Computer Graphics

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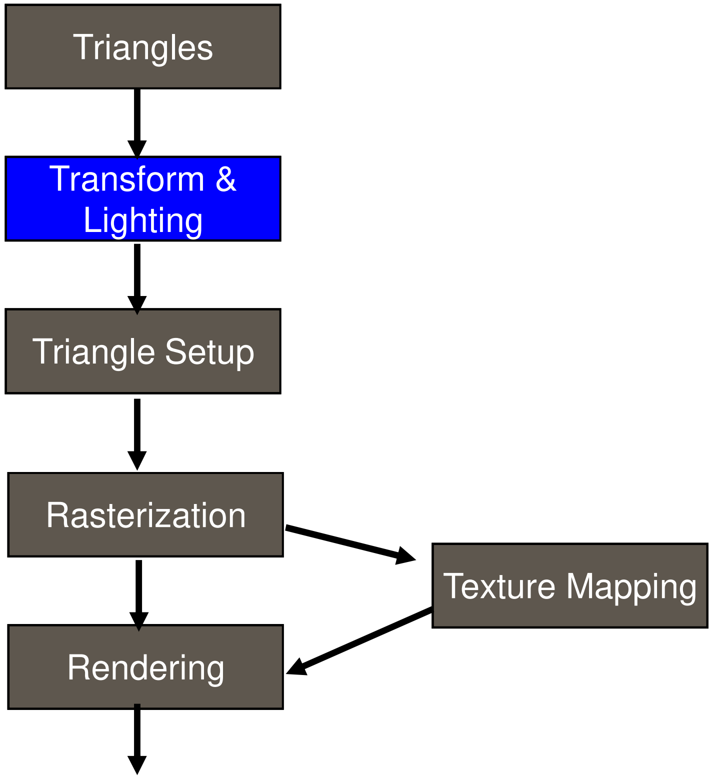
Wrap up from last course

- Geometrical Transformation
- Viewing Transformation
- Projection Transformation
- Viewport Transformation

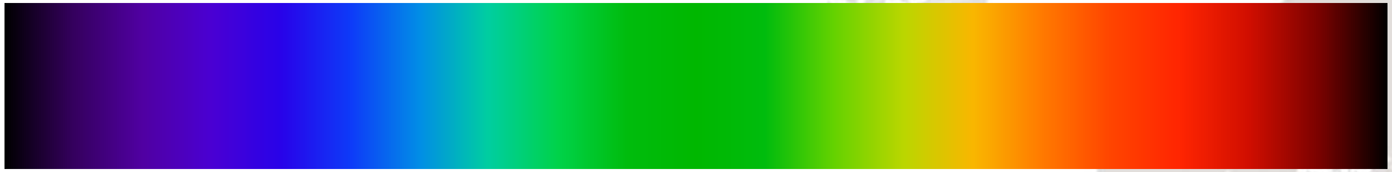
Part I:
Conventional 3D Graphics Pipeline



Lighting

Color Model
Illumination Model
Polygon Shading

Color

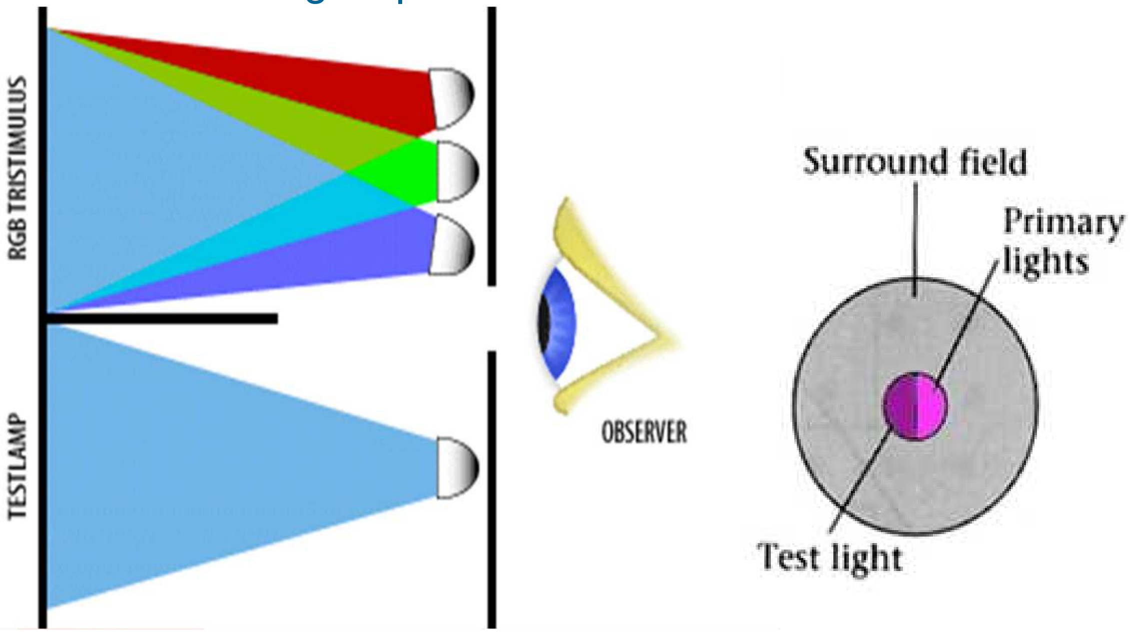


- **Stimulation of cone cells in the human eye by electromagnetic radiation in the visible spectrum**

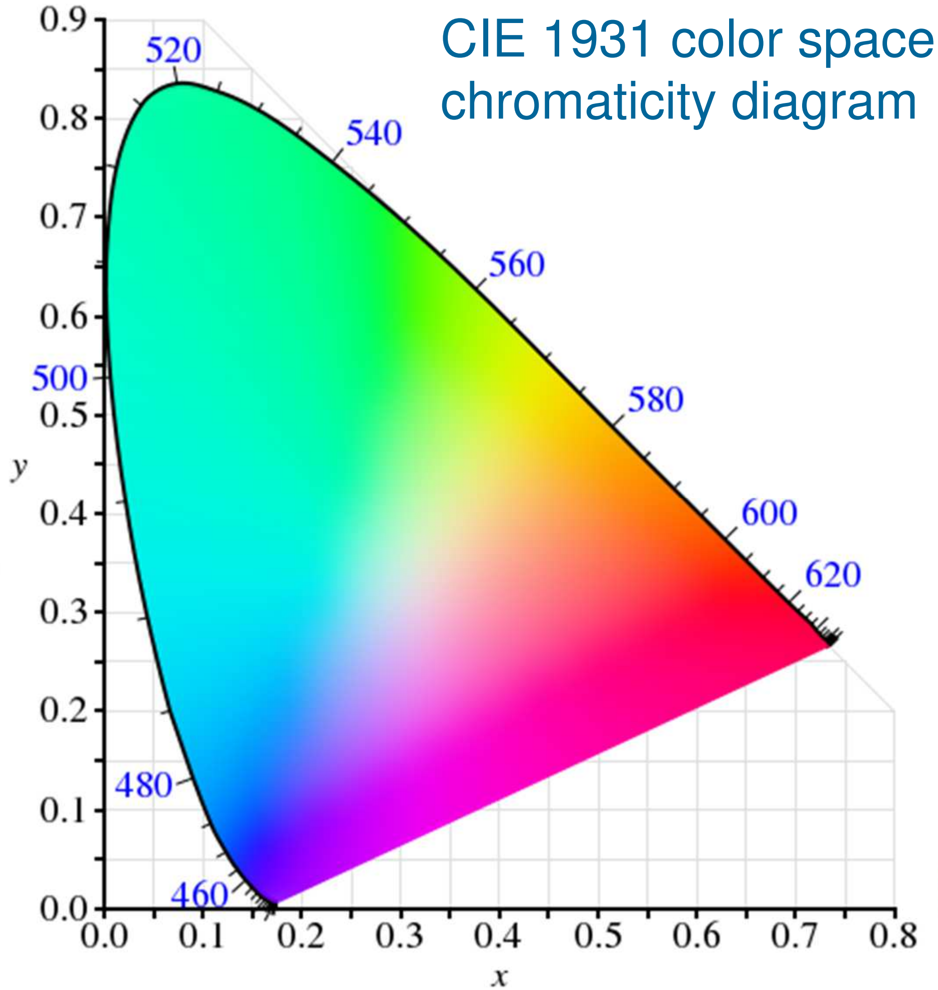
Color = (brightness, chromaticity)

$\underbrace{\text{brightness}}_{\text{amplitude}}$, $\underbrace{\text{chromaticity}}_{\text{frequency}}$

Color Matching Experiment

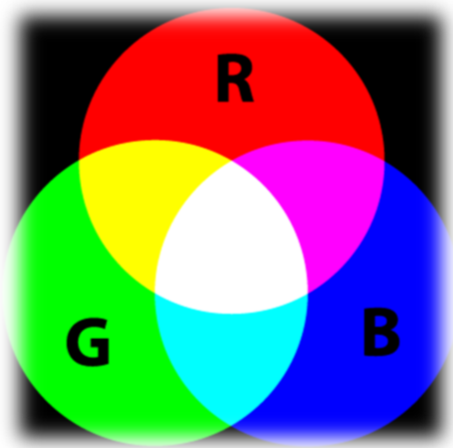


CIE 1931 color space chromaticity diagram



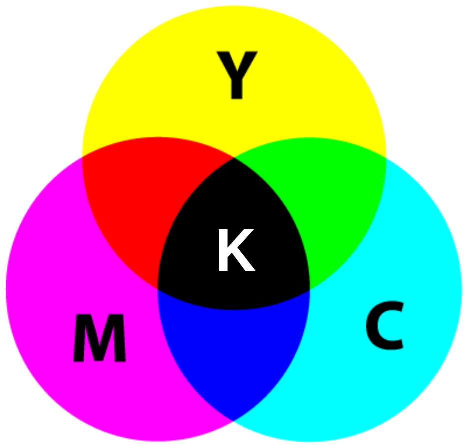
Color Model

- An abstract model to describe colors
- Representation: A tuple (three or four values/components) is used to represent a specific color, such as (*r*, *g*, *b*) in RGB color model or (*c*, *m*, *y*, *k*) in CMYK color model



- Additive color model
- Used in sensing, representation, and display of images in electronic systems, such as TV and computer monitor

RGB Color Model

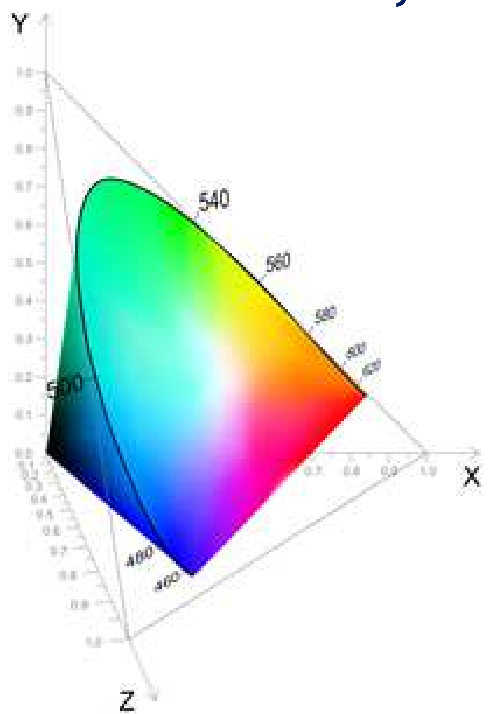


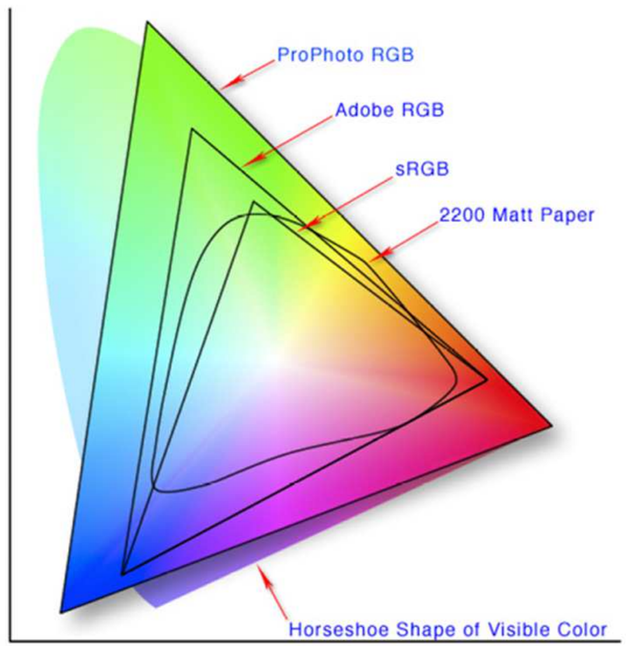
- Subtractive color model
- Used in color printing

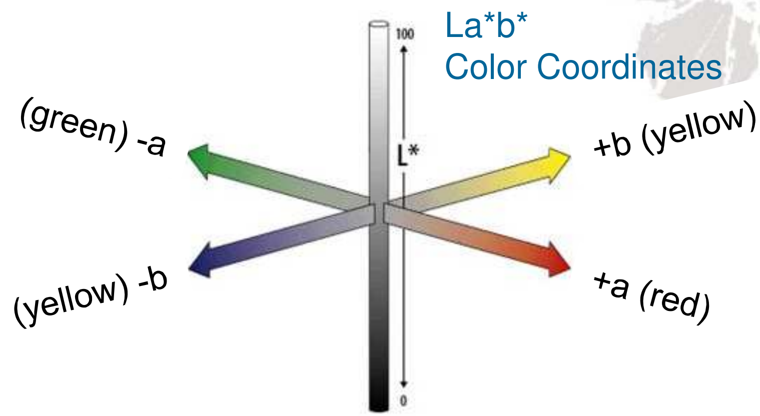
CMYK Color Model

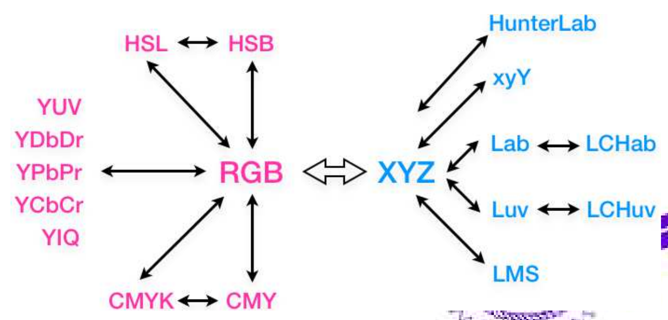
Color Space

- Define the range and tones of colors
  - Device-invariant: human visible colors
    - CIE-RGB, CIE-XYZ, …
  - Device-variant: device producible colors
    - sRGB, Adobe RGB, CMYK, …



