

## CENTER OF MASS, MOMENTUM & COLLISION

### KEY CONCEPTS

#### CENTRE OF MASS :

For a system of particles centre of mass is that point at which its total mass is supposed to be concentrated. The centre of mass of an object is a point that represents the entire body and moves in the same way as a point mass having mass equal to that of the object, when subjected to the same external forces that act on the object.

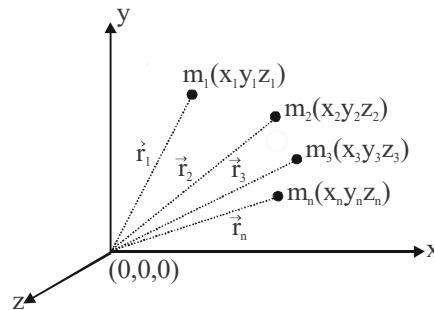
#### Centre of mass of system of discrete particles

Total mass of the body :  $M = m_1 + m_2 + \dots + m_n$

$$\text{Then } \vec{R}_{CM} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + m_3 \vec{r}_3 + \dots}{m_1 + m_2 + m_3 + \dots} = \frac{1}{M} \sum m_i \vec{r}_i$$

co-ordinates of centre of mass :

$$x_{cm} = \frac{1}{M} \sum m_i x_i, y_{cm} = \frac{1}{M} \sum m_i y_i \text{ and } z_{cm} = \frac{1}{M} \sum m_i z_i$$



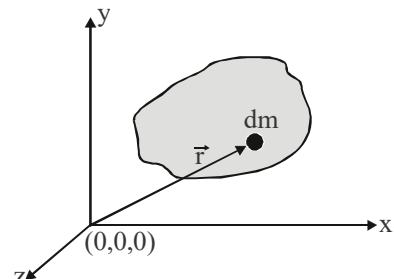
#### Centre of mass of continuous distribution of particles

If the system has continuous distribution of mass, treating the mass element  $dm$  at position  $\vec{r}$  as a point mass and replacing summation by integration.  $\vec{R}_{CM} = \frac{1}{M} \int \vec{r} dm$ .

$$\text{So that } x_{cm} = \frac{1}{M} \int x dm, \quad y_{cm} = \frac{1}{M} \int y dm \text{ and } z_{cm} = \frac{1}{M} \int z dm$$

If co-ordinates of particles of mass  $m_1, m_2, \dots$  are  $(x_1, y_1, z_1), (x_2, y_2, z_2), \dots$

then position vector of their centre of mass is



$$\begin{aligned} \vec{R}_{CM} &= x_{cm} \hat{i} + y_{cm} \hat{j} + z_{cm} \hat{k} \\ &= \frac{m_1(x_1 \hat{i} + y_1 \hat{j} + z_1 \hat{k}) + m_2(x_2 \hat{i} + y_2 \hat{j} + z_2 \hat{k}) + m_3(x_3 \hat{i} + y_3 \hat{j} + z_3 \hat{k}) + \dots}{m_1 + m_2 + m_3 + \dots} \\ &= \frac{(m_1 x_1 + m_2 x_2 + \dots) \hat{i} + (m_1 y_1 + m_2 y_2 + \dots) \hat{j} + (m_1 z_1 + m_2 z_2 + \dots) \hat{k}}{m_1 + m_2 + m_3 + \dots} \end{aligned}$$

$$\text{So, } x_{cm} = x_{cm} = \left( \frac{m_1 x_1 + m_2 x_2 + \dots}{m_1 + m_2 + m_3 + \dots} \right), \quad y_{cm} = \left( \frac{m_1 y_1 + m_2 y_2 + \dots}{m_1 + m_2 + \dots} \right), \quad z_{cm} = \left( \frac{m_1 z_1 + m_2 z_2 + \dots}{m_1 + m_2 + \dots} \right)$$

#### The centre of mass after removal of a part of a body

If a portion of a body is taken out, the remaining portion may be considered as,

Original mass ( $M$ ) – mass of the removed part ( $m$ ) = {original mass ( $M$ )} + {–mass of the removed part ( $m$ )}

The formula changes to :  $x_{CM} = \frac{Mx - mx'}{M - m}$ ,  $y_{CM} = \frac{My - my'}{M - m}$ ,  $z_{CM} = \frac{Mz - mz'}{M - m}$

Where  $x'$ ,  $y'$  and  $z'$  represent the coordinates of the centre of mass of the removed part.

## MOTION OF CENTRE OF MASS

As for a system of particles, position of centre of mass is  $\vec{R}_{CM} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + m_3 \vec{r}_3 + \dots}{m_1 + m_2 + m_3 + \dots}$

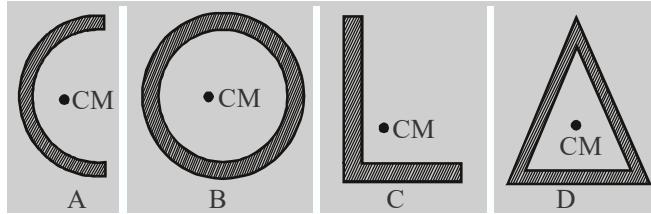
So  $\frac{d}{dt}(\vec{R}_{CM}) = \frac{m_1 \frac{d\vec{r}_1}{dt} + m_2 \frac{d\vec{r}_2}{dt} + m_3 \frac{d\vec{r}_3}{dt} + \dots}{m_1 + m_2 + m_3 + \dots} \Rightarrow \vec{V}_{CM} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2 + m_3 \vec{v}_3 + \dots}{m_1 + m_2 + m_3 + \dots}$

We can write  $M\vec{V}_{CM} = m_1 \vec{v}_1 + m_2 \vec{v}_2 + \dots = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \dots$  [since  $\vec{p} = m\vec{v}$ ]

$$M\vec{V}_{CM} = \vec{p}_{CM} [\because \sum \vec{p}_i = \vec{p}_{CM}]$$

Similarly acceleration  $\vec{a}_{CM} = \frac{d}{dt}(\vec{V}_{CM}) = \frac{m_1 \vec{a}_1 + m_2 \vec{a}_2 + \dots}{m_1 + m_2 + \dots}$

important points



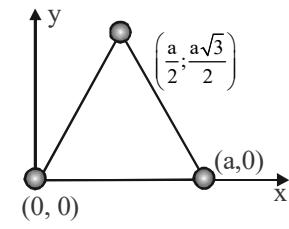
- There may or may not be any mass present physically at centre of mass (See Figure A, B, C)
- Centre of mass may be inside or outside of the body (See figure A, B, C)
- Position of centre of mass depends on the shape of the body. (See figure A, B, C)
- For a given shape it depends on the distribution of mass of within the body and is closer to massive part. (See figure A,C)
- For symmetrical bodies having homogeneous distribution of mass it coincides with centre of symmetry of geometrical centre. (See figure B,D).
- If we know the centre of mass of parts of the system and their masses, we can get the combined centre of mass by treating the parts as point particles placed at their respective centre of masses.
- It is independent of the co-ordinate system, e.g., the centre of mass of a ring is at its centre whatever be the co-ordinate system.
- If the origin of co-ordinate system is at centre of mass, i.e.,  $\vec{R}_{CM} = 0$ , then by definition.

$$\frac{1}{M} \sum m_i \vec{r}_i = 0 \Rightarrow \sum m_i \vec{r}_i = 0$$

The sum of the moments of the masses of a system about its centre of mass is always zero.

- Ex.** Three bodies of equal masses are placed at  $(0, 0)$ ,  $(a, 0)$  and at  $\left(\frac{a}{2}, \frac{a\sqrt{3}}{2}\right)$ .

Find out the co-ordinates of centre of mass.

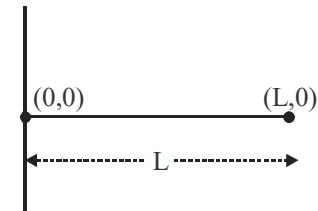


**Sol.**  $x_{CM} = \frac{0 \times m + a \times m + \frac{a}{2} \times m}{m + m + m} = \frac{a}{2}$ ,  $y_{CM} = \frac{0 \times m + 0 \times m + \frac{a\sqrt{3}}{2} \times m}{m + m + m} = \frac{a\sqrt{3}}{6}$

- Ex.** Calculate the position of the centre of mass of a system consisting of two particles of masses  $m_1$  and  $m_2$  separated by a distance  $L$  apart, from  $m_1$ .

- Sol.** Treating the line joining the two particles as x axis

$$\frac{m_1 \times 0 + m_2 \times L}{m_1 + m_2} = \frac{m_2 L}{m_1 + m_2}, y_{CM} = 0, z_{CM} = 0$$

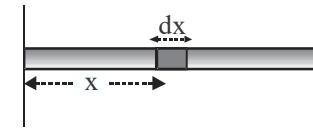


- Ex.** If the linear density of a rod of length  $L$  varies as  $\lambda = A + Bx$ , compute position of its centre of mass.

- Sol.** Let the x-axis be along the length of the rod and origin at one of its end as shown in figure. As rod is along x-axis, for all points on it y and z will be zero so,  $y_{CM} = 0$  and  $z_{CM} = 0$  i.e., centre of mass will be on the rod.

Now consider an element of rod of length  $dx$  at a distance  $x$  from the origin, mass of this element  $dm = \lambda dx = (A + Bx)dx$

$$\text{so, } x_{CM} = \frac{\int_0^L x dm}{\int_0^L dm} = \frac{\int_0^L x(A+Bx) dx}{\int_0^L (A+Bx) dx} = \frac{\frac{AL^2}{2} + \frac{BL^3}{3}}{AL + \frac{BL^2}{2}} = \frac{L(3A+2BL)}{3(2A+BL)}$$

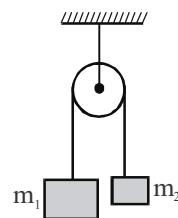


**Note :** (i) If the rod is of uniform density then  $\lambda = A = \text{constant}$  &  $B = 0$  then  $X_{CM} = L/2$

(ii) If the density of rod varies linearly with x, then  $\lambda = Bx$  and  $A = 0$  then  $X_{CM} = 2L/3$

- Ex.** Two bodies of masses  $m_1$  and  $m_2$  ( $< m_1$ ) are connected to the ends of a massless cord and allowed to move as shown in. The pulley is both massless and frictionless. Calculate the acceleration of the centre of mass.

- Sol.** If  $\vec{a}$  is the acceleration of  $m_1$ , then  $-\vec{a}$  is the acceleration of  $m_2$  then



$$\text{acceleration of each body } a = \frac{\text{Net force}}{\text{Net mass}} = \left( \frac{m_1 - m_2}{m_1 + m_2} \right) g$$

$$\vec{a}_{cm} = \frac{m_1 \vec{a} + m_2 (-\vec{a})}{m_1 + m_2} = \left( \frac{m_1 - m_2}{m_1 + m_2} \right) \vec{a}$$

$$\text{But } \vec{a} = \left( \frac{m_1 - m_2}{m_1 + m_2} \right) \vec{g} \text{ so } \vec{a}_{cm} = \left( \frac{m_1 - m_2}{m_1 + m_2} \right)^2 \vec{g}$$

- Ex.** A circle of radius  $R$  is cut from a uniform thin sheet of metal. A circular hole of radius  $\frac{R}{2}$  is now cut out of the circle, with the hole tangent to the rim. Find the distance of centre of mass from the centre of the original uncut circle to the centre of mass.

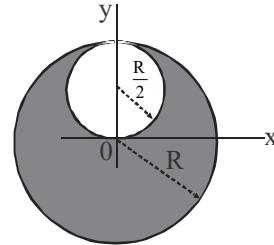
- Sol.** We treat the hole as a 'negative mass' object that is combined with the original uncut circle. (When the two are added together, the hole region then has zero mass). By symmetry, the CM lies along the +y-axis in figure, so  $x_{CM} = 0$ . With the origin at the centre of the original circle whose mass is assumed to be  $m$ .

Mass of original uncut circle

$$m_1 = m \text{ & } (0, 0)$$

Mass of hole of negative mass :  $m_2 = \frac{m}{4}$  ; Location of CM  $\left(0, \frac{R}{2}\right)$

$$\text{Thus } y_{CM} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = \frac{m(0) + \left(-\frac{m}{4}\right) \frac{R}{2}}{m + \left(-\frac{m}{4}\right)} = \frac{R}{6}$$



So the centre of mass is at the point  $\left(0, \frac{R}{6}\right)$

- Ex.** Two particles of mass 1 kg and 0.5 kg are moving in the same direction with speeds of 2 m/s and 6 m/s, respectively, on a smooth horizontal surface. Find the speed of centre of mass of the system.

- Sol.** Velocity of centre of mass of the system  $\vec{v}_{cm} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2}$  Since the particles  $m_1$  and  $m_2$  are moving in same direction,  $m_1 \vec{v}_1$  and  $m_2 \vec{v}_2$  are parallel.  $\Rightarrow |m_1 \vec{v}_1 + m_2 \vec{v}_2| = m_1 v_1 + m_2 v_2$

$$\text{Therefore, } v_{cm} = \frac{|m_1 \vec{v}_1 + m_2 \vec{v}_2|}{m_1 + m_2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{(1)(2) + \left(\frac{1}{2}\right)(6)}{\left(1 + \frac{1}{2}\right)} = 3.33 \text{ ms}^{-1}$$

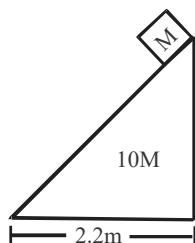
- Ex.** Two particles of masses 2 kg and 4 kg are approaching towards each other with acceleration 1 m/s<sup>2</sup> and 2 m/s<sup>2</sup>, respectively, on a smooth horizontal surface. Find the acceleration of centre of mass of the system.

- Sol.** The acceleration of centre of mass of the system  $\vec{a}_{cm} = \frac{m_1 \vec{a}_1 + m_2 \vec{a}_2}{m_1 + m_2} \Rightarrow a_{cm} = \frac{|m_1 \vec{a}_1 + m_2 \vec{a}_2|}{m_1 + m_2}$

$$\text{Since } \vec{a}_1 \text{ and } \vec{a}_2 \text{ are anti-parallel, so } a_{cm} = \frac{|m_1 a_1 - m_2 a_2|}{m_1 + m_2} = \frac{|(2)(1) - (4)(2)|}{2 + 4} = 1 \text{ ms}^{-2}$$

Since  $m_2 a_2 > m_1 a_1$  so the direction of acceleration of centre of mass will be directed in the direction of  $a_2$ .

- Ex.** A block of mass  $M$  is placed on the top of a bigger block of mass  $10M$  as shown in figure. All the surfaces are frictionless. The system is released from rest. Find the distance moved by the bigger block at the instant the smaller block reaches the ground.



**Sol.** If the bigger block moves towards right by a distance ( $X$ ), the smaller block will move towards left by a distance  $(2.2 - X)$  (taking the two blocks together as the system). The horizontal position of CM remains same  $\Rightarrow M(2.2 - X) = 10MX \Rightarrow X = 0.2\text{ m}$ .

### MOMENTUM

The total quantity of motion possessed by a moving body is known as the momentum of the body. It is the product of the mass and velocity of a body i.e. momentum  $\vec{p} = m\vec{v}$

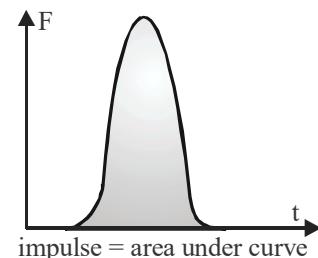
### IMPULSE

When a large force acts for an extremely short duration, neither the magnitude of the force nor the time for which it acts is important. In such a case, the total effect of force is measured. The total effect of force is called impulse (measure of the action of force). This type of force is generally variable in magnitude and is sometimes called impulsive force.

If a large force acts on a body or particle for a small time then

$$\text{Impulse} = \text{product of force with time.}$$

Suppose a force  $\vec{F}$  acts for a short time  $dt$  then impulse =  $\vec{F}dt$



For a finite interval of time from  $t_1$  to  $t_2$  then the impulse =  $\int_{t_1}^{t_2} \vec{F}dt$

If constant force  $\vec{F}$  acts for an interval  $\Delta t$  then Impulse =  $\vec{F}\Delta t$

### Impulse – Momentum theorem :

Impulse of a force is equal to the change of momentum

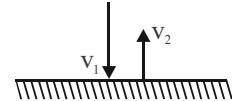
$$\vec{F}\Delta t = \Delta\vec{p}$$

<p>A large force acting for a short time and a small force acting for a long time may result in the same impulse, as indicated by the equal areas under the force-versus time curves.</p>	<p>The force <math>F(t)</math> involved in the collision between a ball and bat. The interaction occurs during the time <math>\Delta t</math>. The impulse is equal in magnitude to the shaded area under the curve.</p>	<p>The constant force <math>F_{av}</math> acting for the same time interval <math>\Delta t</math> produces the same impulse as in figure since the areas under the two graphs are the same.</p>
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**Ex.** A ball of mass 50 g is dropped from a height  $h = 10$  m. It rebounds losing 75 percent of its total mechanical energy. If it remains in contact with the ground for 0.01s, find the impulse of the impact force.

**Sol.** Impulse = change in momentum =  $m(v_1 + v_2)$

$$\text{Here } v_1 = \sqrt{2gh} \text{ and for } v_2, \frac{1}{2}mv_2^2 = \frac{1}{2}mv_1^2 \left(1 - \frac{75}{100}\right) \Rightarrow v_2 = \frac{v_1}{2}$$



$$\text{So impulse} = m \left( v_1 + \frac{v_1}{2} \right) = \frac{3mv_1}{2} = \frac{3}{2} m \times \sqrt{2gh} = \frac{3}{2} \times 50 \times 10^{-3} \times \sqrt{2 \times 9.8 \times 10} = 1.05 \text{ N-s}$$

## LAW OF CONSERVATION OF LINEAR MOMENTUM

According to Newton's Second law of motion the rate of change of momentum is equal to the applied force.

$$\vec{F} = \frac{d\vec{p}}{dt} \quad \text{if } \vec{F} = \vec{0} \text{ then } \frac{d\vec{p}}{dt} = \vec{0} \text{ i.e. } \vec{p} = \text{constant}$$

This leads to the law of conservation of momentum which is "In the absence of external force, the total momentum of the system is conserved."

### IMPORTANT POINT

- For an isolated system, the initial momentum of the system is equal to the final momentum of the system. If the system consists of  $n$  bodies having momentum

$$\vec{p}_1, \vec{p}_2, \vec{p}_3, \dots, \vec{p}_n, \text{ then } \vec{p}_1, \vec{p}_2, \vec{p}_3 + \dots + \vec{p}_n = \text{constant}$$

- As linear momentum depends on frame of reference. Observers in different frames would find different values of linear momentum of a given system but each would agree that his own value of linear momentum does not change with time provided. But the system should be isolated and closed, i.e., law of conservation of linear momentum is independent of frame of reference though linear momentum depends on frame of reference.
- Conservation of linear momentum is equivalent to Newton's III law of motion for a system of two particles in absence of external force by law of conservation of linear momentum.

$$\Rightarrow \vec{p}_1 + \vec{p}_2 = \text{constant} \quad \text{i.e. } m_1 \vec{v}_1 + m_2 \vec{v}_2 = \text{constant}$$

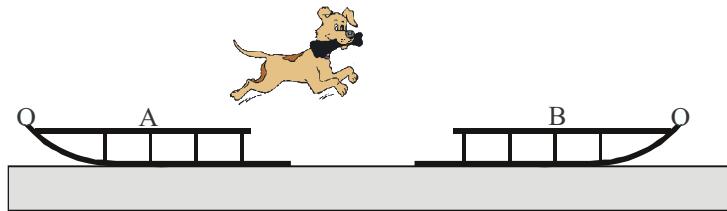
$$\text{Differentiating above with respect to time } m_1 \frac{d\vec{v}_1}{dt} + m_2 \frac{d\vec{v}_2}{dt} = \vec{0} \quad [\text{as } m \text{ is constant}]$$

$$\Rightarrow m_1 \vec{a}_1 + m_2 \vec{a}_2 = \vec{0} \quad [:\vec{d}\vec{v} = \vec{a}] \Rightarrow \vec{F}_1 + \vec{F}_2 = \vec{0} \quad [:\vec{F} = m\vec{a}] \Rightarrow \vec{F} = -\vec{F}_2$$

i.e., for every action there is equal and opposite reaction which is Newton's III law of motion.

- This law is universal, i.e., it applies to body macroscopic as well as microscopic systems.

**Ex.** Two 22.7 kg ice sleds A and B are placed a short distance apart, one directly behind the other, as shown in figure. A 3.63 kg dog, standing on one sled, jumps across to the other and immediately back to the first. Both jumps are made at a speed of  $3.05 \text{ ms}^{-1}$  relative to the ice. Find the final speeds of the two sleds.



**Sol.** Total momentum imparted to B  $p_B = 2 \times 3.63 \times 3.05 \text{ kg ms}^{-1}$ .

$$\text{Velocity of B} = \frac{p_B}{m_B} = \frac{2 \times 3.63 \times 3.05}{22.7} = 0.975 \text{ ms}^{-1}$$

$$\text{Velocity of A when the dog jumps away from A} = \frac{p_A}{m_A} = \frac{3.63 \times 3.05}{22.7} = 0.4877 \text{ ms}^{-1}$$

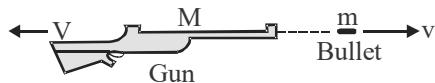
$$\text{When the dog comes back to A, Velocity of A} = \frac{22.7 \times 0.4877 + 3.63 \times 3.05}{22.7 + 3.63} = 0.841 \text{ ms}^{-1}$$

## APPLICATION OF CONSERVATION OF LINEAR MOMENTUM

### Firing a Bullet from a Gun :

- If the bullet is the system, the force exerted by trigger will be external and so the linear momentum of the bullet will change from 0 to  $mv$ . This is not the violation of law of conservation of linear momentum as linear momentum is conserved only in absence of external force.
- If the bullet and gun is the system, the force exerted by trigger will be internal so.

$$\text{Total momentum of the system} \quad \vec{p}_s = \vec{p}_B + \vec{p}_G = \text{constant.}$$



Now as initially both bullet and gun are at rest so  $\vec{p}_B + \vec{p}_G = \vec{0}$  From this it is evident that :

- $\vec{p}_G = -\vec{p}_B$ , i.e., if bullet acquires forward momentum, the gun will acquire equal and opposite (backward) momentum.
- As  $\vec{p} = m\vec{v} + M\vec{V} = \vec{0}$ , i.e.,  $\vec{V} = -\frac{m}{M}\vec{v}$  i.e, if the bullet moves forward, gun 'recoils' or 'kicks' backward. Heavier the gun lesser will be the recoil velocity V.
- Kinetic energy  $K = \frac{p^2}{2m}$  and  $|\vec{p}_B| = |\vec{p}_G| = p$  Kinetic energy of gun  $K_G = \frac{p^2}{2M}$ ,  
Kinetic energy of bullet  $K_B = \frac{p^2}{2M}$   $\therefore \frac{K_G}{K_B} = \frac{m}{M} < 1$  ( $\because M \gg m$ ) Thus kinetic energy of gun is smaller than bullet i.e., kinetic energy of bullet and gun will not be equal.
- Initial kinetic energy of the system is zero as both are at rest initially.  
Final kinetic energy of the system  $[(1/2)(mv^2 + MV^2)] > 0$ .  
So, here kinetic energy of the system is not constant but increases. If PE is assumed to be constant then Mechanical energy = (kinetic energy + potential energy) will also increase. However, energy is always conserved. Here chemical energy of gun powder is converted into KE.

**Ex.** A bullet of mass 100g is fired by a gun of 10kg with a speed 2000 m/s. Find recoil velocity of gun.

**Sol.** According to conservation of momentum  $mv + MV = 0$ .

$$\text{Velocity of gun} = -\frac{mv}{M} = -\frac{0.1 \times 2000}{10} = -20 \text{ m/s}$$

### Block Bullet System :

#### (a) When bullet remains in the block

Conserving momentum of bullet and block  $mv + 0 = (M+m)V$

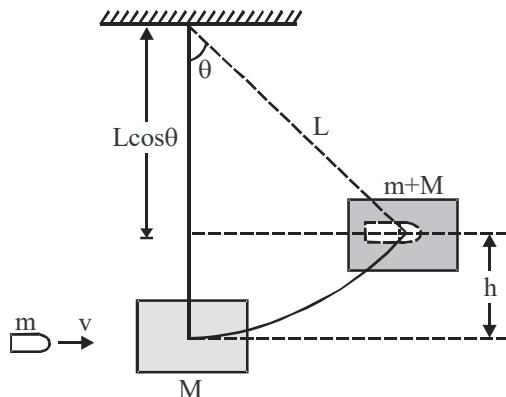
$$\text{Velocity of block } V = \frac{mv}{M+m} \quad \dots(\text{i})$$

By conservation of mechanical energy

$$\frac{1}{2}(M+m)V^2 = (M+m)gh \Rightarrow V = \sqrt{2gh} \quad \dots(\text{ii})$$

$$\text{From eqn. (i) and eqn. (ii)} \frac{mv}{M+m} = \sqrt{2gh};$$

$$\text{Speed of bullet } v = \frac{(M+m)\sqrt{2gh}}{m},$$



$$\text{Maximum height gained by block } h = V = \frac{V^2}{2g} = \frac{m^2 v^2}{2g(M+m)^2}$$

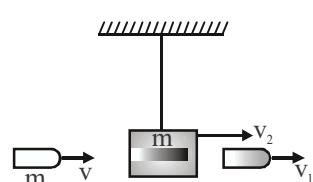
$$h = L - L \cos\theta \quad \therefore \cos\theta = 1 - \frac{h}{L} \Rightarrow \theta = \cos^{-1}\left(1 - \frac{h}{L}\right)$$

#### (b) If bullet moves out of the block

$$\begin{aligned} \text{Conserving momentum } mv + 0 &= mv_1 + Mv_2 \\ m(v - v_1) &= Mv_2 \quad \dots\dots\dots(\text{i}) \end{aligned}$$

$$\text{Conserving energy} \quad \frac{1}{2}mv_2^2 = Mgh \Rightarrow v_2 = \sqrt{2gh} \quad \dots\dots\dots(\text{ii})$$

$$\text{From eqn. (i) \& eqn. (ii)} \quad m(v - v_1) = M\sqrt{2gh} \Rightarrow h = \frac{m^2(v - v_1)^2}{2gM^2}$$



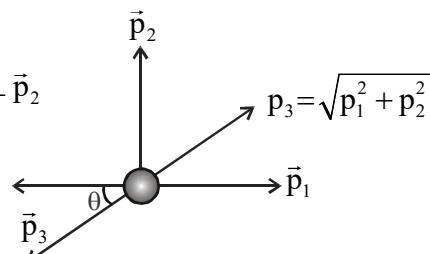
### Explosion of a Bomb at rest

Conserving momentum

$$\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = \vec{0} \Rightarrow \vec{p}_3 = -(\vec{p}_1 + \vec{p}_2) \Rightarrow p_3 = \sqrt{p_1^2 + p_2^2} \quad \text{as } \vec{p}_1 \perp \vec{p}_2$$

Angle made by  $\vec{p}_3$  from  $\vec{p}_1 = \pi + \theta$

Angle made by  $\vec{p}_3$  from  $\vec{p}_1 = \frac{\pi}{2} + \theta$



$$\text{Energy released in explosion} = K_f - K_i = \frac{\vec{p}_1^2}{2m_1} + \frac{\vec{p}_2^2}{2m_2} + \frac{\vec{p}_3^2}{2m_3} - 0 = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + \frac{p_3^2}{2m_3}$$

## Motion of Two Masses Connected to a Spring

Consider two blocks, resting on a frictionless surface and connected by a massless spring as shown in figure. If the spring is stretched (or compressed) and then released from rest ,

Then  $\vec{F}_{\text{ext}} = 0$  so  $\vec{p}_s = \vec{p}_1 + \vec{p}_2 = \text{constant}$

However, initially both the blocks were at rest so,  $\vec{p}_1 + \vec{p}_2 = \vec{0}$



It is clear that :

- $\vec{p}_2 = -\vec{p}_1$ , i.e., at any instant the two blocks will have momentum equal in magnitude but opposite in direction (Though they have different values of momentum at different positions).

