

# Energy efficiency analysis of a series plug-in hybrid electric bus with different energy management strategies and battery sizes



Xiaosong Hu<sup>\*</sup>, Nikolce Murgovski, Lars Johannesson, Bo Egardt

Department of Signals and Systems, Chalmers University of Technology, 41296 Gothenburg, Sweden

## HIGHLIGHTS

- Recuperation and fuel to traction efficiencies are analyzed for a PHEV powertrain.
- Two different energy controls are compared in terms of the two efficiencies.
- Impact of battery downsizing on the two efficiencies is quantified.
- Convex modeling and optimization are used to analyze the powertrain.

## ARTICLE INFO

### Article history:

Received 8 February 2013

Received in revised form 15 May 2013

Accepted 29 June 2013

### Keywords:

Recuperation efficiency

Fuel-to-traction efficiency

TTW analysis

Plug-in hybrid electric vehicle

Convex optimization

## ABSTRACT

This paper is concerned with the tank to wheel (TTW) analysis of a series plug in hybrid electric bus operated in Gothenburg, Sweden. The bus line and the powertrain model are described. The definition and the calculation method of the recuperation and fuel to traction efficiencies are delineated for evaluating the TTW energy conversion. The two efficiencies are quantified and compared for two optimization based energy management strategies, in which convex modeling and optimization are used. The impact of downsizing the battery on the two efficiencies is also investigated.

© 2013 Elsevier Ltd. All rights reserved.

## 1. Introduction

Oil supply uncertainty, growing mobility demand, and increasingly stringent regulations on pollutants and carbon footprint are expediting a paradigm shift towards sustainable transportation [1–3]. As key ingredients, hybrid electric vehicles (HEVs) are being actively developed by automotive companies worldwide to pursue higher fuel economy than conventional internal combustion engine (ICE) vehicles without inducing range anxiety [4–8]. Owing to vehicle to grid (V2G) services, plug in hybrid electric vehicles (PHEVs) potentially can take advantage of renewable energy sources to reduce reliance on fossil fuels and thus are an important solution to reducing carbon dioxide emission in the transportation sector [9,10].

In order to evaluate the total energy consumption and carbon dioxide emission of a vehicle in a broad sense, well to wheel (WTT) analysis is often needed [11–14]. It consists of well to tank

(WTT) and tank to wheel (TTW) analyses. WTT takes the upstream processes into account and focuses on the fuel production, processing, and delivery, such as oil refinery and electricity generation, in which sophisticated chemical and physical processes are involved. TTW is concerned with energy conversion efficiency from the on board fuel/electricity to the mechanical energy demanded at the wheels. For automotive engineers, TTW is often applied to assess the energy saving capability of a vehicle design and to guide the improvement of the vehicle energy efficiency. In this study, the TTW energy analysis is therefore considered.

One lumped energy efficiency ratio of the demanded mechanical energy at the wheels to the needed fuel energy was commonly used for the TTW analysis of a vehicle [12,13,15]. This lumped TTW efficiency is well suited for conventional ICE vehicles. Nevertheless, for HEVs or PHEVs with braking energy regeneration, the efficiency may become arbitrarily large, leading to no significance [16].

Another method to describe the TTW analysis of electrified mobility is to characterize each energy conversion process occurring inside the driveline by an averaged efficiency over the driving cycle. For example, energy conversion efficiencies of hybrid electric vehicles with different topologies were investigated by using this

<sup>\*</sup> Corresponding author. Address: E-building, Hörsalsvägen 11, 41296 Gothenburg, Sweden. Tel.: +46 31 772 1538; fax: +46 31 772 1748.

E-mail address: [xiaosong@chalmers.se](mailto:xiaosong@chalmers.se) (X. Hu).

methodology [17–20]. The energy conversion phenomena in PHEVs were also analyzed by considering the complete energy path [21]. The advantage of the approach is the availability of a thorough understanding of efficiencies of all the energy conversion steps within the powertrain. However, since the method needs a plethora of distinct efficiencies, the large complexity may be undesirable to a system level fuel/electricity energy estimation and analysis for HEVs or PHEVs.

Recently, a simple yet useful concept for the TTW analysis was proposed for HEVs [16]. In order to account for the energy recuperation, the mechanical energy demand at the wheels is viewed as a summation of dissipative energy and circulating energy. Accordingly, the fuel to traction efficiency and recuperation efficiency are defined to evaluate the TTW process, rather than one lumped efficiency. The novel concept is the most simple and straightforward extension of the conventional TTW efficiency, which is meaningful and applicable to vehicles with energy recuperation. Furthermore, it allows automotive engineers to easily assess the energy effectiveness of electrified powertrains and to find possible solutions with enhanced energy efficiencies from a system level.

The fuel to traction and recuperation efficiencies of a parallel hybrid electric passenger car with an optimal control strategy were analyzed in [16]. However, in the literature, there is a lack of discussion on quantification of the two efficiencies for PHEVs, particularly on how PHEV energy management strategy and battery size influence them. The purpose of this paper is to analyze the fuel to traction and recuperation efficiencies for a series plug-in hybrid electric bus operating in Gothenburg, Sweden. The main contribution is twofold: (1) the fuel to traction and recuperation efficiencies of the bus are analyzed and compared for two different optimization based energy management strategies, i.e., the charge depleting and charge sustaining (CD-CS) and blended controls [22–27]. The convex optimization is herein adopted for the two strategies, as it has been certified in our previous work [28–30] that compared to dynamic programming (a benchmark), the convex optimization can accomplish a comparable result with a much lower computational intensity; (2) how downsizing the bus battery affects the two efficiencies is quantified.

The remainder of the paper is organized as follows: Section 2 introduces the bus line and the powertrain model; the definition and the calculation method of the fuel to traction and recuperation efficiencies are depicted in Section 3; the convex optimization based two energy management strategies for the bus are presented in Section 4; the energy efficiency analysis and comparison for the two strategies are discussed in Section 5, in which the impact of downsizing the battery on the bus energy efficiencies is evaluated; Conclusions are finally drawn in Section 6.

## 2. Bus line and powertrain model

The PHEV considered in this study is a bus with a series powertrain topology [28]. The powertrain does not have a direct mechanical link between the ICE and the wheels, as shown in Fig. 1. The wheels are propelled by a 220 kW electrical motor (EM, 2200 rpm, and no EM reduction gear) powered by a lithium ion battery pack and/or a 180 kW engine generator unit (EGU). The bus is driven on a bus line in Gothenburg, Sweden, which is described by demanded velocity and road gradient at each point of time (see Fig. 2). The quasi static modeling methodology is employed to model the powertrain [5]. The velocity and force demands of the bus can be translated into an angular velocity  $\mathbf{x}(t)$  and torque  $T_v(t)$  on the shaft between the EM and the final drive, given the demanded acceleration and speed on the bus line and the known vehicle parameters, such as inertia, aerodynamic drag, rolling resistance, and wheel radius.

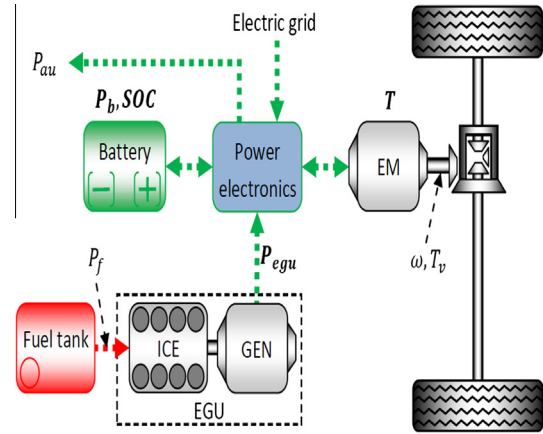


Fig. 1. Powertrain configuration of the plug-in bus.

