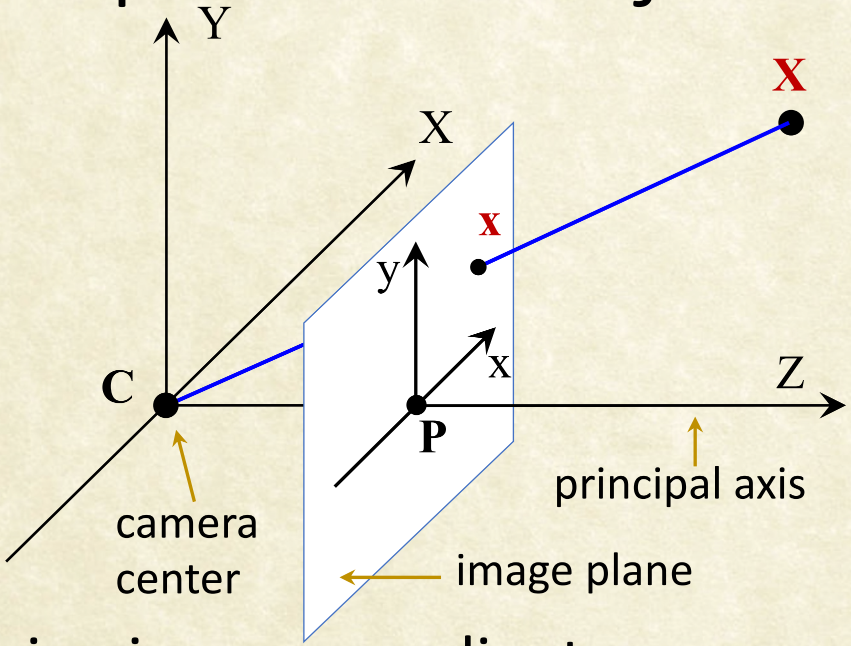
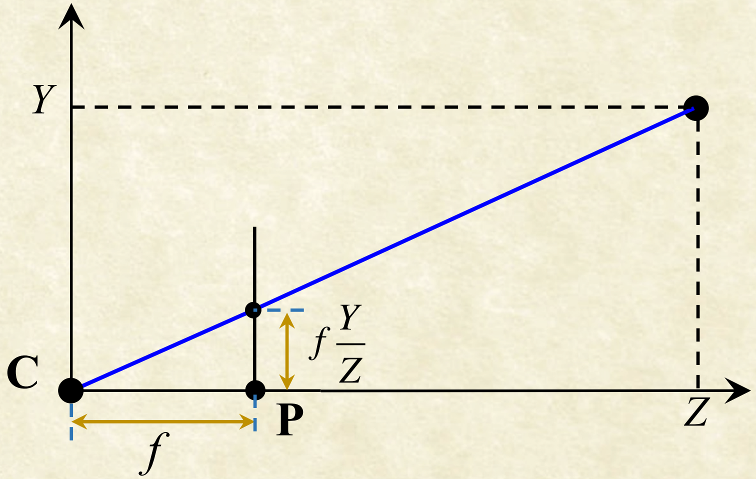
Perspective Projection





- Cartesian image coordinates:

$$x = f\frac{X}{Z}, \qquad y = f\frac{Y}{Z}$$

- In matrix form (homogeneous):

$$\mathbf{x} = \begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \mathbf{PX}$$

General Camera Equation

- Camera and world are related by: $\mathbf{X_c} = \begin{bmatrix} \mathbf{R} & \mathbf{-RC} \\ \mathbf{0} & \mathbf{1} \end{bmatrix} \mathbf{X_w}$
- 2D projection $\mathbf{x}$ of a 3D point $\mathbf{X_w}$ is given by: $\mathbf{x} = \mathbf{PX_w}$
- Camera matrix: $\mathbf{P} = [\mathbf{KR} \mid \mathbf{-KRC}] = [\mathbf{M} \mid \mathbf{p_4}]$
- Common K: General K:

$$\begin{bmatrix} f & 0 & x_0 \\ 0 & f & y_0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} \alpha_x & s & x_0 \\ 0 & \alpha_y & y_0 \\ 0 & 0 & 1 \end{bmatrix}$$