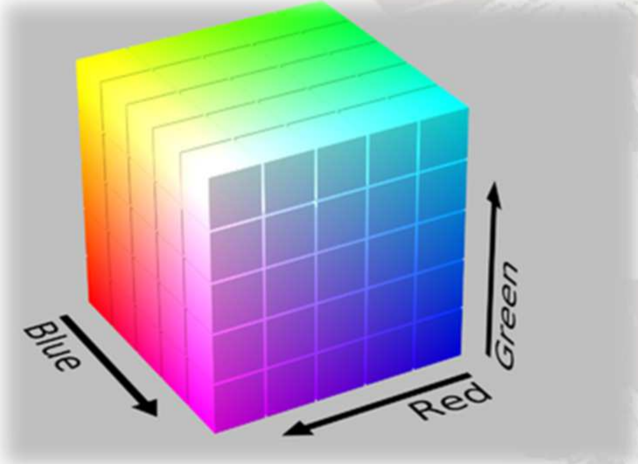


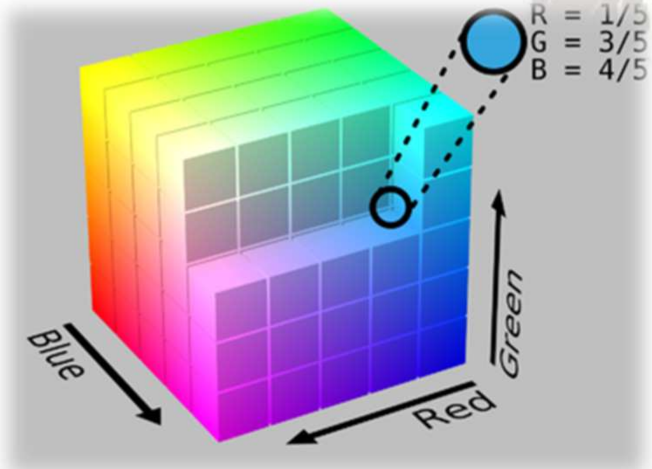




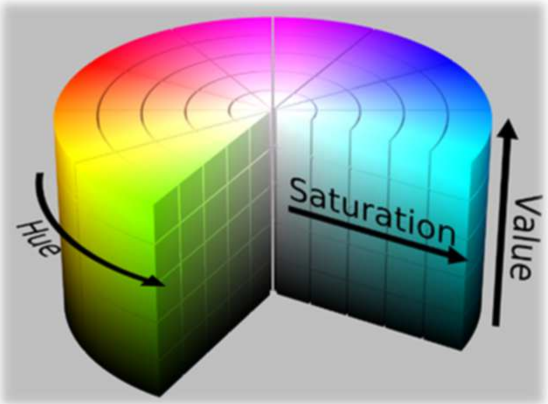
RGB Color Model and Color Space

- **Three additive primary colors, red, green and blue light are added together in various ways to reproduce the colors in the associated color space**
  - **HSV and HSL are transformation of a RGB color space**

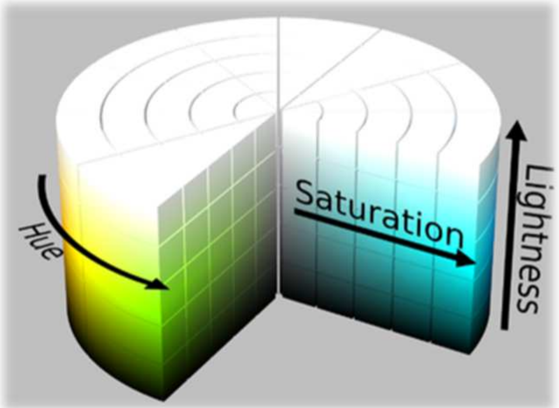




HSV

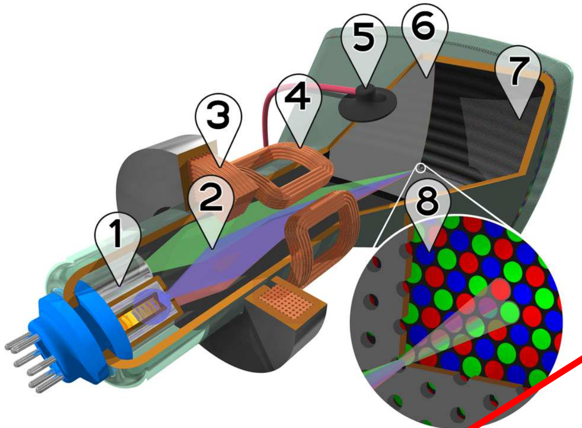


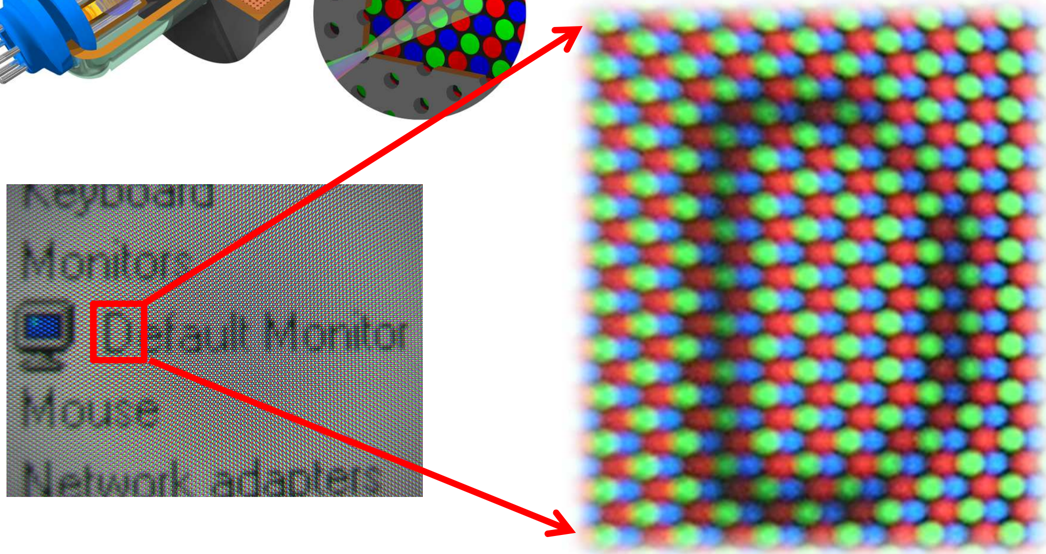
HSL

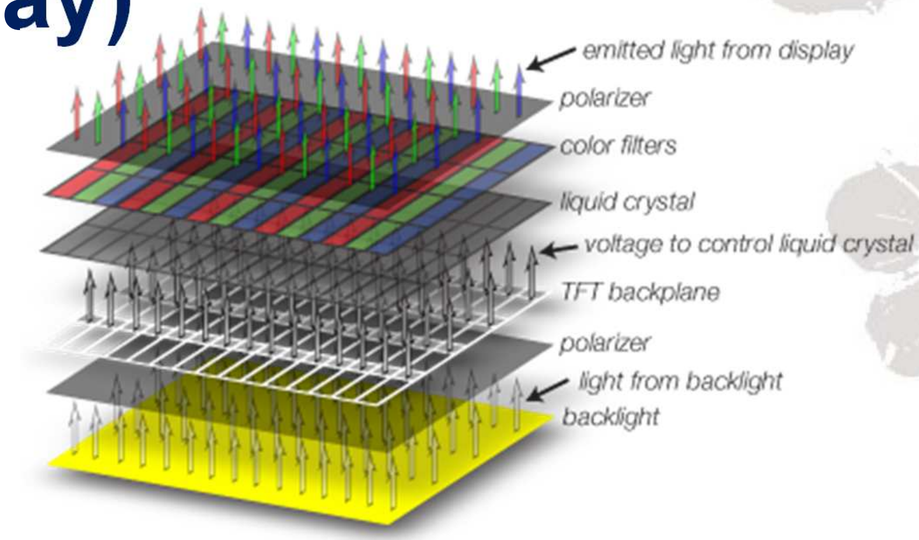


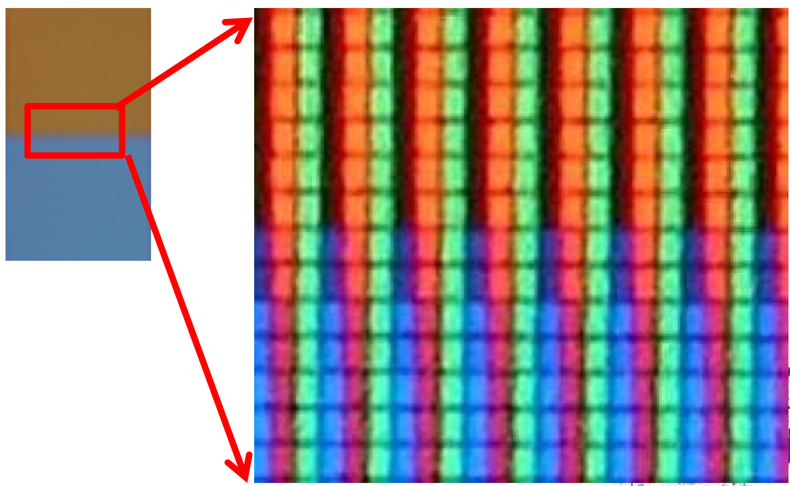
Displays using RGB Color Model

- CRT (cathode ray tube)
- LCD (liquid crystal display)



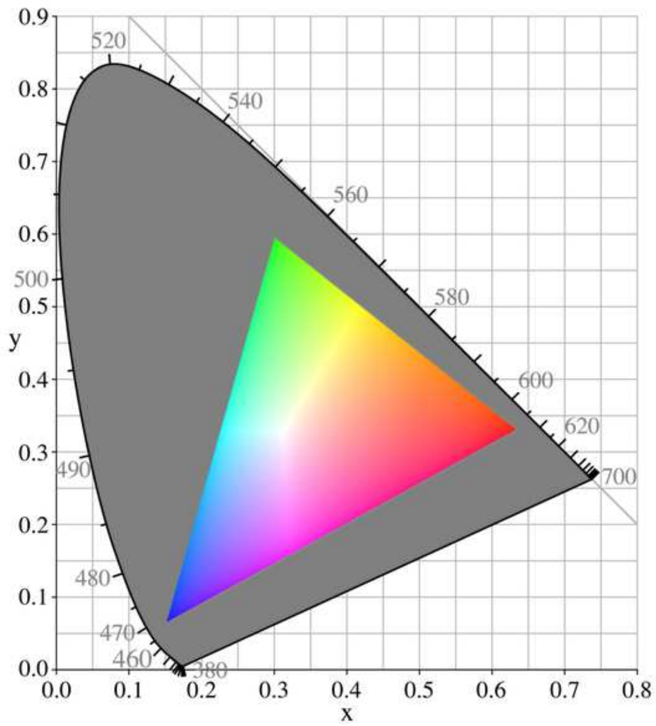






Color Depth

- Gamut: a subset of colors which can be represented in a given color space
- Color depth: the number of bits to indicate the color of a pixel (bpp: bits per pixel)
  - 15/16 bits, 24/32 bits, …
  - The number of colors can be represented for a given color depth d is 2^d
    - e.g., 24-bit (R8G8B8) → 2^{24} colors



sRGB Gamut

Shading

- Shading is a light-material interaction in determining the color for each pixel of rendered objects or scenes

Flat Shading

Smooth Shading

Advanced Shading

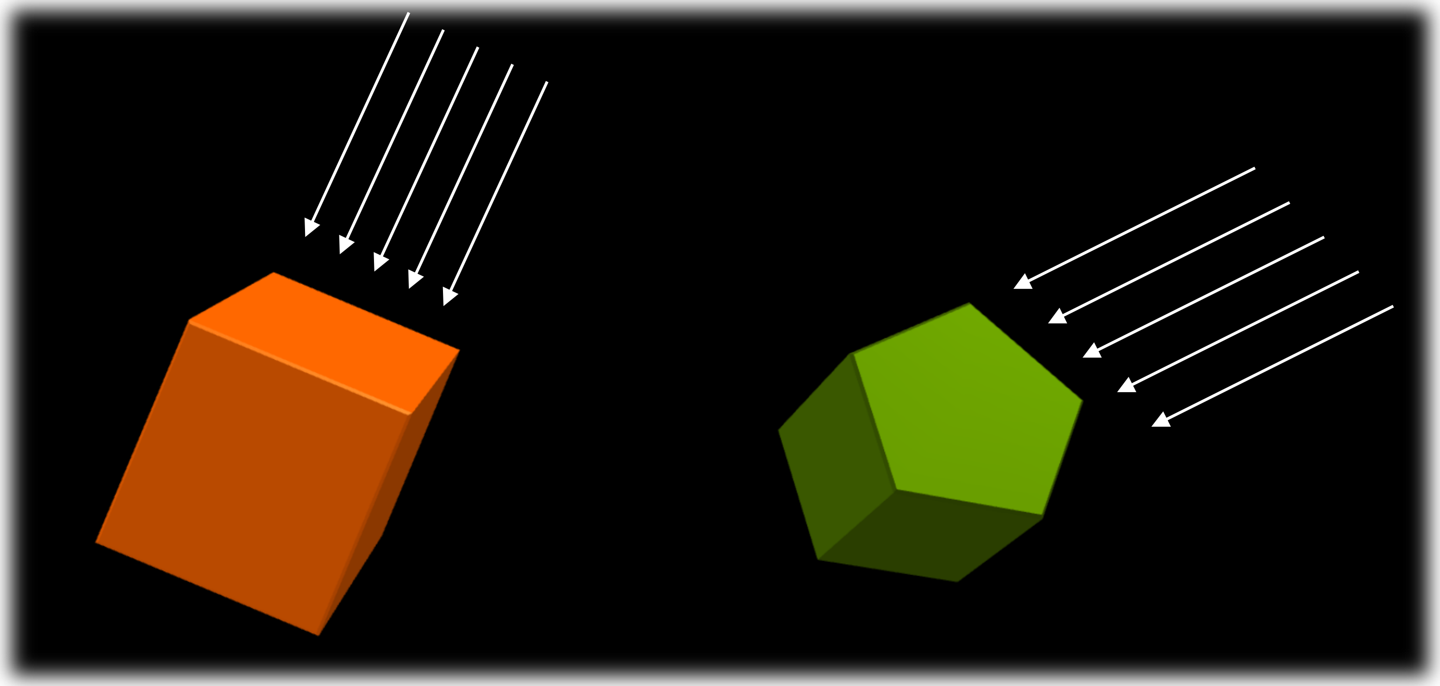
- Global illumination
  - Direct illumination plus indirect illumination
    - Reflection
    - Refraction
    - Shadow