- As momentum  $\vec{p} = m\vec{v}$ ,  $m_1\vec{v}_1 + m_2\vec{v}_2 = \vec{0} \Rightarrow \vec{v}_2 = -\left(\frac{m_1}{m_2}\right)\vec{v}_1$

The two blocks always move in opposite directions with lighter block moving faster.

- Kinetic energy  $KE = \frac{\vec{p}}{2m}$  and  $|\vec{p}_1| = |\vec{p}_2|$ ,  $\frac{KE_1}{KE_2} = \frac{m_2}{m_1}$  or the kinetic energy of two blocks will not be equal but in the inverse ratio of their masses and so lighter block will have greater kinetic energy.
- Initially kinetic energy of the blocks is zero (as both are at rest) but after some time kinetic energy of the blocks is not zero (as both are in motion). So, kinetic energy is not constant but changes. Here during motion of blocks KE is converted into elastic potential energy of the spring and vice-versa but total mechanical energy of the system remain constant.

$$\text{Kinetic energy} + \text{Potential energy} = \text{Mechanical Energy} = \text{Constant}$$

Note – If  $\bar{F}$  is the average of the time varying force during collision and  $\Delta t$  is the duration of collision then impulse  $\bar{I} = \bar{F}\Delta t$ .

## Conservation of Linear Momentum During Impact :

If two bodies of masses  $m_1$  and  $m_2$  collide in air, the total external force acting on the system of bodies ( $m_1 + m_2$ ) is equal to  $\bar{F}_1 + m_1\vec{g} + \bar{F}_2 + m_2\vec{g} \Rightarrow F_{\text{total}} = m_1\vec{g} + m_2\vec{g} + \bar{F}_1 + \bar{F}_2$

During collision the impact forces  $\bar{F}_1$  and  $\bar{F}_2$  are equal in magnitude and opposite in direction.

According to Newton's 3rd law of motion,  $\bar{F}_1 + \bar{F}_2 = 0 \Rightarrow \bar{F}_{\text{total}} = m_1\vec{g} + m_2\vec{g}$

So Impulse =  $\bar{F}_{\text{total}}\Delta t = (m_1\vec{g} + m_2\vec{g})\Delta t$

Since  $\Delta t$  is a very small time interval, the impulse  $F(\Delta t)$  will be negligibly small. As impulse is equal to change in momentum of the system, a negligible impulse means negligible change of momentum. Let the change of momentum of 1 & 2 be  $\Delta\vec{p}_1$  &  $2\vec{p}_2$ , respectively then the total change in momentum of the system  $\Rightarrow \Delta\vec{p} = \Delta\vec{p}_1 + \Delta\vec{p}_2 = \bar{F}_{\text{net}}\cdot dt \approx \vec{0} \Rightarrow \Delta(\vec{p}_1 + \vec{p}_2) = 0 \Rightarrow \vec{p}_1 + \vec{p}_2 = \text{constant}$

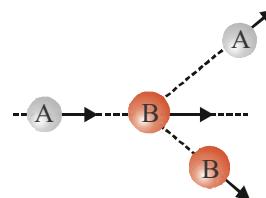
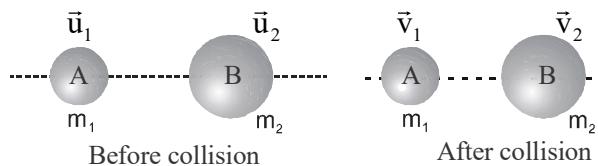
Therefore, the net or total momentum of the colliding bodies remains practically unchanged along the line of action (impact) during the collision. In other words, the momentum of the system remains constant or conserved during the period of impact. Therefore, we can conveniently equate the net momentum of the colliding bodies at the beginning and at the end of the collision (or just before and just after the impact).

**Note :** Remember that the impact force  $F$  is not an external force for the system of colliding bodies. If no

external force acts on the system, its momentum remains constant for all the times including the time of collision. Even if some external forces like gravitation and friction (known as non-impulsive forces in general) are present, we can conserve the momentum of the system during the impact, because the finite external forces cannot change the momentum of the system significantly in very short time. Therefore, the change in position of the system during infinitesimal time of impact can also be neglected.

- Types of collision according to the direction of collision :

(a) **Head on collision** : Direction of velocities of bodies is similar to the direction of collision.



(b) **Oblique collision** : Direction of velocities of bodies is not similar to the direction of collision.

- Types of collision according to the conservation law of kinetic energy :

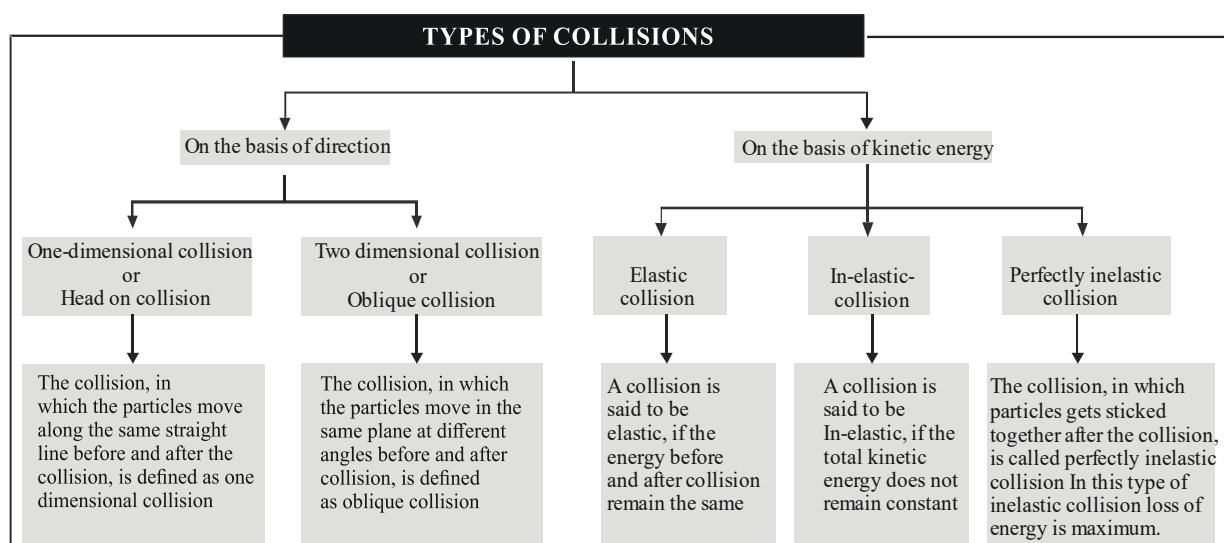
(a) **Elastic collision** :  $KE_{\text{before collision}} = KE_{\text{after collision}}$

(b) **Inelastic collision** : kinetic energy is not conserved.

Some energy is lost in collision  $KE_{\text{before collision}} > KE_{\text{after collision}}$

(c) **Perfectly inelastic collision** : Two bodies stick together after the collision.

momentum remains conserved in all types of collisions.



### Coefficient of restitution (e)

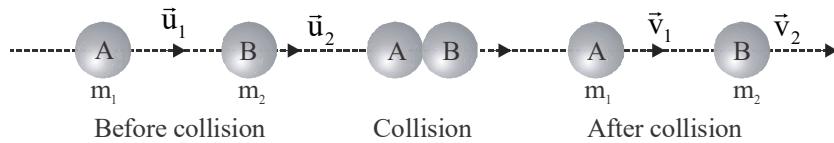
The coefficient of restitution is defined as the ratio of the impulses of recovery and deformation of either body.

$$e = \frac{\text{velocity of separation along line of impact}}{\text{velocity of approach along line of impact}}$$

Value of e is 1 for elastic collision, 0 for perfectly inelastic collision and  $0 < e < 1$  for inelastic collision.

## HEAD ON ELASTIC COLLISION

The elastic collision in which the colliding bodies move along the same straight line path before and after the collision.



Assuming initial direction of motion to be positive and  $u_1 > u_2$  (so that collision may take place) and applying law of conservation of linear momentum

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2 \Rightarrow m_1(u_1 - v_1) = m_2(v_2 - u_2) \quad \dots(i)$$

For elastic collision, kinetic energy before collision must be equal to kinetic energy after collision, i.e.,

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \Rightarrow m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2) \quad \dots(ii)$$

Dividing equation (ii) by (i)  $u_1 + v_1 = v_2 + u_2 \Rightarrow (u_1 - u_2) = (v_2 - v_1) \quad \dots(iii)$

In 1-D elastic collision 'velocity of approach' before collision is equal to the 'velocity of separation' after collision, no matter what the masses of the colliding particles be.

This law is called **Newton's law for elastic collision**

Now if we multiply equation (iii) by  $m_1$  and subtracting it from (i)

$$(m_1 - m_2) u_1 + 2m_2 u_2 = (m_1 + m_2) v_1 \Rightarrow v_1 = \frac{m_1 - m_2}{m_1 + m_2} u_1 + \frac{2m_2}{m_1 + m_2} u_2 \quad \dots(iv)$$

Similarly multiplying equation (iii) by  $m_2$  and adding it to equation (i)

$$2m_1 u_1 + (m_2 - m_1) u_2 = (m_2 + m_1) v_2 \Rightarrow v_2 = \frac{2m_1}{m_1 + m_2} u_1 + \frac{m_2 - m_1}{m_1 + m_2} u_2 \quad \dots(v)$$

### IMPORTANT POINTS

- **If the two bodies are of equal masses :**  $m_1 = m_2 = m$ ,  $v_1 = u_2$  and  $v_2 = u_1$   
Thus, if two bodies of equal masses undergo elastic collision in one dimension, then after the collision, the bodies will exchange their velocities.
- **If two bodies are of equal masses and second body is at rest.**  
 $m_1 = m_2$  and initial velocity of second body  $u_2 = 0$ ,  $v_1 = 0$ ,  $v_2 = u_1$   
When body A collides against body B of equal mass at rest, the body A comes to rest and the body B moves on with the velocity of the body A. In this case transfer of energy is hundred percent e.g.. Billiard's Ball, Nuclear moderation.
- **If the mass of a body is negligible as compared to other.**  
If  $m_1 \gg m_2$  and  $u_2 = 0$  then  $v_1 = u_1$ ,  $v_2 = 2u_1$   
When a heavy body A collides against a light body B at rest, the body A should keep on moving with same velocity and the body B will move with velocity double that of A.  
If  $m_2 \gg m_1$  and  $u_2 = 0$  then  $v_2 = 0$ ,  $v_1 = -u_1$   
When light body A collides against a heavy body B at rest, the body A should start moving with same velocity just in opposite direction while the body B should practically remain at rest.

**Ex.** Two ball of mass 5kg each is moving in opposite directions with equal speed 5m/s. collides head on with each other. Find out the final velocities of the balls if collision is elastic.

**Sol.** Here  $m_1 = m_2 = 5\text{ kg}$ ,  $u_1 = 5 \text{ m/s}$ ,  $u_2 = -5 \text{ m/s}$

In such type of condition velocity get interchange so  $v_2 = u_1 = 5 \text{ m/s}$  &  $v_1 = u_2 = -5 \text{ m/s}$

**Ex.** A ball of 0.1 kg makes an elastic head on collision with a ball of unknown mass that is initially at rest. If the 0.1kg ball rebounds at one third of its original speed. What is the mass of other ball ?

**Sol.** Here  $m_1 = 0.1 \text{ kg}$ ,  $m_2 = ?$ ,  $u_2 = 0$ ,  $u_1 = u$ ,  $v_1 = -u/3$

$$\text{As } v_1 = \left( \frac{m_1 - m_2}{m_1 + m_2} \right) u + \frac{2m_2 u_2}{m_1 + m_2} \Rightarrow \frac{u}{3} = \left( \frac{0.1 - m_2}{0.1 + m_2} \right) u \Rightarrow m_2 = 0.2 \text{ kg}$$

### HEAD ON INELASTIC COLLISION OF TWO PARTICLES

Let the coefficient of restitution for collision is e

(i) Momentum is conserved  $m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$  ... (i)

(ii) Kinetic energy is not conserved.

(iii) According to Newton's law  $\frac{v_2 - v_1}{u_2 - u_1} = -e$  ... (ii)

By solving eq. (i) and (ii)

$$v_1 = \left( \frac{m_1 + em_2}{m_1 + m_2} \right) u_1 + \left( \frac{(1-e)m_2}{m_1 + m_2} \right) u_2, v_2 = \left( \frac{m_2 + em_1}{m_1 + m_2} \right) u_2 + \left( \frac{(1+e)m_1}{m_1 + m_2} \right) u_1$$

### PERFECT INELASTIC COLLISION

In case of inelastic collision, after collision two bodies move with same velocity (or stick together). If two particles of masses  $m_1$  and  $m_2$ , moving with velocity  $u_1$  and  $u_2$  ( $u_2 < u_1$ ) respectively along the same line collide 'head on' and after collision they have same common velocity  $v$ , then by conservation of linear momentum,

$$m_1 u_1 + m_2 u_2 = m_1 v + m_2 v \Rightarrow v = \frac{m_1 u_1 + m_2 u_2}{(m_1 + m_2)} \quad \dots (\text{i})$$

Kinetic energy of the system before collision is  $KE_i = \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2$

And after collision is  $KE_f = \frac{1}{2} (m_1 + m_2) v^2$

### Loss in KE during collision

$$\Delta KE = KE_i - KE_f = \left[ \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 \right] - \frac{1}{2} (m_1 + m_2) v^2 \quad \dots (\text{ii})$$

Substituting the value of  $v$  from eq. (i),

$$\Delta KE = \frac{1}{2} [(m_1 u_1^2 + m_2 u_2^2) - \frac{(m_1 u_1 + m_2 u_2)^2}{(m_1 + m_2)}]$$

$$\Rightarrow \Delta KE = \frac{1}{2} \left[ \frac{m_1 m_2 (u_1^2 + u_2^2 - 2 u_1 u_2)}{(m_1 + m_2)} \right] \Rightarrow \boxed{\Delta KE = \frac{1}{2} \frac{m_1 m_2}{(m_1 + m_2)} (u_1 - u_2)^2}$$

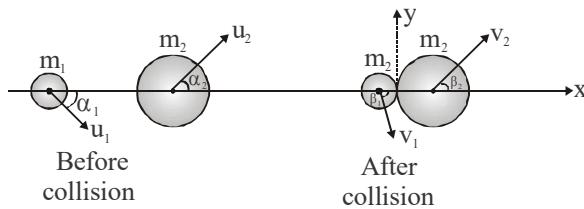
If the target is initially at rest  $u_2 = 0$  and  $u_1 = u$

$$\Delta KE = \frac{m_1 m_2}{2(m_1 + m_2)} u^2, \quad \frac{\Delta KE}{KE_i} = \frac{m_2}{(m_1 + m_2)} [\because KE_i = \frac{1}{2} m_1 u_1^2]$$

Now if target is massive, i.e.,  $m_2 \gg m_1$  then  $\frac{\Delta KE}{KE_i} \approx 1$  so percentage loss in KE = 100%

i.e., if a light moving body strike a heavy target at rest and sticks to it, practically all its KE is lost.

### Oblique Collision



In oblique impact the relative velocity of approach of the bodies doesn't coincide with the line of impact. Conserving the momentum of system in directions along normal (x axis in our case) and tangential (y axis in our case)

$m_1 u_1 \cos \alpha_1 + m_2 u_2 \cos \alpha_2 = m_1 v_1 \cos \beta_1 + m_2 v_2 \cos \beta_2$  and  $m_2 u_2 \sin \alpha_2 - m_1 u_1 \sin \alpha_1 = m_2 v_2 \sin \beta_2 - m_1 v_1 \sin \beta_1$   
Since no force is acting on  $m_1$  and  $m_2$  along the tangent (i.e. y-axis) the individual momentum of  $m_1$  and  $m_2$  remains conserved.  $m_1 u_1 \sin \alpha_1 = m_1 v_1 \sin \beta_1$  &  $m_2 u_2 \sin \alpha_2 = m_2 v_2 \sin \beta_2$

By using Newton's experimental law along the line of impact  $e = \frac{v_2 \cos \beta_2 - v_1 \cos \beta_1}{u_1 \cos \alpha_1 - u_2 \cos \alpha_2}$

### Oblique Impact on a Fixed Plane

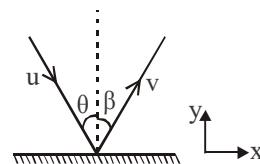
Let a small ball collides with a smooth horizontal floor with a speed  $u$  at an angle  $\theta$  to the vertical as shown in the figure. Just after the collision, let the ball leaves the floor with a speed  $v$  at angle  $\beta$  to vertical.

It is quite clear that the line of action is perpendicular to the floor. Therefore, the impact takes place along the (normal) vertical. Now we can use Newton's experimental law as

$$e = \frac{\text{velocity of separation}}{\text{velocity of approach}}$$

$$\Rightarrow e [\text{velocity of approach}] = \text{velocity of separation}$$

$$\Rightarrow e[u \cos \theta (-\hat{j})] = -[v \cos \beta (\hat{j})] \Rightarrow v \cos \beta = e u \cos \theta \quad \dots(i)$$



Since impulsive force  $N$  acts on the body along the normal, we cannot conserve its momentum. Since along horizontal the component of  $N$  is zero, therefore we can conserve the horizontal momentum of the body.

$$\text{Momentum } (p_x)_{\text{body}} = \text{constant} \Rightarrow (p_x)_{\text{initial}} = (p_x)_{\text{final}} \quad \dots(ii)$$

$$\Rightarrow m u \sin \theta = m v \sin \beta \Rightarrow v \sin \beta = u \sin \theta$$

Squaring equations(i) and (ii) and adding,  $v^2 \cos^2 \beta + v^2 \sin^2 \beta = e^2 u^2 \cos^2 \theta + u^2 \sin^2 \theta$

$$\Rightarrow v^2 = u^2 [e^2 \cos^2 \theta + \sin^2 \theta] \Rightarrow v = u \sqrt{\sin^2 \theta + e^2 \cos^2 \theta}$$

Dividing equation (i) by (ii)

$$\Rightarrow \frac{v \cos \beta}{v \sin \beta} = \frac{eu \cos \theta}{u \sin \theta} \Rightarrow \cot \beta = e \cot \theta \Rightarrow \beta = \cot^{-1}(e \cot \theta)$$

Impulse of the blow = change of momentum of the body

$$\begin{aligned} &= \{(mv \sin \beta) \hat{i} + (mv \cos \beta) \hat{j}\} - \{(mu \sin \theta) \hat{i} - (mu \cos \theta) \hat{j}\} \\ &= (mv \sin \beta - mu \sin \theta) \hat{i} + (mv \cos \beta + mu \cos \theta) \hat{j} \end{aligned}$$

Since  $v \sin \beta = u \sin \theta \Rightarrow$  Impulse =  $m(v \cos \beta + u \cos \theta) \hat{j}$

Putting  $v \cos \beta = eu \cos \theta$  from eq. (i),

$$\text{Impulse} = m(1+e)u \cos \theta \hat{j} \quad \therefore \text{Magnitude of the impulse} = m(1+e)u \cos \theta$$

$$\text{Change in Kinetic energy: } \Delta \text{KE.} = \frac{1}{2} mv^2 - \frac{1}{2} mu^2$$

Putting the value of v we obtain

$$\begin{aligned} \Delta \text{KE.} &= \frac{1}{2} m \left[ \left[ \sqrt{u(\sin^2 \theta + e^2 \cos^2 \theta)} \right]^2 - u^2 \right] = \frac{1}{2} mu^2 [\sin^2 \theta + e^2 \cos^2 \theta - 1] \\ &= -\frac{1}{2} mu^2 [\cos^2 \theta - e^2 \cos^2 \theta] = -\frac{1}{2} (1 - e^2) mu^2 \cos^2 \theta \end{aligned}$$

Negative sign indicates the loss of kinetic energy

### IMPORTANT POINTS

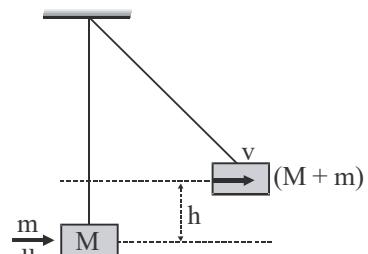
- Momentum remains conserved in all types of collisions.
- Total energy remains conserved in all types of collisions.
- Only conservative forces work in elastic collisions.
- In inelastic collisions all the forces are not conservative.

**Ex.** A simple pendulum of length 1m has a wooden bob of mass 1kg. It is struck by a bullet of mass  $10^{-2}$  kg moving with a speed of  $2 \times 10^2$  m/s. The bullet gets embedded into the bob. Obtain the height to which the bob rises before swinging back.

**Sol.** Applying principle of conservation of linear momentum

$$mu = (M+m)v \Rightarrow 10^{-2} \times (2 \times 10^2) = (1 + .01)v \Rightarrow v = \frac{2}{1.01}$$

KE<sub>i</sub> of the block with bullet in it, is converted into P.E. as it rises through a height h



$$\frac{1}{2} (M+m)v^2 (M+m)gh \Rightarrow v^2 = 2gh \Rightarrow h = \frac{v^2}{2g} = \left( \frac{2}{1.01} \right)^2 \times \frac{1}{2 \times 9.8} = 0.2 \text{ m}$$

- Ex.** A body falling on the ground from a height of 10m, rebounds to a height 2.5m calculate  
 (i) The percentage loss in K.E  
 (ii) Ratio of the velocities of the body just before and just after the collision.

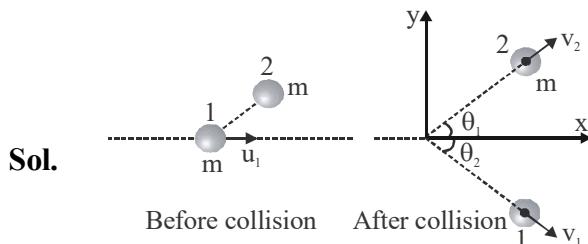
**Sol.** Let  $v_1$  and  $v_2$  be the velocity of the body just before and just after the collision

$$KE_1 = \frac{1}{2}mv_1^2 = mgh_1 \quad \dots(i) \quad \text{and } KE_2 = \frac{1}{2}mv_2^2 = mgh_2 \quad \dots(ii)$$

$$\Rightarrow \frac{v_1^2}{v_2^2} = \frac{h_1}{h_2} = \frac{10}{2.5} = 4 \Rightarrow \frac{v_1}{v_2} = 2$$

$$\text{Percentage loss in KE} = \frac{mg(h_1 - h_2)}{mgh_1} \times 100 = \frac{10 - 2.5}{10} \times 100 = 75\%$$

- Ex.** A body strikes obliquely with another identical stationary rest body elastically. Prove that they will move perpendicular to each other after collision.



Conservation of linear momentum in x-direction gives

$$mu_1 = mv_1 \cos \theta_1 + mv_2 \cos \theta_2 \Rightarrow u_1 = v_1 \cos \theta_1 + v_2 \cos \theta_2 \quad \dots(i)$$

Conservation of linear momentum in y-direction gives

$$0 = mv_1 \sin \theta_1 - mv_2 \sin \theta_2 \Rightarrow 0 = v_1 \sin \theta_1 - v_2 \sin \theta_2 \quad \dots(ii)$$

Conservation of kinetic energy

$$\frac{1}{2}mu_1^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 \Rightarrow u_1^2 = v_1^2 + v_2^2 \quad \dots(iii)$$

$$(i)^2 + (ii)^2$$

$$\Rightarrow u_1^2 + 0 = v_1^2 \cos^2 \theta_1 + v_2^2 \cos^2 \theta_2 + 2v_1 v_2 \cos \theta_1 \cos \theta_2 + v_1^2 \sin^2 \theta_1 + v_2^2 \sin^2 \theta_2 - 2v_1 v_2 \sin \theta_1 \sin \theta_2$$

$$\Rightarrow u_1^2 = v_1^2 (\cos^2 \theta_1 + \sin^2 \theta_1) + v_2^2 (\cos^2 \theta_2 + \sin^2 \theta_2) + 2v_1 v_2 (\cos \theta_1 \cos \theta_2 - \sin \theta_1 \sin \theta_2)$$

$$\Rightarrow u_1^2 = v_1^2 + v_2^2 + 2v_1 v_2 \cos (\theta_1 + \theta_2) \quad \{ \because u_1^2 = v_1^2 + v_2^2 \}$$

$$\Rightarrow \cos (\theta_1 + \theta_2) = 0 \Rightarrow \theta_1 + \theta_2 = 90^\circ$$

- Ex.** A steel ball is dropped on a smooth horizontal plane from certain height  $h$ . Assuming coefficient of restitution of impact as  $e$ , find the average speed of the ball till it stops.