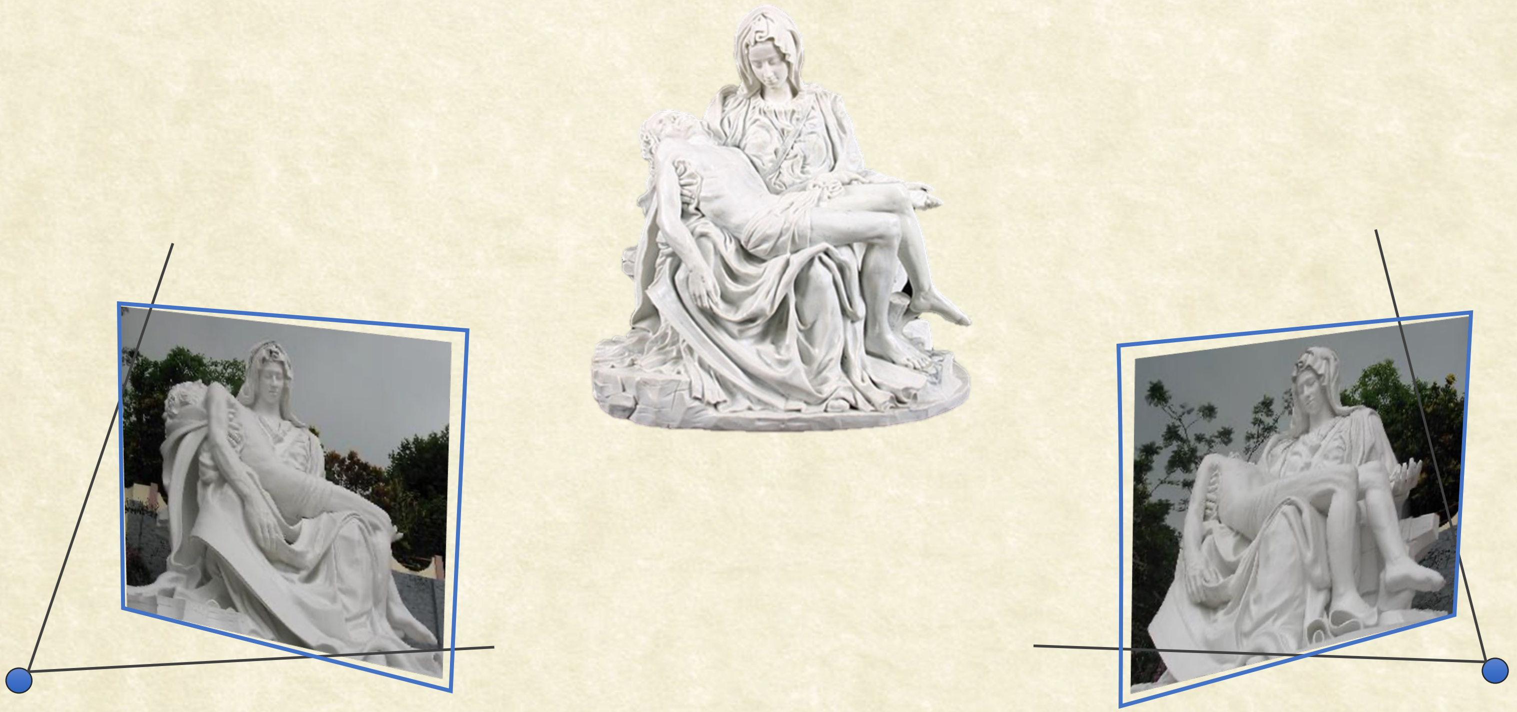
$$\boxed{\mathbf{x} = \mathbf{PX_w}}$$

Camera calibration (code walkthrough)

- LearnOpenCV
- https://learnopencv.com/camera-calibration-using-opencv/

CS7.505: Computer Vision

Spring 2024: Two View Geometry



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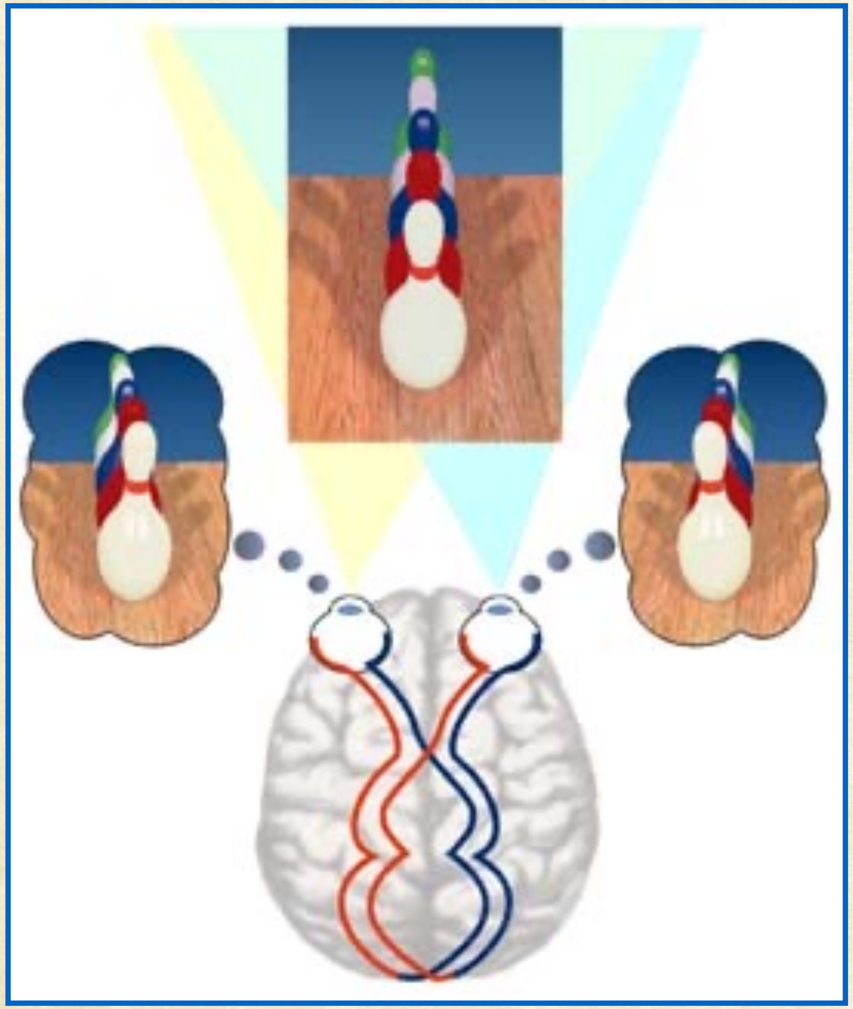
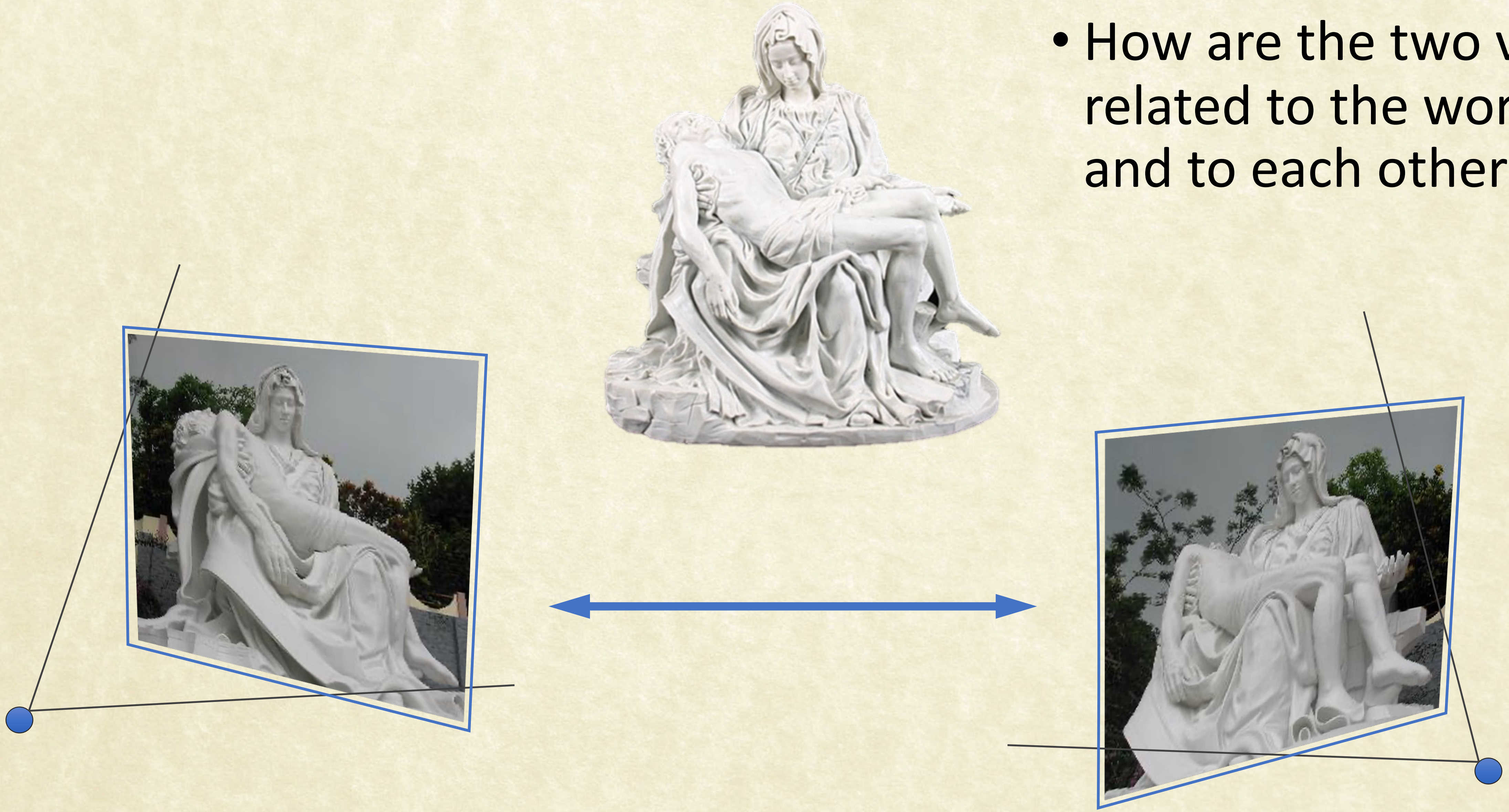
Human **stereo** vision