Factors that affect Shading

- Light sources
  - Ambient light, directional light, positional light, spot light…
- Material properties
  - Ambient reflection, diffuse reflection, specular reflection…
- Location of the viewer
  - Position of perceiving specular highlight
- Surface orientation
  - Surface normal, vertex normal

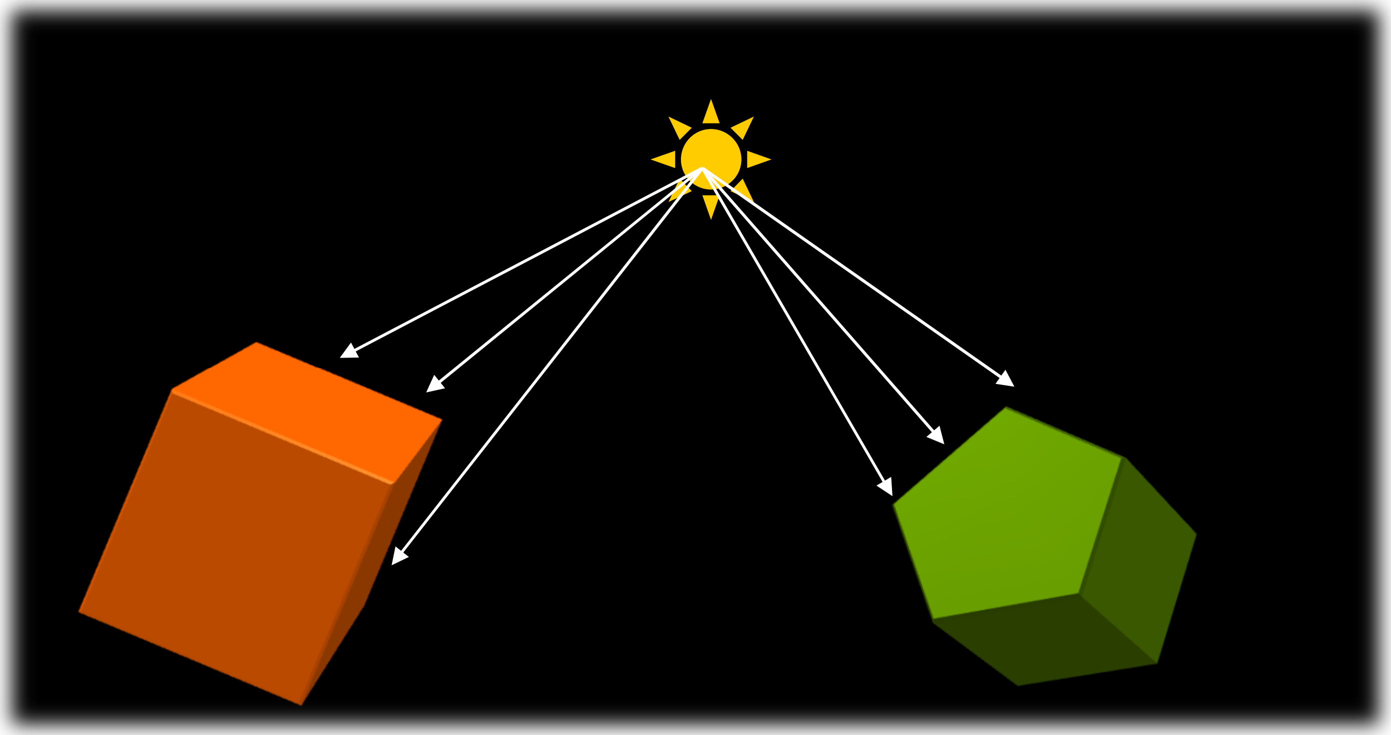
Light Sources

- Directional Light
  - Light source located at infinite far away such as the sun



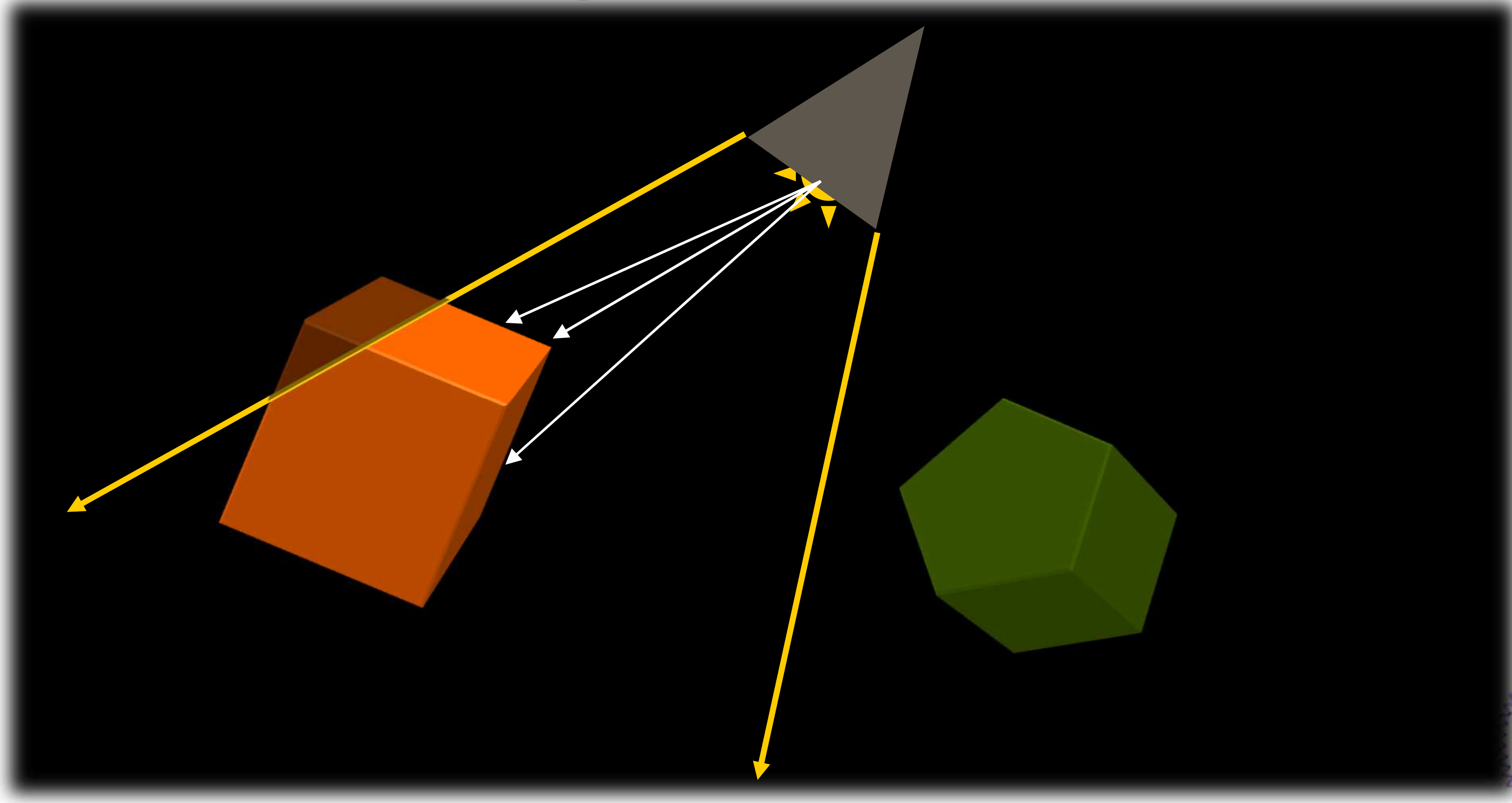
Light Sources

- Positional Light (Point Light)
  - Light source located at a specific position



Light Sources

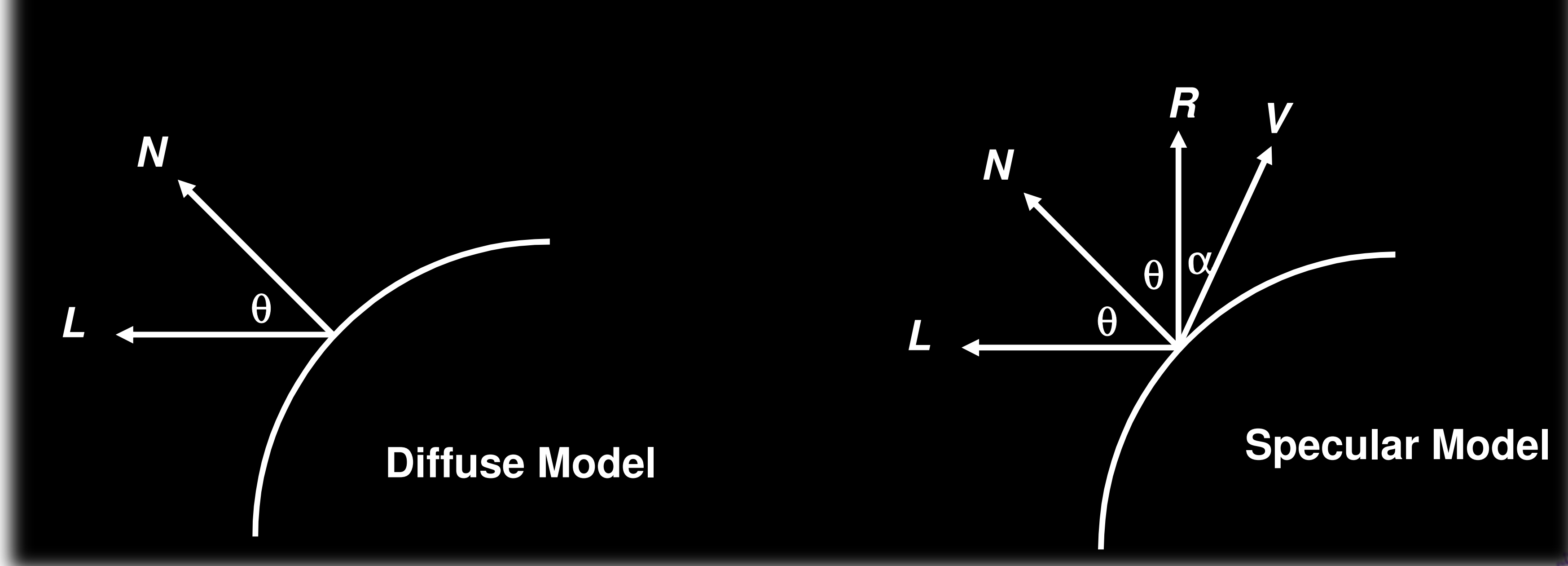
- **Spot Light**
  - **Light source located at a specific position with certain cutoff range**



Lighting Equation

- Intensity = Ambient + Diffuse + Specular

$$I = \boxed{I_a k_a} + \sum_{p=1}^{m} f_p I_p \left(\boxed{k_d (N \cdot L_p)} + \boxed{k_s (R_p \cdot V)^n} \right)$$



