Assignment - 2

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1. T(n) = 3T($\frac{n}{4}$) + $\sqrt(n)$, T(1) = 1

Since this is of the form T(n) = a T($\frac{n}{b}$)+f(b) we can apply master theorem

From the above recurrance relation, we have a = 3, b = 4, f(n) = $\sqrt(n)$

since $n^{(log_3 4)} > n^{\frac{1}{2}}$ case (i) of master theorem applies here

$\implies$ f(n) = $O(n^{(log_3 4)\ -\ \epsilon})$ for some $\epsilon > 0$

T(n) = $\theta(n^{(log_3 4)})$

2. T(n) = 9T($\frac{n}{3}$) + $5n^2$, T(1) = 1

Since this is of the form T(n) = a T($\frac{n}{b}$)+f(b) we can apply master theorem

From the above recurrance relation, we have a = 9, b = 3 and f(n) =$5n^2$

since $n^{(log_3 9)} = n^2$ we have f(n) = $\theta(n^{log_b a})$

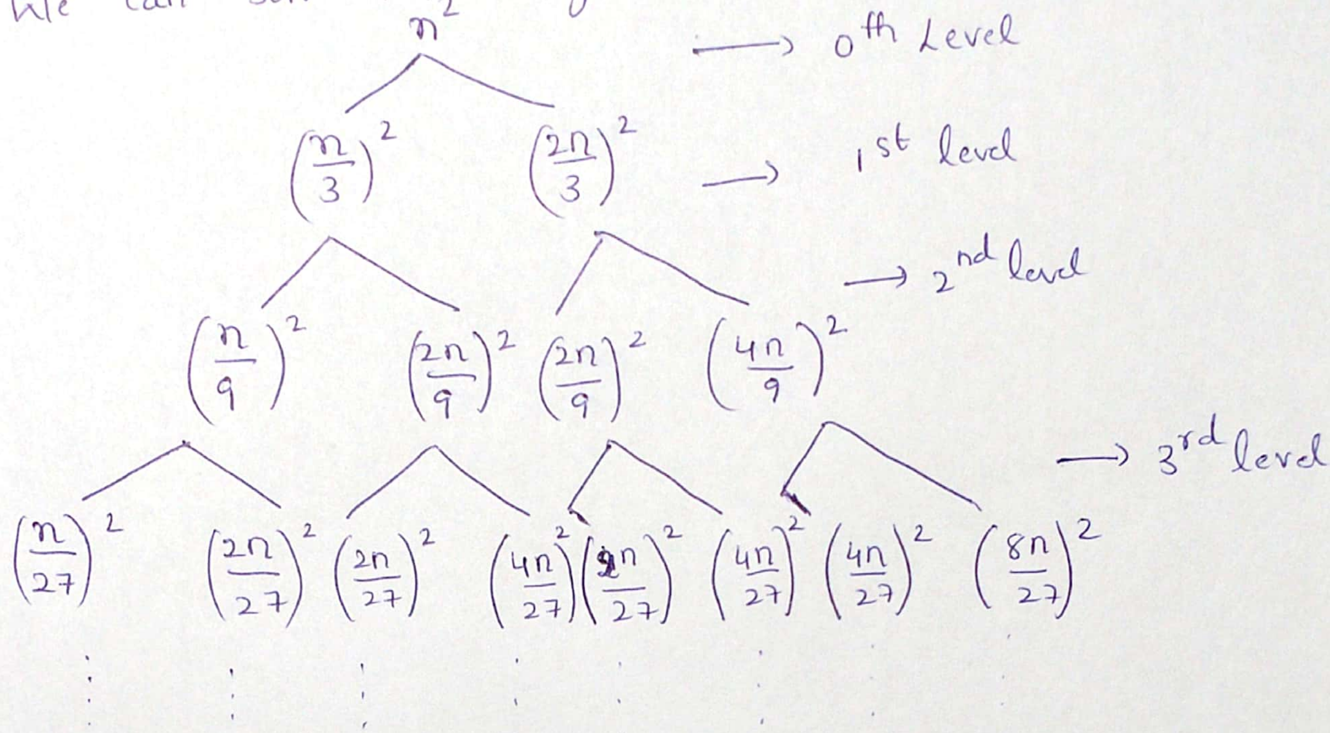
case (ii) of master theorem applies here

$\mathbf{T(n)} = \theta(n^{log_3 9} \log n)$

$\implies \mathbf{T(n)} = \theta(n^2 \ \log n)$

1) c) Given $T(n) = T(n/3) + T\left(\frac{2n}{3}\right) + cn^2$

We can solve this using recursion tree method



for 0^{th} level : n^2

1^{st} level $= \left(\frac{n}{3}\right)^2 + \left(\frac{2n}{3}\right)^2 = \frac{5n^2}{3^2}$

2^{nd} level $= \left(\frac{n}{9}\right)^2 + \left(\frac{2n}{9}\right)^2 + \left(\frac{2n}{9}\right)^2 + \left(\frac{4n}{9}\right)^2 = \frac{25n^2}{81} = \frac{5^2n^2}{3^4}$

3^{rd} level $= \left(\frac{n}{27}\right)^2 + \left(\frac{2n}{27}\right)^2 + \left(\frac{2n}{27}\right)^2 + \left(\frac{4n}{27}\right)^2 + \left(\frac{2n}{27}\right)^2 + \left(\frac{4n}{27}\right)^2 + \left(\frac{4n}{27}\right)^2 + \left(\frac{8n}{27}\right)^2$

$= \frac{125n^2}{(27)^2} = \frac{125n^2}{(3^3)^2} = \frac{5^3n^2}{3^6}$

By using above results, we can generalise it as below

$T(n) = n^2 + \frac{5n^2}{3^2} + \frac{5^2n^2}{3^4} + \frac{5^3n^2}{3^6} + \cdots$

$= n^2\left[1 + \frac{5}{3^2} + \frac{5^2}{3^4} + \frac{5^3}{3^6} + \cdots\cdots\right]$

This is a Geometric progression series with

$a = 1$ $r = \frac{\frac{5^2}{3^4}}{\frac{5}{3^2}} = \frac{5}{3^2}$ & $n = n$

Infinite Geometric progression series can be solved using the formula:

$$S_n = \frac{a}{1-r} = \frac{1}{1-\frac{5}{3^2}} = \frac{1}{\frac{9-5}{9}} = \frac{9}{4}$$

$$\therefore T(n) = n^2\left(\frac{9}{4}\right) = \frac{9n^2}{4}$$

$$\Rightarrow T(n) = O(n^2).$$