http://www.vision3d.com/stereo.html

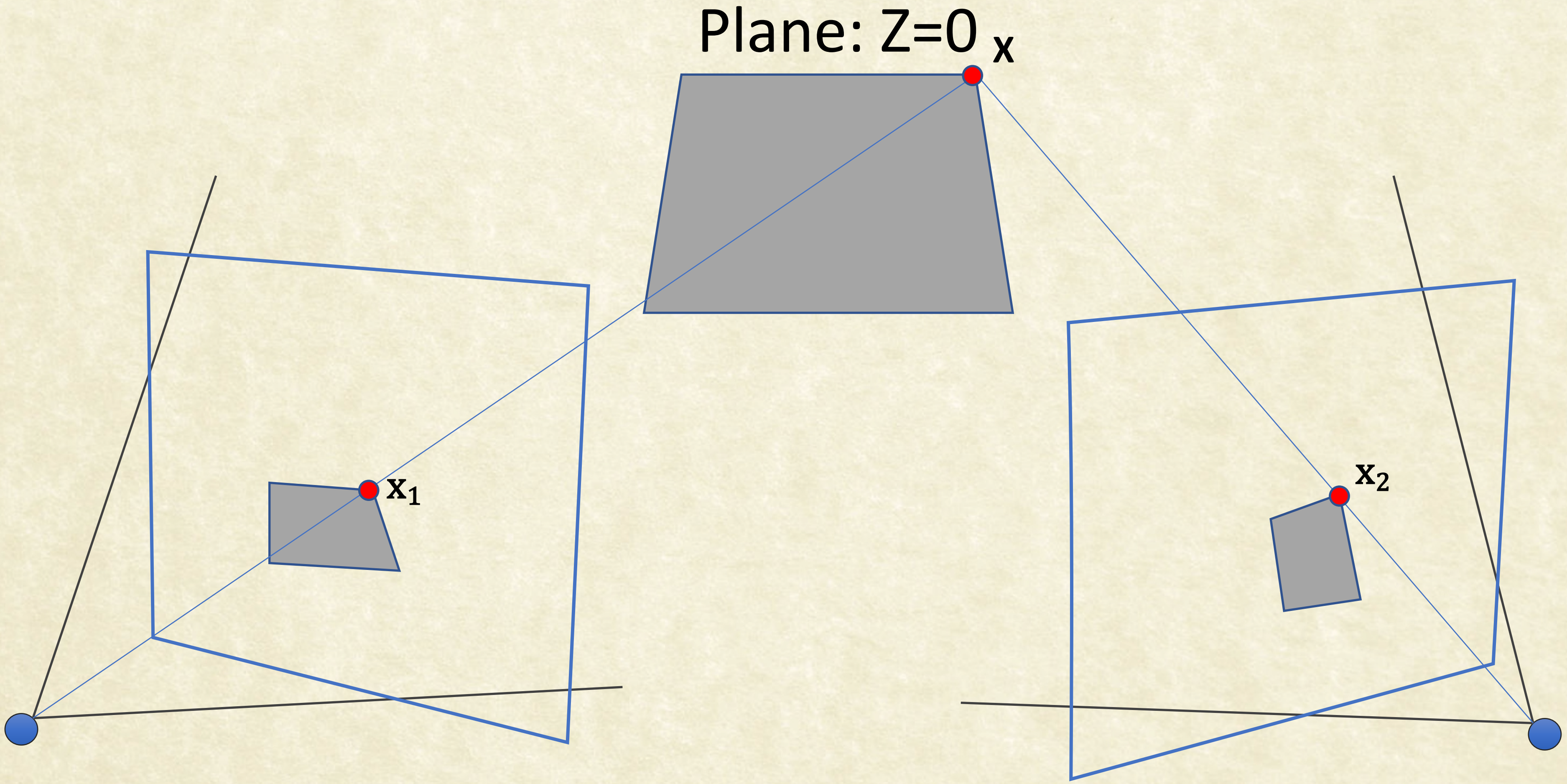
1. We see a slightly different image of the world through the two eyes.
2. The shift in image is proportional to the the distance to the object.
3. Roadside trees are left behind, while the farther mountains follow you.

