Computer Graphics

by Ruen-Rone Lee
ICL/ITRI

Wrap up from last week

- Shadow
  - Shadow Volume
  - Shadow Map
  - Ambient Occlusion
  - Ray Tracing

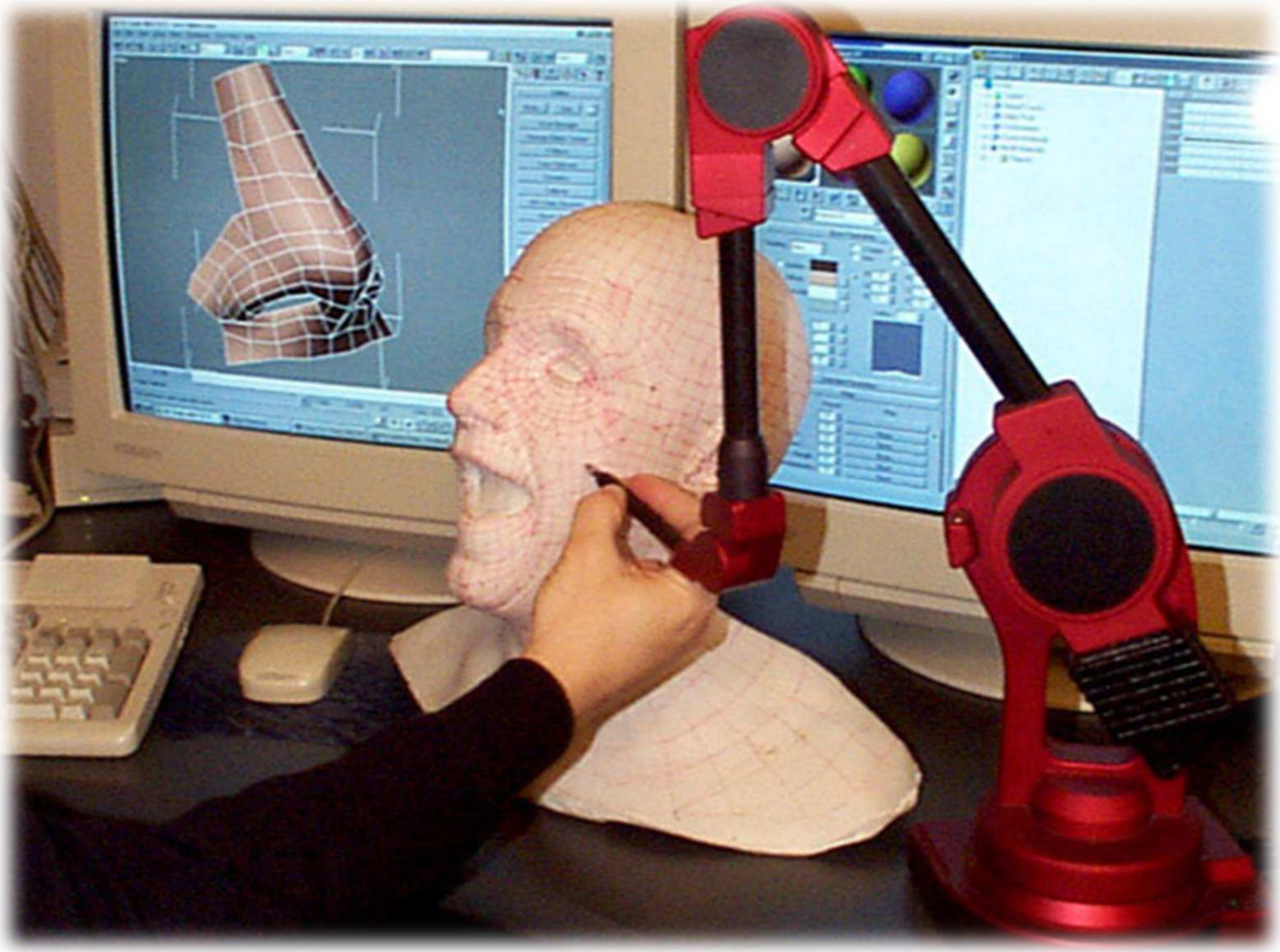
3D Modeling

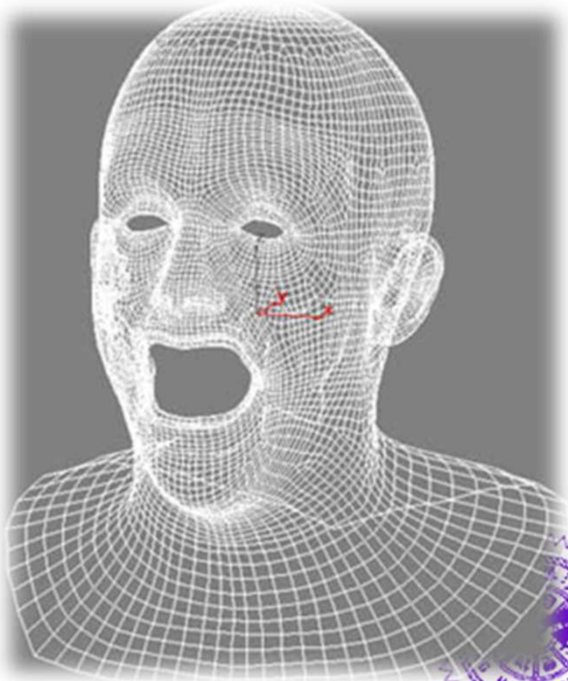
3D Reconstruction

Solid Modeling

Curve and Surface Modeling

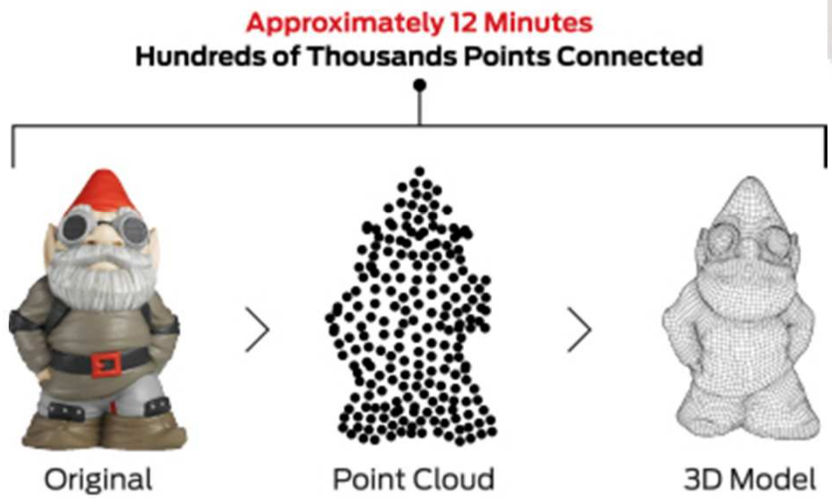
3D Digitizer





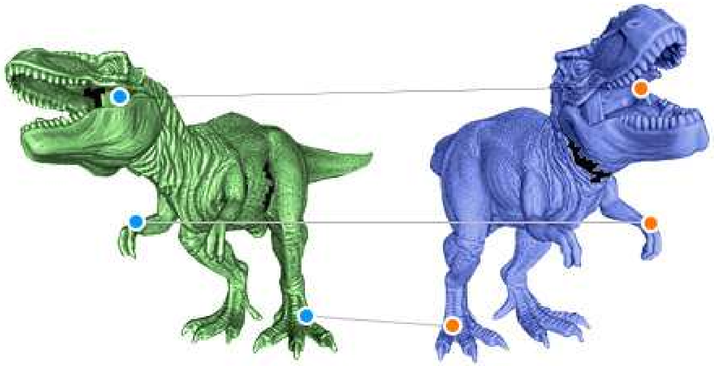
Source: Ghost 3D

3D Digitizer

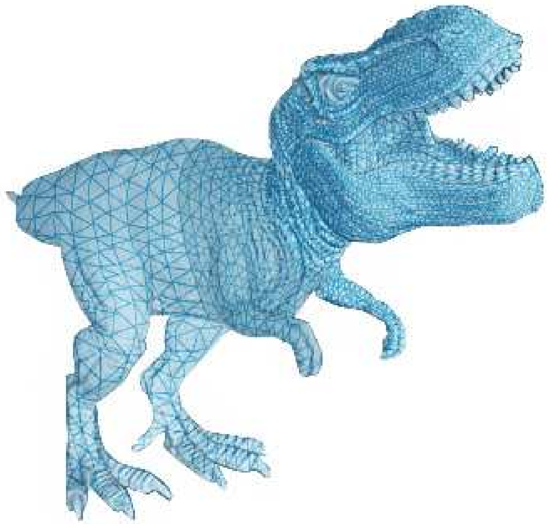


Source: MarkerBot

3D Scanner



Align Scans



Smooth and optimize

Fuse scans into one single 3D model

Texturize

Source: Artec 3D

3D Scanner

Source: Ten 24

3D Scanner


The Stanford
Large Statue
Scanner
truss extensions
for tall statues
laser, range camera,
white light, and color camera

