# Camera Calibration

Tutorial 1

Computer Vision CS7.505

January 30, 2024

## Outline-1

- 1. Homogeneous Co-ordinates
  - 1.1. Relation with Cartesian co-ordinates
  - 1.2. Infinities in HC
  - 1.3. Transforms
- 2. Calibration
  - 2.1. Camera Model
  - 2.2. Intrinsic Matrix
  - 2.3. Extrinsic Matrix
  - 2.4. DLT

# Homogeneous co-ordinates

• Push point to a higher dimensional space.

$$\begin{bmatrix} x \\ y \end{bmatrix} \equiv \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} kx \\ ky \\ k \end{bmatrix}$$
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \equiv \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} kx \\ ky \\ kz \\ k \end{bmatrix}$$

• WHY ?

## **Handling Infinities**

• Point at infinity

$$P_{\infty} = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$$

Represents a point at infinity in direction  $\begin{bmatrix} x \\ y \end{bmatrix}$ 

Translation

$$\begin{bmatrix} 1 & 0 & t_0 \\ 0 & 1 & t_1 \\ 0 & 0 & 1 \end{bmatrix}$$

Scaling/ Stretching (along axes

$$\begin{bmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

e Shear

$$\begin{bmatrix} 1 & s & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$egin{array}{cccc} r_{11} & r_{12} & 0 \ r_{21} & r_{22} & 0 \ 0 & 0 & 1 \ \end{array}$$

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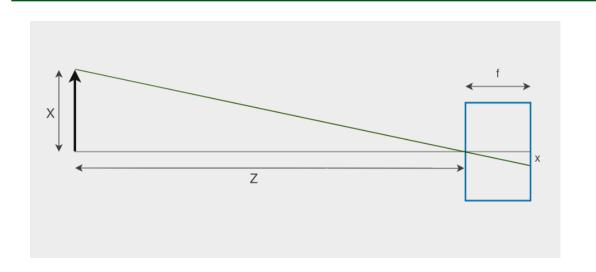
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$$\begin{bmatrix} 1 & s & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} r_{11} & r_{12} & 0 \\ r_{21} & r_{22} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

## **Outline-2**

- 1. Homogeneous Co-ordinates
  - 1.1. Relation with Cartesian co-ordinates
  - 1.2. Infinities in HC
  - 1.3. Transforms
- 2. Calibration
  - 2.1. Camera Model
  - 2.2. Intrinsic Matrix
  - 2.3. Extrinsic Matrix
  - 2.4. DLT



$$x = f_x \cdot \frac{X}{Z} \qquad \qquad y = f_y \cdot \frac{Y}{Z}$$

$$x = f_x \cdot \frac{X}{Z} + o_x \qquad \qquad y = f_y \cdot \frac{Y}{Z} + o_x$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} f_x & 0 & o_x & 0 \\ 0 & f_y & o_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

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$$x = f_x \cdot \frac{X}{Z}$$
  $y = f_y \cdot \frac{Y}{Z}$   $y = f_x \cdot \frac{X}{Z} + o_x$   $y = f_y \cdot \frac{Y}{Z} + o_y$ 

• As a transform ?

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & 0 & o_x & 0 \\ 0 & f_y & o_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}^{(i)}$$

$$M_{int} = \begin{bmatrix} f_x & 0 & o_x & 0 \\ 0 & f_y & o_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$x = f_x \cdot \frac{X}{Z}$$
  $y = f_y \cdot \frac{Y}{Z}$   $y = f_x \cdot \frac{X}{Z} + o_x$   $y = f_y \cdot \frac{Y}{Z} + o_y$ 

• As a transform ?

