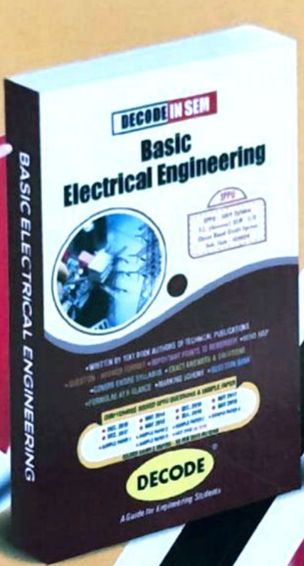
Basic Electrical Engineering

Sub. Code : 103004

- CHAPTERWISE SOLVED SPPU QUESTIONS DEC.1997 to MAY 2019
- SOLVED SAMPLE PAPERS AS PER 2019 PATTERN (IN SEM & END SEM)
- MIND MAP FOR EACH CHAPTER
- FORMULAE AT A GLANCE

FREE DECODE IN SEM WITH THIS BOOK



SPPU

CBCS SCHEME - COURSE 2019
F.E. (COMMON) SEM - I/II

SOLVED SPPU PAPERS
OCT. 2019 (IN SEM) & DEC. 2019 (END SEM)

TECHNICAL PUBLICATIONS
SINCE 1993
An Up-Thrust for Knowledge

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SUBJECT CODE : 103004

As per Revised Syllabus of

SAVITRIBAI PHULE PUNE UNIVERSITY

F.E. (Common to All Branches) Semester - I/II

BASIC ELECTRICAL ENGINEERING

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Unit - I
Electromagnetism

Syllabus : *Review : resistance, emf, current, potential, potential difference and Ohm's law* ***Electromagnetism** : Magnetic effect of an electric current, cross and dot conventions, right hand thumb rule, nature of magnetic field of long straight conductor, solenoid and toroid. Concept of mmf, flux, flux density, reluctance, permeability and field strength, their units and relationships. Simple series magnetic circuit, Introduction to parallel magnetic circuit(Only theoretical treatment), comparison of electric and magnetic circuit, force on current carrying conductor placed in magnetic field, Fleming's left hand rule. Faradays laws of electromagnetic induction, Fleming's right hand rule, statically and dynamically induced e.m.f., self and mutual inductance, coefficient of couplings. Energy stored in magnetic field.*

UNIT - I

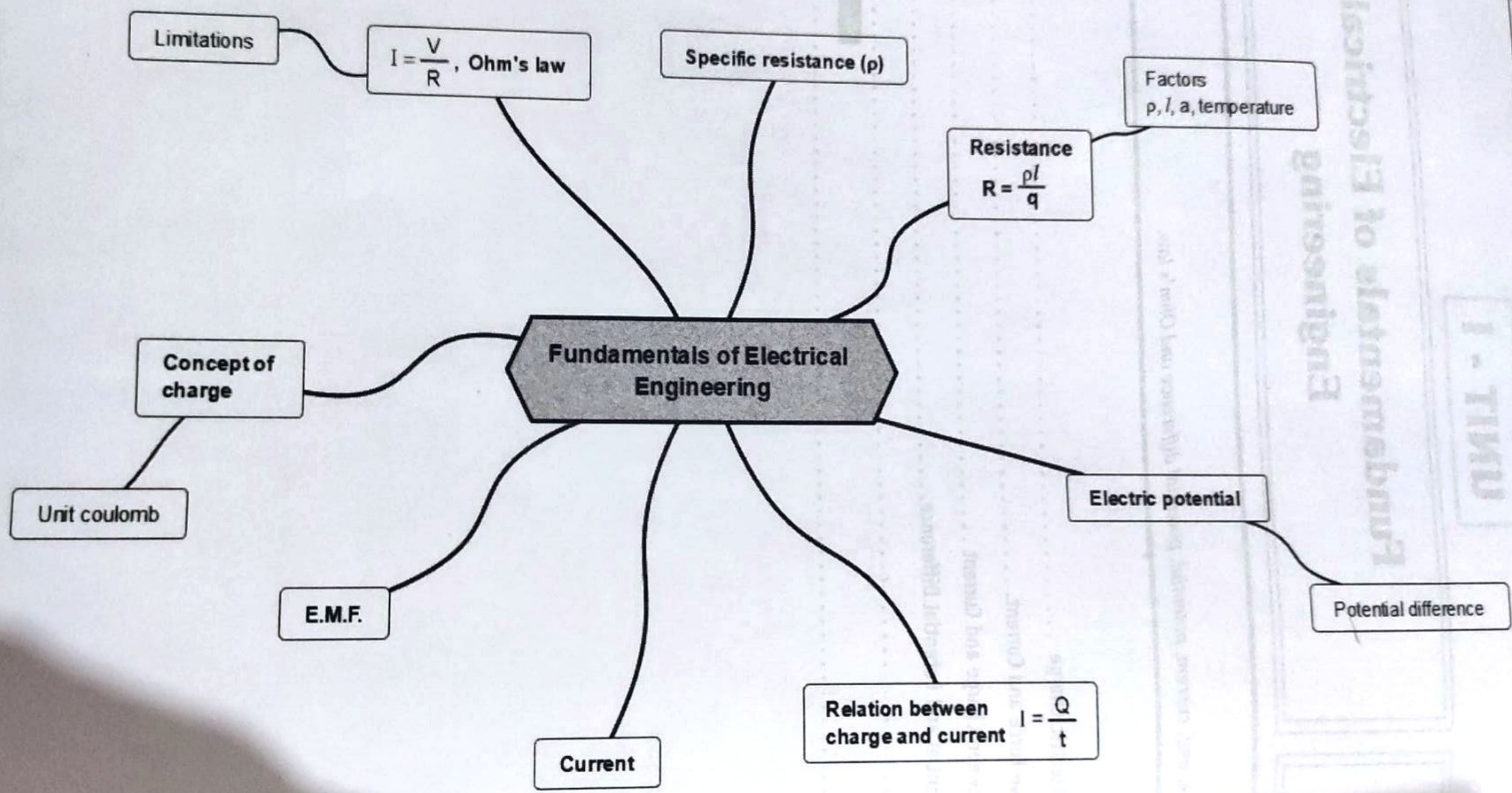
0 Fundamentals of Electrical Engineering

Syllabus

Review : Resistance, emf, current, potential, potential difference and Ohm's law.

Contents

Mind Map - Fundamentals of Electrical Engineering



0.1 : Concept of Electric Charge

- The matter on the earth which occupies the space may be solid, liquid or gaseous and is made up of many atoms which are of similar nature.
- According to modern electron theory, atom is composed of the three fundamental particles, which are invisible to bare eyes. These are the **neutron**, the **proton** and the **electron**.
- The proton is positively charged while the electron is negatively charged. The neutron is electrically neutral i.e. possessing no charge.
- Atom as a whole is electrically neutral as the number of protons is always equal to the number of electrons.
- If by some means electron is removed from an atom, it will loose negative charge and will become positively charged. Such positively charged atom is called **cation**. As against this, if excess electron is added to an atom, it will become negatively charged. Such negatively charged atom is called **anion**.

*This total deficiency or addition of excess electrons in an atom is called as its **charge** and the atom is said to be **charged**. The unit of charge is **coulomb**.*

- The charge on one electron is 1.602×10^{-19} C. Hence one coulomb of charge means the total charge possessed by $\frac{1}{1.602 \times 10^{-19}}$ electrons i.e. 6.24×10^{18} number of electrons.

Thus, $1 \text{ coulomb} = \text{Charge on } 6.24 \times 10^{18} \text{ electrons}$

Expected Question

1. *Explain the concept of an electric charge. State the unit of charge.*

0.2 : Electromotive Force and Current

- A conductor is one which has abundant free electrons. The free electrons in such a conductor are always moving in random directions as shown in the Fig. 0.2.1.

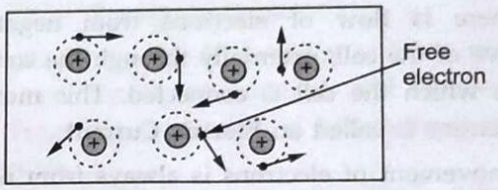


Fig. 0.2.1 Inside the piece of a conductor

- The small electrical effort, externally applied to such conductor makes all such free electrons to drift along the metal in a definite particular direction.
- This direction depends on how the external electrical effort is applied to the conductor. Such an electrical effort may be an electrical cell, connected across the two ends of a conductor. Such physical phenomenon is represented in the Fig. 0.2.2.

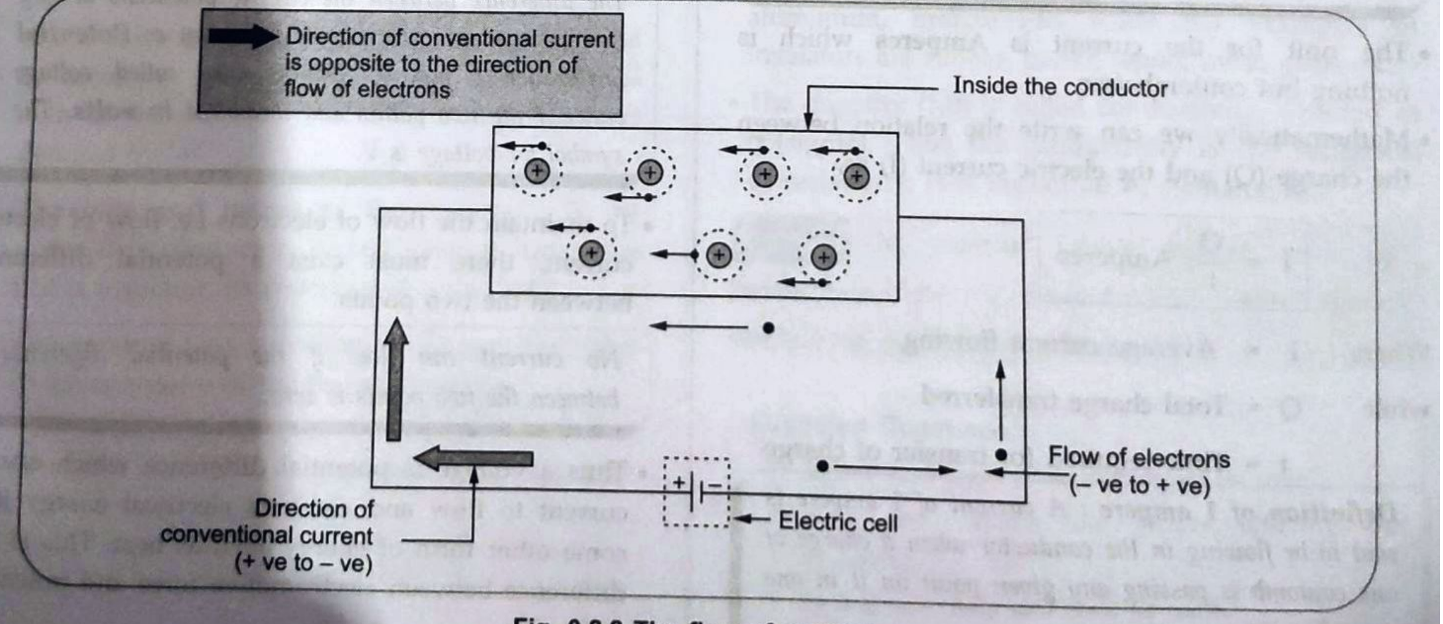


Fig. 0.2.2 The flow of current

An electrical effort required to drift the free electrons in one particular direction, in a conductor is called ***Electromotive Force*** *(e.m.f.).* ***It is denoted as E and measured in volts.*** *Thus e.m.f. is an electromotive force which converts any other form of energy into an electrical energy.*

- When free electron gets dragged towards positive from an atom it becomes positively charged ion. Such positive ion drags a free electron from the next atom. This process repeats from atom to atom along the conductor.
- So there is flow of electrons from negative to positive of the cell, externally through the conductor across which the cell is connected. This movement of electrons is called an **Electric Current**.

The movement of electrons is always from negative to positive while movement of current is always assumed as from positive to negative. This is called **direction of conventional current**.

Expected Question

1. *Explain the concept of electromotive force and current.*

3 : Relation between Charge and Current

The current can be defined as ***rate of flow of charge in an electric circuit*** *or in any medium in which charges are subjected to an external electric field.*

The unit for the current is **Amperes** which is nothing but coulombs/sec.

Mathematically we can write the relation between the charge (Q) and the electric current (I) as,

$$I = \frac{Q}{t} \text{ Amperes}$$

where I = Average current flowing

Q = Total charge transferred

t = Time required for transfer of charge.

Definition of 1 ampere : *A current of 1 ampere is said to be flowing in the conductor when a charge of one coulomb is passing any given point on it in one second.*

Expected Question

1. *State the relation between charge and current. Define unit of a current.*

0.4 : Electric Potential and Potential Difference

- When two similarly charged particles are brought near, they try to repel each other while dissimilar charges try to attract each other. This means, every charged particle has a tendency to do work.

This ability of a charged particle to do the work is called its ***electric potential****. The unit of electric potential is* ***volt.***

- The electric potential at a point due to a charge is one volt if one joule of work is done in bringing a unit positive charge i.e. positive charge of one coulomb from infinity to that point.
- Mathematically it is expressed as,

$$\text{Electrical Potential} = \frac{\text{Work done}}{\text{Charge}} = \frac{W}{Q}$$

- It is well known that, flow of water is always from higher level to lower level, flow of heat is always from a body at higher temperature to a body at lower temperature. Such a level difference which causes flow of water, heat and so on, also exists in electric circuits. In electric circuits, flow of current is always from higher electric potential to lower electric potential.

The difference between the electric potentials at any two given points in a circuit is known as ***Potential Difference (p.d.)****. This is also called voltage between the two points and measured in* ***volts.*** *The symbol for voltage is V.*

- To maintain the flow of electrons i.e. flow of electric current, there must exist a potential difference between the two points.

No current can flow if the potential difference between the two points is zero.

- Thus a voltage is potential difference which causes current to flow and converts electrical energy into some other form of energy such as heat. This is the difference between electromotive force and potential difference.

Expected Questions

1. *Define electric potential. State its unit.*
2. *Define potential difference and mention its unit.*

0.5 : Resistance

SPPU : May-04,10

The property of an electric circuit opposing the flow of current and at the same time causes electrical energy to be converted to heat is called resistance.

- Higher the availability of the free electrons, lesser will be the opposition to the flow of current and lesser is the resistance. A conductor having high number of free electrons offer less resistance to the flow of current.
- The resistance is denoted by the symbol 'R' and is measured in **Ohm** symbolically represented as Ω . We can define unit ohm as below :

Definition : 1 ohm : *The resistance of a circuit, in which a current of 1 ampere generates the heat at the rate of one joules per second is said to be 1 ohm.*

4.186 joules = 1 calorie and 1 joule = 0.24 calorie

- Thus unit 1 ohm can be defined as that resistance of the circuit if it develops 0.24 calories of heat, when one ampere current flows through the circuit for one second.

1. **Length of the material :** The resistance of a material is directly proportional to the length. Length is denoted by '*l*'.
2. **Cross-sectional area :** The resistance of a material is inversely proportional to the cross-sectional area of the material. The cross sectional area is denoted by 'a'.
3. **The type and nature of the material :** If the material is conductor, its resistance is less while if it is insulator, its resistance is very high.
4. **Temperature :** As temperature changes, the value of the resistance of the material changes.

- So for a certain material at a certain constant temperature we can write a mathematical expression as,

$$R \propto \frac{l}{a}$$

- The effect of nature of material is considered through the constant of proportionality denoted by ρ (rho) called **resistivity** or **specific resistance** of the material.

So finally,

$$R = \frac{\rho l}{a}$$

Where l = Length in metres,

a = Cross-sectional area in square metres

ρ = Resistivity in ohms-metres,

R = Resistance in ohms

- The resistivity or specific resistance of a material depends on nature of material and denoted by ρ(rho). From the expression of resistance it can be expressed as,

$$\rho = \frac{Ra}{l} \quad \text{i.e.} \quad \frac{\Omega - m^2}{m} \quad \text{i.e.} \quad \Omega - m$$

- It is measured in Ω - m.

Definition : *The resistance of a material having unit length and unit cross-sectional area is known as its specific resistance or resistivity.*

Key Point *A material with highest value of resistivity is the best insulator while with poorest value of resistivity is the best conductor.*

- The examples of conductors are copper, gold, aluminium, bronze etc. while the examples of insulators are rubber, paper, wood, mica, glass etc.
- The quantity (1/ρ) is called **conductivity**, denoted as σ (sigma). **Thus the conductivity is the reciprocal of resistivity. It is measured in siemens/ m.**

Key Point *The material having highest value of conductivity is the best conductor while having poorest conductivity is the best insulator.*

Expected Questions

1. *Define resistance. State and define its unit.*
2. *State and explain the factors affecting the resistance.*
3. *Define resistivity and state its units.*

4. *How materials are classified as conductors and insulators ? Give two examples of each.* **May- 04, 10, Marks 4**

5. *Define conductivity and state its units.*

0.6 : Ohm's Law

- The Ohm's law gives relationship between the potential difference (V), the current (I) and the resistance (R) of a d.c. circuit.
- It states that, the current flowing through the electric circuit is directly proportional to the potential difference across the circuit and inversely proportional to the resistance of the circuit, provided the temperature remains constant.
- Mathematically, $I \propto \frac{V}{R}$ Where I is the current

flowing in amperes, the V is the voltage applied and R is the resistance of the conductor, as shown in the Fig. 0.6.1.

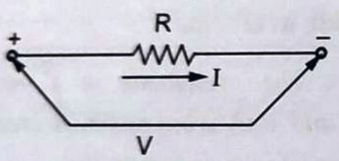


Fig. 0.6.1 Ohm's law

Then $$I = \frac{V}{R}$$

- The unit of potential difference is defined in su way that the constant of proportionality is unity.

Ohm's Law is, $I = \frac{V}{R}$ *amperes* *or* $V = IR$ vo

or $\frac{V}{I}$ = *Constant* = *R* Ohms

- The Ohm's law can be defined as, the ratio potential difference (V) between any two points conductor to the current (I) flowing between th is constant, provided that the temperature of conductor remains constant.
- The **limitations of the Ohm's law** are,
  1) It is not applicable to the nonlinear devi such as diodes, zener diodes, voltage regulat etc.
  2) It does not hold good for non-metal conductors such as silicon carbide. The law f such conductors is given by,

$$V = k\, I^{m} \quad \text{where k, m are constants}$$

Expected Question

1. *State Ohm's law and its limitations.*

UNIT - I

1 Electromagnetism

Syllabus

Magnetic effect of an electric current, cross and dot conventions, right hand thumb rule, nature of magnetic field of long straight conductor, solenoid and toroid. Concept of mmf, flux, flux density, reluctance, permeability and field strength, their units and relationships. Simple series magnetic circuit, Introduction to parallel magnetic circuit(Only theoretical treatment), comparison of electric and magnetic circuit, force on current carrying conductor placed in magnetic field, Fleming's left hand rule. Faradays laws of electromagnetic induction, Fleming's right hand rule, statically and dynamically induced e.m.f., self and mutual inductance, coefficient of couplings. Energy stored in magnetic field.

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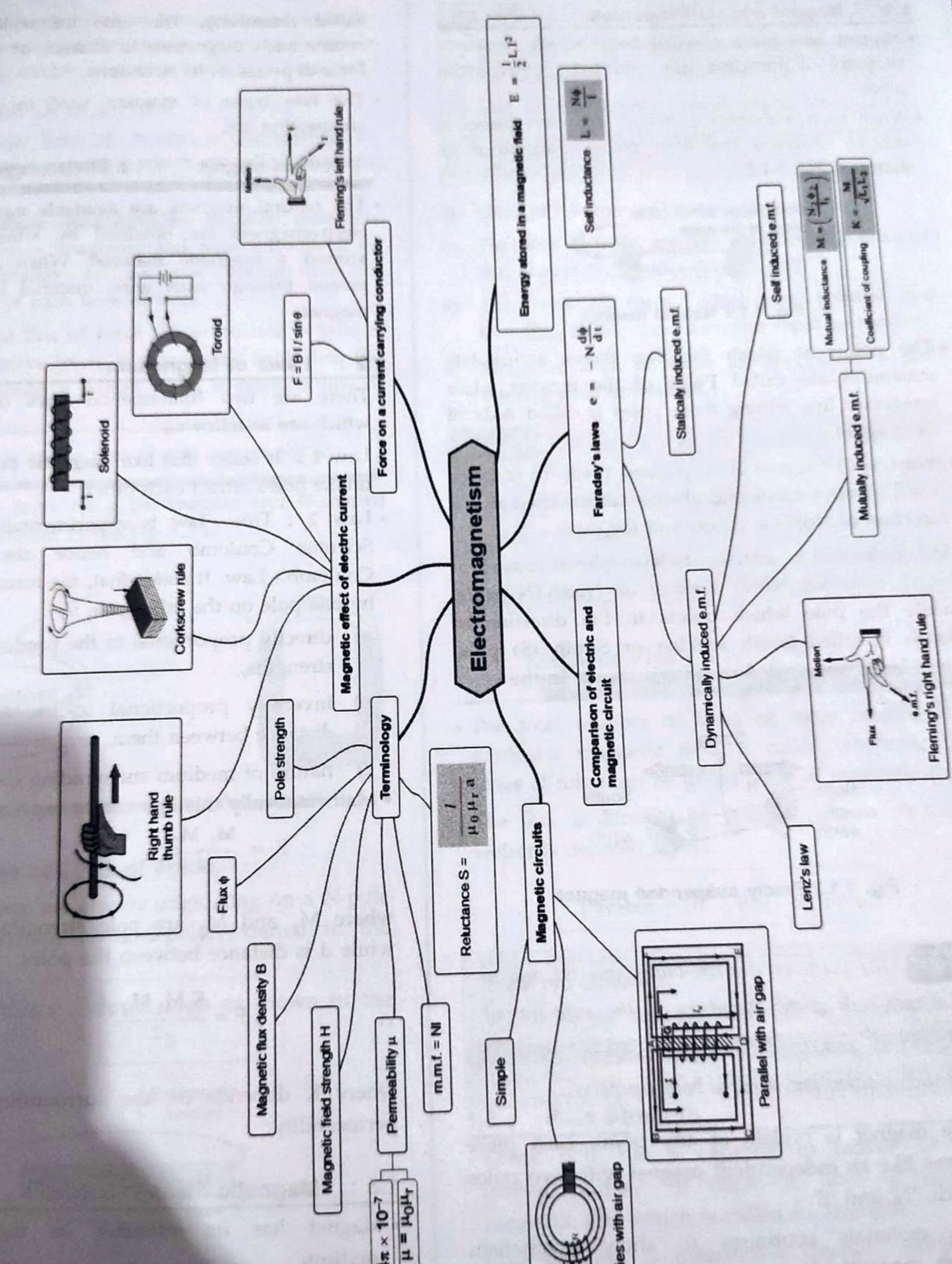

Electromagnetism
Magnetic effect of electric current
Solenoid
Toroid
Corkscrew rule
Right hand thumb rule
Force on a current carrying conductor
F = BI l sin θ
Fleming's left hand rule
Terminology
Pole strength
Flux φ
Magnetic flux density B
Magnetic field strength H
Permeability μ
$\mu_0 = 4\pi \times 10^{-7}$
$\mu = \mu_0\mu_r$
m.m.f. = NI
Reluctance S = $\frac{l}{\mu_0 \mu_r a}$
Magnetic circuits
Simple
Series with air gap
Parallel with air gap
Comparison of electric and magnetic circuit
Faraday's laws $e = -N\frac{d\phi}{dt}$
Statically induced e.m.f.
Self induced e.m.f.
Mutually induced e.m.f.
Mutual inductance $M = \left(\frac{N_2\phi_2}{I_1}\right)$
Coefficient of coupling $K = \frac{M}{\sqrt{L_1L_2}}$
Self inductance $L = \frac{N\phi}{I}$
Energy stored in a magnetic field $E = \frac{1}{2}LI^2$
Dynamically induced e.m.f.
Fleming's right hand rule
Motion
Flux
e.m.f.
Lenz's law

1.1 : Magnet and its Properties

- Magnet is a piece of solid body which possesses property of attracting iron and some other metal pieces.
- When such a magnet is rolled into iron pieces it will be observed that iron pieces cling to it as shown in Fig. 1.1.1

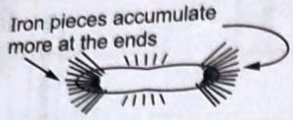


Fig. 1.1.1 Natural magnet

- The points at which the iron pieces accumulate maximum are called **Poles** of the magnet while imaginary line joining these poles is called **Axis** of the magnet.
- When such magnet is suspended freely by a piece of silk fibre, it turns and always adjusts itself in the direction of North and South of the earth.
- The pole which adjusts itself in the direction of North is called North seeking or **North (N) pole,** while the pole which points in the direction of South is called South seeking or **South (S) pole**. Such freely suspended magnet is shown in the Fig. 1.1.2.

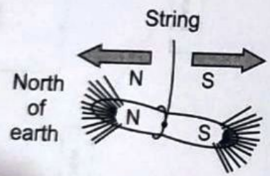


Fig. 1.1.2 Freely suspended magnet

Key Point *Like poles repel each other and the unlike poles attract each other. Repulsion is the sure test of magnetism as ordinary piece of magnetic material always shows attraction towards both the poles.*

If the magnet is broken at any point, each piece behaves like an independent magnet with two poles to each, 'N' and 'S'.

Some materials continues to show magnetism though magnetizing force is removed. They retain magnetism for some time. The power of retaining magnetism after the magnetizing force is removed is called **retentivity.** The time for which material retains such magnetism in absence of magnetizing force depends on its retentivity.

- The two types of magnets used in an electrical engineering are,

1. Natural magnet	2. Electromagnet

- The natural magnets are available naturally while electromagnets are obtained by winding a wire around a magnetic material. When a current is passed through such wire, material behaves as a magnet.

1.2 : Laws of Magnetism

- There are two fundamental laws of magnetism which are as follows :
- **Law 1 :** It states that like magnetic poles repel and unlike poles attract each other.
- **Law 2 :** This law is experimentally proved by Scientist Coulomb and hence also known as Coulomb's Law. It states that, the force (F) exerted by one pole on the other pole is,
  a) directly proportional to the product of the pole strengths,
  b) inversely proportional to the square of the distance between them, and
  c) nature of medium surrounding the poles.
- Mathematically this law can be expressed as,

$$F \propto \frac{M_1 M_2}{d^2}$$

where M_1 and M_2 are pole strengths of the poles while d is distance between the poles.

$$\therefore \quad \boxed{F = \frac{K M_1 M_2}{d^2}}$$

where K depends on the surroundings and called permeability.

1.3 : Magnetic Field

- Magnet has its influence on the surrounding medium.
- The region around a magnet within which the influence of the magnet can be experienced is called **magnetic field**.

- Existence of such field can be experienced with the help of compass needle, iron or pieces of metals or by bringing another magnet in vicinity of a magnet.

1.3.1 Magnetic Lines of Force

- The magnetic field of magnet is represented by imaginary lines around it which are called **magnetic lines of force.**
- Note that these lines have no physical existence, these are purely imaginary and were introduced by **Michael Faraday** to get the visualization of distribution of such lines of force.
- **Definition of line of force :** If an isolated N pole is allowed to move freely in a magnetic field, then the line along which it would move is called a line of force.

Its direction is always from N-pole towards S-pole.

- The lines of force for a bar magnet and U-shaped magnet are shown in the Fig. 1.3.1.

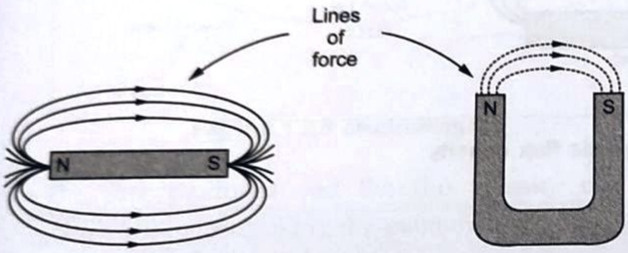


Fig. 1.3.1 Concept of magnetic flux density

1.3.2 Properties of Lines of Force

1) Lines of force are always originating on a N-pole and terminating on a S-pole, external to the magnet.
2) Each line forms a **closed loop** as shown in the Fig. 1.3.2.