- Sol.** Since the ball falls through a height  $h$ , just before the first impact its speed  $v$  will be given as  $v = \sqrt{2gh}$ . Let its speed the  $v_1$  just after the first impact. Then, Newton's experimental formula yields,

$$\frac{0 - v_1}{v} = e \Rightarrow v_1 = ev$$

Similarly, its speed just before 2nd impact,  $v_1 = ev = e\sqrt{2gh}$

Speed just after  $n^{\text{th}}$  impact,  $v_n = e^n v = e^n \sqrt{2gh}$

The maximum height attained after 1<sup>st</sup> impact =  $h_1 = \frac{v^2}{2g} = (e \sqrt{2gh})^2 = e^2 h$ . Similarly, the maximum

height attained after 2nd impact,  $h^2 = e^4 h$ . Hence, the maximum height attained after  $n^{\text{th}}$  impact =  $e^{2n} h$

The ball experiences infinite impacts till it becomes stationary.  $\Rightarrow$  The total distance covered,

$$d = h + 2h_1 + 2h_2 + \dots d = h + 2e^2 h + 2e^4 h + \dots = h[1 + 2(e^2 + e^4 + e^6 + \dots)]$$

$$= \left[ 1 + 2 \left( \frac{e^2}{1-e^2} \right) \right] h = \left[ \left( \frac{1+e^2}{1-e^2} \right) \right] h.$$

The total time taken by the ball till it stops bounding  $T = \sqrt{\frac{2h}{g}} + 2\sqrt{\frac{2h_1}{g}} + 2\sqrt{\frac{2h_2}{g}} + \dots$

$$\text{Putting } h_1 = e^2 h, h_2 = e^4 h, T = \sqrt{\frac{2h}{g}} + 2\sqrt{\frac{2e^2 h}{g}} + 2\sqrt{\frac{2e^4 h}{g}} + \dots$$

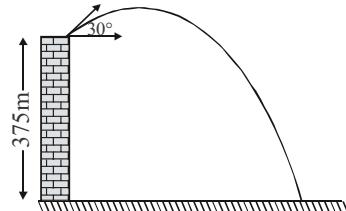
$$\Rightarrow T = \sqrt{\frac{2h}{g}} [1 + 2(e + e^2 + \dots)] = \sqrt{\frac{2h}{g}} \left[ 1 + \frac{2e}{1-e} \right] = \frac{1+e}{1-e} \sqrt{\frac{2h}{g}}$$

Therefore, average speed of the ball for its total time of motion,  $\bar{v} = \frac{\text{Total distance}}{\text{total time}} = \frac{d}{T}$

$$\text{Putting the values of } d \text{ and } T, \text{ we obtain } \bar{v} = \frac{1+e^2}{(1+e)^2} \sqrt{\frac{gh}{2}}$$

- Ex.** A particle of mass 1 kg is projected from a tower of height 375m with initial velocity  $100 \text{ ms}^{-1}$  at an angle  $30^\circ$  with the horizontal. Find out its kinetic energy in joule just after collision with ground if collision is

$$\text{inelastic with } e = \frac{1}{2} \quad (g = 10 \text{ ms}^{-2})$$



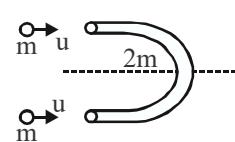
$$\text{Sol. } v_y^2 = u_y^2 + 2gh \Rightarrow v_y = \sqrt{(50)^2 + 2 \times 10 \times 375} = 100 \text{ m/s}^2$$

$$\text{Horizontal velocity just after collision} = 50\sqrt{3} \text{ ms}^{-1}$$

$$\text{Vertical velocity just after collision} = 100 \times \frac{1}{2} = 50 \text{ ms}^{-1}$$

$$\text{Kinetic energy just after collision} = \frac{1}{2} \times 1 \times \left[ (50\sqrt{3})^2 + (50)^2 \right] = 5000 \text{ J}$$

- Ex.** A U shaped tube of mass 2m is placed on a horizontal surface. Two spheres each of diameter d (just less than the inner diameter of tube) and mass m enter into the tube with a velocity u as shown in figure. Taking all collisions to be elastic and all surfaces smooth. Match the following-



**Column-I**

- (A) The speed of the tube with respect to ground, when spheres are just about to collide inside the tube.
- (B) The speed of spheres when spheres are just about to collide.
- (C) The speed of the spheres when they comes out the tube.
- (D) The speed of the tube when spheres comes out

**Column-II**

- (p) u
- (q)  $u/2$
- (r)  $\frac{\sqrt{3}}{2}u$
- (s) zero

**Sol.** For (A) From conservation of linear momentum  $2mu = (m+m)v \Rightarrow v = \frac{u}{2}$

For (B) Let  $v_1$  be the velocity of spheres w.r.t. tube when they are just about to collide then by using

$$\text{conservation of kinetic energy } \frac{1}{2}(2m)u^2 = \frac{1}{2}(4m)\left(\frac{u}{2}\right)^2 + 2\frac{1}{2}mv_1^2 \Rightarrow v_1 = \frac{u}{\sqrt{2}}$$

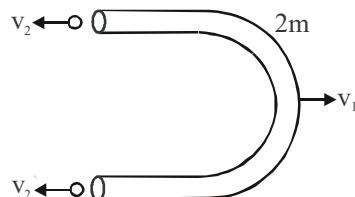
$$\Rightarrow \text{Required speed of spheres} = \sqrt{\left(\frac{u}{2}\right)^2 + \left(\frac{u}{\sqrt{2}}\right)^2} = \sqrt{\frac{u^2}{4} + \frac{u^2}{2}} = \frac{\sqrt{3}u}{2}$$

For (C)  $2mu = 2mv_1 - 2mv_2$

$$2 \times \frac{1}{2}mu^2 = 2 \times \frac{1}{2}mv_2^2 + \frac{1}{2}(2m)v_1^2$$

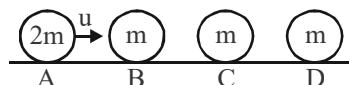
$$\Rightarrow u^2 = v_1^2 + v_2^2 - 2v_1v_2 \quad \& \quad u^2 = v_1^2 + v_2^2$$

$$\Rightarrow v_1v_2 = 0 \text{ but } v_1 \neq 0 \text{ so } v_2 = 0$$



For (D) Speed of tube  $v_1 = u$

**Ex.** Four balls A, B, C and D are kept on a smooth horizontal surface as shown in figure. Ball A is given velocity  $u$  towards B-



**Column-I**

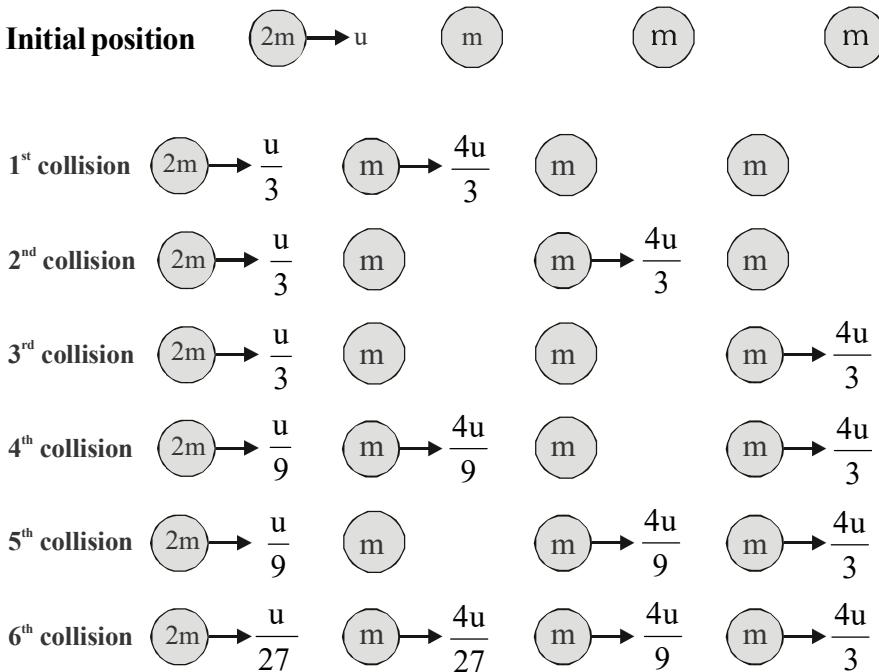
- (A) Total impulse of all collisions on A (p)  $\frac{4mu}{9}$
- (B) Total impulse of all collisions on B (q)  $\frac{4mu}{27}$
- (C) Total impulse of all collision on C (r)  $\frac{4mu}{3}$
- (D) Total impulse of all collisions on D (s)  $\frac{52}{27}mu$

**Column-II**

**Sol.** In 1<sup>st</sup> collision between A & B

$$2mu = 2mv_A + 2mv_B \quad \& \quad e = 1 = \frac{v_B - v_A}{u} \Rightarrow v_A = \frac{u}{3}, v_B = \frac{4u}{3}$$

Situation of all collisions is shown in figure.



**For (A)** Total impulse on A =  $2m \left( u - \frac{u}{27} \right) = \frac{52}{27} mu$

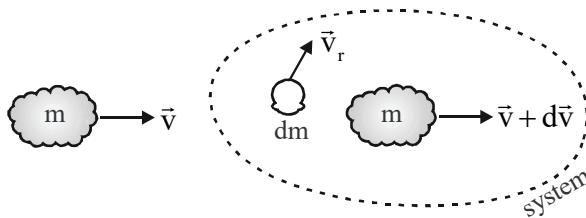
**For (B)** Total impulse on B =  $m \left( \frac{4u}{27} - 0 \right) = \frac{4}{27} mu$

**For (C)** Total impulse on C =  $m \left( \frac{4u}{9} - 0 \right) = \frac{4}{9} mu$

**For (D)** Total impulse on D =  $m \left( \frac{4u}{3} - 0 \right) = \frac{4}{3} mu$

### Variable mass system:

In previous discussion of the conservation of linear momentum, we assume that system's mass remains constant. Now we are consider those system whose mass is variable i.e. those in which mass enters or leaves the system. Suppose at some moment  $t = t$  mass of a body is  $m$  and its velocity is  $\vec{v}$ . After some time at  $t = t + dt$  its mass becomes  $(m - dm)$  and velocity becomes  $\vec{v} + d\vec{v}$ . The mass  $dm$  is ejected with relative velocity  $\vec{v}_r$ .



If no forces are acting on the system then the linear momentum of the system will remain conserved.

$$\Rightarrow \vec{F}_{\text{ex}} dt = (m - dm)(\vec{v} + d\vec{v}) + dm(\vec{v}_r + \vec{v} + d\vec{v}) - m\vec{v}$$

$$\therefore F_{\text{ex}} = 0 \Rightarrow md\vec{v} = -\vec{v}_r dm \Rightarrow m\left(\frac{d\vec{v}}{dt}\right) = \vec{v}_r\left(-\frac{dm}{dt}\right)$$

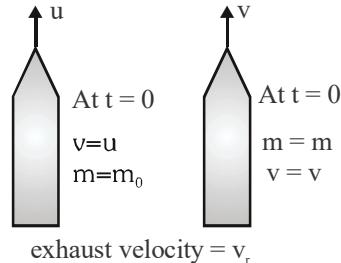
### Rocket propulsion :

$$\text{Thrust force on the rocket} = v_r\left(-\frac{dm}{dt}\right)$$

$$\text{So for motion of rocket } m \frac{dv}{dt} = v_r\left(-\frac{dm}{dt}\right) - mg$$

$$\Rightarrow dv = v_r\left(-\frac{dm}{dt}\right) - gdt \quad \Rightarrow \int_{u_0}^v dv = -v_r \int_{m_0}^m \frac{dm}{m} - g \int_0^t dt$$

$$\Rightarrow v - u = v_r \ln\left(\frac{m_0}{m}\right) - gt \Rightarrow v = u - gt + v_r \ln\left(\frac{m_0}{m}\right)$$



**Ex.** An open topped rail road car of mass  $M$  has an initial velocity  $v_0$  along a straight horizontal frictionless track. It suddenly starts raining at time  $t = 0$ . The rain drops fall vertically with velocity  $u$  and add a mass  $m$  kg/sec of water. Find the velocity of car after  $t$  second (assuming that it is not completely filled with water).

**Sol.** According to law of conservation of momentum,  $Mv_0 = (M + m \times t) v$ . Where  $m$  is the mass of water added per second and  $v$  is the velocity of the car after  $t$  second.  $\therefore v = \frac{Mv_0}{M + mt}$

**Ex.** A uniform chain of mass  $m$  and length  $l$  hangs on a thread and touches the surface of a table by its lower end. Find the force exerted by the chain on the table when half of its length has fallen on the table. The fallen part does not form heap.

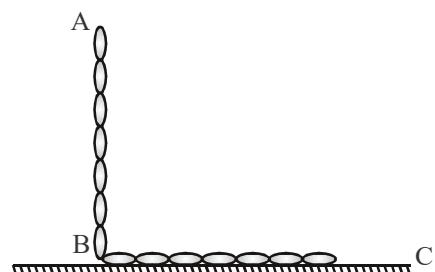
**Sol.** At given condition force exerted by the chain on the table consists of two parts

$$(i) \text{Weight of portion BC} = \frac{mg}{2}$$

$$(ii) \text{Thrust force} = v_r\left(-\frac{dm}{dt}\right) = v\left(\frac{m}{\ell}v\right) = \frac{m}{\ell}v^2$$

$$\text{but } v = \sqrt{2g\left(\frac{\ell}{2}\right)} = \sqrt{g\ell}$$

$$\Rightarrow \text{Thrust force} = \frac{m}{\ell}(g\ell) = mg$$



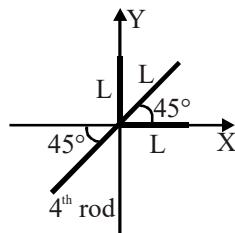
$$\therefore \text{Net force exerted by falling chain} = \frac{mg}{2} + mg = \frac{3mg}{2}$$

## EXERCISE

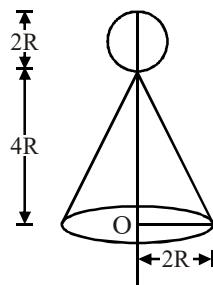
### EXERCISE (S-1)

#### HCV Worked out Examples(Chapter No. 9 :- 1,2,3)

- Four particles of mass 5, 3, 2, 4 kg are at the points (1, 6), (-1, 5), (2, -3), (-1, -4). Find the coordinates of their centre of mass.
- A rigid body consists of a 3 kg mass connected to a 2 kg mass by a massless rod. The 3kg mass is located at  $\vec{r}_1 = (2\hat{i} + 5\hat{j})$  m and the 2 kg mass at  $\vec{r}_2 = (4\hat{i} + 2\hat{j})$  m. Find the length of rod and the coordinates of the centre of mass.
- Three identical uniform rods of the same mass M and length L are arranged in xy plane as shown in the figure. A fourth uniform rod of mass 3M has been placed as shown in the xy plane. What should be the value of the length of the fourth rod such that the center of mass of all the four rods lie at the origin?



- From a circle of radius a, an isosceles right angled triangle with the hypotenuse as the diameter of the circle is removed. The distance of the centre of gravity of the remaining position from the centre of the circle is
- A man has constructed a toy as shown in figure. If density of the material of the sphere is 12 times of the cone compute the position of the centre of mass. [Centre of mass of a cone of height h is at height of  $h/4$  from its base.]



- The linear mass density of a ladder of length  $\ell$  increases uniformly from one end A to the other end B,
  - Form an expression for linear mass density as a function of distance x from end A where linear mass density  $\lambda_0$ . The density at one end being twice that of the other end.
  - Find the position of the centre of mass from end A.
- Mass centers of a system of three particles of masses 1, 2, 3 kg is at the point (1 m, 2 m, 3 m) and mass center of another group of two particles of masses 2 kg and 3 kg is at point (-1 m, 3 m, -2 m). Where a 5 kg particle should be placed, so that mass center of the system of all these six particles shifts to mass center of the first system?

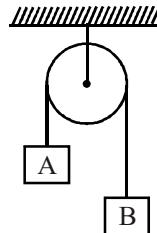
#### HCV Exercises (Chapter No. 9 :- 1 to 6, 11)

HCV Worked out Examples(Chapter No. 9 :- 4,5,6,8,9)

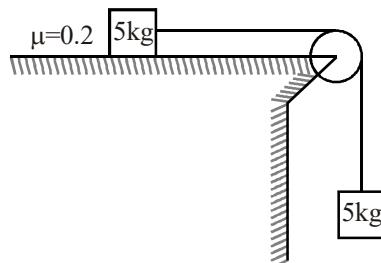
8. The figure shows the positions and velocities of two particles. If the particles move under the mutual attraction of each other, then find the position of centre of mass at  $t = 1$  s.



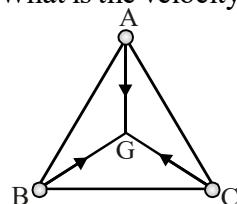
9. In the arrangement shown in the figure,  $m_A = 2\text{ kg}$  and  $m_B = 1\text{ kg}$ . String is light and inextensible. Find the acceleration of centre of mass of both the blocks. Neglect friction everywhere.



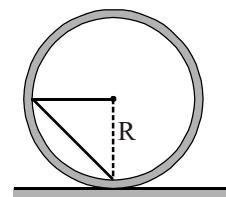
10. The magnitude of acceleration of centre of mass of the system is ( $g = 10 \text{ ms}^{-2}$ )



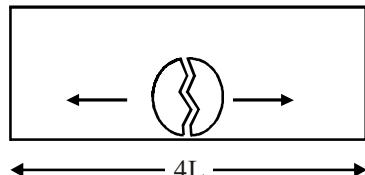
11. Three particles A, B and C of equal mass move with equal speed  $v$  along the medians of an equilateral triangle as shown in fig. They collide at the centroid G of the triangle. After the collision, A comes to rest, B retraces its path with the speed  $v$ . What is the velocity of C ? [JEE - 83]



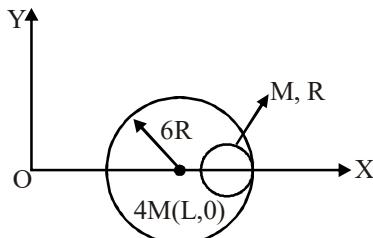
12. Inside a hollow uniform sphere of inner radius  $R$  a uniform rod of length  $R\sqrt{2}$  is released from the state of rest as shown. The mass of the rod is same as that of the sphere. Assume friction to be absent every where. Horizontal displacement of sphere with respect to earth in the time in which the rod becomes horizontal, is



13. A bomb of mass  $3\text{m}$  is kept inside a closed box of mass  $3\text{m}$  and length  $4L$  at its centre. It explodes in two parts of mass  $m$  &  $2m$ . The two parts move in opposite direction and stick to the opposite side of the walls of box. Box is kept on a smooth horizontal surface. What is the distance moved by the box during this time interval.



14. A small sphere of radius  $R$  is held against the inner surface of a larger sphere of radius  $6R$ . The masses of large and small spheres are  $4M$  and  $M$  respectively. This arrangement is placed on a horizontal table. There is no friction between any surfaces of contact. The small sphere is now released. Find the co-ordinates of the centre of the larger sphere when the smaller sphere reaches the other extreme position.

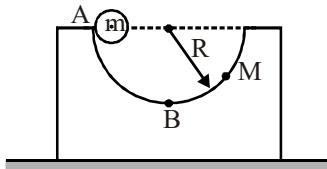


[JEE - 96]

**HCV Exercises (Chapter No. 9 : -7,8,10,12,13)**

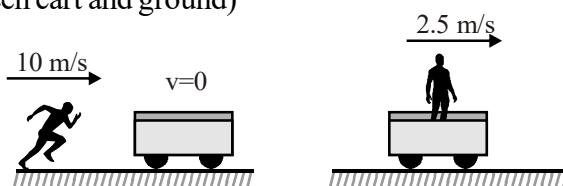
**HCV Worked out Examples(Chapter No. 9 :- 11,12,13,14,15)**

15. A block of mass  $M$  with a semicircular track of radius  $R$ , rests on a horizontal frictionless surface. A uniform cylinder of radius  $r$  and mass  $m$  is released from rest at the top point A (see Fig). The cylinder slips on the semicircular frictionless track. How far has the block moved when the cylinder reaches the bottom (point B) of the track ? How fast is the block moving when the cylinder reaches the bottom of the track?

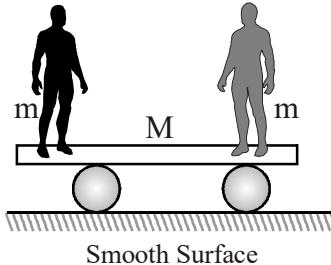


[JEE - 83]

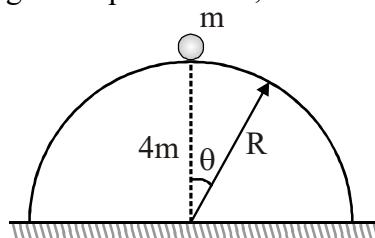
16. Two cars initially at rest are free to move in the  $x$  direction. Car A has mass  $4 \text{ kg}$  and car B has mass  $2 \text{ kg}$ . They are tied together, compressing a spring in between them. When the string holding them together is burned, car A moves off with a speed of  $2 \text{ m/s}$ .
- With what speed does car B leave.
  - How much energy was stored in the spring before it was burned.
17. A  $24 \text{ kg}$  projectile is fired at an angle of  $53^\circ$  above the horizontal with an initial speed of  $50 \text{ m/s}$ . At the highest point in its trajectory, the projectile explodes into two fragments of equal mass, the first of which falls vertically with zero initial speed.
- How far from the point of firing does the second fragment strike the ground? (Assume the ground is level.)
  - How much energy was released during the explosion?
18. A spaceship is moving with constant speed  $v_0$  in gravity free space along  $+Y$ -axis suddenly shoots out one third of its part with speed  $2v_0$  along  $+X$ -axis. Find the speed of the remaining part.
19. A  $50 \text{ kg}$  boy runs at a speed of  $10 \text{ m/s}$  and jumps onto a cart as shown in the figure. The cart is initially at rest. If the speed of the cart with the boy on it is  $2.50 \text{ m/s}$ , what is the mass of the cart? (Assuming friction is absent between cart and ground)



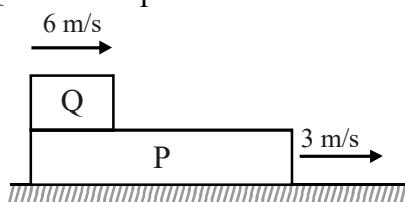
20. Two men of mass  $m$  are standing on the two corners of platform of length ' $L$ ' and mass ' $M$ '. They jump one by one from the platform in either direction with velocity  $u$  with respect to the final state of platform. Then find



- (a) Speed of platform after the first person jumps  
 (b) Speed of the platform after the 2<sup>nd</sup> person jumps to the other side  
 (c) Work done by 1<sup>st</sup> person on the platform after he jumps
21. A hemisphere of radius  $R$  and of mass  $4m$  is free to slide with its base on a smooth horizontal table. A particle of mass  $m$  is placed on the top of the hemisphere. Find the angular velocity of the particle relative to hemisphere as function of angular displacement  $\theta$ , when velocity of hemisphere has become  $v$ .



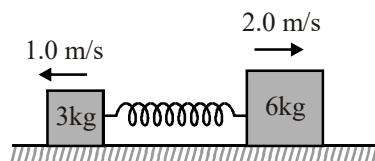
22. A cannon of mass  $5m$  (including a shell of mass  $m$ ) is at rest on a smooth horizontal ground, fires the shell with its barrel at an angle  $\theta$  with the horizontal at a velocity  $u$  relative to cannon. Find the horizontal distance of the point where shell strikes the ground from the initial position of the cannon:
23. A plank  $P$  and block  $Q$  are arranged as shown on a smooth table top. They are given velocities  $3 \text{ m/s}$  and  $6 \text{ m/s}$  respectively. The length of plank is  $1\text{m}$  and block is of negligible size. After some time when the block has reached the other end of plank it stops slipping on plank. Find the coefficient of friction between plank  $P$  and block  $Q$  if mass of plank is double of block).



#### HCV Exercises (Chapter No. 9 :- 14,15,18,19,20,21,22,23,26,28,29,38)

#### HCV Worked out Examples(Chapter No. 9 :- 7,18,19)

24. Two blocks of mass  $3 \text{ kg}$  and  $6 \text{ kg}$  respectively are placed on a smooth horizontal surface. They are connected by a light spring of force constant  $k = 200 \text{ N/m}$ . Initially the spring is unstretched. The indicated velocities are imparted to the blocks. The maximum extension of the spring will be



#### HCV Exercises (Chapter No. 9 :- 41,46,49,50,51,52)

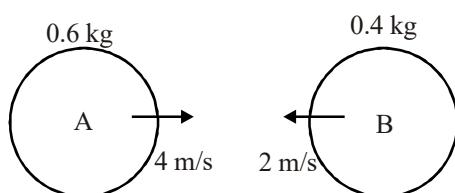
**HCV Worked out Examples(Chapter No. 9 :- 10,17,24)**

25. A bullet of mass  $m$  strikes an obstruction and deviates off at  $60^\circ$  to its original direction. If its speed is also changed from  $u$  to  $v$ , find the magnitude of the impulse acting on the bullet.
26. Bullets of mass 10 g each are fired from a machine gun at rate of 60 bullets/minute. The muzzle velocity of bullets is 100 m/s. The thrust force due to firing bullets experienced by the person holding the gun stationary is \_\_\_\_\_.

**HCV Exercises (Chapter No. 9 :-24,42,57)**

**HCV Worked out Examples(Chapter No. 9 - 16,20,21,22,23)**

27. The velocities of two steel balls before impact are shown. If after head on impact the velocity of ball B is observed to be 3 m/s to the right, the coefficient of restitution is

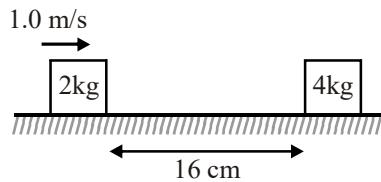


28. Three carts move on a frictionless track with inertias and velocities as shown. The carts collide and stick together after successive collisions.

