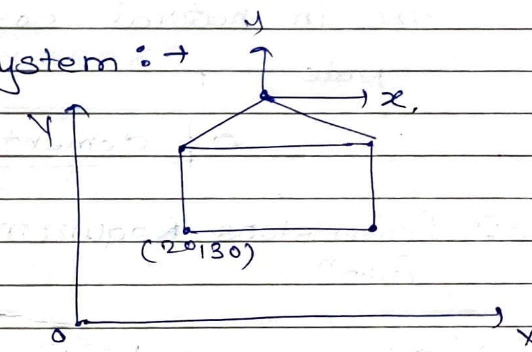
Symmetri Conditions :-



→ If configuratⁿ of body; as well as external condiⁿ are symmetric, we may consider only half of body for finite element problems,

Co-ordinate System :→

1) Global C.S
2) local C.S
3) Natural C.S.



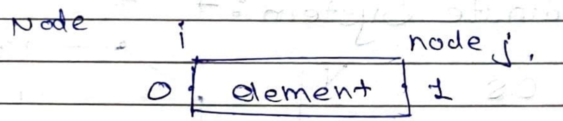
1) Global Co-ordinate System :-
- co-ordinate system used to define points in entire structure is called as Global Co-ordinate system.

2) Local Co-ordinate System:
for each element or component a separate coordinate system is used to deriving element properties. it is called local co-ord. sys.

- however, final eqⁿ are to be formed in global co-ordinate System.

3) Natural Co-ordinate System:
This co-ordinate sys permits the specificatⁿ of a pt. within the element by a set of dimenssionless no, never use magnitude exceeds unity. It is obtained by assigning weightages to nodal co-ordinates in defining the co-ordinate of any pt inside the element.

- The use of natural co-ordinate sys is advantageous in assembling element properties. since closed form integratⁿ formule are available when expression are in natural co-ordinate system.



● Conversions Requirement of Displacement funtⁿ

1) Displacement functⁿ must be continuous & compatible within the element

- continuous means 2^{nd} element should start by taking one edge common of 1^{st} element.
- Compatible means when it deforms, there should not be any discontinuety betⁿ elements, means element must not

separate or overlap and there should not be any sudden change in slope across the inter element boundries,

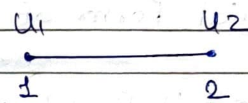
2) Displacement funtⁿ must be capable of Representing constant strain stage within the element.

3) Displacement funtⁿ must be capable of representing rigid body displacement.

* Displacement funtⁿ for 1D element:

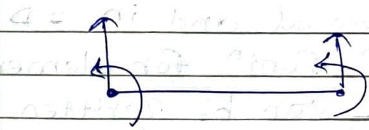
1) Bar Element:

$\boxed{u = \alpha_1 + \alpha_2 x}$



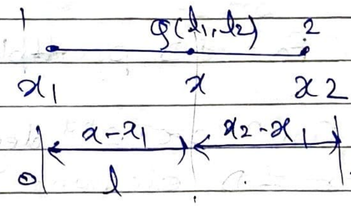
at node 1 $u_1 = \alpha_1 + \alpha_2 x_1$
$u_2 = \alpha_1 + \alpha_2 x_2$

2) Beam Element



$w = \alpha_1 + \alpha_2 x + \alpha_3 x^2 + \alpha_4 + x^3$

Natural C.S for 1D



$L_1 + L_2 = 1$
$L_1 x_1 + L_2 x_2 = 1$
in matrix form

$$\begin{bmatrix} L_1 \\ L_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ x_1 & x_2 \end{bmatrix} \begin{Bmatrix} 1 \\ x \end{Bmatrix}$$

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$$\begin{Bmatrix} L_1 \\ L_2 \end{Bmatrix} = \frac{1}{x_2 - x_1} \begin{bmatrix} x_2 & -1 \\ -x_1 & 1 \end{bmatrix} \begin{Bmatrix} 1 \\ x \end{Bmatrix}$$

$$= \frac{1}{l} \begin{bmatrix} x_2 & -1 \\ -x_1 & 1 \end{bmatrix} \begin{Bmatrix} 1 \\ x \end{Bmatrix}$$

$$L_1 = \frac{x_2 - x}{l} + L_2 \frac{x - x_2}{l}$$

- **Polynomial form of Interpolat^n funt^n:-**

Interpolat^n funt^n also known as shape funt^n are used in numerical methods like FEA to approximate sol^n per element

