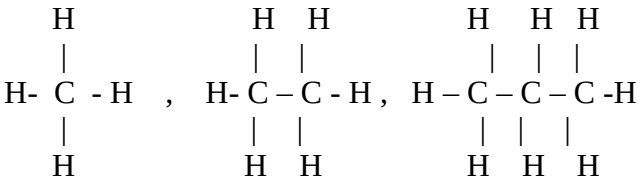
Q3> Algorithm Assignment 6.3

A saturated hydrocarbon is a molecule made of 'k' carbon atoms and 'l' hydrogen atoms. Each carbon atom is connected to 4 other atoms and each hydrogen atom is connected to 1 other atom. Since no sequence of bonds form cycles, this molecule has a tree structure. We know that in a tree, the total number of edges = the total number of nodes in the tree -1

If we consider carbon and hydrogen atoms as nodes and the bonds between atoms as edges then the saturated hydrocarbon forms a tree.Therefore the above equation can be written as Total No of bonds (B) = (k + l -1) -> Equation 1

An saturated hydro carbon molecule is as follows:



From the above representation, we can see that
number of hydrogen atoms (l) = (2* K + 2) => k = (l-2)/2

In order to prove this let's consider the theorem that the sum of degrees of all nodes is equal to twice the number of edges in the tree

This can be proved by induction:

Let's say a graph T(V , E) has V nodes and Edges. We want to prove

$2(E) = \sum_{v \in V} D(v)$ where, D stands for degree of the node v

Base Condition : Let C(0) be a case where no edges exist in the tree in that case total number of edges = 0 = sum of degree of all nodes

Induction step: Let's assume that the above equation holds good for C(n) for a tree with n edges.

i.e. $2(n) = \sum_{v \in V} D(v)$ holds true

Now we need to prove that the above condition holds good even after adding an additional edge to the tree i.e. C(n+1)

So Let T' (V', E') be a tree with V' vertices and E' edges where E' = E +1. So removal of an edge will result in G(V, E)

We need to prove $2(n+1) = \sum_{v \in V'} D(V')$ holds good

E’ is obtained by adding one edge e to E. Each edge contributes two degrees. Therefore total degrees of all nodes in graph G’

$2n + 2 = \sum_{v \in V} D(V)$

⇨ $2(n+1) = \sum_{v \in V} D(V)$

Hence the above equation holds true for C(n+1).

Hence by induction, It holds good for all cases.

Therefore by using the above proof and equation 1 we have,

$4k + l = 2(k+l -1) \Rightarrow 4k + l = 2k+ 2l -2 \Rightarrow 2k = l-2$

⇨ Number of carbon atoms(k) = (l-2) /2

Algorithm:

Input : l -> number of hydrogen atoms in saturated hydrocarbon molecule and
Returns: Integer value which represents the total number of carbon atoms (k) in saturated hydro carbon molecule.

```
FetchHydrogenAtomCount( l )
{
        return (l-2)/2;
}
```