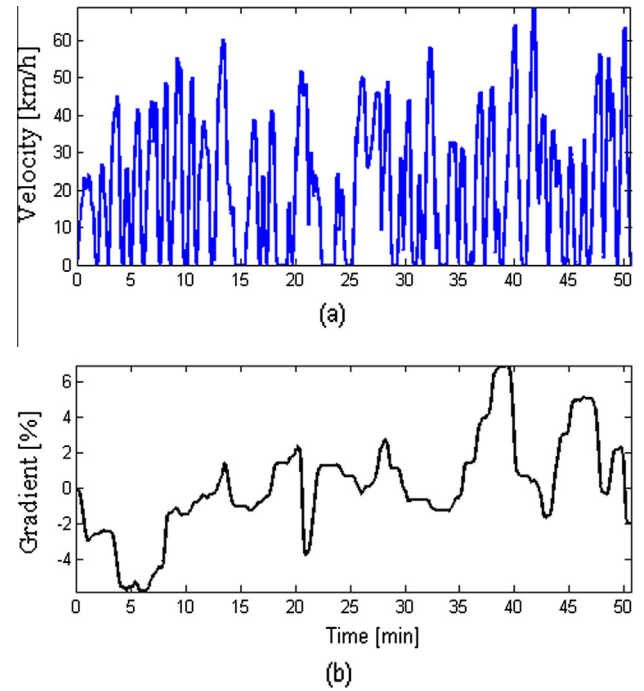


Fig. 2. Driving cycle of the bus line: (a) velocity demand; (b) road gradient.

The EM, which delivers a torque  $T(t)$ , is designed to be able to meet the high torque demands. Moreover, the EM is also used to recuperate braking energy; when either its torque limit  $T_{min}(\mathbf{x}(t))$  or the battery charging limit is met, friction braking is supplemented. The power balance equation is described as

$$P_m(t) + P_{loss,m}(t) + P_{au}(t) = P_b(t)g_1 + P_{egu}(t)g_{con}, \quad (1)$$

where  $P_m$  is the EM power on the shaft between the EM and the final drive calculated by

$$P_m(t) = T(t)\mathbf{x}(t), \quad (2)$$

and  $P_{loss,m}$  is the EM power loss characterized by a quadratic function of the motor torque

$$P_{loss,m} = b_0(\mathbf{x})T^2 + b_1(\mathbf{x})T + b_2(\mathbf{x}), \quad (3)$$

in which  $b_0$ ,  $b_1$ , and  $b_2$  are nonnegative speed dependent coefficients.  $P_{au}$ ,  $P_b$ , and  $P_{egu}$  are the auxiliary power, battery power, and EGU power, respectively.  $g_{con}$  is the efficiency of the power con

verter (here the averaged efficiency is used, since our focus is on the system level behavior of the powertrain [5,28]), and

$$\begin{cases} g_1 & g_{con}, & P_b \geq 0 \\ g_1 & \frac{1}{g_{con}}, & P_b < 0. \end{cases} \quad (4)$$

In order to preserve the optimization problem convexity (described later in Section 4), Eq. (1) can be rewritten as

$$P_m(t) + P_{loss,m}(t) + P_{au}(t) \quad \min \left( P_b(t)g_{con}, \frac{P_b(t)}{g_{con}} \right) + P_{egu}(t)g_{con}. \quad (5)$$

The EGU power loss is modeled as a quadratic function of  $P_{egu}$ , thereby yielding the fossil fuel (diesel) power

$$P_f(t) = a_0 P_{egu}^2(t) + (a_1 + 1)P_{egu}(t) + a_2 e(t) \quad (6)$$

with  $a_i \geq 0$ ,  $i \in \{0, 2\}$ . For better readability, we represent all the optimization variables in bold. The binary signal  $e(t)$  allows for zero diesel power by removing the idling loss  $a_2$  when the engine is off.  $e(t)$  is herein determined by heuristics that turn the engine on if the vehicle power exceeds a threshold  $P_{on}^*$ , i.e.,

$$e(t) = \begin{cases} 1, & T_v(t)X(t) \geq P_{on}^* \\ 0, & \text{otherwise.} \end{cases} \quad (7)$$

The optimal power threshold  $P_{on}^*$  is found by iteratively solving the convex optimization problem for several values of  $P_{on}$  within the power range of the vehicle. The detailed procedure can be found in [29], where it has been shown that these heuristics ensure a solution close to the global optimum. The efficiencies of the EM and EGU are illustrated in Fig. 3. The quadratic efficiency functions, i.e., Eqs. (3) and (6), are used to preserve the optimization problem convexity, and details on the validity of using quadratic losses for these components can be found in [29–31]. The battery pack consists of  $n_c$  consistent cells, where each cell is modeled as a simple resistive circuit. Therefore, the pack power can be described by

$$P_b(t) = (V_{oc}(t)I(t) - I^2(t)R)n_c, \quad (8)$$

where  $V_{oc}$  is the cell open circuit voltage (OCV),  $I$  is the cell current, and  $R$  is the internal resistance (assumed constant). According to Eq. (8), the current  $I$  can be expressed as

$$I(t) = \frac{1}{2R} \left( V_{oc}(t) \pm \sqrt{V_{oc}^2(t) - \frac{4RP_b(t)}{n_c}} \right) \in [I_{min}, I_{max}], \quad (9)$$

where  $[I_{min}, I_{max}]$  is the cell current limits. Additionally, the pack power is limited by

$$P_b(t) \leq \frac{V_{oc}^2(t)n_c}{4R}. \quad (10)$$

The cell OCV  $V_{oc}(t)$  is modeled as an affine function of State of Charge (SOC)

$$V_{oc}(t) = d_0 \text{SOC}(t) + d_1, \quad (11)$$

which is a rational approximation within the cell SOC range  $[\text{SOC}_{min}, \text{SOC}_{max}]$  in PHEVs applications, especially for lithium iron phosphate cells, such as A123's ANR26650m1 cell (see Fig. 4). The cell dynamics is described by

$$\frac{d\text{SOC}(t)}{dt} = \frac{I(t)}{Q_n}, \quad (12)$$

where  $Q_n$  is the cell nominal capacity. A constraint is imposed on the initial SOC

$$\text{SOC}(t_0) = \text{SOC}_0. \quad (13)$$

The main specification of the cell (A123's ANR26650m1) is listed in Table 1, where the internal resistance is an average in the usable SOC range [29,31,40]. Cell packaging and circuitry are assumed to account for 12.3% of the total mass of the battery pack [32]. The main vehicle parameters are shown in Table 2. For simplicity, we assume that there is the constant number of passengers on board. Note that since the bus operates on a specific route and experiences a relatively long time downward slope (see Fig. 2) at the beginning, the battery initial SOC is specified to be 60% rather than 80% to be able to recuperate the initial braking energy.

