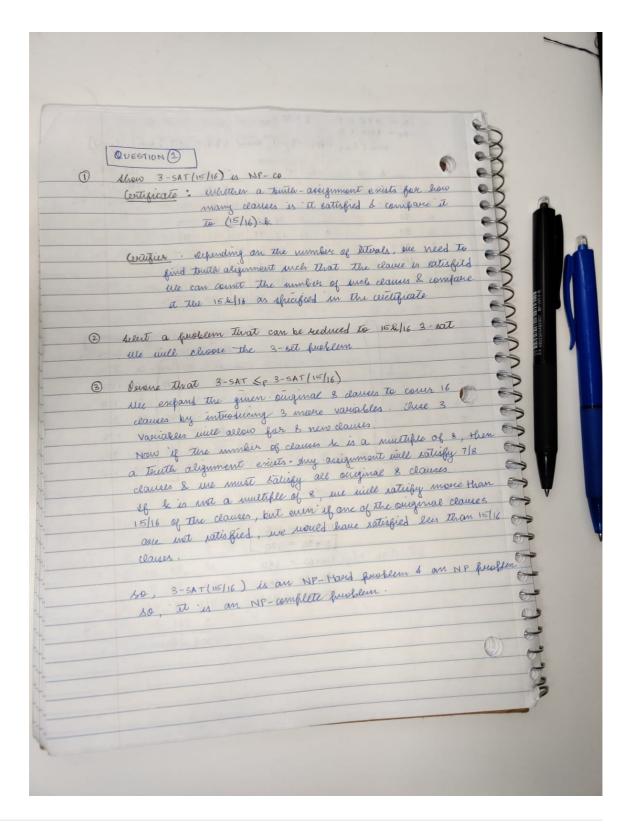


▼ Question 1

1. (20 pts) Consider the partial satisfiability problem, denoted as $3\text{-Sat}(\alpha)$. We are given a collection of k clauses, each of which contains exactly three literals, and we are asked to determine whether there is an assignment of true/false values to the literals such that at least αk clauses will be true. Note that 3-Sat(1) is exactly the 3-SAT problem from lecture.

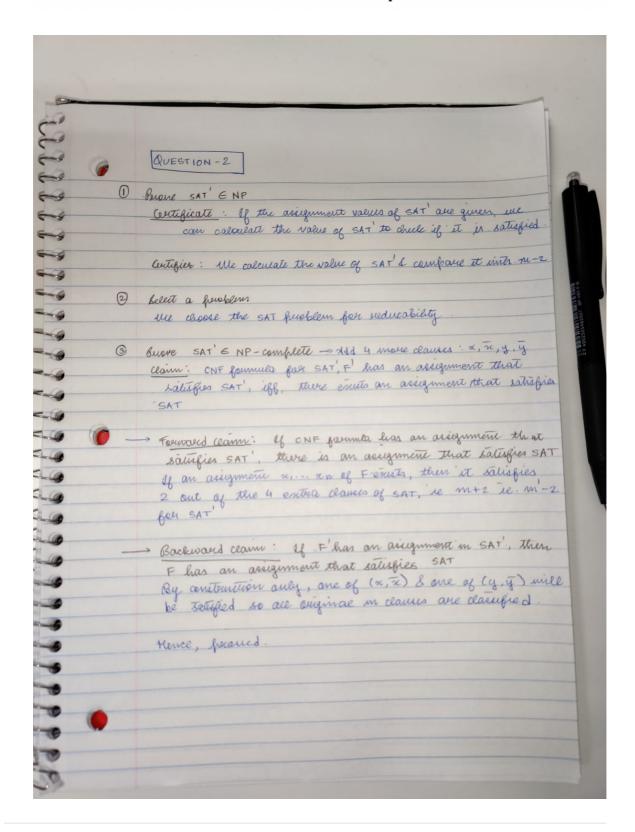
Prove that 3-Sat(15/16) is NP-complete.

Hint: If x, y, and z are literals, there are eight possible clauses containing them: $(x \lor y \lor z)$, $(!x \lor y \lor z)$, $(x \lor !y \lor z)$, $(x \lor !y \lor z)$, $(!x \lor !y \lor z)$, $(!x \lor !y \lor z)$, $(!x \lor !y \lor !z)$, $(!x \lor !y \lor !z)$



▼ Question 2

2. (20 pts) Consider modified SAT problem, SAT' in which given a CNF formula having m clauses in n variables x_1, x_2, \ldots, x_n , the output is YES if there is an assignment to the variables such that exactly m – 2 clauses are satisfied, and NO otherwise. Prove that SAT' is also NP-Complete.



▼ Question 3

3. (20 pts) Given a graph G=(V,E) and two integers k, m, the *Dense Subgraph Problem* is to find a subset V' of V, whose size is at most k and are connected by at least m edges. Prove that the *Dense Subgraph Problem is* NP-Complete.

Solution

1. Prove that Dense Subgraph Problem is an NP problem

Certificate: Given a graph G(V, E) and 2 integers k, m

Certifier: We can find if a subgraph exists with k vertices and m edges in

polynomial time

2. Chose a problem to reduce it to Dense Subgraph Problem

We will make use of the Independent set problem

3. Prove that Independency Set <p Dense Subgraph Problem

Construction: Take the complement of the graph G, which will be Gc.

Claim: There exists an independent set of size k in graph G iff there exists a subgraph of graph Gc with nodes k and edges m = k(k - 1)/2

Forward Claim: If there exists a clique of graph Gc of size k, then there exists a subgraph of Gc of size atleast k and edges at most $k(k - 1)/2 \rightarrow By$ definition, a clique of size k will automatically have edges at most k(k - 1)/2

Backward Claim: If there exists a subgraph of Gc with vertices k and edges atmost k(k-1)/2, then there exists a clique of size atleast $k \to for$ a subset to have k(k-1) edges, there has to be k edges. So this subset of k vertices forms a clique of size k in Gc.