


Decoding and Encoding using CRC

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Cyclic Redundancy Check (CRC)

- This is one of the most popular Error Detecting Codes being used today.
- Basic approach used for error detection is the use of redundancy, where additional bits are added to facilitate detection and correction of errors.
- CRC is based on binary division. In CRC, a sequence of redundant bits, called cyclic redundancy check bits, are appended to the end of data unit so that the resulting data unit becomes exactly divisible by a second, predetermined binary number.
- CRC is designed for and used to protect against common errors in communication channels but not against intentional alteration of data.

Characteristics

- This is the most powerful type of checksum which is able to detect accidental errors in data that is transmitted/stored between computers.
- According to the remainder of a polynomial division of the contents of the blocks, a short check value is attached to them.
- On the retrieval side, the calculation is repeated and, if the check values do not match, error correction has to be done in order to retrieve data. CRCs can be used for error correction.
- CRC uses Generator Polynomial which is represented by a key. Key is known to both sender and receiver side. Eg: Generator polynomial : $x^3 + x + 1$, key 1011
- Cyclic Redundancy Codes are among the best checksums available to detect and/or correct errors in communications transmissions.

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- **Encoding:** Encoding is the process of applying a specific code, such as letters, symbols and numbers, to data for conversion into an equivalent cipher.
 - **Decoding:** Decoding is the process of converting code into plain text or any format that is useful for subsequent processes. Decoding is the reverse of encoding.
 - CRCs are most efficiently calculated in dedicated hardware. CRC algorithms are long division algorithms in disguise. They are readily implemented using shift-registers.
 - But the modulo-2 binary division doesn't map particularly well to the instruction sets off-the-shelf processors. The reasons are:
 - (i) Generally no registers are available to hold the very long bit sequence(numerator)
 - (ii) Modulo-2 binary division is not the same as ordinary division. So even a processor with a division instruction can't help.

Modulo 2 Division

- The process of modulo 2 division is the same as the familiar division process used for decimal numbers except that there is a small change here.
- Instead of subtraction as in normal division here we use xor instead in modulo 2 division.
- In each step, a copy of the divisor(or data) is XORed with the k bits of the dividend.
- The result of the XOR operation (remainder) is $(n-1)$ bits, which is used for the next step after 1 extra bit is pulled down to make it n bits long.
- When there are no bits left to pull down, we have a result. The $(n-1)$ -bit remainder which is appended at the sender side.

At the Sender side

- Let k : Number of bits in data to be sent from sender side and
 $n-k$: Number of bits in the key obtained from generator polynomial.
- Then we'll create bit sequence of degree ' n ' by multiplying polynomial representation of k bits, $i(x)$ with x^{n-k}
- The **codeword $b(x)$** is finally produced by adding additional checkbits, i.e sequence $r(x)$ which is the remainder of division of $i(x)$ (dividend) by $g(x)$ (divisor).

Note: Modulo 2 Division is used while dividing $i(x)$ by $g(x)$

At the Sender side (contd..)

Hence,

$$x^{n-k} \cdot i(x) = q(x)g(x) + r(x)$$

$$b(x) = x^{n-k} \cdot i(x) + r(x)$$

$$b(x) = q(x)g(x) + r(x) + r(x)$$

- The remainder can have a maximum degree of $n-k-1$
- The most significant 'k' bits in the codeword correspond to information sequence being sent, and least significant $n-k$ bits correspond to remainder respectively(crc check bits)

Generator polynomial: $g(x) = x^3 + x + 1$
 Information: $(1,1,0,0) \rightarrow i(x) = x^3 + x^2$
 Encoding: $x^3 i(x) = x^6 + x^5$

$$\begin{array}{r} x^3 + x^2 + x \\ \hline x^3 + x + 1 \overline{)x^6 + x^5} \\ x^6 + \quad x^4 + x^3 \\ \hline x^5 + x^4 + x^3 \\ x^5 + \quad x^3 + x^2 \\ \hline x^4 + \quad x^2 \\ x^4 + \quad x^2 + x \\ \hline x \end{array}$$

Transmitted codeword:

$$b(x) = x^6 + x^5 + x \\ \rightarrow \underline{b} = (1,1,0,0,0,1,0)$$

$$\begin{array}{r} 1110 \\ \hline 1011 \overline{)1100000} \\ 1011 \\ \hline 1110 \\ 1011 \\ \hline 1010 \\ 1011 \\ \hline 010 \end{array}$$

Here the generator key is **g** :1011
 Information to be passed is **i** : 1100
 Multiply the information by **x³** then we
 get **x^{n-k}.i** = 1100000.