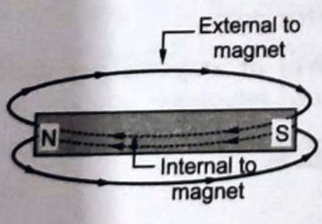


Fig. 1.3.2 Lines of force complete the closed path

Key Point *This means that a line emerging from N-pole, continues upto S-pole **external to the magnet** while it is assumed to continue from S-pole to N-pole **internal to the magnet** completing a closed loop. Such lines internal to the magnet are called as **lines of induction.***

3) Lines of force never intersect each other.
4) The lines of force, are like stretched rubberbands and always try to take shortest path.
5) The lines of force, which are parallel and travelling in the same direction repel each other.
6) Magnetic lines of force always prefer a path offering least opposition.

Key Point *The opposition by the material to the flow of lines of force is called reluctance. Air has more reluctance while magnetic materials like iron, steel etc. have low reluctance. Thus magnetic lines of force can easily pass through iron or steel but cannot pass easily through the air.*

1.4 : Magnetic Flux (ϕ)

SPPU : May-04,06,07,08,11,12,14, Dec.-05,06,07

- The total number of lines of force existing in a particular magnetic field is called **magnetic flux.** Lines of force can be called **lines of magnetic flux.**
- The flux is denoted by symbol (ϕ) and its unit is **weber** is denoted as **Wb.**

$$\therefore \quad \boxed{1 \text{ weber} = 10^8 \text{ lines of force}}$$

Expected Question

1. *Define magnetic flux and state its units.*

SPPU : May-04,06,07,08,11,12,14, Dec.-05,06,07, Marks 2

1.5 : Pole Strength

- Every pole has a capacity to radiate or accept certain number of magnetic lines of force i.e. magnetic flux which is called its **strength.**
- Pole strength is measurable quantity assigned to poles which depends on the force between the poles. If two poles are exerting equal force on one other, they are said to have equal pole strengths.

- Unit of pole strength is **weber** as pole strength is directly related to flux i.e. lines of force.

Key Point *A unit pole may be defined as that pole which when placed from an identical pole at a distance of 1 metre in free space experiences a force of $\frac{10^7}{16\pi^2}$ newtons.*

- So when we say Unit N-pole, it means a pole is having a pole strength of 1 weber.

1.6 : Magnetic Flux Density (B)

SPPU : May-04,06,07,08,11,12,14, Dec.-05,06,07

- It can be defined as 'The flux per unit area (a) in a plane at right angles to the flux is known as '**flux density**'. Mathematically,

$$\therefore \quad \boxed{B = \frac{\phi}{a} \ \frac{Wb}{m^2} \text{ or Tesla}}$$

- It is shown in the Fig. 1.6.1.

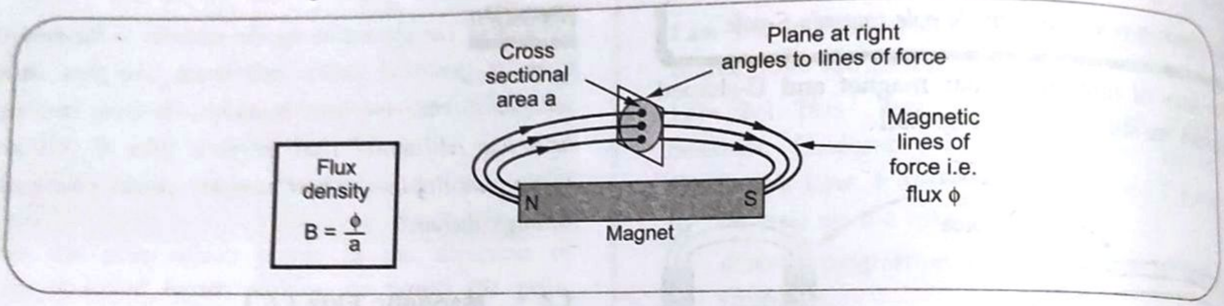


Fig. 1.6.1 Concept of magnetic flux density

Key Point *The unit of flux density is Wb/m², also called **tesla** denoted as T.*

Expected Question

1. *Define magnetic flux density and state its unit.* SPPU : May-04,06,07,08,11,12,14, Dec.-05,06,07, Marks 2

1.7 : Magnetic Field Strength (H)

SPPU : May-04,08,11,12,14, Dec.-05,07

- This gives quantitative measure of strongness or weakness of the magnetic field. Note that pole strength and magnetic field strength are different.
- The force experienced by a unit N-pole (i.e. N pole with 1 Wb of pole strength) when placed at any point in a magnetic field is known as **magnetic field strength** at that point.
- It is denoted by H and its unit is **newtons per weber i.e. (N/Wb) or amperes per metre** (A/m) or **ampere turns per metre (AT/m).**
- The mathematical expression for calculating magnetic field strength is,

$$\therefore \quad \boxed{H = \frac{\text{Ampere turns}}{\text{Length}} = \frac{NI}{l} \ \text{AT/m}}$$

Key Point *More the value of 'H', more stronger is the magnetic field. This is also called **magnetic field intensity**.*

Expected Question

1. *Define magnetic field strength and state its units.*

SPPU : May-04,08,11,12,14, Dec.-05,07, Marks 2

1.8 : Magnetic Effect of an Electric Current (Electromagnets)

- When a coil or a conductor carries a current, it produces the magnetic flux around it. Then it starts behaving as a magnet. Such a current carrying coil or conductor is called an **electromagnet.** This is due to magnetic effect of an electric current.
- The Fig. 1.8.1 shows the concept of an electromagnet.

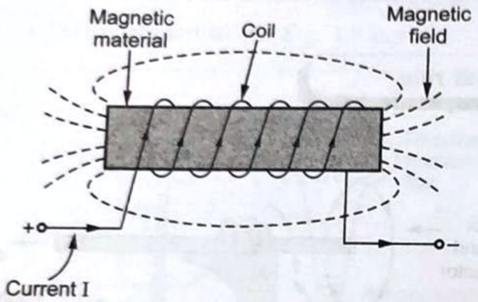


Fig. 1.8.1 An electromagnet

- The flux produced and the flux density can be controlled by controlling the magnitude the current.
- The direction and shape of the magnetic field around the coil or conductor depends on the direction of current and shape of the conductor through which it is passing.

1.8.1 Magnetic Field due to Straight Conductor

- When a straight conductor carries a current, it produces a magnetic field all along its length. The lines of force are in the form of concentric circles in the planes right angles to the conductor. This can be demonstrated by a small experiment.
- Consider a straight conductor carrying a current, passing through a sheet of cardboard as shown in the Fig. 1.8.2.

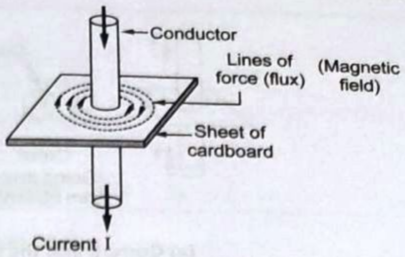


Fig. 1.8.2 Magnetic field due to a straight conductor

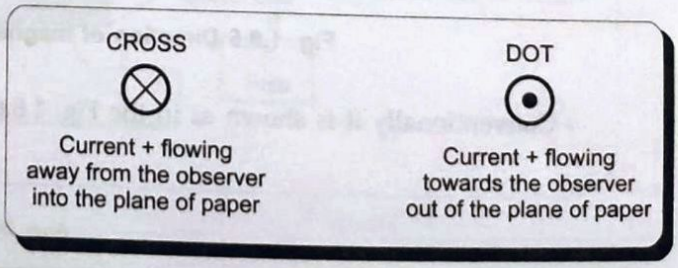
- Sprinkle iron fillings on the cardboard. Small tapping on the cardboard causes the iron filling to set themselves, in the concentric circular pattern.
- The direction of the magnetic flux can be determined by placing compass needle near the conductor. This direction depends on the direction of the current passing through the conductor.
- For the current direction shown in the Fig. 1.8.2 i.e. from top to bottom the direction of flux is clockwise around the conductor.
- Conventionally such current carrying conductor is represented by small circle, (top view of conductor shown in the Fig. 1.8.2). Then current through such conductor will either come out of paper or will go into the plane of the paper.

Key Point *When current is going into the plane of the paper, i.e. away from observer, it is represented by a 'cross', inside the circle indicating the conductors.*

- The cross indicates rear view of feathered end of an arrow.

Key Point *The current flowing towards the observer i.e.* ***coming out of the plane of the paper*** *is represented by a 'dot' inside the circle.*

- The dot indicates front view i.e. tip of an arrow. This is shown in the Fig. 1.8.3.
(See Fig. on next page)



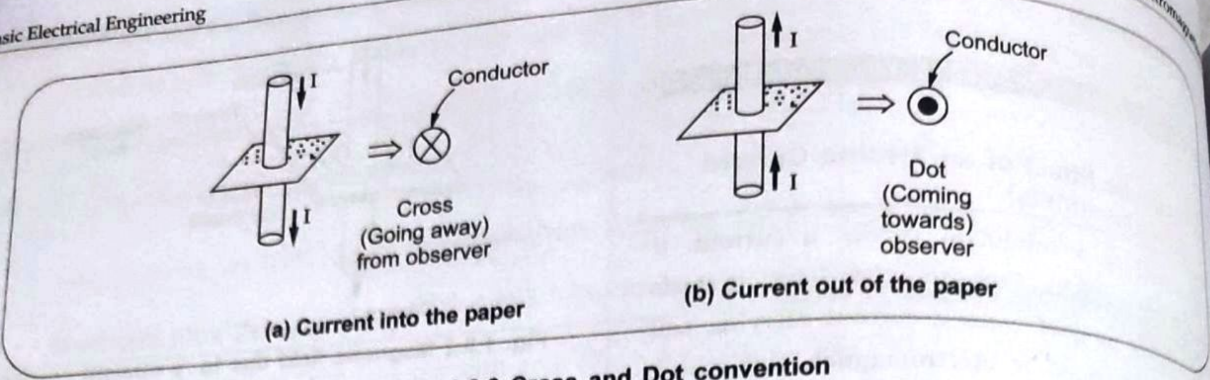


Fig. 1.8.3 Cross and Dot convention

1.8.2 Rules to Decide Direction of Magnetic Field

- The rules to decide the direction of magnetic field due to the current flowing through the conductor are,

1. Right hand thumb rule	2. Corkscrew rule	3. End rule

1) Right Hand Thumb Rule :

- It states that, hold the current carrying conductor in the right hand such that the thumb pointing in the direction of current and parallel to the conductor, then curled fingers gives the direction of the magnetic field or flux around it.
- The Fig. 1.8.4 explains the rule.
- Let us apply this rule to the conductor passing through card sheet considered earlier. This can be explained by the Fig. 1.8.5.

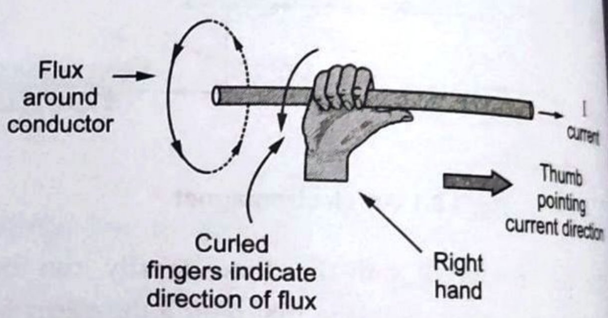


Fig. 1.8.4 Right hand thumb rule

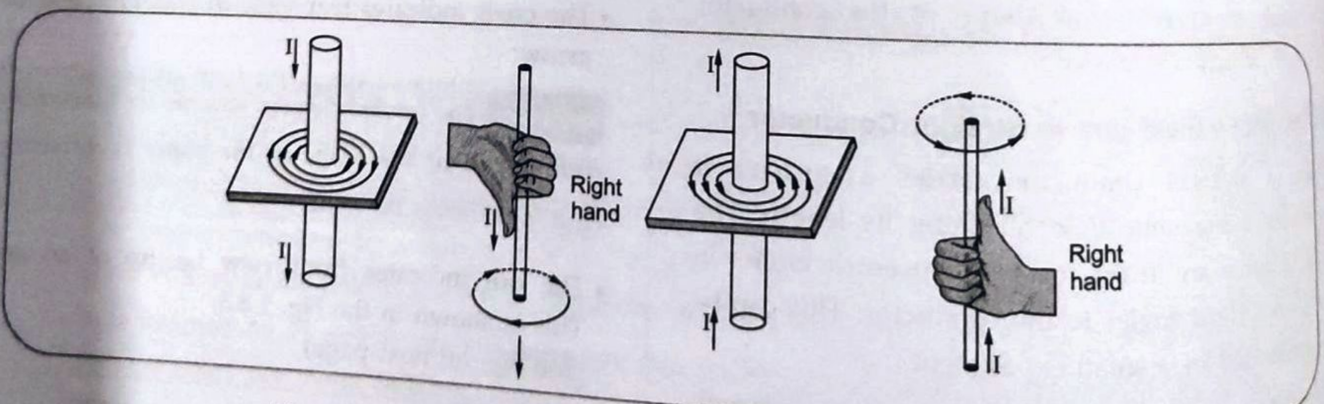


Fig. 1.8.5 Direction of magnetic lines by right hand thumb rule

- Conventionally it is shown as in the Fig. 1.8.6.

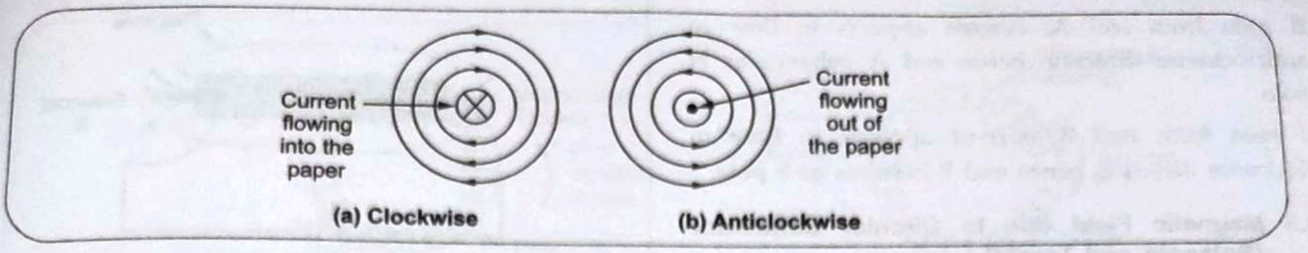


Fig. 1.8.6 Representation of direction of flux

2) Corkscrew Rule :

- Imagine a right handed screw to be along the conductor carrying current with its axis parallel to the conductor and tip pointing in the direction of the current flow.
- Then the direction of the magnetic field is given by the direction in which the screw must be turned so as to advance in the direction of the current.
- This is shown in the Fig. 1.8.7.

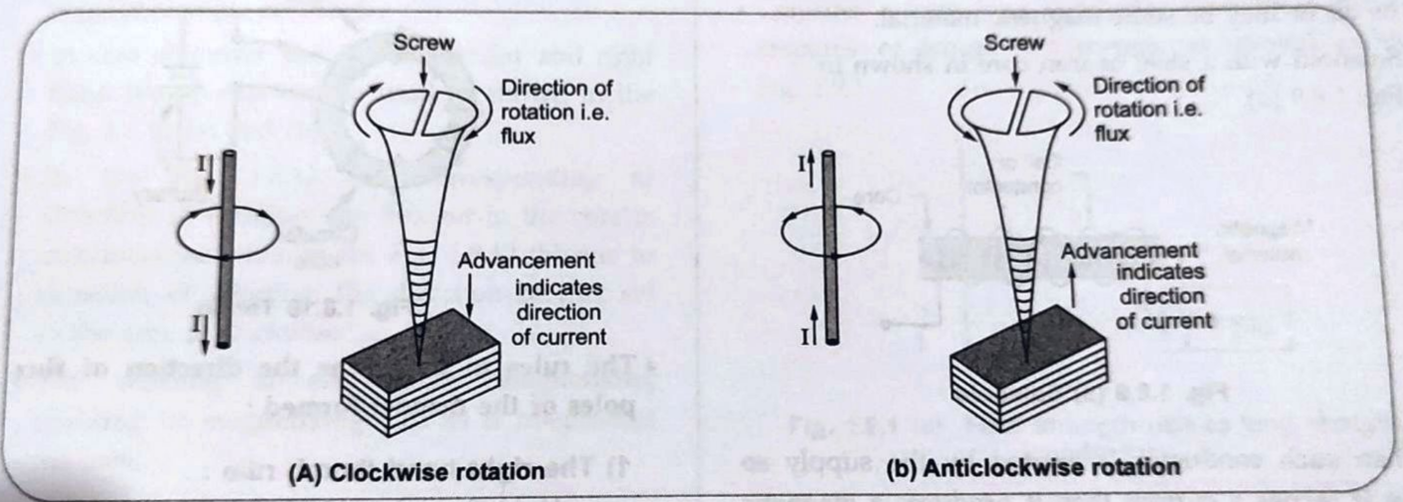


Fig. 1.8.7 Corkscrew rule

3) End Rule :

- If an electromagnet is observed from any one end then direction of magnetic field can be decided from the direction of current.
- Consider an electromagnet shown in the Fig. 1.8.8.

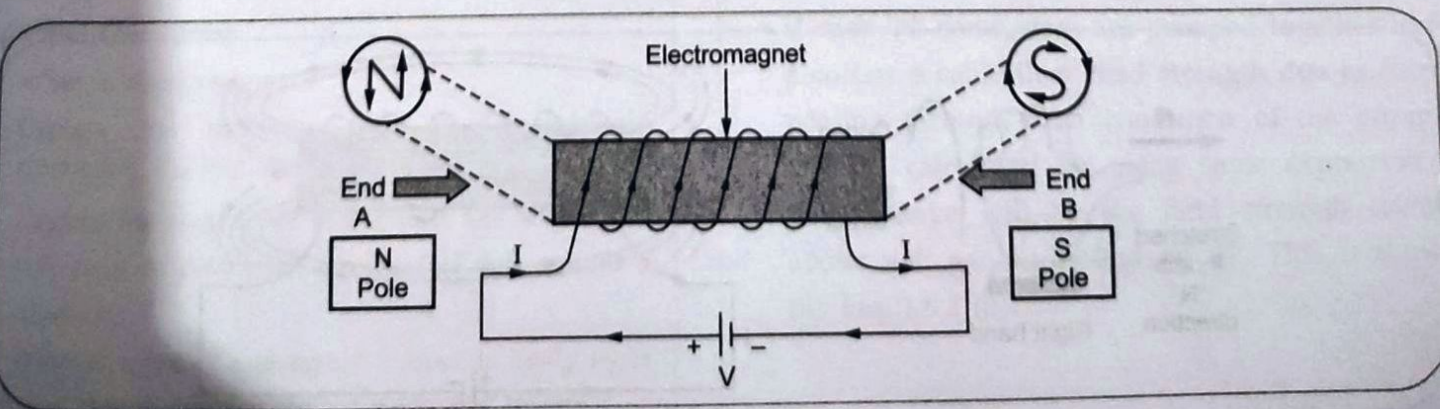


Fig. 1.8.8 End rule

- If seen from end A, current appears to flow in anticlockwise direction hence end A behaves as N pole.
- If seen from end B, current appears to flow in clockwise direction hence end B behaves as S pole.

1.8.3 Magnetic Field due to Circular Conductor (Solenoid and Toroid)

1. Solenoid

- A **solenoid** is an arrangement in which long conductor is wound with number of turns close together to form a coil. The axial length of conductor is much more than the diameter of turns.
- The part or element around which the conductor is wound is called as **core** of the solenoid. Core may be air or may be some magnetic material.
- Solenoid with a steel or iron core in shown in Fig. 1.8.9 (a).

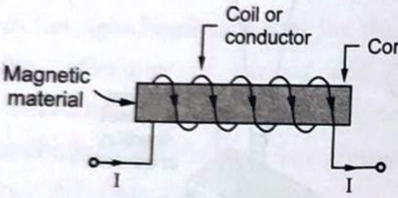


Fig. 1.8.9 (a) Solenoid

- When such conductor is excited by the supply so that it carries a current then it produces a magnetic field which acts through the coil along its axis and also around the solenoid.
- The pattern of the flux around the solenoid is shown in the Fig. 1.8.9 (b).

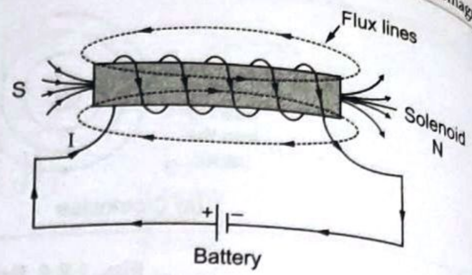


Fig. 1.8.9 (b) Flux around a solenoid

2. Toroid

- Instead of using a straight core to wound the conductor, a circular core also can be used to wound the conductor. In such case the resulting solenoid is called **Toroid**.

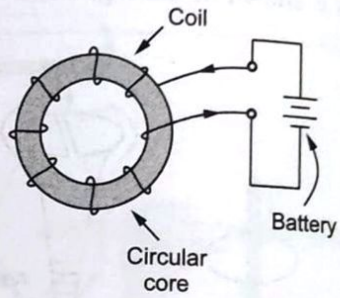


Fig. 1.8.10 Toroid

- **The rules to determine the direction of flux and poles of the magnet formed :**

1) The right hand thumb rule :

- Hold the solenoid in the right hand such that curled fingers point in the direction of the current through the curled conductor, then the outstretched thumb along the axis of the solenoid point to the **North pole** of the solenoid or point the direction of flux lines **inside the core.** This is shown in Fig. 1.8.11.

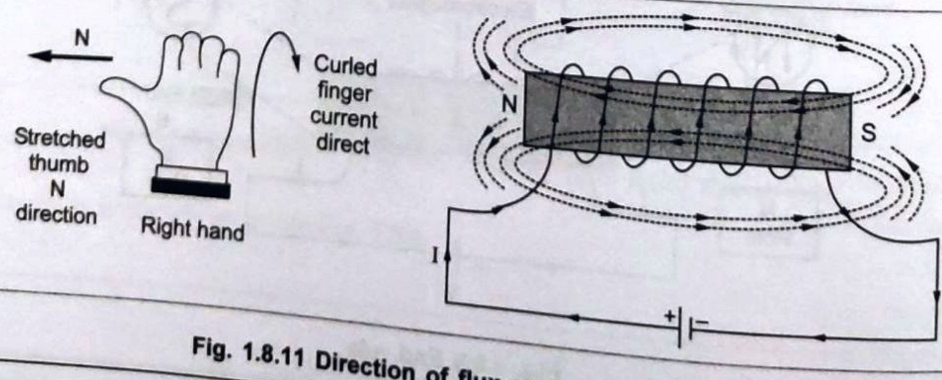


Fig. 1.8.11 Direction of flux around a solenoid

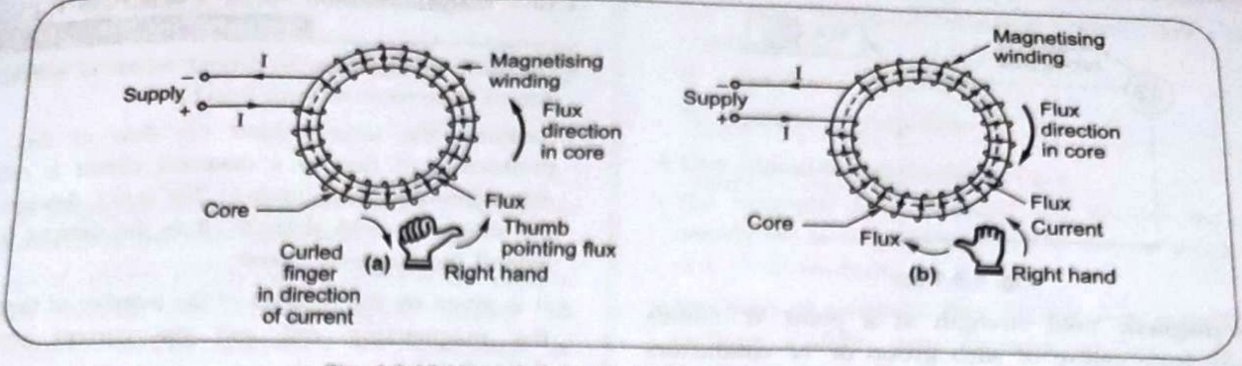


Fig. 1.8.12 Use of right hand thumb rule for toroid

- If the direction of winding or direction of current is reversed then the positions of N and S poles also get reversed.
- In case of toroid, the core is circular and right hand thumb rule can be used as shown in the Fig. 1.8.12 (a) and (b).
- In the Fig. 1.8.12 (a), corresponding to direction of winding, the flux set in the core is anticlockwise while in the Fig. 1.8.12 (b) due to direction of winding, the direction of flux set in the core is clockwise.
- The winding is also called **magnetising winding** or **magnetising coil** as it magnetises the core.

2) Corkscrew rule :

- If axis of the screw is placed along the axis of the solenoid and if screw is turned in the direction of the current, then it travels towards the **N-pole** or in the direction of the magnetic field **inside** the solenoid.

Expected Questions

1. *What is electromagnet ?*
2. *Explain the magnetic field due to straight conductor. Explain the cross and dot convention.*
3. *Explain the right hand thumb rule and cork screw rule used to determine direction of flux around a conductor*
4. *What is solenoid and toroid ? How to apply right hand thumb rule for the solenoids and toroids ?*

1.9 : Nature of Magnetic Field of Long Straight Conductor

- Consider a very long conductor carrying current I amperes of length 'l' meters, as shown in the Fig. 1.9.1.

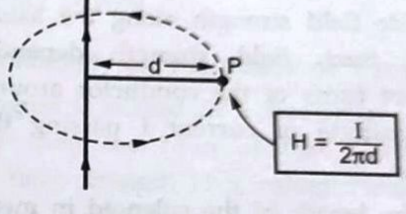
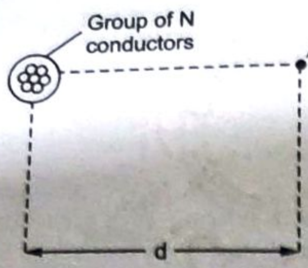


Fig. 1.9.1 (a) Field strength due to long straight conductor

- The field strength at any point P within magnetic field at a distance 'd' from the centre of a long conductor is given by,

$$\boxed{H = \frac{I}{2\pi d} \quad \text{A/m}}$$

- If such 'N' conductors are grouped together to form a coil or a cable then field strength due to current I passing through each conductor of the group can also be calculated by using same expression. The only change will be the field strength calculated above will get multiplied by 'N'. This is shown in the Fig. 1.9.1 (b).



$$H = \frac{NI}{2\pi d}$$

Fig. 1.9.1 (b)

- So magnetic field strength at a point 'd' metres away from centre of such group of 'N' conductors each carrying current I amperes is

$$\boxed{H = \frac{NI}{2\pi d} \quad \text{AT/m}}$$

- AT/m means ampere turns per metre.

1.9.1 Magnetic Field Strength due to a Long Solenoid

- Similar to the case of long straight conductor, we can decide field strength along the axis of a long solenoid. Such field strength depends on the number of turns of the conductor around the core and magnitude of current I passing through the conductor.
- If 'l' is the length of the solenoid in metres then H can be determined as,

$$\boxed{H = \frac{NI}{l} \text{ AT/m}}$$

Ex 1.9.1 : ***A current of 2 amp is flowing through each of the conductors in a coil containing 15 such conductors. If a point pole of unit strength is placed at a perpendicular distance of 10 cm from the coil, determine the field intensity at that point.***

Sol. : Given : I = 2 A, N = 15, d = 10 cm = 0.1 m.

To find : H

$$H = \frac{NI}{2\pi d} = \frac{15\times 2}{2\times\pi\times 0.1} = \mathbf{47.74\ AT/m}$$

Expected Question

1. *Explain the nature of magnetic field of long straight conductor.*

1.10 : Magnetomotive Force (M.M.F. or F)

SPPU : Dec.-06, May-10, 11, 14

- The flow of electrons is current which is basically due to electromotive force (e.m.f.).
- Similarly the force behind the flow of flux or production of flux in a magnetic circuit is called magnetomotive force (m.m.f.) The m.m.f. determines the magnetic field strength. It is the driving force behind the magnetic circuit.
- It is given by the product of the number of turns of the magnetizing coil and the current passing through it.
- Mathematically it can be expressed as,

$$\boxed{\text{m.m.f.} = N\,I \quad \text{ampere turns}}$$

where N = Number of turns of magnetising coil

and I = Current through coil

- Its unit is **ampere turns (AT) or amperes (A).**
- It is also defined as the work done in joules on a unit magnetic pole in taking it once round a closed magnetic circuit.

