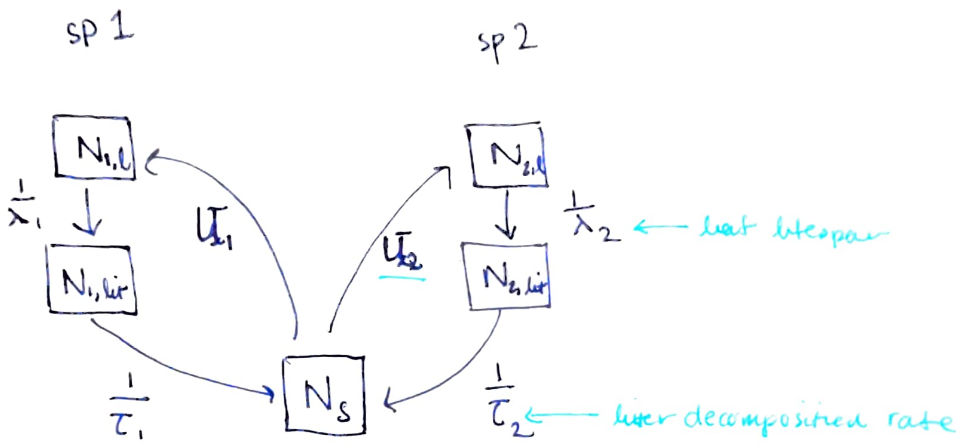
MODEL REVIEW 2023-05-24

* track N only ~ write down system of eq'ns
* review Ensheng model analytical results



U_i are N uptake functions limited either by

a) N_s, or

b) carbon uptake of the plant ~ existing biomass

$$U_i = f(N_{soil}, N_{i,leaf}) \quad \left[\frac{gN}{day}\right]$$ units for total flux of N into leaves

uptake depends on the amount of plant biomass (expressed as $N_{i,leaf}$ in gN units w/ conversions) ~ and the amount of carbon that plant can uptake/produce.

If the plant cannot fix enough carbon with it's existing leaves, it will not uptake more nitrogen until leaf N reaches a sustainable level (or the plant dies out).

To determine U_i, you need to answer two questions

→ How much N is in the soil? (N_{soil}) available for uptake? What is the conversion efficiency between N_{soil} and N_{leaf}?
  * assume 100% to start

→ How does uptake depend on existing biomass (N_{leaf})?
  If the plant can uptake enough carbon in the same timestep w/ existing leaf area to build leaves from N from the soil it will take that N. If it does not have enough C, it will not

Consider first the complete Nitrogen cycle w/ one sp. only

$$
\begin{tikzcd}
\boxed{N_{leaf}} \arrow[d, "\frac{1}{\lambda}"'] & \\
\boxed{N_{litter}} \arrow[r, "\frac{1}{\tau}"'] & \boxed{N_{soil}} \arrow[ul, bend right, "U"']
\end{tikzcd}
$$

$$N_{TOT} = N_{leaf} + N_{litter} + N_{soil}$$

$$
\begin{cases}
\dfrac{dN_{leaf}}{dt} = U(N_{leaf}, N_{soil}) - \dfrac{N_{leaf}}{\lambda} \\[2ex]
\dfrac{dN_{litter}}{dt} = \dfrac{N_{leaf}}{\lambda} - \dfrac{N_{litter}}{\tau} \\[2ex]
\dfrac{dN_{soil}}{dt} = \dfrac{N_{litter}}{\tau} - U(N_{leaf}, N_{soil})
\end{cases}
$$

$\downarrow$ collapse to 2 eq$^{\underline{n}}$s $\quad N_{litter} = N_{tot} - N_{leaf} - N_{soil}$

$$
\begin{cases}
\dfrac{dN_{leaf}}{dt} = U(N_{leaf}, N_{soil}) - \dfrac{N_{leaf}}{\lambda} \\[2ex]
\dfrac{dN_{soil}}{dt} = \dfrac{(N_{TOT} - N_{leaf} - N_{soil})}{\tau} - U(N_{leaf}, N_{soil})
\end{cases}
$$

1 g N will make $\frac{1}{n}$ $\frac{\text{g C leaf}}{\text{g N leaf}}$ g C of leaf.

This means UPTAKE of 1 g N_{soil} requires the plant to be able to produce $\frac{1}{n}$ g C from photosynthesis (for now not considering storage)

How much C can a plant fix per day? (or arbitrary timestep)

$$C_{new} = \underbrace{\frac{V}{K}\left(1 - e^{-KL}\right)}_{\text{new carbon gained per unit leaf area}} - \underbrace{(R \cancel{+MG})L}_{\text{respiration from existing leaves}} \cancel{+ \left[\text{carbon cost of constructing new leaves}\right]}$$

light limitation

recall $L = \frac{N_{leaf}}{n\sigma}$, and so we can write

$$C_{new} = \frac{V}{K}\left(1 - e^{-K\frac{N_{leaf}}{n\sigma}}\right) - R\left[\frac{N_{leaf}}{n\sigma}\right] \qquad \left[\frac{g\,C}{day}\right]$$

$\downarrow$

this is equivalent to $\underline{C_{new}\left[\frac{g\,C}{day}\right]} \cdot n \left[\frac{\text{g N leaf}}{\text{g C leaf}}\right]$ of

NEW LEAF N (e.g. $N_{leaf} + N'_{leaf}$)

<u>HOWEVER</u>, there is a CARBON COST to constructing new leaves

$$\sigma_{new} = \frac{\text{cm}^2 \text{ new leaf area}}{\text{g C for construction}} \Rightarrow \sigma_{new} = \frac{\text{g C for construction}}{\text{cm}^2 \text{ new leaf area}}$$

Leaf area $\rightarrow$ leaf N : $L(n\sigma) = N_{leaf}$