1) Linear Interpolat^n funt^n:
- first degree polynomial and in 1D; linear interpolat^n funt^n for element with node x_1 & x_2 can be written as

$$N_1(x) = \frac{x_2 - x}{x_2 - x_1} \; ; \quad N_2 = \frac{x - x_1}{x_2 - x_1}$$

2) Quadratic interpolat^n: For 1D problem:-

Quadratic interpolat^n funt^n are 2nd degree polynominal, & for 1D element with nodes at x_1, x_2, x_3

$$N_1(x) = \frac{(x-x_2)(x-x_3)}{(x_1-x_2)(x_1-x_3)}$$

$$N_2(x) = \frac{(x-x_1)(x-x_3)}{(x_2-x_1)(x_2-x_3)}$$

$$N_3(x) = \frac{(x-x_1)(x-x_2)}{(x_3-x_1)(x_3-x_2)}$$

(3) Cubic Interpolat^n funt^n:-

- 3rd degree polynomial; and in 1D for an element with nodes at x_1, x_2, x_3 and x_4

$x_1 \quad x_2 \quad x_3 \quad x_4$

$$N_1(x) = \frac{(x-x_2)(x-x_3)(x-x_4)}{(x-x_2)(x_1-x_3)(x_1-x_4)}$$

$$N_2(x) = \frac{(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_1)(x_2-x_3)(x_2-x_4)}$$

$$N_3(x) = (x-x_1)(x-x_2)(x-x_4)$$

$$N_4(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_1)(x_4-x_2)(x_4-x_3)}$$

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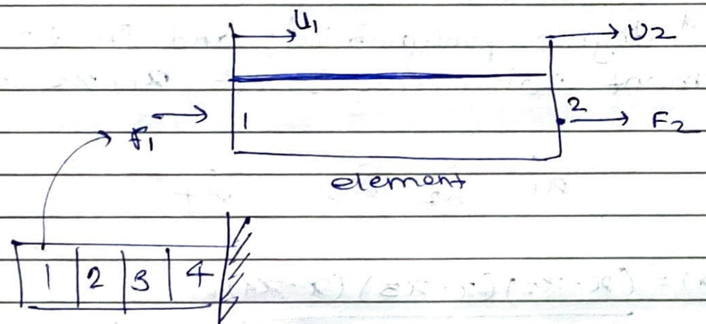
- **Properties of Shape funtn :-**

- It posseses several imp properties that ensure their utility in FEA.

1) Compatibility
2) Completeness

3) Interpolatn property.
4) Patition of unit
5) Non-negativity.

- **Stiffness matrix :→**



l = length of Element
A = c/s area of —"—
E = modulus of Elasticity of material
f_1 = force acting on ~~load~~ Node 1
f_2 = —"— 2
u_1 = displacement of node 1
u_2 = —"— 2

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Total degree of freedom
= DOF per node × No. of nodes.

1 D = 1×2 = 2

$\boxed{F = KU}$ → Stiffness Eqn.

for element;

$f_1 = k(u_1 - u_2) = -k(u_2 - u_1)$

$f_2 = k(u_2 - u_1) = k(u_2 - u_1)$

$\boxed{k = \frac{AE}{l}}$

$$\begin{bmatrix} -k & k \\ k & -k \end{bmatrix} \begin{Bmatrix} u_2 \\ u_1 \end{Bmatrix} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

$$\begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix} = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

$$= k \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

$$\begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix} = \frac{AE}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

{Element force vector} = [element stiffness matrix] {element displacement vector}

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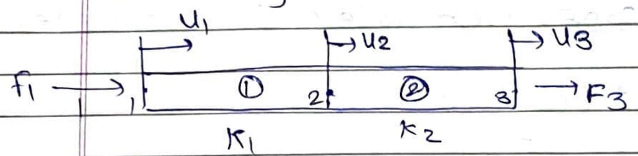
★ Assembly of Stiffness Matrix :-



for element 1

$$\begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix} = k_1 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}.$$

for element 2;

$$\underbrace{\begin{Bmatrix} f_2 \\ f_3 \end{Bmatrix}}_{\text{load}} = \underbrace{k_2 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}}_{\text{stiffness}} \overbrace{\begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix}}^{\text{displacement}}.$$

Tot DOF = 1 × 3 = 3

$$= \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1+1 & 0-1 \\ 0 & 0-1 & 0+1 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \\ 3 \end{array}$$

$$= \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \ldots$$

Global Stiffness Matrix.