Ambient Light

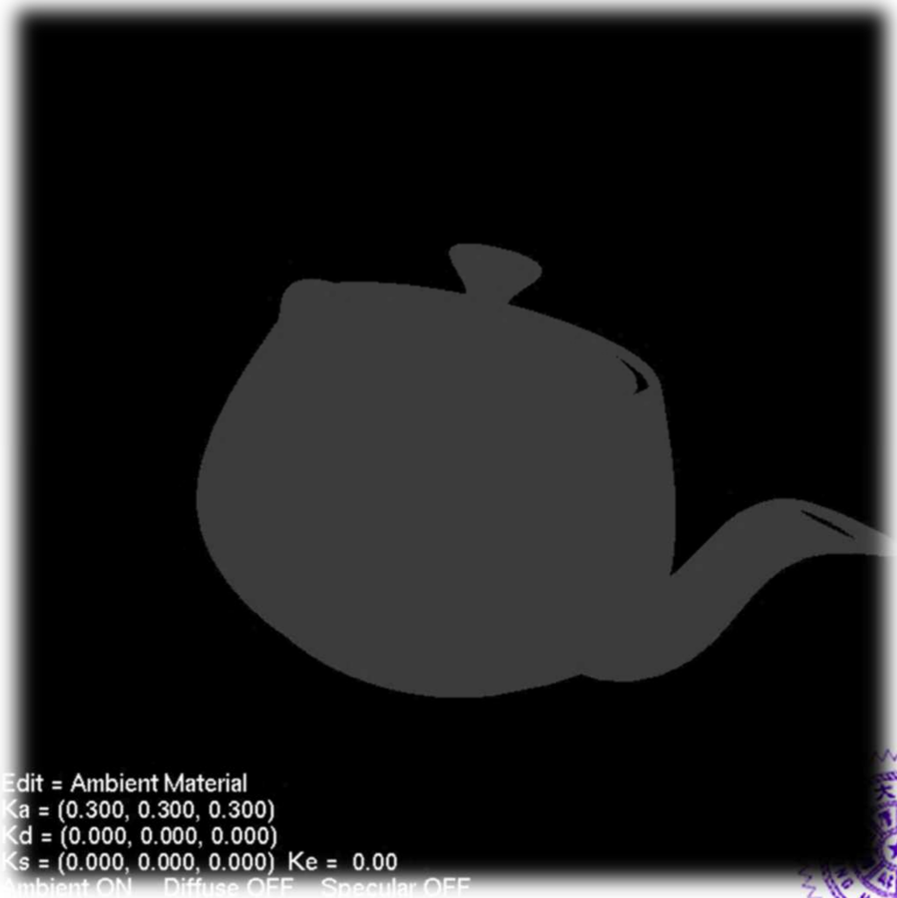
- **Illumination surrounding a scene without providing any specific light source**

$$I = I_a k_a$$

I: resulting intensity

I_a: ambient light intensity

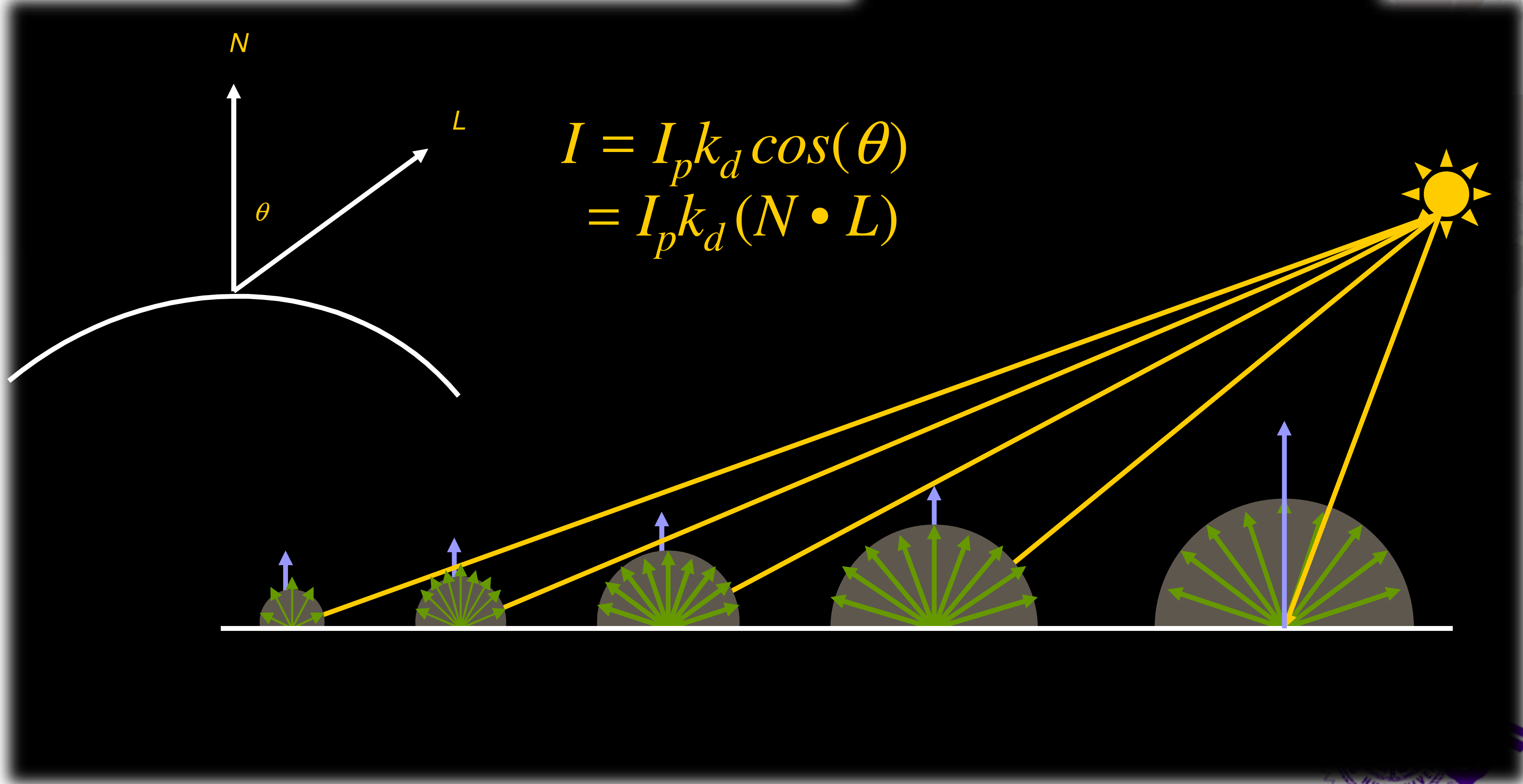
k_a : ambient reflection coefficient



Diffuse Reflection

- Lambert's Cosine Law

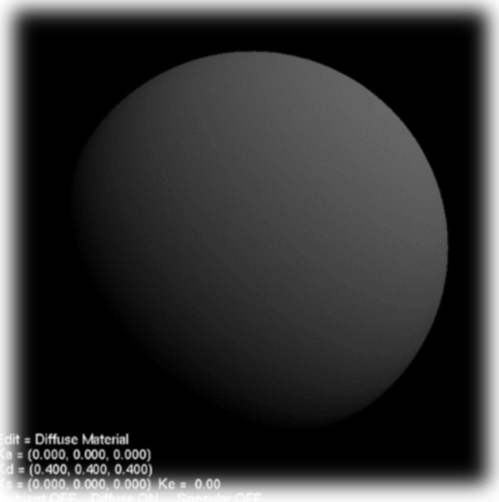
I_p: point light source intensity
k_d: diffuse reflection coefficient
N: normalized normal vector
L: normalized light direction vector



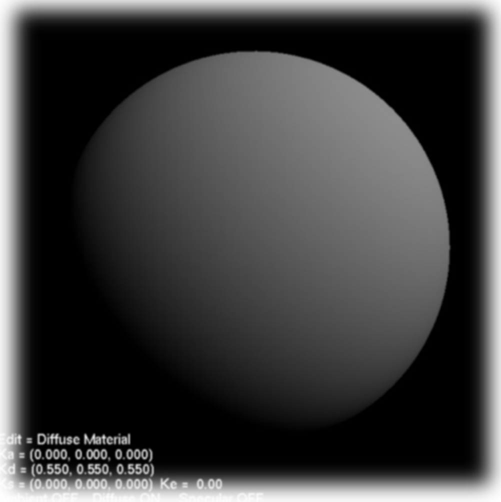
$$I = I_p k_d \cos(\theta) = I_p k_d (N \bullet L)$$

Diffuse Reflection

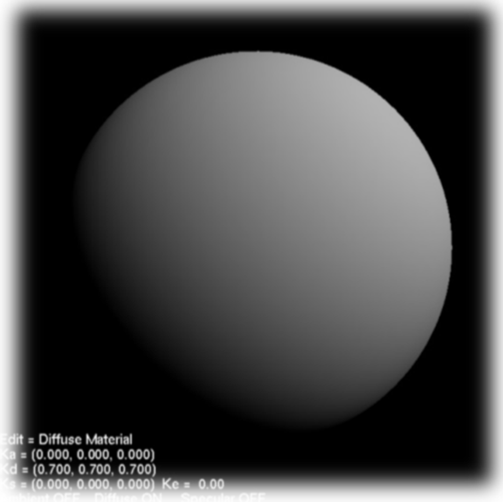
- **Example**
  - **Fixed point light source at (1.0, 1.0, 1.0)**