Expected Questions

1. *Explain the nature of magnetic field of long straight conductor.*
2. *Explain the concept of magnetomotive force and state its units.*

SPPU : Dec.-06, May-10, 11, 14, Marks 2

1.11 : Reluctance (S)

SPPU : May- 04,06,07,08,10,11,14, Dec.-06,07

- In an electric circuit, current flow is opposed by the resistance of the material, similarly there opposition by the material to the flow of flux which is called **reluctance**
- It is defined as the resistance offered by the material to the flow of magnetic flux through it. is denoted by 'S'.
- It is directly proportional to the length of magnetic circuit while inversely proportional to area of cross-section.

$$S \propto \frac{l}{a} \qquad \text{where } 'l' \text{ in } 'm' \text{ while } 'a' \text{ in } 'm^2'$$

$$\therefore \quad S = \frac{Kl}{a}$$

where K = Constant

= Reciprocal of absolute permeability of material = $\frac{1}{\mu}$

$$\therefore \quad \boxed{S = \frac{l}{\mu a} = \frac{l}{\mu_0 \mu_r a} \quad A/Wb}$$

- It is measured in **amperes per weber (A/Wb)** or **ampere-turns per weber (AT/Wb).**
- As in the electric circuit the resistance is the ratio of e.m.f. and the current, in magnetic circuit the reluctance can be expressed as the ratio of magnetomotive force to the flux produced.

$$\therefore \quad \text{Reluctance} = \frac{\text{m.m.f}}{\text{flux}}$$

$$\text{i.e.} \quad \boxed{S = \frac{NI}{\phi} \quad AT/Wb \text{ or } A/Wb}$$

1.11.1 Permeance

- The **permeance** of the magnetic circuit is defined as the reciprocal of the reluctance.
- It is defined as the property of the magnetic circuit due to which it allows flow of the magnetic flux through it.

$$\therefore \quad \boxed{\text{Permeance} = \frac{1}{\text{Reluctance}}}$$

- It is measured in **weber per amperes (Wb/A).**

Expected Question

1. *What is reluctance ? Obtain its expression and state its units.*

SPPU : May- 04,06,07,10,11,14, Dec.-06, 07, Marks 2

1.12 : Permeability

SPPU : Dec.-05,06,07, May-04,06,07,08,10,11,12,14

- The flow of flux produced by the magnet not only depends on the magnetic field strength but also related to the medium in which magnet is placed. The force exerted by one magnetic pole on other depends on the medium in which magnets are placed.

Key Point *The **permeability** is defined as the ability or ease with which the magnetic material forces the magnetic flux through a given medium.*

- For any magnetic material, there are two permeabilities,
  i) Absolute permeability
  ii) Relative permeability.

1.12.1 Absolute Permeability (μ)

- The magnetic field strength (H) decides the flux density (B) to be produced by the magnet around it, in a given medium.
- The ratio of magnetic flux density *B* in a particular medium (other than vacuum or air) to the magnetic field strength H producing that flux density is called **absolute permeability** of that medium.
- It is denoted by μ and mathematically can be expressed as,

$$\therefore \quad \boxed{\mu = \frac{B}{H} \quad \text{i.e.} \quad B = \mu H}$$

The permeability is measured in units **henries per metre** denoted as **H/m.**

1.12.2 Permeability of Free Space or Vacuum (μ_0)

- If the magnet is placed in a free space or vacuum or in air then the ratio of flux density B and magnetic field strength H is called **Permeability of free space** or **Vacuum** or **air.**
- It is denoted as μ_0 and measured in **H/m.** It denotes the ease with which the magnetic flux permeates the free space or vacuum or air.
- It is experimentally found that this μ_0 i.e. ratio of B and H in vacuum remains constant every where in the vacuum and its value is $4\pi \times 10^{-7}$ H/m.

$$\therefore \quad \boxed{\begin{aligned} \mu_0 &= \frac{B}{H} \text{ in vacuum} = 4\pi \times 10^{-7} \text{ H/m} \\ &= 1.256 \times 10^{-6} \text{ N/m} \end{aligned}}$$

Key Point *For a magnetic material, the absolute permeability μ is not constant. This is because B and H bears a nonlinear relation in case of magnetic materials. If magnetic field strength is increased, there is change in flux density B but not exactly proportional to the increase in H. The ratio B to H is constant only for **free space, vacuum or air .***

1.12.3 Relative Permeability (μ_r)

- Generally the permeability of different magnetic materials is defined relative to the permeability of free space (μ_0).
- The **relative permeability** is defined as the ratio of flux density produced in a medium (other than free space) to the flux density produced in free space, under the influence of same magnetic field strength and under identical conditions.
- Thus if the magnetic field strength is H which is producing flux density B in the medium while flux density B_0 in free space then the relative permeability is defined as,

$$\boxed{\mu_r = \frac{B}{B_0} \quad \text{where H is same.}}$$

- **It is dimensionless and has no units.**
- For free space, vacuum or air, $\mu_r = 1$
- According to definition of absolute permeability we can write for given H,

$$\mu = \frac{B}{H} \quad \text{in medium} \quad \ldots(1.12.1)$$

and $$\mu_0 = \frac{B_0}{H} \quad \text{in free space} \quad \ldots(1.12.2)$$

Dividing (1.12.1) and (1.12.2) ,

$$\frac{\mu}{\mu_0} = \frac{B}{B_0}$$

but $$\frac{B}{B_0} = \mu_r$$

i.e. $$\frac{\mu}{\mu_0} = \mu_r$$

$\therefore$ $$\boxed{\mu = \mu_0 \mu_r \ \text{H/m}}$$

- The relative permeability of metals like iron, steel varies from 100 to 100,000

Key Point *If we require maximum flux production for the lesser magnetic field strength then the value of the relative permeability of the core material should be as high as possible.*

Expected Questions

1. *Define permeance and state its units.* **SPPU : Dec.-05,07, May-06,07,08, 10, Marks 2**
2. *Define permeability.* **SPPU : Dec.-05,06, May-07, 10, 11, Marks 2**
3. *Define absolute permeability. State its units.* **SPPU : May-06,08, Marks 2**
4. *Define permeablity of free space and state its units.* **SPPU : May-04,12,14, Marks 2**
5. *What is relative permeability ?* **SPPU : May-04,07,08, Dec.-07, Marks 2**

1.13 Magnetic Circuits

SPPU : Dec.-03, 08, 09, 13, 14, 15, 17, May-05, 09, 11, 12, 13, 14, 17, 18

Definition

- The **magnetic circuit** can be defined as, the closed path traced by the magnetic lines of force i.e. flux. Such a magnetic circuit is associated with different magnetic quantities as m.m.f., flux, reluctance, permeability etc.

- Consider simple magnetic circuit shown in the Fig. 1.13.1 (a). This circuit consists of an iron core with cross-sectional area of 'a' m^2 with a mean length of 'l' m. (This is mean length of the magnetic path which flux is going to trace.)
- A coil of N turns is wound on one of the sides of the square core which is excited by a supply. This supply drives a current I through the coil.

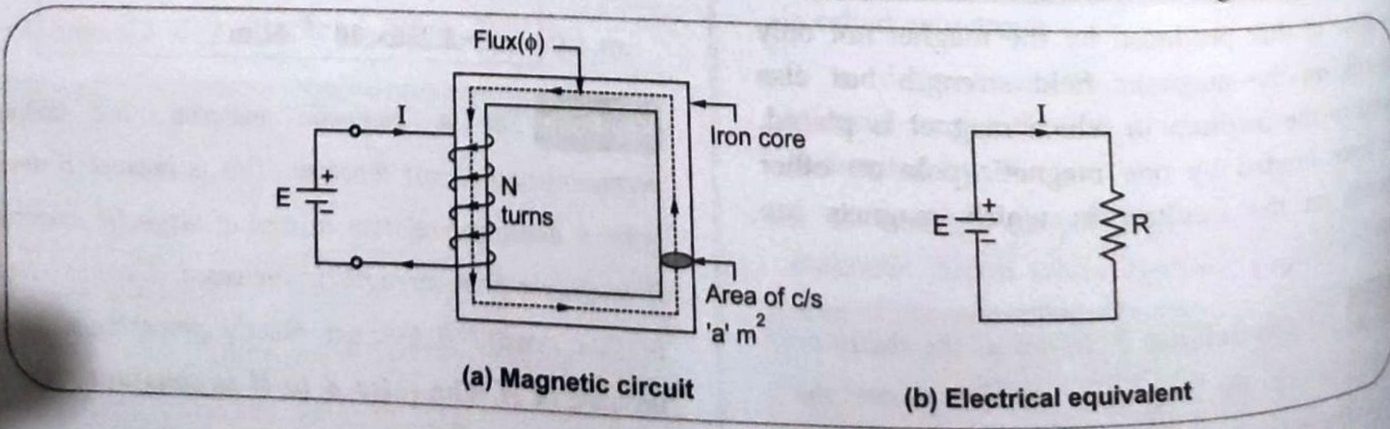


Fig. 1.13.1

- This current carrying coil produces the flux (ϕ) which completes its path through the core as shown in the Fig. 1.13.1 (a).
- This is analogous to simple electric circuit in which a supply i.e. e.m.f. of E volts drives a current I which completes its path through a closed conductor having resistance R. This analogous electrical circuit is shown in the Fig. 1.13.1 (b).

1.13.1 Relationship between M.M.F., Flux and Reluctance

I = Current flowing through the coil,

N = Number of turns,

ϕ = Flux in webers

B = Flux density in the core,

μ = Absolute permeability of the magnetic material

μ_r = Relative permeability of the magnetic material

- Magnetic field strength inside the solenoid is given by,

$$H = \frac{NI}{l} \text{ AT/m} \quad \ldots(1.13.1)$$

Now flux density is,

$$B = \mu H = \frac{\mu_0 \mu_r N I}{l} \text{ Wb/m}^2 \quad \ldots(1.13.2)$$

Now as area of cross-section is 'a' m^2, total flux in core is,

$$\phi = B a = \frac{\mu_0 \mu_r N I a}{l} \text{ Wb} \quad \ldots(1.13.3)$$

Thus,

$$\boxed{\phi = \frac{NI}{\dfrac{l}{\mu_0 \mu_r a}} = \frac{\text{m.m.f.}}{\text{reluctance}}}$$

where NI = Magnetomotive force m.m.f. in AT

$S = \dfrac{l}{\mu_0 \mu_r a}$ = Reluctance offered by the magnetic path.

- This expression of the flux is very much similar to expression for current in electric circuit.

Key Point *So current is analogous to the flux, e.m.f. is analogous to the m.m.f. and resistance is analogous to the reluctance.*

Ex. 1.13.1 : ***An iron ring having mean diameter 20 cm is wound with 600 turns and carries a current of 1.5 A. The area of cros-section of the ring is 20 cm^2. The relative permeability of iron is 500. Find i) mmf ii) Reluctance iii) Flux***

Sol. : Given : D = 20 cm = 0.2 m

$l = \pi D = 3.14 \times 0.2 = 0.628$ m, N = 600, I = 1.5 A

$\mu_r = 500$, $a = 20 \text{ cm}^2 = 20 \times 10^{-4} \text{ m}^2$

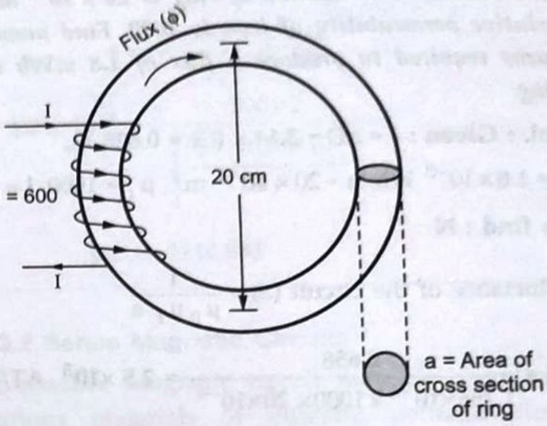


Fig. 1.13.2

To find : mmf, S, ϕ

i) mmf = NI = 600 × 1.5 = **900** AT

ii) Reluctance (S) = $\dfrac{l}{\mu_0 \mu_r a}$

$$= \frac{0.628}{1.256 \times 10^{-6} \times 500 \times 20 \times 10^{-4}}$$

$$S = \mathbf{5 \times 10^5} \text{ AT/Wb}$$

Note : $\mu_0 = 4\pi \times 10^{-7}$ H/m $= 4 \times 3.14 \times 10^{-7}$

$= \mathbf{1.256 \times 10^{-6}}$ H/m

iii) Flux (ϕ) = $\dfrac{\text{mmf (NI)}}{\text{Reluctance (S)}} = \dfrac{900}{5 \times 10^5} = \mathbf{1.8 \times 10^{-3}}$ **Wb**

Ex. 1.13.2 : ***In Ex. 1.13.1 if the relative permeability of iron used is μ_r = 1000, find new, reluctance and flux.***

Sol. : i) As number of turns and current is same ; mmf will be same i.e. NI = 600 × 1.5 = **900** AT

ii) Reluctance (S) = $\dfrac{l}{\mu_0 \mu_r \times a}$

$$= \frac{0.628}{1.256 \times 10^{-6} \times 1000 \times 20 \times 10^{-4}}$$

$$S = \mathbf{2.5 \times 10^5} \text{ AT}$$

- It is found that reluctance become just half the original value.

$$S \propto \frac{1}{\mu}$$

iii) Flux $(\phi) = \frac{NI}{S} = \frac{600 \times 1.5}{2.5 \times 10^5} = 3.6 \times 10^{-3}$ Wb = 3.6 mWb

Flux is doubled $\left[\phi \propto \frac{1}{S}\right]$

Ex. 1.13.3 : ***An iron ring having mean diameter 20 cm is wound with N turns and carry current of 1.5 A. The area of cross section of ring is 20×10^{-4} m^2 and relative permeability of iron is 1000. Find number of turns required to produce a flux of 1.8 mWb in the ring.***

Sol. : Given : $l = \pi D = 3.14 \times 0.2 = 0.628$ m,

$\phi = 1.8 \times 10^{-3}$ Wb, $a = 20 \times 10^{-4}$ m^2, $\mu_r = 1000$, I = 1.5 A

To find : N

Reluctance of the circuit $(S) = \frac{l}{\mu_0 \mu_r a}$

$$S = \frac{0.658}{1.256 \times 10^{-6} \times 1000 \times 20 \times 10^{-4}} = \mathbf{2.5 \times 10^5} \text{ AT/Wb}$$

Now, $\phi = \frac{NI}{S}$ i.e. $NI = \phi S$

$$N \times 1.5 = 1.8 \times 10^{-3} \times 2.5 \times 10^5$$

$$N = \mathbf{300}$$

[Thus, as relative permeability is doubled ; number of turns required is halved (i.e. half)]

∴ Thus, less copper material required, coil and equipment will be cheap and can be easily sell in the today's competitive market.

Ex. 1.13.4 : ***In the above Ex. 1.13.3 find the mmf required to maintain the flux = 1.8 mWb with $\mu_r = 1000$.***

Sol. : NI = mmf = $\phi \times S$

$= 1.8 \times 10^{-3} \times 2.5 \times 10^5 =$ **450 AT** (Just half)

Now, mmf = NI, either N or I can be decreased.

If N = 600 ; I = **0.75 A**

Ex. 1.13.5 : ***A coil of 500 turns is uniformly wound on iron ring having mean circumference 50 cm, carrying current of 3 A. The iron ring has square cross section of 2 cm × 2 cm. The relative permeability of iron is 1500, find i) mmf ii) reluctance iii) flux***

Sol. : Given : N = 500 ; I = 3 A, l = 50 cm = 0.5 m

$a = 2 \text{ cm} \times 2 \text{ cm} = 4 \text{ cm}^2 = 4 \times 10^{-4} \text{ m}^2$, $\mu_r = 1500$

To find : mmf, S, ϕ

i) mmf = NI = 500×3 = 1500 AT

ii) Reluctance(S) $= \frac{l}{\mu_0 \mu_r a} = \frac{0.5}{1.256 \times 10^{-6} \times 1500 \times 4 \times 10^{-4}}$

$$S = \mathbf{663481.95 \text{ AT/Wb}}$$

iii) Flux $(\phi) = \frac{NI}{S} = \frac{1500}{663481.95} = 2.26 \times 10^{-3}$ wb

$= \mathbf{2.26 \text{ mWb}}$

Ex. 1.13.6 : ***A purely inductive coil of 600 turns and external resistance of 20 Ω is uniformly would on steel ring of mean circumference 30 cm and cross sectional area 9 cm^2. It is connected to 40 V DC supply. If the relative permeability of steel ring is 1600. Calculate :***

i) Magnetic field intensity ii) Reluctance of the path

iii) mmf and iv) Flux

v) If voltage is increased to 60 V, find new value of flux.

Sol. : Given : l = 30 cm = 0.3 m, R = 20 Ω, V = 40 V

$a = 9 \text{ cm}^2 = 9 \times 10^{-4} \text{ m}^2$, $\mu_r = 1600$

To find : H, S, m.m.f., ϕ

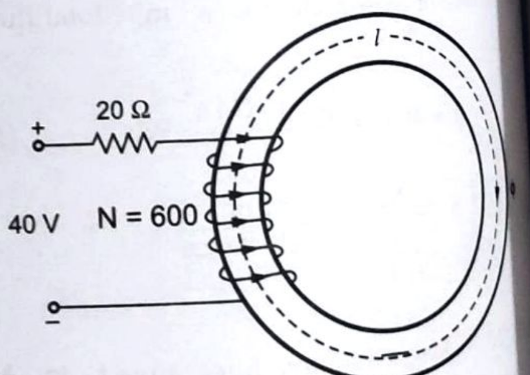


Fig. 1.13.3

i) $I = \frac{V}{R} = \frac{40}{20} = 2$ A

∴ $H = \frac{NI}{l} = \frac{600 \times 2}{0.3}$

$= \mathbf{4000 \text{ AT/m}}$ (Magnetizing field intensity)

ii) mmf = NI = 600×2 = **1200 AT**

iii) Reluctance (S) = $\frac{l}{\mu_0 \mu_r a}$

$= \frac{0.3}{1.256\times 10^{-6} \times 1600 \times 9\times 10^{-4}} = \mathbf{1.81\times 10^{+6}\ AT/Wb}$

iv) flux (ϕ) = $\frac{\text{mmf}}{\text{Reluctance}} = \frac{1200}{1.81\times 10^{+6}} = 6.63\times 10^{-4}$ Wb

$= 0.663 \times 10^{-3}$ Wb = **0.663 mWb**

v) If the voltage applied is 60 V

$I = \frac{60}{20} = 3$ A

$\phi = \frac{NI}{S} = \frac{600\times 3}{1.81\times 10^6} = 0.994\times 10^{-3}$ Wb = **0.994 mWb**

Note : Reluctance is independent of voltage or current.

Ex. 1.13.7 : ***A coil is wound with 300 turns on steel ring of relative permeability of 900. It has mean circumference of 40 cm and a cross sectional area of 5 cm². If the coil has resistance of 100 Ω and it is connected to 250 V DC supply, calculate -***
a) Coil mmf b) Field strength c) Total flux
d) Reluctance of ring e) Permeance of ring.

SPPU : Dec.-17, Marks 6

Sol. : Given : N = 300, μ_r = 900, l_i = 40 cm , a = 5 cm²

To find : m.m.f., H, ϕ, permeance.

a) $I = \frac{V}{R} = \frac{250}{100} = 2.5$ A

$\therefore$ Coil m.m.f. = NI = 300 × 2.5 = **750 AT**

b) $H = \frac{NI}{l_i} = \frac{750}{40\times 10^{-2}}$ = **1875 AT/m**

c) $B = \mu_0\mu_r H = 4\pi\times 10^{-7}\times 900\times 1875 = 2.1205\ Wb/m^2$

$\therefore$ $\phi = B\times a = 2.1205\times 5\times 10^{-4}$ = **1.0602 mWb**

d) $S = \frac{\text{m.m.f.}}{\phi} = \frac{750}{1.0602\times 10^{-3}} = \mathbf{707.414\times 10^3\ AT/Wb}$

e) Permeance = $\frac{1}{S} = \mathbf{1.4136\times 10^{-6}\ Wb/AT}$

Ex. 1.13.8 : ***An iron ring with mean circumference of 140 cm and cross section of 12 cm² is wound with 500 turns of wire. What is the relative permeability of the iron if exciting current of 2 amp flowing in coil, produces flux of 1.2 mWb ?***

SPPU : Dec.-14, May-11, 13, Marks 6

Sol. : Given : l = 140 cm , a = 12 cm², N = 500 ,
I = 2 A , ϕ = 1.2 mWb

To find : μ_r

$$S = \frac{l}{\mu_0\mu_r a} = \frac{140\times 10^{-2}}{4\pi\times 10^{-7}\times\mu_r\times 12\times 10^{-4}}$$

$$= \frac{928.4038\times 10^6}{\mu_r}$$

$$\phi = \frac{\text{m.m.f.}}{S} = \frac{NI}{S}$$

i.e. $$1.2\times 10^{-3} = \frac{500\times 2}{\left[\frac{928.4038\times 10^6}{\mu_r}\right]}$$

$\therefore$ μ_r = **1114.084**

1.13.2 Series Magnetic Circuits

- In practice magnetic circuit may be composed of various materials of different permeabilities, of different lengths and of different cross-sectional areas. Such a circuit is called **composite** magnetic circuit. When such parts are connected one after the other the circuit is called **series magnetic circuit**.
- Consider a circular ring made up of different materials of lengths l_1, l_2 and l_3 and with cross-sectional areas a_1, a_2 and a_3 with absolute permeabilities μ_1, μ_2 and μ_3 as shown in the Fig. 1.13.4.

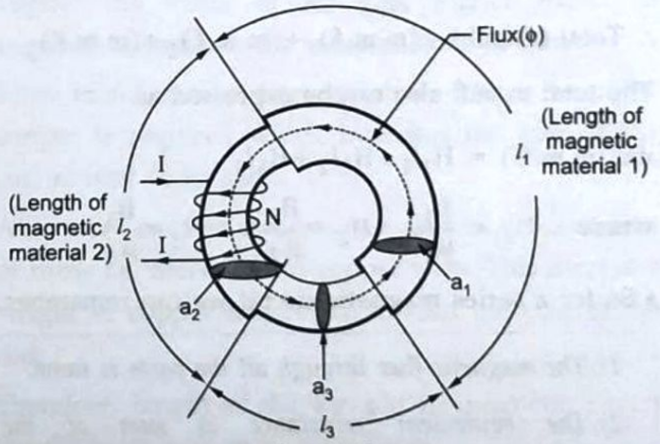


Fig. 1.13.4 A series magnetic circuit

- Let coil wound on ring has N turns carrying a current of I amperes.
- The total m.m.f. available is = NI AT
- This will set the flux 'ϕ' which is same through all the three elements of the circuit.
- This is similar to three resistances connected in series in electrical circuit and connected to e.m.f. carrying same current 'I' through all of them.
- Its analogous electric circuit can be shown as in the Fig. 1.13.5.

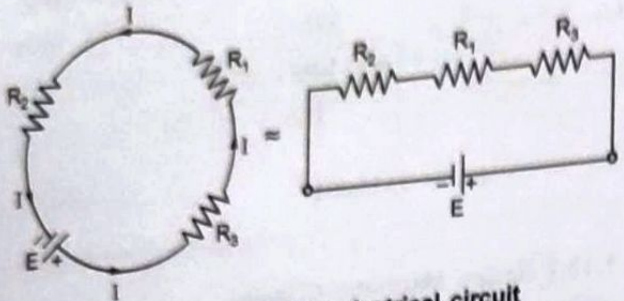


Fig. 1.13.5 Equivalent electrical circuit

- The total resistance of the electric circuit is $R_1 + R_2 + R_3$. Similarly the total reluctance of the magnetic circuit is,

$$\text{Total } S_T = S_1 + S_2 + S_3 = \frac{l_1}{\mu_1 a_1} + \frac{l_2}{\mu_2 a_2} + \frac{l_3}{\mu_3 a_3}$$

$$\therefore \text{ Total } \phi = \frac{\text{Total m.m.f.}}{\text{Total reluctance}} = \frac{NI}{S_T} = \frac{NI}{(S_1 + S_2 + S_3)}$$

$$\therefore \quad NI = S_T \phi = (S_1 + S_2 + S_3)\phi$$

$$\text{i.e.} \quad NI = S_1\phi + S_2\phi + S_3\phi$$

$$\therefore \text{ Total (m.m.f.)} = (\text{m.m.f.})_1 + (\text{m.m.f.})_2 + (\text{m.m.f.})_3$$

The total m.m.f. also can be expressed as,

$$\text{Total (m.m.f.)} = H_1 l_1 + H_2 l_2 + H_3 l_3$$

$$\text{where} \quad H_1 = \frac{B_1}{\mu_1}, \quad H_2 = \frac{B_2}{\mu_2}, \quad H_3 = \frac{B_3}{\mu_3}$$

- So for a **series magnetic** circuit we can remember,

> 1) *The magnetic flux through all the parts is same.*
>
> 2) *The equivalent reluctance is sum of the reluctances of different parts.*
>
> 3) *The resultant m.m.f. necessary is sum of the m.m.f.s in each individual part.*

1.13.3 Series Circuit with Air Gap

- The series magnetic circuit can also have a short air gap.

Key Point *This is possible because we have seen earlier that flux can pass through air also.*

- Such air gap is not possible in case of electric circuit.
- Consider a ring having mean length of iron part as 'l_i' as shown in the Fig. 1.13.6.

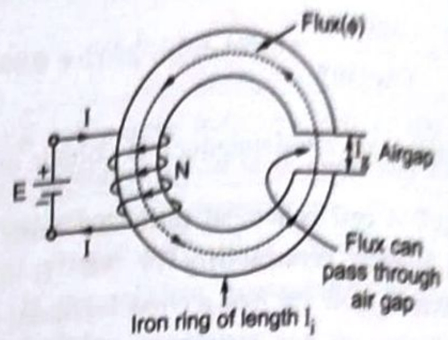


Fig. 1.13.6 A ring with an air gap

$$\text{Total m.m.f} = N\,I\ \text{AT}$$

$$\boxed{\text{Total reluctance } S_T = S_i + S_g}$$

where S_i = Reluctance of iron path

S_g = Reluctance of air gap

$$\therefore \quad \boxed{S_i = \frac{l_i}{\mu a_i} \quad \text{and} \quad S_g = \frac{l_g}{\mu_0 a_i}}$$

Key Point *The absolute permeability of air $\mu = \mu_0$ as $\mu_r = 1$ for air.*

- The cross-sectional area of air gap is assumed to be equal to area of the iron ring.

$$\therefore \quad S_T = \frac{l_i}{\mu a_i} + \frac{l_g}{\mu_0 a_i}$$

$$\text{and} \quad \phi = \frac{\text{m.m.f.}}{\text{Reluctance}} = \frac{NI}{S_T}$$

or Total m.m.f. = m.m.f. for iron + m.m.f for air gap

$$\therefore \quad \boxed{NI = S_i\phi + S_g\phi \quad \text{AT for ring.}}$$

1.13.3.1 Salient Features of Series Magnetic Circuit

- The flux (ϕ) remains same through the series magnetic circuit.

$$\therefore \quad \boxed{\phi_i = \phi_g = \phi}$$

where ϕ_i = Flux flowing through iron part

ϕ_g = Air gap flux called useful flux

- The reluctance of iron part and air gap are different denoted as S_i and S_g respectively.

$$\boxed{S_T = S_i + S_g}$$

- As discussed above, the total m.m.f., is sum of the m.m.f.s required for the iron part and air gap.

$$\therefore \quad (AT)_T = (AT)_i + (AT)_g$$

$$\therefore \quad NI = \phi S_T = \phi(S_i + S_g)$$

$$\therefore \quad \boxed{NI = S_i\phi + S_g\phi}$$

where, $(AT)_T$ = NI = Total m.m.f

- Using the expressions of S_i and S_g,

$$NI = \phi\left[\frac{l_i}{\mu_0\mu_r a} + \frac{l_g}{\mu_0 a}\right]$$

$$\therefore \quad \boxed{NI = \frac{\phi}{\mu_0 a}\left[\frac{l_i}{\mu_r} + l_g\right]}$$

- The result can be expressed in terms of magnetic field strength H as m.m.f. is the product of H and length l.

$$\therefore \quad (AT)_i = H_i\, l_i \quad \text{and} \quad (AT)_g = H_g\, l_g$$

$$\therefore \quad \boxed{NI = H_i\, l_i + H_g\, l_g}$$

- As area of cross-section 'a' and flux 'ϕ' is same for iron part and air gap, the flux density is same for iron part and air gap.

$$\therefore \quad \boxed{B_i = B_g = B = \frac{\phi}{a}\ \text{Wb/m}^2}$$

- The flux density B and H are related as,

$$B_i = \mu H_i \quad \text{and} \quad B_g = \mu_0 H_g$$

$$\therefore \quad H_i = \frac{B_i}{\mu} = \frac{B}{\mu_0\mu_r} \quad \text{and} \quad H_g = \frac{B_g}{\mu_0} = \frac{B}{\mu_0}$$

- Using in the expression of NI,

$$\boxed{NI = \frac{B}{\mu_0\mu_r}\, l_i + \frac{B}{\mu_0}\, l_g} \quad \text{but, } B = \frac{\phi}{a}$$

$$\therefore \quad \boxed{NI = \frac{\phi}{\mu_0 a}\left[\frac{l_i}{\mu_r} + l_g\right]}$$

Observations :

- The total m.m.f. (NI) required to force the magnetic flux in the magnetic circuit is,
  1. Directly proportional to the length of air gap.
  2. Inversely proportional to the relative permeability of the magnetic material.
- Higher the value of relative permeability of the magnetic material, better is the magnetic material and vice versa.
  3. Inversely proportional to the area of cross-section of magnetic circuit i.e. the area of cross-section of the iron ring.
- Higher the area of cross-section, lesser is the m.m.f. required i.e. either less number of turns or less current is required.