### 3. Fuel-to-traction and recuperation efficiencies for plug-in bus

#### 3.1. Dissipative energy and circulating energy

The traction power  $P(t)$  that is used to propel the wheels and auxiliary devices of the bus is calculated by

$$P(t) = P_{ae}(t) + P_{rol}(t) + P_{au}(t) + P_{ac}(t) + P_{gr}(t), \quad (14)$$

where  $P_{ae}(t) = q_{air}A_f C_d v^3(t)/2$  is the power to overcome the aerodynamic drag force, with  $v$  being the vehicle velocity;  $P_{rol}(t) = (m_v + m_p)g \cos(a(t))v(t)$  is the power to overcome the rolling resistance, in which  $a$  is the slope, and  $m_p$  is the pack mass;  $P_{ac}(t) = (m_v + m_p)v(t) \frac{dv(t)}{dt}$  and  $P_{gr}(t) = (m_v + m_p)g \sin(a(t))v(t)$  are

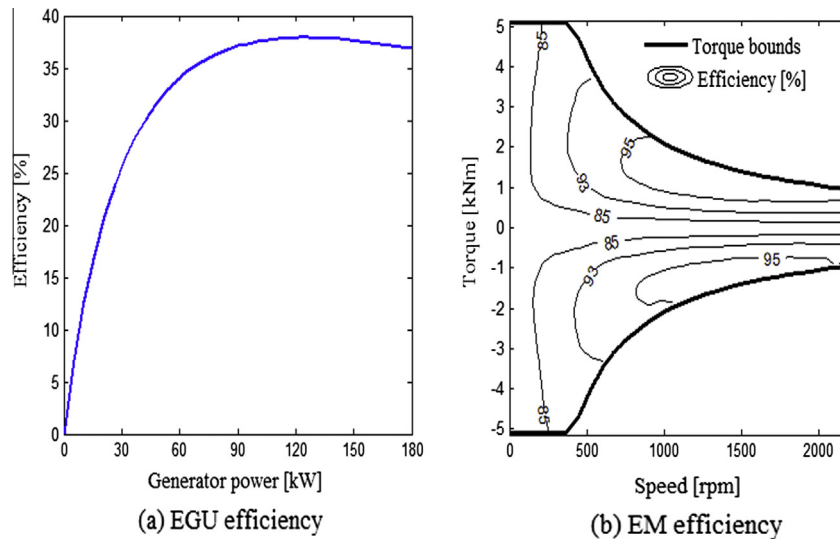


Fig. 3. Efficiencies of the EGU and EM: (a) EGU; (b) EM.

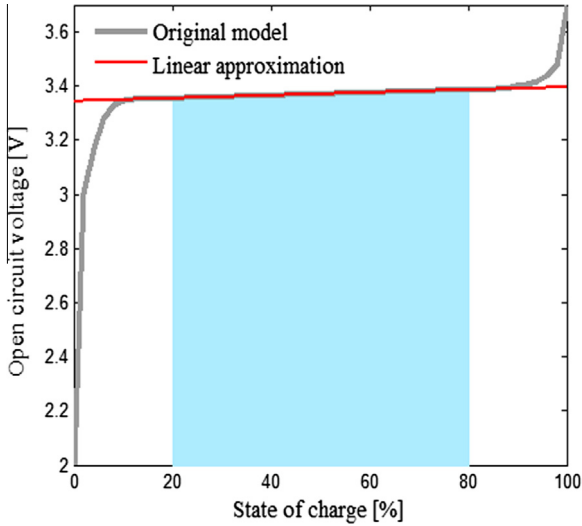


Fig. 4. Cell OCV approximation (A123's ANR26650m1 cell).

**Table 1**  
Li-ion cell specification.

Nominal capacity	$Q_n = 8280 \text{ As}$
Nominal voltage	$\bar{V}_{oc} = 3.3 \text{ V}$
Maximum discharging current	$I_{max} = 70 \text{ A}$
Maximum charging current	$I_{min} = -45 \text{ A}$
Internal resistance	$R = 0.01 \text{ X}$
Mass	$m_{bc} = 0.07 \text{ kg}$
Coefficient for OCV approximation	$d_0 = 0.5 \text{ V}$
Coefficient for OCV approximation	$d_1 = 2.95 \text{ V}$

**Table 2**  
Vehicle parameters.

Parameter	Value	Parameter	Value
Frontal area	$A_f = 7.54 \text{ m}^2$	Averaged converter efficiency	$\eta_{con} = 98\%$
Aerodynamic drag coefficient	$c_d = 0.7$	Final drive efficiency	$\eta_{fd} = 97\%$
Air density	$\rho_{air} = 1.184 \text{ kg/m}^3$	Diesel price	$c_f = 3.47 \times 10^{-8} \text{ €/J}$
Gravity	$g = 9.81 \text{ m/s}^2$	Electricity price	$c_{el} = 2.78 \times 10^{-8} \text{ €/J}$
Rolling resistance coefficient	$c_r = 0.007$	Minimum allowable pack SOC	$SOC_{min} = 20\%$
Wheel radius	$R_w = 0.509 \text{ m}$	Maximum allowable pack SOC	$SOC_{max} = 80\%$
Final gear	$c = 4.7$	Initial SOC	$SOC_0 = 60\%$
Vehicle Mass excluding the pack	$m_v = 14.5 \text{ ton}$	Base number of cells	$n_c = 1000$

the powers for acceleration/deceleration and driving uphill/down hill, respectively.  $P(t)$  can be split up into two groups: one is the dissipative power  $P_{dis}(t) = P_{ae}(t) + P_{rol}(t) + P_{au}(t)$ ; the other is the conservative power  $P_{cons}(t) = P_{ac}(t) + P_{gr}(t)$  [16]. Since the initial velocity and altitude of the bus are the same as those at the final time in the driving cycle, the integration of the conservative power over the duration of the cycle is equal to zero. If there is a perfect recuperation, the traction energy demand per distance  $E_{trac}$  would equal the dissipative energy  $E_{dis}$  per distance

$$E_{trac} = \frac{\int_{t_0}^{t_f} P(t) dt}{d} = \frac{\int_{t_0}^{t_f} P_{dis}(t) dt}{d} = E_{dis}, \quad (15)$$

where  $t_0$ ,  $t_f$ , and  $d$  are the initial time, final time, and the driving distance in the cycle, respectively. If there is no recuperation (i.e., ICE vehicles),

$$E_{trac} = \frac{\int_{P(t) \geq 0} P(t) dt}{d} = \frac{\int_{t_0}^{t_f} P_{dis}(t) dt}{d} + \frac{\int_{P(t) < 0} P(t) dt}{d} = E_{dis} + \frac{\int_{P(t) < 0} P(t) dt}{d}, \quad (16)$$

where the term  $\frac{\int_{P(t) < 0} P(t) dt}{d}$  is the circulating energy per distance  $E_{cir}$  that is a temporal vehicle energy circulating in the form of kinetic or potential energy and is ultimately dissipated during friction braking. Therefore, Eq. (16) for vehicles without recuperation functionality can be abbreviated as

$$E_{trac} = E_{dis} + E_{cir}. \quad (17)$$

In the case of a real recuperation, the following energy balance equation holds

$$E_{trac} = E_{dis} + E_{cir} - E_{rec}, \quad (18)$$

where  $E_{rec}$  is the net energy recuperated that is usable for traction. According to Eqs. (15) and (18), it can be found that the perfect recuperation is  $E_{rec} = E_{cir}$ .