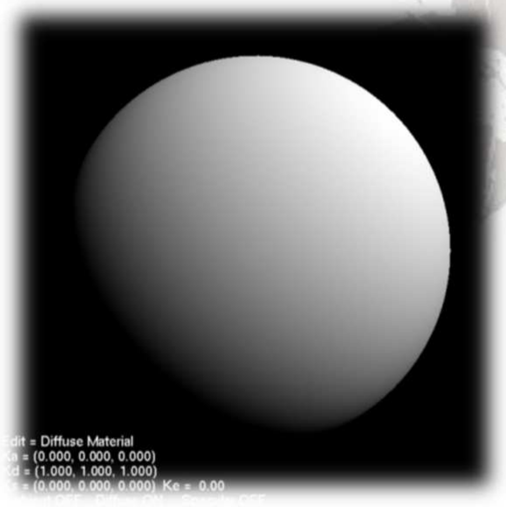
$k_d = 0.4$



$k_d = 0.55$



$k_d = 0.7$

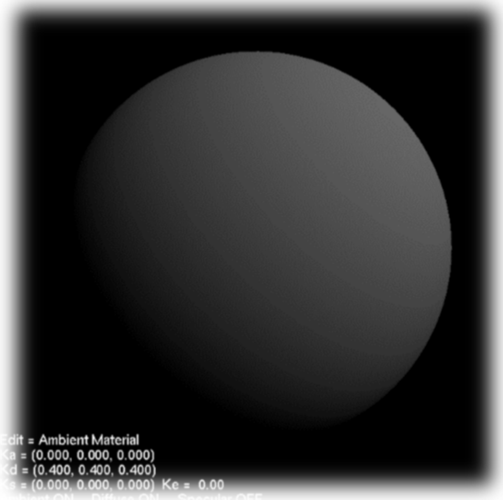


$k_d = 1.0$

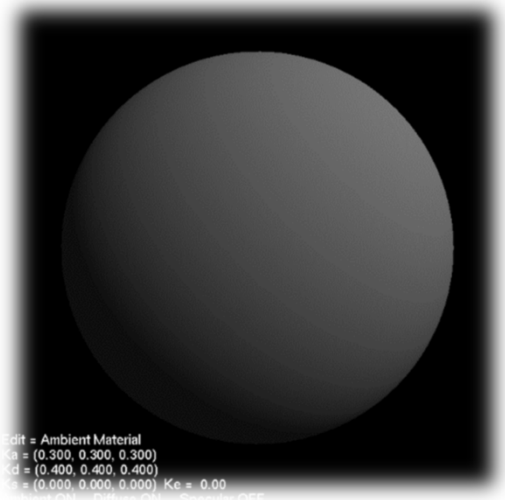
Ambient + Diffuse Reflection

- **Example**
  - $k_d = 0.4$

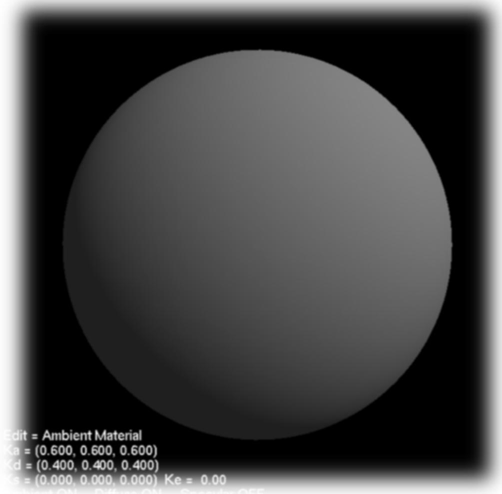
$$I = I_a k_a + I_p k_d (N \bullet L)$$



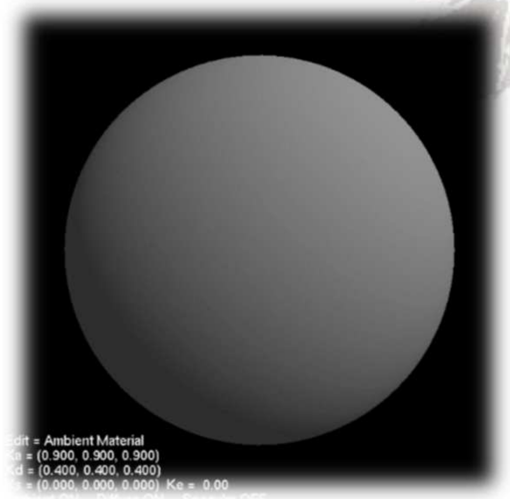
$k_a = 0.0$



$k_a = 0.3$



$k_a = 0.6$



$k_a = 0.9$

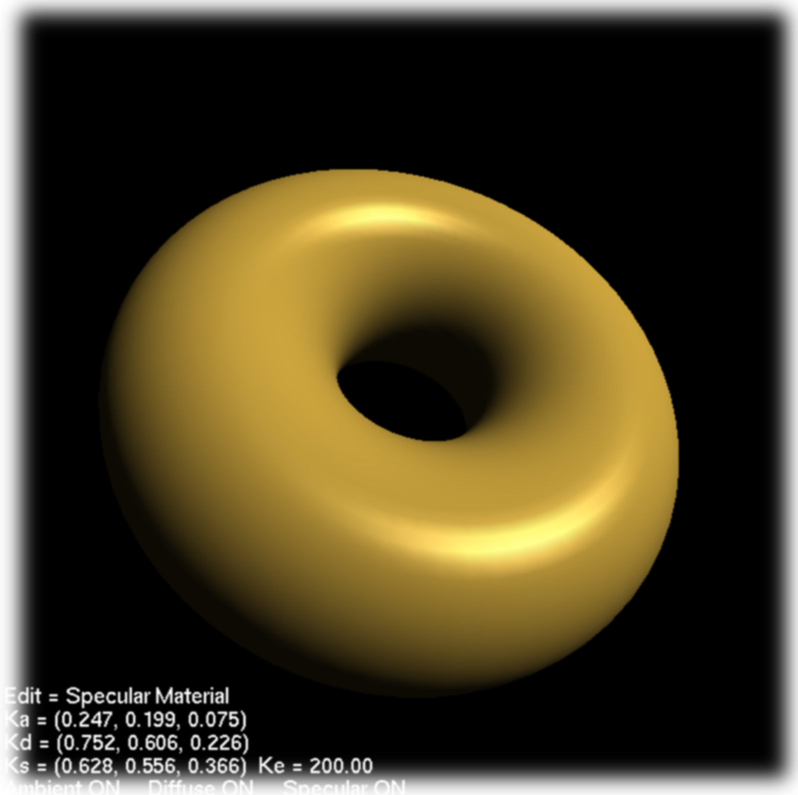
Light Source Attenuation

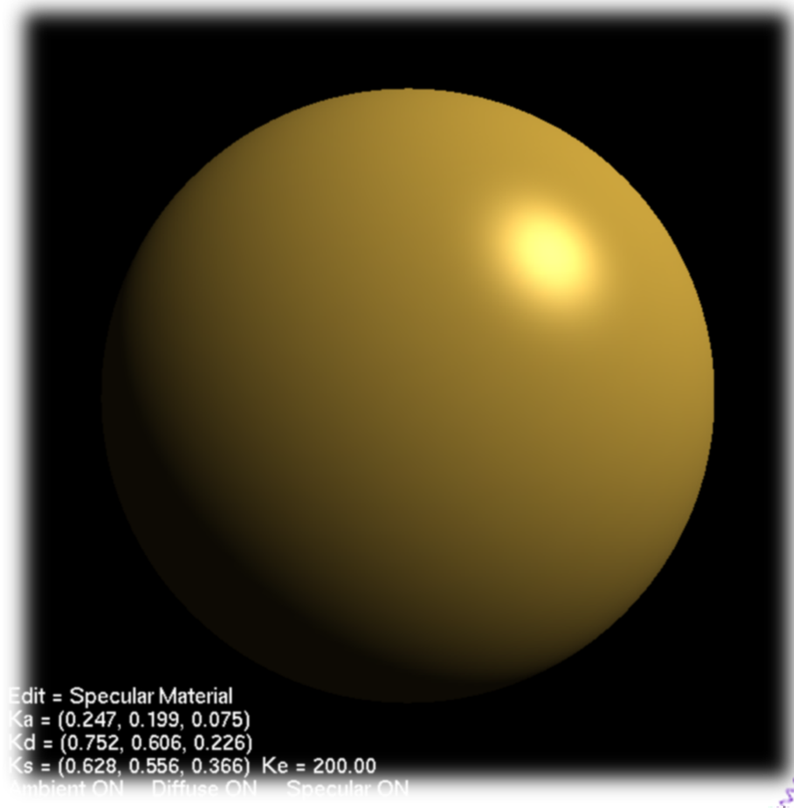
- **Light source intensity will attenuate with respect to the distance between the light source and the object**

$$I = I_a k_a + f_{att} I_p k_d (N \bullet L) \qquad f_{att} = \min\left(\frac{1}{c_1 + c_2 d_L + c_3 d_L^2}, 1\right)$$

Specular Highlight

- Specular highlight is the bright spot of light on object being illuminated

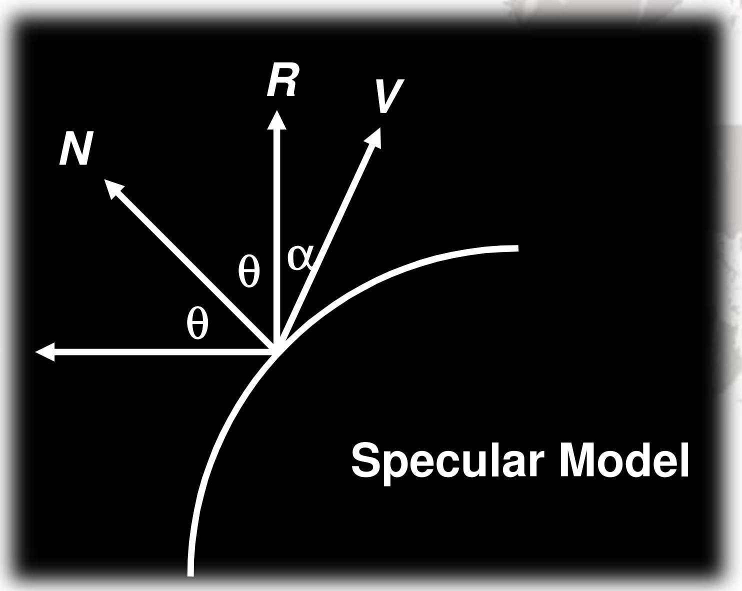




Specular Highlight

$$I_s = I_p k_s \cos^n \alpha = I_p k_s (R_p \cdot V)^n$$

$$I = I_a k_a + f_p I_p (k_d (N \cdot L_p) + k_s (R_p \cdot V)^n)$$



I: Intensity of final illumination

I_a: Intensity of ambient light

k_a: Ambient reflection coefficient

f_p: Attenuation function of point light source *p*

k_d: Diffuse reflection coefficient

N: Normalized normal vector

L_p: Normalized light source direction of point light source *p*

k_s: Specular reflection coefficient

R_p: Normalized light source reflection vector of point light source *p*

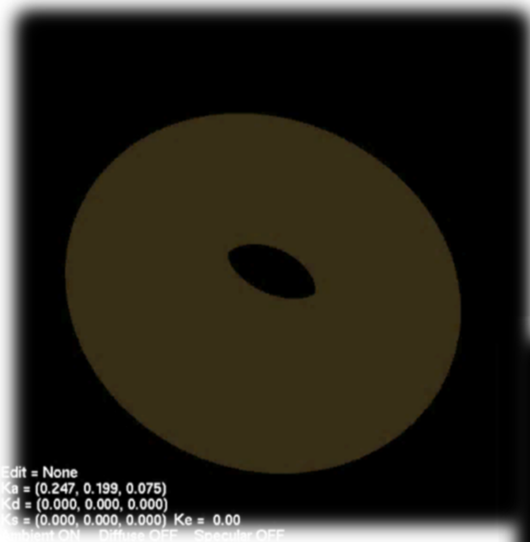
V: Normalized viewpoint direction vector

n: Material's specular reflection exponent

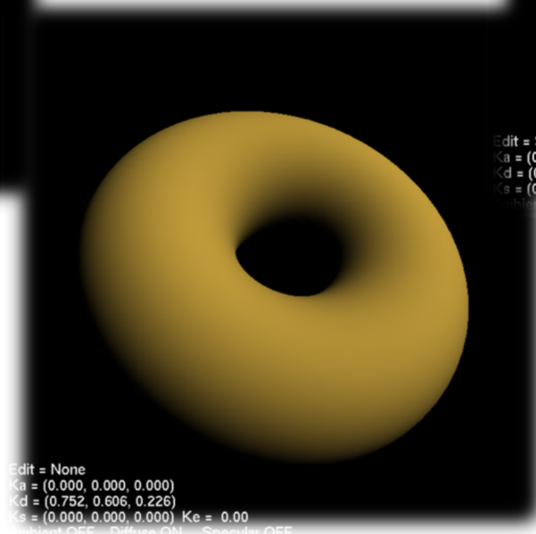
Specular Highlight

- **Phong Reflection Model**

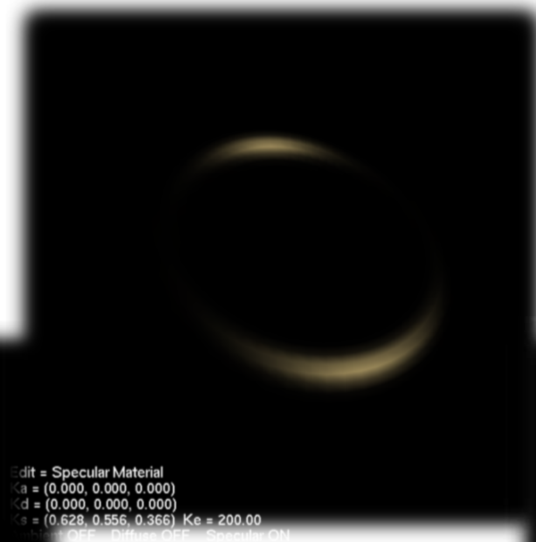
$$I = I_a k_a + f_p I_p (k_d (N \cdot L_p) + k_s (R_p \cdot V)^n)$$



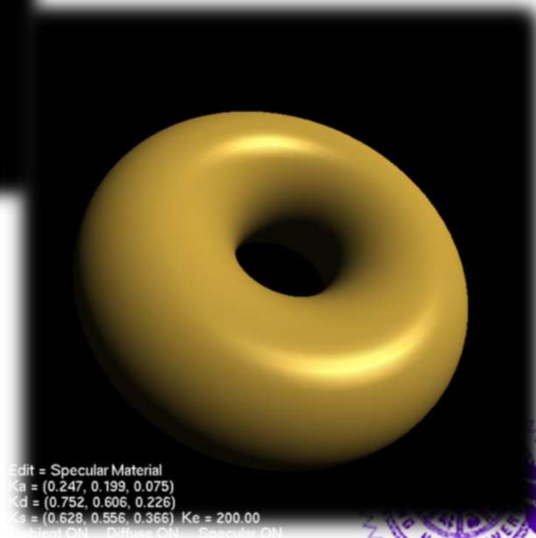
Ambient



Diffuse



Specular



Final illumination

Modified Phong Reflection Model

- **Also called Blinn-Phong Reflection Model**
  - **Original Phong model requires to calculate the reflection vector and view vector for each point**
  - **Blinn suggested to use an approximated way to calculated the specular reflection term by introducing the halfway vector**

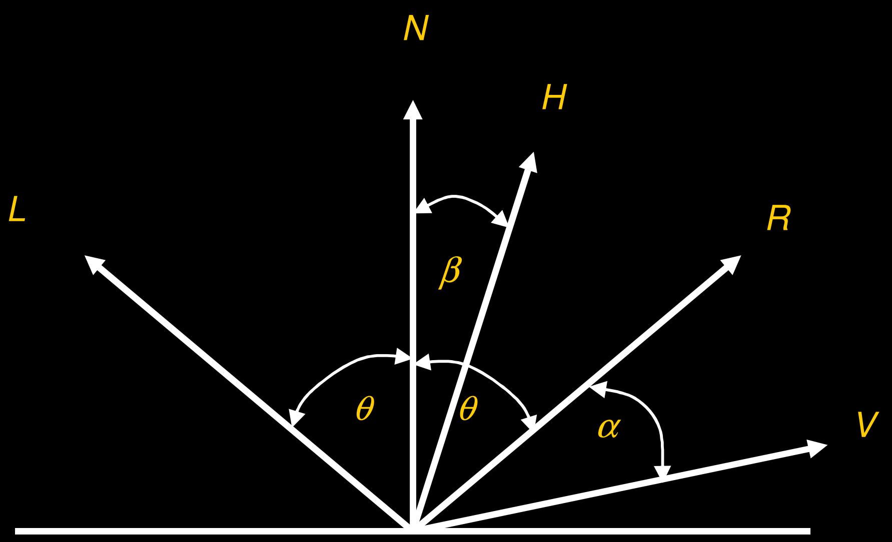
The Halfway Vector

- **The halfway vector is the normalized vector halfway between the viewpoint and the light vector**

$$H = \frac{L+V}{|L+V|}$$

$$\theta + \beta = \theta - \beta + \alpha$$

$$\beta = \frac{1}{2}\alpha$$



Multiple Light Sources

$$I = I_a k_a + \sum_{p=1}^{m} f_p I_p (k_d (N \cdot L_p) + k_s (N \cdot H_p)^{n'})$$

I: Intensity of final illumination

I_a: Intensity of ambient light

k_a: Ambient reflection coefficient

f_p: Attenuation function of point light source *p*

k_d: Diffuse reflection coefficient

N: Normalized normal vector

L_p: Normalized light source direction of point light source *p*

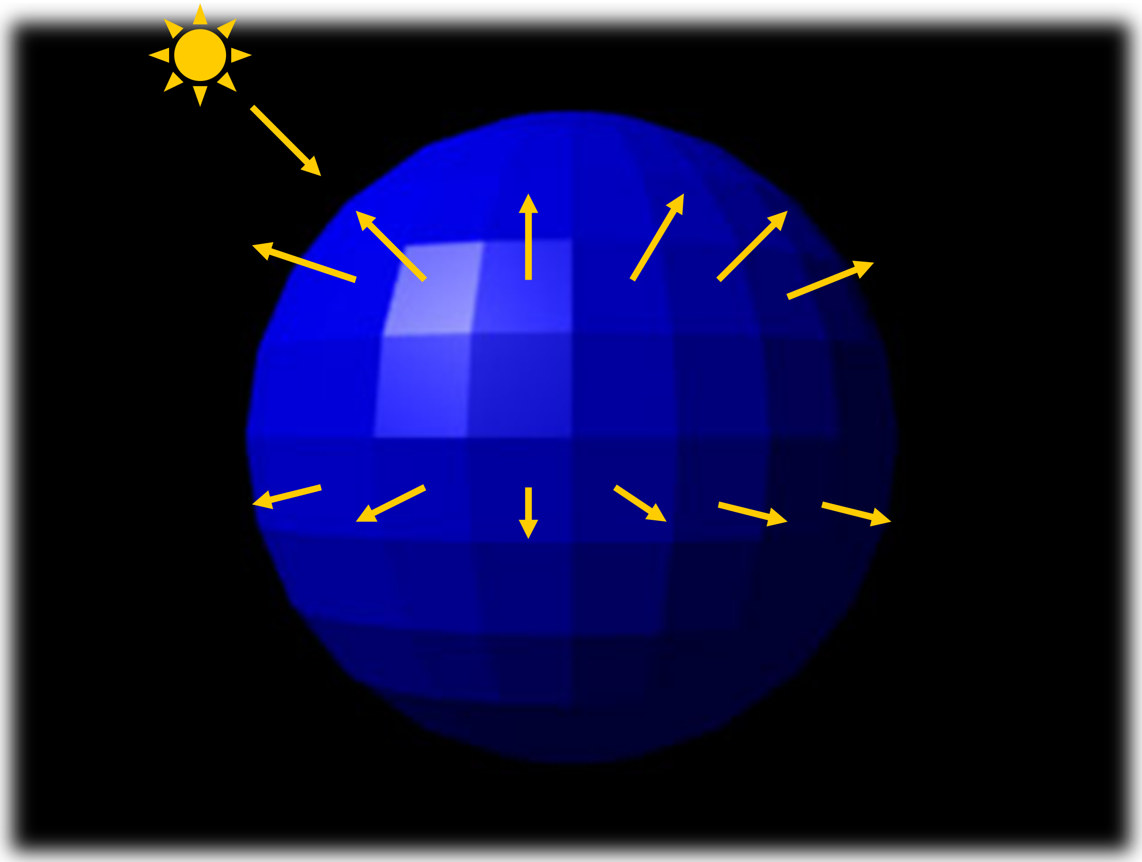
k_s: Specular reflection coefficient

H_p: Normalized half vector between viewpoint and the point light source *p*

n': Material's specular reflection exponent (Blinn-Phong Reflection Model)