When this is divided by g(x) the
 remainder r = 010

After adding r(x) we get b = 1100010

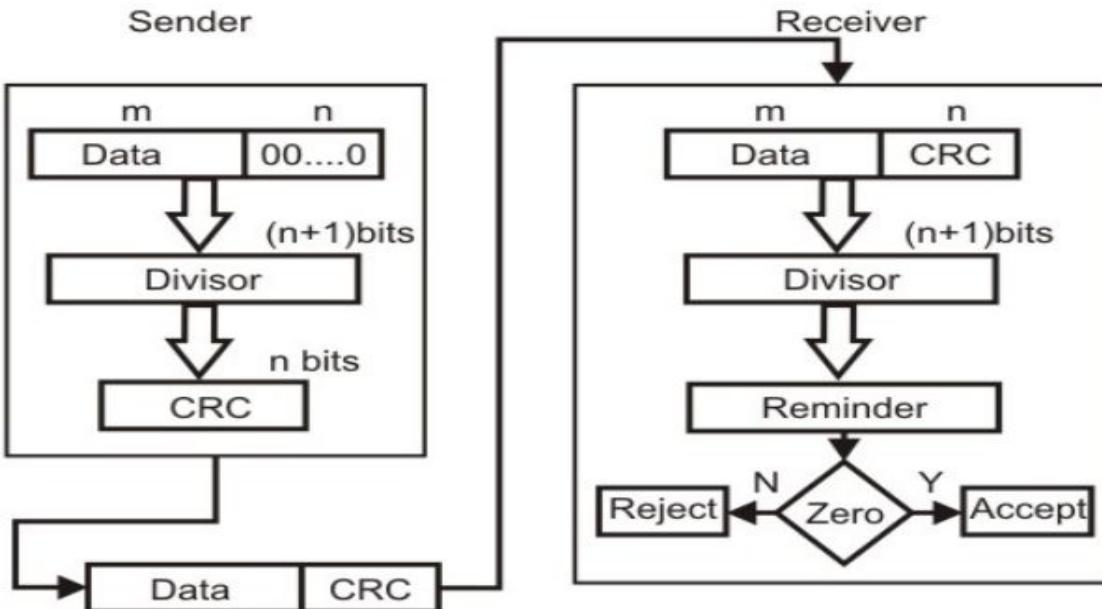
At the Receiver

From the calculations at the sender it's clear that the code word sent is a multiple of generator $g(x)$ as $b(x)$ where we have used the fact that in modulo 2 arithmetic $r(x) + r(x) = 0$.

$$b(x) = x^{n-k} \cdot i(x) + r(x) = q(x)g(x) + r(x) + r(x) = q(x)g(x) \text{ i.e a multiple of } g(x)$$

- The codeword received is divided by $g(x)$,
 - If the remainder is non zero
 - Then an error has been detected, Receiver rejects the data and asks the sender for retransmission.
 - If the remainder is zero
 - Receiver assumes that no error occurred in the data during the transmission. Receiver accepts the data.

Block diagram



If a k bit message is to be transmitted, the transmitter generates an r -bit sequence, known as Frame Check Sequence (FCS) so that the $(k+r)$ bits are actually being transmitted. Now this r -bit FCS is generated by dividing the original number, appended by r zeros, by a predetermined number. This number, which is $(r+1)$ bit in length, can also be considered as the coefficients of a polynomial, called Generator Polynomial. The remainder of this division process generates the r -bit FCS. On receiving the packet, the receiver divides the $(k+r)$ bit frame by the same predetermined number and if it produces no remainder, it can be assumed that no error has occurred during the transmission.