1.13.3.2 Why Air Gap is Kept Minimum ?

- Practically in electrical circuits, air gap is kept as minimum as possible.
- As m.m.f. is directly proportional to the length of the air gap, NI $\propto l_g$.
- Higher the value of air gap, higher m.m.f. is required to produce the same amount of flux.
- More m.m.f. means more number of turns i.e. more copper is required which increases the size of the coil as well as its cost.
- Also more current means more area of cross-section of turns i.e. more size of copper wire. This increases the weight of copper and hence in turn the cost of the coil.
- Therefore, length of the air gap in magnetic circuit should be as minimum as possible.

Ex. 1.13.9 : *A coil of 500 turns is uniformly wound on iron ring of mean circumference 30 cm and having area of cross section 15 cm^2. The current through the coil is 2 A; the μ_r = 400, find*

a) mmf, reluctance and flux in the iron ring.

b) If an air gap of 1 mm is cut in the ring, find reluctance of air gap flux.

Sol. : Given : N = 500, I = 2 A,

$a = 15 \text{ cm}^2 = 15 \times 10^{-4} \text{ m}^2, \mu_r = 400, l = 30 \text{ cm} = 0.3 \text{ m}.$

To find : m.m.f., S, ϕ

i) mmf = NI = 500 × 2 = **1000 AT**

ii) Reluctance (S) = $\dfrac{l}{\mu_0 \mu_r a}$

$$= \frac{0.3}{1.256 \times 10^{-6} \times 400 \times 15 \times 10^{-4}}$$

$$S = 398089.17 \Rightarrow \mathbf{3.98 \times 10^5} \text{ AT/Wb}$$

iii) Flux (ϕ) = $\dfrac{NI}{S} = \dfrac{1000}{3.98 \times 10^5}$

$$= \mathbf{2.51 \times 10^{-3}\ Wb} = \mathbf{2.51}\ \text{mWb}$$

b) When an air gap of 1 mm is cut in it

$$l_i = 0.3 - 1 \times 10^{-3} = \mathbf{0.299\ m}$$

$$l_g = 1 \times 10^{-3} = \mathbf{0.001}\ \text{m}$$

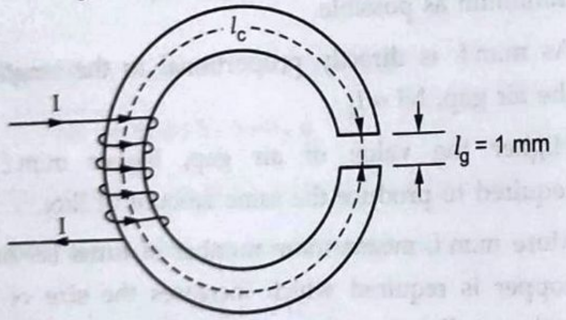


Fig. 1.13.7

As N and I are as it is, mmf remains same

$$NI = 500 \times 2 = \mathbf{1000\ AT}$$

ii) Now total reluctance $(S_T) = S_i + S_g$

$$S_i = \frac{l_i}{\mu_0 \mu_r a} = \frac{0.299}{1.256 \times 10^{-6} \times 400 \times 15 \times 10^{-4}}$$

$$= 396762.2 \text{ AT/Wb}$$

Similarly, $S_g = \dfrac{l_g}{\mu_0 a} = \dfrac{0.001}{1.256 \times 10^{-6} \times 15 \times 10^{-4}}$

$$= \mathbf{530785.56\ AT/Wb}$$

[Here μ_r = 1 for air]

$\therefore$ Total S_T = 396762.2 + 530785.56 = 927547.7...

$$= \mathbf{9.275 \times 10^5\ AT/Wb}$$

$\therefore$ New value of flux (ϕ)

$$\phi = \frac{NI}{S_T} = \frac{1000}{9.275 \times 10^5} = \mathbf{1.078\ mWb}$$

Comments on results :

i) When air gap of only **1 mm** is cut in the ring flux reduces from **2.5 mWb** to **1.078** mWb.

ii) An air gap of 1 mm offers reluctance of ... higher than iron path of **29.9 cm**. This is ... of iron is good magnetic conductor whereas ... is magnetic insulator.

We can also obtain the flux directly as,

$$NI = \phi \cdot S_T$$

$$NI = \phi \cdot (S_i + S_g) = \phi \left[\frac{l_i}{\mu_0 \mu_r a} + \frac{l_g}{\mu_0 a} \right]$$

$$NI = \frac{\phi}{\mu_0\ a} \left[\frac{l_i}{\mu_r} + \frac{l_g}{1} \right]$$

$$500 \times 2 = \frac{\phi}{1.256 \times 10^{-6} \times 15 \times 10^{-4}} \left[\frac{0.299}{400} + \frac{0.001}{1} \right]$$

$$\phi = \mathbf{1.078\ mWb}$$

Ex. 1.13.10 : *In A coil having 300 turns is uniformly wound on iron ring with mean circumference 30 ... The relative permeability of ring is 500. Find*

a) Exciting current required to produce flux density 1.2 T in ring.

b) If air gap of 1 mm is cut in the ring, find ... value of current to maintain same air gap ... density [as in (a)].

Sol. : Given : N = 300 ; B = 1.2 T, μ_r = 500,

l = 30 cm = 0.30 m

To find : I

a) We know that,

$$\text{mmf (NI)} = Hl \text{ as } H = \frac{NI}{l}$$

$$NI = \frac{B}{\mu_0 \mu_r} \cdot l \qquad \ldots B = \mu_0 \mu_r H$$

$$300 \times I = \frac{1.2}{1.256 \times 10^{-6} \times 500} \times 0.3$$

$$I = \mathbf{1.91\ A}$$

b) When air gap of 1 mm is cut in the ring Length of iron path $(l_i) = l_c - l_g$

$$l_c - l_g = 0.30 - 1\times10^{-3} = \mathbf{0.299\ m}$$

$$l_g = 1\ \text{mm} = 1\times10^{-3}\ \text{m} = \mathbf{0.001\ m}$$

Here, $B = B_i = B_g$

Let new current is I_2 :

$\therefore$ Total $AT = AT_i + AT_g$

$$NI_2 = H_i\, l_i + H_g\, l_g$$

$$NI_2 = \frac{B}{\mu_0}\left[\frac{l_i}{\mu_r} + \frac{l_g}{1}\right] \qquad \ldots \mu_r = 1 \text{ for air}$$

$$300 \times I_2 = \frac{1.2}{1.256 \times 10^{-6}}\left[\frac{0.299}{500} + \frac{0.001}{1}\right]$$

$$I_2 = \mathbf{5.089\ A}$$

Note that due to air gap, the current required to maintain same flux density, increases.

Ex. 1.13.11 : ***Iron ring of mean diameter 20 cm have area of cross section 20 cm^2. It is wound with 'N' turns. An air gap of 1 mm is cut in it. It is found that current of 1.5 A produces a flux of 1.8 mWb in the air gap. The relative permeability of iron is 500. Find a) Number of turns b) Total mmf***

c) With same air gap flux, if number of turns = 600, find new value of μ_r.

Sol. : Given : Mean diameter (D) = 20 cm = 0.2 m

Mean length of path $(l_c) = \pi D = 3.14 \times 0.2 = \mathbf{0.628\ m}$

$l_g = 1\ \text{mm} = 1\times10^{-3}$ m ; l_i = Length of iron path

$l_i = \pi D - l_g = 0.628 - 1\times10^{-3} = \mathbf{0.627\ m}$

$a = 20\ \text{cm}^2 = 20\times10^{-4}\ \text{m}^2$; $I = 1.5$ A ; $\mu_r = 500$.

To find : N, m.m.f., μ_r

a) $$S_T = S_i + S_g$$

$$NI = \phi \cdot S_T$$

$$NI = \frac{\phi}{\mu_0 \cdot a}\left[\frac{l_i}{\mu_r} + \frac{l_g}{1}\right]$$

$$NI = \frac{1.8\times10^{-3}}{1.256\times10^{-6}\times20\times10^{-4}}\left[\frac{0.627}{500} + \frac{0.001}{1}\right]$$

$\therefore$ mmf = NI = **1615.12 AT**

b) $N \times 1.5 = 1615.12$

$$N = 1076.75 \cong \mathbf{1077}$$

c) With same air gap flux $\phi_g = \phi = 1.8\times10^{-3}$ Wb

and, new turns = 600

New mmf = $600 \times 1.5 = \mathbf{900}$

$\therefore$ mmf $= \phi \cdot S_T$

$$900 = 1.8\times10^{-3} \times S_T$$

$\therefore$ $S_T = \mathbf{5\times10^5}$ **AT/Wb**

$$S_T = S_i + S_g$$

But, $$S_g = \frac{l_g}{\mu_0\, a} = \frac{0.001}{1.256\times10^{-6}\times20\times10^{-4}}$$

$$= \mathbf{398089}\ \text{AT/Wb}$$

$$S_i = S_T - S_g = 5\times10^{-5} - 398089 = 101911\ \text{AT/Wb}$$

$$S_i = 101911 = \frac{l_i}{\mu_0\, \mu_r\, a} = \frac{0.627}{1.256\times10^{-6}\times20\times10^{-4}\times\mu_r}$$

$\therefore$ New μ_r = **2449.22**

Comment : When less number of turns are used, we have to use magnetic material having higher relative permeability i.e. good quality of magnetic material.

Ex. 1.13.12 : ***A coil of N turns is wound on a cast iron ring which has mean length of 50 cm and its cross-section is of 4 cm diameter. The current flowing through the coil is 2 Amp which produces a flux of 6 mWb in the air gap of 2 mm length. If the relative permeability of iron is 1000, calculate no. of turns N.***

SPPU : May-09, 14, Dec.-09, 13, Marks 10

Sol. : Given : d = 4 cm, l_i = 50 cm

To find : N

$$a = \frac{\pi}{4}d^2 = \frac{\pi}{4}\times4^2 = 12.5663\ \text{cm}^2$$

$$S_i = \frac{l_i}{\mu_0 \mu_r a} = \frac{50\times10^{-2}}{4\pi\times10^{-7}\times1000\times12.5663\times10^{-4}}$$

$$= 316.6304\times10^3\ \text{AT/Wb}$$

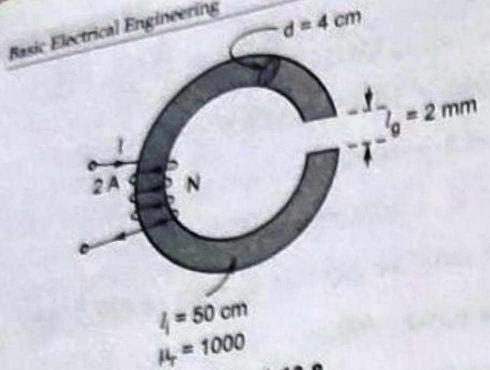


Fig. 1.13.8

$$\ldots \mu_r = 1 \text{ for air}$$

$$S_g = \frac{l_g}{\mu_0 a}$$

$$= \frac{2\times10^{-3}}{4\pi\times10^{-7}\times12.5663\times10^{-4}} = 1.2665\times10^{6} \text{ AT/Wb}$$

$$\therefore S_T = S_i + S_g = 1.58315\times10^{6} \text{ AT/Wb}$$

$$\phi = \frac{\text{m.m.f.}}{\text{Reluctance}} = \frac{NI}{S_T} \quad \ldots \phi = 6 \text{ mWb (given)}$$

$$\therefore 6\times10^{-3} = \frac{N\times2}{1.58315\times10^{6}}$$

i.e. $\quad N = \mathbf{4749.456 \approx 4750}$

Ex. 1.13.13 : ***Derive the formula of total reluctance and total mmf in following circuit if relative permeability are μ_{r1} and μ_{r2} respectively.*** **SPPU : May-18, Marks 6**

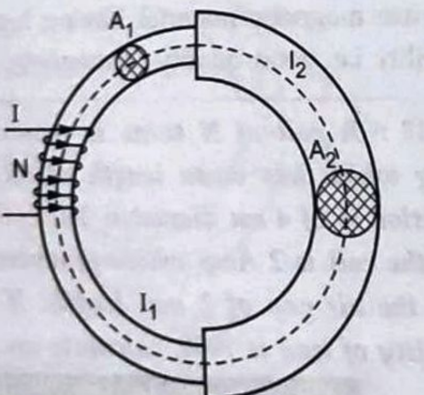


Fig. 1.13.9

Sol. : It is series magnetic circuit with two materials having permeabilities μ_{r1} and μ_{r2} and with cross-section areas A_1 and A_2. The lengths of these materials are l_1 and l_2.

- The reductances of two materials are,

$$S_1 = \frac{l_1}{\mu_0 \mu_{r1} A_1} \text{ and } S_2 = \frac{l_2}{\mu_0 \mu_{r2} A_2}$$

$$\therefore \boxed{\begin{aligned} S_T &= \text{Total reluctance} = S_1 + S_2 \\ &= \frac{1}{\mu_0}\left[\frac{l_1}{\mu_{r1} A_1} + \frac{l_2}{\mu_{r2} A_2}\right] \text{AT/Wb}\end{aligned}}$$

- The current is I and number of turns N.

$\therefore$ Total m.m.f. $= NI = \phi S_T = \phi[S_1 + S_2]$

- The m.m.f. is addition of the m.m.f.s in each part with flux ϕ constant.

$$\therefore \quad NI = (\text{m.m.f.})_1 + (\text{m.m.f.})_2 = \phi[S_1 + S_2]$$

$$\therefore \quad \boxed{NI = \phi\times\frac{1}{\mu_0}\left[\frac{l_1}{\mu_{r1} A_1} + \frac{l_2}{\mu_{r2} A_2}\right]}$$

Ex. 1.13.14 : ***A steel ring of mean diameter of 50 cm is wound with 500 turns on it. A flux density of 1 Tesla is produced in the ring by mmf of 4000 AT/m. Calculate magnetising current. Also find the current when an air gap of 1 mm is cut in it, keeping B = 1 Tesla in the ring.*** **SPPU : May-12, Dec.-03, 15, Marks 6**

Sol. : Given :

Mean length $= l_i = \pi\times d_{mean} = \pi\times 50 = 157.08$ cm

N = 500, B = 1 T, H = 4000 AT/m

To find : I

$$B = \mu H \quad \text{i.e.} \quad \mu = \frac{B}{H} = \frac{1}{4000} \quad \text{i.e.} \quad \mu_0\mu_r = \frac{1}{4000}$$

$$\therefore \quad \mu_r = \frac{1}{4000\times4\pi\times10^{-7}} = 198.94$$

$$\text{Total AT} = H\times l_i = 4000\times1.5708 = 6283.2 \text{ AT}$$

$$\text{Total AT} = N I \quad \text{i.e.} \quad I = \frac{6283.2}{500} = 12.567 \text{ A}$$

Air gap of 1 mm is cut i.e. $l_g = 1 \text{ mm} = 1\times10^{-3}$ m

$$\therefore \quad l_i = 1.5708 - 1\times10^{-3}$$

$$= 1.5698 \text{ m} \quad \ldots \text{New iron length}$$

$$\therefore S_T = S_i + S_g = \frac{l_i}{\mu_0\mu_r a} + \frac{l_g}{\mu_0 a} \quad \ldots \text{For air gap } \mu_r = 1$$

$$\therefore S_T = \frac{1}{\mu_0 a}\left[\frac{1.5698}{198.94} + 1\times10^{-3}\right] = \frac{8.8908\times10^{-3}}{\mu_0 a} \text{ AT/Wb}$$

$$\therefore \quad \phi = B\times a = 1\times a = a \quad \ldots a = \text{Area of cross-section}$$

$$\phi = \frac{\text{m.m.f.}}{S_T}$$

$$\text{i.e. } \phi = \frac{NI}{\left[\dfrac{8.8908\times 10^{-3}}{\mu_0 a}\right]}$$

$$\therefore\ I = \frac{\mu_0 \times 500 \times 1}{8.8908\times 10^{-3}} \quad \text{i.e. } \mathbf{I = 14.15\ A}$$

Ex. 1.13.15 : ***An iron ring has magnetic circuit of cross section 10 cm^2 and radial air gap of 1 mm cut in it the ring is uniformly wound with 1000 turns and current of 2 A produces a flux of 1 mWb in the air gap. Calculate the reluctance of magnetic path.***

SPPU : Dec.-08, May-17, Marks 6

Sol. : Given : $l_g = 1$ mm, $a = 10$ cm^2, $N = 1000$

$I = 2$ A, $\phi = 1$ mWb

To find : S_i

$$\phi = \frac{\text{m.m.f}}{S_T} = \frac{NI}{S_T} \quad \text{i.e. } 1\times 10^{-3} = \frac{2\times 1000}{S_T}$$

$$\therefore\ S_T = 2\times 10^6 \text{ AT/Wb} = \text{Total reluctance} = S_i + S_g$$

$$\therefore\ 2\times 10^6 = S_i + \frac{l_g}{\mu_0 a} \qquad \ldots \mu_r = 1 \text{ for air}$$

$$\text{i.e. } 2\times 10^6 = S_i + \frac{1\times 10^{-3}}{4\pi\times 10^{-7}\times 10\times 10^{-4}}$$

$$\therefore\ S_i = \mathbf{1.2042\times 10^6\ AT/Wb}$$

... Reluctance of magnetic path

Ex. 1.13.16 : ***For the magnetic circuit shown below, find the value of current I_3 which will make flux in the core zero. The relative permeability of core is 1600, $N_1 = 300$, $I_1 = 1$ A, $N_2 = 250$, $I_2 = 2$ A, $N_3 = 200$, mean length of magnetic path = 40 cm Also calculate the flux through the circuit when $I_3 = 1$ A.***

Sol. : Flux $\phi = \dfrac{\text{mmf}}{\text{Reluctance}}$

i) To have flux = 0, Net mmf must be zero.

Using **right hand thumb rule or end rule.**

Flux due to I_1 is clockwise direction, flux due to I_2 is also clockwise but flux due to I_3 is anticlockwise (ccw) direction.

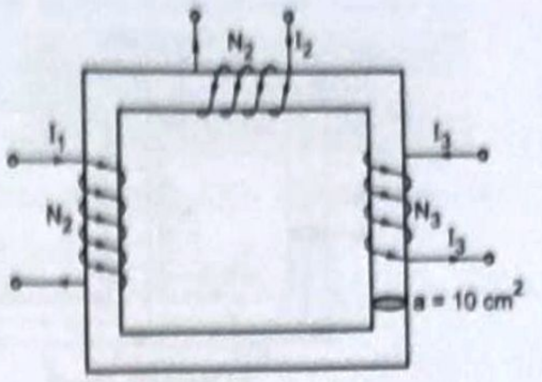


Fig. 1.13.10

$$\therefore\ \text{Total (Net) mmf} = N_1I_1 + N_2I_2 - N_3I_3 = 0$$

$$\therefore\ 300\times 1 + 250\times 2 = 200\times I_3$$

$$I_3 = 800/200 = \mathbf{4\ A}$$

ii) When $I_3 = 1$ A,

$$\text{Total mmf acting} = N_1I_1 + N_2I_2 - N_3I_3 = 0$$

$$= 300\times 1 + 250\times 2 - 200\times 1 = \mathbf{600\ AT}$$

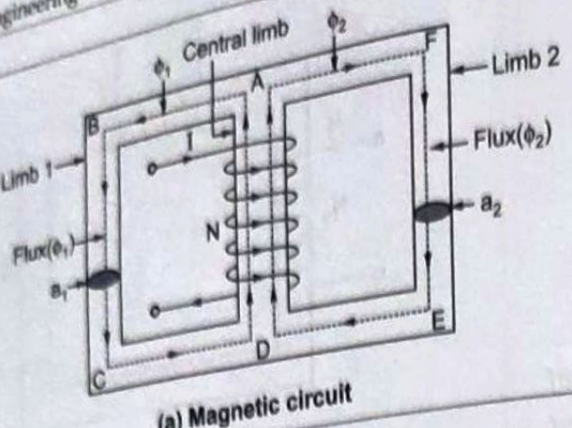
$$\text{Reluctance (S)} = \frac{l}{\mu_0 \mu_r a}$$

$$= \frac{0.4}{1.256\times 10^{-6}\times 1600\times 10\times 10^{-4}} = 199044.58 \text{ AT/Wb}$$

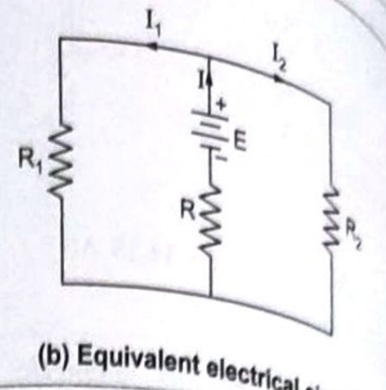
$$\therefore\ \phi = \frac{\text{mmf}}{S} = \mathbf{3.014\ mWb}$$

1.13.4 Parallel Magnetic Circuits

- In case of electric circuits, resistances can be connected in parallel. Current through each of such resistances is different while voltage across all of them is same. Similarly different reluctances may be in parallel in case of magnetic circuits.
- A magnetic circuit which has more than one path for the flux is known as a **parallel magnetic circuit.**
- Consider a magnetic circuit shown in the Fig. 1.13.11 (a). (Refer Fig. 1.13.11 on next page)
- At point A the total flux ϕ, divides into two parts ϕ_1 and ϕ_2 hence $\phi = \phi_1 + \phi_2$
- The fluxes $\phi_1$and ϕ_2 have their paths completed through ABCDA and AFEDA respectively.
- This is similar to division of current in case of parallel connection of two resistances in an electric circuit. The analogous electric circuit is shown in the Fig. 1.13.11 (b).



(a) Magnetic circuit



(b) Equivalent electrical circuit

Fig. 1.13.11 A parallel magnetic circuit

The mean length of path ABCD = l_1 m

The mean length of the path AFED = l_2 m

The mean length of the path AD = l_c m

The reluctance of the path ABCD = S_1

The reluctance of path AFED = S_2

The reluctance of path AD = S_c

The total m.m.f. produced = NI AT

$$\text{flux} = \frac{\text{m.m.f.}}{\text{Reluctance}}$$

i.e. m.m.f. = $\phi \times S$

$\therefore$ For path ABCDA,

$NI = \phi_1 S_1 + \phi S_c$ while For path AFEDA,

$NI = \phi_2 S_2 + \phi S_c$

where $S_1 = \frac{l_1}{\mu a_1}$, $S_2 = \frac{l_2}{\mu a_2}$ and $S_c = \frac{l_c}{\mu a_c}$

Generally $a_1 = a_2 = a_c$ = Area of cross-section same

- For parallel circuit,

Total m.m.f. = m.m.f. required by central limb + m.m.f. required by any one of outer limbs

$$\boxed{\begin{aligned} NI &= (NI)_{AD} + (NI)_{ABCD} \text{ or } (NI)_{AFED} \\ &= \phi S_c + [\phi_1 S_1 \text{ or } \phi_2 S_2] \end{aligned}}$$

- **As in the electric circuit e.m.f. across parallel branches is same, in the magnetic circuit the m.m.f. across parallel branches is same.**
- Thus same m.m.f. produces different fluxes in the two parallel branches.

- For such parallel branches,

$$\boxed{\phi_1 S_1 = \phi_2 S_2}$$

- Hence while calculating total m.m.f., the m.m.f. of **only one** of the two parallel branches must be considered.

1.13.5 Parallel Magnetic Circuit with Air Gap

- Consider a parallel magnetic circuit with air gap in the central limb as shown in the Fig. 1.13.12.

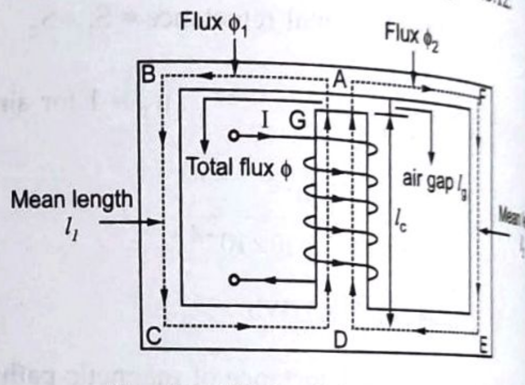


Fig. 1.13.12 Parallel circuit with air gap

- The analysis of this circuit is exactly similar to the parallel circuit discussed above.
- The only change is the analysis of central limb. The central limb is series combination of iron path and air gap.
- The central limb is made up of,

path GD = iron path = l_c

path GA = air gap = l_g

The total flux produced is ϕ. It gets divided at A into ϕ_1 and ϕ_2.

$$\therefore \quad \phi = \phi_1 + \phi_2$$

The reluctance of central limb is,

$$S_c = S_i + S_g = \frac{l_c}{\mu a_c} + \frac{l_g}{\mu_0 a_c}$$

Hence m.m.f. of central limb is,

$$(\text{m.m.f.})_{AD} = (\text{m.m.f.})_{GD} + (\text{m.m.f.})_{GA}$$

Hence the total m.m.f. can be expressed as,

$$\boxed{(NI)_{total} = (NI)_{GD} + (NI)_{GA} + (NI)_{ABCD} \text{ or } (NI)_{AFED}}$$

Thus the electrical equivalent circuit for such case becomes as shown in the Fig. 1.13.13.

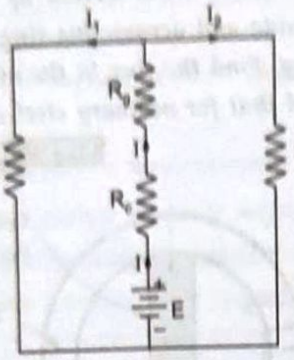


Fig. 1.13.13 Electrical equivalent circuit

- Similarly there may be air gaps in the side limbs but the method of analysis remains the same.