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8) Sol^n of simultaneous eq^n

9) Computat^n of element strain & stress.

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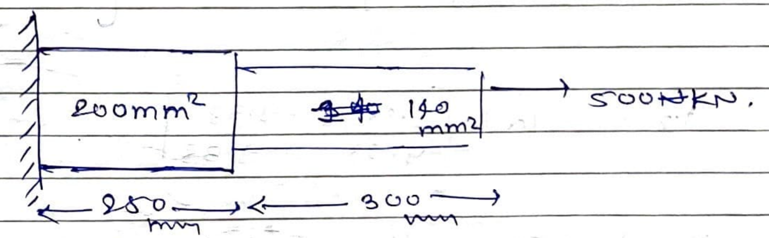
● Basic Steps in FEA:-

1) Discretizat^n of Body
2) Format^n of element stiffness matrix
3) format^n Global stiffness Matrix
4) format^n of Global Load Vector.
5) —— 1 —— Nodal displacement vector
6) Assembly of Global Stiffness - Nodal displacement, load eq^n.
7) Incorporat^n of specified B.C.

ex - A stepped metallic bar with circular cross sect^n consists of 2 segments, 1st seg is of length 250 mm & cross sect^n Area 200 mm^2. 2nd segment is of length 300 mm & cross sect^n area 140 mm^2. Modulous of elasticity of bar material is 200 GPa. If bar is fixed at 1 end of bigger cross sect^n & subjected to tensile force of 500 kN at opposite end of smallest sect^n. Determine :

1) Nodal displacement
2) Element Stress
3) Support React^n



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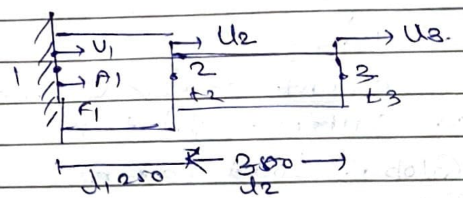
→ Discretization :-

1)



2) formatⁿ of Element Stiffness Matrix :-

$$k_1 = \frac{A_1 E_1}{l_1}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \frac{200 \times 200 \times 10^3}{250}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

~~for E2~~;

$$k_1 = 10^4 \begin{array}{c c} \begin{array}{cc} 1 & 2 \end{array} & \\ \begin{bmatrix} 16 & -16 \\ -16 & 16 \end{bmatrix} & \begin{array}{c} 1 \\ 2 \end{array} \end{array}$$

for E2;

$$k_2 = \frac{A_2 E_2}{l_2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \frac{140 \times 200 \times 10^3}{300}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$k_2 = 10^4 \begin{array}{c c} \begin{array}{cc} 2 & 3 \end{array} & \\ \begin{bmatrix} 9.33 & -9.33 \\ -9.33 & 9.33 \end{bmatrix} & \begin{array}{c} 2 \\ 3 \end{array} \end{array}$$

3) Global Stiff ~~matix~~ matrix :

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$$k = k_1 + k_2 = 10^4 \begin{array}{c c} \begin{array}{ccc} 1 & 2 & 3 \end{array} & \\ \begin{bmatrix} 16 & -16 & 0 \\ -16 & 16+9.33 & 0-9.33 \\ 0 & 0-9.33 & 0+9.33 \end{bmatrix} & \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \end{array}$$

$$= 10^4 \begin{bmatrix} 16 & -16 & 0 \\ -16 & 25.33 & -9.33 \\ 0 & -9.33 & 9.33 \end{bmatrix}$$

4) Global Load Vector :-

$$\begin{Bmatrix} R \\ 0 \\ F_3 \end{Bmatrix} = \begin{Bmatrix} R \\ 0 \\ 800 \times 10^3 \end{Bmatrix}$$

Reactⁿ

F_2

5) Nodal displacement

$$\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}$$

6) Assembly of Global Stiffness Matrix :-

$$[K]\{u\} = \{F\}$$

$$10^4 \begin{bmatrix} 16 & -16 & 0 \\ -16 & 25.33 & -9.33 \\ 0 & -9.33 & 9.33 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} R \\ 0 \\ 500 \times 10^3 \end{Bmatrix}$$

$u_1 \to 0$; bcz dis = 0 ; bcz its fixed (Node I)

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7)