### 3.2. Recuperation efficiency

The recuperation efficiency is defined as follows:

$$\eta_{rec} = \frac{E_{rec}}{E_{cir}}. \quad (19)$$

Since  $E_{cir}$  can be easily achieved from the driving cycle, the key task is to calculate  $E_{rec}$ . The recuperation energy involves two flow ways (i.e., the input and output ways), as shown in Fig. 5. The recuperation capability is governed by the loss of each step, the motor torque limitation, and the lithium ion battery current and charge limitations.

Firstly, we find the time set  $S$  such that

$$S = \{t | t \in [t_0, t_f], P(t) - P_{au}(t) < 0, P_b(t) < 0\}. \quad (20)$$

Note that the auxiliary power demand  $P_{au}$  is physically loaded at the converter. The absolute input energy for the set  $S$  at the wheels per distance,  $E_{w,in}$ , is calculated by

$$E_{w,in} = \frac{\int_S |P(t) - P_{au}(t)| dt}{d}. \quad (21)$$

The associated net energy stored in the battery

$$E_{b,in} = \frac{\int_S |P_b(t) + I(t)^2 R_{nc}| dt}{d}. \quad (22)$$

The resulting energy loss

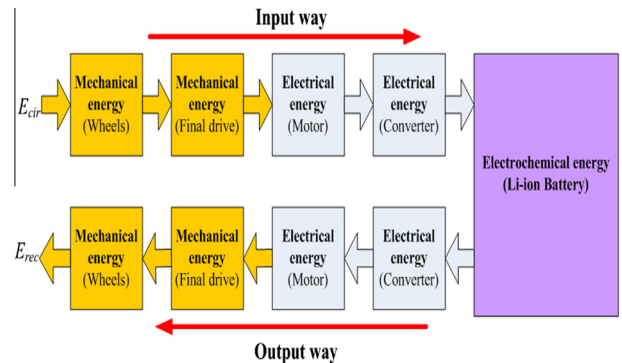


Fig. 5. Schematic of recuperation energy flow of the bus.

$$E_{loss,in} = E_{w,in} - E_{b,in} - \frac{\int_S P_{au}(t)dt}{d}. \quad (23)$$

The cycle averaged wheel to battery energy efficiency can thus be obtained by

$$g_{wb,in} = 1 - \frac{E_{loss,in}}{E_{w,in}}, \quad (24)$$

yielding the net recuperation energy stored in the battery

$$E_{rec,in} = E_{cir} g_{wb,in}. \quad (25)$$

Then, we find the time set  $D$  such that

$$D = \{t | t \in [t_0, t_f], P(t) - P_{au}(t) \geq 0, P_b(t) \geq 0\}. \quad (26)$$

The associated motor energy loss in propelling condition

$$E_{loss,m} = \frac{\int_D P_{loss,m}(t)dt}{d}, \quad (27)$$

and the averaged motor efficiency is

$$g_{m,out} = 1 - \frac{E_{loss,m}d}{\int_D P_b(t)dt} + P_{egu}(t)g_{con} - P_{au}(t)dt. \quad (28)$$

The battery energy loss in the time set  $D$

$$E_{loss,b} = \frac{\int_D I(t)^2 R_n dt}{d}, \quad (29)$$

and the corresponding battery efficiency in the output way is

$$g_{b,out} = 1 - \frac{E_{loss,b}d}{\int_D P_b(t) + I(t)^2 R_n dt}. \quad (30)$$

The cycle averaged battery to wheel energy efficiency can thus be obtained by

$$g_{bw,out} = g_{b,out} g_{con} g_{m,out} g_{fd}, \quad (31)$$

where  $g_{fd}$  is the efficiency of the final drive. According to Eqs. (25) and (31), we can acquire the net recuperation energy for traction

$$E_{rec} = E_{rec,in} g_{bw,out}, \quad (32)$$

and the cycle averaged recuperation efficiency

$$g_{rec} = \frac{E_{rec}}{E_{cir}} = g_{wb,in} g_{bw,out}. \quad (33)$$

### 3.3. Fuel to traction efficiency

The fuel to traction efficiency is defined as

$$g_{ft} = \frac{E_{trac}}{E_{ef}} = \frac{E_{dis} + (1 - g_{rec})E_{cir}}{E_{ef}}, \quad (34)$$

where  $E_{ef}$ , termed as the equivalent fuel energy, is the sum of the consumed diesel energy and electric energy per distance.  $g_{ft}$  is the cycle averaged conversion efficiency from the total consumed energy (diesel and electricity) to the mechanical energy at the wheels and the electrical energy for the auxiliaries. Compared to the electrical and mechanical paths, the ICE has a far lower efficiency, leading to the dominant restriction for  $g_{ft}$ . Given Eq. (34), the equivalent fuel energy  $E_{ef}$  is obtained by

$$E_{ef} = \frac{E_{dis} + (1 - g_{rec})E_{cir}}{g_{ft}}. \quad (35)$$

Then, based on the initial and final values of the battery SOC,  $E_{ef}$  can be simply split to achieve the individual diesel energy and electricity.

## 4. Energy management strategies based on convex optimization

### 4.1. Convex modeling

A convex optimization problem can be written as follows:

$$\begin{aligned} & \text{minimize} \quad f_0(\mathbf{x}) \\ & \text{subject to} \quad f_i(\mathbf{x}) \leq 0, \quad i = 1, \dots, m \\ & \quad \quad \quad h_j(\mathbf{x}) = 0, \quad j = 1, \dots, p \\ & \quad \quad \quad \mathbf{x} \in \mathbf{v}, \end{aligned} \quad (36)$$

where  $\mathbf{v} \in \mathbb{R}^n$  is a convex set,  $f_i(\mathbf{x}), i = 0, \dots, m$ , are convex functions, and  $h_j(\mathbf{x})$  are affine functions in the optimization vector  $\mathbf{x}$  [33]. In our studied problem, the optimization variables are  $\mathbf{T}(t)$ ,  $\mathbf{P}_b(t)$ ,  $\mathbf{P}_{egu}(t)$ , and  $\mathbf{SOC}(t)$ , and the constraints are vehicle power balance, i.e., Eq. (5), and the battery constraints Eqs. (9)–(13). It is easy to notice that Eqs. 5, 9, and 10 do not comply with the definition of a convex problem. Eq. (5) can be written as convex, by relaxing the equality to inequality; it has been suggested in [34] that Eqs. (9) and (10) can be reformulated as convex functions by introducing a new variable

$$\mathbf{E}(t) = \frac{CV_{oc}^2(t)n}{2} \quad (37)$$

with  $C = \frac{2Q_n}{V_{oc}}$ . The resulting convex constraints are then given as

$$P_m(t) + P_{loss,m}(t) + P_{au}(t) \leq \min \left( \mathbf{P}_b(t)g_{con}, \frac{\mathbf{P}_b(t)}{g_{con}} \right) + \mathbf{P}_{egu}(t)g_{con}, \quad (38)$$

$$\mathbf{P}_b(t) \leq \sqrt{\frac{2\mathbf{E}(t)n_c}{C}} - R_{min}^2 n_c, \quad (39)$$

$$\mathbf{P}_b(t) \leq \frac{\mathbf{E}(t)}{2RC}, \quad (40)$$

$$\mathbf{E}(t) \leq \sqrt{\mathbf{E}^2(t) - 2RCE(t)\mathbf{P}_b(t)} \leq I_{max}R\sqrt{2CE(t)n_c}, \quad (41)$$