3D Scanner

Source: Artec 3D

3D Modeling Tools

- **AutoCAD, Maya, 3ds Max, Softimage, Blender, Cinema4D, Houdini, …**

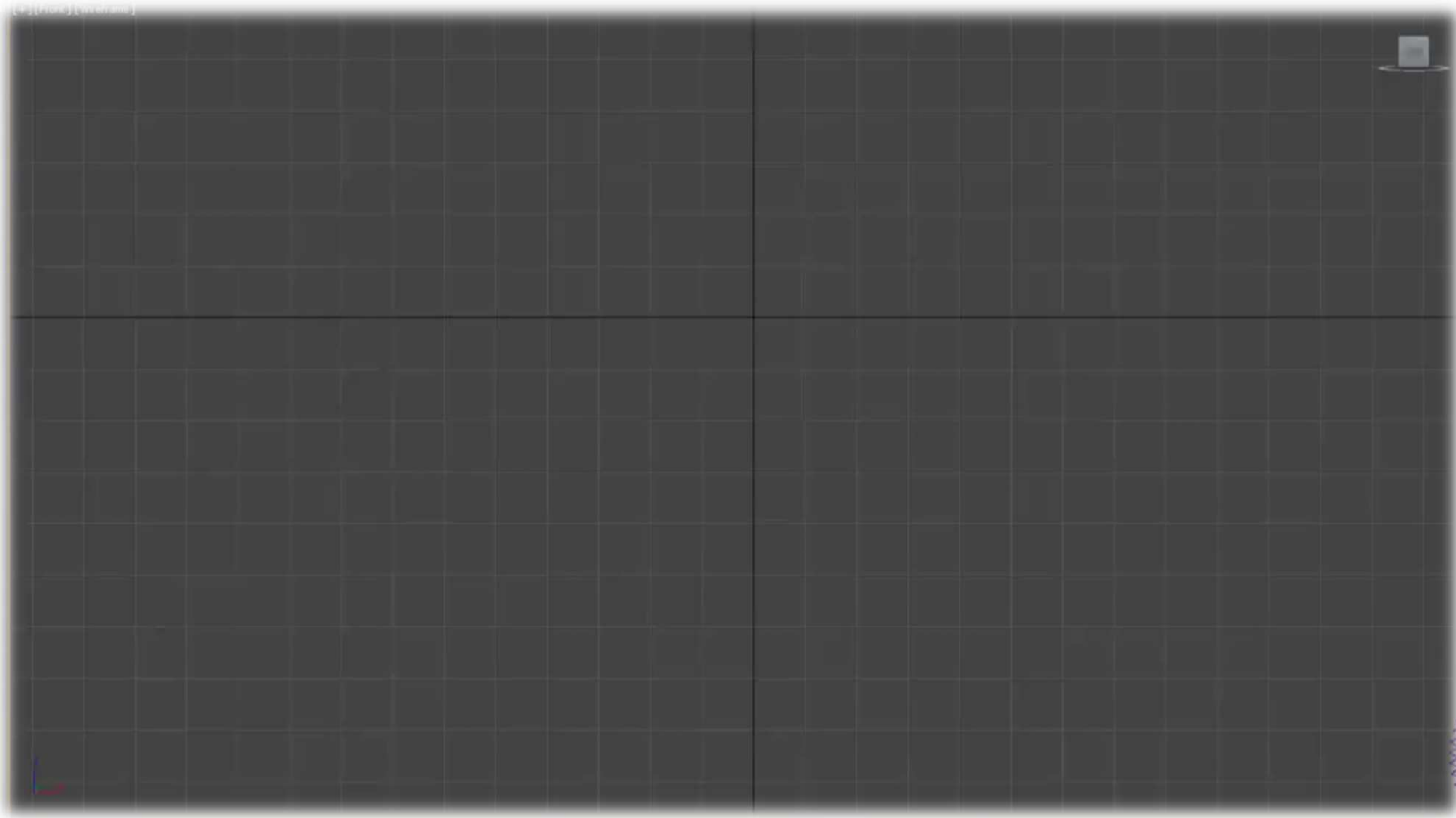


Image-Based Modeling

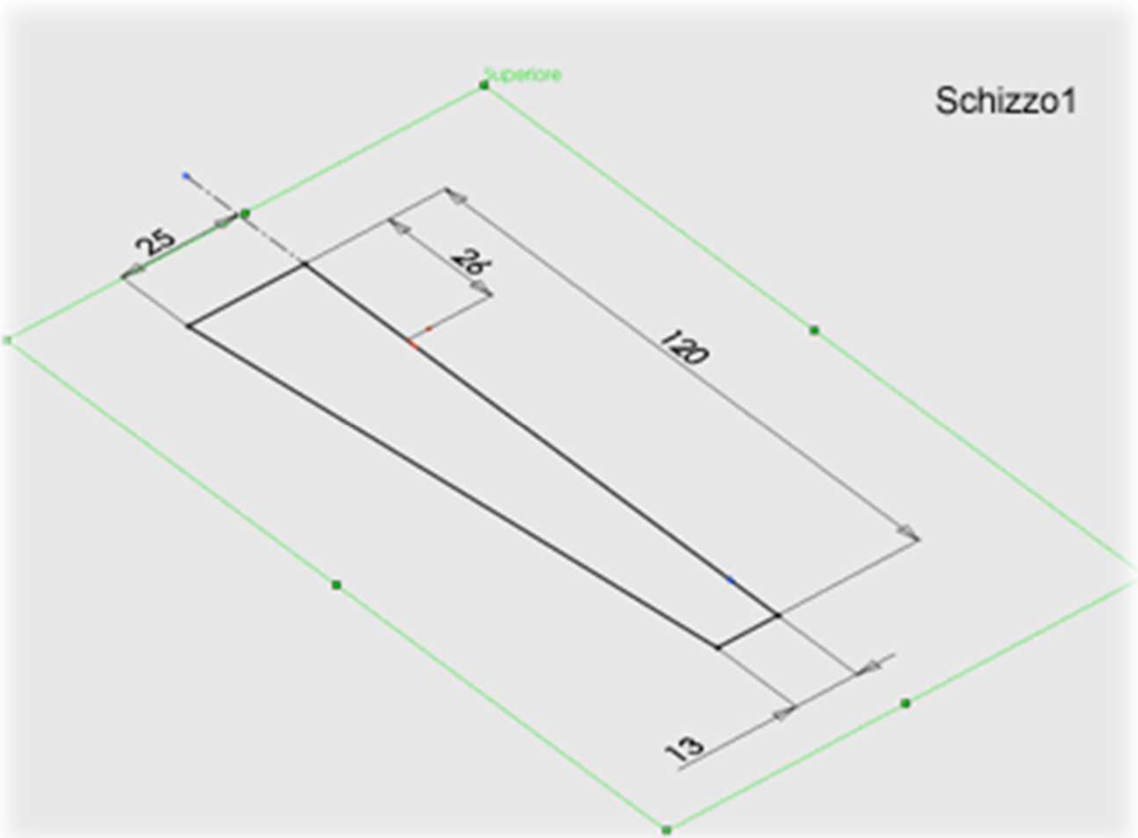
- Image-based modeling and rendering differs from traditional graphics in that both the geometry and appearance of the scene are derived from real photographs

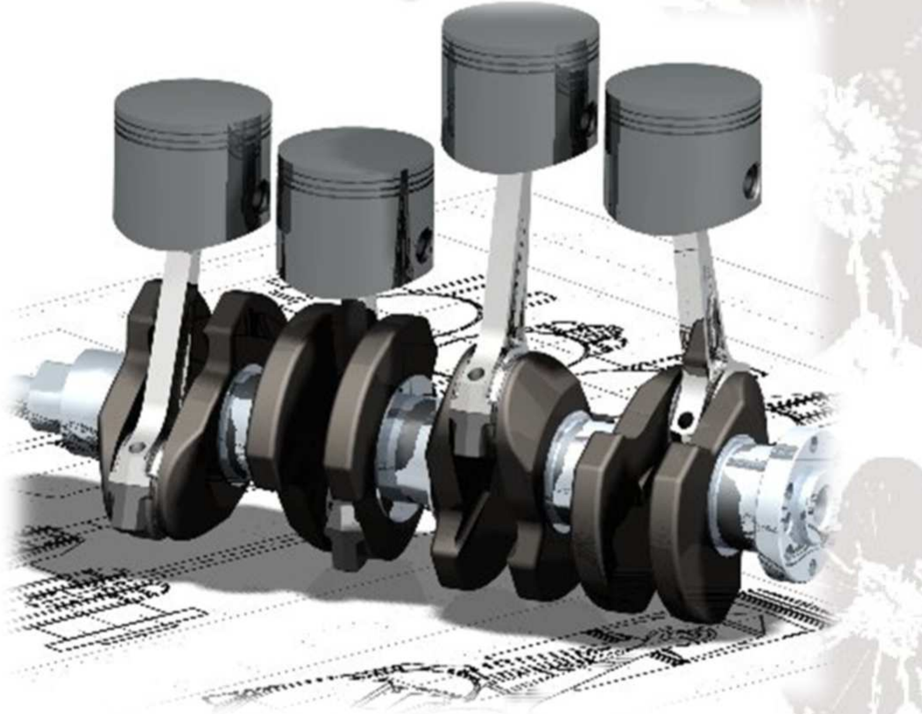
Image-Based Modeling

Image-Based Modeling

Image-Based Modeling

Solid Modeling





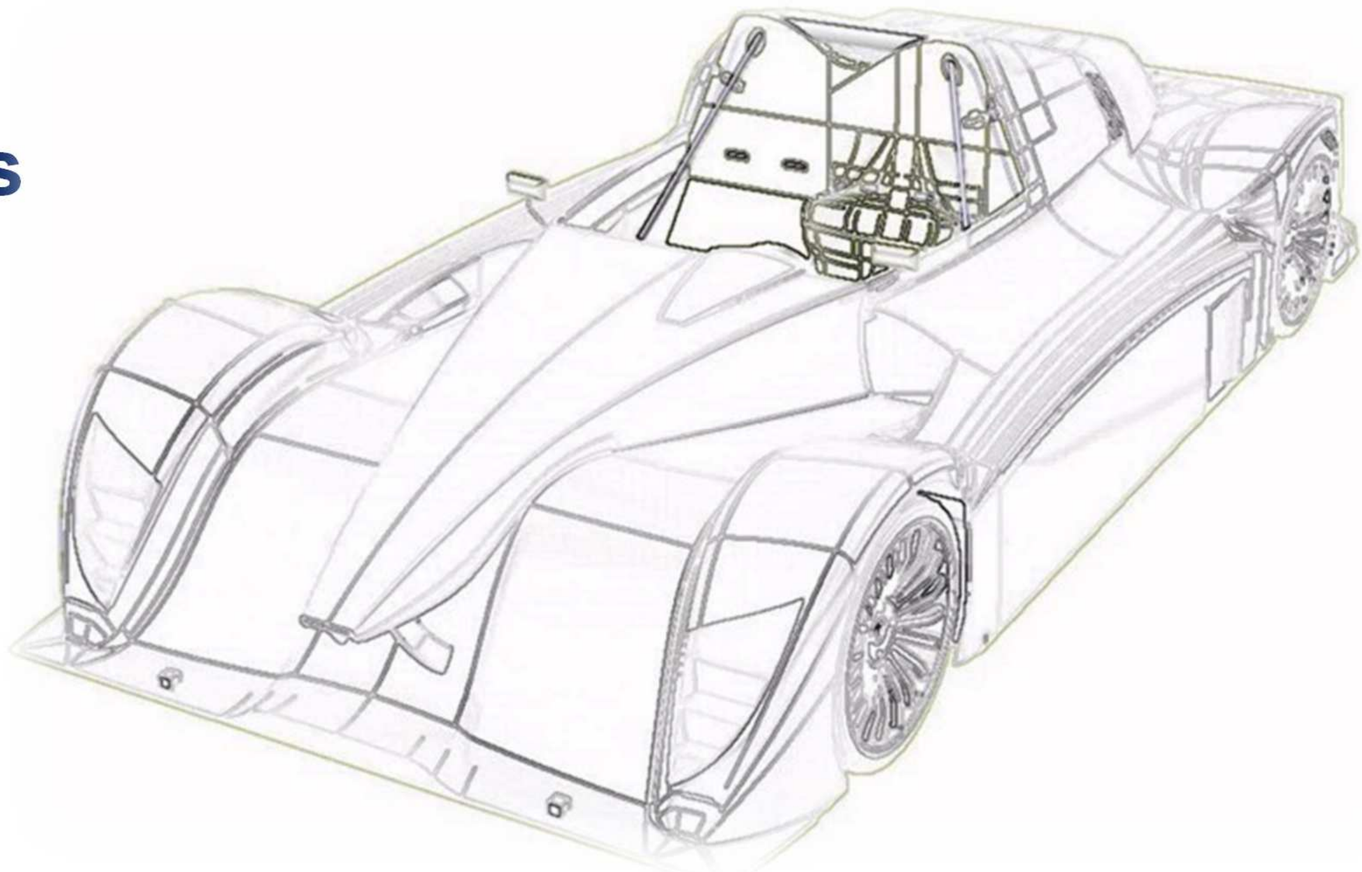
Boundary Representation

Constructive Solid Geometry

Voxel Based

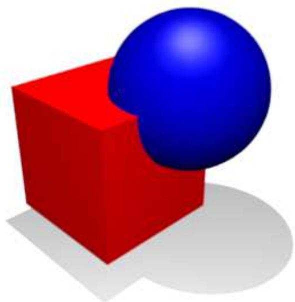
Solid Modeling

- Boundary Representation
  - Faces
  - Edges
  - Vertices

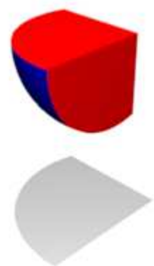


Solid Modeling

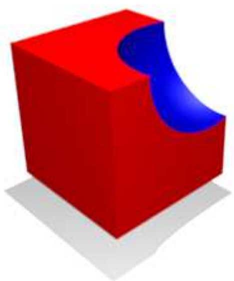
- **Constructive Solid Geometry**
  - **Boolean construction of primitives**