At Node 1; Rigid suppot is there

$\therefore u_1 = 0$

By using row eliminatn;

$$10^4 \begin{bmatrix} 25.33 & -9.33 \\ -9.33 & 9.33 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = 10^3 \begin{Bmatrix} 0 \\ 500 \cancel{\times 10^3} \end{Bmatrix}$$

$$10^4 [25.33 \times u_2 - 9.33 \times u_3] = 0$$

$$10^4 [-9.33 u_2 + 9.33 u_3] = 500 \times 10^3.$$

8) By adding $\left(\frac{9.33}{25.33}\right) \times$ Row I to Row II.

$$= 10^4 \begin{bmatrix} 25.33 & -9.33 \\ 0 & 5.894 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = 10^3 \begin{Bmatrix} 0 \\ 500 \end{Bmatrix}$$

$$10^4 [0 \times u_2 + 5.894 \times u_3] = 10^3 \times 500$$

$$10^4 [5.894 \, u_3] = 10^3 \times 500$$

$$5.894 \, u_3 = 50$$

$$\boxed{\therefore u_3 = 8.483.}$$

$$10^4 [25.33 \, u_2 - 9.33 \times 8.483] = 0$$

$$25.33 u_2 = 79.1463$$

$$\boxed{\therefore u_2 = 3.124}$$

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$$\{u_x\} = \begin{Bmatrix} 0 \\ 3.125 \\ 8.482 \end{Bmatrix}$$

Stress:-

$$\sigma = E [B] \{u_N\}$$

$$\sigma_x = \frac{E_x}{l} [-1 \quad 1] \{u_N\}$$

$$\sigma_1 = \frac{E_1}{l_1} [-1 \quad 1] \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

$$= \frac{200 \times 10^3}{250} [-1 \times 0 + 1 \times 3.125]$$

$$\sigma_1 = 2500 \text{ N/mm}^2$$

$$\sigma_2 = \frac{E_2}{l_2} [-1 \quad 1] \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix}$$

$$\sigma_2 = \frac{200 \times 10^3}{300} [-1 \times 3.125 + 1 \times 8.483]$$

$$\sigma_2 = 3571.3 \text{ N/mm}^2.$$

$$10^4 \begin{bmatrix} 16 & -16 \\ -16 & 16 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} R \\ 0 \end{Bmatrix}$$

$$10^4 [16 \times u_1 - 16 \times u_2] = R.$$

$u_1 \to 0$

$$\boxed{R = -500 \times 10^3 \text{ N}.}$$

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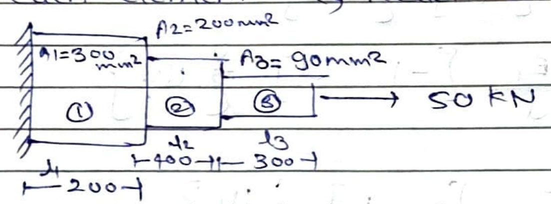
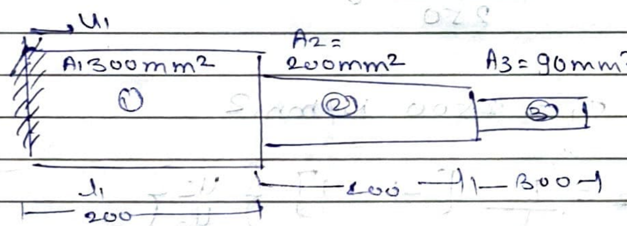
ex - Axial step bar shown in fig. is subjected to an axial pull of 50 kN. If material for the bar is uniform and as modulous of elasticity 200 GPa. Find: (1) Nodal displacement. (2) Stress in each element. (3) Reactⁿ at support.

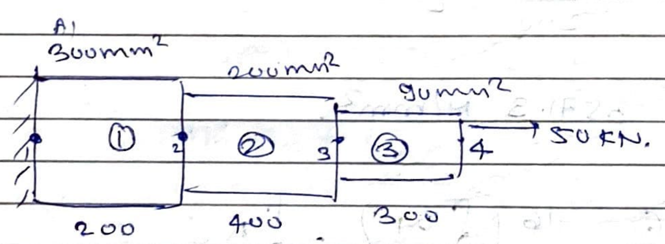


$E_1 = E_2 = E_3 = 200 GPa$

→ 1) Discritizatⁿ:



Give nodes:



1) Total DOF :- DOF at each node × no. of element

$= 1 \times 4$

$= 4$

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2) Formatⁿ of Element Stiffness Matrix :-

for element 1;

$$\boxed{k_1 = \frac{A_1 E_1}{l_1}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}}$$

$$= \frac{300 \times 200 \times 10^3}{200}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = 300 \times 10^4$$

$$k_1 = 10^4 \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} 30 & -30 \\ -30 & 30 \end{bmatrix} \end{array}$$