Algorithm

Server:

- Our aim here is to pass string from sender side to receiver side and detect if any accidental errors occurred.
- The string $i(x)$ is converted into binary string data.
- Augment $n-k-1$ zeros at the end of the binary data (i.e multiply by x^{n-k}).
- Divide it by generator polynomial $g(x)$ using modulo 2 division.
- Add the remainder $r(x)$ to the binary data to get the codeword.
- Now the data (encoded using the CRC code using the key) is sent to the receiver.

Algorithm

Receiver:

- The receiver receives the codeword from the sender.
- Now the receiver has to decode in order to find any accidental errors that occurred in the transmission.
- Divide the received codeword again by the key of generator polynomial using modulo 2 division to check if it's a multiple of generator polynomial.
- **Zero remainder:** This implies that no errors have occurred in the data during transmission. ACK is sent to the Sender.
- **Non-zero remainder:** This implies that error has occurred in the data during transmission. NACK is sent to the Sender so that it can retransmit the data again.

Standardized Polynomial Codes

The generator polynomials which are highly endorsed,

CRC type	Polynomial	Used in
CRC-8	$x^8 + x^2 + x + 1$	ATM header error check
CRC-10	$x^{10} + x^9 + x^5 + x^4 + x + 1$	ATM AAL CRC
CRC-12	$x^{12} + x^{11} + x^3 + x^2 + x + 1 \\ = (x + 1)(x^{11} + x^2 + 1)$	Bisync
CRC-16	$x^{16} + x^{15} + x^2 + 1 \\ = (x + 1)(x^{15} + x + 1)$	Bisync
CCITT-16	$x^{16} + x^{12} + x^5 + 1$	HDLC, XMODEM, V.41
CCITT-32	$x^{32} + x^{26} + x^{23} + x^{22} + x^{16} + x^{12} + \\ x^{11} + x^{10} + x^8 + x^7 + x^5 + x^4 + x^2 + x \\ + 1$	IEEE 802, DoD, V.42, AAL5

Error detecting capability

Cases where error detection fails

1. Single errors: $e(x) = x^i \quad 0 \leq i \leq n - 1$

If $g(x)$ has more than one term, it cannot divide $e(x)$.

2. Double errors: $e(x) = x^i + x^j \quad 0 \leq i \leq j \leq n - 1$
 $= x^i(1 + x^{j-i})$

If $g(x)$ is primitive, it will not divide $(1 + x^{j-i})$
for $j - i \leq 2^{n-k} - 1$.

3. Odd number of errors: $e(1) = 1$ if number of errors is odd.

If $g(x)$ has $(x + 1)$ as a factor, then $g(1) = 0$ and all codewords have an even number of 1s.

$$X^n \cdot M(X)/P(X) = Q(X) + R(X) / P(X)$$

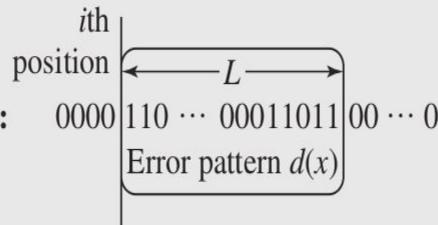
Single errors: if $e(x) = x^i$, $g(x)$ has at least has 2 coefficients . Then $e(x)$ is not divisible by $g(x)$.So all single errors can be detected.

Double errors: $e(x) = x^i (1 + x^{j-i})$ can be detected if $g(x)$ (of degree $n-k$) is primitive as it can't divide $(1 + x^{j-i})$ unless $j-i > 2^{n-k}$.

Odd no. of errors: $e(1)=1$, if $(x+1)$ is a factor of $g(x)$ then $g(1) = 0$ and hence can't divide $e(x)$.