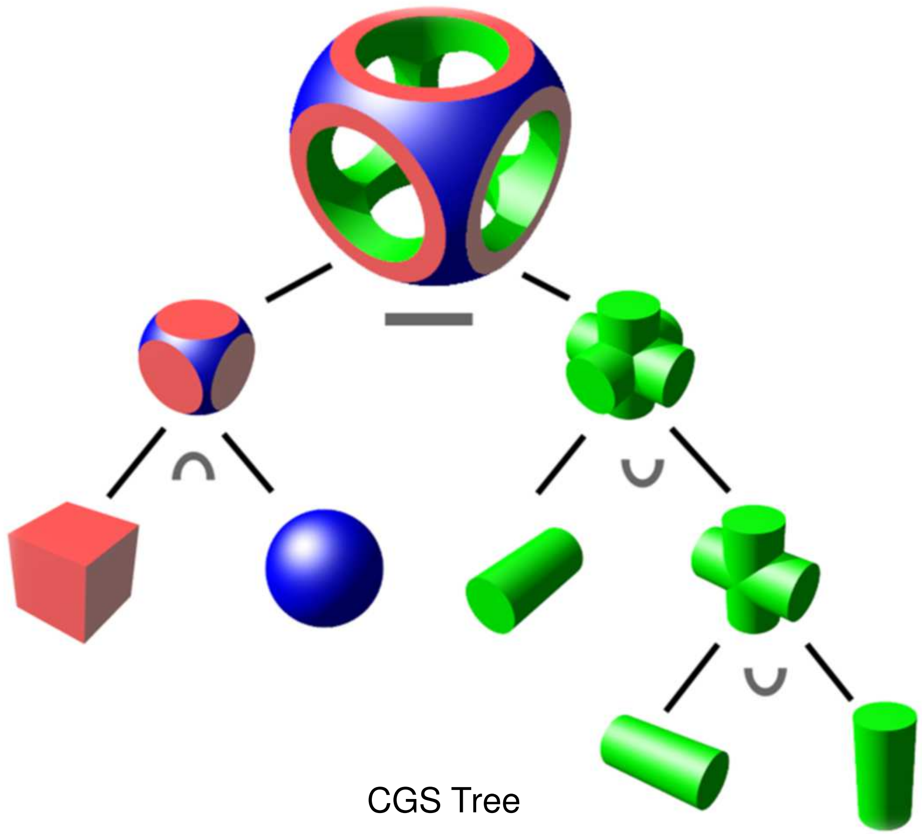
Union



Intersection



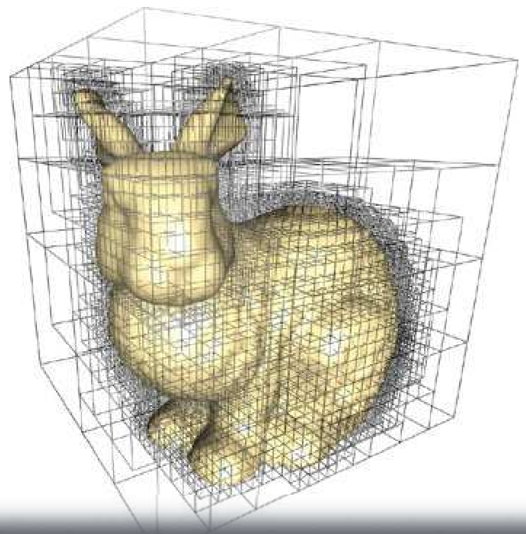
Difference



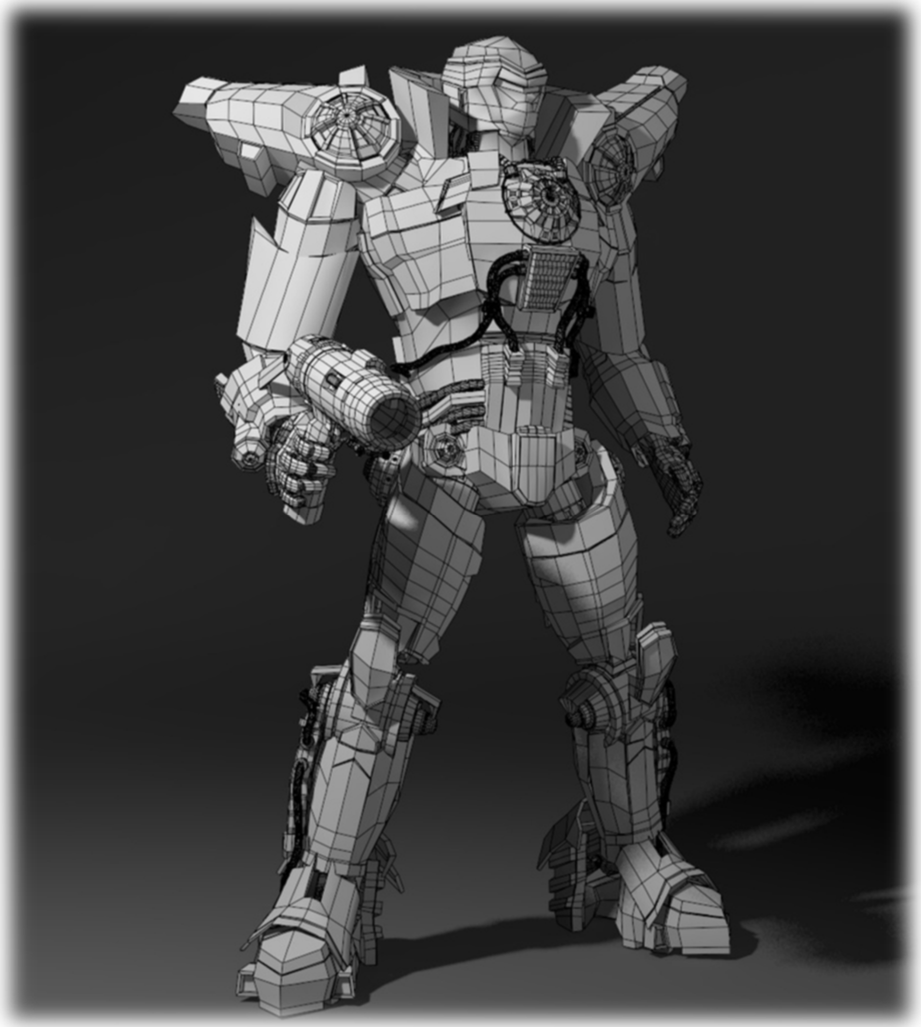
CGS Tree

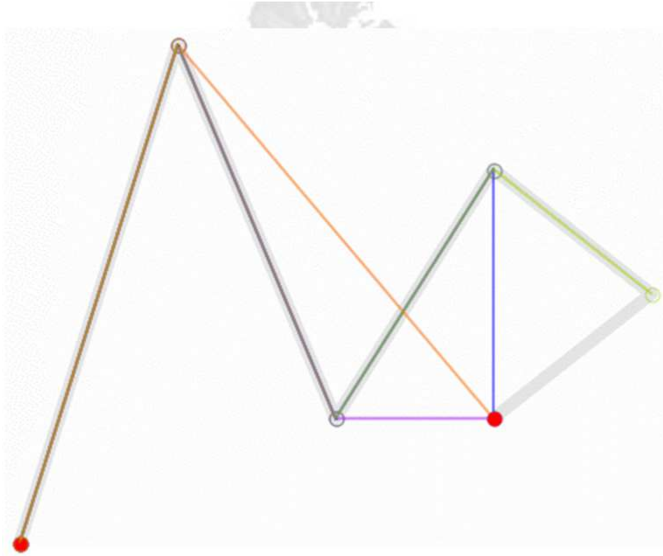
Solid Modeling

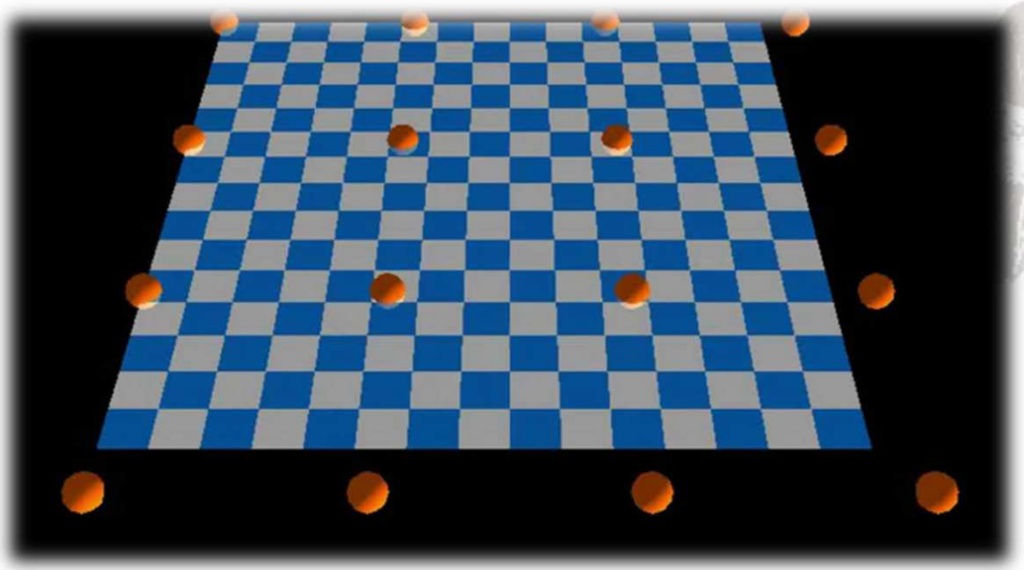
- Spatial Occupancy Enumeration
  - Voxel representation
  - Octree representation



