Homework #3 Cheat Sheet

CS231

1 Language of Booleans and Integers

1.1 Syntax

```
t  ::=  true | false | if t then t else t
     |  n | t + t | t > t
n  ::=  integer constant
v  ::=  true | false | n
T  ::=  Bool | Int
```

1.2 Small-Step Operational Semantics

$$\frac{}{\texttt{if true then } t_2 \texttt{ else } t_3 \longrightarrow t_2} \quad \text{(E-IfTrue)}$$

$$\frac{}{\texttt{if false then } t_2 \texttt{ else } t_3 \longrightarrow t_3} \quad \text{(E-IfFalse)}$$

$$\frac{t_1 \longrightarrow t_1'}{\texttt{if } t_1 \texttt{ then } t_2 \texttt{ else } t_3 \longrightarrow \texttt{if } t_1' \texttt{ then } t_2 \texttt{ else } t_3} \quad \text{(E-If)}$$

$$\frac{t_1 \longrightarrow t_1'}{t_1 \texttt{ + } t_2 \longrightarrow t_1' \texttt{ + } t_2} \quad \text{(E-Plus1)}$$

$$\frac{t_2 \longrightarrow t_2'}{v_1 \texttt{ + } t_2 \longrightarrow v_1 \texttt{ + } t_2'} \quad \text{(E-Plus2)}$$

$$\frac{n = n_1 \,[\![+]\!]\, n_2}{n_1 \texttt{ + } n_2 \longrightarrow n} \quad \text{(E-PlusRed)}$$

$$\frac{t_1 \longrightarrow t_1'}{t_1 \texttt{ > } t_2 \longrightarrow t_1' \texttt{ > } t_2} \quad \text{(E-GT1)}$$

$$\frac{t_2 \longrightarrow t_2'}{v_1 \texttt{ > } t_2 \longrightarrow v_1 \texttt{ > } t_2'} \quad \text{(E-GT2)}$$

$$\frac{v = n_1 \,[\![>]\!]\, n_2}{n_1 \texttt{ > } n_2 \longrightarrow v} \quad \text{(E-GTRed)}$$

1.3 Static Type System

$$\frac{}{\texttt{true : Bool}} \quad \text{(T-True)} \qquad \frac{}{\texttt{false : Bool}} \quad \text{(T-False)}$$

$$\frac{t_1 : \text{Bool} \quad t_2 : T \quad t_3 : T}{\text{if } t_1 \text{ then } t_2 \text{ else } t_3 : T} \tag{T-IF}$$

$$\frac{}{n : \text{Int}} \tag{T-NUM}$$

$$\frac{t_1 : \text{Int} \quad t_2 : \text{Int}}{t_1 + t_2 : \text{Int}} \tag{T-PLUS}$$

$$\frac{t_1 : \text{Int} \quad t_2 : \text{Int}}{t_1 > t_2 : \text{Bool}} \tag{T-GT}$$

2 Simply-Typed Lambda Calculus

2.1 Syntax

```
t  ::=  x | function x:T -> t | t t
v  ::=  function x:T -> t
T  ::=  T1 → T2
```

2.2 Substitution

$[x \mapsto v]x = v$
$[x \mapsto v]x' = x'$, where $x \neq x'$
$[x \mapsto v]\texttt{function } x\texttt{:T -> } t_0 = \texttt{function } x\texttt{:T -> } t_0$
$[x \mapsto v]\texttt{function } x_0\texttt{:T -> } t_0 = \texttt{function } x_0\texttt{:T -> } [x \mapsto v]t_0$, where $x \neq x_0$
$[x \mapsto v]t_1\ t_2 = [x \mapsto v]t_1\ [x \mapsto v]t_2$

2.3 Small-Step Operational Semantics

$$\frac{}{((\texttt{function } x\texttt{:T -> } t)\ v) \longrightarrow [x \mapsto v]t} \tag{E-APPBETA}$$

$$\frac{t_1 \longrightarrow t_1'}{t_1\ t_2 \longrightarrow t_1'\ t_2} \tag{E-APP1}$$

$$\frac{t_2 \longrightarrow t_2'}{v_1\ t_2 \longrightarrow v_1\ t_2'} \tag{E-APP2}$$

2.4 Static Type System

Γ is a finite function from variable names to types.

$$\frac{\Gamma(x) = T}{\Gamma \vdash x : T} \tag{T-VAR}$$

$$\frac{\Gamma, x\texttt{:}T_1 \vdash t : T_2}{\Gamma \vdash \texttt{function } x\texttt{:}T_1 \texttt{ -> } t : T_1 \to T_2} \tag{T-FUN}$$

$$\frac{\Gamma \vdash t_1 : T_2 \to T \qquad \Gamma \vdash t_2 : T_2}{\Gamma \vdash t_1\ t_2 : T} \tag{T-APP}$$