SIMPLEX ALGORITHM

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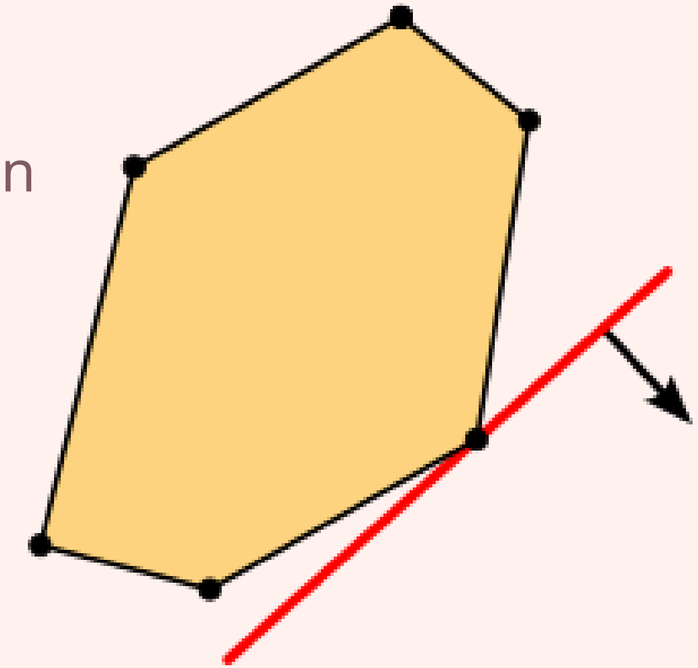
THE ALGORITHM

- Linear Programming
- Constraints and Optimal Function
- Similar to solving linear systems of equations in middle school



TERMINOLOGY

1. Objective Function

$Z = 8x + 10y + 7z$

To maximize or minimize

Should include all variables in constraints

Should be bound by constraints

TERMINOLOGY

1. Objective Function

$$Z = 8x + 10y + 7z$$

2. Constraints

$$x + 3y + 2z \leq 10$$
$$-x - 5y - z \geq -8$$

Assumed that variables are all positive

Constraints should bind region and solution should exist

TERMINOLOGY

1. Objective Function

$$Z = 8x + 10y + 7z$$

2. Constraints

$$x + 3y + 2z \leq 10$$
$$-x - 5y - z \geq -8$$

3. Slack Variables

Unique variables added to each constraint to convert from inequality to equality form

STANDARD FORM

1) Must be maximization function
   Can convert by finding the maximum of the dual

2) Constraints are $\leq$ inequality
   Convert by multiplying by -1 and reversing sign

3) All variables are non-negative
   This assumption is taken and not provided as a separate constraint in this implementation

$$\text{Max } Z = 8x + 10y + 7z$$
$$x + 3y + 2z \leq 10$$
$$-x - 5y - z \geq -8$$
$$x, y, z \geq 0$$

$$\text{Max } Z = 8x + 10y + 7z$$
$$x + 3y + 2z + e1 \leq 10$$
$$x + 5y + z + e2 \leq 8$$
$$x, y, z \geq 0$$

Max Z = 8x + 10y + 7z

x + 3y + 2z + e1 ≤ 10

x + 5y + z + e2 ≤ 8

x, y, z ≥ 0

INITIALIZE

x	y	z	e1	e2	b
1	3	2	1	0	10
1	5	1	0	1	8
-8	-10	-7	0	0	0

Inverse of Objective Function

IMPLEMENTATION

```
Objective Function: 8.0x1 + 10.0x2 + 7.0x3
Simplex Matrix:
   x1    x2    x3    e1    e2  |     b
 1.00  3.00  2.00  1.00  0.00  | 10.00
 1.00  5.00  1.00  0.00  1.00  |  8.00
-----------------------------------------
-8.00 -10.00 -7.00  0.00  0.00  |  0.00

Solution (Rounded) [x1: 0, x2: 0, x3: 0, e1: 10, e2: 8]
Solution [x1: 0.00, x2: 0.00, x3: 0.00, e1: 10.00, e2: 8.00]
Z (Optimal) = 0.0
```