$$\frac{d\mathbf{E}(t)}{dt} \leq \frac{d_0}{RQ_n}(\mathbf{E}(t) - \sqrt{\mathbf{E}^2(t) - 2RCE(t)\mathbf{P}_b(t)}), \quad (42)$$

$$\mathbf{E}(t) \in \frac{C}{2}[V_{oc}^2(SOC_{min}), V_{oc}^2(SOC_{max})]n_c, \quad (43)$$

$$\mathbf{E}(t_0) \leq \frac{C}{2}V_{oc}^2(SOC_0)n_c. \quad (44)$$

### 4.2. Energy management strategies

As opposed to HEVs, the battery charge in PHEVs is used. The optimal energy control strategy for PHEVs is often defined to minimize the equivalent fuel cost (i.e., liquid fuel and electricity). For trips shorter than the all electric range (AER), PHEVs are equivalent to battery electric vehicles (BEVs), and the associated optimal energy control charge depleting (CD) mode is deployed, since electricity is considerably cheaper than liquid fuel [35]. The optimal strategy in this case is trivial [35–37]. The discussed plug-in hybrid electric bus has a fixed driving route longer than the AER, which gives a degree of freedom concerning the optimal power split between the battery and the EGU. Otherwise, it will become a battery electric bus that typically needs considerably larger and more expensive battery pack. Two feasible energy management strategies, based on convex optimization, are considered. One is the CD-CS strategy for which the bus firstly operates in CD mode until the battery charge arrives at a pre-specified threshold and then switches to convex optimization based CS mode; the other is the blended strategy, implying that the optimal power allocation between the battery and the EGU is attained throughout the route by convex optimization. The two strategies are formulated in Table 3. In order to guarantee a fair comparison, the final battery SOC for the two strategies should be identical. The natural and straightforward form of the convex optimization problem is automatically parsed by a tool, CVX [33,38], so as to derive a general



**Table 3**

Two energy management strategies for the plug-in bus.

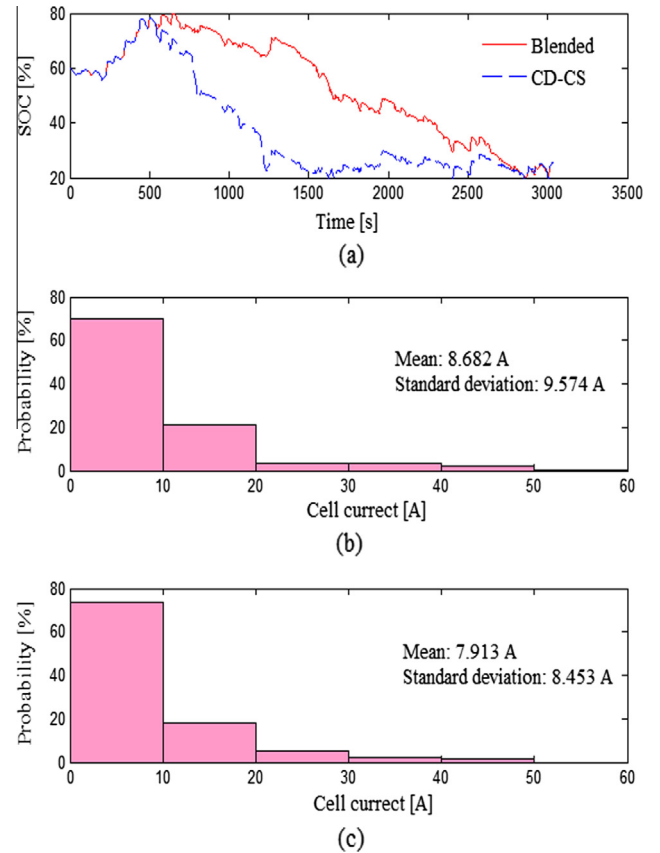
Convex-optimization-based energy management
<p>1. CD-CS strategy</p> <p>(1) CD Phase (no optimization)</p> <p>For <math>k = 1, \dots, N_{cd}</math>, with <math>SOC(N_{cd} + 1)</math> corresponding to the switching SOC (25% in our case)</p> <p>(a) Solve the cell current <math>I_0(k)</math> algebraically, according to the calculated battery power <math>P_{b,0}</math> without considering the cell current and charge limitations:</p> $T(k) \in [T_{min}(\mathbf{x}(k)), T_{max}(\mathbf{x}(k))], P_m(t) + P_{loss,m}(t) + P_{au}(t) \leq \min \left( P_{b,0}(t) g_{con}, \frac{P_{b,0}(t)}{g_{con}} \right),$ $I_0(k) = \frac{V_{oc}(k) \sqrt{\frac{V_{oc}^2(k)}{2R} - \frac{4P_{b,0}(k)}{g_{con}}}}{2R}.$ <p>(b) Achieve the correct cell current <math>I(k)</math>, considering the cell current and charge limitations:</p> $I(k) \in [I_{min}, I_{max}], SOC(k+1) = SOC(k) - \frac{I(k)Dt}{Q_n}, SOC(k) \in [SOC_{min}, SOC_{max}], Dt \text{ is the time interval.}$ <p>(c) Calculate the correct battery power <math>P_b(k) = V_{oc}(k)I(k)n_c - I^2(k)Rn_c</math>, <math>SOC(k)</math>, <math>E(k)</math>, and <math>T(k)</math>. Note that the above limitations are only active when the friction braking is needed.</p> <p>(2) CS Phase (convex optimization)</p> <p>For <math>k = N_{cd} + 1, \dots, N</math>, with <math>N</math> corresponding to the final time step</p> <p>Variables: <math>P_b(N - N_{cd}), P_{egu}(N - N_{cd}), E(N + 1 - N_{cd}), T(N - N_{cd})</math> (the number in parentheses is the vector length)</p> <p>Expressions: <math>P_f(N - N_{cd}), P_m(N - N_{cd}), P_{loss,m}(N - N_{cd}), J_c = \sum_{k=N_{cd}+1}^N c_f P_f(k)Dt</math></p> <p>Minimize <math>J_c</math></p> <p>Subject to Eqs. (38)–(43), <math>E(N_{cd} + 1) = E(N + 1)</math>, <math>T(k) \in [T_{min}(\mathbf{x}(k)), T_{max}(\mathbf{x}(k))]</math>, <math>P_{egu}(k) \in [0, P_{max,egu}]</math>, <math>P_{egu}(e_{off}) = 0</math> (<math>e_{off}</math> is an index vector of <math>e(k) = 0</math>, indicating the engine-off steps.)</p> <p>2. Blended strategy (convex optimization)</p> <p>For <math>k = 1, \dots, N</math>, with <math>N</math> corresponding to the final time step</p> <p>Variables: <math>P_b(N), P_{egu}(N), E(N + 1), T(N)</math></p> <p>Expressions: <math>P_f(N), P_m(N), P_{loss,m}(N), J_c = \sum_{k=1}^N c_f P_f(k)Dt + c_{et}(SOC_0 - SOC_f)Q_n \bar{V}_{oc} n_c</math></p> <p>Minimize <math>J_c</math></p> <p>Subject to Eqs. (38)–(44), <math>E(N + 1) = \frac{1}{2} V_{oc}^2(SOC_f) n_c</math>, <math>T(k) \in [T_{min}(\mathbf{x}(k)), T_{max}(\mathbf{x}(k))]</math>, <math>P_{egu}(k) \in [0, P_{max,egu}]</math>, <math>P_{egu}(e_{off}) = 0</math></p>

semi definite program (SDP) form. The bold variables are parameters to optimize (see Section 4.1). Note that the so called expressions in the table are not equality constraints, but are merely used for better readability. CVX substitutes the expressions with the corresponding mathematical operations. After these, the problem is passed to the solver, SeDuMi [39,40], to obtain the optimal variables with high computational efficiency. Since the optimization problem is convex, instead of trapping into local minima, a global optimization solution can be rapidly accomplished, with any initialization. Please refer to [33] to find more details on the theoretical and algorithmic properties of convex optimization.