Surface Modeling





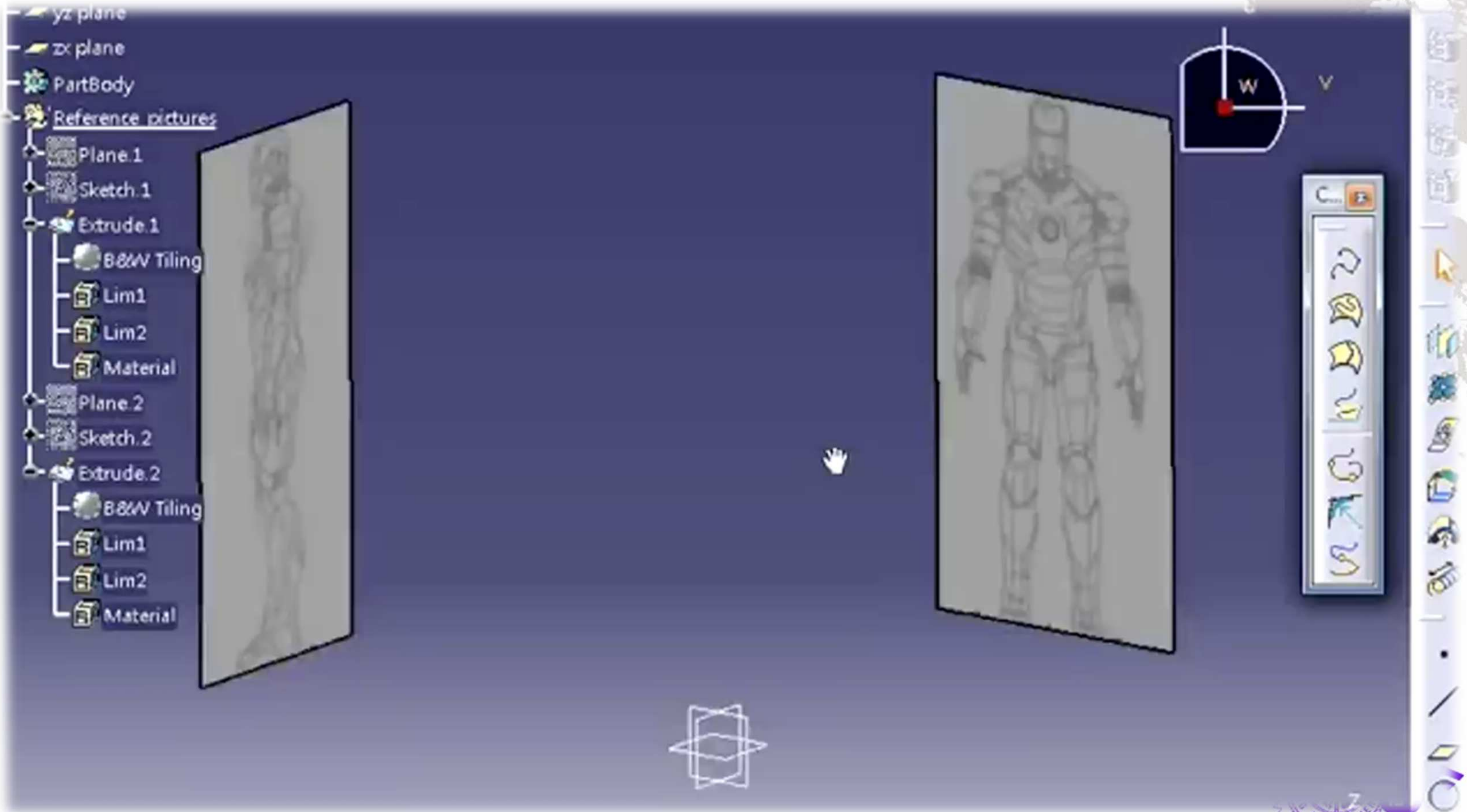


Parametric Curves

Parametric Surfaces

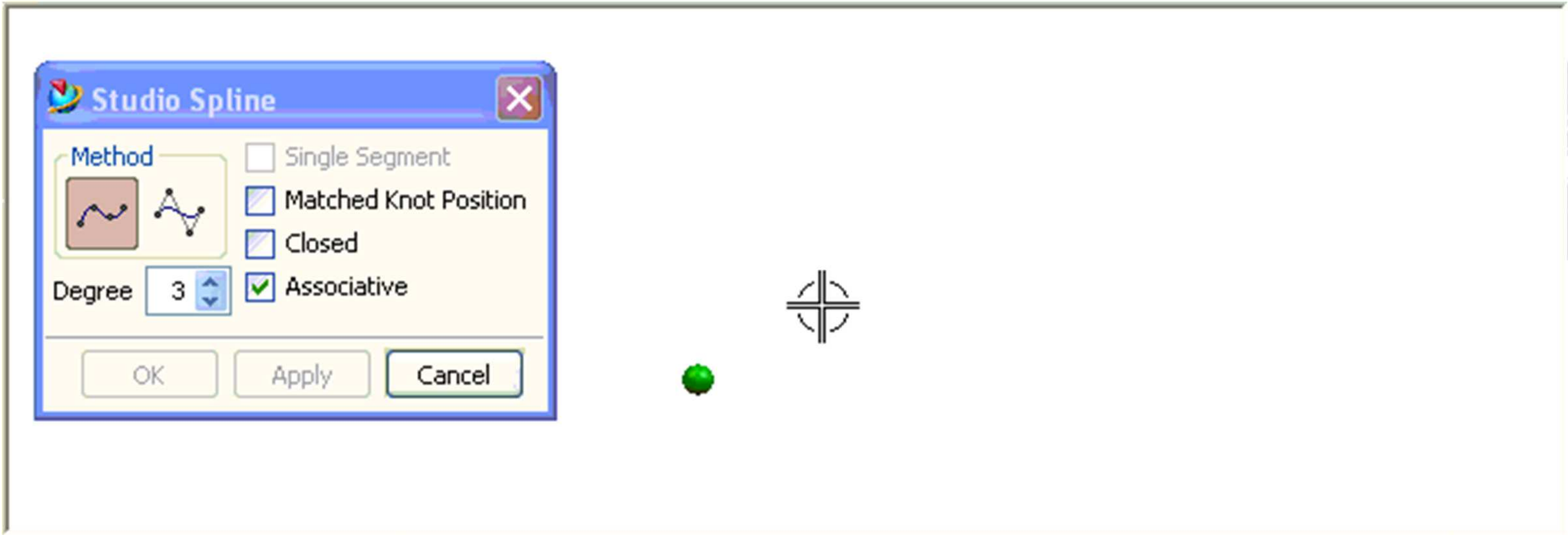
Surface Modeling

Source: CATiA



Curves

- Example



Representation

- **Explicit Form**

  $y = f(x)$ and $z = g(x)$

- **Implicit Form**

  $f(x, y, z) = 0$

Representation

- Parametric Form

  $x = x(t)$, $y = y(t)$, and $z = z(t)$

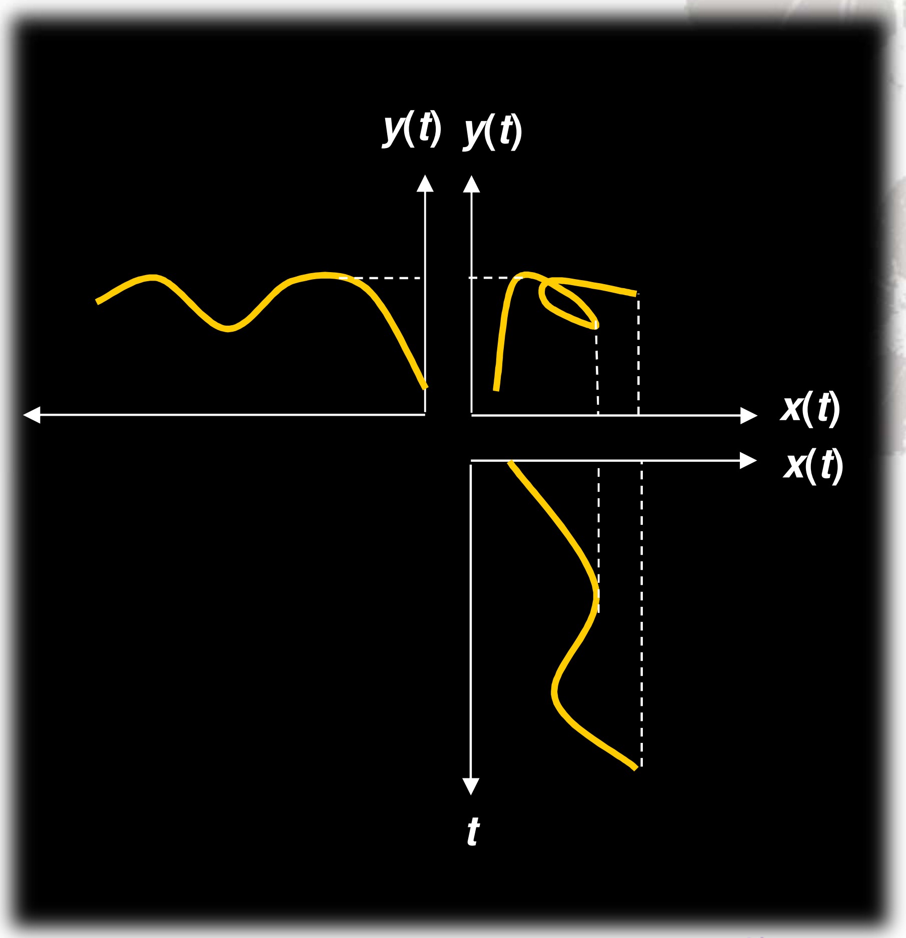
  e.g.

  $$x(t) = a_x t^3 + b_x t^2 + c_x t + d_x$$

  $$y(t) = a_y t^3 + b_y t^2 + c_y t + d_y$$

  $$z(t) = a_z t^3 + b_z t^2 + c_z t + d_z,$$

  $$0 \le t < 1$$



Parametric Form with Control Points

- **General Form**

$$p(t) = \sum_{i=0}^{n} p_i f_i(t)$$

  - **Bézier Spline (Bernstein Polynomial)**

$$f_i(t) = B_{i,n}(t) = \frac{n!}{i!(n-i)!} t^i (1-t)^{n-i} \qquad 0 \le t \le 1$$

  - **B-Spline**

$$f_i(t) = N_{i,k}(t) = \frac{(t-t_i)N_{i,k-1}(t)}{t_{i+k-1}-t_i} + \frac{(t_{i+k}-t)N_{i-1,k-1}(t)}{t_{i+k}-t_{i+1}}$$

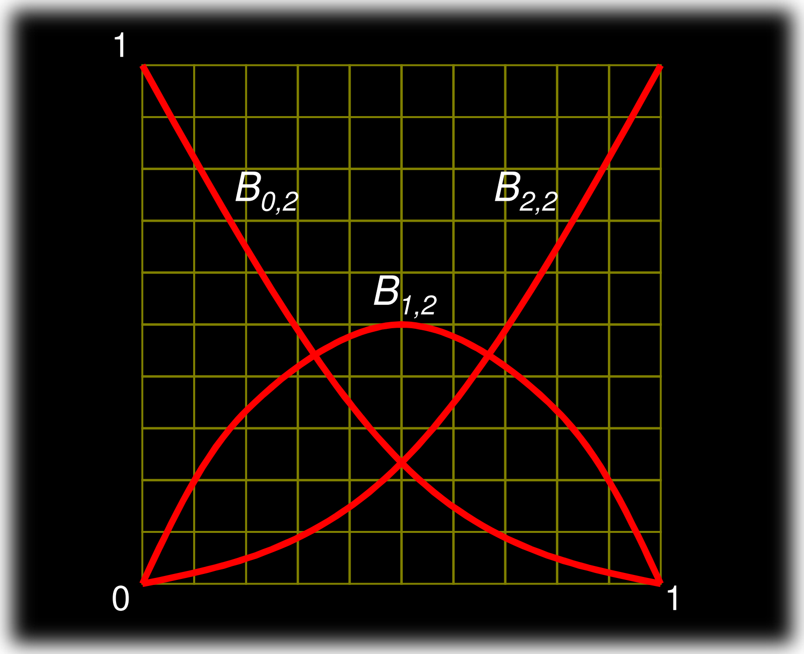
$$N_{i,1}(t) = 1 \quad \text{if } t_i \le t < t_{i+1}$$

$$= 0 \quad \text{otherwise}$$

Bernstein Polynomial

- $n = 2$

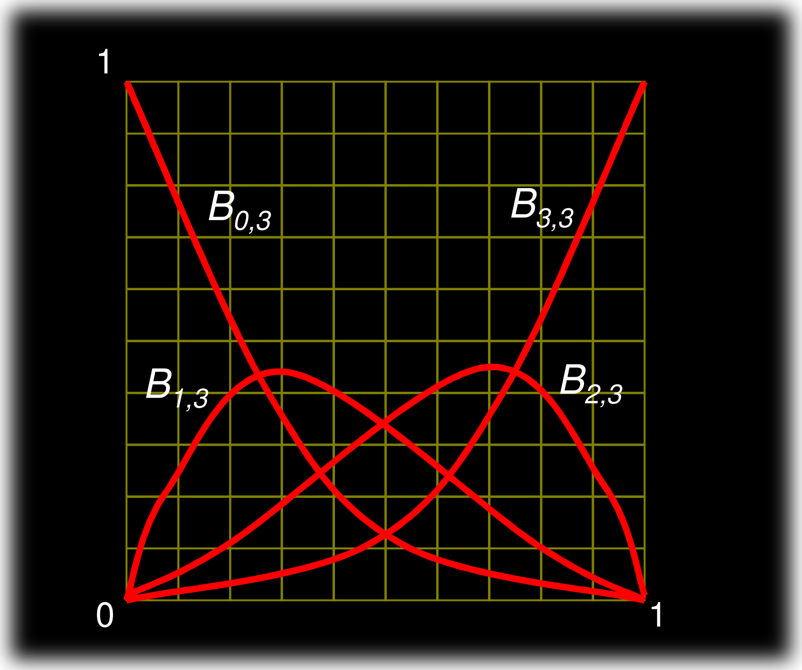
$$\boxed{p(t) = (1-t)^2 p_0 + 2t(1-t)\,p_1 + t^2 p_2}$$



Bernstein Polynomial

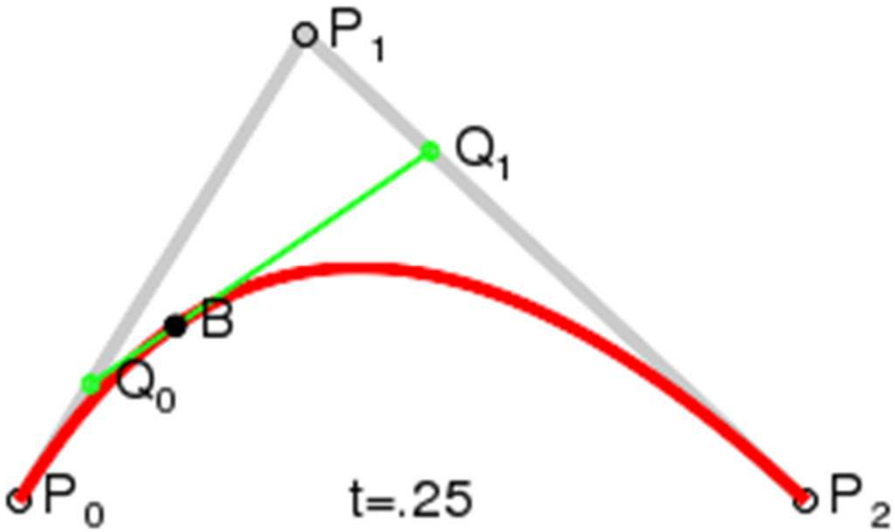
- $n = 3$

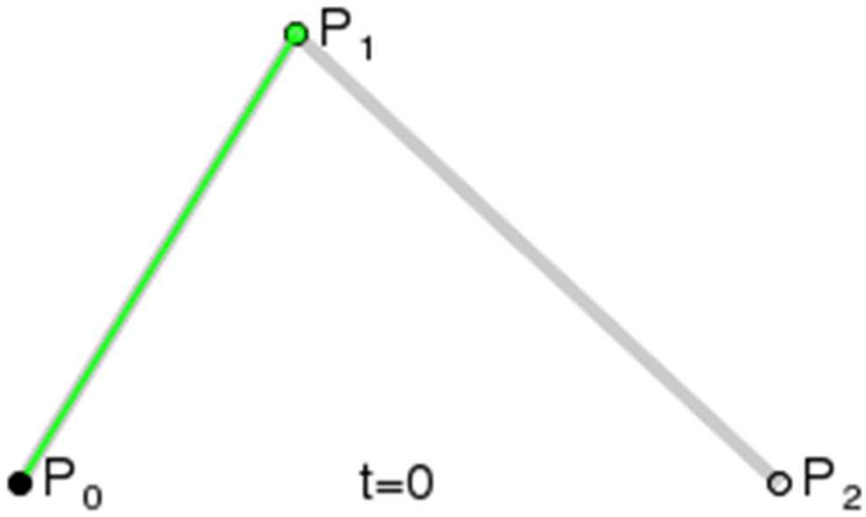
$$p(t) = (1-t)^3 p_0 + 3t(1-t)^2 p_1 + 3t^2(1-t)p_2 + t^3 p_3$$