~~$k_2 = 10^4$~~

$$k_2 = \frac{A_2 E_2}{l_2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \frac{200 \times 200 \times 10^3}{400}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\therefore k_2 = 10 \times 10^4 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = 10^4 \begin{array}{c} \begin{array}{cc} 2 & 3 \end{array} \\ \begin{bmatrix} 10 & -10 \\ -10 & 10 \end{bmatrix} \end{array}$$

$$k_3 = \frac{A_3 E_3}{l_3}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \frac{90 \times 200 \times 10^3}{300}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\therefore k_3 = 60 \times 10^3 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = 10^4 \begin{array}{c} \begin{array}{cc} 3 & 4 \end{array} \\ \begin{bmatrix} 6 & -6 \\ -6 & 6 \end{bmatrix} \end{array}$$

$$k = k_1 + k_2 + k_3 = 10^4 \begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 30 & -30 & 0 & 0 \\ -30 & 30+10 & -10 & 0 \\ 0 & -10 & 10+6 & -6 \\ 0 & 0 & -6 & 6 \end{bmatrix} \end{array}$$

1 2 3 4

Step-3]

$$\therefore k = 10^4 \begin{bmatrix} 30 & -30 & 0 & 0 \\ -30 & 40 & -10 & 0 \\ 0 & -10 & 16 & -6 \\ 0 & 0 & -6 & 6 \end{bmatrix}$$

$$= 2\times10^4 \begin{bmatrix} 15 & -15 & 0 & 0 \\ -15 & 20 & -5 & 0 \\ 0 & -5 & 8 & -3 \\ 0 & 0 & -3 & 3 \end{bmatrix}$$

Step④ Global load vector:-

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix} \begin{Bmatrix} R \\ 0 \\ 0 \\ P_4 \end{Bmatrix} \begin{Bmatrix} R \\ 0 \\ 0 \\ 50\times10^3 \end{Bmatrix}$$

load acting at 1, 2, 3, 4.

$R \to$ for support (जिथे fixed असेल तीथे R)

- Step⑤: Nodal Displacement:

$$\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix}$$

- Step⑥: Assembly of Stiffness matrix:

$$[k]\{u\} = \{F\}$$

$$2\times10^4 \begin{bmatrix} 15 & -15 & 0 & 0 \\ -15 & 20 & -5 & 0 \\ 0 & -5 & 8 & -3 \\ 0 & 0 & -3 & 3 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} R \\ 0 \\ 0 \\ 50\times10^3 \end{Bmatrix}$$

7) incorporatn of specified BC:

at node 1; $u_1 = 0$

using row eliminatn:

$$2\times10^4 \begin{bmatrix} 20 & -5 & 0 \\ -5 & 8 & -3 \\ 0 & -3 & 3 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 50\times10^3 \end{Bmatrix}$$

$$2\times10^4 [20\times u_2 - 5u_3] = 0$$

$$2\times10^4 [-5u_2 + 8u_3 - 3u_4] = 0$$

$$2\times10^4 [0u_2 - 3u_3 + 3u_4] = 50\times10^3$$

$$\therefore \left(\frac{3}{8}\right)\times 8 + R_3 \quad \to \frac{3}{8}\times 8 + R_3$$

$$3 + (-3)$$

$$3 - 3 = 0$$

$$\frac{3}{8}\times(-3) + R_3$$

$$\frac{-9}{8} + \frac{3\times8}{8}$$

$$\frac{-9+24}{8} = \frac{-15}{8}$$

$$2\times10^4 \begin{bmatrix} 20 & -5 & 0 \\ -5 & 8 & -3 \\ 0 & 0 & \frac{-15}{8} \end{bmatrix}$$

$$2\times10^4 \begin{bmatrix} 20 & -5 & 0 \\ -5 & 8 & -3 \\ 0 & 0 & -15/8 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 50\times10^3 \end{Bmatrix}$$

$$2\times10^4 [-15/8\times u_4] = 50\times10^3$$

$$\frac{-15}{8} u_4 =$$

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Multiplying: $\frac{5}{20} \times$ Row I + Row II

$$2\times10^4 \begin{bmatrix} 20 & -5 & 0 \\ -5 & 8 & -3 \\ 0 & -3 & 3 \end{bmatrix}$$

We need zero here.

That's why we need zero here, so that row below won't change.

$\therefore$ Multiply $\frac{5}{20}$ to Row I & add to row ②.