Ex. 1.13.17 : *A magnetic circuit made of steel is as shown. Calculate current to set up a flux of 1.3 mWb in central limb.*

l_g = 1 mm = 0.001 m

$A'_c = l_c$ = Length of central limb = 120 mm = 0.12 m

$l_{ABC} = l_{ADC}$ = 300 mm = 0.3 m

Area of cross section of central limb

$a_c = 800\ mm^2 = 800 \times 10^{-6}\ m^2$

Area of cross section of each of side limb

$a_{side\ limb} = 500\ mm^2 = 500 \times 10^{-6}\ m^2$

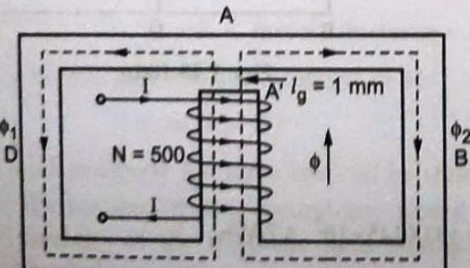


Fig. 1.13.14

Magnetic characteristics of steel material is

H (AT/m)	4000	900	500
B (Tesla)	1.625	1.3	1

Sol. : Total AT = NI = $AT_c + AT_g$ + AT either for l_{ABC} or l_{ADC}

i) AT for central limb : $AT_c = H_c \cdot l_c$

Now $\phi_c = 1.3 \times 10^{-3}$ Wb

$$B_c = \frac{\phi}{a_c} = \frac{1.3 \times 10^{-3}}{800 \times 10^{-6}}$$

$$B_c = 1.625\ \text{T}$$

From magnetic characteristics table, corresponding H_c is,

$$H_c = 4000\ \text{AT/m}$$

$$\therefore \quad AT_c = H_c \times l_c = 4000 \times 0.12 = 480$$

ii) AT_g : AT for air gap

As central limb and air gap forms series magnetic circuit

$$\phi_c = \phi_g = 1.3 \times 10^{-3}\ \text{wb}$$

$$\therefore \quad B_g = B_c = 1.625\ \text{T}$$

[**Note :** We cannot take H from table for B_g = 1.625 T because it is for steel material and not for air]

For air gap $\mu_r = 1$

$$\therefore\ H_g = \frac{B_g}{\mu_0} = \frac{1.625}{1.256 \times 10^{-6}}$$

$$\therefore AT_g = H_g \cdot l_g = \frac{1.625}{1.256 \times 10^{-6}} \times 1 \times 10^{-3} = \mathbf{1293.79}$$

iii) AT_{ABC} : $H_{ABC} \cdot l_{ABC}$

As both the side limbs have equal area of cross - section, length and same material, their reluctances are equal $\left(S = \frac{l}{\mu_0 \mu_r a}\right)$ Therefore, **flux in central limb will be equally divided into both the side limbs.**

$\therefore$ Flux in the path ABC = $\phi_1 = \phi_c / 2$

$$= \frac{1.3 \times 10^{-3}}{2} = \mathbf{0.65 \times 10^{-3}}$$

$$\therefore \text{Flux density in one side limb} = B_{ABC} = \frac{0.65\times10^{-3}}{500\times10^{-6}} = 1.3 \text{ T}$$

Corresponding H = 900 AT/m from steel characteristics table.

$$\therefore \quad \text{AT for ABC} = 900\times0.3 = 270$$

$$\therefore \quad \text{Total AT} = AT_c + AT_g + AT_{ABC} \text{ or } AT_{ADC}$$

$$\therefore \quad NI = 480 + 1293.79 + 270$$

$$500\times I = 2043.79$$

$$I = 4.0876 \text{ A}$$

Ex. 1.13.18 : *A ring of cast steel has an external diameter of 25 cm and a square cross-section of 4 cm side. An ordinary steel bar 17 cm × 4 cm × 0.5 cm is fitted with negligible gap inside and across this ring. A coil of 500 turns and carrying a D.C. current of 1.5 A is placed on one half of the ring. Find the flux in the other half of the ring. Neglect leakage. Assume relative permeability of cast steel as 850 and that for ordinary steel as 700.*

SPPU : Dec.08

Sol. : The ring is shown in the Fig. 1.13.15 (a).

Inner diameter of ring = length of steel bar = 17 cm.

$$\therefore \text{Mean diameter of ring} = \frac{\text{outer}+\text{inner}}{2} = \frac{25+17}{2} = 21 \text{ cm}$$

$$\therefore \quad \text{Mean circumference} = \pi\times21 = 65.9734 \text{ cm.}$$

$$\therefore \text{Length of half section of ring} = l_{i1} = \frac{65.9734}{2} = 32.9867 \text{ cm}$$

$$\therefore \text{Length of other section of ring} = l'_{i1} = l_{i1} = 32.9867 \text{ cm}$$

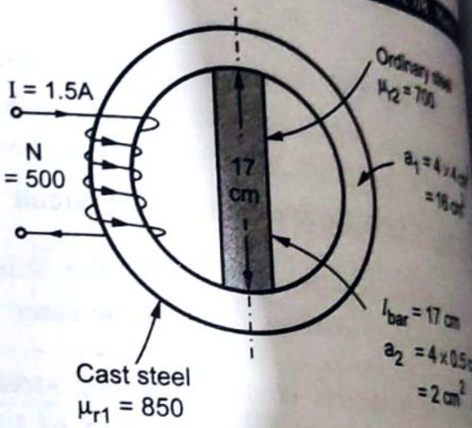


Fig. 1.13.15(a)

This is a parallel magnetic circuit as shown in the Fig. 1.13.15 (b).

$$\therefore \quad \text{Total m.m.f.} = NI = 500\times1.5 = 750 \text{ AT}$$

Now $\phi_T = \phi_1 + \phi_2$

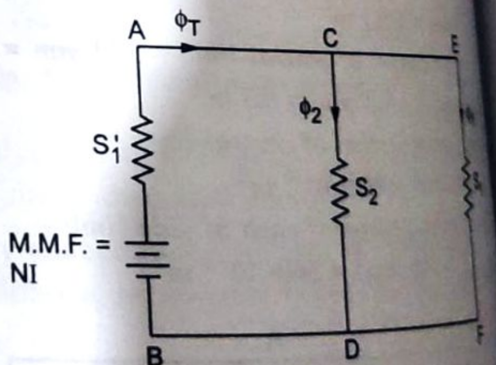


Fig. 1.13.15(b)

Key Point *M.M.F across CD and EF is same as both are in parallel.*

$\therefore$ Total m.m.f= m.m.f. for path AB + m.m.f. for path CD or path EF.

$$\text{m.m.f. for path AB} = \phi_T\times S'_1 \quad \text{while}$$

$$\text{m.m.f. for path CD} = \phi_2\times S_2 = \phi_1\times S_1$$

$$\therefore \quad \text{Total m.m.f.} = \phi_T\times S'_1 + \phi_1\times S_1$$

$$S_1 = \frac{l_{i1}}{\mu_0\mu_{r1}a_1} = \frac{32.9867\times10^{-2}}{4\pi\times10^{-7}\times850\times16\times10^{-4}} = 193.0145\times10^3 \text{ AT/Wb}$$

$$S'_1 = \frac{l'_{i1}}{\mu_0\mu_{r1}a_1} \quad \text{but } l'_{i1} = l_{i1} \text{ hence } S'_1 = 193.0145\times10^3 \text{ AT/Wb}$$

$\therefore$ Total m.m.f. $= (\phi_1 + \phi_T)193.0145 \times 10^3$

But $\phi_2 S_2 = \phi_1 S_1$ and $S_2 = \dfrac{l_{bar}}{\mu_0 \mu_{r2} a_2}$ ... (1)

$$\therefore \quad S_2 = \frac{17 \times 10^{-2}}{4\pi \times 10^{-7} \times 700 \times 2 \times 10^{-4}} = 966.297 \times 10^3 \text{ AT/Wb}$$

$\therefore \quad \phi_2 \times 966.297 \times 10^3 = \phi_1 \times 193.0145 \times 10^3$ i.e. $\phi_2 = 0.19974\,\phi_1$ ... (2)

But $\phi_T = \phi_1 + \phi_2$ hence using in (1),

Total m.m.f. $= (\phi_1 + \phi_1 + \phi_2) \times 193.0145 \times 10^3$ i.e. $750 = (2\,\phi_1 + 0.19974\,\phi_1) \times 193.0145 \times 10^3$

$\therefore \quad \phi_1 = 1.7664$ **mWb** ... **Flux through other half of ring**

Expected Questions

1. *What is magnetic circuit ? Derive the relation between m.m.f., flux and reluctance from it.* **SPPU : May-05, Marks 4**
2. *Write a note on series magnetic circuit with air gap.* **SPPU : May-12, Marks 4**
3. *How to analyse the series magnetic circuit with air gap ?*
4. *Explain the analysis of parallel magnetic circuit.*
5. *Explain a sample series magnetic circuit.*

1.14 : Kirchhoff's Laws for Magnetic Circuit

- Similar to the electrical circuit Kirchhoff's laws can be used to analyse complex magnetic circuit. The laws can be stated as below :

1.14.1 Kirchhoff's Flux Law

- The total magnetic flux arriving at any junction in a magnetic circuit is equal to the total magnetic flux leaving that junction.

At a junction, $\boxed{\sum \phi = 0}$

1.14.2 Kirchhoff's M.M.F. Law

- The resultant m.m.f. around a closed magnetic circuit is equal to the algebraic sum of the products of the flux and the reluctance of each part of the closed circuit i.e. for a closed magnetic circuit.

$$\boxed{\sum \text{m.m.f.} = \sum \phi S}$$

As $\phi \times S = \text{flux} \times \text{reluctance} = \text{m.m.f.}$

- M.M.F. also can be calculated as $H \times l$ where H is field strength and 'l' is mean length

$\therefore \quad \text{m.m.f.} = Hl$

- Alternatively the same law can be stated as :

The resultant m.m.f. around any closed loop of a magnetic circuit is equal to the algebraic sum of the products of the magnetic field strength and the length of each part of the circuit i.e. for a closed magnetic circuit.

$$\boxed{\sum \text{m.m.f.} = \sum H.l}$$

Expected Question

1. *State Kirchhoff's laws for magnetic circuit.*

1.15 : Comparison of Magnetic and Electric Circuits

SPPU : Dec.-05, 06,07,09,10,11, May-05,07,08,09,11,12,14,16

- Similarities between electric and magnetic circuits are listed below :

Sr. No.	Electric Circuit	Magnetic Circuit
1.	Path traced by the current is called electric circuit.	Path traced by the magnetic flux is defined as magnetic circuit.
2.	E.M.F. is the driving force in electric circuit, the unit is **volts**.	M.M.F. is the driving force in the magnetic circuit, the unit of which is **ampere turns**.
3.	There is current I in the electric circuit measured in amperes.	There is flux ϕ in the magnetic circuit measured in webers.
4.	The flow of electrons decides the current in conductor.	The number of magnetic lines of force decides the flux.
5.	Resistance oppose the flow of the current. Unit is ohm.	Reluctance is opposed by magnetic path to the flux. Unit is ampere turn/weber.
6.	$R = \rho \frac{l}{a}$. Directly proportional to l. Inversely proportional to 'a'. Depends on nature of material.	$S = \frac{l}{\mu_0 \mu_r a}$. Directly proportional to l. Inversely proportional to $\mu = \mu_0 \mu_r$. Inversely proportional to area 'a'.
7.	The current $I = \frac{\text{e.m.f.}}{\text{resistance}}$	The flux $\phi = \frac{\text{m.m.f.}}{\text{reluctance}}$
8.	The current density $\delta = \frac{I}{a}$ A/m^2	The flux density $B = \frac{\phi}{a}$ Wb/m^2
9.	Conductivity is reciprocal of the resistivity. Conductance $= \frac{1}{R}$	Permeance is reciprocal of the reluctance. Permeance $= \frac{1}{S}$
10.	Kirchhoff's current and voltage law is applicable to the electric circuit.	Kirchhoff's m.m.f. law and flux law is applicable to the magnetic circuit.

- There are few dissimilarities between the two which are listed below :

Sr. No.	Electric Circuit	Magnetic Circuit
1.	In the electric circuit the current actually flows i.e. there is movement of electrons.	Due to m.m.f. flux gets established and does not flow in the sense in which current flows.
2.	There are many materials which can be used as insulators i.e. air, P.V.C., synthetic resin etc, from which current cannot pass.	There is no magnetic insulator as flux can pass through all the materials, even through the air as well.
3.	Energy must be supplied to the electric circuit to maintain the flow of current.	Energy is required to create the magnetic flux, but is not required to maintain it.
4.	The resistance and the conductivity are independent of current density (δ) under constant temperature. But may change due to the temperature.	The reluctance, permeance and permeability are dependent on the flux density.
5.	Electric lines of flux are not closed. They start from positive charge and end on negative charge.	Magnetic lines of flux are closed lines. They flow from N pole to S pole externally while S pole to N pole internally.
6.	There is continuous consumption of electrical energy.	Energy is required to create the magnetic flux and not to maintain it.

Expected Question

1. *Compare electric and magnetic circuits clearly stating similarities and dissimilarities between them.* **SPPU : Dec.-05, 06,07,09,10,11, May-05,07,08,09,11, 12,14,16, Marks 8**

1.16 : Magnetic Leakage and Fringing

SPPU : May-09,Dec.-97, 09

- Most of the applications which are using magnetic effects of an electric current, are using flux in air gap for their operation. Such devices are generators, motors, measuring instruments like ammeter, voltmeter etc. Such devices consist of magnetic circuit with an air gap and flux in air gap is used to produce the required effect.

- Such flux which is available in air gap and is utilised to produce the desired effect is called **useful flux** denoted by ϕ_u .
- It is expected that whatever is the flux produced by the magnetizing coil, it should complete its path through the iron and air gap. So all the flux will be available in air gap.
- In actual practice it is not possible to have entire flux available in air gap. This is because, we have already seen that there is no perfect insulator for the flux.
- So part of the flux completes its path through the air or medium in which coil and magnetic circuit is placed.

Key Point *Such flux which leaks and completes its path through surrounding air or medium instead of the desired path is called the leakage flux.*

- The Fig. 1.16.1 shows the useful and leakage flux.

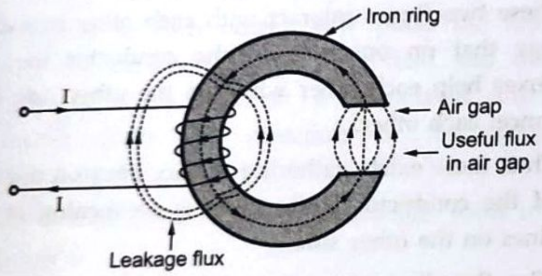


Fig. 1.16.1 Leakage and useful flux

1.16.1 Leakage Coefficient or Hopkinson's Coefficient

- The ratio of the total flux (ϕ_T) to the useful flux (ϕ_u) is defined as the **leakage coefficient** of **Hopkinson's coefficient** or **leakage factor** of that magnetic circuit.
- It is denoted by λ .

$$\therefore \quad \boxed{\lambda = \frac{\text{Total flux}}{\text{Useful flux}} = \frac{\phi_T}{\phi_u}}$$

- The value of 'λ' is always greater than 1 as ϕ_T is always more than ϕ_u.
- It generally varies between 1.1 and 1.25. Ideally its value should be 1.

1.16.2 Magnetic Fringing

- When flux enters into the air gap, it passes through the air gap in terms of parallel flux lines.
- There exists a force of repulsion between the magnetic lines of force which are parallel and having same direction.
- Due to this repulsive force there is tendency of the magnetic flux to bulge out (spread out) at the edge of the air gap.

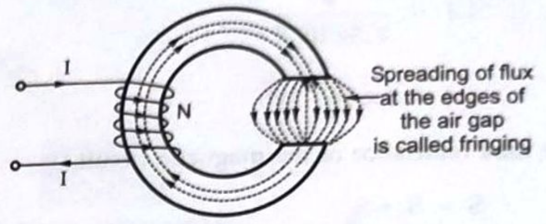


Fig. 1.16.2 Magnetic fringing

- This tendency of flux to bulge out at the edges of the air gap is called **magnetic fringing.**
- **It has following two effects :**
  1) It increases the effective cross-sectional area of the air gap.
  2) It reduces the flux density in the air gap.

So leakage, fringing and reluctance, in practice should be as small as possible.

Key Point *This is possible by choosing good magnetic material and making the air gap as narrow as possible.*

Ex. 1.16.1 : ***A soft iron ring of 20 cm mean diameter and circular cross-section of 4 cm diameter is wound with a magnetising coil. A current of 5 A flowing in the coil produces flux of 2.5 mWb in the air gap which is 2.2 mm wide. Taking relative permeability to be 1000 at this flux density and allowing for a leakage coefficient of 1.2, find the number of the turns on the coil.*** **SPPU : Dec.-97**

Sol. : d_{mean} = 20 cm, d = 4 cm, I = 5 A,

ϕ_g = 2.5 mWb, l_g = 2.2 mm, λ = 1.2

$\therefore$ mean length $l = \pi \times d_{mean} = \pi \times 20 \times 10^{-2} = 0.6283$ m

Cross section diameter = 4 cm

$\therefore \quad a = \frac{\pi}{4}d^2 = \frac{\pi}{4}\times(4)^2$

$= 12.566 \text{ cm}^2 = 12.566\times10^{-4} \text{ m}^2$

l_g = length of air gap

$= 2.2 \text{ mm} = 2.2\times10^{-3}$ m

$\therefore \quad l_i$ = length of iron path $= l - l_g = 0.6261$ m

Now $\quad \lambda = \frac{\text{total flux}}{\text{air gap flux}} = \frac{\phi}{\phi_g}$

i.e. $\quad 1.2 = \frac{\phi}{2.5\times10^{-3}}$

$\therefore \quad \phi = 3\times10^{-3}$ Wb.

The total reluctance of the magnetic circuit,

$$S = S_i + S_g$$

Now $\quad S_i = \frac{l_i}{\mu_0\mu_r a} = \frac{0.6261}{4\pi\times10^{-7}\times1000\times12.566\times10^{-4}}$

$= 396494.15$ AT/Wb

While $\quad S_g = \frac{l_g}{\mu_0 a} = \frac{2.2\times10^{-3}}{4\pi\times10^{-7}\times12.566\times10^{-4}}$

$= 1393207.4$ AT/Wb

Now $\quad \phi = \frac{\text{m.m.f.}}{\text{reluctance}} = \frac{NI}{Si+Sg}$

$\phi_g = \frac{\text{m.m.f. for air gap}}{S_g}$

i.e. $\quad 2.5\times10^{-3} = \frac{\text{m.m.f. for air gap}}{1393207.4}$

$\therefore$ m.m.f. for air gap = 3483.01

$\phi = \frac{\text{m.m.f. for iron}}{S_i}$

i.e. $\quad 3\times10^{-3} = \frac{\text{m.m.f. for iron}}{396494.15}$

$\therefore$ m.m.f. for iron = 1189.4825

Hence the total m.m.f. can be obtained as :

Total = m.m.f. for air gap + m.m.f. for iron

m.m.f. = 3483.01 + 1189.48 = **4672.501 AT/Wb**

Now m.m.f = N × I i.e. 4672.501 = N × 5

$\therefore \quad N = \frac{4672.501}{5} = 934.5$

Hence the number of turns on the coil required is approximately 935.

Expected Questions

1. *Write a note on magnetic leakage and fringing.* **SPPU : May-09, Dec.-09, Marks 5**
2. *Define leakage coeficient and state its impotance.*

1.17 : Force on a Current Carrying Conductor in a Magnetic Field

SPPU : May-05

- Consider a current carrying conductor placed in a magnetic field produced by a permanent magnet.
- Any current carrying conductor produces its own magnetic filed around it. The direction of this magnetic field can be obtained by right hand thumb rule.
- Thus there exists two fluxes, one produced by the permanent magnet while the other produced by current carrying conductor.
- These two fluxes interact with each other in such a way that on one side of the conductor the two fluxes help each other while on the other side they cancel each other.
- Thus there exists gathering of flux lines on one side of the conductor while there is weakening of flux lines on the other side.
- The flux lines act as stretched rubber bands and hence exert a force from high flux region towards low flux region.
- The entire process is shown in the Fig. 1.17.1 (a), (b), (c) and (d). (Refer fig. 1.17.1 on next page)

Thus we can conclude that current carrying conductor placed in the magnetic field, experiences a mechanical force, due to interaction of two fluxes.

1.17.1 Fleming's Left Hand Rule

- The direction of the force experienced by the current carrying conductor placed in magnetic field can be determined by a rule called 'Fleming's left hand rule'.

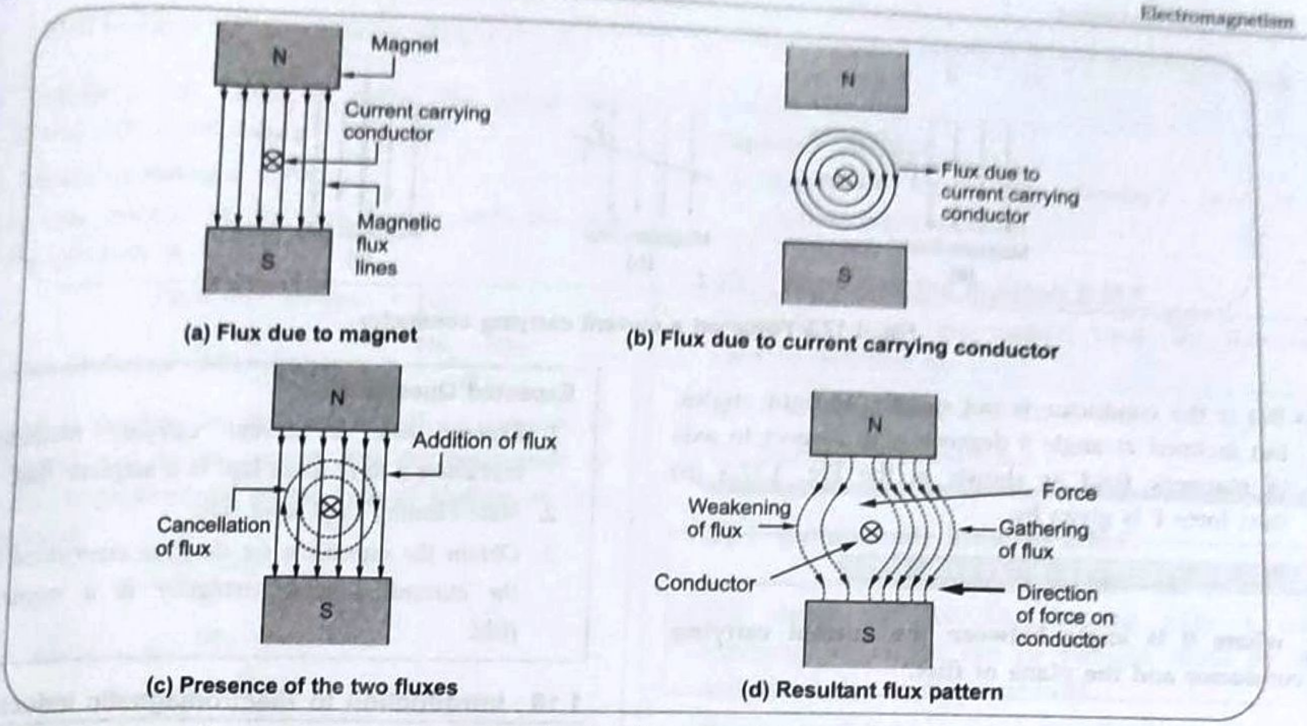


Fig. 1.17.1 Interaction of the two flux lines to produce force on conductor

- The rule states that, 'Outstretch the three fingers of the left hand namely the first finger, middle finger and thumb such that they are mutually perpendicular to each other. Now point the first finger in the direction of magnetic field and the middle finger in the direction of the current then the thumb gives the direction of the force experienced by the conductor'.
- The rule is explained in the diagrammatic form in the Fig. 1.17.2.

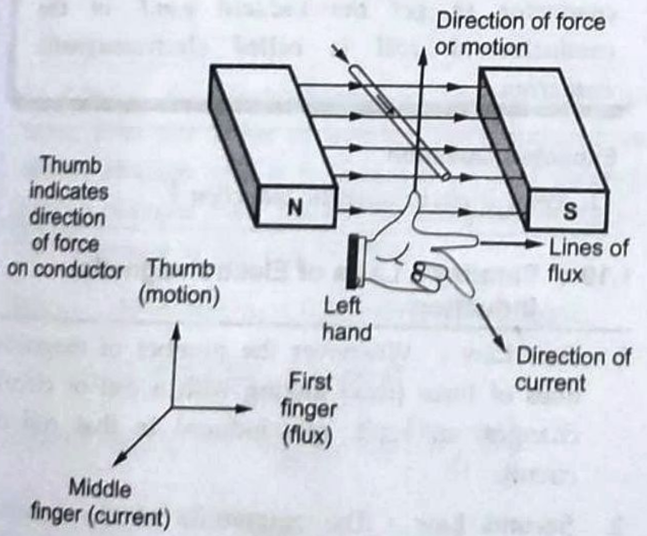


Fig. 1.17.2 Fleming's left hand rule

Key Point *The direction of such force can be reversed either by changing the direction of current or by changing the direction of the flux lines in which it is kept. If both are reversed, the direction of force remains unchanged.*

1.17.2 Magnitude of Force Experienced by the Conductor

- The magnitude of the force experienced by the conductor depends on :
  1) Flux density (B) of the magnetic field in which the conductor is placed measured in Wb/m^2 i.e. Tesla.
  2) Magnitude of the current I passing through the conductor in Amperes.
  3) Active length 'l' of the conductor in metres.
- The **active length** of the conductor is that part of the conductor which is actually under the influence of magnetic field.
- If the conductor is at right angles to the magnetic field as shown in Fig. 1.17.3 (a) then force F is given by,

$$F = B I l \text{ Newtons}$$

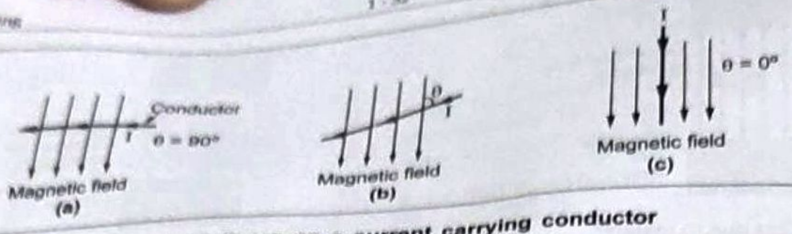


Fig. 1.17.3 Force on a current carrying conductor

- But if the conductor is not exactly at right angles, but inclined at angle θ degrees with respect to axis of magnetic field as shown in the Fig. 1.17.3 (b) then force F is given by,

$$F = B\,I\,l \sin\theta \text{ Newtons}$$

where θ is angle between the current carrying conductor and the plane of flux.

- As shown in the Fig. 1.17.3 (c), if conductor is kept along the lines of magnetic field then $\theta = 0^\circ$ and as $\sin 0^\circ = 0$, the force experienced by the conductor is also zero.

Ex. 1.17.1 : ***A conductor of length 10 cm carrying 5 A is placed in a uniform magnetic field of flux density 1.25 tesla. Find the force acting on the conductor, if it is placed i) along the lines of magnetic flux, ii) perpendicular to the lines of flux, and ii) at 30° to the flux.*** **SPPU : May-05, Marks 6**

Sol. : $l = 10 \text{ cm} = 10 \times 10^{-2}$ m, $I = 5$ A, $B = 1.25$ T

Case 1 : Along lines of magnetic flux

θ = Angle between conductor and axis of magnetic field = 0°

$$\therefore F = B.I.l. \sin\theta = 1.25 \times 5 \times 10 \times 10^{-2} \sin 0^\circ = \mathbf{0\ N}$$

Case 2 : Perpendicular to lines of flux i.e. $\theta = 90^\circ$

$$\therefore \quad F = B.I.l. \sin 90$$

$$= B.I.l. = 1.25 \times 5 \times 10 \times 10^{-2} = \mathbf{0.625\ N}$$

Case 3 : At 30° to the flux i.e. $\theta = 30^\circ$

$$\therefore \quad F = B.I.l. \sin 30^\circ$$

$$= 1.25 \times 5 \times 10 \times 10^{-2} \times \frac{1}{2} = \mathbf{0.3125\ N.}$$

Expected Questions

1. *Explain how a current carrying conductor expriences a force when kept in a magnetic field.*
2. *State Fleming's left hand rule.*
3. *Obtain the expression for the force experienced by the current carrying conductor in a magnetic field.*

1.18 : Introduction to Electromagnetic Induction

- The phenomenon by which e.m.f. is obtained from flux is called **electromagnetic induction.**
- E.M.F. can be induced in a coil by moving a coil in the fixed magnetic field or keeping the coil fixed in a moving magnetic field, by creating relative motion between flux and coil.
- If there is **change of flux lines** with respect to **conductor** i.e. there is cutting of the flux lines by the conductor then e.m.f. gets induced in the conductor.