Geometry of Two Views



- How are the two views related to the world and to each other?

Case 1: Planar World



What is the relation between $\mathbf{x}_1$ and $\mathbf{x}_2$?

Case 1: Planar World

- Projection equation of points on a plane:

$$\mathbf{x} = \mathbf{K}\begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix}\mathbf{X} = \mathbf{K}\begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{r}_3 & \mathbf{t} \end{bmatrix}\begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix} = \mathbf{K}\begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix}\begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = \mathbf{H}\mathbf{X},$$

where H is a 3 × 3 non-singular matrix

- Now if we consider two different views of the same world point, we get:

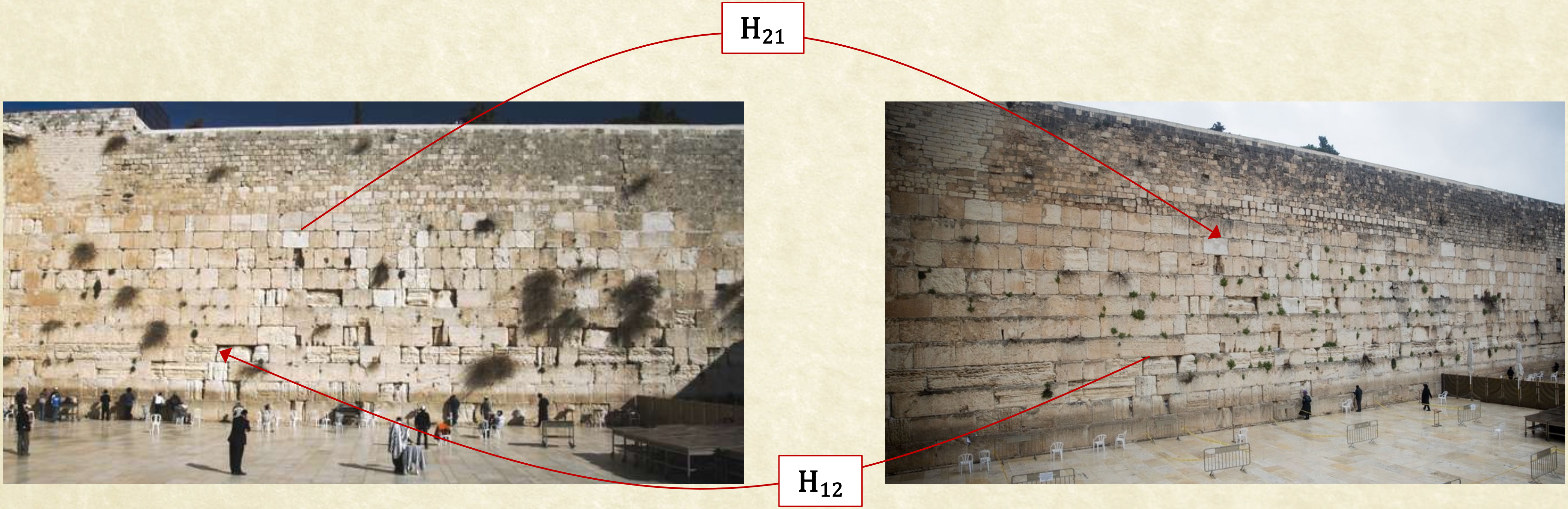
$$\mathbf{x}_1 = \mathbf{H}_1\mathbf{X} \qquad \mathbf{x}_2 = \mathbf{H}_2\mathbf{X}$$

$$\mathbf{x}_2 = \mathbf{H}_2\mathbf{H}_1{}^{-1}\mathbf{x}_1 = \mathbf{H}_{21}\mathbf{x}_1$$

$$\boxed{\mathbf{x}_1 = \mathbf{H}_{12}\mathbf{x}_2; \qquad \mathbf{x}_2 = \mathbf{H}_{21}\mathbf{x}_1}$$

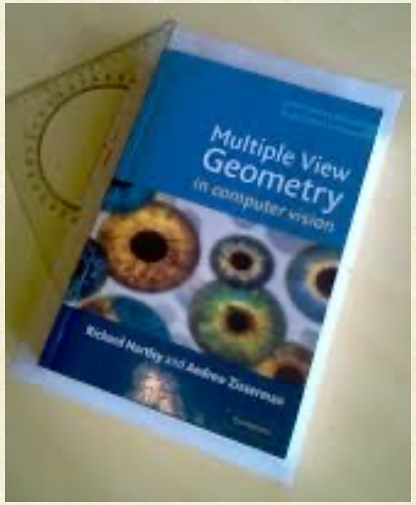
Planar Homography

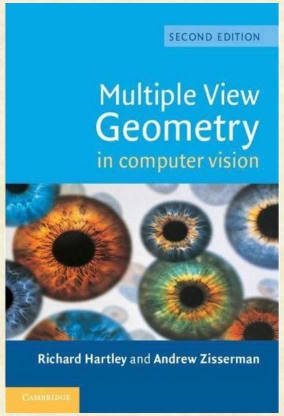
- Given two images of a planar world, every pixel in an image can be computed from the other.   Its location is given by $\mathbf{x}_a = \mathbf{H}\mathbf{x}_b$