$\therefore \frac{5}{20}$ Row ① + Row ②;

1) $20 \times \frac{5}{20} + -5 \rightarrow 0$

2) $-5 \times \frac{5}{20} + 8 \Rightarrow 9$

$\frac{5}{20} \times (-5) + 8 = \frac{-5}{4} + \frac{8 \times 4}{1 \times 4} = \frac{32-5}{4} = 6.75$

$$2\times10^4 \begin{bmatrix} 20 & -5 & 0 \\ 0 & 6.75 & -3 \\ 0 & -3 & 3 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 50\times10^3 \end{Bmatrix}$$

zero

R3: $\Rightarrow \frac{3}{6.75} \times R_2 + R_3 : \rightarrow$

1) $\frac{3}{6.75} \times 6.75 + (-3) = 0$

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2) $\frac{3}{6.75} \times -3 + 3 = -1.33 + 3 = 1.67.$

$$2\times10^4 \begin{bmatrix} 20 & -5 & 0 \\ 0 & 6.75 & -3 \\ 0 & 0 & 1.67 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 50\times10^3 \end{Bmatrix}$$

$2\times10^4 \{1.67 \times u_4\} = 50\times10^3$

$1.67\, u_4 = \frac{5}{2}$

$\therefore u_4 = 1.49700$

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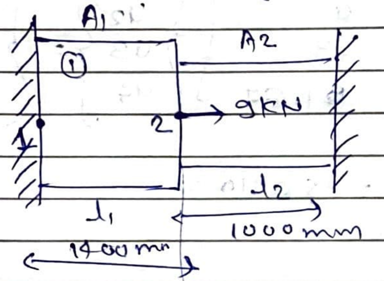
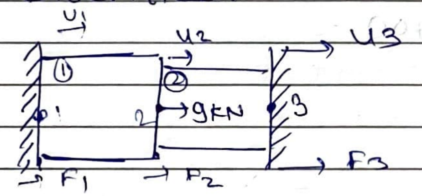
Que. 2 bar 1 of Aluminium & other of Steel joined to together; & subjected to load as shown in fig. Determine displacement at common joint and member forced.



$A_1 = 2000\ mm^2$
$A_2 = 400\ mm^2$
$E_1 = 70\ KN/mm^2$
$E_2 = 200\ KN/mm^2$.

→ 1) Discritizatⁿ:



$u_1 = u_3 = 0$

Total Dof $= 1 \times 3 = 3$,

2) Formatⁿ of Element Stiffness Matrix:

$$k_1 = \frac{A_1E_1}{l_1}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \frac{2000 \times 70 \times 10^3}{1400}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$k_1 = \cancel{10^4}\ 10^5 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \cancel{10^4 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}}$$

$$k_1 = 10^4 \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} 10 & -10 \\ -10 & 10 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \end{array}$$

$$k_2 = \frac{A_2E_2}{l_2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \frac{400 \times 200 \times 10^3}{1000}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= 8 \times 10^4 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = 10^4 \begin{array}{c} \begin{array}{cc} 2 & 3 \end{array} \\ \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix} \end{array} \begin{array}{c} 2 \\ 3 \end{array}$$

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3) Global Stiffness matrix:

$$K = K_1 + K_2 = 10^4 \begin{bmatrix} 10 & -10 & 0 \\ -10 & 10+8 & -8 \\ 0 & -8 & 8 \end{bmatrix}$$

$$K = 10^4 \begin{bmatrix} 10 & -10 & 0 \\ -10 & 18 & -8 \\ 0 & -8 & 8 \end{bmatrix}$$

4) Global Load Vector:

$$\begin{Bmatrix} R_1 \\ 9 \times 10^3 \\ R_2 \end{Bmatrix}$$

5) Nodal displacement: $\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ u_2 \\ 0 \end{Bmatrix}$

6) Assembly of Global Stiffness Matrix:

$$[K]\{u\} = \{F\}$$

$$10^4 \begin{bmatrix} 10 & -10 & 0 \\ -10 & 18 & -8 \\ 0 & -8 & 8 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} R_1 \\ 9 \times 10^3 \\ R_2 \end{Bmatrix}$$

7) incorporatⁿ of Specified B.C. :- at node 1. $u_1 = 0$, Using row eliminatⁿ. (Support removed row)

$$\cancel{\begin{bmatrix} 18 & -8 \\ -8 & 8 \end{bmatrix}}$$

$$10^4 (18\ u_2) = 9 \times 10^3$$

$$\therefore u_2 = 0.5 \times 10^3 = \frac{500}{10^4} = 0.05$$

$u_2 = 0.05$

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$$\begin{Bmatrix} u_x \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0.05 \\ 0 \end{Bmatrix}$$