Flat Shading

- Using face normal to derive polygon color

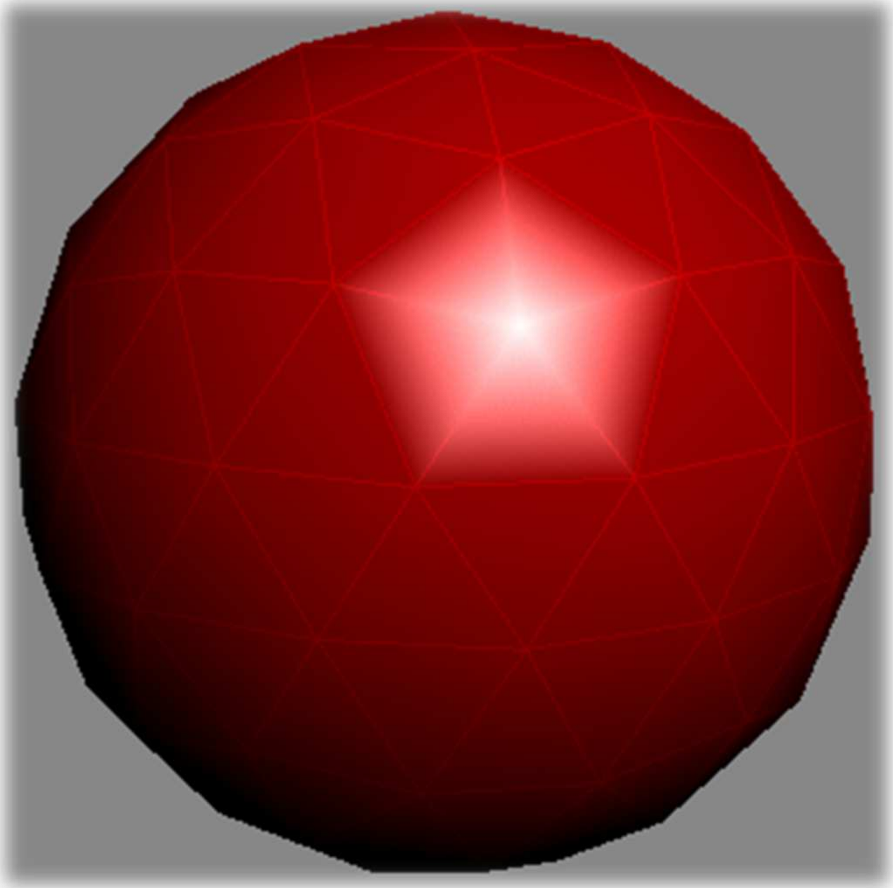


Smooth Shading

- The colors for the interior of the polygon are interpolated between vertex colors

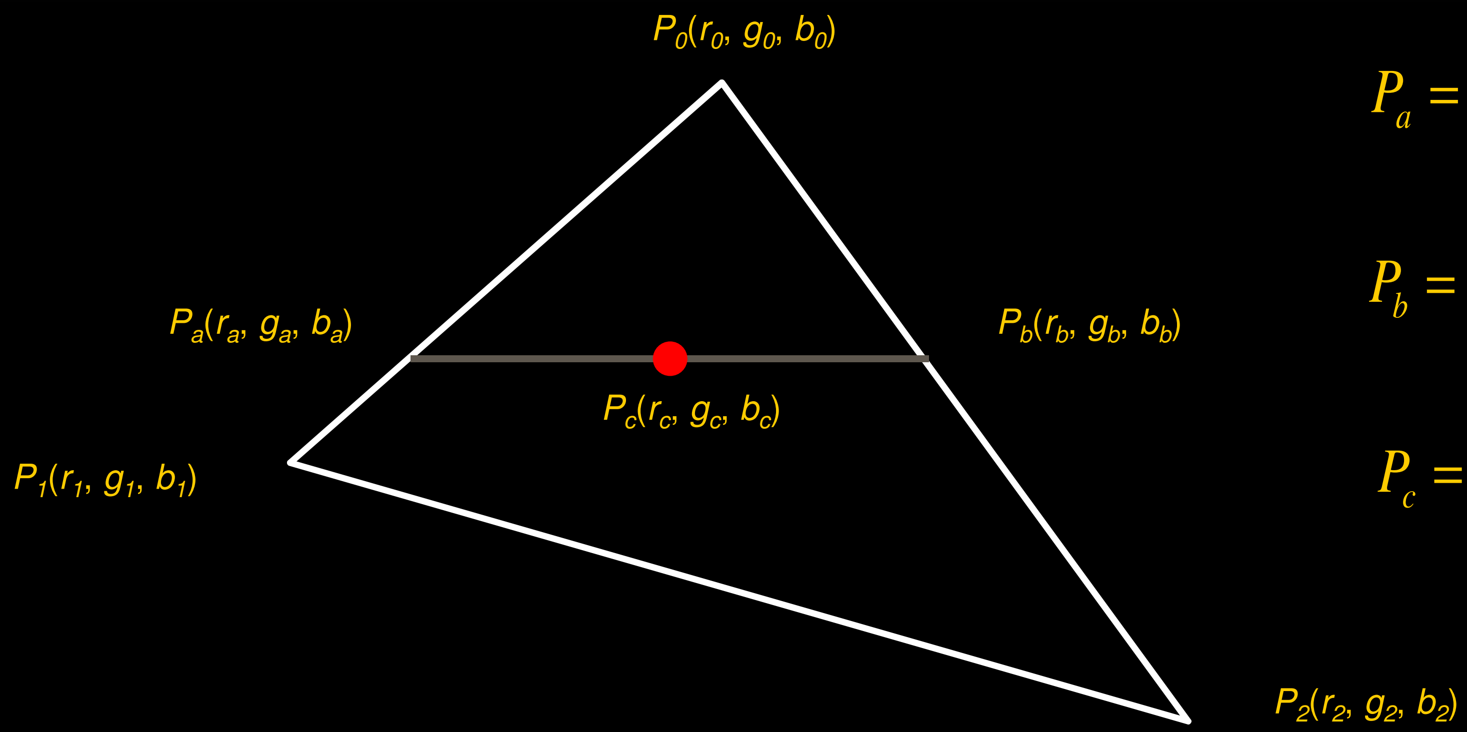
Smooth Shading

- Gouraud Shading



Smooth Shading

- **Gouraud Shading**
  - Compute colors for each vertices respectively
  - Interpolate pixel colors through vertex colors



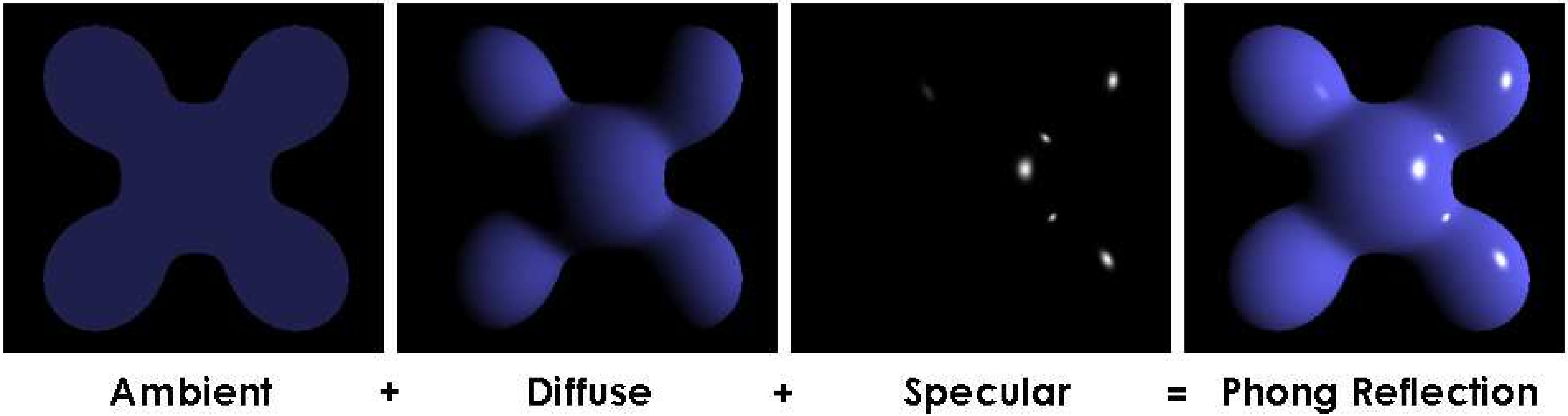
$$P_a = P_0\left(1-\frac{y_a - y_0}{y_1 - y_0}\right) + P_1\frac{y_a - y_0}{y_1 - y_0}$$

$$P_b = P_0\left(1-\frac{y_b - y_0}{y_2 - y_0}\right) + P_2\frac{y_b - y_0}{y_2 - y_0}$$

$$P_c = P_a\left(1-\frac{x_c - x_a}{x_b - x_a}\right) + P_b\frac{x_c - x_a}{x_b - x_a}$$

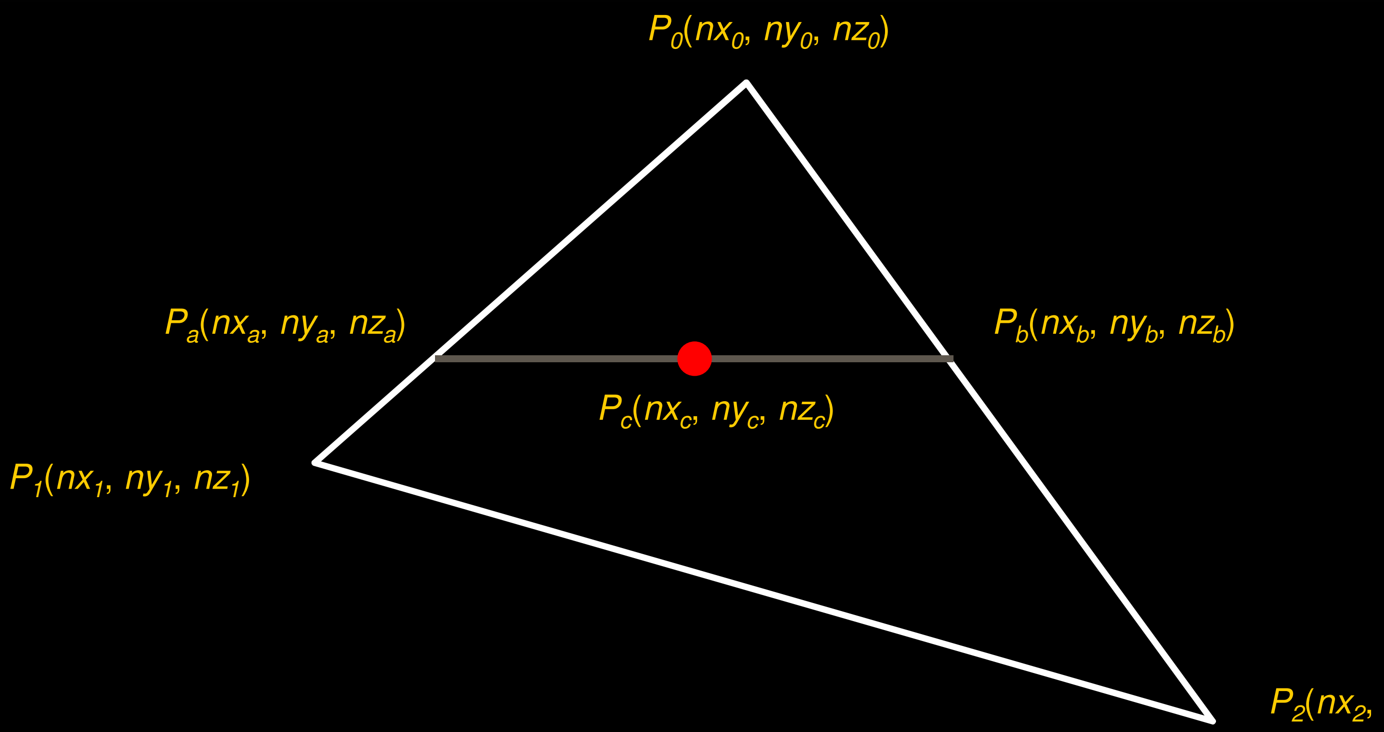
Smooth Shading

- Phong Shading



Smooth Shading

- **Phong Shading**
  - **Interpolate pixel normal through vertex normals**
  - **Compute pixel color with derived pixel normal**