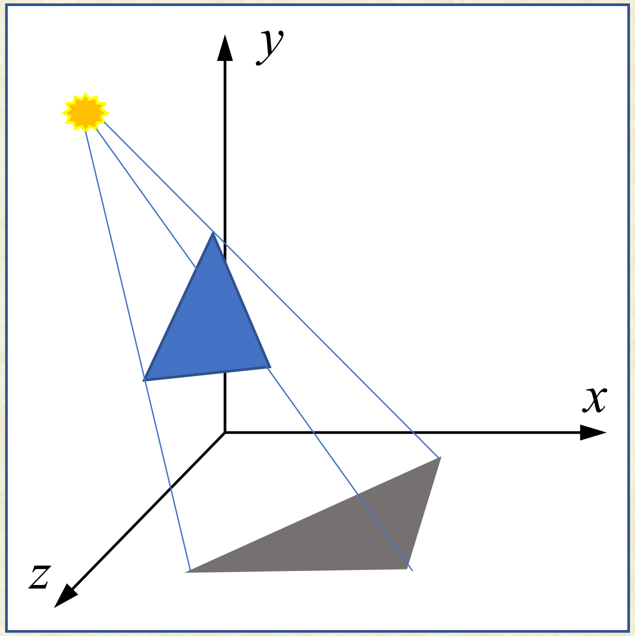


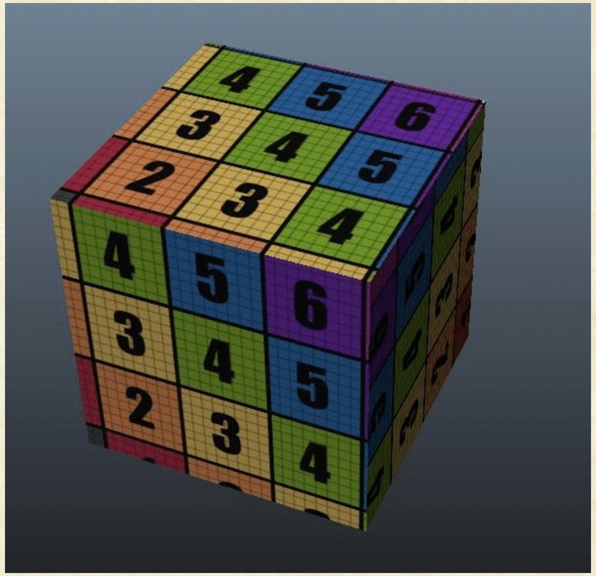
Planar Homography: Applications

- Removing perspective distortion
- Rendering planar textures
- Rendering planar shadows
- Estimating Camera Pose; AR

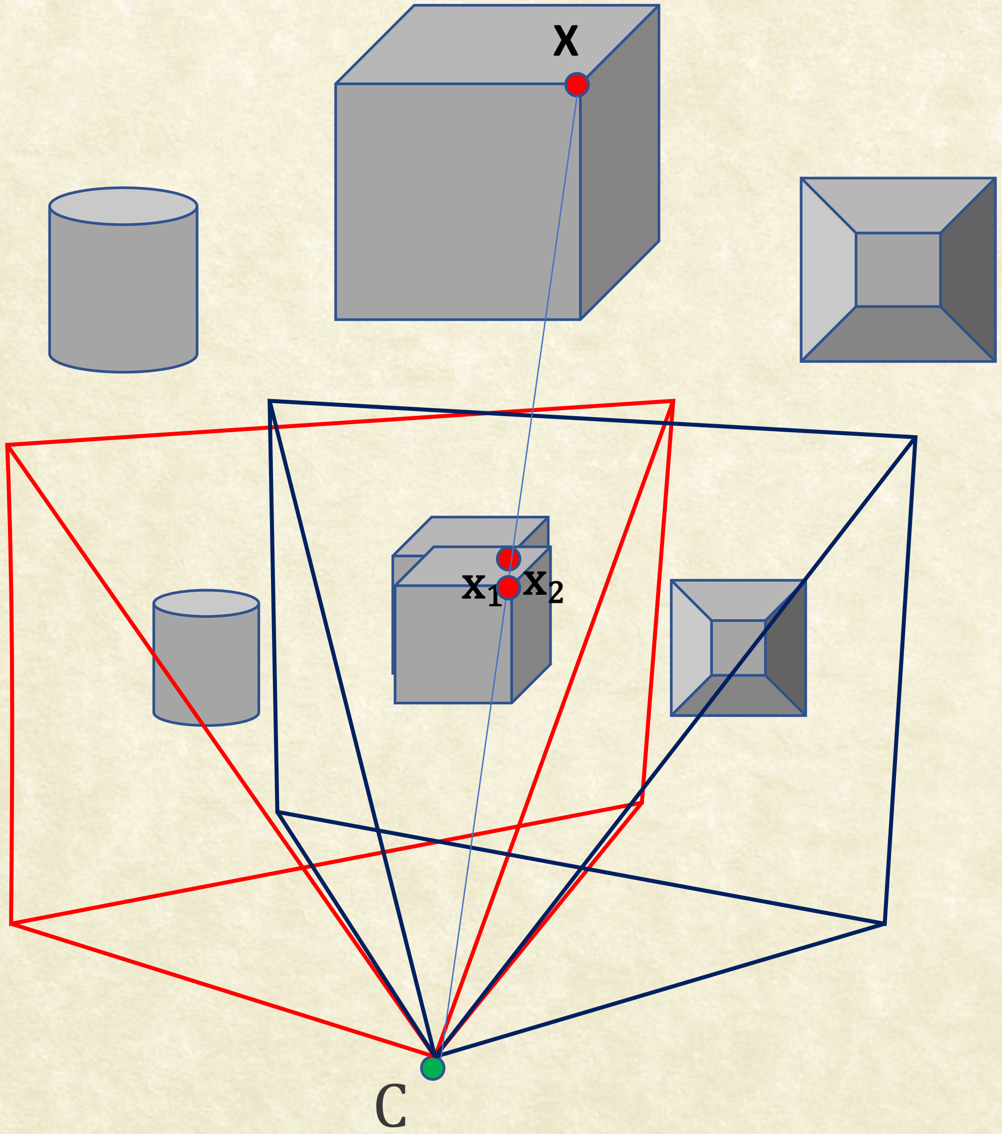








Case 2: Same Camera Center



Arbitrary world

What is the relation between $\mathbf{x}_1$ and $\mathbf{x}_2$?

Case 2: Same Camera Center

- Projection equation for two cameras with same C:

$$\mathbf{x}_1 = \mathbf{K}_1\mathbf{R}_1\begin{bmatrix} \mathbf{I} & \mathbf{-C} \end{bmatrix}\mathbf{X}$$

$$\begin{aligned}\mathbf{x}_2 &= \mathbf{K}_2\mathbf{R}_2\begin{bmatrix} \mathbf{I} & \mathbf{-C} \end{bmatrix}\mathbf{X} \\ &= \mathbf{K}_2\mathbf{R}_2\left(\mathbf{K}_1\mathbf{R}_1\right)^{-1}\mathbf{K}_1\mathbf{R}_1\begin{bmatrix} \mathbf{I} & \mathbf{-C} \end{bmatrix}\mathbf{X} \\ &= \mathbf{K}_2\mathbf{R}_2\left(\mathbf{K}_1\mathbf{R}_1\right)^{-1}\mathbf{x}_1 \\ &= \mathbf{H}_{21}\mathbf{x}_1\end{aligned}$$

where H is a 3 × 3 non-singular matrix

$$\boxed{\mathbf{x}_1 = \mathbf{H}_{12}\mathbf{x}_2; \quad \mathbf{x}_2 = \mathbf{H}_{21}\mathbf{x}_1}$$

Homography: Applications

- Image Mosaicing
- Detecting camera translation
- Multi-frame super-resolution

...