$m_1 = 2 \text{ kg}$	$m_2 = 1 \text{ kg}$	$m_3 = 2 \text{ kg}$	→ +ve
$v_1 = 1 \text{ m/s}$	$v_2 = 1 \text{ m/s}$	$v_3 = 2 \text{ m/s}$	

- (i) Find loss of mechanical energy when B & C stick together.  
(ii) Find magnitude of impulse experienced by A when it sticks to combined mass (B & C).

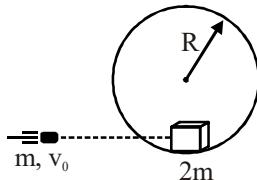
29. The friction coefficient between the horizontal surface and each of the block shown in the figure is 0.2. The collision between the blocks is perfectly elastic. Find the separation (in cm) between them when they come to rest.



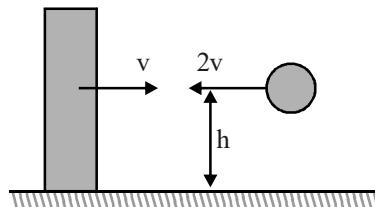
30. A body is thrown vertically upwards from ground with a speed of 10 m/s. If coefficient of restitution of ground,  $e = 1/2$ . Find  
(i) the total distance travelled by the time it almost stops.  
(ii) time elapsed (after the ball has been thrown) when it is at its subsequent maximum height for the third time.

31. A small block of mass  $2m$  initially rests at the bottom of a fixed circular, vertical track, which has a radius of  $R$ . The contact surface between the mass and the loop is frictionless. A bullet of mass  $m$  strikes the block horizontally with initial speed  $v_0$  and remains embedded in the block as the block and the bullet circle the loop. Determine each of the following in terms of  $m$ ,  $v_0$ ,  $R$  and  $g$ .

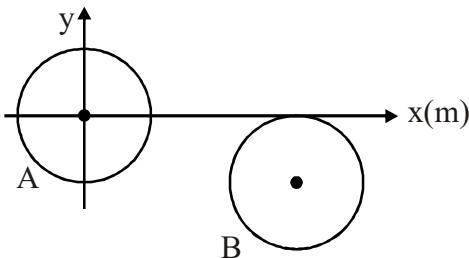
- The speed of the masses immediately after the impact.
- The minimum initial speed of the bullet if the block and the bullet are to successfully execute a complete ride on the loop



32. A ball collides elastically with a massive wall moving towards it with a velocity  $v$ . The collision occurs at a height of  $h$  above ground level and the velocity of the ball just before collision is  $2v$  in horizontal direction as shown in figure. Then the distance from the foot of the wall and the point on the ground where the ball lands (at the instant the ball lands) is :



33. Two smooth balls A and B, each of mass  $m$  and radius  $R$ , have their centres at  $(0,0,R)$  and at  $(5R,-R,R)$  respectively, in a coordinate system as shown. Ball A, moving along positive x axis, collides with ball B. Just before the collision, speed of ball A is  $4 \text{ m/s}$  and ball B is stationary. The collision between the balls is elastic. Find Velocity of the ball A just after the collision and impulse of the force exerted by A on B during the collision.

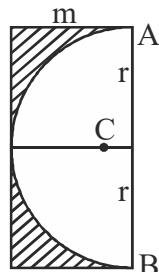


34. A ball of mass  $100 \text{ g}$  is dropped from a height  $h = 5\text{m}$  on an inelastic floor. If the coefficient of restitution is  $e = 0.5$  then. Find
- Time of entire motion in seconds
  - Average speed of the entire motion in  $\text{ms}^{-1}$

**HCV Exercises (Chapter No. 9 :- 25,31,32,33,34,36,37,39,43,44,45,47,48,53-56,58 to 64)**

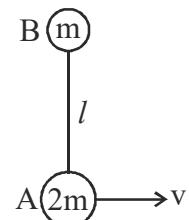
### EXERCISE (S-2)

1. Determine the centre of gravity of a thin homogeneous plate having the form of a rectangle with sides  $r$  and  $2r$  from which a semicircle with a radius  $r$  is cut out of figure.

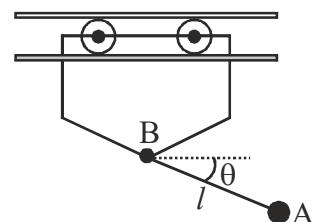


2. Two balls of equal masses are projected upward simultaneously, one from the ground with speed  $50 \text{ m/s}$  and other from a  $40 \text{ m}$  high tower with initial speed  $30 \text{ m/s}$ . Find the maximum height attained by their centre of mass.
3. A man whose mass is  $m \text{ kg}$  jumps vertically into air from a sitting position in which his centre of mass is at a height  $h_1$  from the ground. When his feet are just about to leave the ground his centre of mass is  $h_2$  from the ground and finally rises to  $h_3$  when he is at the top of the jump. What is the average upward force exerted by the ground on him?

4. Two masses  $A$  and  $B$  connected with an inextensible string of length  $\ell$  lie on a smooth horizontal plane.  $A$  is given a velocity of  $v \text{ m/s}$  along the ground perpendicular to line  $AB$  as shown in figure. Find the tension in string during their subsequent motion.

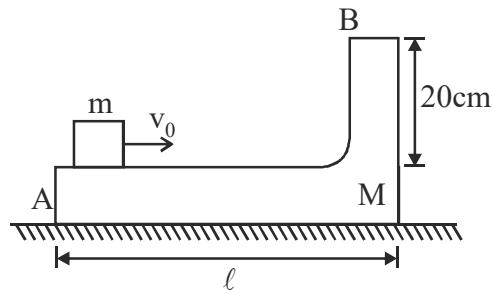


5. The simple pendulum  $A$  of mass  $m_A$  and length  $\ell$  is suspended from the trolley  $B$  of mass  $m_B$ . If the system is released from rest at  $\theta = 0$ , determine the velocity  $v_B$  of the trolley and tension in the string when  $\theta = 90^\circ$ . Friction is negligible.



6. Two bodies of same mass tied with an inelastic string of length  $l$  lie together. One of them is projected vertically upwards with velocity  $\sqrt{6gl}$ . Find the maximum height up to which the centre of mass of system of the two masses rises.
7. Two particles, each of mass  $m$ , are connected by a light inextensible string of length  $2\ell$ . Initially they lie on a smooth horizontal table at points  $A$  and  $B$  distant  $\ell$  apart. The particle at  $A$  is projected across the table with velocity  $u$ . Find the speed with which the second particle begins to move if the direction of  $u$  is :- (i) along  $BA$ . (ii) at an angle of  $120^\circ$  with  $AB$ . (iii) perpendicular to  $AB$ .
- In each case calculate (in terms of  $m$  &  $u$ ) the impulsive tension in the string.

8. A wedge of mass  $M=2m$  rests on a smooth horizontal plane. A small block of mass  $m$  rests over it at left end A as shown in figure. A sharp impulse is applied on the block, due to which it starts moving to the right with velocity  $v_0 = 6 \text{ m/s}$ . At highest point of its trajectory, the block collides with a particle of same mass  $m$  moving vertically downwards with velocity  $v = 2 \text{ m/s}$  and gets stuck with it. If the combined body lands at the end point A of body of mass  $M$ , calculate length  $\ell$ . Neglect friction ( $g = 10 \text{ m/s}^2$ )

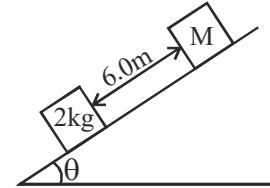


9. Two persons A and B each of mass  $100 \text{ kg}$  are on a frictionless horizontal surface. Person A at rest is holding a block of mass  $50 \text{ kg}$ , suddenly pushes the block with some velocity ( $v$ ) towards the person B approaching at a velocity of  $5 \text{ m/s}$ . B catches the block & slows down. Now the separation between A and B becomes constant. Find the speed  $v$  (in m/s).



10. A massive vertical wall is approaching a man at a speed  $u$ . When it is at a distance of  $10 \text{ m}$ , the man throws a ball with speed  $10 \text{ m/s}$  at an angle of  $37^\circ$  which after completely elastic rebound reaches back directly into his hands. Find the velocity  $u$  of the wall.

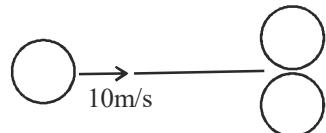
11. Two blocks of mass  $2\text{kg}$  and  $M$  are at rest on an inclined plane and are separated by a distance of  $6\text{m}$  as shown. The coefficient of friction between each of the blocks and the inclined plane is  $0.25$ . The  $2\text{kg}$  block is given a velocity of  $10 \text{ m/s}$  up the inclined plane. It collides with  $M$ , comes back and has a velocity of  $1 \text{ m/s}$  when it reaches its initial position. The other block  $M$  after the collision moves  $0.5 \text{ m}$  up and comes to rest. Calculate the coefficient of restitution between the blocks and the mass of the block  $M$ .



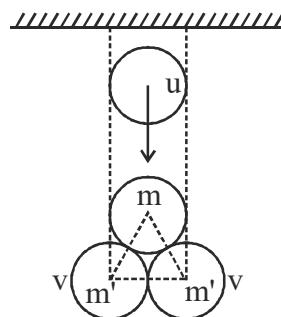
[Take  $\sin\theta \approx \tan\theta = 0.05$  and  $g = 10 \text{ m/s}^2$ ]

[JEE-99]

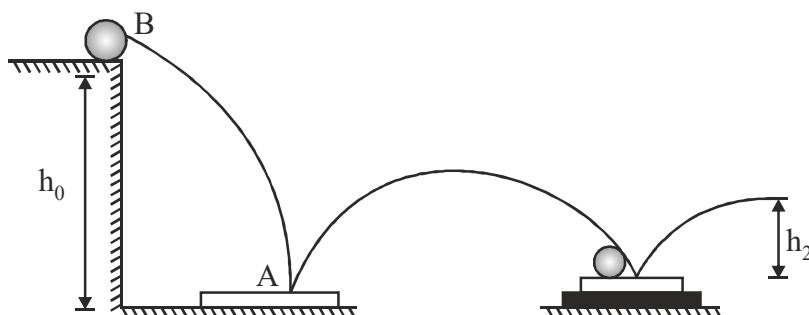
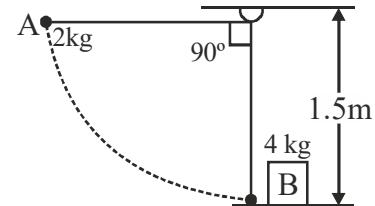
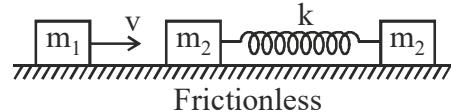
12. A ball with initial speed of  $10 \text{ m/s}$  collides elastically with two other identical ball whose centres are on a line perpendicular to the initial velocity and which are initially in contact with each other. All the three ball are lying on a smooth horizontal table. The first ball is aimed directly at the contact point of the other two balls. All the balls are smooth. Find the velocities of the three balls after the collision.



13. Two equal spheres of mass  $m'$  are suspended by vertical strings so that they are in contact with their centers at same level. A third equal spheres of mass  $m$  falls vertically and strikes elastically the other two simultaneously so that their centres at the instant of impact form an equilateral triangle in a vertical plane. If  $u$  is the velocity of  $m$  just before impact, find the velocities just after impact and the impulse of tension of the strings.

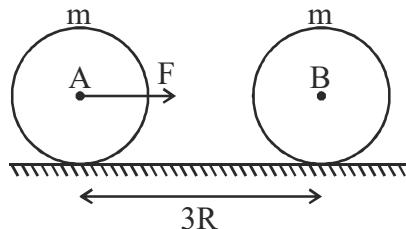


14. Mass  $m_1$  hits & sticks with  $m_2$  while sliding horizontally with velocity  $v$  along the common line of centres of the three equal masses ( $m_1 = m_2 = m_3 = m$ ). Initially masses  $m_2$  and  $m_3$  are stationary and the spring is unstretched. Find the
- velocities of  $m_1$ ,  $m_2$  and  $m_3$  immediately after impact.
  - maximum kinetic energy of  $m_3$ .
  - minimum kinetic energy of  $m_2$ .
  - maximum compression of the spring.
15. A sphere of mass  $m$  is moving with a velocity  $4\hat{i} + 3\hat{j} - 5\hat{k}$  when it hits a smooth wall and rebounds with velocity  $2\hat{i} + 2\hat{j} + 3\hat{k}$ . Find the impulse it receives. Find also the coefficient of restitution between the sphere and the wall.
16. A sphere A is released from rest in the position shown and strikes the block B which is at rest. If  $e = 0.75$  between A and B and  $\mu_k = 0.5$  between B and the support, determine
- the velocity of A just after the impact
  - the maximum displacement of B after the impact.
17. Two particles A and B of mass  $2m$  and  $m$  respectively are attached to the ends of a light inextensible string of length  $4a$  which passes over a small smooth peg at a height  $3a$  from an inelastic table. The system is released from rest with each particle at a height  $a$  from the table. Find
- The speed of B when A strikes the table.
  - The time that elapses before A first hits the table.
  - The time for which A is resting on the table after the first collision & before it is first jerked off.
18. A cylindrical solid of mass  $10^{-2}$  kg and cross-sectional area  $10^{-4}$  m<sup>2</sup> is moving parallel to its axis (the x-axis) with a uniform speed of  $10^3$  m/s in the positive direction. At  $t = 0$ , its front face passes the plane  $x = 0$ . The region to the right of this plane is filled with the dust particle of uniform density  $10^{-3}$  kg/m<sup>3</sup>. When a dust particles collides with the face of the cylinder, it sticks to its surface. Assuming that the dimensions of the cylinder remain practically unchanged and that the dust sticks only to the front face of the cylinder find the x-coordinate of the front of the cylinder at  $t = 150$  s. [JEE-98]
19. A 70 g ball B dropped from a height  $h_0 = 9$  m reaches a height  $h_2 = 0.25$  m after bouncing twice from identical 210 g plates. Plate A rests directly on hard ground, while plate C rests on a foam-rubber mat. Determine.

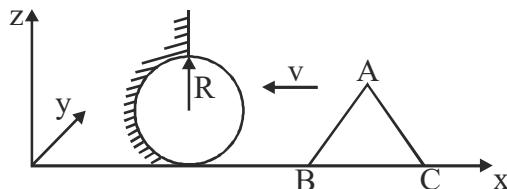


- the coefficient of restitution between the ball and the plates.
- the height  $h_1$  of the ball's first bounce.

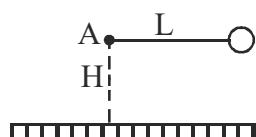
20. Two identical spheres A & B of mass  $50 \text{ kg}$  and radius  $1 \text{ m}$  each lie on a smooth horizontal surface as shown in figure. The initial distance between their centres is  $3 \text{ m}$ . Starting from  $t = 0$  a constant force  $F = 100 \text{ N}$  acts on the sphere A towards right. The coefficient of restitution is  $e = 0.5$ . Find the time interval (in sec) between the first collision and the instant after which collision between both the spheres cease.



21. A uniform wedge of mass  $m$  and triangular cross-section ( $AB = BC = CA = 2R$ ) is moving with a constant velocity  $-v\hat{i}$  towards a sphere of radius  $R$  fixed on a smooth horizontal table as shown in the figure. The wedge makes an elastic collision with the fixed sphere and returns along the same path without any rotation. Neglect all friction and suppose that the wedge remains in contact with the sphere for a time  $\Delta t$ . Find the average force  $\bar{F}$  and also the average normal reaction  $\vec{N}$  exerted by the table on the wedge during the time  $\Delta t$ .

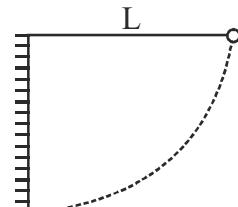


22. A is a fixed point at a height  $H$  above a perfectly inelastic smooth horizontal plane. A light inextensible string of length  $L (> H)$  has one end attached to A and other is to a heavy particle. The particle is held at the level of A with string just taut and released from rest. Find the height of the particle above the plane when it is next instantaneously at rest.

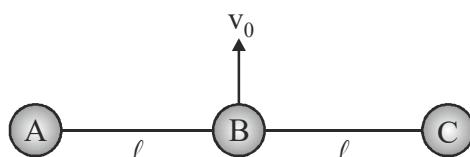


23. A simple pendulum is suspended from a peg on a vertical wall. The pendulum is pulled away from the wall to a horizontal position (see figure) and released. The ball hits the wall, the coefficient of restitution being

$\frac{2}{\sqrt{5}}$ . What is the minimum number of collisions after which the amplitude of oscillations becomes less than  $60^\circ$ . [JEE-87]

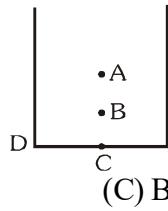


24. Three identical balls each of mass  $m = 0.5 \text{ kg}$  are connected with each other as shown in figure and rest over a smooth horizontal table. At moment  $t = 0$ , ball B is imparted a horizontal velocity  $v_0 = 9 \text{ ms}^{-1}$ . Calculate velocity of A just before it collides with ball C.



### EXERCISE (O-1)

1. A thick uniform wire is bent into the shape of the letter "U" as shown. Which point indicates the location of the center of mass of this wire? A is the midpoint of the line joining mid points of two parallel sides of 'U' shaped wire.



(A) D

(B) A

(C) B

(D) C

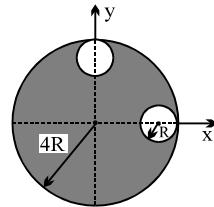
2. From the circular disc of radius  $4R$  two small disc of radius  $R$  are cut off. The centre of mass of the new structure will be:

$$(A) i \frac{R}{5} + j \frac{R}{5}$$

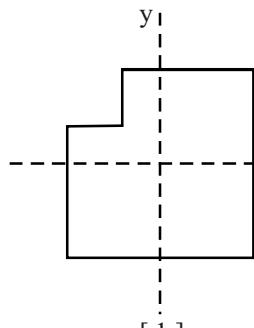
$$(B) -i \frac{R}{5} + j \frac{R}{5}$$

$$(C) \frac{-3R}{14}(\hat{i} + \hat{j})$$

(D) None of these

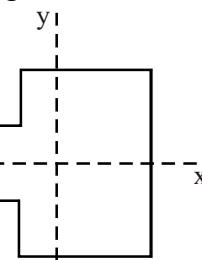


3. A machinist starts with three identical square plates but cuts one corner from one of them, two corners from the second, and three corners from the third. Rank the three plates according to the x-coordinate of their centers of mass, from smallest to largest.

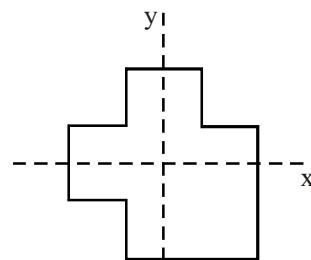


(A) 3, 1, 2

(B) 1, 3, 2

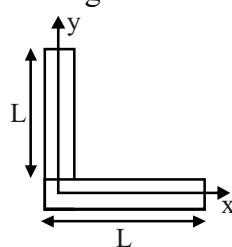


[2] (C) 3, 2, 1



[3] (D) 1 and 3 tie, then 2

4. Centre of mass of two thin uniform rods of same length but made up of different materials & kept as shown, can be, if the meeting point is the origin of co-ordinates



(A)  $(L/2, L/2)$

(B)  $(2L/3, L/2)$

(C)  $(L/3, L/3)$

(D)  $(L/3, L/6)$

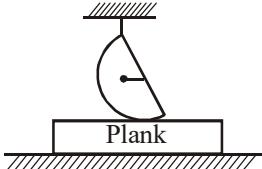
5. Seven identical birds are flying south together at constant velocity. A hunter shoots one of them, which immediately dies and falls to the ground. The other six continue flying south at the original velocity. After the one bird has hit the ground, the centre of mass of all seven birds

(A) continues south at the original speed, but is now located some distance behind the flying birds

(B) continues south, but at  $6/7$  the original velocity

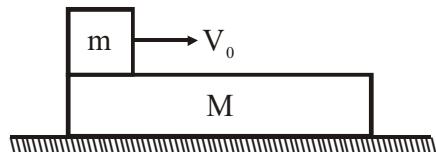
(C) continues south, but at  $1/7$  the original velocity

(D) stops with the dead bird

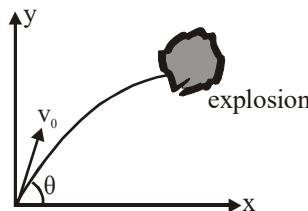
6. A man weighing 80 kg is standing at the centre of a flat boat and he is 20 m from the shore. He walks 8 m on the boat towards the shore and then halts. The boat weight 200 kg. How far is he from the shore at the end of this time?
- (A) 11.2 m      (B) 13.8 m      (C) 14.3 m      (D) 15.4 m
7. There are some passengers inside a stationary railway compartment. The track is frictionless. The centre of mass of the compartment itself (without the passengers) is  $C_1$ , while the centre of mass of the 'compartment plus passengers' system is  $C_2$ . If the passengers move about inside the compartment along the track.
- (A) both  $C_1$  and  $C_2$  will move with respect to the ground.  
 (B) neither  $C_1$  nor  $C_2$  will move with respect to the ground.  
 (C)  $C_1$  will move but  $C_2$  will be stationary with respect to the ground.  
 (D)  $C_2$  will move but  $C_1$  will be stationary with respect to the ground.
8. A non-zero external force acts on a system of particles. The velocity and acceleration of the centre of mass are found to be  $v_0$  and  $a_c$  respectively at any instant t. It is possible that
- (i)  $v_0=0, a_c=0$       (ii)  $v_0 \neq 0, a_c=0$       (iii)  $v_0=0, a_c \neq 0$       (iv)  $v_0 \neq 0, a_c \neq 0$
- Then
- (A) (iii) and (iv) are true.      (B) (i) and (ii) are true.  
 (C) (i) and (iii) are true.      (D) (ii), (iii) and (iv) are true.
9. **Lower surface of a plank is rough and lying at rest on a rough horizontal surface. Upper surface of the plank is smooth and has a smooth hemisphere placed over it through a light string as shown in the figure. After the string is burnt, trajectory of centre of mass of the sphere is :-**
- 
- A a circle      B an ellipse      C a straight line      D a parabola**
10. Three interacting particles of masses 100 g, 200 g and 400 g each have a velocity of 20 m/s magnitude along the positive direction of x-axis, y-axis and z-axis. Due to force of interaction the third particle stops moving. The velocity of the second particle is  $(10\hat{i} + 5\hat{k})$ . What is the velocity of the first particle?
- (A)  $20\hat{i} + 20\hat{j} + 70\hat{k}$       (B)  $10\hat{i} + 20\hat{j} + 8\hat{k}$       (C)  $30\hat{i} + 10\hat{j} + 7\hat{k}$       (D)  $15\hat{i} + 5\hat{j} + 60\hat{k}$
11. A system of N particles is free from any external forces.
- (i) Which of the following is true for the magnitude of the total momentum of the system?
- (A) It must be zero  
 (B) It could be non-zero, but it must be constant  
 (C) It could be non-zero, and it might not be constant  
 (D) The answer depends on the nature of the internal forces in the system
- (ii) Which of the following must be true for the sum of the magnitudes of the momenta of the individual particles in the system?
- (A) It must be zero      (B) It could be non-zero, but it must be constant  
 (C) It could be non-zero, and it might not be constant  
 (D) It could be zero, even if the magnitude of the total momentum is not zero

12. The coefficient of friction between the block and plank is  $\mu$  and ground is smooth. The value of  $\mu$  is such that block becomes stationary with respect to plank before it reaches the other end. Then which of the following statement is **incorrect**.

- (A) The work done by friction on the block is negative.
- (B) The work done by friction on the plank is positive.
- (C) The net work done by friction is negative.
- (D) Net work done by the friction is zero.



13. **A projectile is projected in x-y plane with velocity  $v_0$ . At top most point of its trajectory projectile explodes into two identical fragments. Both the fragments land simultaneously on ground and stick there. Taking point of projection as origin and R as range of projectile if explosion had not taken place. Which of the following can not be position vectors of two pieces, when they land on ground.**



- A**  $\frac{R}{2}\hat{i}, \frac{3R}{2}\hat{i}$
- B**  $0\hat{i}, 2R\hat{i}$
- C**  $R\hat{i} - R\hat{k}, R\hat{i} + R\hat{k}$
- D**  $2R\hat{i} + \frac{R}{2}\hat{k}, R\hat{i} - \frac{R}{2}\hat{k}$

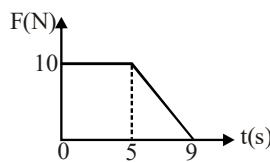
14. Ball A of mass 5.0 kilograms moving at 20 m/s collides with ball B of unknown mass moving at 10 m/s in the same direction. After the collision, ball A moves at 10 m/s and ball B at 15 m/s, both still in the same direction. What is the mass of ball B?