Cases where error detection fails

4. Errors bursts of length b :



$e(x) = x^i d(x)$ where $\deg(d(x)) = L - 1$
 $g(x)$ has degree $n - k$;
 $g(x)$ cannot divide $d(x)$ if $\deg(g(x)) > \deg(d(x))$

- $L = (n - k)$ or less: all will be detected
- $L = (n - k + 1)$: $\deg(d(x)) = \deg(g(x))$
i.e. $d(x) = g(x)$ is the only undetectable error pattern,
fraction of bursts that are undetectable = $1/2^{L-2}$
- $L > (n - k + 1)$: fraction of bursts that are undetectable
= $1/2^{n-k}$

$$X^n \cdot M(X)/P(X) = Q(X) + R(X) / P(X)$$

1. Error bursts of length b : $e(x) = x^i d(x)$ If the error burst involves L consecutive bits, then the degree of $d(x)$ is $L - 1$. $e(x)$ is divisible by $g(x)$ only if $d(x)$ is divisible by $g(x)$.

2. If the burst error has length $L = n - k + 1$, that is, degree of $d(x) = \deg(g(x))$, then $d(x)$ is divisible by $g(x)$ only if $d(x) = g(x)$. probability that error goes undetected is $1/2^{n-k-1}$.

3. In the case of $L > n - k + 1$ the fraction of bursts that is undetectable is $1/2^{n-k}$.

Improve Efficiency :

- The most common way to improve efficiency of the CRC calculation is to throw memory at the problem.
- For a given input remainder and generator polynomial, the output remainder always be the same i.e for a given dividend and divisor the remainder is always the same.
- So we can precompute the output remainder for each of the possible byte-wide input remainders and store the results in a lookup table.
- The lookup table can be used to speed up the CRC calculations for a given message. The speedup is realized because the message can now be processed byte by byte, rather than bit by bit.

CRC 3: Key: 1010 does not detect most of the errors whereas for the same strings key 1011 detects errors in data

CRC 3: Key: 1001 does not detect most of the errors whereas for the same strings key 1011 detects errors in data

CRC 3: Key: 1100 does not detect most of the errors whereas for the same strings key 1011 detects errors in data

Standard crc, 1011 detects errors while 1000 and 0100 don't

```
sathya@sathya-Inspiron-5559:~/cn2$ python2 c.py
```

```
Enter data you want to send->iaygdcisiucjsdcviydsdifhvdvusydgcihiufhviguysytffdyfsyudgvctsdfcyfcutvcqtdfycuagi8sftgvatsffcxa
```

```
sgcysdcdyu
```

```
sgcyusdtsucsvycusdvutfdyfcysvdgjdscfcsdytcdsidhgwyugwefcygeisfcusifhcudgfuygvduysdgyics
```

```
110100111000011110011100100110001110100111100101110001111010101110010011000111101101111001110010011110011110010011110011110
```

```
0111110100111001101100100110100011101001001100011110101111001111100111001001111000111010011110011110100011010011110101110011
```

```
01101000111011010011100111110101111100111110010011100111110011101001101111001110010011110011110011110011111001111010111
```

```
0010011001111101101100011110100111001001100011111001110011111001111010011001101111001110010011110011111010011001001101110
```

```
0111000111110101110000110011111010011100011100111100110110010011000111101001110011110011011001101100011111000110001111001111
```

```
10110111100011010001100111110011110100111100011110011011001001110011111010011100111100111110011110011111001001100011111001111001100
```

```
11110010011101011111001111001111100011111001111010111100100111001111100101110011111001001110011111001111001111100111100111110011110011011
```

```
11001111001001110101111010011100100111100110110001111100111100111110011110101100100111001111100111100111110011110011111001101110
```

```
011110010011110011110010011100100111100100111100100111100100111100100111100100111100100111100100111100100111100100111100100111100101
```

```
1101001111001101100011110100111100100111100100111100100111100100111100100111100100111100100111100100111100100111100100111100101111
```

```
00111100100111100111100100111100100111100100111100100111100100111100100111100100111100100111100100111100100111100100111100100111100101111
```

```
length of data
```

```
1483
```

```
Error in data
```

```
sathya@sathya-Inspiron-5559:~/cn2$
```