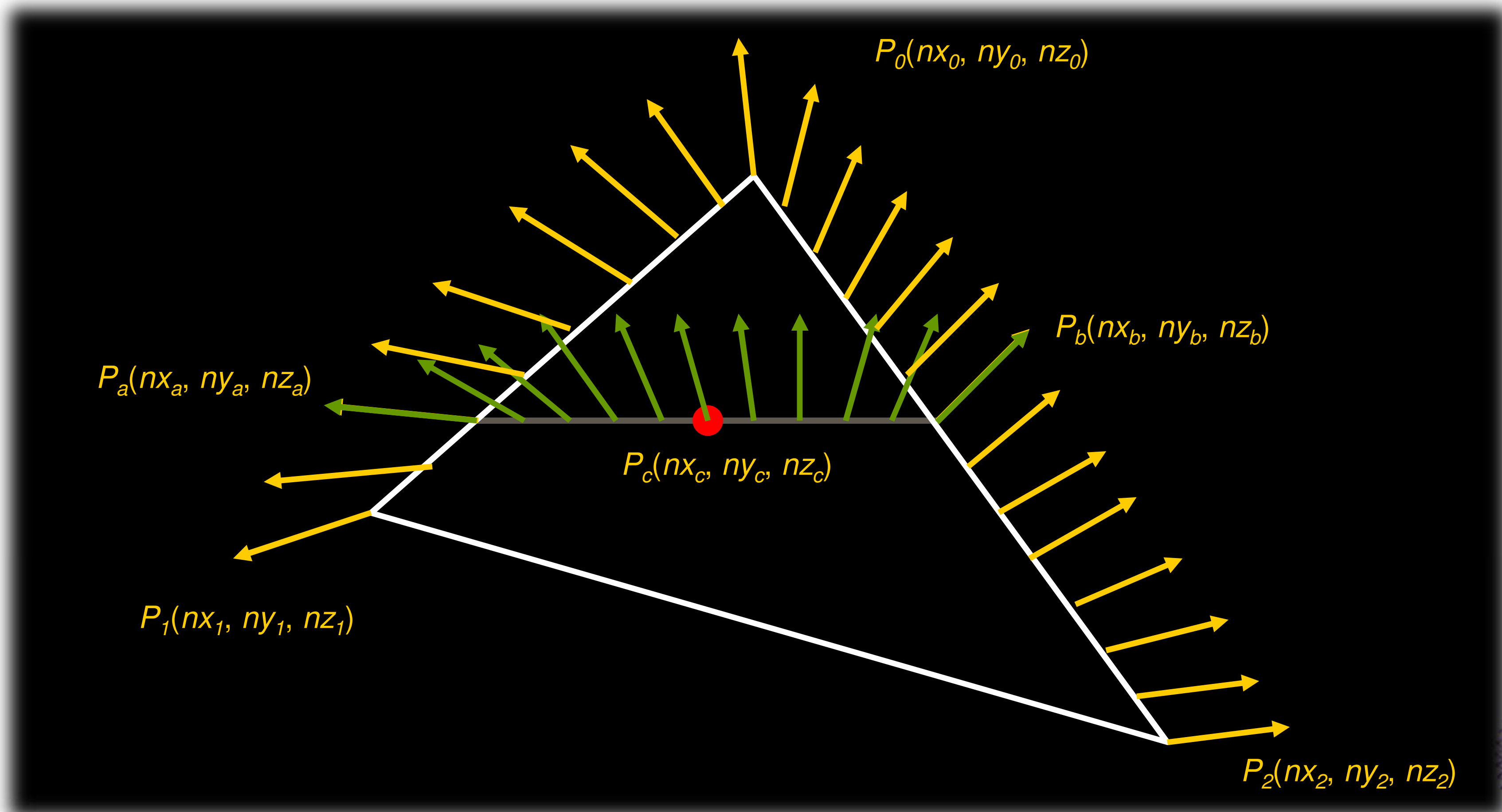
$$P_a = P_0\left(1 - \frac{y_a - y_0}{y_1 - y_0}\right) + P_1 \frac{y_a - y_0}{y_1 - y_0}$$

$$P_b = P_0\left(1 - \frac{y_b - y_0}{y_2 - y_0}\right) + P_2 \frac{y_b - y_0}{y_2 - y_0}$$

$$P_c = P_a\left(1 - \frac{x_c - x_a}{x_b - x_a}\right) + P_b \frac{x_c - x_a}{x_b - x_a}$$

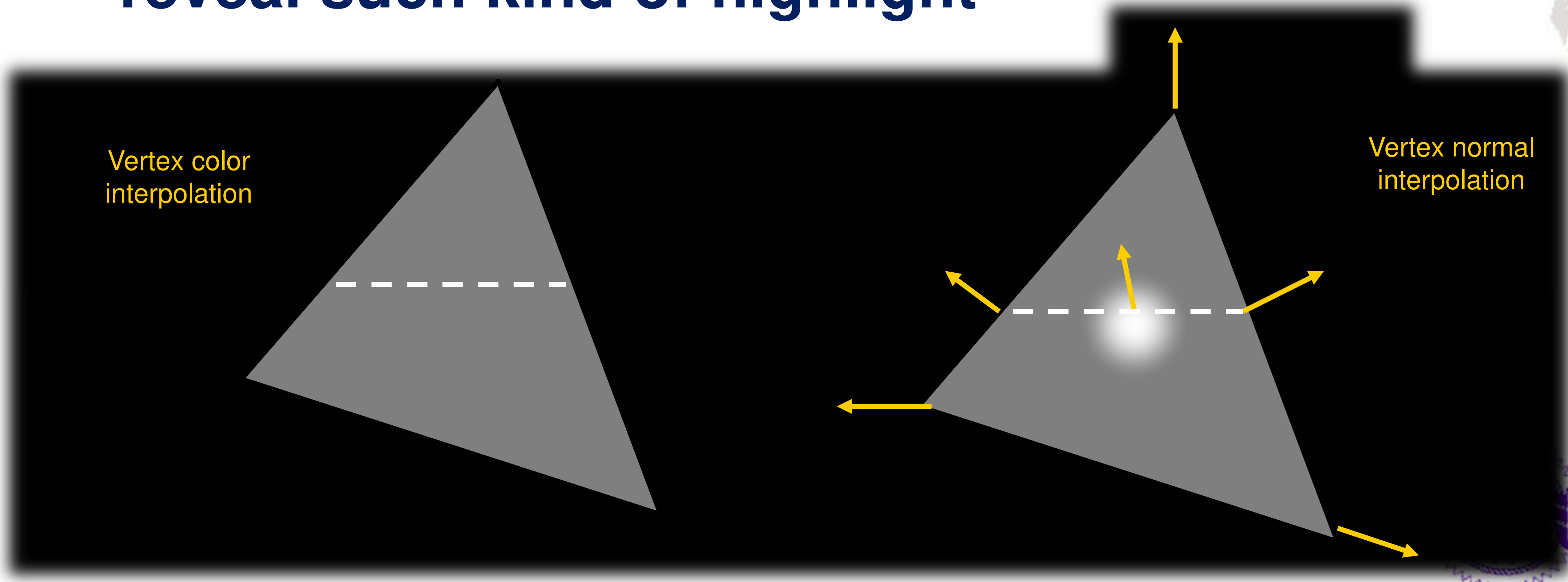
Smooth Shading

- Phong Shading
  - Interpolate pixel normal through vertex normals



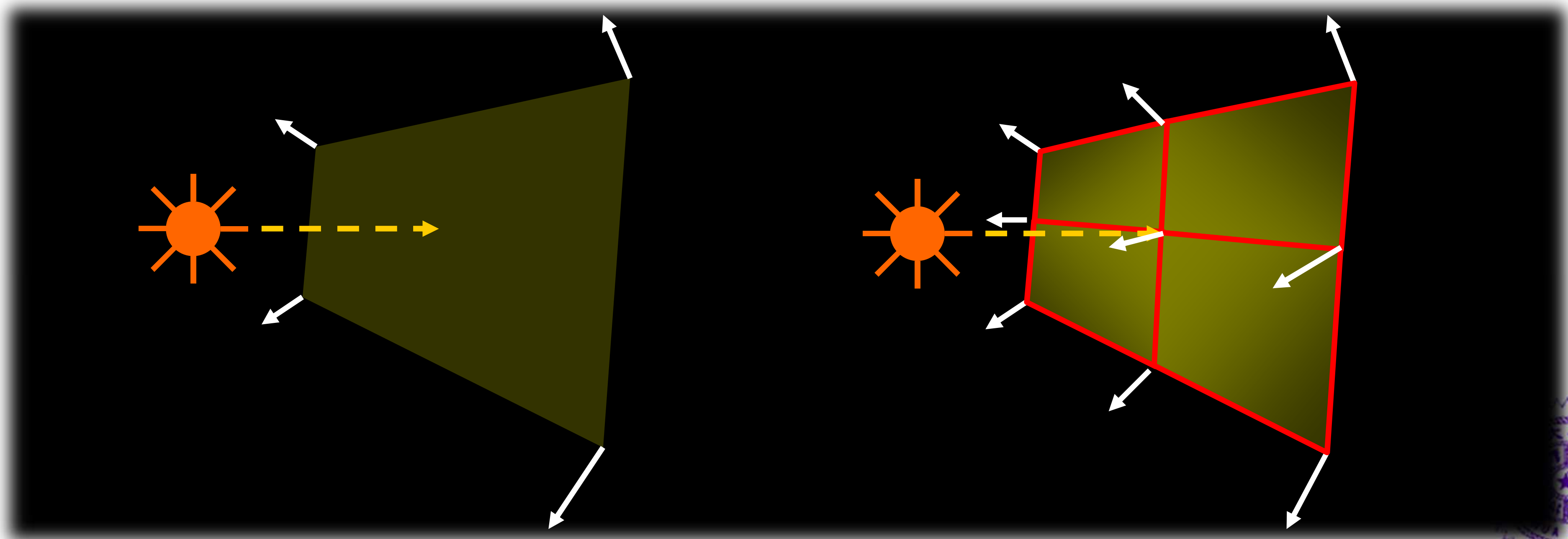
Smooth Shading

- Difference between Gouraud shading and Phong shading
  - If specular highlight is located inside the triangle only, then Gouraud shading cannot reveal such kind of highlight



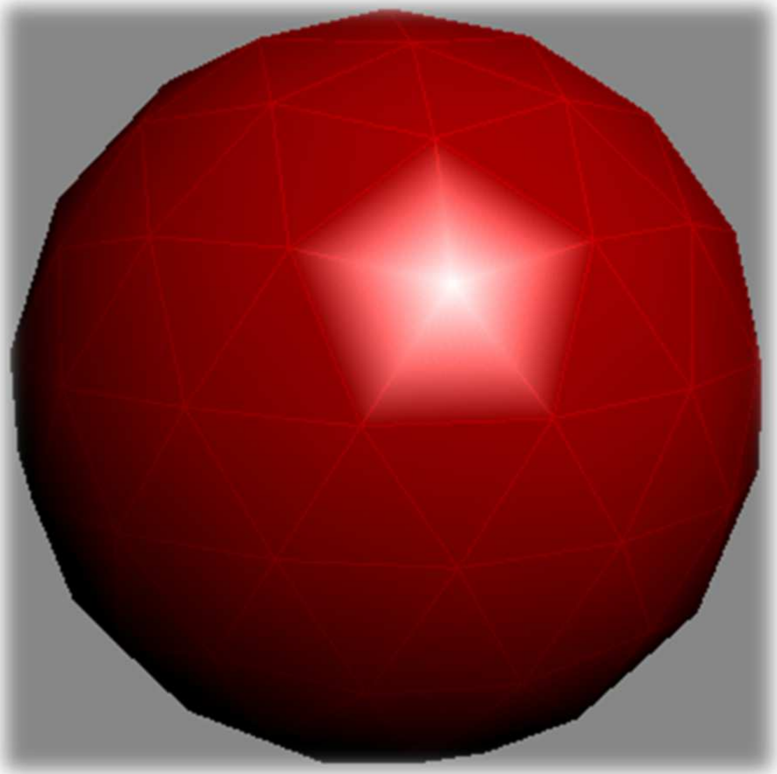
Polygon Size Mattered

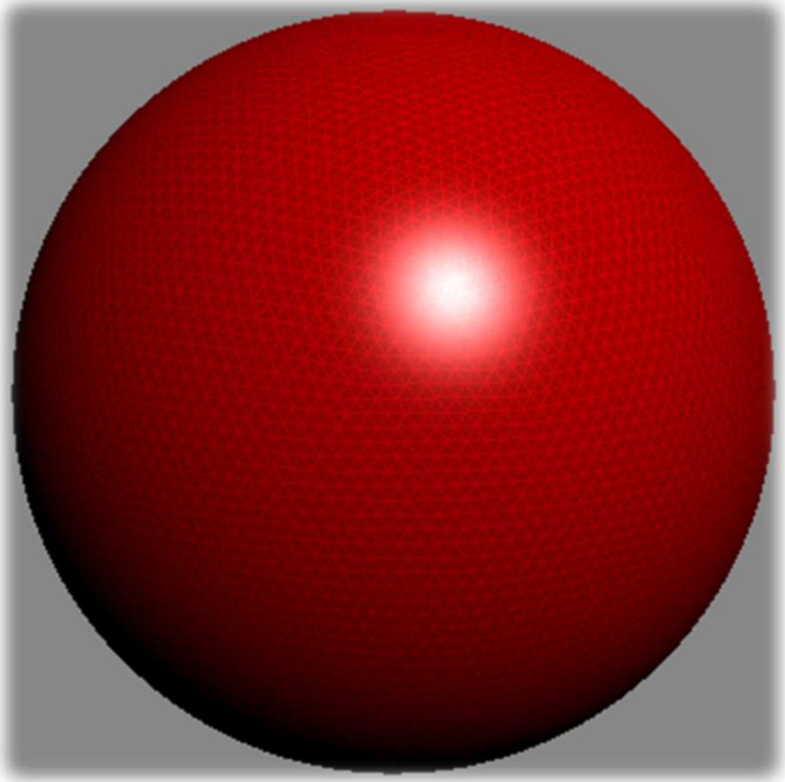
- Large polygon size might degrade the lighting quality
  - Darker if the light source is closer to the polygon
  - No specular highlight can be perceived in the middle of a polygon