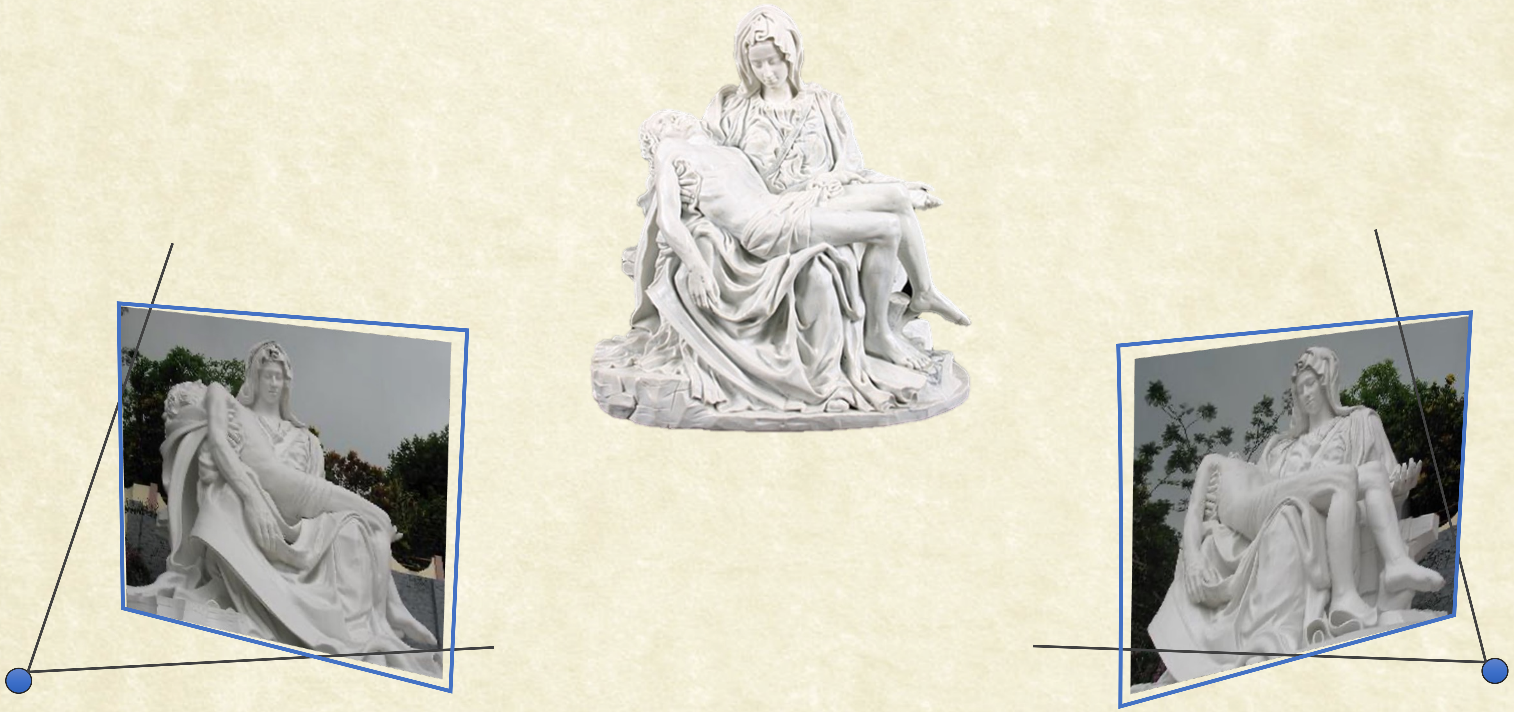
...

...

Questions?

CS7.505: Computer Vision

Spring 2024: Epipolar Geometry



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Cross Product: A Recap

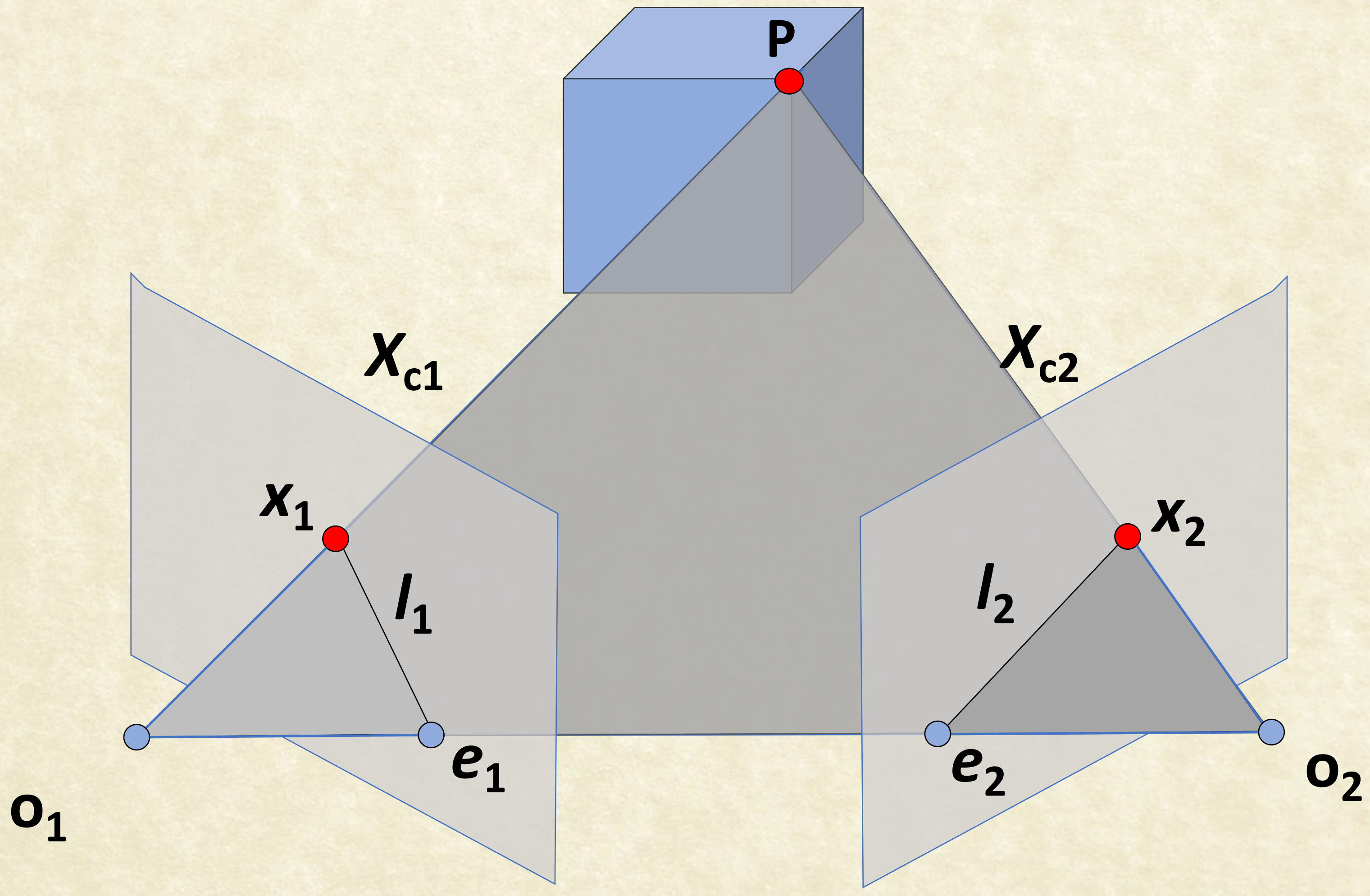
- Consider $\mathbf{A} = \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix}$; and Let $\widehat{\mathbf{A}} = \begin{bmatrix} 0 & -A_z & A_y \\ A_z & 0 & -A_x \\ -A_y & A_x & 0 \end{bmatrix}$

