MGS-9920

HOMEWORK-10.
HYPOTHESIS TESTING II

39 a $H_0 : p = 0.7$

$H_1 : p \neq 0.7$

This is a two-tailed test

39 b $\alpha = 0.05$ (Significance Level)

$\boxed{\text{Wisconsin}}$: 252 out of 350

Sample Proportion, $\hat{p} = \frac{x}{n} = \frac{252}{350} = 0.72$

Claimed Proportion $= 0.7$

Significance Level $= 0.05$

Standard Deviation of $\hat{p}$ $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$

$$\sigma_{\hat{p}} = \sqrt{\frac{0.7 \times (1-0.7)}{350}} \approx 0.0245$$

Test Statistic, $Z_{observed} = \frac{\hat{p} - 0.7}{\sigma_{\hat{p}}}$

$$= \frac{0.72 - 0.7}{0.0245}$$

$$= 0.816 \approx 0.82$$

So Test Statistic : $z = 0.82$

Critical Value Approach :

For $\alpha/2 = \frac{0.05}{2} = 0.025$, $z_{0.025} = 1.96$

Reject H_0 if $z \le -1.96$ or $z \geq 1.96$

But; 0.82 lies between -1.96 and 1.96

So, we fail to reject H_0

ONCLUSION : Exercise rate of people surveyed in Wisconsin is not significantly different from 70 %

$\boxed{\text{California}}$ → 189 out of 300

$$\hat{p} = \frac{x}{n} = \frac{189}{300} = 0.63$$

$$\sigma_{\hat{p}} = \sqrt{\frac{0.7(1-0.7)}{300}} = 0.0264$$

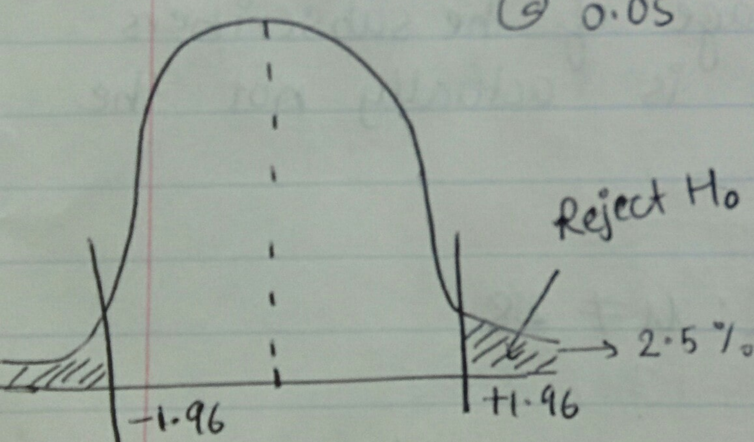
Test statistic; $z_{obs} = \dfrac{\hat{p} - 0.7}{\sigma_{\hat{p}}}$

$$= \frac{0.63 - 0.7}{0.0264}$$

$$= \frac{-0.07}{0.0264}$$

$$\boxed{z_{obs} = -2.65}$$

Critical Value Approach : $z_{0.025} = 1.96$
@ 0.05



If; $z_{obs} \leq -1.96$

or $z_{obs} \geq 1.96$

then Reject Ho

Here; $z_{obs} = -2.65$ is not between -1.96 and $+1.96$

<u>So; Reject Ho</u>

CONCLUSION : There is a possibility that exercise rate of people surveyed in Wisconsin is significantly different from 70 %.

(50) (a) Type-II error = When null is false and we fail to reject it

$H_0: \mu = 28$

$H_A: \mu \neq 28$

In this case, making a Type-II error would mean that we falsely believe that the mean age of the subscribers is 28 when it is actually not the case.

(50) (b) $H_0: \mu = 28$, $H_a: \mu \neq 28$

$\alpha = 0.05$, $z_{\alpha = 0.025} = +/- 1.96$

$$\bar{x} = \mu \pm z\left(\frac{\sigma}{\sqrt{n}}\right)$$

$= 28 \pm 1.96(0.6) = 29.176$ OR 26.8

So, we fail to reject Ho when

$\Rightarrow \quad 26.824 \leq x \leq 29.176$

So;

When; $\mu = 30$, $z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}} = -1.373, -5.29$, p-value $= 1 - 0.915$
$= 0.085$

When $\mu = 26$, $z = +1.373, +5.29$, p-value $= 0.085$

when $\mu = 27$, $z = -0.293, 3.62$, p-value $= 0.38$

when $\mu = 29$, $z = -3.626, 0.293$, p-value $= 0.38$

50 (c) Power $= 1 - \beta$
when $\mu = 26$

Power $= 1 - \beta = 1 - 0.085 = 0.915$

This means, there is 91.5% probability of correctly rejecting Null when it is false.

(54) $H_0 : \mu \geq 10$

$H_a : \mu < 10$

$n = 120$, $\mu_0 = 10$, $\alpha = 0.05$, $\sigma = 5$, $\mu_a = 9$

$Z_{0.05} = 1.645$ (Area in upper tail of std. Normal Distribⁿ)

$Z_{0.10} = 1.28$ (when reducing Type-II error to 0.10)

Sample size for a one tailed hypothesis test about a population mean is given by :

$$n = \frac{(Z_\alpha + Z_\beta)^2 \sigma^2}{(\mu_0 - \mu_a)^2}$$

$$n = \frac{(1.645 + 1.28)^2 (5)^2}{(10-9)^2}$$

$$n = \underline{213.89}$$

So, to reduce the probability of Type-II Error to 0.10, the sample size required is at least 214.