This phenomenon of cutting of flux lines by the conductor to get the induced e.m.f. in the conductor or coil is called electromagnetic induction.

Expected Question

1. *What is electromagnetic induction ?*

1.19 : Faraday's Laws of Electromagnetic Induction

1. **First Law :** Whenever the number of magnetic lines of force (flux) linking with a coil or circuit changes, an e.m.f. gets induced in that coil or circuit.
2. **Second Law :** The magnitude of the induced e.m.f. is directly proportional to the rate of change of flux linkages (flux × turns of coil).

Flux linkages = Flux × Number of turns of coil

- Consider a coil having N turns. The initial flux linking with a coil is ϕ_1.

$\therefore$ Initial flux linkages = $N\phi_1$

- In time interval dt, the flux linking with the coil changes from ϕ_1 to ϕ_2.

$\therefore$ Final flux linkages = $N\phi_2$

$\therefore$ Rate of change of flux linkages = $\dfrac{N\phi_2 - N\phi_1}{dt}$

- Now as per the first law, e.m.f. will get induced in the coil and as per second law the magnitude of e.m.f. is proportional to the rate of change of flux linkages.

$$\therefore \quad e \propto \frac{N\phi_2 - N\phi_1}{dt}$$

$$\text{i.e.} \quad e = K \times \frac{N\phi_2 - N\phi_1}{dt}$$

$$\therefore \quad e = N\frac{d\phi}{dt} \qquad (d\phi = \phi_2 - \phi_1)$$

- With K as unity to get units of e as volts, $d\phi$ is change in flux, dt is change in time hence ($d\phi/dt$) is rate of change of flux.
- As per Lenz's law, the induced e.m.f. sets up a current in such a direction so as to oppose the very cause producing it. Mathematically this opposition is expressed by a negative sign.
- Thus such an induced e.m.f. is mathematically expressed alongwith its sign as,

$$\boxed{e = -N\frac{d\phi}{dt} \quad \text{volts}}$$

Ex. 1.19.1 : ***An electromagnet is wound with 800 turns. Find the value of average e.m.f. induced and current through coil, if it is moved to that magnetic field is changed from 1 mWb to 0.25 mWb in 0.2 sec. The resistance of the coil is 500 Ω.***

Sol. : N = 800, ϕ_2 = 0.25 mWb, ϕ_1 = 1 mWb,

t = 0.2 sec., R = 500 Ω

$$\text{Induced e.m.f } e = -N\frac{d\phi}{dt} = -800\left[\frac{\phi_2 - \phi_1}{dt}\right]$$

$$= -\left[\frac{0.25\times10^{-3} - 1\times10^{-3}}{0.2}\right] = 3 \text{ volts}$$

$$\therefore \quad \text{Current } I = \frac{\text{e.m.f.}}{R} = \frac{3}{500} = 6\times10^{-3} \text{ A} = 6 \text{ mA}$$

Expected Question

1. *State and explain the Faraday's laws of electromagnetic induction.*

1.20 : Nature of the Induced E.M.F.

- Depending upon the method used, the induced e.m.f. is classified as,

1) Dynamically induced e.m.f. and
2) Statically induced e.m.f.

1.21 : Dynamically Induced E.M.F.

SPPU : May-04, 09, 11, 15, Dec.-03, 08, 10

- The change in the flux linking with a coil, conductor or circuit can be brought about by its motion relative to magnetic field.
- This is possible by moving flux with respect to coil conductor or circuit or it is possible by moving conductor, coil, circuit with respect to stationary magnetic flux.

*An induced e.m.f. which is due to **physical movement** of coil, conductor with respect to flux or movement of magnet with respect to stationary coil, conductor is called **dynamically induced e.m.f. or motional induced e.m.f.***

1.21.1 Expression for Dynamically Induced E.M.F.

- Consider a conductor of length l metres moving in the air gap between the poles of the magnet.
- If plane of the motion of the conductor is parallel to the plane of the magnetic field then there is no cutting of flux lines and there cannot be any induced e.m.f. in the conductor.

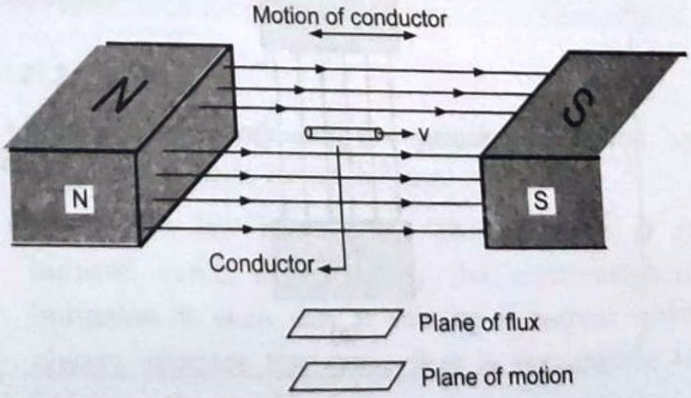


Fig. 1.21.1 (a) No cutting of flux

- If the velocity direction i.e. motion of conductor is perpendicular to the flux then whole length of conductor cuts the flux line and there is maximum possible induced e.m.f. in the conductor. **Under such condition plane of the flux and plane of motion are perpendicular to each other.**

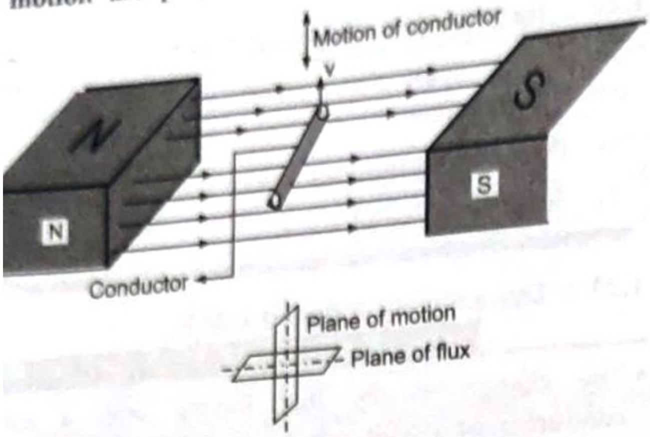


Fig. 1.21.1 (b) Maximum cutting of flux

Consider a conductor moving with velocity v m/s such that its plane of motion or direction of velocity is perpendicular to the direction of flux lines as shown in Fig. 1.21.2 (a).

B = Flux density in Wb/m^2

l = Active length of conductor in metres.

(This is the length of conductor which is actually responsible for cutting of flux lines.)

v = Velocity in m/sec.

Let this conductor is moved through distance dx in a small time interval dt, then

Area swept by conductor = $l \times dx \ m^2$

Flux cut by conductor=Flux density × Area swept

i.e.

$$d\phi = B \times l \times dx \ Wb$$

- According to Faraday's law, magnitude of induced e.m.f. is proportional to the rate of change of flux.

$$\therefore \quad e = \frac{\text{Flux cut}}{\text{Time}} = \frac{d\phi}{dt}$$

$$= \frac{B\,l\,dx}{dt} \quad \text{[Here N = 1 as single conductor]}$$

But $\frac{dx}{dt}$ = Rate of change of displacement

= Velocity of the conductor = v

$$\therefore \quad e = B\,l\,v \ \text{ volts}$$

- This is the induced e.m.f. when plane of motion is **exactly perpendicular to the plane of flux.** This is maximum possible e.m.f. as plane of motion is at right angles to plane of the flux.
- But if conductor is moving with a velocity v but at a certain angle θ measured with respect to direction of the field (plane of the flux) as shown in the Fig. 1.21.2 (b) then component of velocity which is $v\sin\theta$ is perpendicular to the direction of flux and hence responsible for the induced e.m.f..
- The other component $v\cos\theta$ is parallel to the plane of the flux and hence will not contribute to the dynamically induced e.m.f.
- Under this condition magnitude of induced e.m.f. is given by,

$$\boxed{e = B\,l\,v \sin\theta \ \text{ volts}}$$

where θ is measured with respect to plane of flux

- The dynamically induced e.m.f. is used in the machines like d.c. generators, alternators, d.c. motors etc.

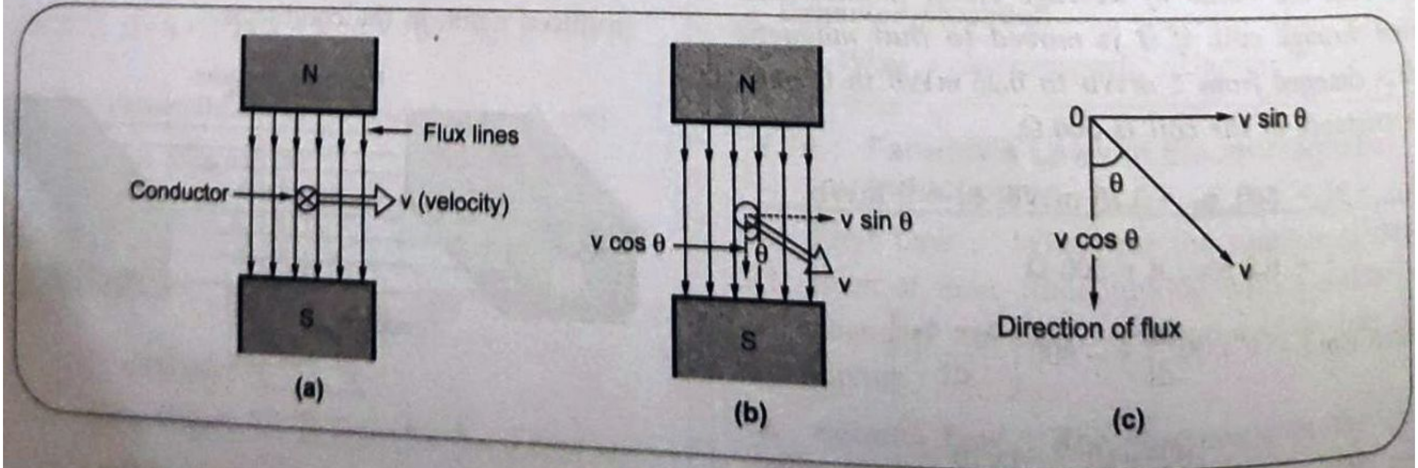


Fig. 1.21.2

Ex. 1.21.1 : ***An electric conductor of effective length of 0.3 metre is made to move with a constant velocity of 5 metre per second perpendicular to a magnetic field of uniform flux density 0.5 tesla. Find the e.m.f. induced in it. If this e.m.f. is used to supply a current of 25 A, find the force on the conductor. Find the power required to keep the conductor moving across the field.*** **SPPU : Dec.-03, Marks 5**

Sol. : l = 0.3 m, v = 5 m/s, B = 0.5 T

$$e = Blv = 0.3\times 5\times 0.5 = \mathbf{0.75\ V}$$

Now $I = 25$ A

$$\therefore \quad F = BIl = 0.5\times 25\times 0.3 = \mathbf{3.75\ N}$$

$\therefore$ The power required to keep the conductor moving is,

$$P = e\times I = 0.75\times 3.75 = \mathbf{2.8125\ W}$$

Ex. 1.21.2 : ***A straight conductor 1.5 m long lies in a plane perpendicular to a uniform magnetic field of flux density 1.2 tesla. When a current of 'I' ampere is passed through it, it makes the conductor move across the magnetic field with a velocity of 1 m/s. Ignoring resistance of the conductor and friction, find the current 'I', if the power of the moving conductor is 90 watt. Find the e.m.f. induced in the conductor and the force on it. State the sense of the force w.r.t. the velocity, and sense of the e.m.f. induced w.r.t. current.*** **SPPU : May-04, Marks 6**

Sol. : l = 1.5 m, B = 1.2 T, v = 1 m/s, P = 90 W

$$e = Blv = 1.2\times 1.5\times 1 = 1.8\ V$$

$$P = e\times I \quad \text{i.e.} \quad 90 = 1.8\times I \quad \text{i.e.} \quad I = \mathbf{50\ A}$$

$$\therefore \quad F = BIl = 1.2\times 50\times 1.5 = \mathbf{90\ N}$$

The force is so as to oppose the velocity while the sense of e.m.f. is so as to oppose the current.

1.21.2 Fleming's Right Hand Rule (Direction of Dynamically Induced E.M.F.)

- Fleming's Right Hand Rule is used to get direction of induced e.m.f. when conductor is moving in a magnetic field.
- According to Fleming's right hand rule, outstretch the three fingers of right hand namely the thumb, fore finger and the middle finger, perpendicular to each other. Arrange the right hand so that first finger point in the direction of flux lines (from N to S) and thumb in the direction of motion of conductor with respect to the flux then the middle finger will point in the direction of the induced e.m.f. (or current).
- Consider the conductor moving in a magnetic field as shown in the Fig. 1.21.3 (a).

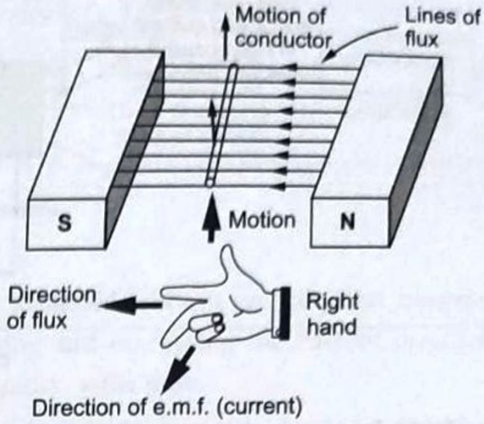


Fig. 1.21.3 (a)

- It can be verified using Fleming's right hand rule that the direction of the current due to the induced e.m.f. is coming out.
- Symbolically this is shown in the Fig. 1.21.3 (b).

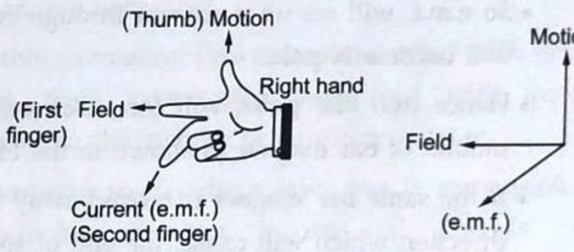


Fig. 1.21.3 (b)

Key Point *In practice though magnet is moved keeping the conductor stationary, while application of rule, thumb should point in the direction of relative motion of conductor with respect to flux, assuming the flux stationary.*

1.21.2.1 Lenz's Law

- This rule is based on the principles derived by German Physicist Heinrich Lenz.
- The Lenz's law states that, **'The direction of an induced e.m.f. produced by the electromagnetic induction is such that it sets up a current which always opposes the cause that is responsible for inducing the e.m.f.'**

- In short the induced e.m.f. always opposes the cause producing it, which is represented by a negative sign, mathematically in its expression.

$$\therefore \quad e = -N\frac{d\phi}{dt}$$

- Consider a solenoid as shown in the Fig. 1.21.4.

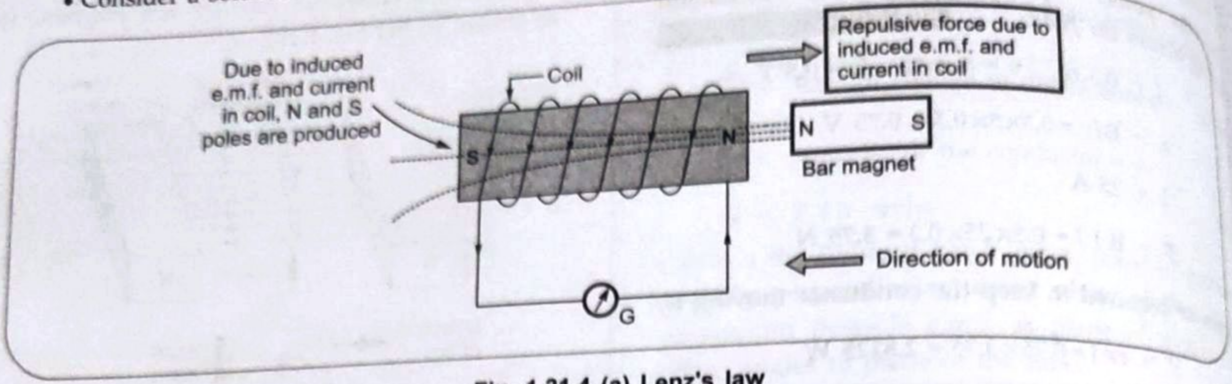


Fig. 1.21.4 (a) Lenz's law

- Let a bar magnet is moved towards coil such that N-pole of magnet is facing a coil.
- Due to cutting of flux lines, e.m.f. gets induced in the coil which will circulate the current through the coil.
- According to Lenz's law, the direction of current due to induced e.m.f. is so as to oppose the cause producing it. The cause is motion of bar magnet towards the coil.
- So e.m.f. will set up a current through coil in such a way that the end of solenoid facing bar magnet will become N-pole.
- Hence two like poles will face each other experiencing force of repulsion which is opposite to the motion of bar magnet as shown in the Fig. 1.21.4 (a).
- If the same bar magnet is moved away from the coil, then induced e.m.f. will set up a current in the direction which will cause, the end of solenoid facing bar magnet to behave as S-pole.
- Because of this, two unlike poles will face each other and there will be force of attraction which is opposite to the direction of movement of magnet as shown in the Fig. 1.21.4 (b).

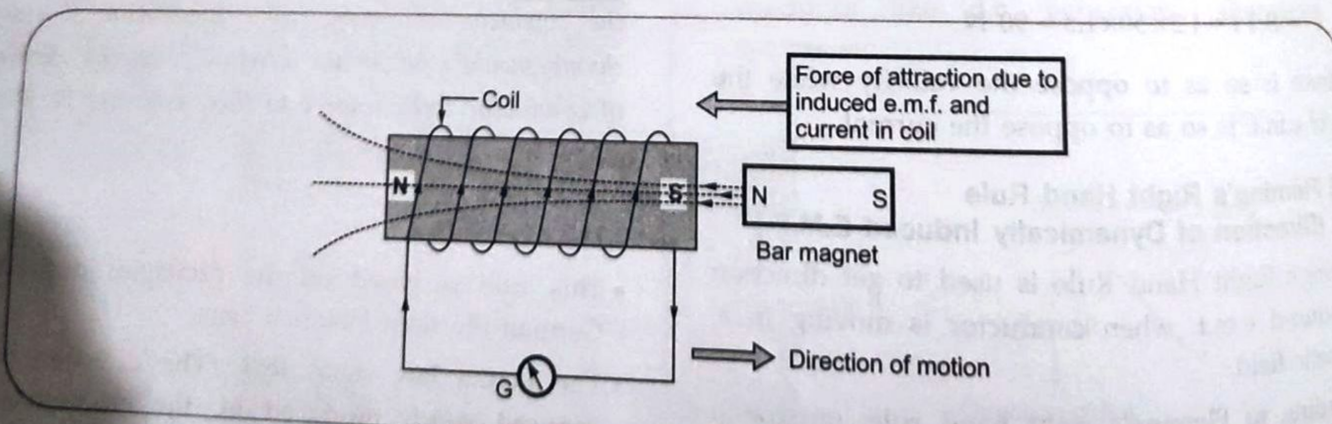


Fig. 1.21.4 (b) Lenz's law

- **In any case the induced e.m.f. always opposes the cause producing it, which is the Lenz's law.**

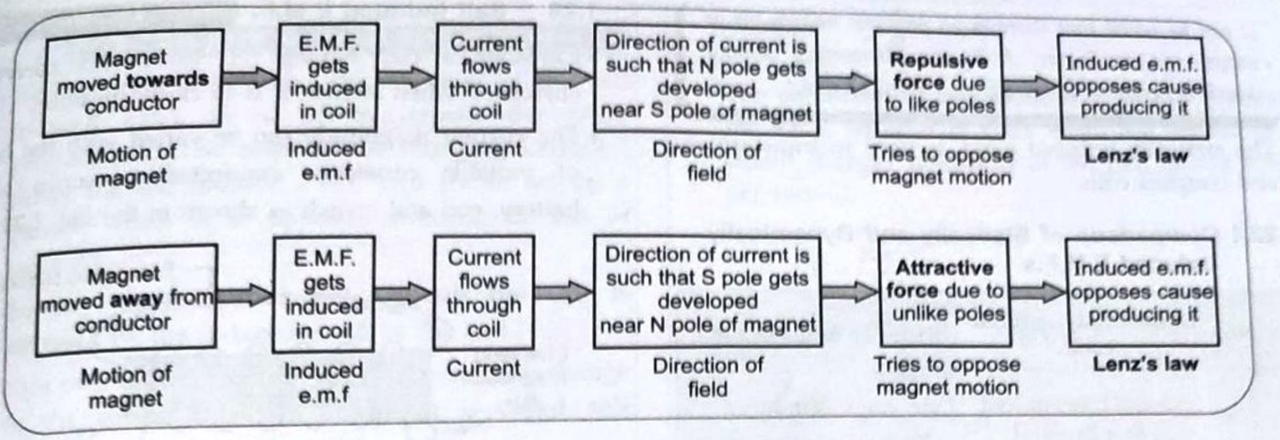


Fig. 1.21.5 Concept of Lenz's law

Expected Questions

1. *What is dynamically induced e.m.f. ? Derive the expression for its magnitude.*

SPPU : May-09,11,15, Marks 3, Dec.-08,10 Marks 4

2. *State Fleming's right hand rule.*
3. *State and explain Lenz's law.*

1.22 : Statically Induced E.M.F.

SPPU : May-09,11,15, Dec.-10,17

The change in flux lines with respect to coil can be achieved without physically moving the coil or the magnet. Such induced e.m.f. in a coil which is without physical movement of coil or a magnet is called ***statically induced e.m.f.***

- To have an induced e.m.f. there must be change in flux associated with a coil. Such a change in flux can be achieved without any physical movement by increasing and decreasing the current producing the flux rapidly, with time.
- Let current through the coil of an electromagnet producing the flux be an alternating one.
- Such alternating current means **it changes its magnitude periodically with time.**
- This produces the flux which is also alternating i.e. changing with time.
- If this alternating flux comes in contact with another coil, there exists $d\phi/dt$ associated with the coil placed in the vicinity of an electromagnet.
- According to Faraday's law, this is responsible for producing an e.m.f. in the coil. This is called statically induced e.m.f.

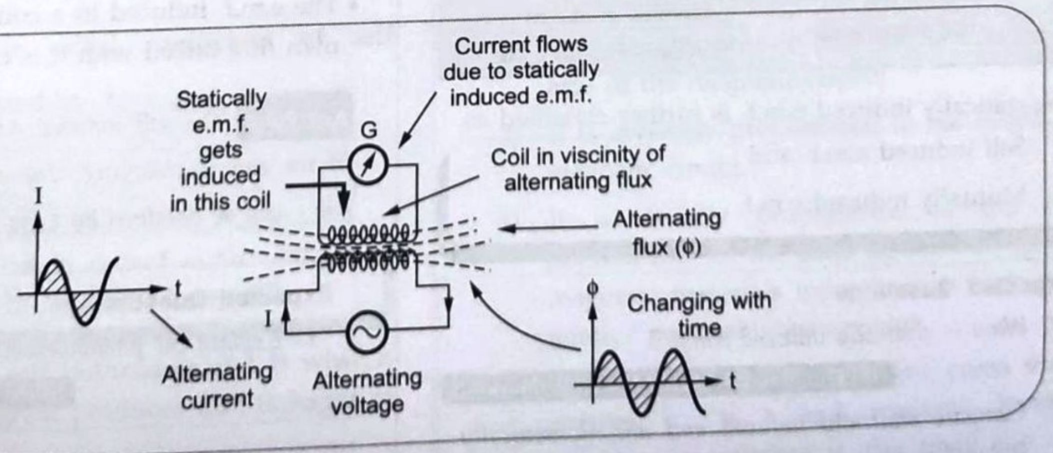


Fig. 1.22.1 Concept of statically induced e.m.f.

It can be noted that there is no physical movement of magnet or conductor, it is the alternating supply which is responsible for such an induced e.m.f.

- The statically induced e.m.f. is used in transformers and coupled coils.

1.22.1 Comparison of Statically and Dynamically Induced E.M.F.s

Sr. No.	Dynamically induced e.m.f.	Statically induced e.m.f.
1.	An e.m.f. is induced due to a physical movement of either conductor or flux.	An e.m.f. is induced due to change in flux associated with conductor without any physical movement.
2.	Either conductor is physically rotating or flux is physically rotating.	Conductor remains stationary and flux linking with it is changed. The flux is alternating.
3.	Its magnitude is given by the relation, $e = Bl v \sin\theta$	Its magnitude is given by the relations, $e = L\frac{di}{dt}$ or $e = M\frac{di}{dt}$
4.	The current producing the flux may be constant.	The current producing the flux must be varying so as to produce varying flux.
5.	It is used in the devices like alternators, generators, motors, measuring instruments etc.	It is used in the devices like transformers, coupled coil, filter choke in rectifier circuits, choke in fluorescent tube etc.

- The statically induced e.m.f. is further classified as,

1) Self induced e.m.f. and
2) Mutually induced e.m.f.

Expected Questions

1. *What is statically induced e.m.f. ?* **SPPU : May-09,11,15, Dec.-10, Marks 3**
2. *Compare statically induced emf and dynamically induced emf.* **SPPU : Dec.-17, Marks 6**

1.23 : Self Induced E.M.F. SPPU : Dec.-06, May-10

- Consider a coil having 'N' turns and carrying current 'I' when switch 'S' is in closed position.
- The current magnitude can be varied with the help of variable resistance connected in series with battery, coil and switch as shown in the Fig. 1.23.1.

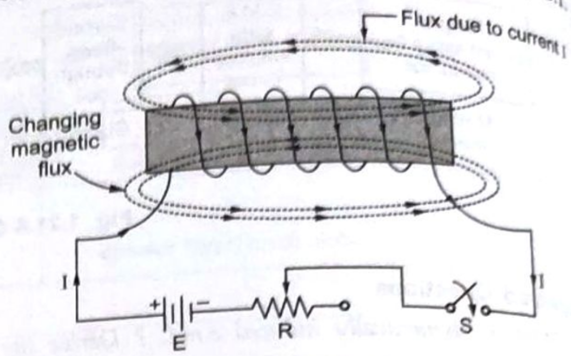


Fig. 1.23.1

- The flux produced by the coil links with the coil itself.
- The total flux linkages of coil will be $N\phi$ Wb-turns.
- Now if the current 'I' is changed with the help of variable resistance R, then flux produced will also change, due to which flux linkages will also change.
- Hence according to Faraday's law, due to rate of change of flux linkages there will be induced e.m.f. in the coil.
- So without physically moving coil or flux there is induced e.m.f. in the coil. The phenomenon is called **self induction.**
- The e.m.f. induced in a coil due to the change of its own flux linked with it is called **self induced e.m.f.**

Key Point *The self induced e.m.f. lasts till the current in the coil is changing. The direction of such induced e.m.f. can be obtained by Lenz's law.*

Expected Question

1. *Explain the phenomenon of self induced e.m.f.* **SPPU : Dec.-06, May-10, Marks 4**

1.24 : Self Inductance

SPPU : Dec.-06,08,09, May-04,06,08,10,11,12,15,19

- According to Lenz's law the direction of the self induced e.m.f. is so as to oppose the cause producing it. The cause is the changing current I hence the self induced e.m.f. will try to set up a current which is in opposite direction to that of current I.
- **So any change in current through the coil is opposed by the induced e.m.f. in the coil.**
- This property of the coil which opposes any change in the current passing through it is called **self inductance or only inductance.**
- It is analogous to electrical inertia or electromagnetic inertia.