- (A) 6.0 kg
- (B) 10 kg
- (C) 2.0 kg
- (D) 12 kg

15. A boy hits a baseball with a bat and imparts an impulse  $J$  to the ball. The boy hits the ball again with the same force, except that the ball and the bat are in contact for twice the amount of time as in the first hit. The new impulse equals:

- (A) half the original impulse
- (B) the original impulse
- (C) twice the original impulse
- (D) four times the original impulse

16. A body of mass 4 kg is acted on by a force which varies as shown in the graph below. The momentum acquired is



- (A) 280 N-s
- (B) 140 N-s
- (C) 70 N-s
- (D) 210 N-s

17. Two balls of same mass are dropped from the same height  $h$ , on to the floor. The first ball bounces to a height  $h/4$ , after the collision & the second ball to a height  $h/16$ . The impulse applied by the first & second ball on the floor are  $I_1$  and  $I_2$  respectively. Then

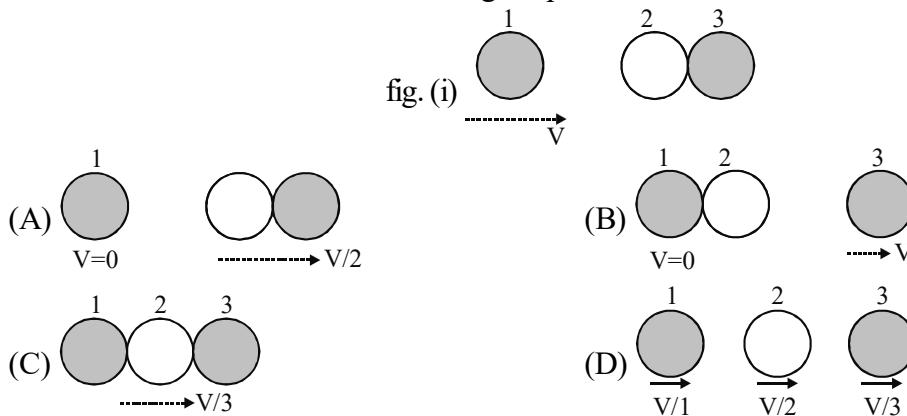
- (A)  $5I_1 = 6I_2$
- (B)  $6I_1 = 5I_2$
- (C)  $I_1 = 2I_2$
- (D)  $2I_1 = I_2$

18. A smooth small spherical ball of mass  $m$ , moving with velocity  $u$  collides head on with another small spherical ball of mass  $m$ , which was initially at rest. One-third of the initial kinetic energy of the system is lost. The coefficient of restitution between the spheres is

- (A)  $\frac{1}{3}$
- (B)  $\frac{1}{\sqrt{3}}$
- (C)  $\frac{1}{2}$
- (D) zero

19. A ball strikes a smooth horizontal ground at an angle of  $45^\circ$  with the vertical. What cannot be the possible angle of its velocity with the vertical after the collision. (Assume  $e \leq 1$ ).  
 (A)  $45^\circ$       (B)  $30^\circ$       (C)  $53^\circ$       (D)  $60^\circ$

20. Two identical ball bearings in contact with each other and resting on a frictionless table are hit head-on by another ball bearing of the same mass moving initially with a speed  $V$  as shown in figure (i). If the collision is elastic, which of the following is a possible result after collision?

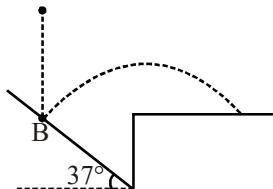


21. A ball is projected from ground with a velocity  $V$  at an angle  $\theta$  to the vertical. On its path it makes an elastic collision with a vertical wall and returns to ground. The total time of flight of the ball is

(A)  $\frac{2v \sin \theta}{g}$       (B)  $\frac{2v \cos \theta}{g}$       (C)  $\frac{v \sin 2\theta}{g}$       (D)  $\frac{v \cos \theta}{g}$

22. A ball is thrown downwards with initial speed = 6 m/s, from a point at height = 3.2 m above a horizontal floor. If the ball rebounds back to the same height then coefficient of restitution equals to  
 (A) 1/2      (B) 0.75      (C) 0.8      (D) None

23. A small ball is dropped from a height  $h$  onto a rigid frictionless plate at B and rebounds. Find the maximum height reached after rebound if the coefficient of restitution between the ball and the plate is  $e = 0.75$ :

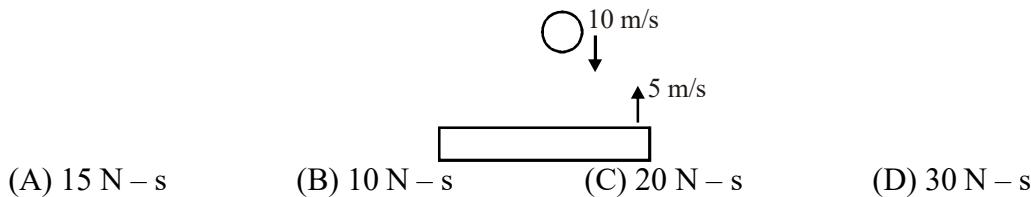


(A) 0      (B)  $3h/25$       (C)  $3h/125$       (D)  $9h/625$

24. A particle is projected from a smooth horizontal surface with velocity  $v$  at an angle  $\theta$  from horizontal. Coefficient of restitution between the surface and ball is  $e$ . The distance of the point where ball strikes the surface second time from the point of projection is

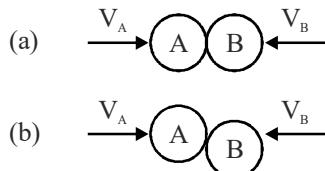
(A)  $\frac{v^2 \sin 2\theta(1+e^2)}{g}$       (B)  $\frac{v^2 \sin 2\theta(1+e^4)}{g}$   
 (C)  $\frac{v^2 \sin 2\theta(1+e^3)}{g}$       (D)  $\frac{v^2 \sin 2\theta(1+e)}{g}$

25. A ball of mass 1kg strikes a heavy platform, elastically, moving upwards with a velocity of 5m/s. The speed of the ball just before the collision is 10m/s downwards. Then the impulse imparted by the platform on the ball is



- (A)  $15 \text{ N-s}$       (B)  $10 \text{ N-s}$       (C)  $20 \text{ N-s}$       (D)  $30 \text{ N-s}$

26. Two bodies, A and B, collide as shown in figures a and b below :



Circle the true statement :

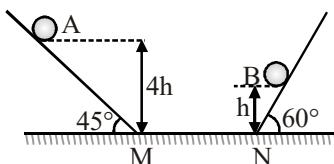
- (A) They exert equal and opposite forces on each other in (a) but not in (b)  
 (B) They exert equal and opposite force on each other in (b) but not in (a)  
 (C) They exert equal and opposite force on each other in both (a) and (b)  
 (D) The forces are equal and opposite to each other in (a), but only the components of the forces parallel to the velocities are equal in (b).

27. A mass ‘ $m$ ’ moves with a velocity ‘ $v$ ’ and collides in elastically with another identical mass at rest. After collision the 1<sup>st</sup> mass moves with velocity  $\frac{v}{\sqrt{3}}$  in a direction perpendicular to the initial direction of motion. Find the speed of the 2<sup>nd</sup> mass after collision

- (A)  $\frac{2v}{\sqrt{3}}$       (B)  $\frac{v}{\sqrt{3}}$       (C)  $v\sqrt{\frac{2}{3}}$

- (D) the situation of the problem is not possible without external impulse

28. Two identical balls are released from positions as shown. They collide elastically on horizontal surface. Ratio of heights attained by A & B after collision is (All surfaces are smooth, neglect energy loss at M & N)

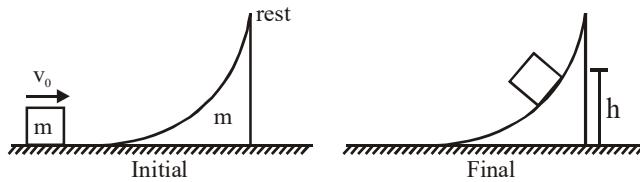


- (A)  $1 : 4$       (B)  $2 : 1$       (C)  $4 : 13$       (D)  $2 : 5$

#### MULTIPLE CORRECT TYPE QUESTIONS

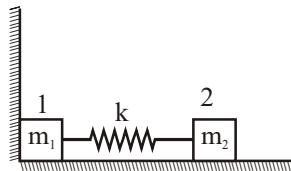
29. Two charges moving under their only own mutual attraction separated by large distance initially. Then choose the correct statement(s)
- (A) If both are free, mechanical energy is conserved.  
 (B) If one is fixed and other is free, mechanical energy is conserved.  
 (C) If one is fixed and other is free, momentum is conserved.  
 (D) If both are free momentum is conserved.

30. In the arrangement shown, horizontal surface is smooth, but friction is present between the block and the surface of the wedge. Block is given velocity  $v_0$  at  $t = 0$ . After achieving height 'h' on the wedge, block comes to rest with respect to wedge at  $t = t_0$ . Then from  $t = 0$  to  $t = t_0$ :



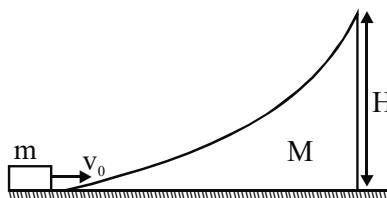
- (A) Work done by friction on the block is negative
- (B) Work done by friction on the wedge is negative
- (C) Work done by block on the wedge is positive
- (D) Work done by wedge on the block is positive

31. Two bars connected by a weightless spring of stiffness  $k$  rest on a smooth horizontal plane as shown in figure. Bar 2 is shifted a small distance  $x$  to the left and then released. The velocity of the centre of mass of the system after bar 1 breaks off the wall is -



$$\text{A } \frac{x\sqrt{m_2k}}{m_1+m_2} \quad \text{B } x\sqrt{\frac{k}{m_1+m_2}} \quad \text{C zero} \quad \text{D } \frac{\sqrt{m_1k}}{m_1+m_2}$$

32. Figure shows a block of mass  $m$  projected with velocity  $v_0$  towards a wedge. Consider all the surfaces to be smooth. Block does not have sufficient energy to negotiate (over come) wedge. Mark the correct option(s)



- (A) when block is at the maximum height on wedge, block and wedge have velocity equal to velocity of centre of mass of block wedge system
- (B) wedge acquires maximum speed with respect to ground when block returns to lowest point on wedge.
- (C) momentum of wedge and block is conserved at all times
- (D) centre of mass of wedge and block remains stationary

#### **COMPREHENSION BASED QUESTIONS**

##### **Paragraph for Question 33 to 35**

2 kg and 3 kg blocks are placed on a smooth horizontal surface and connected by spring which is unstretched initially. The blocks are imparted velocities as shown in the figure.



33. The maximum energy stored in the spring in the subsequent motion will be  
 (A)  $5v_0^2$       (B)  $15v_0^2$       (C) zero      (D)  $10v_0^2$
34. Maximum speed of 3 kg block in the subsequent motion will be  
 (A)  $v_0$       (B)  $2v_0$       (C)  $3v_0$       (D)  $4v_0$

35. Maximum speed of 2 kg block in the subsequent motion will be  
 (A)  $v_0$       (B)  $2v_0$       (C)  $3v_0$       (D)  $4v_0$

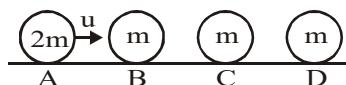
**Paragraph for Question 36 to 37**

A projectile of mass "m" is projected from ground with a speed of 50 m/s at an angle of  $53^\circ$  with the horizontal. It breaks up into two equal parts at the highest point of the trajectory. One particle coming to rest immediately after the explosion.

36. The ratio of the radii of curvatures of the moving particle just before and just after the explosion are:  
 (A) 1 : 4      (B) 1 : 3      (C) 2 : 3      (D) 4 : 9
37. The distance between the pieces of the projectile when they reach the ground are:  
 (A) 240      (B) 360      (C) 120      (D) none

**MATRIX MATCH TYPE QUESTIONS**

38. Four balls A,B,C and D are kept on a smooth horizontal surface as shown in figure. Ball A is given velocity  $u$  towards B

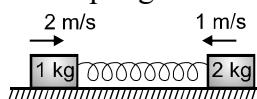


- | <b>Column-I</b>                          | <b>Column-II</b>      |
|--|-----------------------|
| (A) Total impulse of all collisions on A | (P) $\frac{4mu}{9}$   |
| (B) Total impulse of all collisions on B | (Q) $\frac{4mu}{27}$  |
| (C) Total impulse of all collision on C  | (R) $\frac{4mu}{3}$   |
| (D) Total impulse of all collisions on D | (S) $\frac{52}{27}mu$ |
39. In Column-I, 4 situations are depicted and in column-II, 4 possible kinds of collision are listed. Match the situation with type of collision.

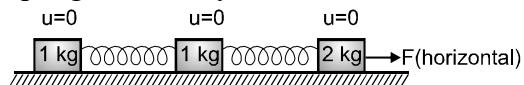
- | <b>Column-I</b> | <b>Column-II</b>              |
|-----------------|-------------------------------|
| <b>Before</b>   | <b>After</b>                  |
| (A)             | (P) Elastic                   |
| (B)             | (Q) Perfectly Inelastic       |
| (C)             | (R) Partially elastic         |
|                 | (S) Collision is not possible |

40. In all cases in **column-I**, the blocks/plank/trolley are placed on the smooth horizontal surface.

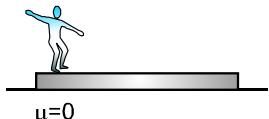
- | <b>Column-I</b>  | <b>Column-II</b>   |
|--|--|
| (A) The initial velocities given to the blocks when spring is relaxed are as shown | (P) Centre of mass of the complete system shown will not move horizontally |



- (B) A constant force is applied on 2 kg block.  
Springs are initially relaxed



- (C) Initially system is at rest. Man starts moving on a large plank with constant velocity.



- (D) A man standing on one of the trolleys (initially at rest) jumps to the other with relative velocity of 4 m/s horizontally



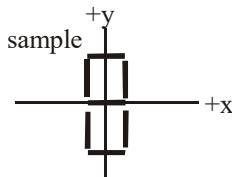
- (Q) Centre of mass of the complete system shown will move horizontally

- (R) Mechanical energy of the system will be conserved

- (S) Mechanical energy of the system will increase

- (T) Linear momentum of the complete system will always remain constant

41. On the left are statements about the location of the center of mass of the objects depicted on the right. The objects on the right are symbols constructed out of sticks of equal length and mass. The location of the center of mass is described using the coordinate system depicted in the sample.



The centre of mass lies at  $x = 0, y = 0$

### **Column I**

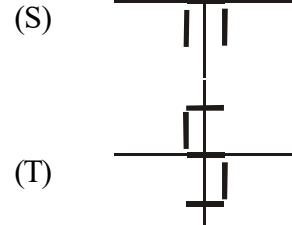
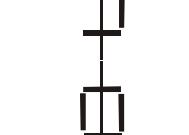
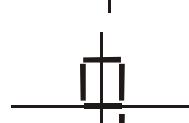
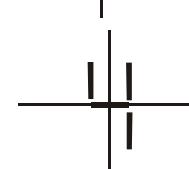
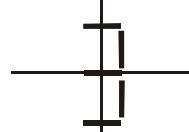
- (A) The center of mass is at  $x > 0$  and  $y = 0$

- (B) The center of mass is at  $x = 0$  and  $y > 0$

- (C) The center of mass is at  $x > 0$  and  $y > 0$

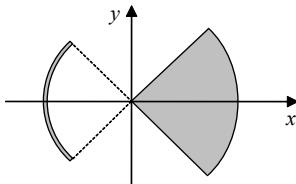
- (D) The center of mass is at  $x = 0$  and  $y = 0$

## Column II



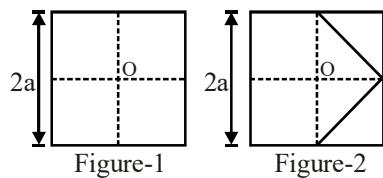
### EXERCISE (O-2)

1. A sector cut from a uniform disk of radius 12 cm and a uniform rod of the same mass bent into shape of an arc are arranged facing each other as shown in the figure. If center of mass of the combination is at the origin, what is the radius of the arc?



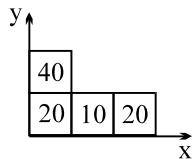
- (A) 8 cm      (B) 9 cm      (C) 12 cm      (D) 18 cm

2. A piece of paper (shown in figure-1) is in form of a square. Two corners of this square are folded to make it appear like figure-2. Both corners are put together at centre of square 'O'. If O is taken to be (0, 0), the centre of mass of new system will be at



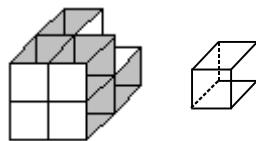
- (A)  $\left(\frac{-a}{8}, 0\right)$       (B)  $\left(\frac{-a}{6}, 0\right)$       (C)  $\left(\frac{a}{12}, 0\right)$       (D)  $\left(\frac{-a}{12}, 0\right)$

3. Four cubes of side "a" each of mass 40gm, 20gm, 10gm and 20gm are arranged in XY plane as shown in figure. The coordinates of centre of mass of the combination with respect to O, are :



- (A)  $19a/18, 17a/18$       (B)  $17a/18, 11a/18$       (C)  $17a/18, 13a/18$       (D)  $13a/18, 17a/18$

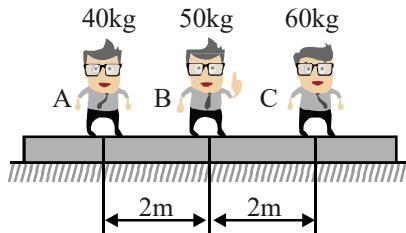
4. Eight solid uniform cubes of edge  $l$  are stacked together to form a single cube with center O. One cube is removed from this system. Distance of the centre of mass of remaining 7 cubes from O is



- (A)  $\frac{7\sqrt{3}l}{16}$       (B)  $\frac{\sqrt{3}l}{16}$       (C)  $\frac{\sqrt{3}l}{14}$       (D) zero

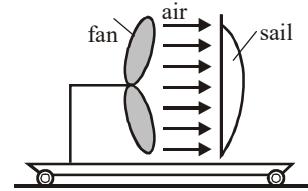
5. An isolated particle of mass  $m$  is moving in horizontal plane ( $x-y$ ), along the  $x$ -axis, at a certain height above the ground. It suddenly explodes into two fragment of masses  $\frac{m}{4}$  and  $\frac{3m}{4}$ . An instant later, the smaller fragment is at  $y=+15$  cm. The larger fragment at this instant is at :  
 (A)  $y=-5$  cm      (B)  $y=+20$  cm      (C)  $y=+5$  cm      (D)  $y=-20$  cm

6. Three man A, B & C of mass 40 kg, 50 kg & 60 kg are standing on a plank of mass 90 kg, which is kept on a smooth horizontal plane. If A & C exchange their positions then mass B will shift





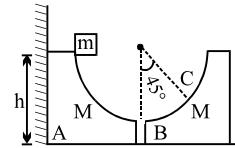
7. A fan and a sail are mounted vertically on a cart that is initially at rest on a horizontal table as shown in the diagram. When the fan is turned on, an air stream is blown towards the right and is incident on the sail. The cart is free to move with negligible resistance forces. After the fan has been turned on the cart will



- (A) move to the right and then to the left      (B) remain at rest  
(C) move towards the right      (D) move towards the left

8. Two identical blocks having mass  $M$  each are smoothly conjugated and placed on a smooth horizontal floor as shown in figure. On the left of block A there is a wall. A small block of mass  $m$  is released from the position. Then velocity of block B is maximum

- (A) when m is at highest position on B  
 (B) when m is at lowest position and moving towards left w.r.t B  
 (C) when m is at point C  
 (D) is equal to  $\frac{m\sqrt{2gh}}{m+M}$



9. Two identical carts constrained to move on a straight line, on which sit two twins of same mass, are moving with same velocity. At some time snow begins to drop uniformly vertically downward. Ram, sitting on one of the trolleys, throws off the falling snow sideways and in the second cart shyam is asleep. (Assume that friction is absent)

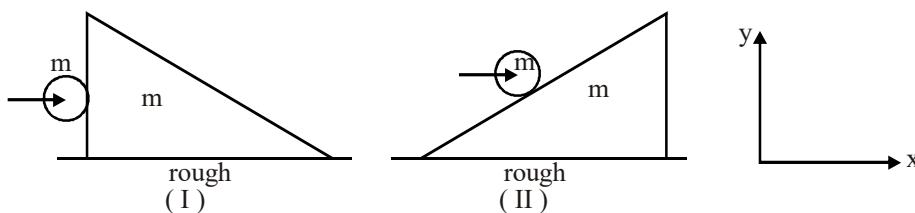
- (A) Cart carrying Ram will speed up while cart carrying shyam will slow down
  - (B) Cart carrying Ram will remain at the same speed while cart carrying shyam will slow down
  - (C) Cart carrying Ram will speed up while cart carrying shyam will remain at the same speed
  - (D) Cart carrying Ram as well as shyam will slow down

- 10.** Figure shows a block A of mass 5 kg kept at rest on a horizontal smooth surface. A spring ( $K = 200 \text{ N/m}$ ) which is compressed by 10 cm and tied with the help of a string to maintain the compression is attached to block A as shown in figure. Block B also of mass 5 kg moving with 2 m/s collides with A, as shown. During the collision the string breaks and after the collision the spring is in its natural state. Assume the bodies to be elastic and let the velocities of A and B be  $v_1$  and  $v_2$  respectively assuming positive direction towards right, after collision. Then

- (A)  $v_1 + v_2 > 2$   
 (B) Initial kinetic energy of system = final kinetic energy of system  
 (C)  $v_1^2 + v_2^2 = 4.4 \text{ (m/s)}^2$   
 (D)  $v_1 - v_2 = 2$



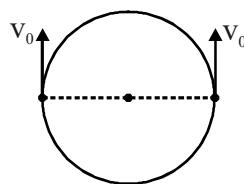
11. An open water tight railway wagon of mass  $5 \times 10^3$  kg coasts at an initial velocity 1.2 m/s without friction on a railway track. Rain drops fall vertically downwards into the wagon. The velocity of the wagon after it has collected  $10^3$  kg of water will be  
 (A) 0.5 m/s      (B) 2 m/s      (C) 1 m/s      (D) 1.5 m/s
12. A rocket of mass 4000 kg is set for vertical firing. How much gas must be ejected per second so that the rocket may have initial upwards acceleration of magnitude  $19.6 \text{ m/s}^2$ . [Exhaust speed of fuel = 980 m/s.]  
 (A)  $240 \text{ kg s}^{-1}$       (B)  $60 \text{ kg s}^{-1}$       (C)  $120 \text{ kg s}^{-1}$       (D) None
13. A ball of mass  $m$  collides horizontally with a stationary wedge on a rough horizontal surface, in the two orientations as shown. Neglect friction between ball and wedge. Two student comment on system of ball and wedge in these situations



**Saurav :** Momentum of system in x-direction will change by significant amount in both cases.

**Rahul :** There are no impulsive external forces in y-direction in both cases hence the total momentum of system in y-direction can be treated as conserved in both cases.

- (A) **Saurav** is wrong and **Rahul** is correct      (B) **Saurav** is correct and **Rahul** is wrong  
 (C) Both are correct      (D) Both are wrong
14. Two particles of mass  $m$ , constrained to move along the circumference of a smooth circular hoop of equal mass  $m$ , are initially located at opposite ends of a diameter and given equal velocities  $v_0$  shown in the figure. The entire arrangement is located in gravity free space. Their velocity just before collision is



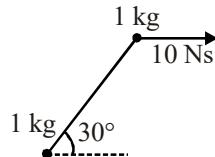
(A)  $\frac{1}{\sqrt{3}}v_0$       (B)  $\frac{\sqrt{3}}{2}v_0$       (C)  $\frac{2}{\sqrt{3}}v_0$       (D)  $\frac{\sqrt{7}}{3}v_0$

15. A uniform rope of linear mass density  $\lambda$  and length  $\ell$  is coiled on a smooth horizontal surface. One end is pulled up with constant velocity  $v$ . Then the average power applied by the external agent in pulling the entire rope just off the ground is :



(A)  $\frac{1}{2}\lambda\ell v^2 + \frac{\lambda\ell^2 g}{2}$       (B)  $\lambda\ell gv$       (C)  $\frac{1}{2}\lambda v^3 + \frac{\lambda\ell gv}{2}$       (D)  $\lambda\ell gv + \frac{1}{2}\lambda v^3$

16. Two balls of masses 1 kg each are connected by an inextensible massless string. The system is resting on a smooth horizontal surface. An impulse of 10 Ns is applied to one of the balls at an angle  $30^\circ$  with the line joining two balls in horizontal direction as shown in the figure. Assuming that the string remains taut after the impulse, the magnitude of impulse of tension is :

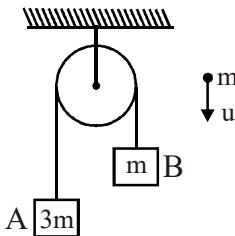


- (A) 6Ns      (B)  $\frac{5}{2}\sqrt{3}$ Ns      (C) 5 Ns      (D)  $\frac{5}{\sqrt{3}}$  Ns

17. A force exerts an impulse  $I$  on a particle changing its speed from  $u$  to  $2u$ . The applied force and the initial velocity are oppositely directed along the same line. The work done by the force is

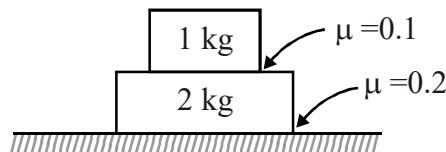
- (A)  $\frac{3}{2} I u$       (B)  $\frac{1}{2} I u$       (C)  $I u$       (D)  $2 I u$

18. A system of two blocks A and B are connected by an inextensible massless strings as shown. The pulley is massless and frictionless. Initially the system is at rest when, a bullet of mass 'm' moving with a velocity 'u' as shown hits the block 'B' and gets embedded into it. The impulse imparted by tension force to the block of mass  $3m$  is



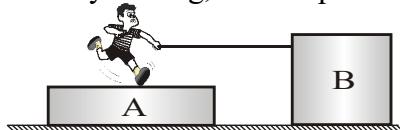
- (A)  $\frac{5mu}{4}$       (B)  $\frac{4mu}{5}$       (C)  $\frac{2mu}{5}$       (D)  $\frac{3mu}{5}$

19. If both the blocks as shown in the given arrangement are given together a horizontal velocity towards right. If  $a_{cm}$  be the subsequent acceleration of the centre of mass of the system of blocks then  $a_{cm}$  equals



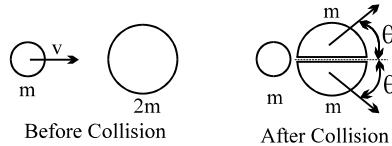
- (A)  $0 \text{ m/s}^2$       (B)  $\frac{5}{3} \text{ m/s}^2$       (C)  $\frac{7}{3} \text{ m/s}^2$       (D)  $2 \text{ m/s}^2$

20. In adjacent figure a boy, on a horizontal platform A, kept on a smooth horizontal surface, holds a rope attached to a box B. Boy pulls the rope with a constant force of 50N. The coefficient of friction between boy and platform is 0.5. (Mass of boy = 80 kg, mass of platform = 120kg and mass of box = 100 kg)

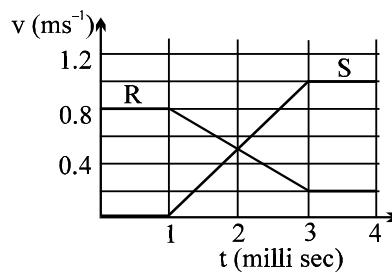


- (A) Velocity of platform relative to box after 4 sec. is 2m/s  
 (B) Velocity of boy relative to platform after 4sec is 2m/s  
 (C) Friction force between boy and platform is 30N  
 (D) Friction force between boy and platform is 50N

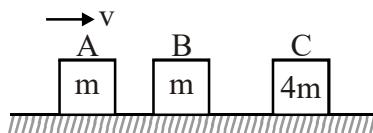
21. A particle of mass  $m$  is moving along the  $x$ -axis with speed  $v$  when it collides with a particle of mass  $2m$  initially at rest. After the collision, the first particle has come to rest, and the second particle has split into two equal-mass pieces that move at equal angles  $\theta > 0$  with the  $x$ -axis, as shown in the figure. Which of the following statements correctly describes the speeds of the two pieces?