STEPS OF SIMPLEX

1. Check if current optimal solution
2. Identify Pivot Variable
3. Pivot Table
4. Update Variables

STEP 1

Check for negative

x	y	z	e1	e2	b
1	3	2	1	0	10
1	5	1	0	1	8
-8	-10	-7	0	0	0

Pick column with most negative value

STEP 2

Pick Pivot Value

x	y	z	e1	e2	b	
1	3	2	1	0	10	e1
1	5	1	0	1	8	e2
-8	-10	-7	0	0	0	

Pick row with smallest ratio

STEP 2

Pick Pivot Value

x	y	z	e1	e2	b		
1	3	2	1	0	10	e1	10/3
1	5	1	0	1	8	e2	8/5
-8	-10	-7	0	0	0		

STEP 3

Pivot

Convert pivot value to 1

x	y	z	e1	e2	b	
1	3	2	1	0	10	e1
1	5	1	0	1	8	e2
-8	-10	-7	0	0	0	

STEP 3

Pivot

x	y	z	e1	e2	b	
1	3	2	1	0	10	e1
1	5	1	0	1	8	e2
-8	-10	-7	0	0	0	

1	5	1	0	1	8	

Divide by 5!

1/5	1	1/5	0	1/5	8/5	

STEP 3

Pivot

x	y	z	e1	e2	b	
1	3	2	1	0	10	e1
1/5	1	1/5	0	1/5	8/5	e2
-8	-10	-7	0	0	0	

Convert all other values in column that was chosen to 0

STEP 3

Pivot

x	y	z	e1	e2	b	
1	3	2	1	0	10	e1
1/5	1	1/5	0	1/5	8/5	e2
-8	-10	-7	0	0	0	

1	3	2	1	0	10

Add to Row 2 * -3

-3/5	-3	-3/5	0	-3/5	-24/5
2/5	0	7/5	0	-3/5	26/5

STEP 3

Pivot

x	y	z	e1	e2	b	
2/5	0	7/5	0	-3/5	26/5	e1
1/5	1	1/5	0	1/5	8/5	e2
-8	-10	-7	0	0	0	

-8	-10	-7	0	0	0	

Add to Row 2 * 10

2	10	2	0	2	16	
-6	0	-5	0	2	16	

STEP 3

Pivot

x	y	z	e1	e2	b	
2/5	0	7/5	0	-3/5	26/5	e1
1/5	1	1/5	0	1/5	8/5	e2
-6	0	-5	0	2	16	

STEP 4

Swap Variables

x	y	z	e1	e2	b	
2/5	0	7/5	0	-3/5	26/5	e1
1/5	1	1/5	0	1/5	8/5	e2
-6	0	-5	0	2	16	

STEP 4

Swap Variables

x	y	z	e1	e2	b	
2/5	0	7/5	0	-3/5	26/5	e1
1/5	1	1/5	0	1/5	8/5	y
-6	0	-5	0	2	16	

Current Solution:

x = 0
y = 1.6
z = 0
e1 = 5.2
e2 = 0

IMPLEMENTATION

```
Objective Function: 8.0x1 + 10.0x2 + 7.0x3
Simplex Matrix:
   x1    x2    x3    e1    e2  |     b
 0.40  0.00  1.40  1.00 -0.60  |  5.20
 0.20  1.00  0.20  0.00  0.20  |  1.60
-------------------------------------------
-6.00  0.00 -5.00  0.00  2.00  | 16.00

Solution (Rounded) [x1: 0, x2: 2, x3: 0, e1: 5, e2: 0]
Solution [x1: 0.00, x2: 1.60, x3: 0.00, e1: 5.20, e2: 0.00]
Z (Optimal) = 16.0
```