0

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & 0 & o_x & 0 \\ 0 & f_y & o_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}^{(C)}$$

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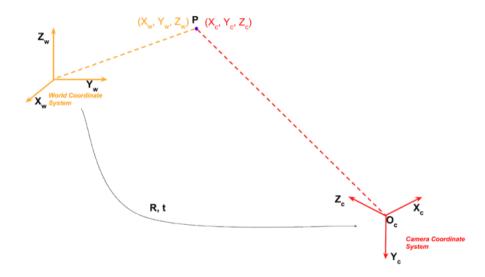
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•

$$M_{int} = \begin{bmatrix} f_x & 0 & o_x & 0 \\ 0 & f_y & o_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



• Transform from world co-ordinate system to camera co-ordinate system.

• just an affine transform!

$$\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}^{(C)} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}^{(W)}$$

$$M_{ext} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Transform from world co-ordinate system to camera co-ordinate system.
- just an affine transform !!

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$$\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}^{(C)} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}^{(W)}$$

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•

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•

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & 0 & o_x & 0 \\ 0 & f_y & o_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$
(C)

0

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & 0 & o_x & 0 \\ 0 & f_y & o_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}^{(W)}$$

$$x^{(I)} = M_{int} \cdot M_{ext} \cdot X^{(W)}$$

•

$$x^{(I)} = P \cdot X^{(W)}$$

•

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•

$$x^{(I)} = M_{int} \cdot M_{ext} \cdot X^{(W)}$$

•

$$x^{(I)} = P \cdot X^{(W)}$$

- Given pairs of points  $(X_i^W, x_i^{(I)})$  solve for P.
- We know

$$\begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}^{(I)} \equiv \begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \cdot \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix}^{(W)}$$

(

$$x_i = \frac{p_{11}X_i + p_{12}Y_i + p_{13}Z_i + p_{14}}{p_{31}X_i + p_{32}Y_i + p_{33}Z_i + p_{34}}$$
$$y_i = \frac{p_{21}X_i + p_{22}Y_i + p_{23}Z_i + p_{24}}{p_{31}X_i + p_{32}Y_i + p_{32}Z_i + p_{34}}$$

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$$x_{i} = \frac{p_{11}X_{i} + p_{12}Y_{i} + p_{13}Z_{i} + p_{13}}{p_{31}X_{i} + p_{32}Y_{i} + p_{33}Z_{i} + p_{33}}$$
$$y_{i} = \frac{p_{21}X_{i} + p_{22}Y_{i} + p_{23}Z_{i} + p_{23}}{p_{31}X_{i} + p_{32}Y_{i} + p_{33}Z_{i} + p_{34}}$$

- Given pairs of points  $(X_i^W, x_i^{(I)})$  solve for P.
- We know

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Alternatively...

$$\begin{bmatrix} X_i & 0 \\ Y_i & 0 \\ Z_i & 0 \\ 1 & 0 \\ 0 & X_i \\ 0 & X_i \\ 0 & Z_i \\ 0 & Z_i \\ 0 & 1 \\ -x_iX_i & -y_iX_i \\ -x_iY_i & -y_iY_i \\ -x_iZ_i & -y_iZ_i \\ -x_i & -y_i \end{bmatrix} \begin{bmatrix} p_{11} \\ p_{12} \\ p_{13} \\ p_{14} \\ p_{21} \\ p_{22} \\ p_{23} \\ p_{23} \\ p_{31} \\ p_{32} \\ p_{33} \\ p_{34} \end{bmatrix} =$$

- Need at least 6 points.
- SVD ?
- Direct Linear Transform!

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•

$$Ap = 0$$

ullet p has 11 degrees of freedom.

$$||p||^2 = 1$$

$$\min_{p} ||Ap||^2$$
 s.t.  $||p||^2 = 1$ 

$$\min_{p} p^{T} A^{T} A p \qquad s.t. \quad p^{T} p = 1$$

•

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We solve the Langrage Dual

$$\min_{p} p^{T} A^{T} A p - \lambda (p^{T} p - 1)$$

Equating gradient to zero:

$$2A^T A p - 2\lambda p = 0$$

p: eigen vector corresponding to the smallest Eigen value !

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## Points in a Plane

ullet What if A is rank deficient  $\ref{eq:continuous}$ ?

# Questions?