## 5. Results and discussion

### 5.1. Efficiency analysis and comparison of the two energy control algorithms

Given the driving cycle and the optimal variables extracted by the foregoing algorithms, the recuperation and fuel to traction efficiencies can be calculated, as Section 3 describes. Before discussing the two efficiencies, it is important to investigate the operating states of the power sources (the battery and the EGU) in the powertrain. The cell SOC and current distribution for the two strategies are shown in Fig. 6. It is clear that both the average and maximum absolute cell currents in the blended algorithm are smaller than those in the CD-CS algorithm, leading to less battery energy loss. Observing the SOC trajectory, we can further conclude that the current difference is mainly existent in the discharging process. The round trip battery efficiency and the recuperation efficiencies in the input and output directions are shown in Fig. 7. Compared to the CD-CS strategy, the blended scenario has obviously higher averaged battery efficiency (by 1.73%) and recuperation efficiency in the output way (by 1.32%), because of much smaller discharging current. A slightly better recuperation efficiency in the input way is also observed for the blended algorithm, because cells have a slightly higher OCV (see Figs. 4 and 6) during charging and thus a little smaller energy loss, given the same charging power. The EGU efficiencies for the two strategies are similar, as shown in



**Fig. 6.** (a) SOC trajectories, (b) absolute current distribution in the CD-CS strategy, and (c) absolute current distribution in the blended strategy.

Fig. 8, since the convex optimization is used to determine the optimal EGU power in both strategies. The recuperation efficiency, fuel to traction efficiency, and energy consumption per kilometer are shown in Fig. 9. The recuperation efficiency for the CD-CS

strategy is 64.86%, while that for the blended strategy is 66.13% an improvement of 1.27% mainly resulting from the battery current deviation. The fuel to traction efficiency for the CD CS strategy is 35.83%, while that for the blended strategy is 36.13% an improvement of 0.3%. The fuel to traction efficiency is mainly limited by the EGU efficiency (see Fig. 8) and also affected by energy losses occurring in other electrical and mechanical paths, e.g., battery, power converter, electric motor, and final drive. The energy consumption per kilometer for the CD CS strategy is 9.02 MJ, while that for the blended strategy is 8.87 MJ an improvement of 0.15 MJ. Since both strategies consume the same amount of electricity, the blended strategy can save diesel energy of 0.15 MJ per kilometer, which is attributed to the superior recuperation and fuel to traction efficiencies. It is worth mentioning that since the transit bus runs approximately 50,000 km per year (much longer than standard passenger cars), the energy saving caused by the more advanced blended strategy is significant.

## 5.2. Impact of battery downsizing on the bus energy efficiency

In addition to a better TTW energy conversion, Section 5.1 shows that the blended energy control algorithm needs less battery discharging power, inducing a potential of reducing the battery size. As lithium ion battery is still highly costly in the market and is the most expensive component in the bus power train, downsizing is of great significance for decreasing the purchase cost of the bus. However, it is necessary to assess the influence of battery downsizing on the recuperation and fuel to traction efficiencies of the plug in bus. Given the reduced battery sizes, the round trip battery efficiency and the recuperation efficiencies in the input and output directions are shown for the optimal blended strategy in Fig. 10. It is notable that all the three efficiencies become lower with the battery downsizing. In particular, the recuperation efficiency in the input way has a dramatic

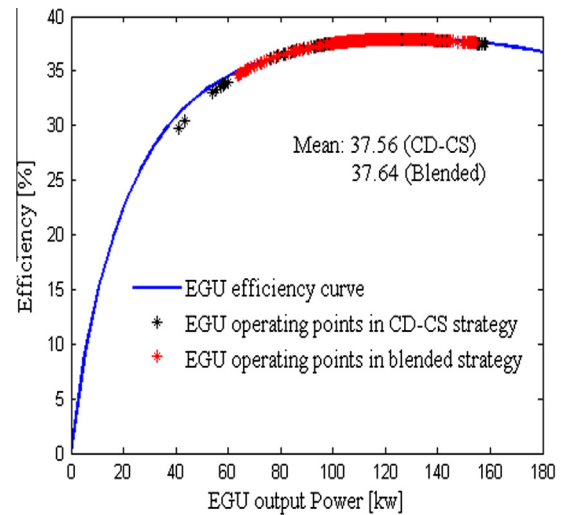


Fig. 8. EGU efficiencies.

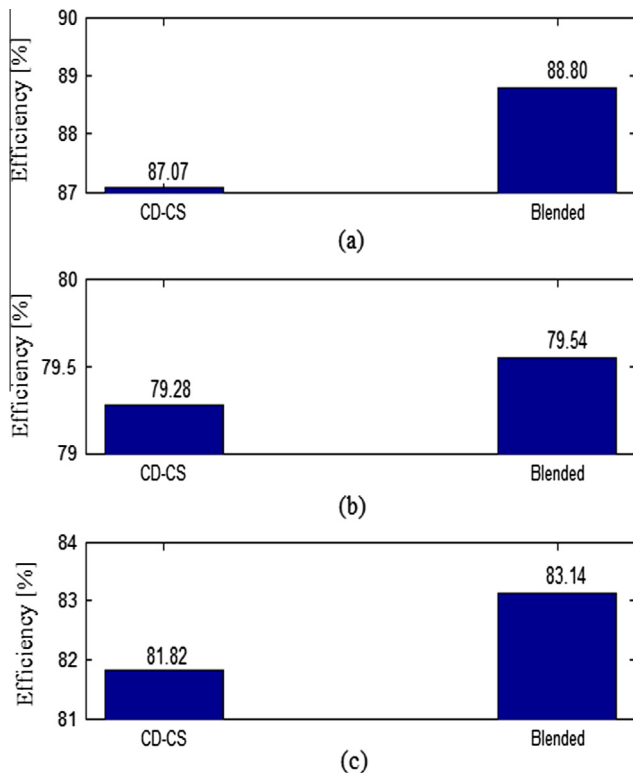


Fig. 7. (a) Round-trip battery efficiency, (b) recuperation efficiency in the input way, and (c) recuperation efficiency in the output way.