1000 doesn't detect error

0100 doesn't detect error

```
sathya@sathya-Inspiron-5559:~/cn2$ python2 c.py
Enter data you want to send->iaygdciucijsdcviydsdifhvdvusydgcihiufhviguysytfyfdyfsyudgvctsdfcyfcutvcgtdfycuagi8sftgvatsffcxasvxhgsvfcytcf
Udsgcysdcduygcusdtsucdsyvcsdvutfdyfcysvdqjdsctsdihgyugwefcygeisfcusifhcudgfuyqvduysdgyics
1101001110000111100111001001111010011110011110100111100111101011100111100100110001111011011001111100111001001110011110
0111110100111001101100100111011001001100011110110111100111110011100100110011110001111010011110011110100110011110101110011
01101000111011011001110011111010111100111001111100100111001111100111101001100111110011100110011110011111001111010111
00100110011111011011000111110100111001110010011000111110011100110011111010111010011100111110010011001101110
0111000111110101110000111001111101001111000111001101110010011110011111001111010011110011011001100111110001100001111001111
1011011110001101000110011111100111110101101100111110011111001001100111101011100100111100111110011111001111001001100
111100100111010111110011110011111001111100111110010011101001111101011100100111100111100111100111100111100111101011
1100111100100111010111100100110011001111001111100111100111110011110101100100111110101011001001111001111001111001101110
01111001001111001111001001100011110010011100111001001100011111100111100101110011111001011100111110010111001101100011111001111001
1101001111001111001101100011111010111100111100100111100101110010011001111001101111001111001111001111001111001111001101111
1001111001111001001100111111001111101001111001111100100111100111110011110101110010011111001111001111001111001111001111001111
length of data
1483
THANK you Data ->11010011100001111001110011110010011000111101001111010011110101110001111010101110011110010011000111110110110011111001
11001001110011110011110100111001100100110001110110110010011000111110101110011111001110010011001111100011110100111100111101000110
10011110101110011011010001110110100111100111110101111100111001111100100111001111101001100110111001110011100110111001
111110011111010111001001100111110110110001111101001111001001111001111100111001101100011111010111010011111010101100111110011110011
00100110011100111000111101001111001001111001110011001111001001111001110011001111000111101001111001110011011001111001111001111001111
0110000111100111110101111000110100011001111100110110011011000111110011110100110001111001101100101110010011100111100011111001111
100111100100110001111001001111010111100111100111100011111001111010111100100111001111001011100100111001111001111001111001111001111
011100011111010111100111100111100111100011111001111010111100111100100111001111001011100100111001111001111001111001111001111001111
sathya@sathya-Inspiron-5559:~/cn2$
```

CRC 4: Key: 10000 does not detect any errors whereas for the same strings key 10011 detects errors in data

CRC 4: Key: 10110 does not detect most of the errors whereas for the same strings key 10011 detects errors in data

CRC4, when key is 10011 error is detected, but when key is 10100 no error was found

```
sathya@sathya-Inspiron-5559:~/cn2$ python2 client_encoder.py 10011 1118
String to be sent:q3efhbwoufgwyoeufygruvcasbdhcdukfvalhdvbcWUDHIQUWEDCBWUDHVBDSJKBCaudsfaeufbvceijvcbauhvfbaljvbhljvhbahdfkvagbdlfvbha sdlj
vbhla
vbdasldhvbjarhrgrthy
('The length of the string is', 1062)
('The key is', '10011')
1110001110011110010111011000110010001011101111101011100110110011111100111101111100101110101110011011110011110111100111100111110
0101110101110110001110000111001111000101100100110001110001011001001110101110101110011011000011101100110011000110010011001110110
1100010110001110101111010110001001001000100100101011010111100010110001001000011100000101010111101011100010010010101101000010100
010010100111001010100101110000101000011110000111101011100100111001111001101101100011100101111010111001101100010111011011000111100110
11101010111010111000111100010110000111101011101000111010110011011000011101101011100110110110001011010001101101011101101100011
000101100001110100011001001100110101111101011000111001111000101100100110110011001101101011101000110110001100000110011110010
011011001101011110110100010110100011110011110100011101100101100011100110011011000111001100110110001110011001101100011100110
000011100101101000111001011001111100101100011100111001100011100110001110011000111001100011100110001110011000111001100011100110
sathya@sathya-Inspiron-5559:~/cn2$ python2 client_encoder.py 10100 1119
String to be sent:q3efhbwoufgwyoeufygruvcasbdhcdukfvalhdvbcWUDHIQUWEDCBWUDHVBDSJKBCaudsfaeufbvceijvcbauhvfbaljvbhljvhbahdfkvagbdlfvbha sdlj
vbhla
vbdasldhvbjarhrgrthy
('The length of the string is', 1062)
('The key is', '10100')
11100011100111100101110110001100100010111011111010111001101100111111001111011111001011101011100110111100111100111110
0101110101110110001110000111001111000101100100110001110001011001001110101110101110011011000011101100110011000110010011001110110
1100010110001110101111010110001001001000100100101011010111100010110001001000011100000101010111101011100010010010101101000010100
010010100111001010100101110000101000011110000111101011100100111001111001101101100011100101111010111001101100010111011011000111100110
11101010111010111000111100010110000111101011101000111010110011011000011101101011100110110110001011010001101101011101101100011
000101100001110100011001001100110101111101011000111001111000101100100110110011001101100010110010001110001110011000111001100011100110
0110110011010111101101000101101000111100111101000111011001011000111001100110110001110011001101100011100110011011000111001100011100110
0000111001011010001110010110011111001011000111001110011000111001100011100110001110011000111001100011100110001110011000111001100011100110
Thankyou, DATA RECEIVED,remainder is 0000
sathya@sathya-Inspiron-5559:~/cn2$ █ x=(0x1a|0x1b|0x1c|0x1d)
```