- (A) Each piece moves with speed  $v$
  - (B) One of the pieces moves with speed  $v$ , the other moves with speed less than  $v$
  - (C) Each piece moves with speed  $v/2$
  - (D) Each piece moves with speed greater than  $v/2$
22. The diagram shows the velocity-time graph for two masses R and S that collided elastically. Which of the following statements is true?



- (I) R and S moved in the same direction after the collision.
  - (II) Kinetic energy of the system (R & S) is minimum at  $t = 2$  milli sec.
  - (III) The mass of R was greater than mass of S.
- (A) I only
  - (B) II only
  - (C) I and II only
  - (D) I, II and III
23. Three blocks are initially placed as shown in the figure. Block A has mass  $m$  and initial velocity  $v$  to the right. Block B with mass  $m$  and block C with mass  $4m$  are both initially at rest. Neglect friction. All collisions are elastic. The final velocity of block A is



- (A)  $0.6v$  to the left
  - (B)  $1.4v$  to the left
  - (C)  $v$  to the left
  - (D)  $0.4v$  to the right
24. Two billiard balls undergo a head-on collision. Ball 1 is twice as heavy as ball 2. Initially, ball 1 moves with a speed  $v$  towards ball 2 which is at rest. Immediately after the collision, ball 1 travels at a speed of  $v/3$  in the same direction. What type of collision has occurred?
- (A) inelastic
  - (B) elastic
  - (C) completely inelastic
  - (D) cannot be determined from the information given

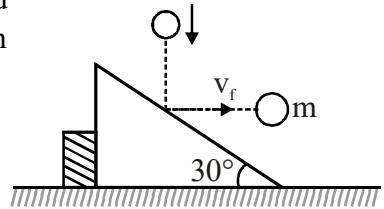
25. As shown in the figure a body of mass  $m$  moving vertically with speed  $3 \text{ m/s}$  hits a smooth fixed inclined plane and rebounds with a velocity  $v_f$  in the horizontal direction. If  $\angle$  of inclined is  $30^\circ$ , the velocity  $v_f$  will be

(A) 3 m/s

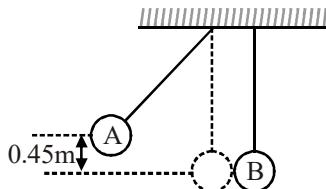
$$(B) \sqrt{3} \text{ m/s}$$

$$(C) \frac{1}{\sqrt{3}} \text{ m/s}$$

(D) this is not possible

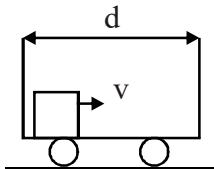


26. Two massless string of length 5 m hang from the ceiling very near to each other as shown in the figure. Two balls A and B of masses 0.25 kg and 0.5 kg are attached to the string. The ball A is released from rest at a height 0.45 m as shown in the figure. The collision between two balls is completely elastic. Immediately after the collision, the kinetic energy of ball B is 1 J. The velocity of ball A just after the collision is



(A)  $5 \text{ ms}^{-1}$  to the right (B)  $5 \text{ ms}^{-1}$  to the left (C)  $1 \text{ ms}^{-1}$  to the right (D)  $1 \text{ ms}^{-1}$  to the left

27. In a smooth stationary cart of length  $d$ , a small block is projected along its length with velocity  $v$  towards front. Coefficient of restitution for each collision is  $e$ . The cart rests on a smooth ground and can move freely. The time taken by block to come to rest w.r.t. cart is



$$(A) \frac{ed}{(1-e)v}$$

$$(B) \frac{ed}{(1+e)v}$$

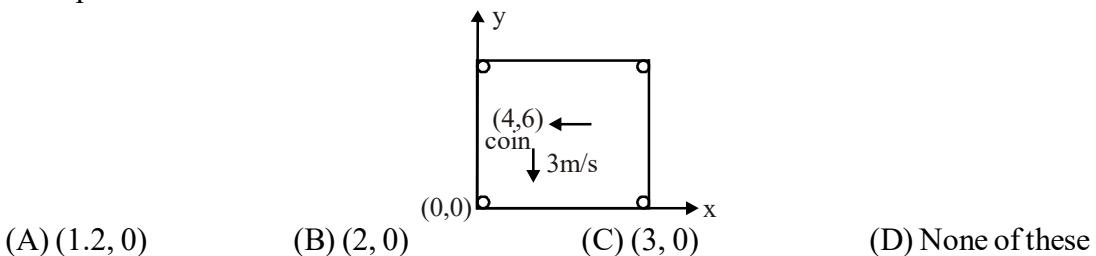
(C)  $\frac{d}{e}$

(D) infinite

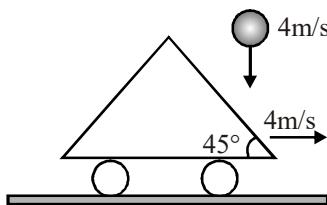
- 28.** A smooth sphere is moving on a horizontal surface with a velocity vector  $(2\hat{i} + 2\hat{j})$  m/s immediately before it hit a vertical wall. The wall is parallel to vector  $\hat{j}$  and coefficient of restitution between the sphere and the wall is  $e = 1/2$ . The velocity of the sphere after it hits the wall is

(A)  $\hat{i} - \hat{j}$       (B)  $-\hat{i} + 2\hat{j}$       (C)  $-\hat{i} - \hat{j}$       (D)  $2\hat{i} - \hat{j}$

- 29.** On a smooth carom board, a coin moving in negative y-direction with a speed of 3 m/s is being hit at the point (4, 6) by a striker moving along negative x-axis. The line joining centres of the coin and the striker just before the collision is parallel to x-axis. After collision the coin goes into the hole located at the origin. Masses of the striker and the coin are equal. Considering the collision to be elastic, the initial and final speeds of the striker in m/s will be

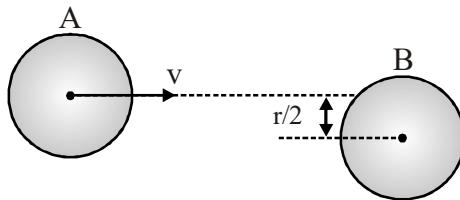


30. A small ball falling vertically downward with constant velocity 4m/s strikes elastically a massive inclined cart moving with velocity 4m/s horizontally as shown. The velocity of the rebound of the ball is



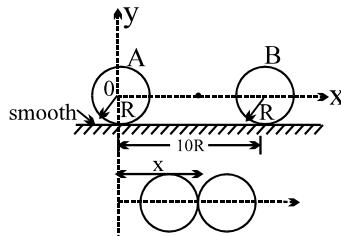
- (A)  $4\sqrt{2}$  m/s      (B)  $4\sqrt{3}$  m/s      (C) 4 m/s      (D)  $4\sqrt{5}$  m/s

31. A disk A of radius  $r$  moving on perfectly smooth surface at a speed  $v$  undergoes an elastic collision with an identical stationary disk B. Find the velocity of the disk B after collision if the impact parameter is  $r/2$  as shown in the figure.



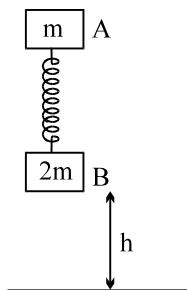
- (A)  $\frac{\sqrt{15}}{4}v$       (B)  $\frac{v}{4}$       (C)  $\frac{v}{2}$       (D)  $\frac{\sqrt{3}v}{2}$

32. Two uniform non conducting balls A & B have identical size having radius  $R$  but made of different density material (density of A = 2 density of B). The ball A is +vely charged & ball B is -vely charged. The balls are released on the horizontal smooth surface at the separation  $10R$  as shown in figure. Because of mutual attraction the balls start moving towards each other. They will collide at a point.



- (A)  $x = \frac{10R}{3}$       (B)  $x = \frac{11R}{3}$       (C)  $x = 5R$       (D)  $x = \frac{7R}{5}$

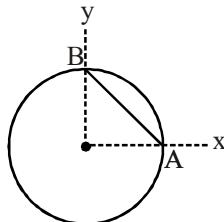
33. From what minimum height  $h$  must the system be released when spring is unstretched so that after perfectly inelastic collision ( $e = 0$ ) with ground, B may be lifted off the ground (Spring constant =  $k$ ).



- (A)  $mg/(4k)$       (B)  $4mg/k$       (C)  $mg/(2k)$       (D) none

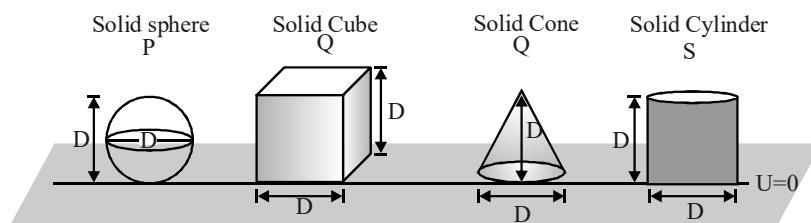
**MULTIPLE CORRECT TYPE QUESTIONS**

34. An object comprises of a uniform ring of radius  $R$  and its uniform chord AB (not necessarily made of the same material) as shown. Which of the following can not be the centre of mass of the object



- (A)  $(R/3, R/3)$       (B)  $(R/3, R/2)$       (C)  $(R/4, R/4)$       (D)  $(R/\sqrt{2}, R/\sqrt{2})$

35. Assuming potential energy 'U' at ground level to be zero.



All objects are made up of same material.

$U_p$  = Potential energy of solid sphere

$U_q$  = Potential energy of solid cube

$U_r$  = Potential energy of solid cone

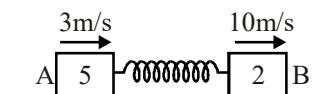
$U_s$  = Potential energy of solid cylinder

- (A)  $U_s > U_p$       (B)  $U_q > U_s$       (C)  $U_p > U_q$       (D)  $U_s > U_r$

36. Two identical balls are interconnected with a massless and inextensible thread of length L. The system is in gravity free space with the thread just taut. Each ball is imparted a velocity  $v$ , one towards the other ball and the other perpendicular to the first, at  $t = 0$ . Then,

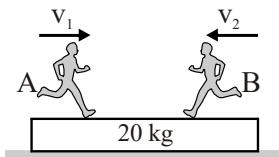
- (A) the thread will become taut at  $t = (L/v)$   
 (B) the thread will become taut at some time  $t < (L/v)$ .  
 (C) the thread will always remain taut for  $t > (L/v)$ .  
 (D) the kinetic energy of the system will always remain  $mv^2$ .

37. Two blocks A (5kg) and B(2kg) attached to the ends of a spring constant  $1120\text{N/m}$  are placed on a smooth horizontal plane with the spring undeformed. Simultaneously velocities of  $3\text{m/s}$  and  $10\text{m/s}$  along the line of the spring in the same direction are imparted to A and B then



- (A) when the extension of the spring is maximum the velocities of A and B are zero.  
 (B) the maximum extension of the spring is  $25\text{ cm}$ .  
 (C) maximum extension and maximum compression occur alternately.  
 (D) the minimum speed of B is 0.

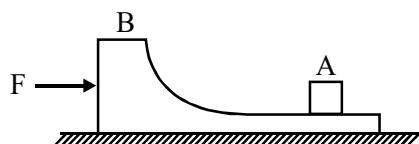
38. In the figure shown the system is at rest initially. Two persons 'A' and 'B' of masses 40 kg each move with speeds  $v_1$  and  $v_2$  respectively towards each other on a plank lying on a smooth horizontal surface as shown in figure. Plank travels a distance of 20 m towards right direction in 5 sec. (Here  $v_1$  and  $v_2$  are given with respect to the plank). Then the possible condition(s) can be



- (A)  $v_1 = 0 \text{ m/s}$ ,  $v_2 = 10 \text{ m/s}$   
 (C)  $v_1 = 10 \text{ m/s}$ ,  $v_2 = 20 \text{ m/s}$

- (B)  $v_1 = 5 \text{ m/s}$ ,  $v_2 = 15 \text{ m/s}$   
 (D)  $v_1 = 2 \text{ m/s}$ ,  $v_2 = 12 \text{ m/s}$

39. Two blocks A & B are placed on a smooth surface as shown, horizontal force F is applied on B when A & B are initially at rest. Friction between A & B is absent (in subsequent motion):



- (A) Kinetic energy of system of A & B increases  
 (B) Momentum of the system of A & B remains constant in frame of center of mass of A & B  
 (C) Kinetic energy of system of A & B remain constant in frame of center of mass of A & B  
 (D) Kinetic energy of A remains constant in B frame

40. A blast breaks a body initially at rest of mass 0.5 kg into three pieces, two smaller pieces of equal mass and the third double the mass of either of small piece. After the blast the two smaller masses move at right angles to one another with equal speed. Find the statements that is/are true for this case assuming that the energy of blast is totally transferred to masses.

- (A) All the three pieces share the energy of blast equally  
 (B) The speed of bigger mass is  $\sqrt{2}$  times the speed of either of the smaller mass  
 (C) The direction of motion of bigger mass makes an angle of  $135^\circ$  with the direction of smaller pieces  
 (D) The bigger piece carries double the energy of either piece.

41. Two balls A and B having masses 1 kg and 2 kg, moving with speeds 21 m/s and 4 m/s respectively in opposite direction, collide head on. After collision A moves with a speed of 1 m/s in the same direction, then correct statements is :

- (A) The velocity of B after collision is 6 m/s opposite to its direction of motion before collision.  
 (B) The coefficient of restitution is 0.2.  
 (C) The loss of kinetic energy due to collision is 200 J.  
 (D) The impulse of the force between the two balls is 40 Ns.

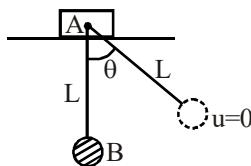
42. In an inelastic collision,

- (A) the velocity of both the particles may be same after the collision  
 (B) kinetic energy is not conserved  
 (C) linear momentum of the system is conserved.  
 (D) velocity of separation will be less than velocity of approach.

43. In a one-dimensional collision between two particles, their relative velocity is  $\vec{v}_1$  before the collision and  $\vec{v}_2$  after the collision
- (A)  $\vec{v}_1 = \vec{v}_2$  if the collision is elastic      (B)  $\vec{v}_1 = -\vec{v}_2$  if the collision is elastic  
 (C)  $|\vec{v}_2| = |\vec{v}_1|$  in all cases      (D)  $\vec{v}_1 = -k\vec{v}_2$  in all cases, where  $k \geq 1$
44. A particle moving with kinetic energy = 3 joule makes an elastic head on collision with a stationary particle which has twice its mass during the impact.
- (A) The minimum kinetic energy of the system is 1 joule.  
 (B) The maximum elastic potential energy of the system is 2 joule.  
 (C) Momentum and total kinetic energy of the system are conserved at every instant.  
 (D) The ratio of kinetic energy to potential energy of the system first decreases and then increases.
45. In a one dimensional collision between two identical particles A and B, B is stationary and A has momentum p before impact. During impact, B gives impulse J to A.
- (A) The total momentum of the 'A plus B' system is p before and after the impact, and  $(p-J)$  during the impact.  
 (B) During the impact A gives impulse of magnitude J to B  
 (C) The coefficient of restitution is  $\frac{2J}{p} - 1$   
 (D) The coefficient of restitution is  $\frac{J}{p} + 1$

#### Paragraph for Question 46 to 48

A small ball B of mass m is suspended with light inelastic string of length L from a block A of same mass m which can move on smooth horizontal surface as shown in the figure. The ball is displaced by angle  $\theta$  from lowest position & then released.



46. The displacement of block when ball reaches the lowest position is
- (A)  $\frac{L \sin \theta}{2}$       (B)  $L \sin \theta$       (C)  $L \cos \theta$       (D)  $\frac{L \cos \theta}{2}$
47. Maximum velocity of block during subsequent motion of the system after release of ball is
- (A)  $[gL(1-\cos\theta)]^{1/2}$       (B)  $[2gL(1-\cos\theta)]^{1/2}$   
 (C)  $[gL \cos \theta]^{1/2}$       (D) information are insufficient to decide
48. The displacement of centre of mass of A + B system till the string becomes vertical is
- (A) zero      (B)  $L(1-\cos\theta)$       (C)  $\frac{L}{2}(1-\sin\theta)$       (D)  $\frac{L}{2}(1-\cos\theta)$

**Paragraph for Question Nos. 49 to 50**

A uniform chain of length  $2L$  is hanging in equilibrium position, if end B is given a slightly downward displacement the imbalance causes an acceleration. Here pulley is small and smooth & string is inextensible



49. The acceleration of end B when it has been displaced by distance  $x$ , is

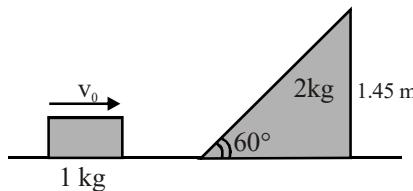
(A)  $\frac{x}{L}g$       (B)  $\frac{2x}{L}g$       (C)  $\frac{x}{2L}g$       (D)  $g$

50. The velocity  $v$  of the chain when it slips out of the pulley (height of pulley from floor  $> 2L$ )

(A)  $\sqrt{\frac{gL}{2}}$       (B)  $\sqrt{2gL}$       (C)  $\sqrt{gL}$       (D) None of these

**Paragraph for Question Nos. 51 to 53**

A block of mass 1 kg is moving towards a movable wedge of mass 2 kg as shown in figure. All surfaces are smooth. When the block leaves the wedge from top, its velocity is making an angle  $\theta = 30^\circ$  with horizontal.



51. The value of  $v_0$  in m/s is

(A) 4      (B) 7      (C) 10      (D) 9

52. To what maximum height will the block rise:

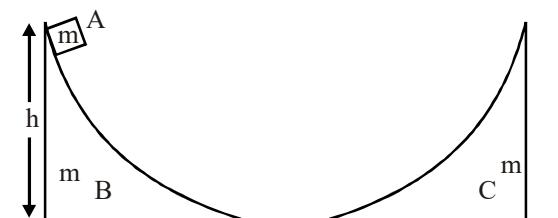
(A) 1.9 m      (B) 2.7 m      (C) 1.6 m      (D) 1.45 m

53. In the whole process let  $J$  be the magnitude of net impulse given to the block by the wedge,  $J_H$  its horizontal component and  $J_V$  its vertical component. Then :

(A)  $J = \frac{8}{\sqrt{3}} \text{ N-s}$       (B)  $J_H = 4 \text{ N-s}$       (C)  $J_V = \frac{4}{\sqrt{3}} \text{ N-s}$       (D) All of these

**Paragraph for Question Nos. 54 to 56**

Block A is placed on wedge B at a height  $h$  above ground. Block and the two wedges are all of same mass  $m$ . Neglect friction every where :



54. Find velocity of B when A has slid down from it :

- (A)  $\sqrt{gh}$       (B)  $\sqrt{\frac{gh}{2}}$       (C)  $\frac{\sqrt{gh}}{2}$       (D) None of these

55. Find maximum height upto which block A rises on wedge C :

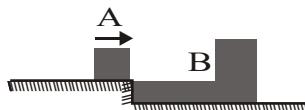
- (A) h      (B)  $\frac{h}{2}$       (C)  $\frac{h}{4}$       (D) None of these

56. Find velocity of A when it has slid down to ground from wedge C :

- (A) 0      (B)  $\sqrt{\frac{gh}{2}}$       (C)  $\sqrt{\frac{gh}{4}}$       (D) None of these

**Paragraph for Question Nos. 57 to 59**

As shown, a block (Object—A) of mass  $m_A$  moves on a frictionless plane at initial speed  $v_0$ , and lands onto a cart (Object—B in the figure, mass  $m_B$ , length L, initially at rest) smoothly. [ $m_A = m_B = m$ ]



Ignore the size of the block. The friction coefficient between A and B is  $\mu$  and the cart is on a frictionless plane. The block collides elastically on the fixed wall w.r.t. cart at the end of the cart and eventually falls off the cart.

57. Find the minimum value of  $v_0$  such that the block can indeed fall off the cart.

- (A)  $\sqrt{2\mu gl}$       (B)  $3\sqrt{2\mu gl}$       (C)  $7\sqrt{2\mu gl}$       (D)  $2\sqrt{2\mu gl}$

58. Suppose the initial speed is larger than the minimum value in (Q. 57), then loss in the kinetic energy of the system is

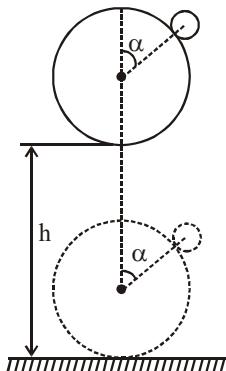
- (A)  $\mu mg l$       (B)  $2\mu mg l$       (C)  $4\mu mg l$       (D)  $9\mu mg l$

59. If the initial speed is less than the minimum value in (Q. 57), find the time the block spends on the cart.

- (A)  $2l/v_0$       (B)  $8l/v_0$       (C)  $3l/v_0$       (D) infinite

**Paragraph for Question Nos. 60 & 61**

A large heavy sphere and a small light sphere are dropped together onto a flat surface from a height  $h$ . The radius of spheres is much smaller than height  $h$ . The large sphere collides with the surface with velocity  $v_0$  and immediately thereafter with the small sphere. The spheres are dropped so that all motion is vertical before the second collision, and the small sphere hits the larger sphere at an angle  $\alpha$  from its uppermost point, as shown in the diagram. All collisions are perfectly elastic and there is no surface friction between the spheres.



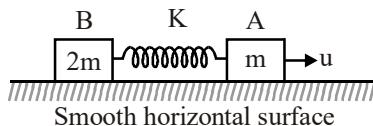
60. The angle made by velocity vector of small sphere with the vertical just after the second collision in the frame of large sphere is.
- (A)  $\alpha$       (B)  $2\alpha$       (C)  $3\alpha/2$       (D) zero
61. Find the vertical velocity of smaller sphere just after the collision with respect to ground.
- (A)  $2v_0 \cos \alpha$       (B)  $2v_0 \cos 2\alpha + v_0$       (C)  $v_0 \cos \alpha + v_0$       (D)  $v_0 \cos 2\alpha + v_0$

### **MATRIX MATCH TYPE QUESTIONS**

62. A particle of mass  $m$ , kinetic energy  $K$  and momentum  $p$  collides head on elastically with another particle of mass  $2m$  at rest. After collision :

<b>Column-I</b>	<b>Column-II</b>
(A) Momentum of first particle	(P) $3/4 p$
(B) Momentum of second particle	(Q) $-K/9$
(C) Kinetic energy of first particle	(R) $-p/3$
(D) Kinetic energy of second particle	(S) $\frac{8K}{9}$
	(T) None

63. Two blocks A and B of mass  $m$  and  $2m$  respectively are connected by a massless spring of spring constant  $K$ . This system lies over a smooth horizontal surface. At  $t = 0$  the block A has velocity  $u$  towards right as shown while the speed of block B is zero, and the length of spring is equal to its natural length at that instant. In each situation of column-I, certain statements are given and corresponding results are given in column II. Match the statements in column I corresponding results in column II.

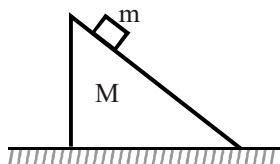


<b>Column I</b>	<b>Column II</b>
(A) The velocity of block A	(P) Can never be zero
(B) The velocity of block B	(Q) may be zero at certain instants of time
(C) The kinetic energy of system of two blocks	(R) is minimum at maximum compression of spring
(D) The potential energy of spring	(S) is maximum at maximum extension of spring

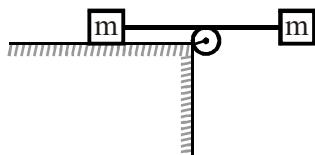
64. In each situation of column-I, a system involving two bodies given. All the strings and pulleys are light and friction is absent every where. Initially each body of every system is at rest. Consider the system in all situation of column I from rest till any collision occurs. Then match the statement in column-I with the corresponding results in column-II.

**Column I**

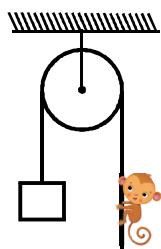
- (A) The block plus wedge system is placed over smooth horizontal surface. After the system is released from rest, the centre of mass of system



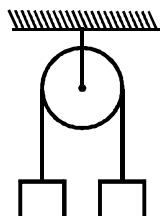
- (B) The string connecting both the blocks of mass  $m$  is horizontal. Left block is placed over smooth horizontal table as shown. After the two block system is released from rest, the centre of mass of system.



- (C) The block and monkey have same mass. The monkey starts climbing up the rope. After the monkey starts climbing up the centre of mass of monkey + block system.



- (D) Both block of mass  $m$  are initially at rest. The left block is given initial velocity  $u$  downwards, then the centre of mass of two block system afterwards.