- $\mathbf{A} \times \mathbf{B} = \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \begin{bmatrix} A_yB_z - B_yA_z \\ B_xA_z - A_xB_z \\ A_xB_y - B_xA_y \end{bmatrix}$

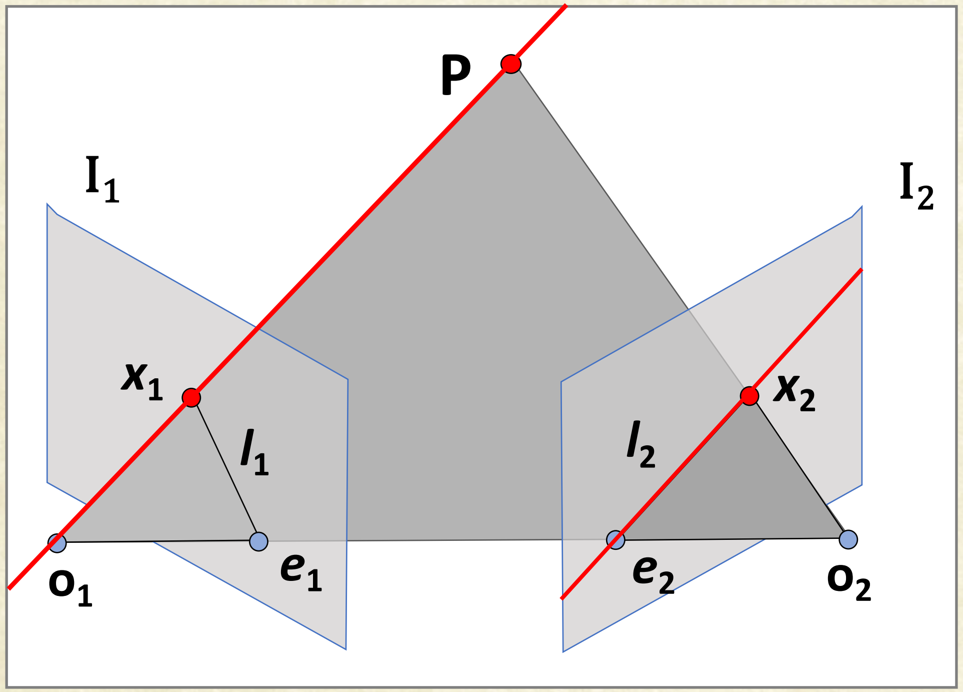
- $\widehat{\mathbf{A}}\mathbf{B} = \begin{bmatrix} 0 & -A_z & A_y \\ A_z & 0 & -A_x \\ -A_y & A_x & 0 \end{bmatrix} \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} A_yB_z - B_yA_z \\ B_xA_z - A_xB_z \\ A_xB_y - B_xA_y \end{bmatrix}$