CRC4, when key is 10011 error is detected, but when key is 10010 no error was found

```
sathya@sathya-Inspiron-5559:~/cn2$ python2 client_encoder.py 10010 1115
String to be sent:ECVRVYEARVaejkrvhbaelrjgtvbaeiugvbaerilgvbaelrivkabdF;ikvabufvikdfhvbd;fkvbadkfjvbadfkhvbadfkhvbadvjhbavljhbavlvjhbadvlhadbfvadhfbva;sdhvfbafhvbadf;hvbd;afhbvadfhbvsjhbvsfluvadhouyaebry
('The length of the string is', 1248)
('The key is', '10010')
1000101100001110101010101010110011000101100000110100101010110000111001011101010111100101110110110001100010110000111001011110
1100011001011010110011110100111101011000101100001110010111010111101101100010110000111001011110010111010011101100110011110110
01100010110000111001011101010011110110110001011110000111001001000110111010111101001110110101111011011000101101011100110111
0110110100111010111100100110011010110001011110000111001001000110111010111101001110110101111011011000101101011100110111
1100001110010011001101011010111101000111001011000011100100110110101111010001110110110001011000011100100111011011010110100010110
0001110010011011001110101011010001100010111010110100011000101110101000110001011000101100110110101100011100100110100011001
0110001011101101100001111011110010011101000111011010011100001110011011010001110011011011000101100001110010011001101111010001110
110110000101100100111011110000111001101000101110101100001110010011011000111001101101000101100001110010011001101111010001110
100110110011101011110000111001000110110100010111010110000111001001101100011100101110001011100101110110011101010110100011100110
Thankyou, DATA RECEIVED, remainder is 0000
sathya@sathya-Inspiron-5559:~/cn2$ python2 client_encoder.py 10011 1116
String to be sent:ECVRVYEARVaejkrvhbaelrjgtvbaeiugvbaerilgvbaelrivkabdF;ikvabufvikdfhvbd;fkvbadkfjvbadfkhvbadfkhvbadvjhbavljhbavlvjhbadvlhadbfvadhfbva;sdhvfbafhvbadf;hvbd;afhbvadfhbvsjhbvsfluvadhouyaebry
('The length of the string is', 1248)
('The key is', '10011')
1000101100001110101010101010110011000101100000110100101010110000111001011101010111100101110110110001100010110000111001011110
1100011001011010110011110100111101011000101100001110010111010111101101100010110000111001011110010111010011101100110011110110
011000101100001110010111010011110110110001011110000111001001000110111010111101001110110101111011011000101101011100110111
0110110010110011011100100110011010110001011110000111001001000110111010111101001110110101111011011000101101011100110111
110000111001001100110101101011110100011100101100001110010011011010111101000111011011000101100001110010011001101111010001110
0001110010011011001110101011010001100010111010110100011000101110101000110001011000101100110110101101000111001001100110111101000110
011000101110110110000111101111001001110100011101101001110000111001101101000111001101101100010110000111001001100110111101000110
11011000010110010011101111000011100110100010111010110000111001001101100011100110110100010110000111001001100110111101000110
100110110011101011110000111001000110110100010111010110000111001001101100011100110110100010110000111001001100110111101000110
Error found, Retransmit data and remainder is 1000
```

