Camera Calibration

Tutorial 1

Computer Vision
CS7.505

January 30, 2024

Outline-1

Homogeneous co-ordinates

- Push point to a higher dimensional space.

$$\begin{bmatrix} x \\ y \end{bmatrix} \equiv \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} kx \\ ky \\ k \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \equiv \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} kx \\ ky \\ kz \\ k \end{bmatrix}$$

- WHY ?

Handling Infinities

- Point at infinity

$$P_\infty = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$$

Represents a point at infinity in direction $\begin{bmatrix} x \\ y \end{bmatrix}$

Transforms

- Translation

$$\begin{bmatrix} 1 & 0 & t_0 \\ 0 & 1 & t_1 \\ 0 & 0 & 1 \end{bmatrix}$$

- Scaling/ Stretching (along axes)

$$\begin{bmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Shear

$$\begin{bmatrix} 1 & s & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Change of basis

$$\begin{bmatrix} r_{11} & r_{12} & 0 \\ r_{21} & r_{22} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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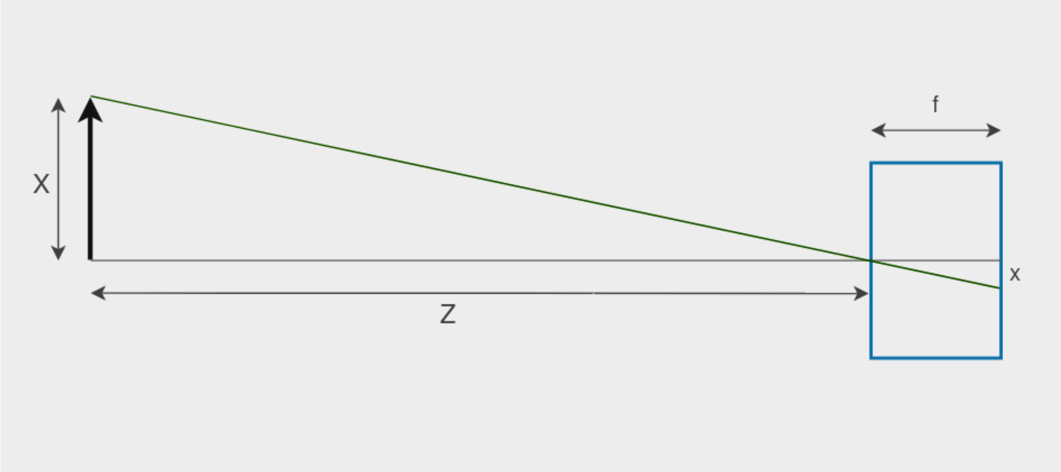
Outline-2

Camera Model



Intrinsic Matrix

$$x = f_x \cdot \frac{X}{Z} \qquad y = f_y \cdot \frac{Y}{Z}$$

$$x = f_x \cdot \frac{X}{Z} + o_x \qquad y = f_y \cdot \frac{Y}{Z} + o_y$$

- As a transform ?

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & 0 & o_x & 0 \\ 0 & f_y & o_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}^{(C)}$$

$$M_{int} = \begin{bmatrix} f_x & 0 & o_x & 0 \\ 0 & f_y & o_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

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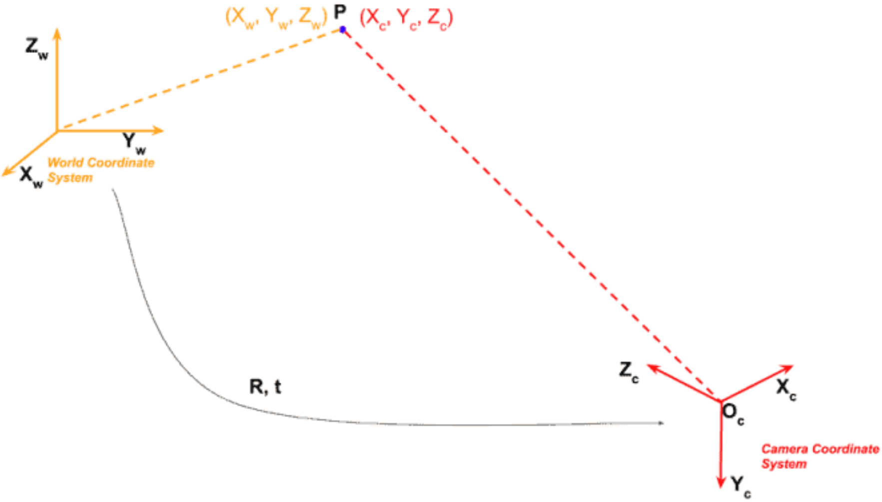
Extrinsic Matrix



Extrinsic Matrix

- Transform from world co-ordinate system to camera co-ordinate system.
- just an affine transform !!
- $$\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}^{(C)} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}^{(W)}$$
- $$M_{ext} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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Camera Model

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- $$x^{(I)} = M_{int} \cdot M_{ext} \cdot X^{(W)}$$

- $$x^{(I)} = P \cdot X^{(W)}$$

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Solving for P

- Given pairs of points $(X_i^W, x_i^{(I)})$ solve for P.
- We know

$$\begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}^{(I)} \equiv \begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \cdot \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix}^{(W)}$$

-

$$x_i = \frac{p_{11}X_i + p_{12}Y_i + p_{13}Z_i + p_{14}}{p_{31}X_i + p_{32}Y_i + p_{33}Z_i + p_{34}}$$

$$y_i = \frac{p_{21}X_i + p_{22}Y_i + p_{23}Z_i + p_{24}}{p_{31}X_i + p_{32}Y_i + p_{33}Z_i + p_{34}}$$

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Solving for P

- Alternatively...

$$\begin{bmatrix} X_i & 0 \\ Y_i & 0 \\ Z_i & 0 \\ 1 & 0 \\ 0 & X_i \\ 0 & Y_i \\ 0 & Z_i \\ 0 & 1 \\ -x_iX_i & -y_iX_i \\ -x_iY_i & -y_iY_i \\ -x_iZ_i & -y_iZ_i \\ -x_i & -y_i \end{bmatrix}^T \cdot \begin{bmatrix} p_{11} \\ p_{12} \\ p_{13} \\ p_{14} \\ p_{21} \\ p_{22} \\ p_{23} \\ p_{24} \\ p_{31} \\ p_{32} \\ p_{33} \\ p_{34} \end{bmatrix} = 0$$

Solving for P

- Need at least 6 points.
- SVD ?
- Direct Linear Transform !!

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DLT

- $$Ap = 0$$
- p has 11 degrees of freedom.
$$||p||^2 = 1$$
- $$\min_p ||Ap||^2 \qquad s.t. \quad ||p||^2 = 1$$
- $$\min_p p^T A^T Ap \qquad s.t. \quad p^T p = 1$$

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DLT

- $$\min_p p^T A^T A p \qquad s.t. \quad p^T p = 1$$
- We solve the Langrnge Dual:
$$\min_p p^T A^T A p - \lambda(p^T p - 1)$$
- Equating gradient to zero:
$$2A^T A p - 2\lambda p = 0$$
- p: eigen vector corresponding to the smallest Eigen value !!

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Points in a Plane

- What if A is rank deficient ??

Questions?