Bézier Curves

- **Quadratic Bézier Curves**

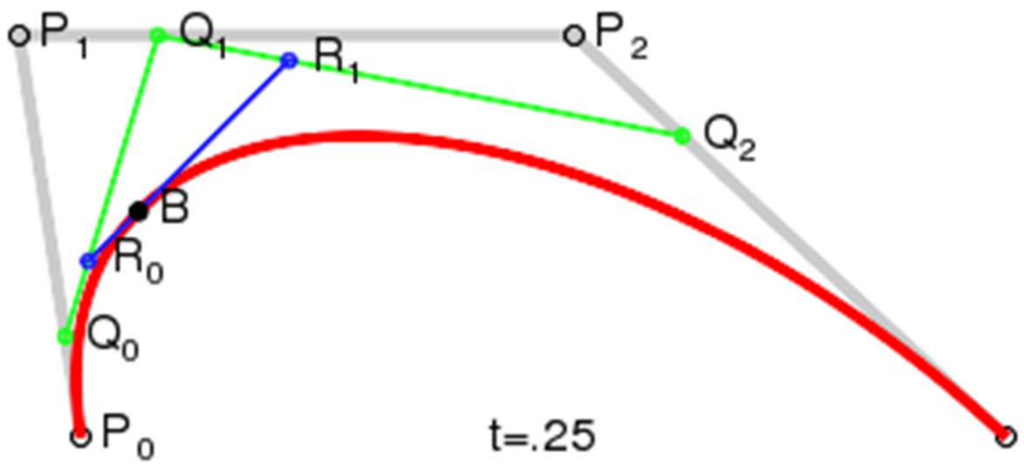


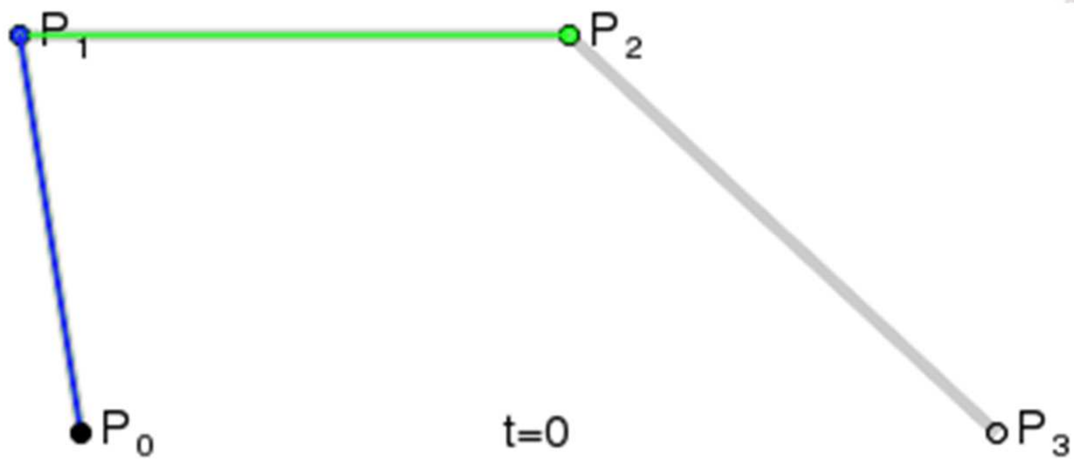


$$
\begin{aligned}
p(t) &= (1-t)^2 p_0 + 2t(1-t)p_1 + t^2 p_2 \\
&= (1-t)\big((1-t)p_0 + tp_1\big) + t((1-t)p_1 + tp_2)
\end{aligned}
$$

Bézier Curves

- Cubic Bézier Curves

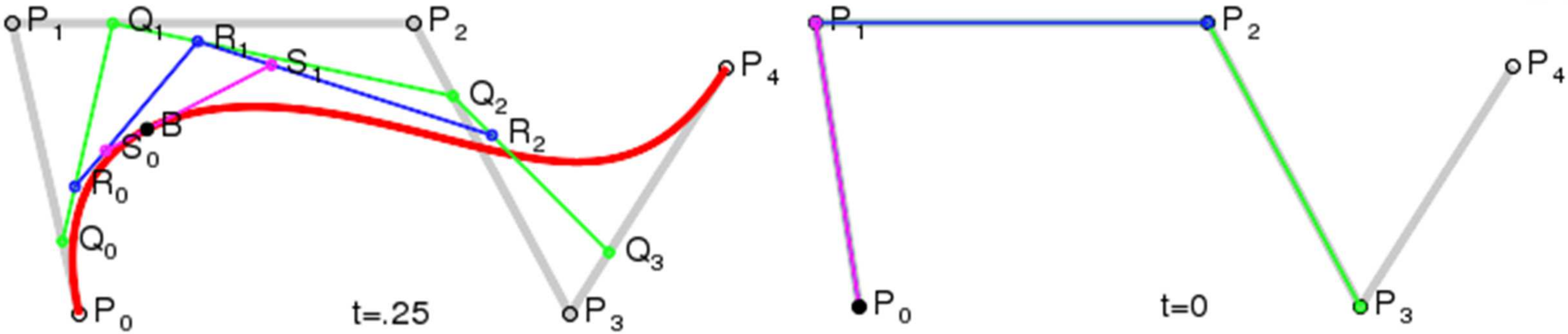




$$
\begin{aligned}
p(t) &= (1-t)^3 p_0 + 3t(1-t)^2 p_1 + 3t^2(1-t)p_2 + t^3 p_3 \\
&= (1-t)\Big((1-t)\big((1-t)p_0 + tp_1\big) + t\big((1-t)p_1 + tp_2\big)\Big) + \\
&\quad t\Big((1-t)\big((1-t)p_1 + tp_2\big) + t\big((1-t)p_2 + tp_3\big)\Big)
\end{aligned}
$$

Bézier Curves

- **Quartic Bézier Curves**



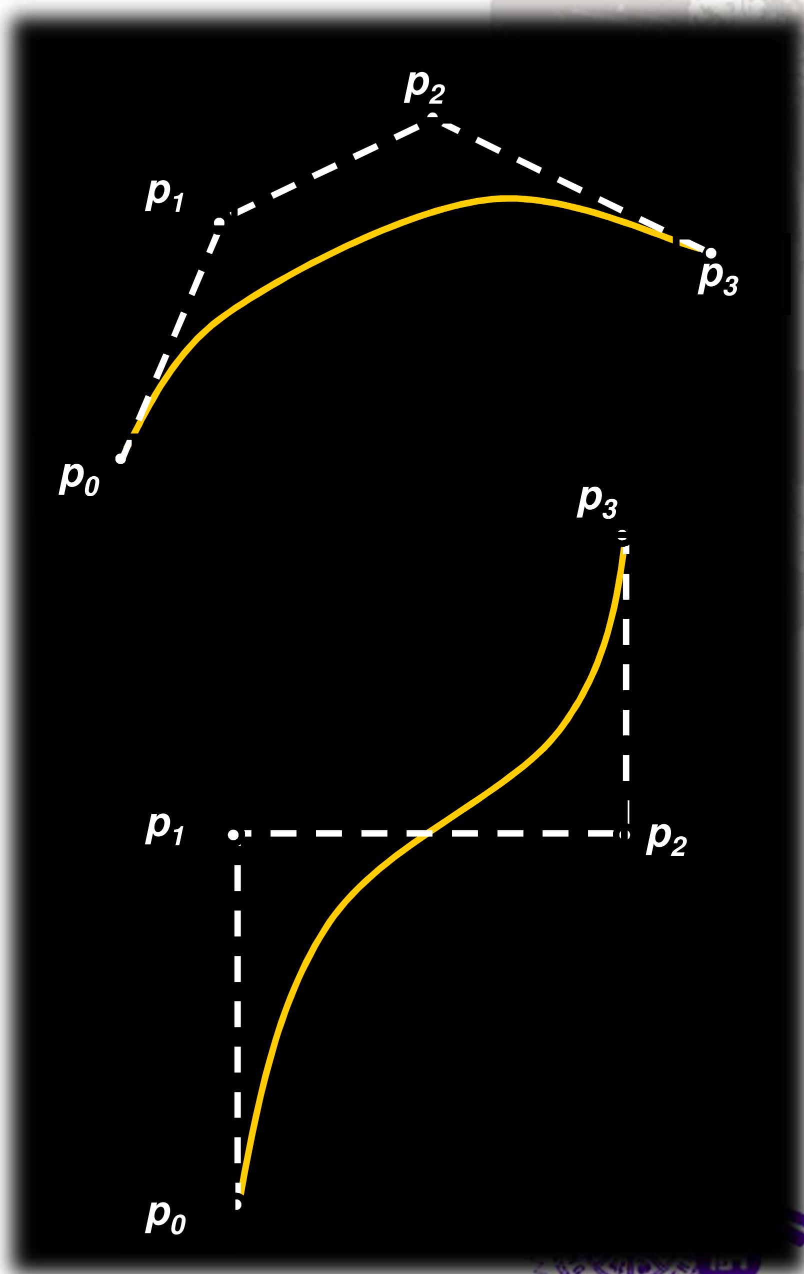
Bézier Curves

◆ Cubic Bézier Curves

$$Q(t) = T \cdot M_B \cdot G_B$$

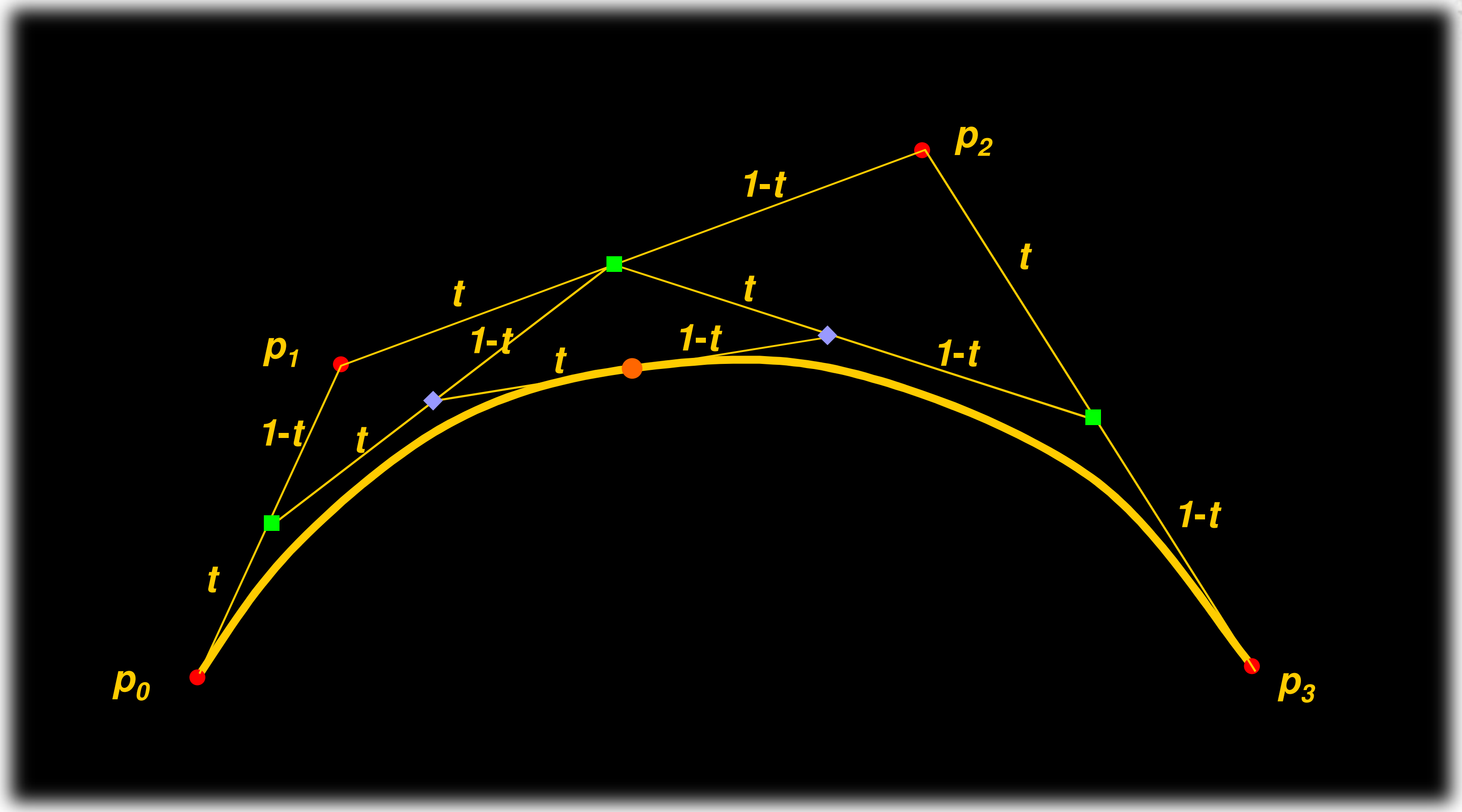
$$= \begin{bmatrix} t^3 & t^2 & t^1 & 1 \end{bmatrix} \cdot \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix}$$