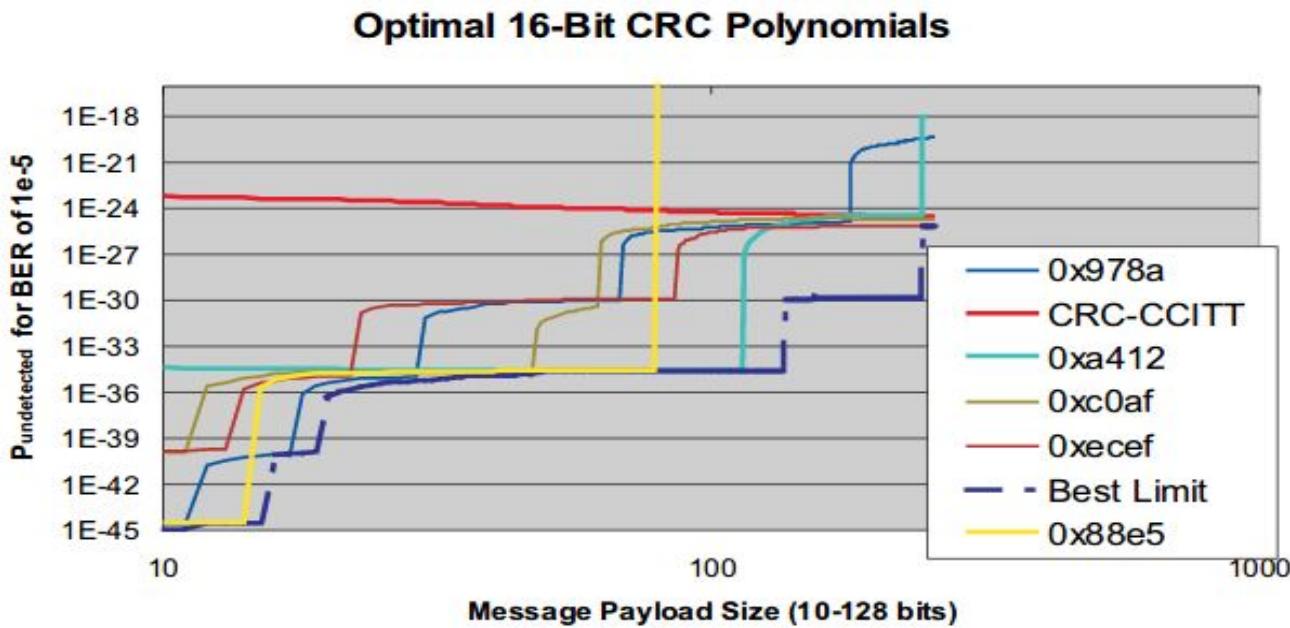
Commonly observed results

Performance

Given that $g(x)$ is at least not divisible by X and is not divisible by $(X+1)$, it's guaranteed all burst errors of a length equal to the degree of polynomial are detected and all burst errors affecting an odd number of bits are detected.

- CRC can detect all burst errors of less than the degree of the polynomial
- CRC detects most of the larger burst errors with a high probability.

Usual Performance of standard CRC16 vs other 16 bit polynomials



As the length of bits in the message transmitted increase, probability of undetected errors increase for all 16 bit generator polynomials except for the standard CRC-CCIT generator polynomial.

This is because of unique properties of CRC-CCIT.

Figure 2. Optimal 16 bit CRC codes for message lengths 10 bits - 128 bits



Thank you.

