MGS-9920 - PROB & STATS - I

$\boxed{\text{HW-8}}$

(45) C.I (Confidence Interval) = $\bar{x} \pm$ Margin of Error

(a)

Margin of Error $= z_{\alpha/2} \times \frac{\sigma}{\sqrt{n}}$; where σ = ~~Sample~~ Populat^n Std. Dev.

$= 1.96 \times \frac{74.5}{\sqrt{64}}$; n = Sample Size

$= 18.25$; $z_{\alpha/2}$ = Confidence Coefficient

$\frac{\sigma}{\sqrt{n}}$ = Standard Error of Mean

So; $\boxed{C.I. = 252.45 \pm 18.25}$

$\Rightarrow$ Lower Boundary $= 252.45 - 18.25 = 234.20$

Upper Boundary $= 252.45 + 18.25 = 270.70$

$\Rightarrow$ $\boxed{\text{C.I is } (\$234.20, \$270.70)}$

(45)

(b) Since the Mean reported by AMA $\boxed{\$215.60}$ is not falling within the Confidence Interval, it can be inferred that the population mean amount spent per day by families visiting Niagara Falls differs from the mean reported by the AMA.

(51) Margin of Error $= Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$ * [$\sigma = 8$ min → Given
$Z_{\alpha/2}$ @ 95% level of confidence is 1.96]

PART-1

So; $2 = 1.96 \times \frac{8}{\sqrt{n}}$

$$\Rightarrow \sqrt{n} = \frac{1.96 \times 8}{2}$$

$$\Rightarrow n = \left(\frac{1.96 \times 8}{2}\right)^2$$

$$\Rightarrow \boxed{n = 62}$$

⇒ Sample size of 62 should be taken if desired margin of error is 2 minutes at a 95% level of confidenc

(51)

PART-2

$\cancel{n = Z_{\alpha/2} \times \sigma}$

$$n = \left(Z_{\alpha/2} \times \frac{\sigma}{\text{Margin of Error}}\right)^2$$

For 99% confidence level, $\alpha/2 = 0.5$

$$n = \left(2.575 \times \frac{8}{2}\right)^2$$

$= 106.15 \sim \boxed{107}$ is the sample size that should be taken for a 99% level of confidence

(57) (a)

Sample Proportion $\bar{p} = 0.3$

So $1-\bar{p} = 0.7$

Margin of Error $= 0.02$

$$\text{Margin of Error} = z_{\alpha/2} \cdot \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

@ 95% Confidence Interval $\Rightarrow$

$$0.02 = 1.96\sqrt{\frac{(0.3)(0.7)}{n}}$$

$$\sqrt{n} = \frac{1.96}{0.02} \times \sqrt{(0.3)(0.7)}$$

$$n = \left(\frac{1.96}{0.02}\right)^2 \times (0.3) \times (0.7)$$

$$n = 2016.9 \sim \boxed{2017}$$ is the required Sample Size.

(57)

(b) If there are 520 smokers out of 2017 smokers, then point estimate or $\bar{p} = \frac{520}{2017} = \boxed{0.258}$

(57)

(c) 95% confidence Interval for the pop^n proportion is found by computing

$$\bar{p} \pm z_{\alpha/2} * \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$0.258 \pm 1.96 * \sqrt{\frac{0.258(1-0.258)}{2017}}$$

$$0.258 \pm 0.019$$

So, I am 95% Confident that the proportion of smokers in the population is between $0.258 - 0.019 = \boxed{0.239}$ and $0.258 + 0.019 = \boxed{0.277}$

59 a Sample Proportion $(\bar{p}) = \frac{110}{200} = 0.55$

95 % Confidence Interval = ?

$\Rightarrow \bar{p} \pm Z_{\alpha/2} \times \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$

$\Rightarrow 0.55 \pm 1.96 \times \sqrt{\frac{(0.55)(0.45)}{200}}$

$\Rightarrow 0.55 \pm 0.068$

$\rightarrow 0.55 + 0.068 = 0.618$

$\rightarrow 0.55 - 0.068 = 0.482$

So, I am 95 % confident that the proportion of people in the population who consider television their major source of entertainment is between 0.482 to 0.618.

59 b Margin of Error $(E) = z_{\alpha/2} \cdot \sqrt{\dfrac{\bar{p}(1-\bar{p})}{n}}$

* $\bar{p} = 0.55$
* $z_{\alpha/2}$ @ 95% confidence is $= 1.96$
* $E = 0.05$

$$0.05 = 1.96 \sqrt{\frac{(0.55)(0.45)}{n}}$$

$$\sqrt{n} = \frac{1.96}{0.05} \times \sqrt{(0.55)(0.45)}$$

$$n = \left(\frac{1.96}{0.05}\right)^2 \times (0.55) \times (0.45)$$

$$n = 380.3 \approx \boxed{381}$$

So, the sample size ought to be 381 to estimate the population proportion with a Margin of Error of 0.05 at 95% Confidence Interval.