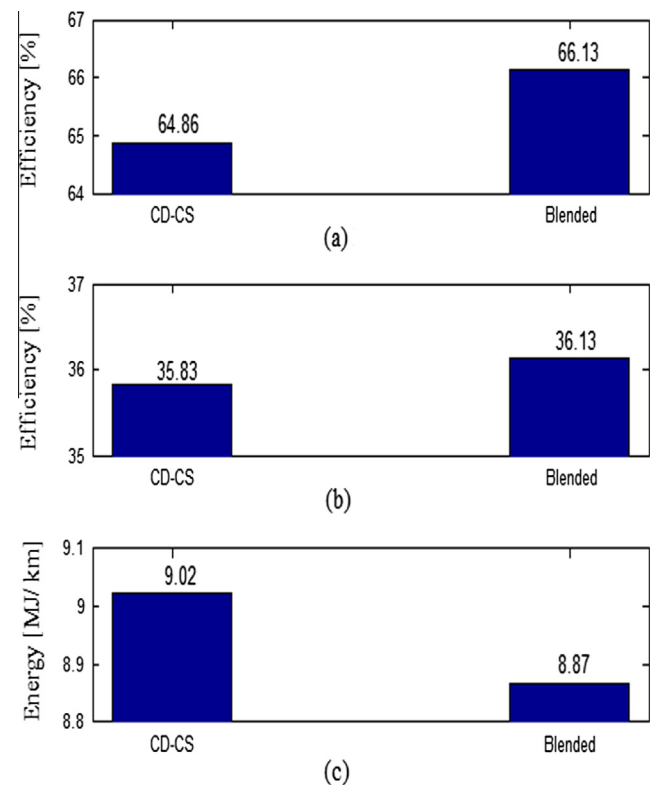
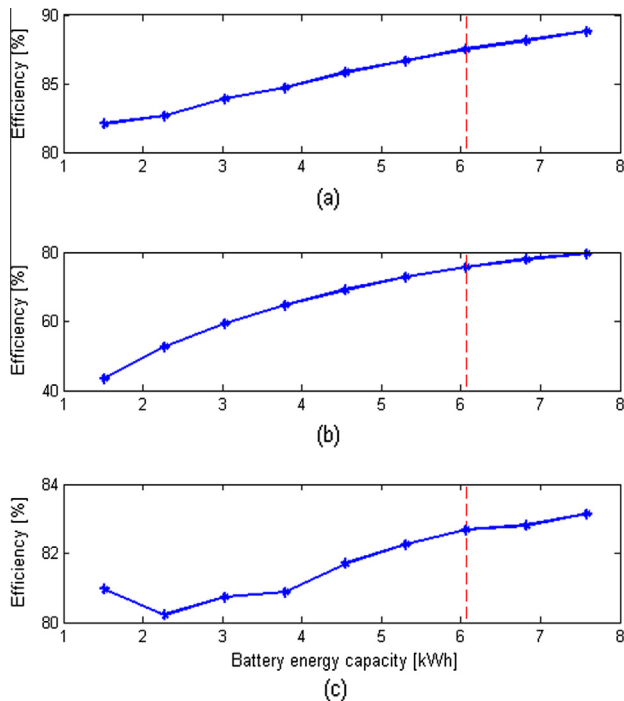


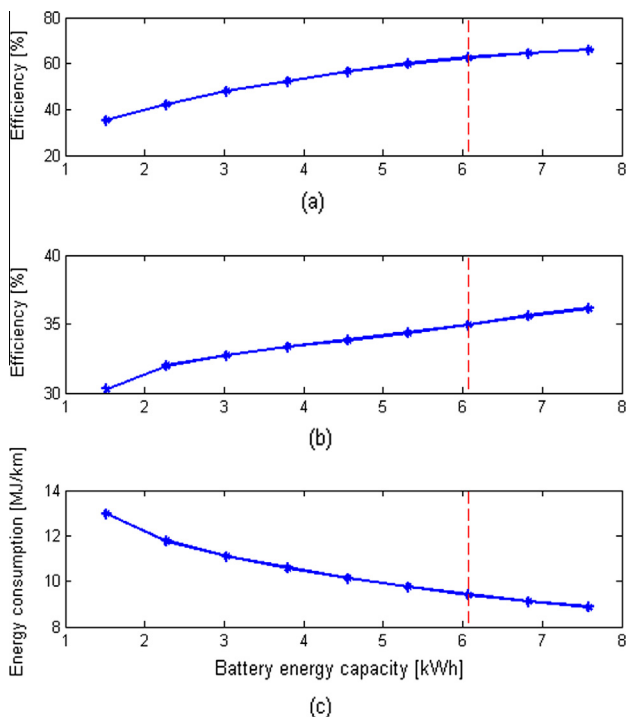
Fig. 9. (a) Recuperation efficiency (two-way), (b) fuel-to-traction efficiency, and (c) energy consumption per distance.

drop, since the battery current has an evident increase during recuperation when the engine is off, and the friction braking is more likely to be used to meet the current and charge limitations.

The associated impact on the recuperation efficiency, fuel to traction efficiency, and energy consumption per kilometer are shown in Fig. 11. It can be found that the battery downsizing has an apparently negative effect on the recuperation efficiency of the bus. The fuel to traction efficiency also decreases with the reduced battery size. Owing to the deterioration of the two efficiencies, the energy consumption per kilometer increases. Therefore, there is a tradeoff between battery downsizing and energy efficiency. The battery should be downsized with a careful consider



**Fig. 10.** Efficiency versus battery size: (a) round-trip battery efficiency; (b) recuperation efficiency in the input way; (c) recuperation efficiency in the output way.



**Fig. 11.** Efficiency versus battery size: (a) recuperation efficiency (two-way); (b) fuel-to-traction efficiency; (c) energy consumption per distance.

ation without sacrificing the two efficiencies to an unacceptable degree. For example, if the recuperation and fuel to traction efficiencies are required not to be less than 63% and 35%, respectively, there will be a possibility of reducing the pack by 1.52 kWh. On the other hand, from a viewpoint of cost, the tradeoff is between

the purchase cost and the operating cost, and the total ownership cost (TCO) should thus be considered. However, it is not trivial to find the best solution, since TCO depends on a variety of factors, such as diesel price, electricity price, battery price, total bus mile age, and battery replacement.

## 6. Conclusions

This paper analyzes the TTW energy conversion efficiency of a series plug in hybrid electric bus operating in Gothenburg, Sweden. The TTW process is characterized by the recuperation and fuel to traction efficiencies, which are quantified and compared for two optimization based energy management strategies, i.e., the CD CS and blended energy controls. The convex modeling of the bus powertrain is performed, and the efficient convex optimization is used for the two strategies. The efficiency analysis indicates that the recuperation efficiencies for the CD CS and blended strategies are 64.86% and 66.13%, respectively; the fuel to traction efficiencies are 35.83% and 36.13%; the diesel energy consumptions per kilometer are 9.02 MJ and 8.87 MJ. The blended algorithm thus leads to a more efficient TTW energy conversion of the plug in bus. Considering the potential of downsizing the battery for the blended algorithm that requires less battery discharging power, we also quantify the impact of battery downsizing on the recuperation and fuel to traction efficiencies and the energy consumption per kilometer. It can be found that as the battery energy capacity decreases, the two efficiencies become worse. There is, therefore, a tradeoff between the battery downsizing and energy efficiencies for the plug in bus with the blended energy management strategy.

## Acknowledgement

This work was supported by Swedish Energy Agency.