8) Stress:-

$$\sigma = E [B] \{u_N\}$$

$$\sigma_1 = \frac{E_1}{l_1} \begin{bmatrix} -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

$$= \frac{70\times10^3}{1400} \begin{bmatrix} -1 & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ 0.05 \end{Bmatrix}$$

$$= \frac{10^4}{200} \; 50 \begin{bmatrix} -1 & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ 0.05 \end{Bmatrix}$$

$$= 50 \left[(-1\times0)+(1\times0.05)\right]$$

$$= 50\times0.05$$

$$= 2.5 \; N/mm^2$$

$$\sigma_2 = \frac{E_2}{l_2} \begin{bmatrix} -1 & 1 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix}$$

$$\sigma_2 = \frac{200\times10^3}{10^3} \begin{bmatrix} -1 & 1 \end{bmatrix} \begin{Bmatrix} 0.05 \\ 0 \end{Bmatrix}$$

$$= 200 \left[-0.05 + 0\right]$$

$$\sigma_2 = -10$$

10) Computation of strain & stress:-

For R1

$$10^4 \begin{bmatrix} 10 & -10 \\ -10 & 10 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} f_1 \\ 0 \end{Bmatrix}$$

Truss:- Axial load acts on truss

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Beam- Transial load acts.

$$10^4 \times \left[10\times u_1 - 10\times u_2\right] = R_1$$

$$10^4 \times \left[10\times0 - 10\times0.05\right] = R_1$$

$$R_1 = 10^4 \times (-0.5)$$

$$R_1 = -5\times10^3 \; N.$$

for R_3;

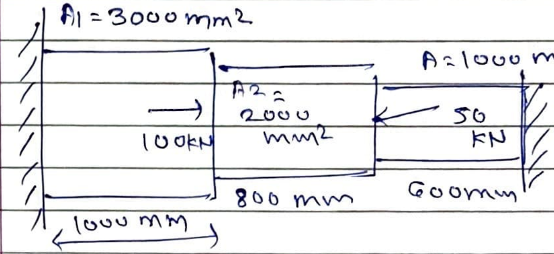
$$10^4 \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ R_3 \end{Bmatrix}$$

$$10^4 \left[(-8\times u_2) + (8\times u_3)\right] = R_3$$

$$R_3 = 10^4 \left[(-8\times0.05) + (8\times0)\right]$$

$$R_3 = -4\times10^3 \; N.$$

Que:-



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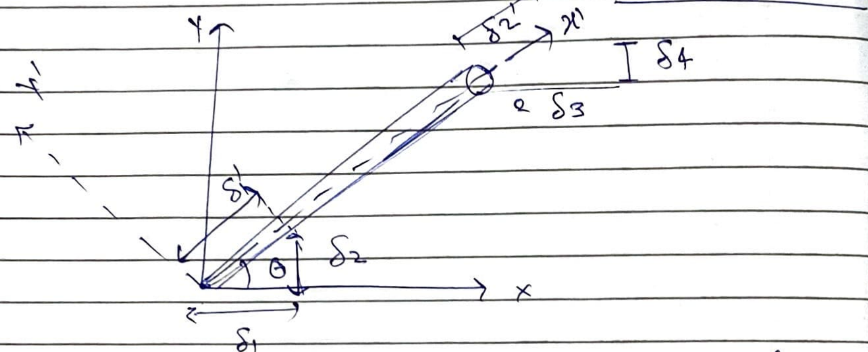
Global Stiffness Matrix for Truss Element:-



In fig, 2 noded bar element for analysis of truss is selected. ∴ The members are subjected to only axial forces, displacement are only axial directⁿ of members. Therefore, nodal displacement vector for bar element is :

$$\{x'\}_e = \begin{Bmatrix} \delta_1' \\ \delta_2' \end{Bmatrix}$$

where δ_1' and δ_2' are displacement in axial directⁿ of element

δ_1' & δ_2' = Axial displacement in local c.s

δ_1 & δ_2 = —"— Global c.s.

Axial directⁿ is not same for all members. Hence, if we select x & y as global co-ordinate sys.

