Massachusetts Institute of Technology
6.046J/18.410J: Introduction to Algorithms
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Handout 9
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Master Theorem Worksheet

This is a worksheet to help you master solving recurrence relations using the Master Theorem. For each recurrence, either give the asympotic solution using the Master Theorem (state which case), or else state that the Master Theorem doesn't apply. You should be able to go through these **25** recurrences in **10** minutes.

Problem 1-1. $T(n) = 3T(n/2) + n^2$

a=3, b=2, f(n) = n^2 c_crit =log base 2 of 3 < c

Case 3: O(n^2)

Problem 1-2. $T(n) = 7T(n/2) + n^2$

a=7, b=2, f(n) = n^2 c_crit=log base 2 of 7 > c

Case 1: O(n^(log 7))

Problem 1-3. $T(n) = 4T(n/2) + n^2$

a=4, b=2, f(n)=n^2 c_crit=log base 2 of 4 = c

Case 2: O(n^2 log n)

Problem 1-4. $T(n) = 3T(n/4) + n \lg n$

a=3, b=4, f(n) = n log n c_crit = log base 4 of 3 < c

Case 3: O(n log n)

Problem 1-5. $T(n) = 4T(n/2) + \lg n$

a=4, b=2, f(n) = log(n) c_crit = log base 2 of 4 > c

Case 1: O(n^2)

Problem 1-6. $T(n) = T(n-1) + n$

Does not apply since n-1 term

Problem 1-7. $T(n) = 4T(n/2) + n^2 \lg n$

a=4, b=2, f(n) = n^2 log(n)

c_crit = log base 2 of 4 = c

Case 2: O(n^2 log log n)

Problem 1-8. $T(n) = 5T(n/2) + n^2 \lg n$

a=5, b=2, f(n) = n^2 log n

c_crit = log base 2 of 5 > c

Case 1: O(n^log(5))

Problem 1-9. $T(n) = 3T(n/3) + n/\lg n$

Does not apply

Problem 1-10. $T(n) = 2T(n/4) + c$

a=2, b=4, f(n) = c

c_crit = 1/2 > c

Case 1: O(n^1/2)

Problem 1-11. $T(n) = T(n/4) + \lg n$

a=1, b=4, f(n) = log n

c_crit = 0 = c

Case 2: O(log^2 n)

Problem 1-12. $T(n) = T(n/2) + T(n/4) + n^2$

Not applicable

Problem 1-13. $T(n) = 2T(n/4) + \lg n$

a=2, b=4, f(n) = log n

c_crit = 1/2 > c

Case 1: O(n^1/2)

Problem 1-14. $T(n) = 3T(n/3) + n \lg n$

a=3, b=3, f(n) = n log n

c_crit = 1 = c

Case 2: O(n log^2 n)

Problem 1-15. $T(n) = 8T((n - \sqrt{n})/4) + n^2$

Not applicable

Problem 1-16. $T(n) = 2T(n/4) + \sqrt{n}$

a=2, b=4, f(n) = sqrt(n) c_crit = 1/2 = c

Case 2: O(n^1/2 log n)

Problem 1-17. $T(n) = 2T(n/4) + n^{0.51}$

a=2, b=4, f(n) = n^0.51 c_crit = 1/2 < c

Case 3: O(n^0.51)

Problem 1-18. $T(n) = 16T(n/4) + n!$

a=16, b=4, f(n) = n! c_crit = 2 < c

Case 3: O(n!)

Problem 1-19. $T(n) = 3T(n/2) + n$

a=3, b=2, f(n) = n c_crit = log base 2 of 3 > c

Case 1: O(n^log 3)

Problem 1-20. $T(n) = 4T(n/2) + cn$

a=4, b=2, f(n) = cn c_crit = 2 > c

Case 1: O(n^2)

Problem 1-21. $T(n) = 3T(n/3) + n/2$

a=3, b=2, f(n) = n/2 c_crit = 1 = c

Case 2: O(n log n)

Problem 1-22. $T(n) = 4T(n/2) + n/\lg n$

a=4, b=2, f(n) = n / log n c_crit = 2 > c

Case 1: O(n^2)

Problem 1-23. $T(n) = 7T(n/3) + n^2$

a=7, b=3, f(n) = n^2

c_crit = log base 3 of 7 < c

Case 3: O(n^2)

Problem 1-24. $T(n) = 8T(n/3) + 2^n$

a=8, b=3, f(n) = 2^n

c_crit = log base 3 of 8 < c

Case 3: O(2^n)

Problem 1-25. $T(n) = 16T(n/4) + n$

a=16, b=4, f(n) = n

c_crit = 2 > c

Case 1: O(n^2)