## References

- [1] Juul N, Meibom P. Road transport and power system scenarios for Northern Europe in 2030. *Appl Energy* 2012;92:573–82.
- [2] Takeshita T. Assessing the co-benefits of CO<sub>2</sub> mitigation on air pollutants emissions from road vehicles. *Appl Energy* 2012;97:225–37.
- [3] Kudoh Y, Ishitani H, Matsushashi R, Yoshida Y, Morita K, Katsuki S, et al. Environmental evaluation of introducing electric vehicles using a dynamic traffic-flow model. *Appl Energy* 2001;69:145–59.
- [4] Ehsani M, Gao Y, Gay SE, Emadi A. Modern electric, hybrid electric, and fuel cell vehicles—fundamentals, theory, and design. Boca Raton: CRC Press; 2005.
- [5] Guzzella L, Sciarretta A. Vehicle propulsion systems—introduction to modeling and optimization. 2nd ed. Berlin: Springer; 2007.
- [6] Mi C, Masrur MA, Gao DW. Hybrid electric vehicles—principles and applications with practical perspectives. West Sussex: John Wiley & Sons; 2011.
- [7] Zhang X, Mi C. Vehicle power management—modeling, control and optimization. Berlin: Springer; 2011.
- [8] Sheu KB. Simulation for the analysis of a hybrid electric scooter powertrain. *Appl Energy* 2008;85:589–606.
- [9] Doucette RT, McCulloch MD. Modeling the prospects of plug-in hybrid electric vehicles to reduce CO<sub>2</sub> emissions. *Appl Energy* 2011;88:2315–23.
- [10] Samaras C, Meisterling K. Life cycle assessment of greenhouse gas emissions from plug-in hybrid vehicles: implications for policy. *Environ Sci Technol* 2008;42:3170–6.
- [11] Demirdöven N, Deutch J. Hybrid cars now, fuel cell cars later. *Science* 2004;305:974–6.
- [12] Guzzella L. Automobiles of the future and the role of automatic control in those systems. *Annu Rev Control* 2009;33:1–10.
- [13] Williamson SS, Emadi A. Comparative assessment of hybrid electric and fuel cell vehicles based on comprehensive well-to-wheels efficiency analysis. *IEEE Trans Vehicle Technol* 2005;54:856–62.
- [14] Hawkins TR, Gausen OM, Strømman AH. Environmental impacts of hybrid and electric vehicles – a review. *Int J Life Cycle Assess* 2012;17:997–1014.
- [15] Ahman M. Primary energy efficiency of alternative powertrains in vehicles. *Energy* 2000;26:973–89.
- [16] Ott T, Zurbruggen F, Onder C, Guzzella L. Cycle-averaged efficiency of hybrid electric vehicles. *Proc Inst Mech Eng Pt D J Automob Eng* 2013;227:78–86.



- [17] Katrašnik T, Trenc F, Opresnik SR. Analysis of energy conversion efficiency in parallel and series hybrid powertrains. *IEEE Trans Vehicle Technol* 2007;56:3649–59.
- [18] Katrašnik T. Analytical framework for analyzing the energy conversion efficiency of different hybrid electric vehicle topologies. *Energy Convers Manage* 2009;50:1924–38.
- [19] Katrašnik T. Energy conversion efficiency of hybrid electric heavy-duty vehicles operating according to diverse drive cycles. *Energy Convers Manage* 2009;50:2865–78.
- [20] Katrašnik T. Analytical method to evaluate fuel consumption of hybrid electric vehicles at balanced energy content of the electric storage devices. *Appl Energy* 2010;87:3330–9.
- [21] Katrašnik T. Energy conversion phenomena in plug-in hybrid-electric vehicles. *Energy Convers Manage* 2011;52:2637–50.
- [22] Kum D, Peng H, Bucknor NK. Optimal control of plug-in HEVs for fuel economy under various travel distances. In: *Proceedings of the 6th IFAC symposium advances in automotive control*, Munich, Germany, 12–14 July; 2010. p. 258–63.
- [23] Moura SJ, Fathy HK, Callaway DS, Stein JL. A stochastic optimal control approach for power management in plug-in hybrid electric vehicles. *IEEE Trans Control Syst Technol* 2011;19:545–55.
- [24] Keefe MPO, Markel T. Dynamic programming applied to investigate energy management strategies for a plug-in HEV. In: *Proceedings of the 22nd international battery, hybrid and fuel cell electric vehicle*, Yokohama, Japan, 23–28 October; 2006.
- [25] Tulpule P, Marano V, Rizzoni G. Energy management for plug-in hybrid electric vehicles using equivalent consumption minimisation strategy. *Int J Electric Hybrid Vehicle* 2010;2:329–50.
- [26] Stockar S, Marano V, Canova M, Rizzoni G, Guzzella L. Energy-optimal control of plug-in hybrid electric vehicles for real-world driving cycles. *IEEE Tran Vehicle Technol* 2011;60:2949–62.
- [27] Zhang C, Vahidi A. Route preview in energy management of plug-in hybrid vehicles. *IEEE Trans Control Syst Technol* 2012;20:546–53.
- [28] Xu L, Ouyang M, Li J, Yang F, Lu L, Hua J. Optimal sizing of plug-in fuel cell electric vehicles using models of vehicle performance and system cost. *Appl Energy* 2013;103:477–87.
- [29] Murgovski N, Johannesson L, Sjöberg J, Egardt B. Component sizing of a plug-in hybrid electric powertrain via convex optimization. *Mechatronics* 2012;22:106–20.
- [30] Murgovski N, Johannesson L, Hellgren J, Egardt B, Sjöberg J. Convex optimization of charging infrastructure design and component sizing of a plug-in series HEV powertrain. In: *Proceedings of the 18th IFAC World Congress*, Milano, Italy, 28 August – 2 September; 2011. p. 13052–7.
- [31] Pourabdollah M, Murgovski N, Grauers A, Egardt B. Optimal sizing of a parallel PHEV powertrain. *IEEE Trans Vehicle Technol* 2013;62:2469–80.
- [32] Gaines L, Cuenca R. Costs of lithium-ion batteries for vehicles. Technical report 2000, Center for Transportation Research at Argonne National Laboratory, US Department of Energy.
- [33] Boyd S, Vandenberghe L. *Convex optimization*. Cambridge: Cambridge University Press; 2004.
- [34] Murgovski N, Johannesson L, Sjöberg J. Convex modeling of energy buffers in power control applications. In: *Proceedings of IFAC workshop on engine and powertrain control, simulation and modeling*, Paris, France, 23–25 October; 2012.
- [35] Larsson V, Johannesson L, Egardt B. Impact of trip length uncertainty on optimal discharging strategies for PHEVs. In: *Proceedings of the 6th IFAC Symposium Advances in Automotive Control*, Munich, Germany, 12–14 July, 2010; 55–60.
- [36] Larsson V, Johannesson L, Egardt B, Larsson A. Benefit of route recognition in energy management of plug-in hybrid electric vehicles. In: *Proceedings of American control conference*, Montreal, Canada, 27–29 June; 2012. p. 1314–20.
- [37] Pourabdollah M, Larsson V, Johannesson L, Egardt B. PHEV energy management: a comparison of two levels of trip information. SAE Technical paper 2012-01-0745, SAE World Congress & Exhibition, Detroit, USA, 24–26 April, 2012.
- [38] Grant M, Boyd S. CVX: Matlab software for disciplined convex programming, version 1.21. <<http://cvxr.com/cvx>> (May 2010).
- [39] Labit Y, Peaucelle D, Henrion D. SeDuMi interface 1.02: a tool for solving LMI problems with SeDuMi. In: *Proceedings of IEEE international symposium on computer aided control system design*, Glasgow, Scotland, UK, 18–20 September; 2002. p. 272–7.
- [40] Murgovski N. Optimal powertrain dimensioning and potential assessment of hybrid electric vehicles. PhD dissertation, Chalmers University of Technology, Gothenburg, Sweden, 2012.