Calculated Solution:

x = 0

y = 1.6

z = 0

e1 = 5.2

e2 = 0

STEP 1

Check for negative

x	y	z	e1	e2	b	
2/5	0	7/5	0	-3/5	26/5	e1
1/5	1	1/5	0	1/5	8/5	y
-6	0	-5	0	2	16	

Pick column with most negative value

STEP 2

Pick Pivot Value

x	y	z	e1	e2	b
2/5	0	7/5	0	-3/5	26/5
1/5	1	1/5	0	1/5	8/5
-6	0	-5	0	2	16

Pick row with smallest ratio

STEP 2

Pick Pivot Value

x	y	z	e1	e2	b		
2/5	0	7/5	0	-3/5	26/5	e1	13
1/5	1	1/5	0	1/5	8/5	y	8
-6	0	-5	0	2	16		

STEP 3

Pivot

Convert pivot value to 1

x	y	z	e1	e2	b	
2/5	0	7/5	0	-3/5	26/5	e1
1/5	1	1/5	0	1/5	8/5	y
-6	0	-5	0	2	16	

STEP 3

Pivot

x	y	z	e1	e2	b	
2/5	0	7/5	0	-3/5	26/5	e1
1/5	1	1/5	0	1/5	8/5	y
-6	0	-5	0	2	16	

1/5	1	1/5	0	1/5	8/5

Multiply by 5

1	5	1	0	1	8

STEP 3

Pivot

x	y	z	e1	e2	b	
2/5	0	7/5	0	-3/5	26/5	e1
1	5	1	0	1	8	y
-6	0	-5	0	2	16	

STEP 3

Pivot

x	y	z	e1	e2	b	
2/5	0	7/5	0	-3/5	26/5	e1
1	5	1	0	1	8	y
-6	0	-5	0	2	16	

2/5	0	7/5	0	-3/5	26/5

Add to Row 2 * -2/5

-2/5	-2	-2/5	0	-2/5	-16/5
0	-2	1	0	-1	2

STEP 3

Pivot

x	y	z	e1	e2	b	
0	-2	1	0	-1	2	e1
1	5	1	0	1	8	y
-6	0	-5	0	2	16	

STEP 3

Pivot

x	y	z	e1	e2	b	
0	-2	1	0	-1	2	e1
1	5	1	0	1	8	y
-6	0	-5	0	2	16	

-6	0	-5	0	2	16	

Add to Row 2 * 6

6	30	6	0	6	48	
0	30	1	0	8	64	

STEP 3

Pivot

x	y	z	e1	e2	b	
0	-2	1	0	-1	2	e1
1	5	1	0	1	8	y
0	30	1	0	8	64	

STEP 4

Swap Variables

x	y	z	e1	e2	b	
0	-2	1	0	-1	2	e1
1	5	1	0	1	8	y
0	30	1	0	8	64	

STEP 4

Swap Variables

x	y	z	e1	e2	b	
0	-2	1	0	-1	2	e1
1	5	1	0	1	8	x
0	30	1	0	8	64	

Current Solution:

x = 8
y = 0
z = 0
e1 = 2
e2 = 0

IMPLEMENTATION

```
Objective Function: 8.0x1 + 10.0x2 + 7.0x3
Simplex Matrix:
   x1    x2    x3    e1    e2  |     b
 0.00 -2.00  1.00  1.00 -1.00  |  2.00
 1.00  5.00  1.00  0.00  1.00  |  8.00
-------------------------------------------
 0.00 30.00  1.00  0.00  8.00  | 64.00

Solution (Rounded) [x1: 8, x2: 0, x3: 0, e1: 2, e2: 0]
Solution [x1: 8.00, x2: 0.00, x3: 0.00, e1: 2.00, e2: 0.00]
Z (Optimal) = 64.0
```

Current Solution:

x = 8

y = 0

z = 0

e1 = 2

e2 = 0