- Note: The cross product, AxB or ÂB is a vector perpendicular to both A and B

Case 3: Generic World and Cameras



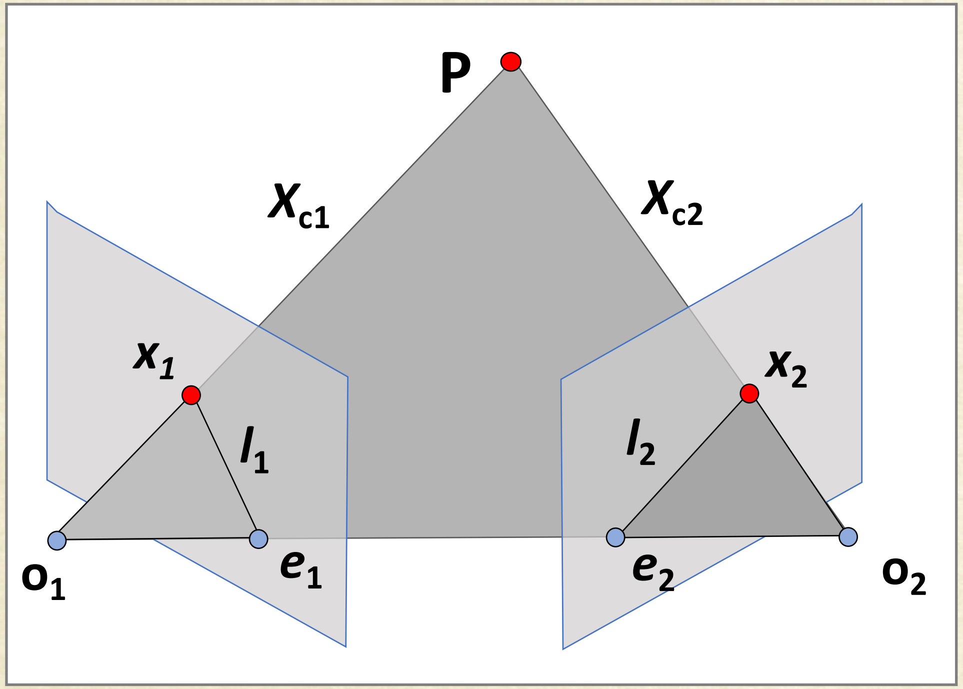
Epipolar Geometry



- All world points $\boldsymbol{X}_{c1}$ that map to $\boldsymbol{x}_1$ in $\mathbf{I}_1$ (pre-image of $\boldsymbol{x}_1$) map to a line $\boldsymbol{l}_2$ in $\mathbf{I}_2$, called an epipolar line.
- $\boldsymbol{l}_1$ is similar!
- $\boldsymbol{e}_2$ is the image of $\boldsymbol{o}_1$ in $\mathbf{I}_2$ is an epipole. (so is $\boldsymbol{e}_1$)

- The plane containing these is called the epipolar plane.
- These result in a set of constraints, which are referred to as the *epipolar constraints* and the resulting geometry is called the *epipolar geometry.*

Epipolar Constraint: Essential Matrix



t : position of right camera in left camera's frame
$\hat{T}$: matrix form, allowing cross-product
R : orientation of left camera in right camera's frame

- Consider **P** in camera 2's coordinates:
  - $\lambda_2 \boldsymbol{X}_{c2} = \mathbf{P}$
- Now, viewing it in camera 1's coordinates:
  - $\lambda_1 \boldsymbol{X}_{c1} = \mathbf{RP} + \mathbf{t}$

$$= \mathbf{R}(\lambda_2 \boldsymbol{X}_{c2}) + \mathbf{t}$$

Pre-multiplying by $\hat{\mathbf{T}}$, and then by $\boldsymbol{X}_{c1}{}^T$,

$$\hat{\mathbf{T}}\lambda_1 \boldsymbol{X}_{c1} = \hat{\mathbf{T}}\mathbf{R}\lambda_2 \boldsymbol{X}_{c2} + \cancel{\hat{\mathbf{T}}\mathbf{T}}$$

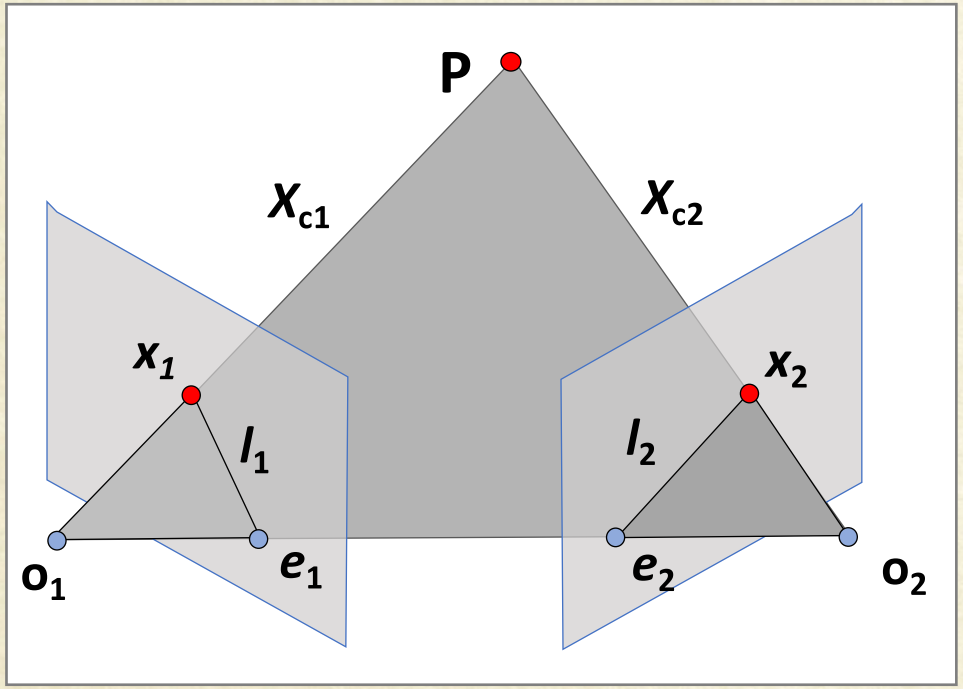
$$\lambda_1 \cancel{\boldsymbol{X}_{c1}{}^T \hat{\mathbf{T}} \boldsymbol{X}_{c1}} = \lambda_2 \boldsymbol{X}_{c1}{}^T \hat{\mathbf{T}}\mathbf{R}\boldsymbol{X}_{c2} + 0$$

$$\boldsymbol{X}_{c1}{}^T \hat{\mathbf{T}}\mathbf{R}\boldsymbol{X}_{c2} = 0$$

$$\boldsymbol{X}_{c1}{}^T \mathbf{E}\boldsymbol{X}_{c2} = 0$$

[Longuet-Higgins 1981]

Epipolar Constraint: Fundamental Matrix



$$\boldsymbol{X}_{c1}{}^{T}\hat{\mathbf{T}}\mathbf{R}\boldsymbol{X}_{c2} = 0$$

$$\boldsymbol{x}_1 = \boldsymbol{K}_1\boldsymbol{X}_{c1}$$

$$\boldsymbol{x}_2 = \boldsymbol{K}_2\boldsymbol{X}_{c2}$$

$$\boldsymbol{x}_1{}^{T}\boldsymbol{K}_2^{-1}\hat{\mathbf{T}}\mathbf{R}\boldsymbol{K}_1^{-1}\boldsymbol{x}_2 = 0$$

$$\boldsymbol{x}_1{}^{T}\boldsymbol{F}\boldsymbol{x}_2 = 0$$

Both Essential and Fundamental matrices are 3x3 and are independent of the world point.