**Column II**

- (P) Shift towards right

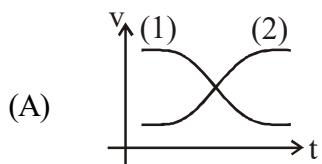
- (Q) Shift downwards

- (R) Shift upwards

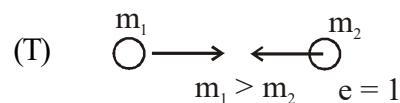
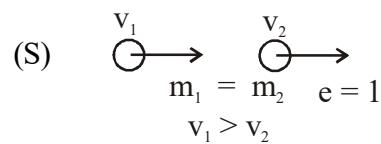
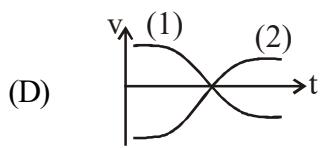
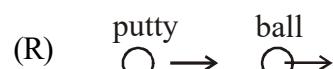
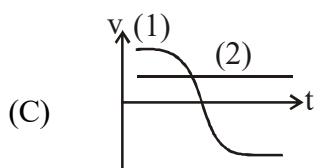
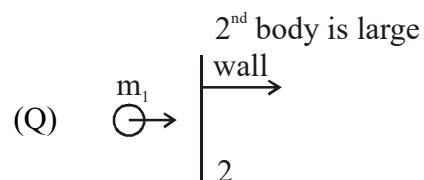
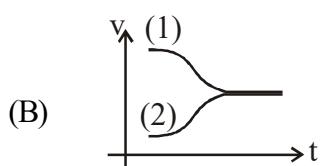
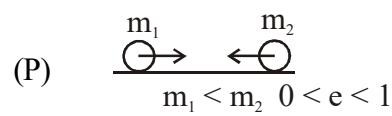
- (S) Does not shift

65. Assume that 2 bodies collide head on. The graph of their velocities with time are shown in column-I  
match them with appropriate situation in column-II

**Column-I**

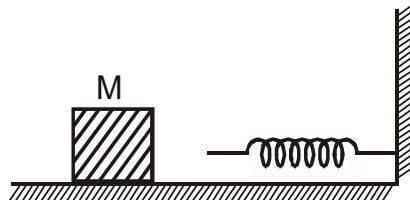


**Column-II**



### EXERCISE (JM)

1. A body A of mass M while falling vertically downwards under gravity breaks into two parts; a body B of mass M and, a body C of mass M. The centre of mass of bodies B and C taken together shifts compared to that of body A towards- [AIEEE - 2005]
  - (1) depends on height of breaking
  - (2) does not shift
  - (3) body C
  - (4) body B
2. The block of mass M moving on the frictionless horizontal surface collides with the spring of spring constant k and compresses it by length L. The maximum momentum of the block after collision is-



- (1)  $\sqrt{MkL}$
- (2)  $\frac{kL^2}{2M}$
- (3) zero
- (4)  $\frac{ML^2}{k}$  [AIEEE - 2005]

3. A bomb of mass 16 kg at rest explodes into two pieces of masses 4 kg and 12 kg. The velocity of the 12 kg mass is  $4 \text{ ms}^{-1}$ . The kinetic energy of the other mass is- [AIEEE- 006]
  - (1) 144 J
  - (2) 288 J
  - (3) 192 J
  - (4) 96 J

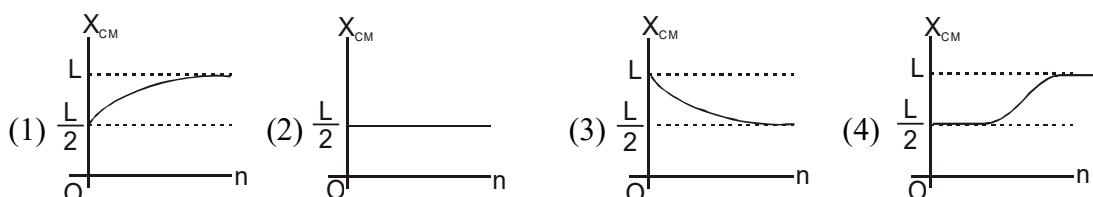
4. Consider a two particle system with particles having masses  $m_1$  and  $m_2$ . If the first particle is pushed towards the centre of mass through a distance d, by what distance should the second particle be moved, so as to keep the centre of mass at the same position ? [AIEEE- 2006]
  - (1)  $\frac{m_2}{m_1}d$
  - (2)  $\frac{m_1}{m_1 + m_2}d$
  - (3)  $\frac{m_1}{m_2}d$
  - (4) d

5. A circular disc of radius R is removed from a bigger circular disc of radius 2R such that the circumferences of the discs coincide. The centre of mass of the new disc is  $\alpha R$  from the centre of the bigger disc. The value of  $\alpha$  is :- [AIEEE - 2007]
  - (1) 1/3
  - (2) 1/2
  - (3) 1/6
  - (4) 1/4

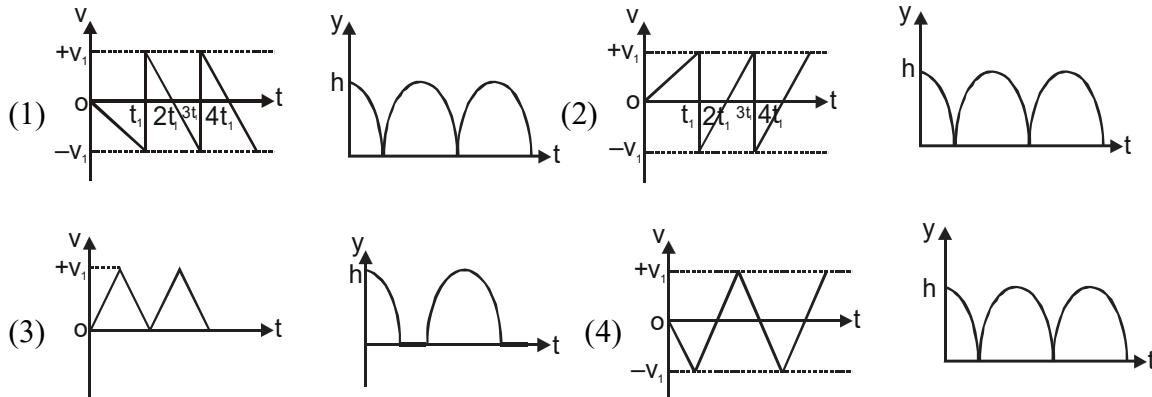
6. A block of mass 0.50 kg is moving with a speed of  $2.00 \text{ ms}^{-1}$  on a smooth surface. It strikes another mass of 1.00 kg and then they move together as a single body. The energy loss during the collision is:- [AIEEE - 2008]
  - (1) 0.16 J
  - (2) 1.00 J
  - (3) 0.67 J
  - (4) 0.34 J

7. A thin rod of length 'L' is lying along the x-axis with its ends at  $x = 0$  and  $x = L$ . Its linear density

(mass/length) varies with x as  $k \left( \frac{x}{L} \right)^n$  where n can be zero or any positive number. If the position  $x_{CM}$  of the centre of mass of the rod is plotted against 'n', which of the following graphs best approximates the dependence of  $x_{CM}$  on n? [AIEEE-2008]



8. Consider a rubber ball freely falling from a height  $h = 4.9 \text{ m}$  onto a horizontal elastic plate. Assume that the duration of collision is negligible and the collision with the plate is totally elastic. Then the velocity as a function of time and the height as a function of time will be :- [AIEEE-2009]



9. Directions : Question number 9 contain Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement-1 : Two particles moving in the same direction do not lose all their energy in a completely inelastic collision.

Statement-2 : Principle of conservation of momentum holds true for all kinds of collisions.

[AIEEE - 2010]

- (1) Statement-1 is true, Statement-2 is false
- (2) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1
- (3) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1
- (4) Statement-1 is false, Statement-2 is true

10. This question has Statement I and Statement II. Of the four choices given after the Statements, choose the one that best describes the two Statements. [JEE Mains-2013]

Statement - I : A point particle of mass  $m$  moving with speed  $v$  collides with stationary point particle of mass  $M$ . If the maximum energy loss possible is given as  $f\left(\frac{1}{2}mv^2\right)$  then  $f = \left(\frac{m}{M+m}\right)$ .

Statement - II : Maximum energy loss occurs when the particles get stuck together as a result of the collision.

- (1) Statement-I is true, Statement-II is true, Statement-II is a correct explanation of Statement-I.
- (2) Statement-I is true, Statement-II is true, Statement-II is a not correct explanation of Statement-I.
- (3) Statement-I is true, Statement-II is false.
- (4) Statement-I is false, Statement-II is true.

11. A particle of mass  $m$  moving in the  $x$  direction with speed  $2v$  is hit by another particle of mass  $2m$  moving in the  $y$  direction with speed  $v$ . If the collision is perfectly inelastic, the percentage loss in the energy during the collision is close to : [JEE Mains-2015]

- (1) 56 %
- (2) 62%
- (3) 44%
- (4) 50%

12. Distance of the centre of mass of a solid uniform cone from its vertex is  $z_0$ . If the radius of its base is  $R$  and its height is  $h$  then  $z_0$  is equal to :- [JEE Mains-2015]

- (1)  $\frac{5h}{8}$
- (2)  $\frac{3h^2}{8R}$
- (3)  $\frac{h^2}{4R}$
- (4)  $\frac{3h}{4}$

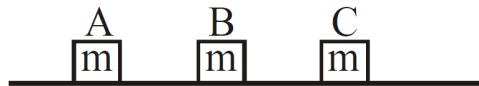
13. In a collinear collision, a particle with an initial speed  $v_0$  strikes a stationary particle of the same mass. If the final total kinetic energy is 50% greater than the original kinetic energy, the magnitude of the relative velocity between the two particles, after collision is : [JEE Mains-2018]

(1)  $\frac{v_0}{\sqrt{2}}$       (2)  $\frac{v_0}{4}$       (3)  $\sqrt{2}v_0$       (4)  $\frac{v_0}{2}$

14. It is found that if a neutron suffers an elastic collinear collision with deuterium at rest, fractional loss of its energy is  $p_d$ ; while for its similar collision with carbon nucleus at rest, fractional loss of energy is  $p_c$ . The values of  $p_d$  and  $p_c$  are respectively : [JEE-Main - 2018]

(1) (0, 1)      (2) (.89, .28)      (3) (.28, .89)      (4) (0, 0)

15. Three blocks A, B and C are lying on a smooth horizontal surface, as shown in the figure. A and B have equal masses,  $m$  while C has mass  $M$ . Block A is given an brutal speed  $v$  towards B due to which it collides with B perfectly in elastically. The combined mass collides with C, also perfectly in elastically  $\frac{5}{6}$  th of the initial kinetic energy is lost in whole process. What is value of  $M/m$ ? [JEE-Main - 2019]



(1) 4      (2) 5      (3) 3      (4) 2

16. A piece of wood of mass 0.03 kg is dropped from the top of a 100 m height building. At the same time, a bullet of mass 0.02 kg is fired vertically upward, with a velocity  $100 \text{ ms}^{-1}$ , from the ground. The bullet gets embedded in the wood. Then the maximum height to which the combined system reaches above the top of the building before falling below is : ( $g=10 \text{ ms}^{-2}$ ) [JEE-Main - 2019]

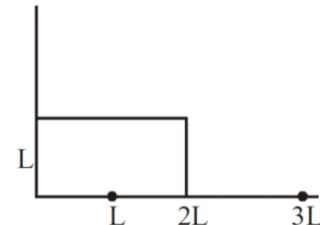
(1) 40 m      (2) 20 m      (3) 10 m      (4) 30 m

17. A particle of mass  $m$  is moving in a straight line with momentum  $p$ . Starting at time  $t = 0$ , a force  $F = kt$  acts in the same direction on the moving particle during time interval  $T$  so that its momentum changes from  $p$  to  $3p$ . Here  $k$  is a constant. The value of  $T$  is : [JEE-Main - 2019]

(1)  $\sqrt{\frac{2k}{p}}$       (2)  $2\sqrt{\frac{k}{p}}$       (3)  $\sqrt{\frac{2p}{k}}$       (4)  $2\sqrt{\frac{p}{k}}$

18. The position vector of the centre of mass  $\vec{r}$  cm of an symmetric uniform bar of negligible area of cross-section as shown in figure is : [JEE-Main-2019]

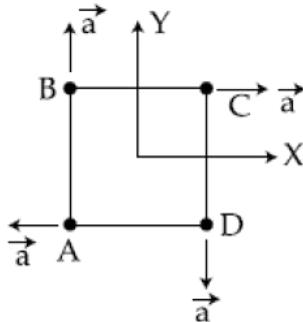
(1)  $\vec{rcm} = \frac{13}{8}L\hat{x} + \frac{5}{8}L\hat{y}$       (2)  $\vec{rcm} = \frac{11}{8}L\hat{x} + \frac{3}{8}L\hat{y}$   
 (3)  $\vec{rcm} = \frac{5}{8}L\hat{x} + \frac{13}{8}L\hat{y}$       (4)  $\vec{rcm} = \frac{3}{8}L\hat{x} + \frac{11}{8}L\hat{y}$



19. An alpha-particle of mass  $m$  suffers 1-dimensional elastic coolision with a nucleus at rest of unknown mass. It is scattered directly backwards losing, 64% of its initial kinetic energy. The mass of the nucleus is :- [JEE-Main-2019]

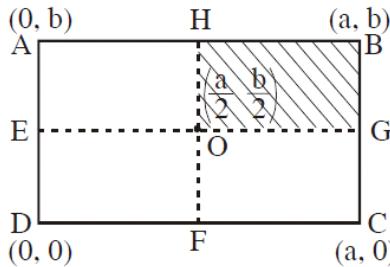
(1) 3.5 m      (2) 2 m      (3) 1.5 m      (4) 4 m

20. Four particles A, B C and D with masses  $m_A = m$ ,  $m_B = 2m$ ,  $m_C = 3m$  and  $m_D = 4m$  are at the corners of a square. They have accelerations of equal magnitude with direction as shown. The acceleration of the centre of mass of the particles is [JEE-Main-2019]



- (1)  $a(\hat{i} + \hat{j})$       (2) Zero      (3)  $\frac{a}{5}(\hat{i} - \hat{j})$       (4)  $\frac{a}{5}(\hat{i} + \hat{j})$

21. A uniform rectangular thin sheet ABCD of mass M has length a and breadth b, as shown in the figure. If the shaded portion HFGO is cut-off, the coordinates of the centre of mass of the remaining portion will be : [JEE-Main-2019]



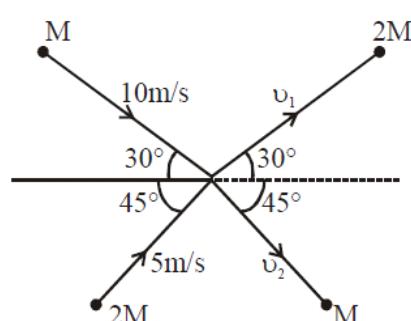
- (1)  $\left(\frac{5a}{3}, \frac{5b}{3}\right)$       (2)  $\left(\frac{3a}{4}, \frac{3b}{4}\right)$       (3)  $\left(\frac{5a}{12}, \frac{5b}{12}\right)$       (4)  $\left(\frac{2a}{3}, \frac{2b}{3}\right)$

22. A body of mass  $m_1$  moving with an unknown velocity of  $v_1\hat{i}$ , undergoes a collinear collision with a body of mass  $m_2$  moving with a velocity  $v_2\hat{i}$ . After collision,  $m_1$  and  $m_2$  move with velocities of  $v_3\hat{i}$  and  $v_4\hat{i}$ , respectively. If  $m_2 = 0.5 m_1$  and  $v_3 = 0.5v_1$ , then  $v_1$  is : [JEE-Main-2019]

- (1)  $v_4 + v_2$       (2)  $v_4 - v_2$       (3)  $v_4 - \frac{v_2}{2}$       (4)  $v_4 - \frac{v_2}{4}$

23. Two particles, of masses M and  $2M$ , moving, as shown, with speeds of 10 m/s and 5 m/s, collide elastically at the origin. After the collision, they move along the indicated directions with speeds  $v_1$  and  $v_2$  are nearly: [JEE-Main-2019]

- (1) 3.2 m/s and 12.6 m/s      (2) 3.2 m/s and 6.3 m/s  
(3) 6.5 m/s and 6.3 m/s      (4) 6.5 m/s and 3.2 m/s

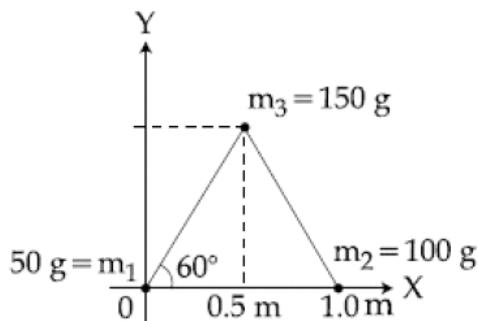


24. A man (mass = 50 kg) and his son (mass = 20 kg) are standing on a frictionless surface facing each other. The man pushes his son so that he starts moving at a speed of  $0.70 \text{ ms}^{-1}$  with respect to the man. The speed of the man with respect to the surface is [JEE-Main-2019]

- (1)  $0.20 \text{ ms}^{-1}$       (2)  $0.28 \text{ ms}^{-1}$       (3)  $0.47 \text{ ms}^{-1}$       (4)  $0.14 \text{ ms}^{-1}$

25. Three particles of masses 50 g, 100 g and 150 g are placed at the vertices of an equilateral triangle of side 1 m (as shown in the figure). The (x, y) coordinates of the centre of mass will be :

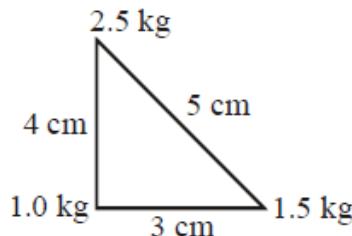
[JEE-Main-2019]



- (1)  $\left(\frac{\sqrt{3}}{8} \text{ m}, \frac{7}{12} \text{ m}\right)$     (2)  $\left(\frac{\sqrt{3}}{4} \text{ m}, \frac{5}{12} \text{ m}\right)$     (3)  $\left(\frac{7}{12} \text{ m}, \frac{\sqrt{3}}{8} \text{ m}\right)$     (4)  $\left(\frac{7}{12} \text{ m}, \frac{\sqrt{3}}{4} \text{ m}\right)$

26. Three point particles of masses 1.0 kg, 1.5 kg and 2.5 kg are placed at three corners of a right angle triangle of sides 4.0 cm, 3.0 cm and 5.0 cm as shown in the figure. The center of mass of the system is at a point:

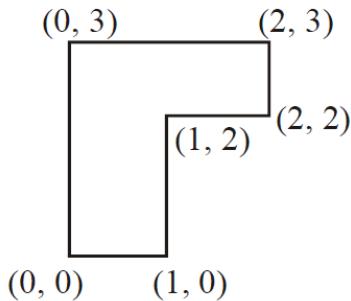
[JEE Main-2020]



- (1) 0.6 cm right and 2.0 cm above 1 kg mass    (2) 2.0 cm right and 0.9 cm above 1kg mass  
 (3) 0.9 cm right and 2.0 cm above 1 kg mass    (4) 1.5 cm right and 1.2 cm above 1 kg mas

27. The coordinates of centre of mass of a uniform flag shaped lamina (thin flat plate) of mass 4 kg. (The coordinates of the same are shown in figure) are:

[JEE Main-2020]

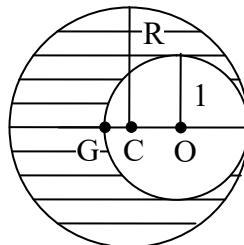


- (1) (1.25 m, 1.50 m)    (2) (1m, 1.75 m)    (3) (0.75 m, 1.75 m)    (4) (0.75 m, 0.75 m)

28. A body A, of mass  $m = 0.1 \text{ kg}$  has an initial velocity of  $3\hat{i} \text{ ms}^{-1}$ . It collides elastically with another body, B of the same mass which has an initial velocity of  $5\hat{j} \text{ ms}^{-1}$ . After collision, A moves with a velocity

$\vec{v} = 4(\hat{i} + \hat{j})$ . The energy of B after collision is written as  $\frac{x}{10} \text{ J}$ . The value of x is \_\_\_\_\_. [JEE Main-2020]

29. As shown in fig. when a spherical cavity (centred at O) of radius 1 is cut out of a uniform sphere of radius R (centred at C), the centre of mass of remaining (shaded) part of sphere is at G, i.e. on the surface of the cavity. R can be determined by the equation : [JEE Main-2020]



(1)  $(R^2 - R - 1)(2 - R) = 1$   
(3)  $(R^2 + R - 1)(2 - R) = 1$

(2)  $(R^2 + R + 1)(2 - R) = 1$   
(4)  $(R^2 - R + 1)(2 - R) = 1$

30. A particle of mass m is dropped from a height h above the ground. At the same time another article of the same mass is thrown vertically upwards from the ground with a speed of  $\sqrt{2gh}$ . If they collide head-

on completely inelastically, the time taken for the combined mass to reach the ground, in units of  $\sqrt{\frac{h}{g}}$  Ls:

[JEE Main-2020]

(1)  $\sqrt{\frac{3}{4}}$       (2)  $\frac{1}{2}$       (3)  $\sqrt{\frac{3}{2}}$       (4)  $\sqrt{\frac{1}{2}}$

31. Two particles of equal mass m have respective initial velocities  $u\hat{i}$  and  $u\left(\frac{\hat{i} + \hat{j}}{2}\right)$ . They collide completely inelastically. The energy lost in the process is : [JEE Main-2020]

(1)  $\sqrt{\frac{2}{3}}mu^2$       (2)  $\frac{1}{8}mu^2$       (3)  $\frac{3}{4}mu^2$       (4)  $\frac{1}{3}mu^2$

32. A rod of length L has non-uniform linear mass density given by  $\rho(x) = a + b\left(\frac{x}{L}\right)^2$  where a and b are constants and  $0 \leq x \leq L$ . The value of x for the centre of mass of the rod is at:

[JEE Main-2020]

(1)  $\frac{4}{3}\left(\frac{a+b}{2a+3b}\right)L$       (2)  $\frac{3}{4}\left(\frac{2a+b}{3a+b}\right)L$       (3)  $\frac{3}{2}\left(\frac{2a+b}{3a+b}\right)L$       (4)  $\frac{3}{2}\left(\frac{a+b}{2a+b}\right)L$

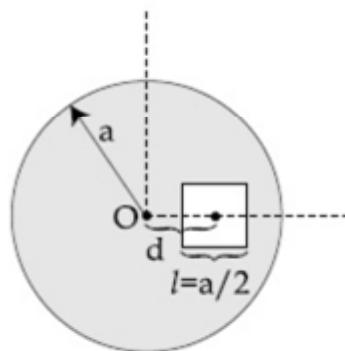
33. A particle of mass  $m$  is projected with a speed  $u$  from the ground at an angle  $\theta = \frac{\pi}{3}$  w.r.t. horizontal (x-axis). When it has reached its maximum height, it collides completely in elastically with another particle of the same mass and velocity  $u\hat{i}$ . The horizontal distance covered by the combined mass before reaching the ground is: [JEE Main-2020]

(1)  $2\sqrt{2} \frac{u^2}{g}$       (2)  $\frac{3\sqrt{3}}{8} \frac{u^2}{g}$       (3)  $\frac{3\sqrt{2}}{4} \frac{u^2}{g}$       (4)  $\frac{5}{8} \frac{u^2}{g}$

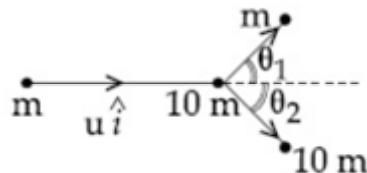
34. A particle of mass  $m$  with an initial velocity  $u\hat{i}$  collides perfectly elastically with a mass  $3m$  at rest. It moves with a velocity  $v\hat{j}$  after collision, then,  $v$  is given by : [JEE Main-2020]

(1)  $v = \frac{1}{\sqrt{6}} u$       (2)  $v = \sqrt{\frac{2}{3}} u$       (3)  $v = \frac{u}{\sqrt{3}}$       (4)  $v = \frac{u}{\sqrt{2}}$

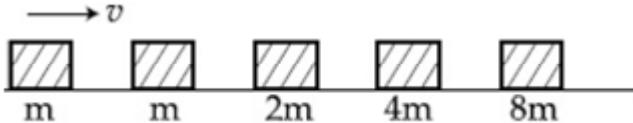
35. A square hole of side  $l = \frac{a}{2}$  is carved out at a distance  $d = \frac{a}{2}$  from the centre 'O' of a uniform circular disk of radius  $a$ . If the distance of the centre of mass of the remaining portion from O is  $-\frac{a}{X}$ , value of X (to the nearest integer) is \_\_\_\_\_. [JEE Main-2020]



36. A particle of mass  $m$  is moving along the A x-axis with initial velocity  $u\hat{i}$ . It collides elastically with a particle of mass  $10m$  at rest and then moves with half its initial kinetic energy (see figure). If  $\sin\theta_1 = \sqrt{n}$   $\sin\theta_2$ , then value of  $n$  is \_\_\_\_\_. [JEE Main-2020]



37. Blocks of masses  $m$ ,  $2m$ ,  $4m$  and  $8m$  are arranged in a line on a frictionless floor. Another block of mass  $m$ , moving with speed  $v$  along the same line (see figure) collides with mass  $m$  in perfectly inelastic manner. All the subsequent collisions are also perfectly inelastic. By the time the last block of mass  $8m$  starts moving the total energy loss is  $p\%$  of the original energy. Value of ' $p$ ' is close to: [JEE Main-2020]



- 38.** A spaceship in space sweeps stationary interplanetary dust. As a result, its mass increases at a rate  $\frac{dM(t)}{dt} = bv^2(t)$ , where  $v(t)$  is its instantaneous velocity. The instantaneous acceleration of the satellite is:

- 39.** Two bodies of the same mass are moving with the same speed, but in different directions in a plane. They have a completely inelastic collision and move together thereafter with a final speed which is half of their initial speed. The angle between the initial velocities of the two bodies (in degree) is. [JEE Main-2020]

40. Particle A of mass  $m_1$  moving with velocity  $(\sqrt{3}\hat{i} + \hat{j}) \text{ ms}^{-1}$  collides with another particle B of mass  $m^2$  which is at rest initially. Let  $\vec{V}_1$  and  $\vec{V}_2$  be the velocities of particles A and B after collision respectively.