$$= (1-t)^3 p_1 + 3(1-t)^2 p_2 + 3t^2(1-t) p_3 + t^3 p_4$$



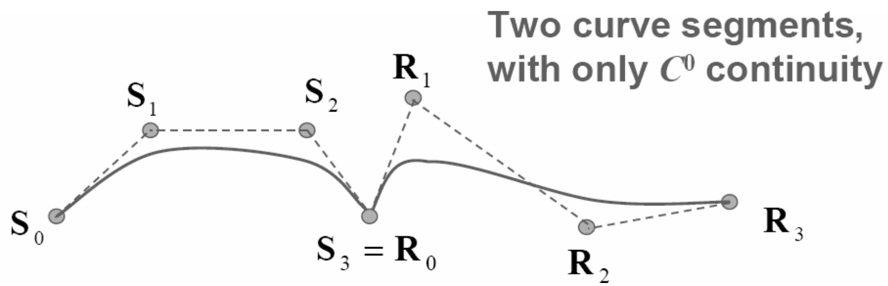
Subdivision Algorithm

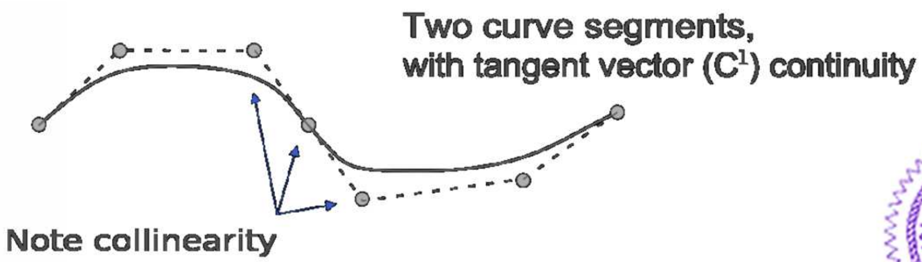
- de Casteljau Algorithm



Parametric Continuity

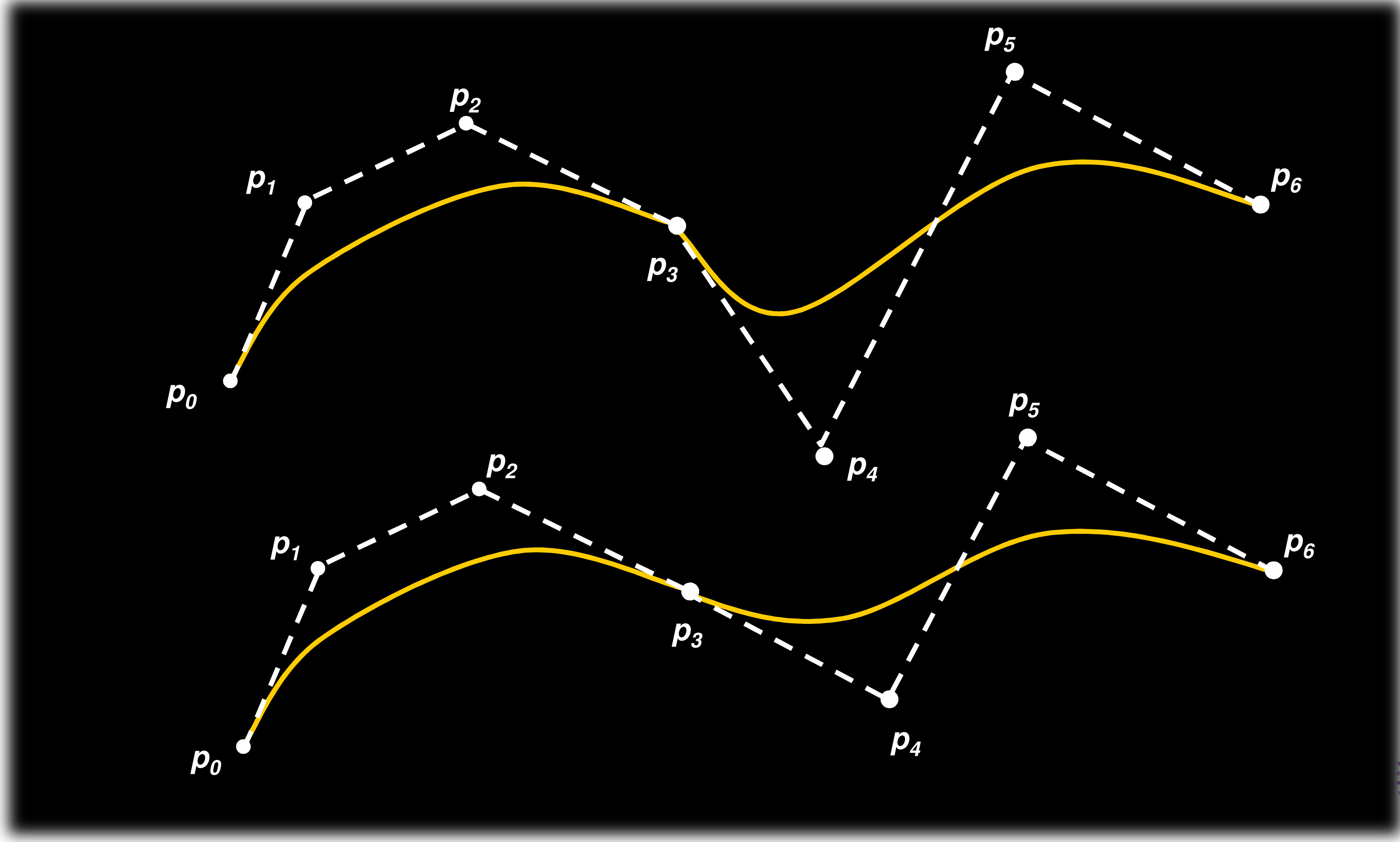
- Describing the smoothness of the parameter's value with distance along the curve
- C^n Continuity
  - C^{-1}: curves include discontinuities
  - C^0: curves are joined
  - C^1: first derivatives are continuous
  - C^2: first and second derivatives are continuous





Composite Bézier Curves

◆ **Continuity Problem**



Various High Order Surfaces

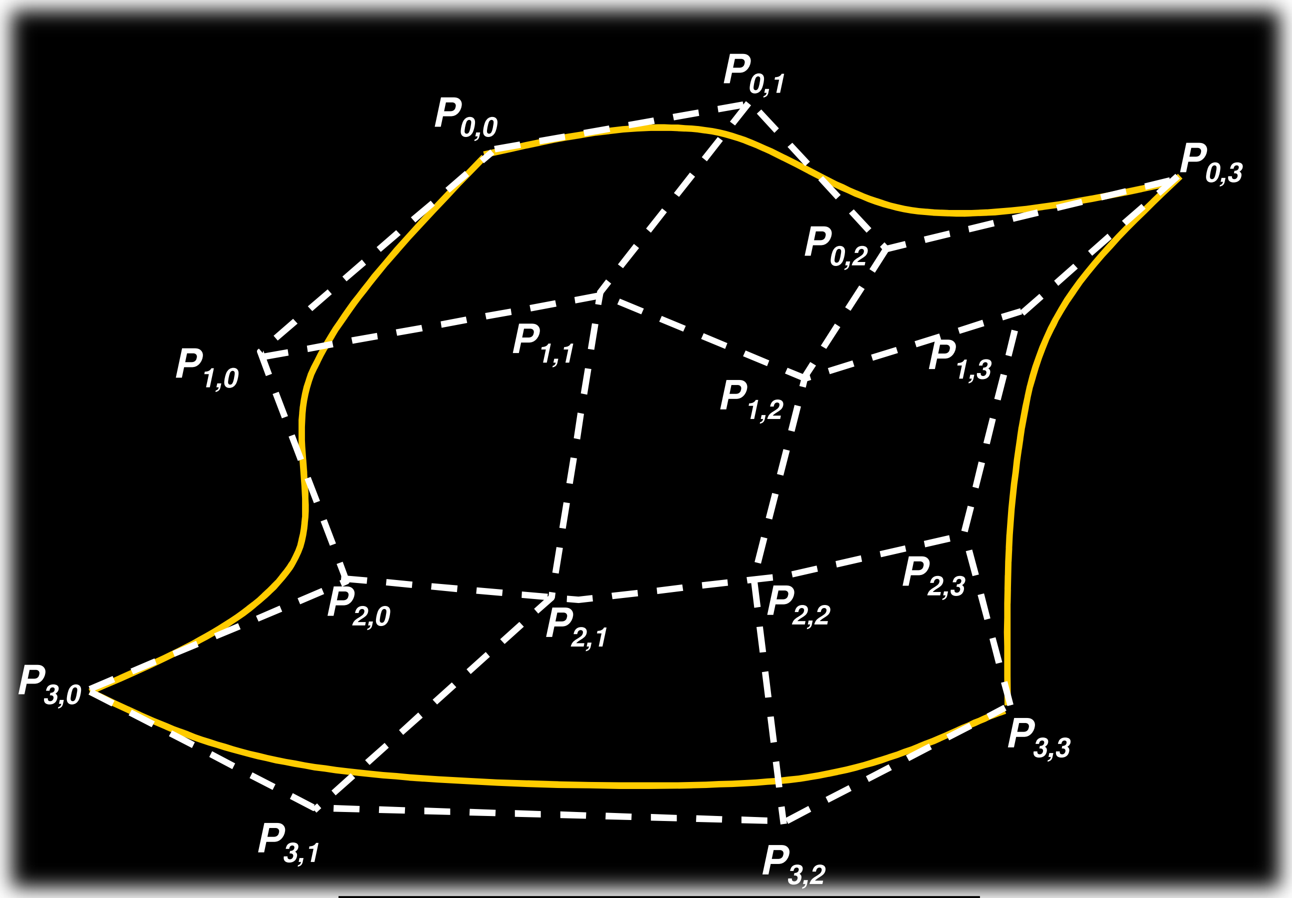
Bezier Surface

B-Spline Surface

Catmull-Rom Surface

Bézier Surface

- Tensor Product Surface



$$\vec{N} = \frac{\partial}{\partial s} Q(s,t) \times \frac{\partial}{\partial t} Q(s,t)$$

$$x(s,t) = S \cdot M_B \cdot G_{B(x)} \cdot M_B^{\mathsf{T}} \cdot T^{\mathsf{T}}$$

$$y(s,t) = S \cdot M_B \cdot G_{B(y)} \cdot M_B^{\mathsf{T}} \cdot T^{\mathsf{T}}$$

$$z(s,t) = S \cdot M_B \cdot G_{B(z)} \cdot M_B^{\mathsf{T}} \cdot T^{\mathsf{T}}$$