Polygon Size Mattered

- Subdivide large polygon into smaller polygons to gain better shading result

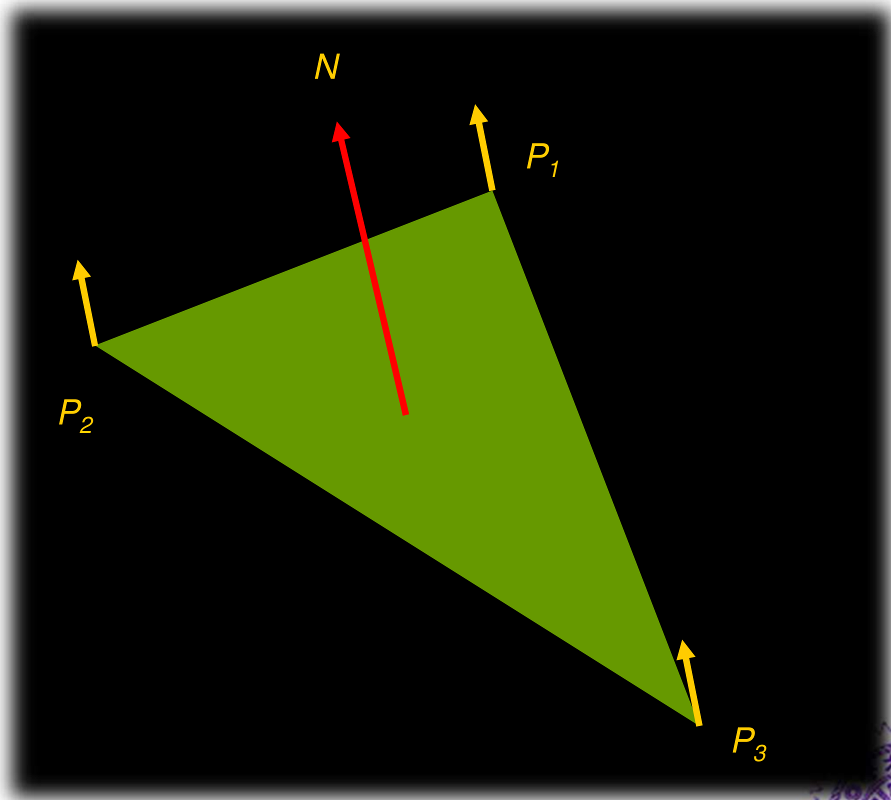




Vertex Normal Derivation

- Flat shading
  - Vertex normal is equal to polygon face normal

$$N = (P_2 - P_1) \times (P_3 - P_1)$$

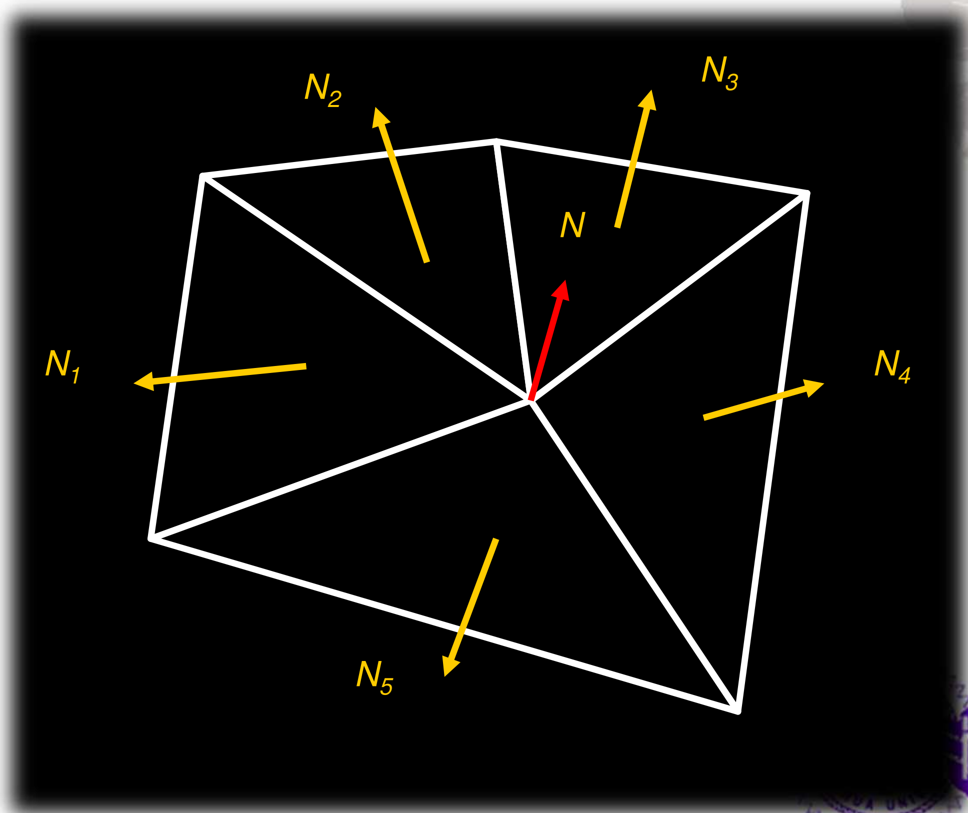


Vertex Normal Derivation

- Smooth shading
  - Vertex normal is equal to the sum of polygon face normals of adjacent polygons

$$N_{sum} = (N_1 + N_2 + \ldots + N_m)$$

$$N = N_{sum} / |N_{sum}|$$

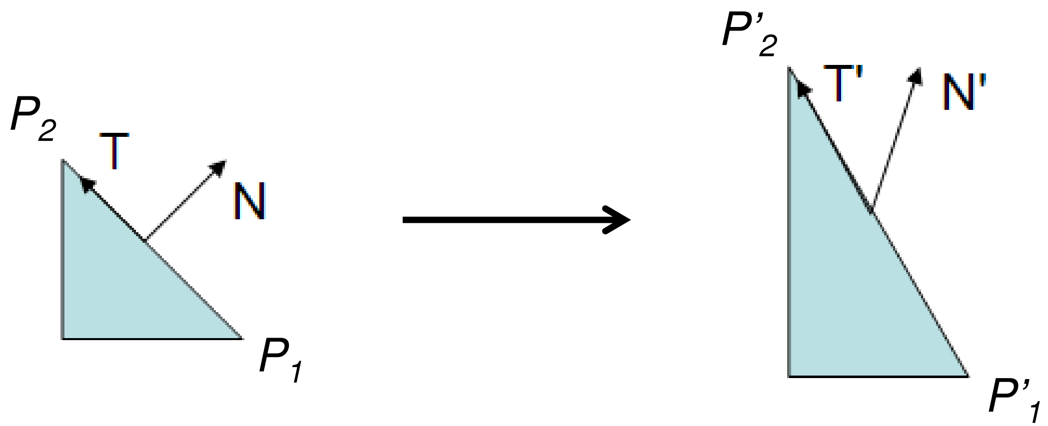


Normal Transformation

- Model transform and viewing transform together can transform a vertex from object space to eye space
- Normal is related to lighting process and the lighting calculation is performed in eye space
- So, normal has to transform to eye space as well
- But, what will happen if we transform normal using the same model-view matrix?

Normal Transformation

- **If we transform normal using model-view matrix M, then…**



$T = P_2 - P_1$
$M \cdot T = M \cdot (P_2 - P_1)$
$T' = M \cdot P_2 - M \cdot P_1 = P'_2 - P'_1$

$N = Q_2 - Q_1$
$M \cdot N = M \cdot (Q_2 - Q_1)$
$N' = M \cdot Q_2 - M \cdot Q_1 = Q'_2 - Q'_1$

$N \cdot T = 0$
But, after normal transformed by model-view matrix M
$N' \cdot T' \neq 0$

Normal Transformation

- **Normal transformation should be taking care if you have done any model and viewing transformations to the geometric data**

A vector in homogeneous coordinate is represented as
$T = (x, y, z, 0) = (x_2, y_2, z_2, 1) – (x_1, y_1, z_1, 1)$

A normal in homogeneous coordinate is represented as $N = (n_x, n_y, n_z, 0)$

Since T and N are orthogonal, thus $N \cdot T = 0$
We also know that after model-view transformation, T' and N' should remain orthogonal.
That is, $N' \cdot T' = 0$.

Normal Transformation

- **Represent the dot product by matrix multiplication and let *M* be the model-view matrix, we have**

$$N \cdot T = \begin{pmatrix} n_x & n_y & n_z & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 0 \end{pmatrix} = 0$$

Normal Transformation

Eye space normal

Object space normal

$$N' \cdot T' = \boxed{\begin{pmatrix} n_x & n_y & n_z & 0 \end{pmatrix} M^{-1}} \boxed{M \begin{pmatrix} x \\ y \\ z \\ 0 \end{pmatrix}} = 0 \Rightarrow \begin{pmatrix} n'_x \\ n'_y \\ n'_z \\ 0 \end{pmatrix} = (M^{-1})^T \begin{pmatrix} n_x \\ n_y \\ n_z \\ 0 \end{pmatrix}$$

Model-view Transformation

Lighting Procedure

- Define the vertex normals
  - Lighting is the interaction between vertex normals and the light source
- Define light sources
  - Light source properties
- Select lighting model
  - Determine which lighting equation is used
- Define material properties
  - Define the percentage of reflectance to the light source

Complete OpenGL Lighting Formula

Object can emit light itself

Global ambient light

Light source contribution

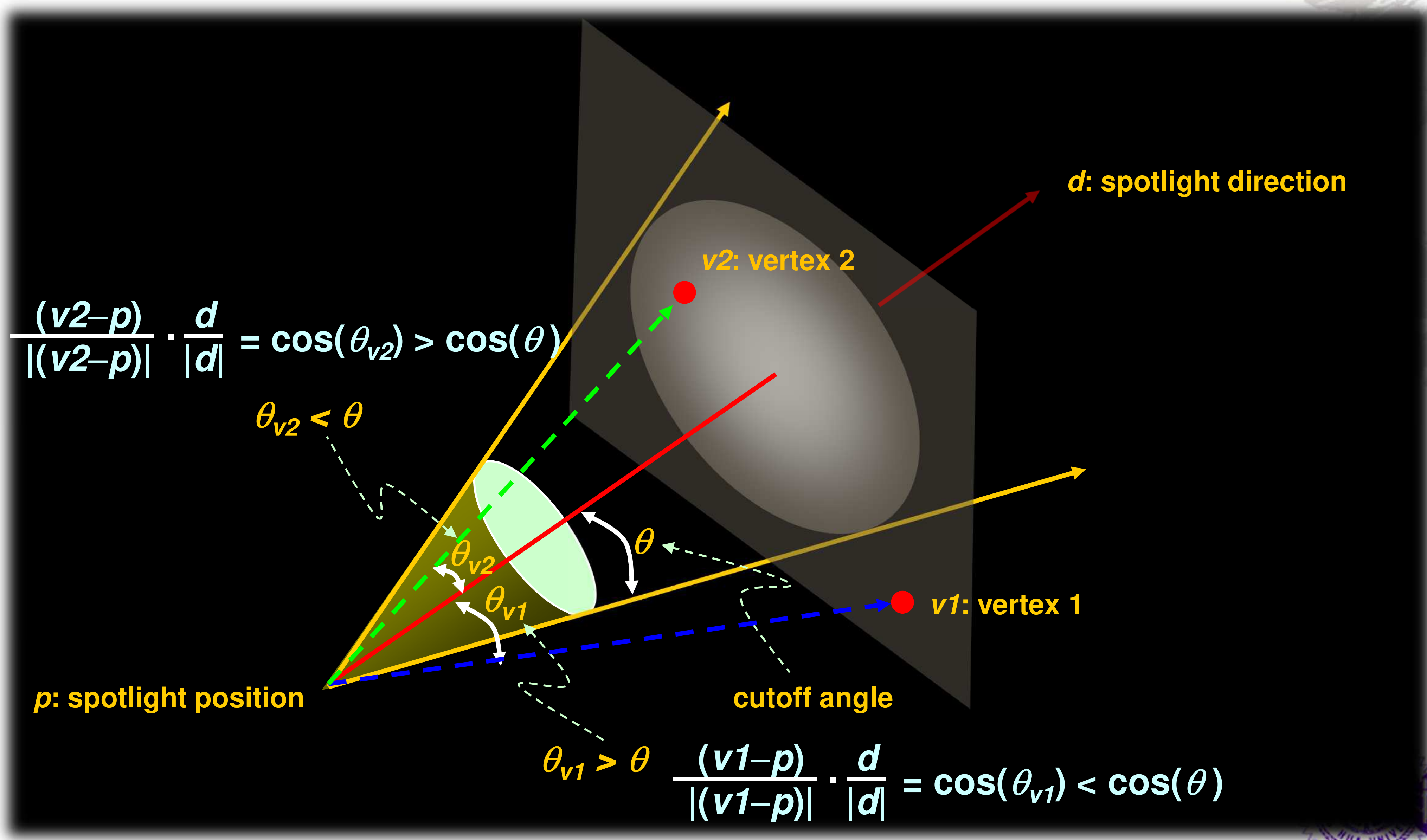
$$\text{vertex color} = \boxed{\text{emission}_{\text{material}}} +$$

$$\boxed{\text{ambient}_{\text{light model}} * \text{ambient}_{\text{material}}} +$$

$$\boxed{\begin{aligned}&\sum_{i=0}^{n-1}\left(\frac{1}{k_c + k_l d + k_q d^2}\right)_i * (\text{spotlight effect})_i * \\ &[\text{ambient}_{\text{light}} * \text{ambient}_{\text{material}} + \\ &(\max\{\mathbf{L}\cdot\mathbf{n}, 0\}) * \text{diffuse}_{\text{light}} * \text{diffuse}_{\text{material}} + \\ &(\max\{\mathbf{s}\cdot\mathbf{n}, 0\})^{\text{shininess}} * \text{specular}_{\text{light}} * \text{specular}_{\text{material}}]_i\end{aligned}}$$

Reference: http://what-when-how.com/opengl-programming-guide/the-mathematics-of-lighting-opengl-programming/

Spotlight Effect



Spotlight Effect

- Spotlight Effect =
  - 1, if the light source is not a spotlight
  - 0, if the light source is a spotlight but the vertex lies outside the cone of illumination produced by the spotlight
  - Otherwise, spotlight effect = $(\max\{v \cdot d, 0\})^{spot\_exp}$
    - v is the unit vector from the spotlight to the vertex
    - d is the spotlight direction

Q&A