∴ There're 2 displacement component at every node. Hence nodal variable vector for an element is

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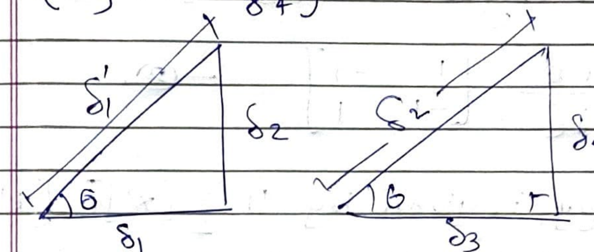
$$\{x\}_e = \begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \end{Bmatrix} = [\delta_1 \ \delta_2 \ \delta_3 \ \delta_4]^T$$



$\delta_1' = \delta_1 \cos\theta + \delta_2 \sin\theta$ — (1)

where,

$[L]$ = Transformatⁿ matrix

$\delta_2' = \delta_3 \cos\theta + \delta_4 \sin\theta$ — (2)

$$[L] = \begin{bmatrix} l & m & 0 & 0 \\ 0 & 0 & l & m \end{bmatrix} \quad \{x'\}_e = \begin{Bmatrix} \delta_1' \\ \delta_2' \end{Bmatrix}$$

$$= \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ 0 & 0 & \cos\theta & \sin\theta \end{bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \end{Bmatrix}$$

$l = \cos\theta, \quad m = \sin\theta$

$$\therefore \{x'\}_e = [L]\{x\}_e$$

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- **Stiffness Matrix :- (Using Strain Energy)**

The matrix of 2 noded bar element used for Analysis of Truss is

$$[K] = \frac{A \cdot E}{L_e} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad — ②$$

- Strain Energy is given by $\cancel{U = \frac{1}{2}[x']^T_e [K']_e}$

$$U = \frac{1}{2} [x']^T_e \, [K]_e \cdot \{x'\}_e$$

$\{x'\}_e \rightarrow$ displacement Vector

-- for local co-ordinate sys.

$$= \frac{1}{2} \cdot \{x\}^T_e \cdot [L]^T \cdot [K]_e \, [L] \, \{x\}_e$$

$$= \frac{1}{2} \{x\}^T_e \cdot [K] \cdot \{x\}_e \quad — ③$$

where,

$$[K] = [L]^T \cdot [K'] \cdot [L] \quad — ④$$

$[K'] \rightarrow$ Stiffness matrix in local co-ordinate sys.
$[K] \rightarrow$ —"— in Global C.S.
$[L] \rightarrow$ Transformat^n Matrix.

eq^n ④ can be written by,

$$[K] = \begin{bmatrix} l & 0 \\ m & 0 \\ 0 & l \\ 0 & m \end{bmatrix} \cdot \frac{AE}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} l & m & 0 & 0 \\ 0 & 0 & l & m \end{bmatrix}$$

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$$[K] = \frac{AE}{L} \begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix}$$

★ **Strain - Displacement Matrix for Bar Element.**

$$\{e\} = \varepsilon_x = \frac{\partial u}{\partial x}$$

$$\varepsilon_x = \frac{\partial}{\partial x} [N] \cdot \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

but, $N_1 = \frac{x_2 - x}{l}$; $\frac{x - x_1}{l}$

derivative w.r.t. x;

$$\frac{\partial N_1}{\partial x} = \frac{-1}{l}, \quad \frac{\partial N_2}{\partial x} = \frac{1}{l}$$

$$\therefore \{\varepsilon\} = \varepsilon_x = \begin{bmatrix} \frac{-1}{l} & \frac{1}{l} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

$$\{\varepsilon\} = [B] \cdot \{a\}$$

We know,

$$\varepsilon_1 = \frac{x - x_2}{l/2} = \frac{2}{l}(x - x_2)$$

$$N_1 = \frac{1 - \varepsilon}{2} \ ; \ N_2 = \frac{1 + \varepsilon}{2}$$

$$\frac{\partial \varepsilon}{\partial x} = \frac{2}{l}, \quad \frac{\partial N_1}{\partial \varepsilon} = \frac{-1}{2}, \quad \frac{\partial N_2}{\partial \varepsilon} = \frac{1}{2}$$

$$\{\varepsilon\} = \varepsilon_x = \frac{2}{l} \begin{bmatrix} \frac{-1}{2} & \frac{1}{2} \end{bmatrix} \cdot \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

$$= \frac{1}{l} \begin{bmatrix} -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

$$\varepsilon = [B] \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \quad \text{where; } [B] = \frac{1}{l} \begin{bmatrix} -1 & 1 \end{bmatrix}$$