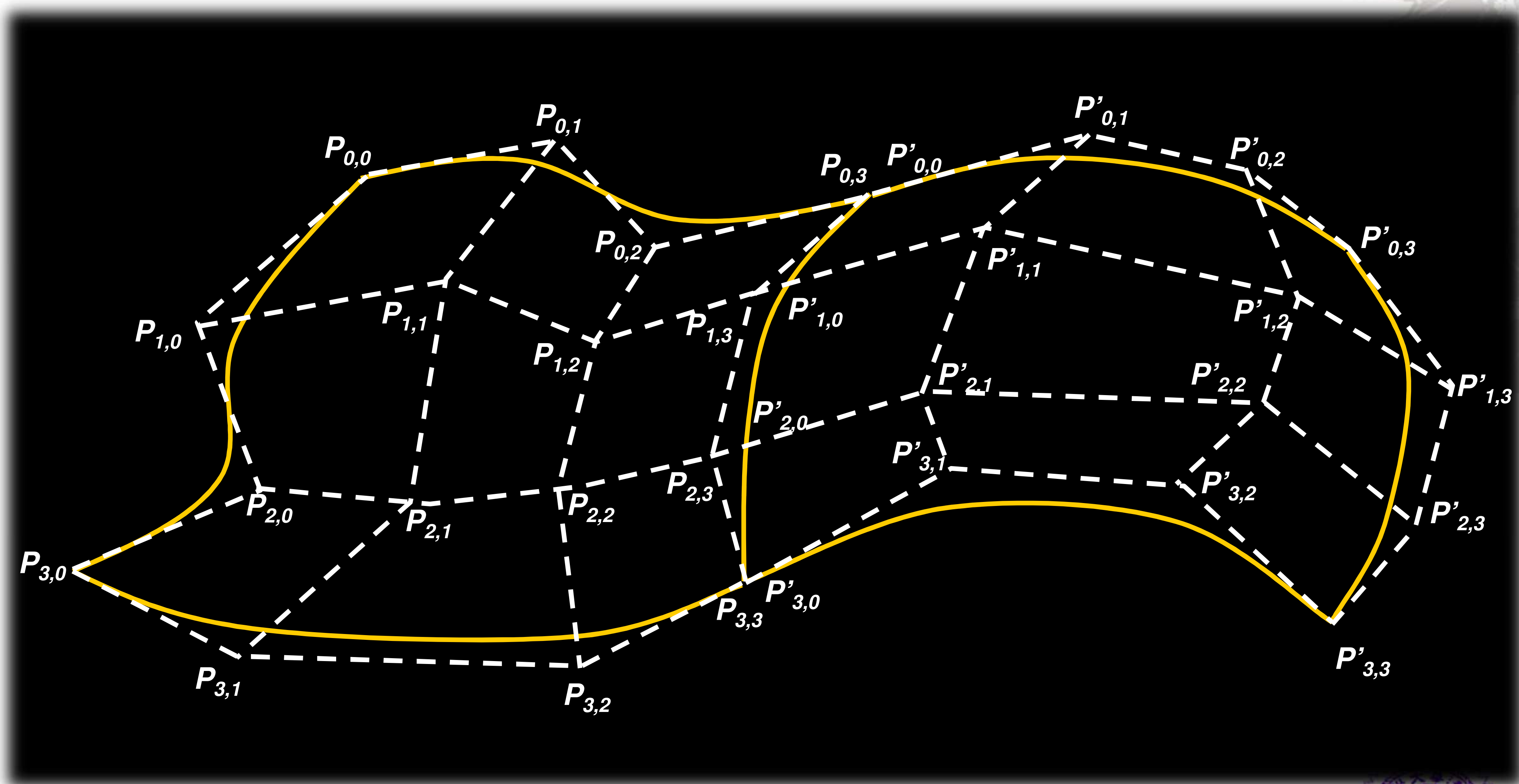
$$S = \begin{pmatrix} s^3 & s^2 & s^1 & 1 \end{pmatrix}$$

$$T = \begin{pmatrix} t^3 & t^2 & t^1 & 1 \end{pmatrix}$$

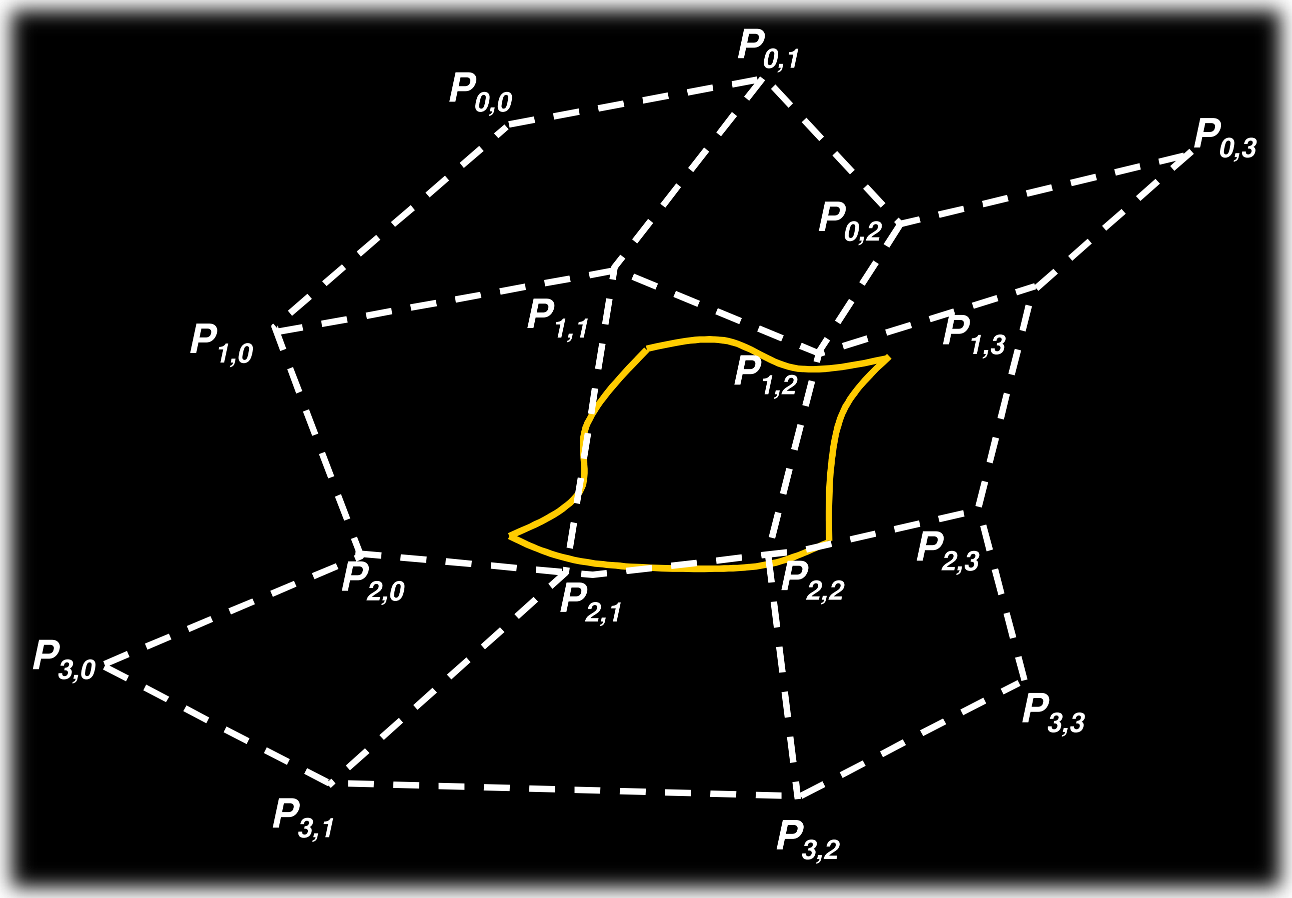
$$M_B = \begin{pmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$G_B = \begin{pmatrix} P_{00} & P_{01} & P_{02} & P_{03} \\ P_{10} & P_{11} & P_{12} & P_{13} \\ P_{20} & P_{21} & P_{22} & P_{23} \\ P_{30} & P_{31} & P_{32} & P_{33} \end{pmatrix}$$

Composite Bézier Surfaces



Uniform B-Spline Surface



$$\vec{N} = \frac{\partial}{\partial s} Q(s,t) \times \frac{\partial}{\partial t} Q(s,t)$$

$$x(s,t) = S \cdot M_{Bs} \cdot G_{Bs(x)} \cdot M_{Bs}^{\mathrm{T}} \cdot T^{\mathrm{T}}$$

$$y(s,t) = S \cdot M_{Bs} \cdot G_{Bs(y)} \cdot M_{Bs}^{\mathrm{T}} \cdot T^{\mathrm{T}}$$

$$z(s,t) = S \cdot M_{Bs} \cdot G_{Bs(z)} \cdot M_{Bs}^{\mathrm{T}} \cdot T^{\mathrm{T}}$$

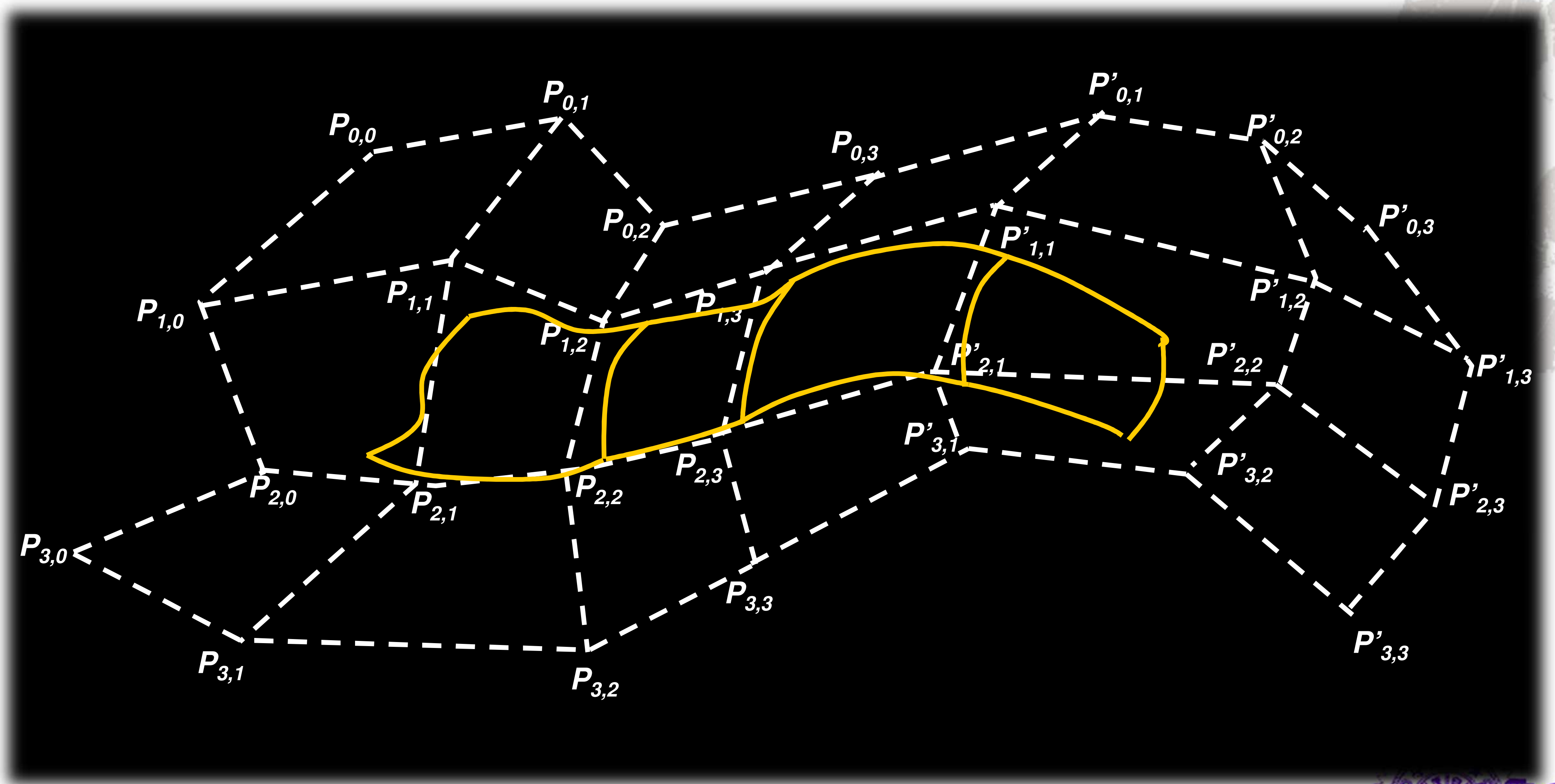
$$S = \begin{pmatrix} s^3 & s^2 & s^1 & 1 \end{pmatrix}$$