1.24.1 Expression of Self Inductance (L)

- From the Faraday's law of electromagnetic induction, self induced e.m.f. can be expressed as,

$$\boxed{e = -N\frac{d\phi}{dt}}$$

- **Negative sign indicates that direction of this e.m.f. is opposing change in current due to which it exists.**

$$\phi = (\text{Flux/ Ampere}) \times \text{Ampere} = \frac{\phi}{I} \times I$$

$$\text{Rate of change of flux} = \frac{\phi}{I} \times \text{Rate of change of current}$$

$$\therefore \quad \frac{d\phi}{dt} = \frac{\phi}{I} \cdot \frac{dI}{dt}$$

$$\text{i.e.} \quad e = -N \cdot \frac{\phi}{I} \cdot \frac{dI}{dt} = -\left(\frac{N\phi}{I}\right)\frac{dI}{dt}$$

- The constant $\frac{N\phi}{I}$ is called **coefficient of self inductance and denoted by 'L'.**

$$\therefore \quad \boxed{L = \frac{N\phi}{I}}$$

> *The self inductance can be defined as flux linkages per ampere current in it. Its unit is* ***henry (H).***

- A circuit possesses **a self inductance of 1 H** when a current of 1 A through it produces flux linkages of 1 Wb-turn in it.

$$\therefore \quad \boxed{e = -L\frac{dI}{dt} \quad \text{volts}}$$

- The coefficient of self inductance is also defined as the e.m.f. induced in volts when the current in the circuit changes uniformly at the rate of one ampere per second.

$$\boxed{L = \frac{N\phi}{I}} \quad \ldots (1.24.1)$$

$$\text{But} \quad \phi = \frac{\text{m.m.f.}}{\text{Reluctance}} = \frac{NI}{S}$$

$$\boxed{L = \frac{N \cdot NI}{I \cdot S} = \frac{N^2}{S} \quad \text{henries}} \quad \ldots (1.24.2)$$

$$\text{• Now} \quad S = \frac{l}{\mu a} \quad \text{i.e.} \quad L = \frac{N^2}{\left(\frac{l}{\mu a}\right)}$$

$$\therefore \quad \boxed{L = \frac{N^2 \mu a}{l} = \frac{N^2 \mu_0 \mu_r a}{l} \quad \text{henries}} \quad \ldots (1.24.3)$$

where l = Length of magnetic circuit

a = Area of cross-section of magnetic circuit

1.24.2 Factors Affecting Self Inductance of a Coil

- It is known that $L = \frac{N^2 \mu_0 \mu_r a}{l}$
- The factors on which self inductance of a coil depends are,

1) It is directly proportional to the square of number of turns of a coil. This means for same length, if number of turns are more then self inductance of coil will be more.
2) It is directly proportional to the cross-sectional area of the magnetic circuit.
3) It is inversely proportional to the length of the magnetic circuit.
4) It is directly proportional to the relative permeability of the core. So for iron and other magnetic materials inductance is high as their relative permeabilities are high.
5) For air cored or non magnetic cored magnetic circuits, μ_r = 1 and constant, hence self inductance coefficient is also small and always constant.

6) Since the relative permeability of iron varies with respect to flux density, the coefficient of self inductance varies with respect to flux density.

Ex. 1.24.1 : ***The field coil is wound with 600 turns. When a current of 6 A flows through it, produces a flux of 2 mWb, find self inductance of the coil.***

Sol. : Given : $N = 600$; $I = 6$ A ;

$\phi = 2$ mWb $= 2 \times 10^{-3}$ Wb.

To find : L

$$L = \frac{N\phi}{I} = \frac{600 \times 2 \times 10^{-3}}{6} = 0.2 \text{ H}$$

Ex. 1.24.2 : ***Iron ring of mean diameter 30 cm has square cross section of 2 cm × 2 cm and is uniformly wound with 500 turns. Assume relative permeability of iron = 800. Calculate i) Self inductance L ii) If number of turns are doubled, keep remaining data as it, find inductance iii) If only μ_r is changed to 1000 find new value of L.***

Sol. : Given : $D = 30$ cm $= 0.3$ m ;

$l = \pi D = 3.14 \times 0.3$ m ; $a = 2$ cm $\times 2$ cm $= 4$ cm^2,

$a = 4 \times 10^{-4}$ m^2 ; $N = 500$; $\mu_r = 800$

To find : L

i) Inductance $L = \dfrac{N^2}{S} = \dfrac{N^2 \mu_0 \mu_r a}{l}$

$$L = \frac{500^2 \times 1.256 \times 10^{-6} \times 800 \times 4 \times 10^{-4}}{0.3 \times 3.14} = \mathbf{0.1067 \text{ H}}$$

ii) When N = 1000 (doubled)

$$L_1 \propto N_1^2 \propto 500^2,\ L_2 \propto N_2^2 \propto 1000^2 \propto 2^2 \times 500^2$$

$$\frac{L_2}{L_1} = \frac{2^2 \times 500^2}{500^2} = 2^2 = 4$$

$$L_2 = 4 \times L_1 = 4 \times 0.1067 = \mathbf{0.427 \text{ H}}$$

OR $$L = \frac{1000^2 \times 1.256 \times 10^{-6} \times 800 \times 4 \times 10^{-4}}{0.3 \times 3.14} = \mathbf{0.427 \text{ H}}$$

iii) When $\mu_{r2} = 1000$, As $L \propto \mu_r$, $\dfrac{L_2}{L_1} = \dfrac{\mu_{r2}}{\mu_{r1}}$

$$L_2 = L_1 \times \frac{1000}{800} = 0.1067 \times \frac{1000}{800} = \mathbf{0.133 \text{ H}}$$

Ex. 1.24.3 : ***The mean diameter of steel ring is 40 cm and flux density of 0.9 Wb/m^2 is produced by 3500 AT/m. If the cross-section of the ring is 15 cm^2 and number of turns 440, calculate: i) the exciting current ii) the self inductance in henry.***

SPPU : Dec.-06, Marks 6

Sol. : Given : $d_{mean} = 40$ cm, $B = 0.9$ Wb/m^2,

$H = 3500$ AT/m, $a = 15$ cm^2, $N = 440$

$l_T = \pi \times d_{mean} = \pi \times 40 = 125.6637$ cm

To find : I, L

i) m.m.f. $= NI = H \times l_T$

$$\therefore \quad I = \frac{3500 \times 125.6637 \times 10^{-2}}{440} = \mathbf{10 \text{ A}}$$

ii) $\phi = B \times a = 0.9 \times 15 \times 10^{-4} = 1.35$ mWb

$$L = \frac{N\phi}{I} = \frac{440 \times 1.35 \times 10^{-3}}{10} = \mathbf{0.0594 \text{ H}}$$

Ex. 1.24.4 : ***If a coil of 150 turns is linked with a flux of 0.01 Wb when carrying a current of 10 A, then calculate the induced emf :***

i) If this current is uniformly reversed in 0.1 second.

ii) If this current is interrupted in 0.05 second.

SPPU : May-04, 15, Marks 6

Sol. : Given : $N = 150$, $\phi = 0.01$ Wb, $I = 10$ A

To find : Induced e.m.f. e

$$\therefore \quad L = \frac{N\phi}{I} = \frac{150 \times 0.01}{10} = 0.15 \text{ H}$$

i) $I_1 = 10$ A and $I_2 = -10$ A, $dt = 0.1$ sec

$$\therefore \quad e = -L\frac{dI}{dt} = -L\frac{[I_2 - I_1]}{dt} = \frac{-0.15\,[-10-10]}{0.1} = 30 \text{ V}$$

ii) $I_1 = 10$ A, $I_2 = 0$ A, $dt = 0.05$ sec

$$\therefore \quad e = -L\frac{[I_2 - I_1]}{dt} = \frac{-0.15\,[0-10]}{0.05} = 30 \text{ V}$$

Ex. 1.24.5 : ***Find the induced emf in the coil having inductance of 0.15 H when***

i) current of 10 A in the coil is switched off in 0.01 sec.

ii) same current is uniformly reversed in 0.01 sec.

SPPU : May-19, Marks 6

Sol. : Given : L = 0.15 H, I = 10 A

To find : Induced e.m.f. e

i) I_1 = 10 A, I_2 = 0A (switched off), dt = 0.01 sec

$$\therefore \quad \frac{dI}{dt} = \frac{I_2 - I_1}{dt} = -1000$$

$$\therefore \quad e = -L\frac{dI}{dt} = -0.15\times(-1000) = \mathbf{150\ V}$$

ii) I_1 = 10 A, I_2 = –10 A (reversed), dt = 0.01 sec

$$\therefore \quad \frac{dI}{dt} = \frac{I_2 - I_1}{dt} = \frac{-10-(10)}{0.01} = -2000$$

$$\therefore \quad e = -L\frac{dI}{dt} = -0.15\times(-2000) = \mathbf{300\ V}$$

Ex. 1.24.6 : *If a coil of 300 turns is linked with a flux of 15 mWb when carrying a current of 5 A. Calculate i) The inductance of the coil. ii) If this current is uniformly reversed in 0.01 second , find induced emf in the coil. iii) If the same current is interrupted in 0.75 second, calculate emf.*

Sol. : Given : N = 300; ϕ = 15 mWb = 0.015 Wb, I = 5 A

i) Inductance $L = \frac{N\phi}{I} = \frac{300\times 0.015}{5} = \mathbf{0.9\ H}$

ii) When current is reversed,

$$dI = -5-5 = -10\ A$$

$$dt = 0.01\ sec$$

$$\therefore \quad e_L = -L\frac{dI}{dt} = -0.9\times\frac{-10}{0.01} = \mathbf{900\ V}$$

iii) When current in interrupted in **0.75** secretary

$$dI = 0-5 = -5\ A$$

$$e_L = -0.9\times\frac{-5}{0.75} = \mathbf{6\ V}$$

Ex. 1.24.7 : *A solenoid is wound with 1000 turns having area of cross-section 25 cm². When 2.5 A current flows through the coil, the flux density is 0.8 Wb/m² and when current is increased to 5 A, the flux density becomes 1.2 Wb/m². Find the average value of self inductance within given current limits. If this change in current is achieved within 0.04 sec., calculate the self induced e.m.f.*

Sol. : Given : N = 1000,

a = 25 cm² = 25 × 10^{-4} m², I_1 = 2.5 A, B_1 = 0.8 Wb/m²

I_2 = 5 A, B_2 = 1.2 Wb/m², t = 0.04 sec.

To find : L, e

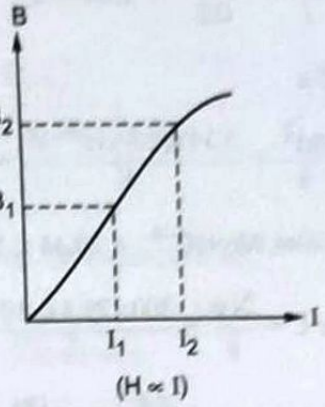


Fig. 1.24.1

$$L = \frac{N\phi}{I} \quad \text{i.e.} \quad L = N\frac{d\phi}{dI}$$

$$L = N\left[\frac{\phi_2 - \phi_1}{I_2 - I_1}\right] \quad \text{as } B = \frac{\phi}{a}$$

$$= Na\left[\frac{\frac{\phi_2}{a} - \frac{\phi_1}{a}}{I_2 - I_1}\right] = Na\left[\frac{B_2 - B_1}{I_2 - I_1}\right]$$

$$\therefore \quad L = 1000\times 25\times 10^{-4}\times\left[\frac{1.2-0.8}{5-2.5}\right] = \mathbf{0.4\ H}$$

$$\text{Now} \quad e = -L\frac{dI}{dt} = -0.4\left[\frac{I_2 - I_1}{dt}\right]$$

$$= -0.4\left[\frac{5-2.5}{0.004}\right] = \mathbf{-25\ volts}$$

Negative sign indicates that it opposes change in current.

Ex. 1.24.8 : *A coil of 300 turns is uniformly wound on iron ring of mean circumference 20 cm and cross section diameter 2.5 cm. When a current of 1 A passed through it, produces a flux density of 0.6 tesla. Calculate.*
i) Self inductance ii) Magnetizing force iii) Flux iv) Induced emf in the coil if this current is interrupted and flux decrease by 10% in 0.01 sec.
v) Relative permeability.

Sol. : Given : $d = 2.5$ cm $= 2.5 \times 10^{-2}$ m, $N = 300$,

$B = 0.6$ T, $I = 1$ A, $l = 0.2$ m

To find : H, ϕ, L, e, μ_r

i) $H = \frac{NI}{l} = \frac{300 \times 1}{0.2} = \mathbf{1500\ AT}$

ii) Flux $\phi = Ba$

$$a = \frac{\pi d^2}{4} = \frac{3.14 \times (2.5 \times 10^{-2})^2}{4} = 4.96 \times 10^{-4}\ m^2$$

$$\phi = 0.6 \times 4.906 \times 10^{-4} = 29.44 \times 10^{-3}\ \text{Wb}$$

iii) Inductance $L = \frac{N\phi}{I} = \frac{300 \times 29.44 \times 10^{-3}}{1} = \mathbf{0.088\ H}$

iv) emf induced $e_L = -N\frac{d\phi}{dt} = -N\frac{[\phi_2 - \phi_1]}{dt}$

But, $\phi_2 = \phi_1 - 10\%$ of $\phi_1 = 0.9\ \phi_1$

$$\therefore \quad e = -N\frac{[0.9\phi_1 - \phi_1]}{dt} = \frac{-300 \times -0.1 \times \phi_1}{0.01}$$

$$= \mathbf{88.32\ V} \qquad ...\phi_1 = 29.44\ \text{mWb}$$

v) Relative permeability of iron $B = \mu_0 \mu_r H$

$$\mu_r = \frac{B}{\mu_0 H} = \frac{0.6}{1.256 \times 10^{-6} \times 1500} = \mathbf{318.47}$$

Expected Questions

1. *Define self inductance.*

 SPPU : Dec.-06, May-10, Marks 2

2. *Derive the expression for the self induced e.m.f. and state its unit.* SPPU : Dec.-06, Marks 4

3. *Derive the equation $L = \frac{N^2}{S}$ where L is the self inductance of coil having N turns and S is the reluctance of the magnetic circuit.*

 SPPU : May-12, Marks 6

4. *State the various expressions for the coefficient of self inductance.*

5. *State the various factors affecting the self inductance of the coil.*

 SPPU : Dec.-06,08,09, May-06,08,11, Marks 4

1.25 : Mutually Induced E.M.F.

SPPU : May-06,07,08,10,16,17,19
Dec.-01,04,08,11,12,15,18

- If the flux produced by one coil is getting linked with another coil and due to change in this flux produced by first coil, there is induced e.m.f. in the second coil, then such an e.m.f. is called **mutually induced e.m.f.**
- Consider two coils which are placed adjacent to each other as shown in the Fig. 1.25.1.

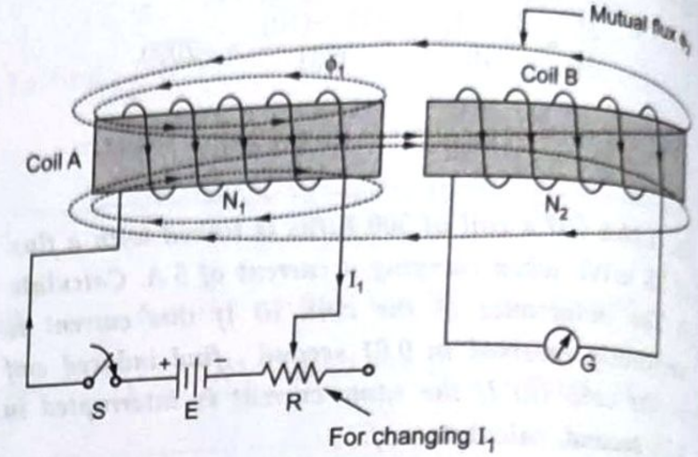


Fig. 1.25.1 Mutually induced e.m.f.

- The coil A has N_1 turns while coil B has N_2 number of turns.
- The coil A has switch S, variable resistance R and battery of 'E' volts in series with it.
- A galvanometer is connected across coil B to sense induced e.m.f. and current because of it.
- Current through coil A is I_1 producing flux ϕ_1.
- Part of this flux will link with coil B i.e. will complete its path through coil B as shown in the Fig. 1.25.1. This is the mutual flux ϕ_2.
- Now if current through coil A is changed by means of variable resistance R, then flux ϕ_1 changes. Due to this, flux associated with coil B, which is mutual flux ϕ_2 also changes.
- Due to Faraday's law there will be induced e.m.f. in coil B which will set up a current through coil B, which will be detected by galvanometer G.

Any change in current through coil A produces e.m.f. in coil B, this phenomenon is called mutual induction and e.m.f. is called mutually induced e.m.f.

1.25.1 Expression of Mutual Inductance (M)

- Let N_1 = Number of turns of coil A,
  N_2 = Number of turns of coil B
  I_1 = Current flowing through coil A
  ϕ_1 = Flux produced due to current I_1
  ϕ_2 = Flux linking with coil B
- According to Faraday's law, the induced e.m.f. in coil B is,

$$e_2 = -N_2 \frac{d\phi_2}{dt}$$

- Negative sign indicates that this e.m.f. will set up a current which will oppose the change of flux linking with it as per the Lenz's law.
- Now $\phi_2 = \frac{\phi_2}{I_1} \times I_1$
- If permeability of the surroundings is assumed constant then $\phi_2 \propto I_1$ and hence ϕ_2 / I_1 is constant.

$\therefore$ Rate of change of $\phi_2 = \frac{\phi_2}{I_1} \times$ Rate of change of current I_1

$$\therefore \quad \frac{d\phi_2}{dt} = \frac{\phi_2}{I_1} \cdot \frac{dI_1}{dt}$$

$$\therefore \quad e_2 = -N_2 \cdot \frac{\phi_2}{I_1} \cdot \frac{dI_1}{dt} = -\left(\frac{N_2 \phi_2}{I_1}\right) \frac{dI_1}{dt}$$

- Here $\left(\frac{N_2 \phi_2}{I_1}\right)$ **is called coefficient of mutual inductance denoted by M.**

$$\therefore \quad \boxed{M = \left(\frac{N_2 \phi_2}{I_1}\right) \text{Henries and } e_2 = -M \frac{dI_1}{dt} \text{ volts}}$$

Coefficient of mutual inductance is defined as the property by which e.m.f. gets induced in the second coil because of change in current through first coil. It is also defined as the flux linkages of the coil per ampere current in other coil.

- Coefficient of mutual inductance is also called mutual inductance. It is measured in henries.
- Two coils which are magnetically coupled are said to have mutual inductance of one henry when a current changing uniformly at the rate of one ampere per second in one coil, induces as e.m.f. of one volt in the other coil.

1) $\boxed{M = \frac{N_2 \phi_2}{I_1}}$ and $M = \frac{N_1 \phi_1}{I_2}$

2) ϕ_2 is the part of the flux ϕ_1 produced due to I_1. Let K_1 be the fraction of ϕ_1 which is linking with coil B.

$$\therefore \quad \phi_2 = K_1 \phi_1$$

$$\therefore \quad \boxed{M = \frac{N_2 K_1 \phi_1}{I_1}}$$

3) The flux ϕ_1 can be expressed as,

$$\phi_1 = \frac{\text{m.m.f.}}{\text{Reluctance}} = \frac{N_1 I_1}{S}$$

i.e. $$M = \frac{N_2 K_1}{I_1}\left(\frac{N_1 I_1}{S}\right)$$

$$\boxed{M = \frac{K_1 N_1 N_2}{S}}$$

- If all the flux produced by coil A links with coil B then $K_1 = 1$.

$$\boxed{M = \frac{N_1 N_2}{S}}$$

4) Now $S = \frac{l}{\mu a}$ and $K_1 = 1$

$$\therefore \quad \boxed{M = \frac{N_1 N_2}{\left(\frac{l}{\mu a}\right)} = \frac{N_1 N_2 a \mu}{l} = \frac{N_1 N_2 a \mu_0 \mu_r}{l}}$$

5) If second coil carries current I_2, producing flux ϕ_2, the part of which links with coil A i.e. ϕ_1 then,

$$\phi_1 = K_2 \phi_2 \quad \text{and} \quad M = \frac{N_1 \phi_1}{I_2}$$

hence $$M = \frac{N_1 K_2 \phi_2}{I_2}$$

Now $$\phi_2 = \frac{N_2 I_2}{S}$$

i.e. $$M = \frac{N_1 K_2 N_2 I_2}{I_2 S}$$

$$\therefore \quad \boxed{M = \frac{K_2 N_1 N_2}{S}}$$

- If entire flux produced by coil B_2 links with coil 1, $K_2 = 1$ hence,

$$\therefore \quad \boxed{M = \frac{N_1 N_2}{S}}$$

1.25.2 Coefficient of Coupling or Magnetic Coupling Coefficient (K)

- We know that,

$$M = \frac{N_2 K_1 \phi_1}{I_1} \quad \text{and} \quad M = \frac{N_1 K_2 \phi_2}{I_2}$$

- Multiplying the two expressions of M,

$$M \times M = \frac{N_2 K_1 \phi_1}{I_1} \times \frac{N_1 K_2 \phi_2}{I_2}$$

i.e. $$M^2 = K_1 K_2 \left(\frac{N_1 \phi_1}{I_1}\right)\left(\frac{N_2 \phi_2}{I_2}\right)$$

- But $\frac{N_1 \phi_1}{I_1}$ = Self inductance of coil 1 = L_1

and $\frac{N_2 \phi_2}{I_2}$ = Self inductance of coil 2 = L_2

$$\therefore \quad M^2 = K_1 K_2 L_1 L_2$$

$$M = \sqrt{K_1 K_2} \cdot \sqrt{L_1 L_2} = K\sqrt{L_1 L_2}$$

where $K = \sqrt{K_1 K_2}$

- *The K is called **coefficient of coupling**.*
- *If entire flux produced by one coil links with other then*

$K = K_1 = K_2 = 1$ and maximum mutual inductance existing between the coil is $M = K\sqrt{L_1 L_2}$.

- *This gives an idea about magnetic coupling between the two coils. When entire flux produced by one coil links with other, this coefficient is maximum i.e. Unity.*
- *It can be defined as the ratio of the actual mutual inductance present between the two coils to the maximum possible value of the mutual inductance.*

$$\therefore \quad \boxed{K = \frac{M}{\sqrt{L_1 L_2}}}$$

- Depending on the value of K, the coils are classified as,

1. Tightly coupled 2. Loosely coupled

Key Point *When K = 1 coils are said to be tightly coupled and if K is a fraction the coils are said to be loosely coupled.*

Ex. 1.25.1 : ***If a coil of 150 turns carries a current of 10 A and flux linked with it is 0.01 Wb, then calculate the inductance of the coil. If the current is uniformly reversed in 0.1 sec., calculate the e.m.f. induced. If a second coil of 100 turns is uniformly wound over the first coil, find the mutual inductance between the coils.*** **SPPU : Dec.-12, May-16, Marks 6**

Sol. : Given : N = 150, I = 10 A, ϕ = 0.01 Wb

To find : e, M

$$\therefore \quad L = \frac{N\phi}{I} = 0.15 \text{ H}$$

Current reversed hence $i_2 = -10$ A, $i_1 = 10$ A

$$\therefore e = -L\frac{di}{dt} = \frac{-L\propto[i_2 - i_1]}{dt} = -\frac{0.15[-10-(-10)]}{0.1} = 30 \text{ V}$$

$$N_2 = 100$$

but $\phi_2 = \phi = 0.01$ Wb, $I_1 = I = 10$ A

$$\therefore \quad M = \frac{N_2 \phi_2}{I_1} = \frac{100 \times 0.01}{10} = 0.1 \text{ H}$$

Ex. 1.25.2 : ***A coil of 100 turns having a mean diameter of 5 cm is placed coaxially at the centre of a solenoid of 50 cm long with 2500 turns and carrying current of 3 A. Calculate the mutual inductance between the two coils.*** **SPPU : Dec.-08, Marks 6**

Sol. : N_1 = 2500 (solenoid),

$N_2 = 100$, $l_1 = 50$ cm = 0.5 m, $I_1 = 3$ A

Now magnetic field strength H at the centre of coil due to solenoid current is

$$H = \frac{N_1 I_1}{l_1} = \frac{2500 \times 3}{0.5} = 15000 \text{ AT/m}$$

$\therefore$ Flux density at centre is,

$$B = \mu_0 H \; (\mu_r = 1)$$

$$\therefore \quad B = 4\pi \times 10^{-7} \times 15000 = 0.01885 \text{ Wb/m}^2$$

$\therefore$ Flux linking with second coil is,

$$\phi_2 = B \times a_2 = 0.01885 \times \frac{\pi}{4} \times (d_2)^2$$

$$= 0.01885 \times \frac{\pi}{4} \times (5 \times 10^{-2})^2 = 3.7011 \times 10^{-5} \text{ Wb}$$

$\therefore$ Mutual inductance between the coils is,

$$M = \frac{N_2 \phi_2}{I_1} = \frac{100 \times 3.7011 \times 10^{-5}}{3} = 1.2337 \times 10^{-3} \text{ H}$$

Ex. 1.25.3 : *Two coils, X of 12000 turns and Y of 15000 turns lie in parallel planes such that 60% of the flux produced by coil X links coil Y. A current of 5 A in coil X produces a flux of 0.05 mWb, while the same current in coil. Y produces a flux of 0.075 mWb. Find : i) self-inductance of each coil ii) mutual inductance iii) coefficient of coupling*

SPPU : Dec.-04, 11, 18, Marks 7

Sol. : Given : $N_X = 12000$, $N_Y = 15000$, $I_X = 5$ A, $\phi_X = 0.05$ mWb, $I_Y = 5$ A, $\phi_Y = 0.075$ mWb

To find : L_X, L_Y, M, K

i) $$L_X = \frac{N_X \phi_X}{I_X} = \frac{12000 \times 0.05 \times 10^{-3}}{5} = 0.12 \text{ H}$$

$$L_Y = \frac{N_Y \phi_Y}{I_Y} = \frac{15000 \times 0.075 \times 10^{-3}}{5} = 0.225 \text{ H}$$

ii) $$\phi_Y = 0.6\, \phi_X$$

$$\therefore \quad M = \frac{N_Y \phi_Y}{I_X} = \frac{N_Y (0.6 \phi_X)}{I_X}$$

$$= \frac{15000 \times 0.6 \times 0.05 \times 10^{-3}}{5} = 0.09 \text{ H}$$

iii) $$K = \frac{M}{\sqrt{L_X L_Y}} = \frac{0.09}{\sqrt{0.12 \times 0.225}} = 0.548$$

Ex. 1.25.4 : *Two coils having turns 1000 and 1500 are placed on common magnetic circuit. A current of 5A in first coil produces a flux of 0.2 mWb and 80% of this flux links to coils 2. Find i) Self inductance of coil 1.*
ii) Mutual inductance between them.
iii) If this current in first coil is interrupted in 0.01 sec. find emf induced in coil 1 and 2.

