Lab 2 - Fourier Transform and LTI systems

Objectives: In this lab, we will

- numerically compute Fourier transform (FT) of some common continuous-time signals we have seen in class and plot them, verify some properties of FT;
- process periodic signals (using FS coefficients) with LTI systems acting as filters (given their frequency response), plot and compare input & output signals.

2.1 Continuous-time Fourier transform

(a) Write a **matlab function** `X = continuousFT(t,xt,a,b,ω)` to numerically compute continuous-time FT of the given signal $x(t)$ which has finite support in [a, b] and is zero outside. The inputs to this function are

- t – symbolic variable
- xt – signal whose FT is to be computed (function of symbolic variable t)
- a,b – the signal is equal to xt in the interval [a, b] and zero outside
- ω – the vector ω contains the values of frequency where FT is to be computed.

>> The function should return a vector X which contains the FT of $x(t)$ for each of the frequencies in the input vector ω.

(b) Write a **matlab script** that calls the function `continuousFT` for a rectangular pulse of unit amplitude in [-T, T] where T = 2 and ω = -5:0.1:5. In a single figure, using `subplot()` commands to get a 2x2 grid of subplots, plot the real part, imaginary part, absolute value and phase of the computed FT as function of ω.
>> For phase use the command `angle()` in matlab. Can you explain each of the observed subplots?

(c) Repeat part (b) for T = 1 and T = 4. Use ω = -5:0.1:5. What FT property supports your observations when T is changed?

(d) Repeat part (b) for $x(t) = \mathrm{e}^{jt}$, and $x(t) = \cos{(t)}$. Limit signals to the interval [-T, T] where T = π and ω = -5:0.1:5. What is the expected FT? What are the shapes you are observing?

(e) Repeat part (b) for a triangle pulse of height 1 and base/support [-1,1]. How would you express xt for this case?
What is the expected FT? Hint: express the triangle pulse as convolution of two signals.

(f) Optional: play with some more signals $x(t)$ to test your function and verify whether standard properties of FT are satisfied as expected.

2.2 Filtering of periodic signals with LTI systems

A continuous-time periodic signal $x(t)$ has Fourier series (FS) coefficients a_k. If $x(t)$ is input to the LTI system with frequency response $H(\omega)$, what are Fourier series coefficients of the output signal? What about the periodicity of the output signal?

Ideal low pass filter (LPF): let $\omega_c > 0$ be its cut-off frequency. We wish to find FS coefficients $b_k, k = -N{:}\,N$, of the output signal when the input signal with coefficients $a_k, k = -N{:}\,N$, is passed through an Ideal LPF.

(a) Write a code for the **matlab function** `B = myLPF(A,w0_FS,wc)` which takes input signal FS coefficents A, frequency of the input periodic signal `w0_FS`, cut-off frequency wc and returns the output signal FS coefficients in the vector B. Note that your code should be written for general N.

Note: the coefficient a_k corresponds frequency $k\omega_0$, use this information while implementing your LPF.

(b) We will now write a **matlab script** file to visualize input and output signals of the filter. For this purpose we will use function '`x = partialfouriersum(A,T,t)`' from last week's lab session.
>> Initialize $\omega_0 = 1$, and FS coefficients A to obtain the signal $x(t) = \cos(t)$
>> Call function `myLPF` with input A and $\omega_c = 2$
>> Use the function `partialfouriersum` to obtain the time domain signals corresponding to A and B and plot them in the same figure. Use the inputs as T = 2π, and t = -2T:0.01:2T
>> What happens when we change cut-off to $\omega_c = 0.5$?

(c) Ideal high pass filter (HPF): let $\omega_c > 0$ be its cut-off frequency.
>> Repeat (a) for an ideal HPF and write **matlab function** `B = myHPF(A,w0_FS,wc)`.
>> Repeat (b) with the ideal LPF replaced by ideal HPF (continue in the same script file).

(d) Non-ideal filter: let the frequency response be $H(\omega) = \frac{G}{a+j\,\omega}$ where G and a are positive real constants. What is the nature of this filter?
>> Write a code to implement this non-ideal filter on FS coefficients A as the **matlab function** `B = NonIdeal(A,w0_FS,G,a)`.
>> Repeat (b) with ideal LPF replaced by the non-ideal filter. Use G = 1, a = 1.
>> How is the complex-valued nature of the LTI system frequency response manifested in the output signal? (to be done in class).

(e) Repeat the script with the input signal as $x(t) = \sin(2t) + \cos(3t)$. Note that you must appropriately modify A, ω_0, and T for this example. For this input set ideal LPF and ideal HPF filter cut-offs to be $\omega_c = 2.5$.

(f) Optional: repeat when A corresponds to FS coefficients of the periodic square wave.