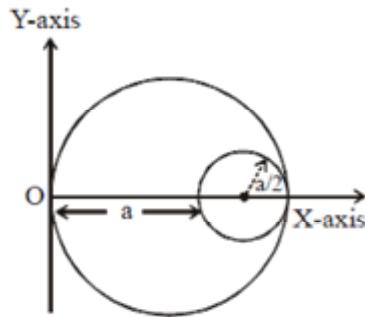
If  $m_1 = 2m_2$  and after collision  $\vec{V}_1 = (\hat{i} + \sqrt{3}\hat{j}) \text{ ms}^{-1}$ , the angle between and is : [JEE Main-2020]

- (1)  $60^\circ$       (2)  $15^\circ$       (3)  $-45^\circ$       (4)  $105^\circ$

41. The centre of mass of a solid hemisphere of radius 8 cm is X cm from the centre of the flat surface. Then value of x is . [JEE Main-2020]

42. A ball will a speed of 9 m/s collides with another identical ball at rest. After the collision, the direction of each ball makes an angle of  $30^\circ$  with the original direction. The ratio of velocities of the balls after collision is  $x : y$ , where  $x$  is \_\_\_\_\_. [JEE Main-2021]

43. A circular hole of radius  $\left(\frac{a}{2}\right)$  is cut out of a circular disc of radius 'a' as shown in figure. The centroid of the remaining circular portion with respect to point 'O' will be : [JEE Main-2021]



- (1)  $\frac{1}{6}a$       (2)  $\frac{10}{11}a$       (3)  $\frac{5}{6}a$       (4)  $\frac{2}{3}a$

44. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

[JEE Main-2021]

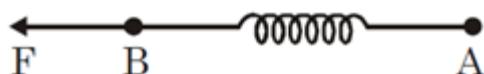
**Assertion A :** Body 'P' having mass M moving with speed 'u' has head-on collision elastically with another body 'Q' having mass 'm' initially at rest. If  $m \ll M$ , body 'Q' will have a maximum speed equal to '2u' after collision.

**Reason R :** During elastic collision, the momentum and kinetic energy are both conserved. In the light of the above statements, choose the most appropriate answer from the options given below :

- (1) A is not correct but R is correct.
- (2) Both A and R are correct but R is NOT the correct explanation of A.
- (3) Both A and R are correct and R is the correct explanation of A.
- (4) A is correct but R is not correct.

45. Two masses A and B, each of mass M are fixed together by a massless spring. A force acts on the mass B as shown in figure. If the mass A starts moving away from mass B with acceleration 'a', then the acceleration of mass B will be :-

[JEE Main-2021]

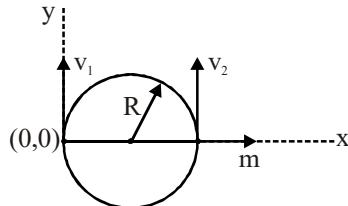


- (1)  $\frac{Ma - F}{M}$       (2)  $\frac{MF}{F + Ma}$       (3)  $\frac{F + Ma}{M}$       (4)  $\frac{F - Ma}{M}$

## EXERCISE (JA)

1. A particle of mass  $m$ , moving in a circular path of radius  $R$  with a constant speed  $v_2$  is located at point  $(2R, 0)$  at time  $t = 0$  and a man starts moving with a velocity  $v_1$  along the positive  $y$ -axis from origin at time  $t = 0$ . Calculate the linear momentum of the particle w.r.t. man as a function of time.

[IIT JEE-2003]



2. **Statement-I :** In an elastic collision between two bodies, the relative speed of the bodies after collision is equal to the relative speed before the collision. [IIT JEE-2007]

**Statement-II :** In a elastic collision, the linear momentum of the system is conserved

- (A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
- (B) Statement-1 is True, Statement-2 is True; Statement-2 is not a correct explanation for Statement-1
- (C) Statement-1 is True, Statement-2 is False
- (D) Statement-1 is False, Statement-2 is True.

3. **Statement I :** If there is no external torque on a body about its centre of mass, then the velocity of the centre of mass remains constant. [IIT JEE-2007]

**Statement II :** The linear momentum of an isolated system remains constant.

- (A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
- (B) Statement-1 is True, Statement-2 is True; Statement-2 is not a correct explanation for Statement-1
- (C) Statement-1 is True, Statement-2 is False
- (D) Statement-1 is False, Statement-2 is True.

4. A particle moves in the X-Y plane under the influence of a force such that its linear momentum is  $\vec{P}(t) = A[\hat{i} \cos(kt) - \hat{j} \sin(kt)]$ , where  $A$  and  $k$  are constants. The angle between the force and the momentum is : [IIT JEE-2007]

- (A)  $0^\circ$
- (B)  $30^\circ$
- (C)  $45^\circ$
- (D)  $90^\circ$

5. Two balls, having linear momenta  $\vec{p}_1 = p\hat{i}$  and  $\vec{p}_2 = -p\hat{i}$ , undergo a collision in free space. There is no external force acting on the balls. Let  $\vec{p}'_1$  and  $\vec{p}'_2$  be their final momenta. The following option(s) is (are) **NOT ALLOWED** for any non-zero value of  $p$ ,  $a_1$ ,  $a_2$ ,  $b_1$ ,  $b_2$ ,  $c_1$  and  $c_2$ . [IIT JEE-2008]

$$(A) \vec{p}'_1 = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$$

$$(A) \vec{p}'_2 = a_2\hat{i} + b_2\hat{j}$$

$$(B) \vec{p}'_1 = c_1\hat{k}$$

$$(B) \vec{p}'_2 = c_2\hat{k}$$

$$(C) \vec{p}'_1 = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$$

$$(C) \vec{p}'_2 = a_2\hat{i} + b_2\hat{j} - c_1\hat{k}$$

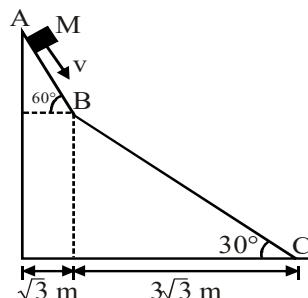
$$(D) \vec{p}'_1 = a_1\hat{i} + b_1\hat{j}$$

$$(D) \vec{p}'_2 = a_2\hat{i} + b_1\hat{j}$$

**Comprehension based questions**

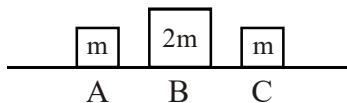
A small block of mass M moves on a frictionless surface of an inclined plane, as shown in figure. The angle of the incline suddenly changes from  $60^\circ$  to  $30^\circ$  at point B. The block is initially at rest at A. Assume that collisions between the block and the incline are totally inelastic ( $g = 10 \text{ m/s}^2$ )

[IIT-JEE 2008]



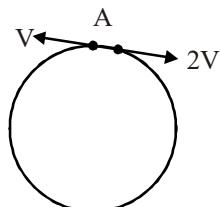
6. The speed of the block at point B immediately after it strikes the second incline is:  
 (A)  $\sqrt{60}$  m/s      (B)  $\sqrt{45}$  m/s      (C)  $\sqrt{30}$  m/s      (D)  $\sqrt{15}$  m/s
  
7. The speed of the block at point C, immediately before it leaves the second incline is:  
 (A)  $\sqrt{120}$  m/s      (B)  $\sqrt{105}$  m/s      (C)  $\sqrt{30}$  m/s      (D)  $\sqrt{75}$  m/s
  
8. If collision between the block and the incline is completely elastic, then the vertical (upward) component of the velocity of the block at point B, immediately after it strikes the second incline is:  
 (A)  $\sqrt{30}$  m/s      (B)  $\sqrt{15}$  m/s      (C) 0      (D)  $-\sqrt{15}$  m/s
  
9. Three objects A, B and C are kept in a straight line on a frictionless horizontal surface. These have masses  $m$ ,  $2m$  and  $m$ , respectively. The object A moves towards B with a speed  $9 \text{ m/s}$  and makes an elastic collision with it. Thereafter, B makes completely inelastic collision with C. All motions occur on the same straight line. Find the final speed (in m/s) of the object C.

[IIT JEE-2009]



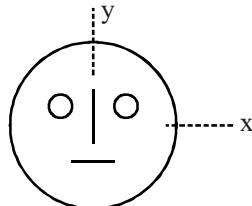
10. Two small particles of equal masses start moving in opposite directions from a point A in a horizontal circular orbit. Their tangential velocities are  $v$  and  $2v$ , respectively, as shown in the figure. Between collisions, the particles move with constant speeds. After making how many elastic collisions, other than at A, these two particles will again reach the point A?

[IIT-JEE 2009]



- (A) 4      (B) 3      (C) 2      (D) 1

11. Look at the drawing given in the figure which has been drawn with ink of uniform line-thickness. The mass of ink used to draw each of the two inner circles, and each of the two line segments in m. The mass of the ink used to draw the outer circle is 6m. The coordinates of the centres of the different parts are : outer circle  $(0, 0)$ , left inner circle  $(-a, a)$ , right inner circle  $(a, a)$ , vertical line  $(0, 0)$  and horizontal line  $(0, -a)$ . The y-coordinates of the centre of mass of the ink in this drawing is: [IIT JEE- 2009]

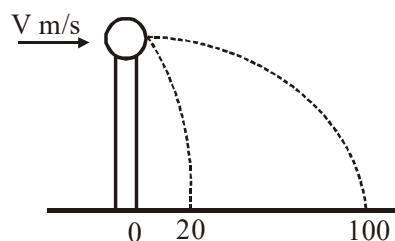


- (A)  $\frac{a}{10}$       (B)  $\frac{a}{8}$       (C)  $\frac{a}{12}$       (D)  $\frac{a}{3}$

12. A point mass of 1kg collides elastically with a stationary point mass of 5 kg. After their collision, the 1 kg mass reverses its direction and moves with a speed of 2 m/s. Which of the following statement(s) is/are correct for the system of these two masses ? [IIT JEE-2010]

- (A) Total momentum of the system is 3 kg m/s  
 (B) Momentum of 5 kg mass after collision is 4 kg m/s  
 (C) Kinetic energy of the centre of mass is 0.75 J  
 (D) Total kinetic energy of the system is 4 J

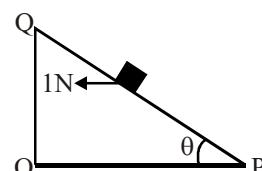
13. A ball of mass 0.2 kg rests on a vertical post of height 5m. A bullet of mass 0.01 kg, traveling with a velocity  $V$  m/s in a horizontal direction, hits the centre of the ball. After the collision, the ball and bullet travel independently. The ball hits the ground at a distance of 20 m and the bullet at a distance of 100 m from the foot of the post. The initial velocity  $V$  of the bullet is [IIT JEE - 2011 ]



- (A) 250 m/s      (B)  $250\sqrt{2}$  m/s      (C) 400 m/s      (D) 500 m/s

14. A small block of mass of 0.1 kg lies on a fixed inclined plane PQ which makes an angle  $\theta$  with the horizontal. A horizontal force of 1 N acts on the block through its center of mass as shown in the figure. The block remains stationary if (take  $g = 10 \text{ m/s}^2$ ) [IIT JEE-2012]

- (A)  $\theta = 45^\circ$   
 (B)  $\theta > 45^\circ$  and a frictional force acts on the block towards P  
 (C)  $\theta > 45^\circ$  and a frictional force acts on the block towards Q  
 (D)  $\theta < 45^\circ$  and a frictional force acts on the block towards

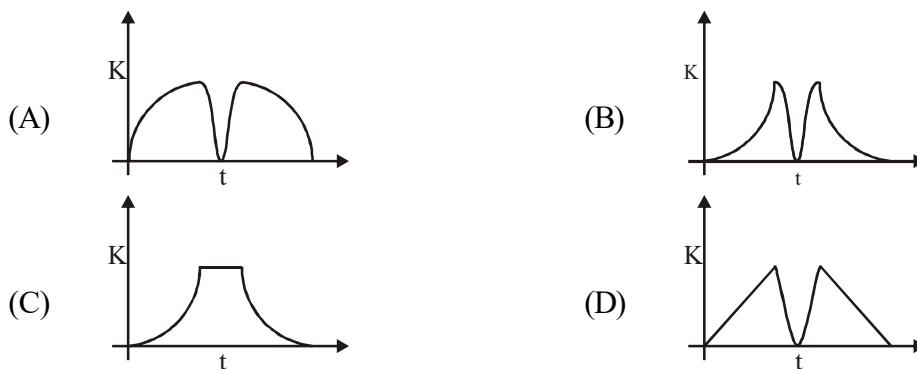


15. A bob of mass  $m$ , suspended by a string of length  $\ell_1$  is given a minimum velocity required to complete a full circle in the vertical plane. At the highest point, it collides elastically with another bob of mass  $m$  suspended by a string of length  $\ell_2$ , which is initially at rest. Both the strings are mass-less and inextensible. If the second bob, after collision acquires the minimum sped required to complete a full circle in the vertical plane, the ratio  $\frac{\ell_1}{\ell_2}$  is

[IIT JEE-2013]

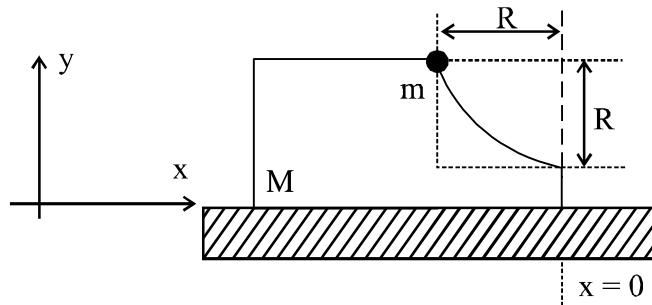
16. A tennis ball is dropped on a horizontal smooth surface. It bounces back to its original position after hitting the surface. The force on the ball during the collision is proportional to the length of compression of the ball. Which one of the following sketches describes the variation of its kinetic energy  $K$  with time  $t$  most appropriately? The figures are only illustrative and not to the scale.

[JEE Adv. 2014]



17. A block of mass  $M$  has a circular cut with a frictionless surface as shown. The block rests on the horizontal frictionless surface of a fixed table. Initially the right edge of the block is at  $x = 0$ , in a coordinate system fixed to the table. A point mass  $m$  is released from rest at the topmost point of the path as shown and it slides down. When the mass loses contact with the block, its position is  $x$  and the velocity is  $v$ . At that instant, which of the following options is/are correct ?

[JEE Adv. 2017]



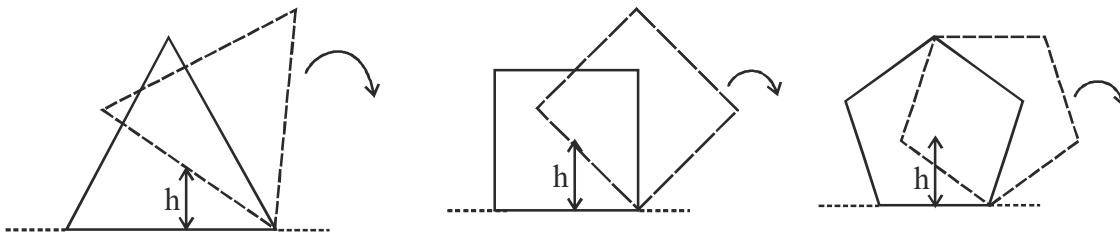
(A) The  $x$  component of displacement of the center of mass of the block  $M$  is :  $-\frac{mR}{M+m}$

(B) The position of the point mass  $m$  is :  $x = -\sqrt{2} \frac{mR}{M+m}$

(C) The velocity of the point mass  $m$  is :  $v = \sqrt{\frac{2gR}{1 + \frac{m}{M}}}$

(D) The velocity of the block  $M$  is :  $V = -\frac{m}{M} \sqrt{2gR}$

18. Consider regular polygons with number of sides  $n = 3, 4, 5 \dots$  as shown in the figure. The center of mass of all the polygons is at height  $h$  from the ground. They roll on a horizontal surface about the leading vertex without slipping and sliding as depicted. The maximum increase in height of the locus of the center of mass for each polygon is  $\Delta$ . Then  $\Delta$  depends on  $n$  and  $h$  as [JEE Adv. 2017]



(A)  $\Delta = h \sin^2\left(\frac{\pi}{n}\right)$

(B)  $\Delta = h \left( \frac{1}{\cos\left(\frac{\pi}{n}\right)} - 1 \right)$

(C)  $\Delta = h \sin\left(\frac{2\pi}{n}\right)$

(D)  $\Delta = h \tan^2\left(\frac{\pi}{2n}\right)$

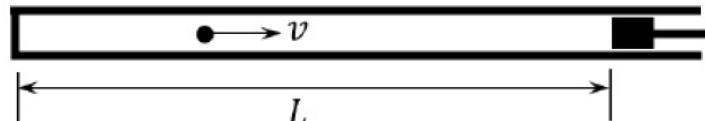
19. A ball is projected from the ground at an angle of  $45^\circ$  with the horizontal surface. It reaches a maximum height of 120 m and returns to the ground. Upon hitting the ground for the first time, it loses half of its kinetic energy. Immediately after the bounce, the velocity of the ball makes an angle of  $30^\circ$  with the horizontal surface. The maximum height it reaches after the bounce, in metres, is \_\_\_\_\_.

[JEE Adv.(P-2) 2018]

20. A small particle of mass  $m$  moving inside a heavy, hollow and straight tube along the tube axis undergoes elastic collision at two ends. The tube has no friction and it is closed at one end by a flat surface while the other end is fitted with a heavy movable flat piston as shown in figure. When the distance of the piston from closed end is  $L = L_0$  the particle speed is  $v = v_0$ . The piston is moved inward at a very low speed

$V$  such that  $V \ll \frac{dL}{L} v_0$ , where  $dL$  is the infinitesimal displacement of the piston. Which of the following statement(s) is/are correct?

[JEE Advanced-2019]



- (1) After each collision with the piston, the particle speed increases by  $2V$
- (2) If the piston moves inward by  $dL$ , the particle speed increases by  $2v \frac{dL}{L}$
- (3) The rate at which the particle strikes the piston is  $v/L$
- (4) The particle's kinetic energy increases by a factor of 4 when the piston is moved inward from  $L_0$  to  $\frac{1}{2}L_0$

## ANSWER KEY

### Exercise (S-1)

1.  $\left(\frac{1}{7}, \frac{23}{14}\right)$

2.  $\sqrt{13}m, \left(\frac{14}{5}, \frac{19}{5}\right)$

3.  $\frac{L(\sqrt{2}+1)}{3}$

4.  $\frac{a}{3(\pi-1)}$

5. 4R from O

6. (i)  $\lambda(x) = \lambda + \frac{\lambda x}{L}$ , (ii)  $\frac{5}{9}L$

7. (3m, 1 m, 8 m)

8.  $x = 6 \text{ m}$

9.  $g/9$  downwards

10.  $2\sqrt{2}$

11.  $\vec{v}_C = -\vec{v}_B$

12.  $\frac{R}{4}$

13.  $\frac{L}{3}$

14.  $(L+2R, 0)$

15.  $\frac{m(R-r)}{M+m}, m\sqrt{\frac{2g(R-r)}{M(M+m)}}$

16. (i) 4 m/s, (ii) 24 J

17. (i) 360 m, (ii) 10800 J

18.  $\frac{\sqrt{13}}{2}v_0$

19. 150 kg

20. (a)  $\frac{mu}{M+2m}$ ; (b)  $mu\left[\frac{1}{M+2m} - \frac{1}{M+m}\right]$ ; (c)  $\frac{1}{2} \frac{m^2 u^2}{(M+2m)^2} (M+m)$

21.  $\frac{5v}{R \cos \theta}$

22.  $\frac{4u^2 \sin 2\theta}{5g}$

23. 0.3

24. 30 cm

25.  $m \times \sqrt{u^2 - uv + v^2}$

26. 1 N

27.  $\frac{7}{18}$

28. (i) 3 J, (ii)  $\frac{12}{5} \text{ N-s}$

29. 005

30. (i)  $\frac{40}{3} \text{ m}$ ; (ii)  $t_e = 3.25 \text{ s}$

31. (i)  $\frac{v_0}{3}$ , (ii)  $3\sqrt{5gR}$

32.  $3v\sqrt{\frac{2h}{g}}$

33.  $(\hat{i} + \sqrt{3}\hat{j}) \text{ m/s}$ ,  $(3m\hat{i} - \sqrt{3}m\hat{j}) \text{ kg-m/s}$

34. (a) 3; (b)  $\frac{25}{9}$

### Exercise (S-2)

1.  $0C = \frac{2r}{3(4-\pi)}$

2.  $100 \text{ m}$

3.  $\frac{mg(h_3 - h_1)}{(h_2 - h_1)}$

4.  $\frac{2mv^2}{3\ell}$

5.  $v_B = \frac{m_A}{m_B} \sqrt{\frac{2gl}{1+m_A/m_B}}, T = 3m_A g + \frac{2m_A^2 g}{m_B}$

6.  $\ell$

7. (i)  $\frac{u}{2}, \frac{mu}{2}$  (ii)  $\frac{\sqrt{13}}{8}u, \frac{\sqrt{13}}{8}mu$  (iii)  $\frac{\sqrt{3}}{4}u, \frac{mu\sqrt{3}}{4}$  8.  $40 \text{ cm}$

9. 4

10.  $13/3 \text{ m/sec}$

11.  $e = \frac{5+\sqrt{3}}{8}, M = \frac{26}{\sqrt{3}} \text{ kg}$

12.  $V_1 = -2 \text{ m/s}, V_2 = 4\sqrt{3} \text{ m/s}, V_3 = 4\sqrt{3} \text{ m/s}$

13.  $\frac{5u}{7}, \frac{2\sqrt{3}u}{7}, \frac{2\sqrt{3}u}{7}, \text{ Impulse} = \frac{6mu}{7}$  14. (i)  $\frac{v}{2}, \frac{v}{2}, 0$  (ii)  $\frac{2mv^2}{9}$  (iii)  $\frac{mv^2}{72}$ , (iv)  $x = \sqrt{\frac{m}{6k}} \cdot v$

15. Impulse =  $m(-2\hat{i} - \hat{j} + 8\hat{k})$ ,  $e = \frac{6}{17}$  16.  $V_A = \sqrt{\frac{g}{12}} \text{ m/s}, S_{\max} = \frac{49}{48} \text{ m}$

17. (i)  $\sqrt{\frac{2ag}{3}}$  (ii)  $\frac{3v}{g}$ , (iii)  $\frac{2v}{g}$

18.  $10^5 \text{ m}$

19. (i) 0.66 (ii) 4m

20. 2 sec

21.  $\vec{F} = \frac{4mv}{\sqrt{3}\Delta t} \left( \frac{\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{k} \right), \vec{N} = \left( \frac{2mg}{\sqrt{3}\Delta t} + mg \right) \hat{k}$

22.  $\frac{H^5}{L^4}$

23. 4

24. 6 m/s

### Exercise (O-1)

1.

C

2.

C

3.

B

4.

D

5.

B

6.

C

7.

C

8.

A

9.

C

10.

A

11.

(i) B; (ii) C

12.

D

13.

D

14.

B

15.

C

16.

C

17.

A

18.

B

19.

B

20.

B

21.

B

22.

C

23.

D

24.

D

25.

D

26.

C

27.

D

28.

A

29.

ABD

30.

AC

31.

A

32.

AB

33.

B

34.

C

35.

D

36.

A

37.

A

38.

(A)→S; (B)→Q; (C)→P; (D)→R

39.

(A)→Q; (B)→S; (C)→P;

40.

(A)→PRT; (B)→QS; (C)→PST; (D)→PST

41.

(A)→P; (B)→S; (C)→QR; (D)→T

### Exercise (O-2)

- |     |  |     |      |     |      |     |     |     |     |
|-----|--|-----|------|-----|------|-----|-----|-----|-----|
| 1.  | A                                      | 2.  | D    | 3.  | A    | 4.  | C   | 5.  | A   |
| 6.  | B                                      | 7.  | B    | 8.  | B    | 9.  | D   | 10. | C   |
| 11. | C                                      | 12. | C    | 13. | D    | 14. | D   | 15. | C   |
| 16. | B                                      | 17. | B    | 18. | D    | 19. | D   | 20. | C   |
| 21. | D                                      | 22. | D    | 23. | A    | 24. | B   | 25. | B   |
| 26. | D                                      | 27. | D    | 28. | B    | 29. | B   | 30. | D   |
| 31. | A                                      | 32. | B    | 33. | B    | 34. | BD  | 35. | ABD |
| 36. | AC                                     | 37. | BCD  | 38. | ABCD | 39. | AB  | 40. | AC  |
| 41. | ABC                                    | 42. | ABCD | 43. | BD   | 44. | ABD | 45. | BC  |
| 46. | A                                      | 47. | A    | 48. | D    | 49. | A   | 50. | C   |
| 51. | B                                      | 52. | C    | 53. | D    | 54. | A   | 55. | C   |
| 56. | A                                      | 57. | D    | 58. | B    | 59. | D   | 60. | B   |
| 61. | B                                      |     |      |     |      |     |     |     |     |
| 62. | (A)→(R); (B)→(T); (C)→(T); (D)→(S)     |     |      |     |      |     |     |     |     |
| 63. | (A)→(Q); (B)→(Q); (C)→(P,R); (D)→(Q,S) |     |      |     |      |     |     |     |     |
| 64. | (A)→Q; (B)→PQ; (C)→R; (D)→S;           |     |      |     |      |     |     |     |     |
| 65. | (A)→S; (B)→R; (C)→ Q; (D)→ P           |     |      |     |      |     |     |     |     |

### Exercise (JM)

1.	2	2.	1	3.	2	4.	3	5.	1	6.	3	7.	3
8.	1	9.	2	10.	4	11.	1	12.	4	13.	3	14.	2
15.	1	16.	1	17.	4	18.	1	19.	4	20.	3	21.	3
22.	3	23.	4	24.	1	25.	4	26.	3	27.	3	28.	(1.00)
29.	3	30.	3	31.	2	32.	2	33.	2	34.	4	35.	23.00
36.	10.00	37.	4	38.	2	39.	120.00	40.	4	41.	3.00	42.	1
43.	3	44.	3	45.	4								

### Exercise (JA)

- |     |   |     |   |     |       |     |     |     |   |  |  |  |  |
|-----|---|-----|---|-----|-------|-----|-----|-----|---|--|--|--|--|
| 1.  | $\vec{P}_{PM} = m\vec{v}_{PM} = -mv_2 \sin \omega t \hat{i} + m(v_2 \cos \omega t - v_1) \hat{j}$ |     |   |     |       |     |     |     |   |  |  |  |  |
| 2.  | B or D  | 3.  | D | 4.  | D     | 5.  | AD  | 6.  | B |  |  |  |  |
| 7.  | B   | 8.  | C | 9.  | 4 m/s | 10. | C   | 11. | A |  |  |  |  |
| 12. | AC  | 13. | D | 14. | AC    | 15. | 5   | 16. | B |  |  |  |  |
| 17. | AC  | 18. | B | 19. | 30    | 20. | 1,4 |     |   |  |  |  |  |