Sol. : Given : $N_1 = 1000$; $N_2 = 1500$, $I_1 = 5$ A, $\phi_1 = 0.2 \times 10^{-3}$ Wb, $K_1 = 0.8$

To find : L_1, M, e_{L1}, e_{M2}

i) Self inductance of coil 1

$$L_1 = \frac{N_1 \phi_1}{I_1} = \frac{1000 \times 0.2 \times 10^{-3}}{5} = 0.04 \text{ H}$$

ii) Mutual inductance

$$M = \frac{N_2 K_1 \phi_1}{I_1} = \frac{1000 \times 0.8 \times 0.2 \times 10^{-3}}{5} = 0.048 \text{ H}$$

iii) Self induced emf in coil 1

$$dI = 0 - 5 = -5 \text{ A}$$

$$e_{L1} = -L_1 \frac{dI}{dt} = \frac{-0.04 \times -5}{0.01} = 20 \text{ V}$$

iv) Mutually induced emf in coil 2

$$e_{M2} = -M \frac{dI_1}{dt} = \frac{-0.048 \times -5}{0.01} = 24 \text{ V}$$

Ex. 1.25.5 : *If a coil of 150 turns is linked with a flux of 0.015 Wb. when carrying a current of 10 A. If now second coil of 100 turn is uniformly wound over the first coil find mutual inductance M. If this current in coil 1 is uniformly reversed in 0.015 sec. find emf induced in coil 1 and coil 2.*

Sol. : Given : $N_1 = 150$, $N_2 = 100$, $\phi = 0.015$, $I_1 = 10$ A

To find : L_1, M, e_{L1}, e_{M2}, e_{M1}

i) $$L_1 = \frac{N_1 \phi_1}{I_1} = \frac{150 \times 0.015}{10} = 0.225 \text{ H}$$

Here assume that coils are tightly coupled K = 1

ii) $$M = N_2 \frac{\phi_1}{I_1} = \frac{100 \times 0.015}{10} = 0.15 \text{ H}$$

iii) When current in uniformely reversed

$$dI = -10 - (10) = -20 \text{ A}, \quad dt = 0.015 \text{ sec.}$$

Self induced emf in coil 1

$$e_{L1} = -L_1 \frac{dI_1}{dt} = -0.225 \times \frac{-20}{0.015} = 300 \text{ V}$$

iv) Mutually induced emf in coil 2

$$e_{M2} = -M \frac{dI_1}{dt} = -0.15 \times \frac{-20}{0.015} = 200 \text{ V}$$

v) If the current in second coil charges at the rate of −50 A/sec, find emf induced in coil 1 by mutual induction.

$$e_{M1} = -M\frac{dI_2}{dt} = -0.15 \times \frac{-50}{1} = 7.5 \text{ V}$$

Ex. 1.25.6 : ***Two windings connected in series are wound on a ferromagnetic ring having cross-sectional a rea of 750 mm² and a mean diameter of 175 mm. The two windings have 250 and 750 turns, while the relative permeability of material is 1500. Assuming no leakage of flux, calculate the self inductances of each winding and the mutual inductance as well. Calculate e.m.f. induced in coil 2 if current is coil 1 in increased uniformly from zero to 5 A in 0.01 sec.***

SPPU : Dec.-01, Marks 8

Sol. : l = Length of magnetic circuit = $\pi \times d_{mean}$

$$l = \pi \times 175 \times 10^{-3} = 0.5497 \text{ m}$$

$$a = 750 \text{ mm}^2 = 750 \times 10^{-6} \text{ m}^2$$

$$= 7.5 \times 10^{-4} \text{ m}^2$$

$$N_1 = 250,\ N_2 = 750,\ \mu_r = 1500$$

Self inductance, $L = \frac{N\phi}{I}$ but $\phi = \frac{NI}{S}$

$$\therefore \quad L = \frac{N\,NI}{IS} = \frac{N^2}{S}$$

We have, $S = \frac{l}{\mu a}$

$$\therefore \quad S = \frac{l}{\mu_0 \mu_r a} = \frac{0.5497}{(4\pi \times 10^{-7})(1500)(7.5 \times 10^{-4})}$$

$$= 388833.2 \text{ AT/Wb}$$

$$\therefore \quad L_1 = \frac{N_1^2}{S} = \frac{(250)^2}{388833.2} = \mathbf{0.1607\ H}$$

$$\therefore \quad L_2 = \frac{N_2^2}{S} = \frac{(750)^2}{388833.2} = \mathbf{1.4466\ H}$$

The mutual inductance between the two windings is given by,

$$M = \frac{N_1 N_2}{S} = \frac{(250)(750)}{38883320} = \mathbf{0.4822\ H}$$

$$\therefore \quad M = \mathbf{0.4822\ H}$$

E.M.F. induced in coil 2 is,

$$e_2 = -M\frac{dI_1}{dt} = -0.4822 \times \frac{(5-0)}{0.01} = \mathbf{-241.1\ V}$$

Expected Questions

1. *Explain the phenomenon of mutually induced e.m.f.* **SPPU : May-10, Marks 4**
2. *Define mutual inductance and state its unit. Derive the expression for mutual inductance.* **SPPU : May-06, Marks 3**
3. *Derive the expression for the magnitude of mutually induced e.m.f.*
4. *Define coefficient of coupling and obtain the relation between self inductances, mutual inductance and coefficient of coupling.* **SPPU : Dec.-04,15, May-07,08,10,16,17,19, Marks 6**
5. *State the various expressions for the mutual inductance.*

1.26 : Energy Stored in a Magnetic Field

SPPU : May-06,07,09,10,11,13,18 Dec.-06,07,09,10,11,12,13

- The energy is required to establish the flux i.e. magnetic field but it is not required to maintain it. This is similar to the fact that the energy is required to raise the water through a certain height but the energy is not required to maintain the water at that height.

> *The energy required to establish magnetic field gets stored into it as a potential energy. This energy can be recovered when magnetic field established gets collapsed.*

- Consider a solenoid, the current through which can be controlled with the help of switch S, resistance R.
- Initially switch 'S' is open, so current through coil, I is zero. When switch is closed, current will try to built its value equal to I. Neglect the resistance of coil.
- It will take some time to increase the current from 'zero' to 'I' say 'dt' seconds. Hence e.m.f. will be induced in a coil.
- Let the induced e.m.f. in a coil be, $e = -L\frac{dI}{dt}$
- This opposes a supply voltage. So supply voltage 'V' supplies energy to overcome this, which ultimately gets stored in the magnetic field.

$$\therefore \quad V = -e = -\left[-L\frac{dI}{dt}\right] = L\frac{dI}{dt}$$

$$\therefore \quad \text{Power supplied} = V \times I = L\frac{dI}{dt} \times I$$

$\therefore$ Energy supplied in time dt is,

$$E = \text{Power} \times \text{Time} = L\frac{dI}{dt} \times I \times dt = L\,dI \times I \text{ joules.}$$

- This is energy supplied for change in current of dI but actually current changes from zero to I.

$\therefore$ Integrating above, the total energy stored is,

$$E = \int_0^I L\,dI\,I = L\int_0^I dI\,I = L\left[\frac{I^2}{2}\right]_0^I = L\left[\frac{I^2}{2} - 0\right]$$

$$\therefore \quad \boxed{E = \frac{1}{2}LI^2 \text{ joules}}$$

1.26.1 Energy Stored per Unit Volume

$$E = \frac{1}{2}LI^2 \text{ joules}$$

Now $\quad L = \dfrac{N\phi}{I}$

hence $\quad E = \dfrac{1}{2}\dfrac{N\phi}{I}I^2 \text{ joules} = \dfrac{1}{2}N\phi I \text{ joules}$

Now $\quad NI = Hl$ ampere-turns and $\phi = Ba$

$\therefore \quad E = \dfrac{1}{2}BaHl$

But $\quad a \times l = \text{Area} \times \text{Length} = \text{Volume of magnetic circuit}$

$\therefore$ Energy stored per unit volume is $= \dfrac{1}{2}BH$ but $B = \mu H$

$$\therefore \quad \boxed{\text{Energy per unit volume} = \frac{1}{2}\mu H^2 = \frac{1}{2}\frac{B^2}{\mu} \text{ joules/m}^3 \quad \text{where } \mu = \mu_0\mu_r}$$

- In case of inductive circuit when circuit is opened with the help of switch then current decays and finally becomes zero. In such case energy stored is recovered and if there is resistance in the circuit, appears in the form of heat across the resistance.
- While if the resistance is not present then this energy appears in the form of an arc across the switch, when switch is opened.
- If the medium is air, $\mu_r = 1$ hence $\mu = \mu_0$ must be used in the above expressions of energy.

Ex. 1.26.1 : ***A coil is wound on an iron core to form a solenoid. A certain current is passed through the coil which is producing a flux of 40 μWb. The length of magnetic circuit is 75 cm while its cross-sectional area is 3 cm². Calculate the energy stored in the circuit. Assume relative permeability of iron as 1500.***

Sol. : Given : $l = 75$ cm $= 0.75$ m, $a = 3$ cm² $= 3\times10^{-4}$ m², $\phi = 40$ μWb $= 40\times10^{-6}$ Wb, $\mu_r = 1500$

To find : E

$$\therefore \quad B = \frac{\phi}{a} = \frac{40\times10^{-6}}{3\times10^{-4}} = 0.133 \text{ Wb/m}^2$$

$\therefore$ Energy stored per unit volume,

$$\frac{1}{2}\frac{B^2}{\mu} = \frac{1}{2}\frac{B^2}{\mu_0\mu_r} = \frac{1}{2}\frac{(0.133)^2}{4\pi\times10^{-7}\times1500} = 4.7157 \text{ J/m}^3$$

$\therefore$ Total energy stored = Energy per unit volume × Volume = $E\times(a\times l) = 4.7157\times(3\times10^{-4}\times0.75) =$ **0.00106 joules**

Ex. 1.26.2 : *An air cored toroid having 25 cm mean diameter and 6.26 cm^2 circular cross sectional area wound uniformly with 1000 turns of wire. Determine*
i) Inductance of toroid,
ii) emf induced in the coil when current is increasing at the rate of 200 A/sec.
iii) Energy stored in its magnetic field when coil carries current of 10 A. **SPPU : May-07, 10, 18, Marks 6**

Sol. : Given : $A = 6.26\,cm^2$, $d = 25$ cm, $N = 1000$

$$\therefore \quad l = \pi \times d = \pi \times 25 = 78.54 \text{ cm.}$$

To find : L, e, E

i) $S = \dfrac{l}{\mu_0 A} = \dfrac{78.54 \times 10^{-2}}{4\,p \times 10^{-7} \times 6.26 \times 10^{-4}}$ ...$\mu_r = 1$ for air

$$= 998.4048 \times 10^6 \text{ AT/Wb}$$

$$\therefore \quad L = \frac{N^2}{S} = \frac{(1000)^2}{998.4048 \times 10^6} = \mathbf{1\ mH}$$

ii) $\dfrac{dI}{dt} = 200$ A/sec

$$\therefore \quad e = -L\frac{dI}{dt} = 1 \times 10^{-3} \times 200 = \mathbf{-0.2\ V}$$

Negative sign indicates that e opposes the increase in current.

iii) $E = \dfrac{1}{2}LI^2 = \dfrac{1}{2} \times 1 \times 10^{-3} \times 10^2 = \mathbf{0.05\ J}$

Ex. 1.26.3 : *An iron ring of 10 cm in diameter and 8 cm^2 in cross-section is wound with 300 turns of wire. For a flux density of 1.2 Wb/m^2 and relative permeability of 500, find the exciting current, the inductance and the energy stored.* **SPPU : May-07, Marks 6**

Sol. : Given : $d = 10$ cm, $a = 8\ cm^2$, $N = 300$, $B = 1.2\ Wb/m^2$, $\mu_r = 500$

To find : I, L, E

$$l = \pi \times d = \pi \times 10 \text{ cm} = 0.3141 \text{ m}$$

$$S = \frac{l}{\mu_0 \mu_r a} = \frac{0.3141}{4p \times 10^{-7} \times 500 \times 8 \times 10^{-4}}$$

$$= 624.882 \times 10^3 \text{ AT/Wb}$$

$$\phi = B \times a = 1.2 \times 8 \times 10^{-4} = 9.6 \times 10^{-4} \text{ Wb}$$

$$\phi = \frac{NI}{S}$$

$$\therefore \quad 9.6 \times 10^{-4} = \frac{300 \times I}{624.882 \times 10^3}$$

$$\therefore \quad I = \mathbf{2\ A}$$

$$L = \frac{N^2}{S} = \frac{(300)^2}{624.882 \times 10^3} = \mathbf{0.14402\ H}$$

$$\therefore \quad E = \frac{1}{2}LI^2 = \frac{1}{2} \times 0.14402 \times (2)^2 = \mathbf{0.288\ J}$$

Expected Questions

1. *Derive an expression for the energy stored in a magnetic field.* **SPPU : Dec.-09, 10, Marks 6**
2. *Obtain the expression for the energy stored per unit volume in an inductor.*

SPPU : May-06, 09, 10, 11, 13, Dec.-06, 07, 11, 12, 13, Marks 6

Formulae At a Glance

- Magnetic flux density, $B = \dfrac{\phi}{a}$ $\dfrac{Wb}{m^2}$ or Tesla
- Magnetic field strength,
  $H = \dfrac{\text{Ampere turns}}{\text{Length}} = \dfrac{NI}{l}$ AT/m
- M. M. F. = N I ampere turns
- Reluctance, $S = \dfrac{l}{\mu a} = \dfrac{l}{\mu_0 \mu_r a}$ A/Wb or AT/Wb

$\therefore$ Reluctance $= \dfrac{\text{m.m.f}}{\text{flux}}$ i.e. $S = \dfrac{NI}{\phi}$ AT/Wb or A/Wb

- Permeance $= \dfrac{1}{\text{Reluctance}}$
- Permeability, $\mu = \dfrac{B}{H}$ i.e. $B = \mu H$

$\therefore$ $\mu_0 = \dfrac{B}{H}$ in vacuum $= 4\pi \times 10^{-7}$ H/m

- **The ratio B to H is constant only for free space, vacuum or air.**
- $\mu = \mu_0 \mu_r$ H/m where μ_r = Relative permeability
- The force experienced by current carrying conductor placed in a magnetic field is,

$$F = B\,I\,l \sin\theta \text{ Newtons}$$

where θ is angle between the current carrying conductor and the plane of flux.

- Induced e.m.f., $e = -N\dfrac{d\phi}{dt}$ volts
- Magnitude of dynamically induced e.m.f. is,

$$e = B\,l\,v \sin\theta \text{ volts}$$

where θ is measured with respect to plane of flux

- Self inductance, $L = \frac{N\phi}{I}$
- Self induced e.m.f., $e = -L\frac{dI}{dt}$ volts
- $L = \frac{N \cdot NI}{I \cdot S} = \frac{N^2}{S} = \frac{N^2 \mu a}{l} = \frac{N^2 \mu_0 \mu_r a}{l}$ henries
- Mutual inductance, $M = \left(\frac{N_2 \phi_2}{I_1}\right)$ Henries and

$e_2 = -M\frac{dI_1}{dt}$ volts

$M = \frac{N_2 K_1 \phi_1}{I_1} = \frac{K_1 N_1 N_2}{S} = \frac{N_1 K_2 \phi_2}{I_2}$

$M = \frac{N_1 N_2}{S} = \frac{N_1 N_2}{\left(\frac{l}{\mu a}\right)} = \frac{N_1 N_2 a \mu}{l}$

$= \frac{N_1 N_2 a \mu_0 \mu_r}{l}$ For K = 1

- Coefficient of coupling, $K = \frac{M}{\sqrt{L_1 L_2}}$
- Energy stored in magnetic field, $E = \frac{1}{2}L I^2$ Jjoules
- Energy per unit volume $= \frac{1}{2}BH = \frac{1}{2}\mu H^2$

$= \frac{1}{2}\frac{B^2}{\mu}$ Joules / m^3 where $\mu = \mu_0 \mu_r$

Examples for Practice

Ex. 1 : *A coil is wound uniformly with 300 turns over a steel of relative permiability 900, having a mean circumference of 40 mm and cross-sectional area of 50 mm^2. If a current of 5 A is passed through the coil, find*

i) m.m.f. ii) reluctance of the ring and

iii) flux **SPPU : Dec.-04**

[Ans. : 1500 AT, 70.7355×10^3 AT/Wb, 21.2057 mWb]

Ex. 2 : *An iron ring has its mean length of flux path as 60 cm and its cross-sectional areas as 15 cm^2. Its relative permeability is 500. Find the current required to be passed, through a coil of 300 turns wound uniformly around it, to produce a flux density of 1.2 tesla. What would be the flux density with the same current, if the iron ring is replaced by air-core ?* **SPPU : May-05**

[Ans. : 3.8197 A, 2.4×10^{-3} T or Wb/m^2]

Ex. 3 : *An iron ring of 20 cm mean diameter and 10 cm^2 cross-section is magnetised by a coil of 500 turns. The current through the coil is 8 A. The relative permeability of iron is 500. Find the flux density inside the ring .*

[Ans. : 4 Wb/m^2]

Ex. 4 : *An iron ring of 100 cm mean circumference is made from round iron of cross-section 10cm^2, it relative permeability is 800. If it is wound with 300 turns, what current is required to produce a flux of 1.1×10^{-3} Wb ?*

[Ans. 3.647 A]

Ex. 5 : *A coil of 300 turns and of resistance 10 Ω is wound uniformly over a steel ring of mean circumference 30 cm, and cross-sectional area 9 cm^2. It is connected to a supply at 20 V d.c. If the relative permeability of the ring is 1500, find :*

i) the magnetising force ; ii) the reluctance ;

iii) the m.m.f. ; and iv) the flux.

[Ans. : 600 AT, 176838.82 AT/Wb, 2000 AT/m, 3.3929 mWb]

Ex. 6 : *A coil is wound uniformly with 300 turns over a steel ring of relative permeability 900 having a mean circumference of 400 mm and cross-sectional area of 500 mm^2. If a current of 25 A is passed through the coil find*

i) m.m.f. ii) reluctance and iii) flux

[Ans. : 7500 AT, 707355.3 AT/Wb, 10.6 mWb]

Ex. 7 : *An iron ring has circular cross-section 4 cm in radius and the average circumference of 100 cm. The ring is uniformly wound with a coil of 700 turns. Calculate,*

i) Current required to produce a flux of 2 mWb in the ring, if relative permeability of the iron is 900.

ii) If a saw cut of 1mm wide is made in the ring, calculate the current which will give same flux as in part (i). Neglect leakage and fringing.

SPPU : Dec.-01, May-03, 06

[Ans. : 0.5025 A, 0.9545 A]

Ex. 8 : *An iron ring 8 cm mean diameter is made up of round iron of diameter 1 cm and permeability of 900, has an air gap of 2 mm wide. It consists of winding with 400 turns carrying a current of 3.5 A. Determine,*

i) m.m.f. ii) total reluctance iii) the flux iv) flux density in ring **SPPU : May-98, Dec.-99**

[Ans. : 1400 AT, 23.0737×10^6 AT / Wb, 6.067×10^{-5} Wb, 0.7725 Wb / m^2]

Ex. 9 : *An iron ring has a mean diameter of 20 cm and a uniform circular cross section of 2.5232 cm diameter with a small brass piece fitted of 1 mm length. Three coils are wound on the ring as shown in the Fig. 1 and carry identical d.c. current of 2 A. If the relative permeability of iron is 800, estimate :- i) the magnetic flux produced in air-gap, ii) self-inductance of the arrangement. iii) net m.m.f. in the ring.* **SPPU : May-01**

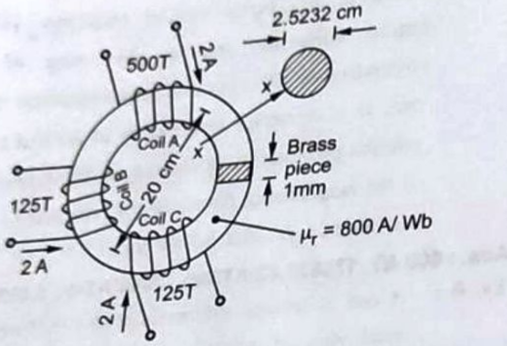


Fig. 1

[Ans. : 1500 AT, 5.282×10^{-4} Wb, 0.198 H]

Ex. 10 : *A magnetic circuit consists of two materials as shown in Fig. 2. The core has uniform cross-section of 6 cm^2. The core carries a winding with 900 turns. The current in the coil is 3 A. Calculate the flux produced in the air gap if the length of the air gap is 1 mm. Relative permeability of material A is 1000 and that for B is 1500. The length of the magnetic circuit for A is 80 cm and for B it is 50 cm.*

[Ans. : 0.9542 mWb]

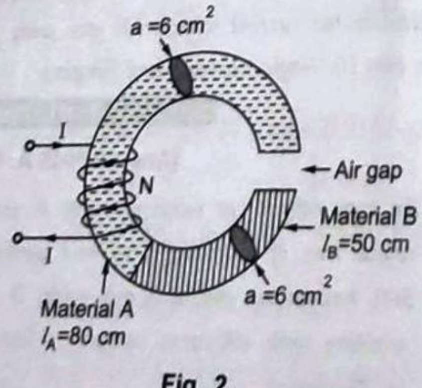


Fig. 2

Ex. 11 : *An iron ring of mean length 50 cm has air gap of 1 mm and a winding of 200 turns. If the relative permeability of iron is 300, find the flux density when a current of 1 amp flows through the coil.* **SPPU : Dec.-07**

[Ans. : 0.0943 Wb/m^2]

Ex. 12 : *An iron ring, cross sectional area of 5 cm^2 and mean length of 100 cm, has an air gap of 2 mm cut in it. Three separate coils having 100, 200 and 300 turns are wound on the ring and carry currents of 1 A, 2.5 A and 3 A respectively such that they produce additive fluxes in the ring. Relative permeability of the ring material is 1000. Calculate the flux in the air gap.* **SPPU : Dec.-98**

[Ans. : 0.3143 mWb]

Ex. 13 : *A ring shaped core is made up of two parts of same material. Part one is a magnetic path of length 25 cm and with cross sectional area 4 cm^2, whereas part two is of length 10 cm and cross sectional area of 6 cm^2. The flux density in part two is 1.5 Tesla. If the current through the coil, wound over core, is 0.5 amp., calculate the number of turns of coil. Assume μ_r is 1000 for material.* **SPPU : Dec.-05**

[Ans. : 1134]

Ex. 14 : *A cast steel structure is made of a rod of square section 2.5 cm ×2.5 cm as shown in the Fig. 3 What is the current that should be passed in a 500 turn coil on the left limb so that a flux of 2.5 mWb is made to pass in the right limb. Assume permeability as 750 and neglect leakage.*

[Ans. : 12.223 A]

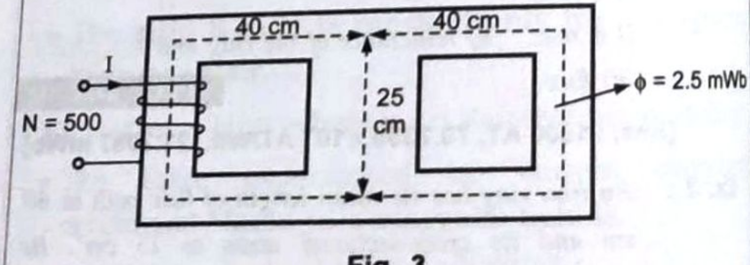


Fig. 3

Ex. 15 : *A ring of cast steel has an external diameter of 24 cm and internal diameter of 18 cm. The area of cross-section is 3 cm × 3 cm. Inside and across the ring an ordinary steel bar 18cm×3 cm × 0.4 cm is fitted with negligible air gap. Calculate the m.m.f. required to produce a flux density of*

1 Wb/m^2 in the other half ABD. Neglect leakage. The B-H curves are given below in table form. Refer the Fig. 4.

For cast steel ring

B Wb/m^2	1	1.193	1.4	1.6
H AT/m	900	800	1750	2000

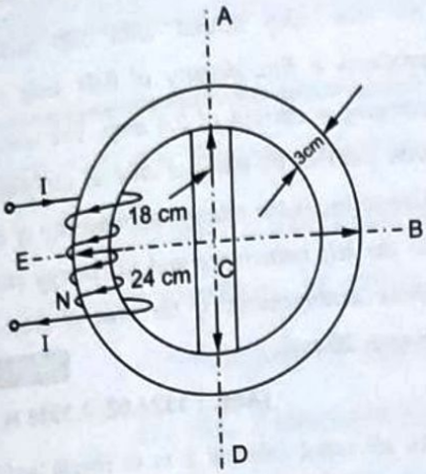


Fig. 4

For steel strip

B Wb/m^2	1.4	1.45	1.5	1.6
H AT/m	1200	1650	1700	2000

[Ans. : 692.706 AT]

Ex.16 : A cast iron ring of 40 cm mean length and circular cross section of 5 cm diameter is wound with a coil. The coil carries a current of 3 A and produces a flux of 3 mWb in the air gap. The length of the air gap is 2 mm. The relative permeability of the cast iron is 800. The leakage coefficient is 1.2. Calculate number of turns of the coil. **[Ans. : 702 turns]**

Ex. 17 : Find the inductance of a coil of 200 turns wound on a paper core tube of 25 cm length and 5 cm radius. **[Ans. : 1.579 mH]**

Ex. 18 : If a coil has 500 turns is linked with a flux of 50 mWb, when carrying a current of 125 A. Calculate the inductance of the coil. If this current is reduced to zero uniformly in 0.1 sec, calculate the self induced e.m.f. in the coil. **[Ans. : 250 volts]**

Ex. 19 : A coil is wound uniformly on an iron core. The relative permeability of the iron is 1400. The length of the magnetic circuit is 70 cm. The cross-sectional area of the core is 5 cm^2. The coil has 1000 turns. Calculate,

i) Reluctance of magnetic circuit

ii) Inductance of coil in henries.

iii) E.M.F. induced in coil if a current of 10 A is uniformly reversed in 0.2 seconds.

[Ans. : 7.957×10^5 AT / Wb, 1.2566 H, 125.66 volts]

Ex. 20 : Two coils A and B are kept in parallel planes, such that 70 % of the flux produced by coil A links with coil B . Coil A has 10,000 turns. Coil B has 12,000 turns. A current of 4 A in coil A produces a flux of 0.04 mWb while a current of 4 A in coil B produces a flux of 0.08 mWb. Calculate,

i) Self inductances L_A and L_B

ii) Mutual inductance M iii) Coupling coefficient.

[Ans. : 0.1 H, 0.24 H, 0.084 H, 0.5422]

Ex. 21 : Two coils having 3000 and 2000 turns are wound on a magnetic ring. 60 % of flux produced in first coil links with the second coil. A current of 3 A produces flux of 0.5 mWb in the first coil and 0.3 mWb in the second coil. Determine the mutual inductance and coefficient of coupling.

SPPU : Dec.-98

[Ans. : 0.2 H, 0.6324]

Ex. 22 : Two identical coils P and Q, each with 1500 turns, are placed in parallel planes near to each other, so that 70 % of the flux produced by current in coil P links with coil Q. If a current of 4 A is passed through any one coil, it produces a flux of 0.04 mWb linking with itself. Find the self inductances of the two coils, the mutual inductance and coefficient of coupling between them.

SPPU : Dec.-93

[Ans. : 15 mH, 15 mH, 10.5 mH, 0.7]

Ex. 23 : If a current of 5 A flowing in coil with 1000 turns wound on a ring of ferromagnetic material produces a flux of 0.5 mWb in the ring. Calculate i) self inductance of coil ii) e.m.f. induced in the coil when current is switched off and reaches zero value in 2 millisec. iii) mutual inductance between the coils, if a second coil with 750 turns is wound uniformly over the first one. **SPPU : May-03**

[Ans. : 0.1 H, 250 V, 0.075 H]

Ex. 24 : *Two coils having 1000 and 300 turns are wound on a common magnetic path with perfect magnetic coupling. The reluctance of the path is 3×10^6 AT/Wb. Find the mutual inductance between them. If the current in 1000 turns coil changes uniformly from 5 A to zero in 10 milliseconds, find the induced e.m.f. in the other coil.*

SPPU : Dec. -99, 2000

[Ans. : 0.1 H, 50 V]

Ex. 25 : *A coil of 200 turns having a mean diameter of 6 cm is placed coaxially at the centre of a solenoid of 50 cm long with 1500 turns and carrying current of 2.5 A. Calculate the mutual inductance between the two coils.* **[Ans. : 2.1318×10^{-3} H]**

Ex. 26 : *An iron cored toroid of relative permeability 980 has a mean length of 120 cm and core area of 100 mm². A current of 0.3 A establishes a flux of 40 μWb, calculate i) the number of turns of coil ii) self inductance iii) energy stored in magnetic field.*

[Ans. : 1300 turns, 0.1733 H, 7.8×10^{-3} joules]

Ex. 27 : *Length of an air cored solenoid is 1.7 m and area of cross-section is 12 cm². The number of turns of coil is 1000.*

Calculate : i) The self inductance. ii) The energy stored in magnetic field when a current of 10 A flows through the coil. **SPPU : Dec.-07**

[Ans. : 88.7 mH, 0.0443 J]

Ex. 28 : *An iron ring wound with 500 turns solenoid produces a flux density of 0.94 tesla in the ring carrying a current of 2.4 amp. The mean length of iron path is 80 cm and that of air gap is 1 mm. Determine i) the relative permeability of iron, ii) the self inductance and iii) energy stored in the above arrangement, if the area of cross-section of ring is 20 cm².* **SPPU : Dec.-05**

[Ans. : 1324.02, 0.3916 H, 1.1278 J]

Ex. 29 : *An air cored solenoid 1 m in length and 10 cm in diameter has 5,000 turns. Calculate :*

i) the self inductance and

ii) the energy stored in the magnetic field when current of 2 A flows in solenoid. **SPPU : Dec.-06**

[Ans. : 0.2467 H, 0.4934 J]

□□□