$$T = \begin{pmatrix} t^3 & t^2 & t^1 & 1 \end{pmatrix}$$

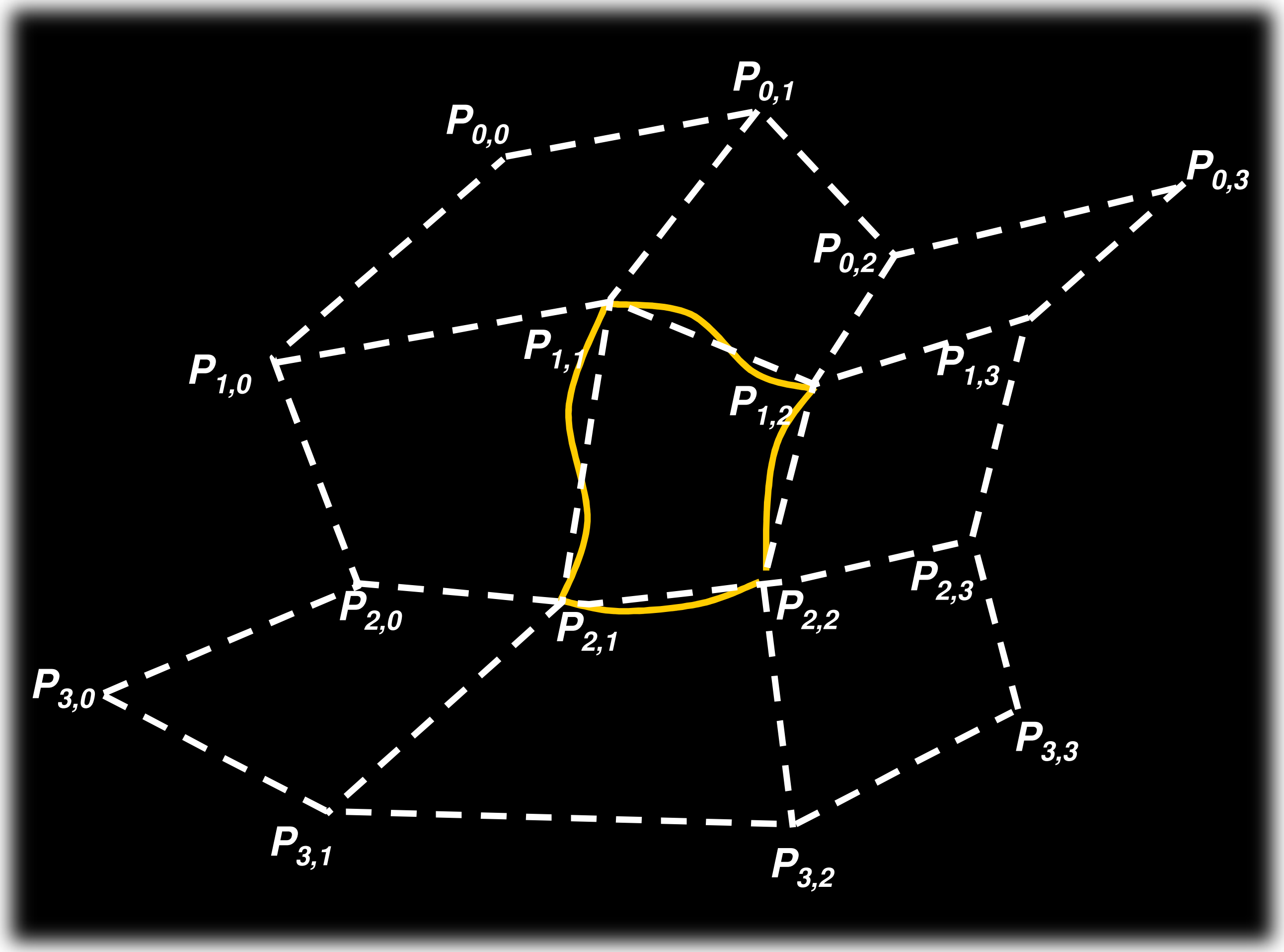
$$M_{Bs} = \frac{1}{6}\begin{pmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{pmatrix}$$

$$G_{Bs} = \begin{pmatrix} P_{00} & P_{01} & P_{02} & P_{03} \\ P_{10} & P_{11} & P_{12} & P_{13} \\ P_{20} & P_{21} & P_{22} & P_{23} \\ P_{30} & P_{31} & P_{32} & P_{33} \end{pmatrix}$$

Composite B-spline Surfaces



Catmull-Rom Surface



$$\vec{N} = \frac{\partial}{\partial s} Q(s,t) \times \frac{\partial}{\partial t} Q(s,t)$$

$$x(s,t) = S \cdot M_{CR} \cdot G_{Bs(x)} \cdot M_{Bs}^{\mathrm{T}} \cdot T^{\mathrm{T}}$$

$$y(s,t) = S \cdot M_{CR} \cdot G_{Bs(y)} \cdot M_{Bs}^{\mathrm{T}} \cdot T^{\mathrm{T}}$$

$$z(s,t) = S \cdot M_{CR} \cdot G_{Bs(z)} \cdot M_{Bs}^{\mathrm{T}} \cdot T^{\mathrm{T}}$$

$$S = \begin{pmatrix} s^3 & s^2 & s^1 & 1 \end{pmatrix}$$

$$T = \begin{pmatrix} t^3 & t^2 & t^1 & 1 \end{pmatrix}$$

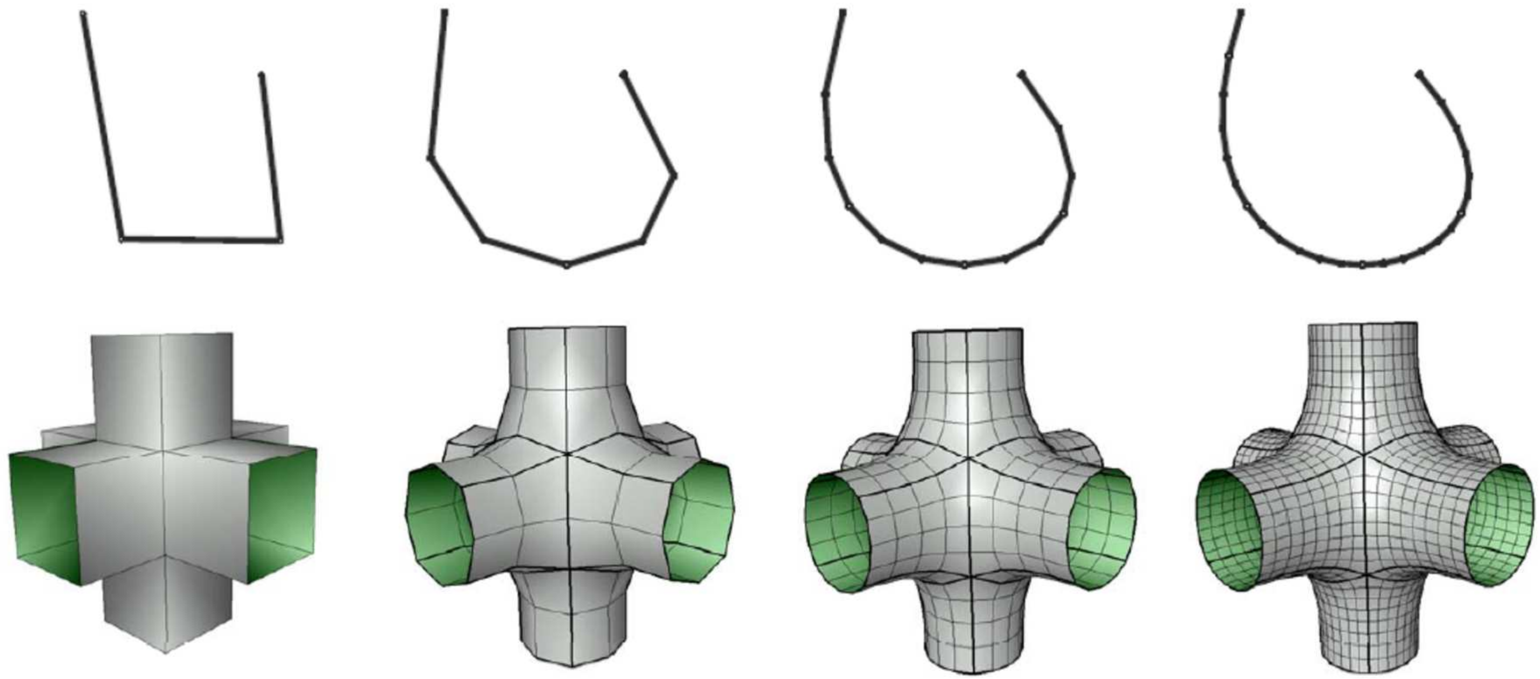
$$M_{CR} = \frac{1}{2} \begin{pmatrix} -1 & 3 & -3 & 1 \\ 2 & -5 & 4 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 \end{pmatrix}$$

$$G_{Bs} = \begin{pmatrix} P_{00} & P_{01} & P_{02} & P_{03} \\ P_{10} & P_{11} & P_{12} & P_{13} \\ P_{20} & P_{21} & P_{22} & P_{23} \\ P_{30} & P_{31} & P_{32} & P_{33} \end{pmatrix}$$

Subdivision Surfaces

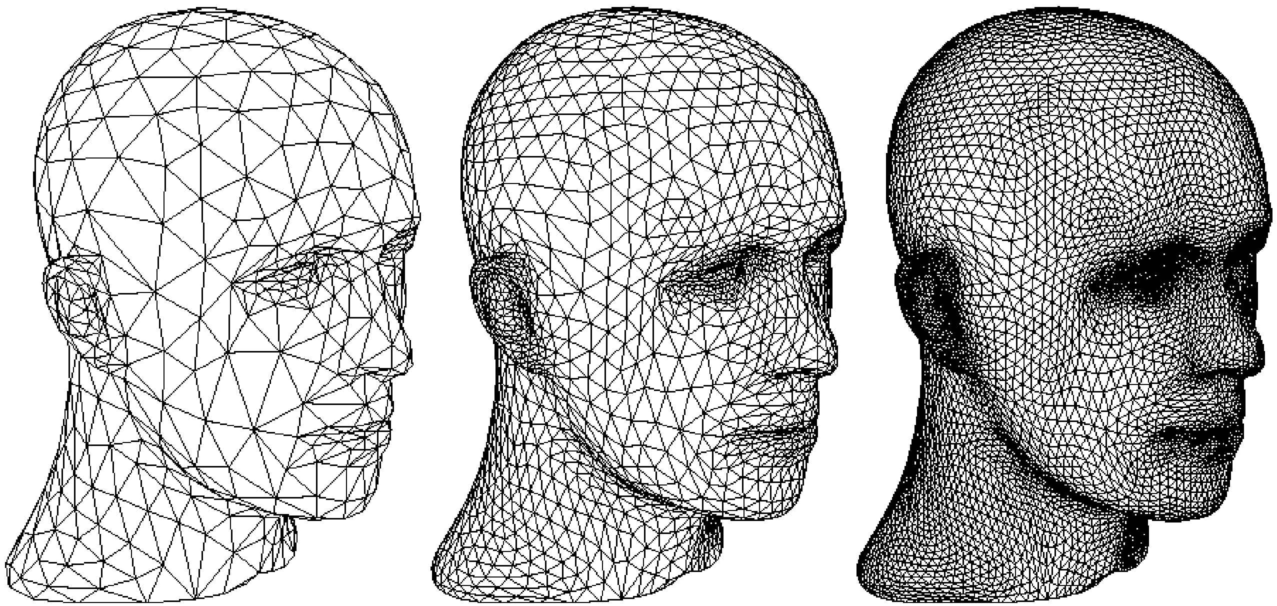
Principle of Subdivision

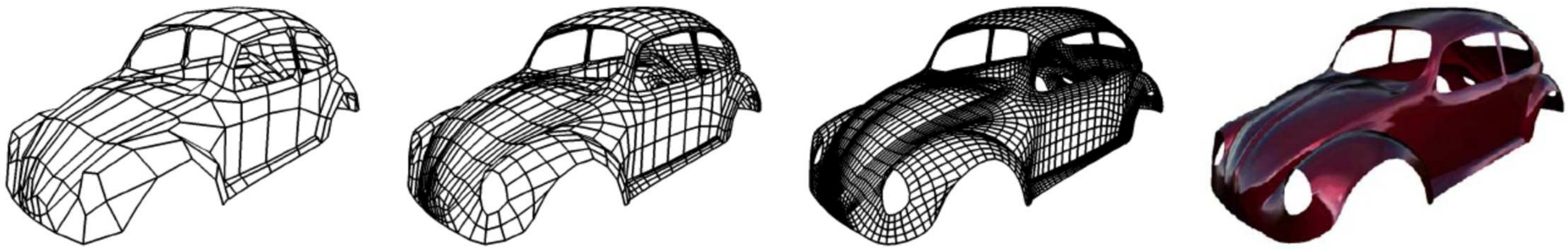
- Subdivision defines a smooth curve or surface as the limit of a sequence of successive refinements



Applications of Subdivision

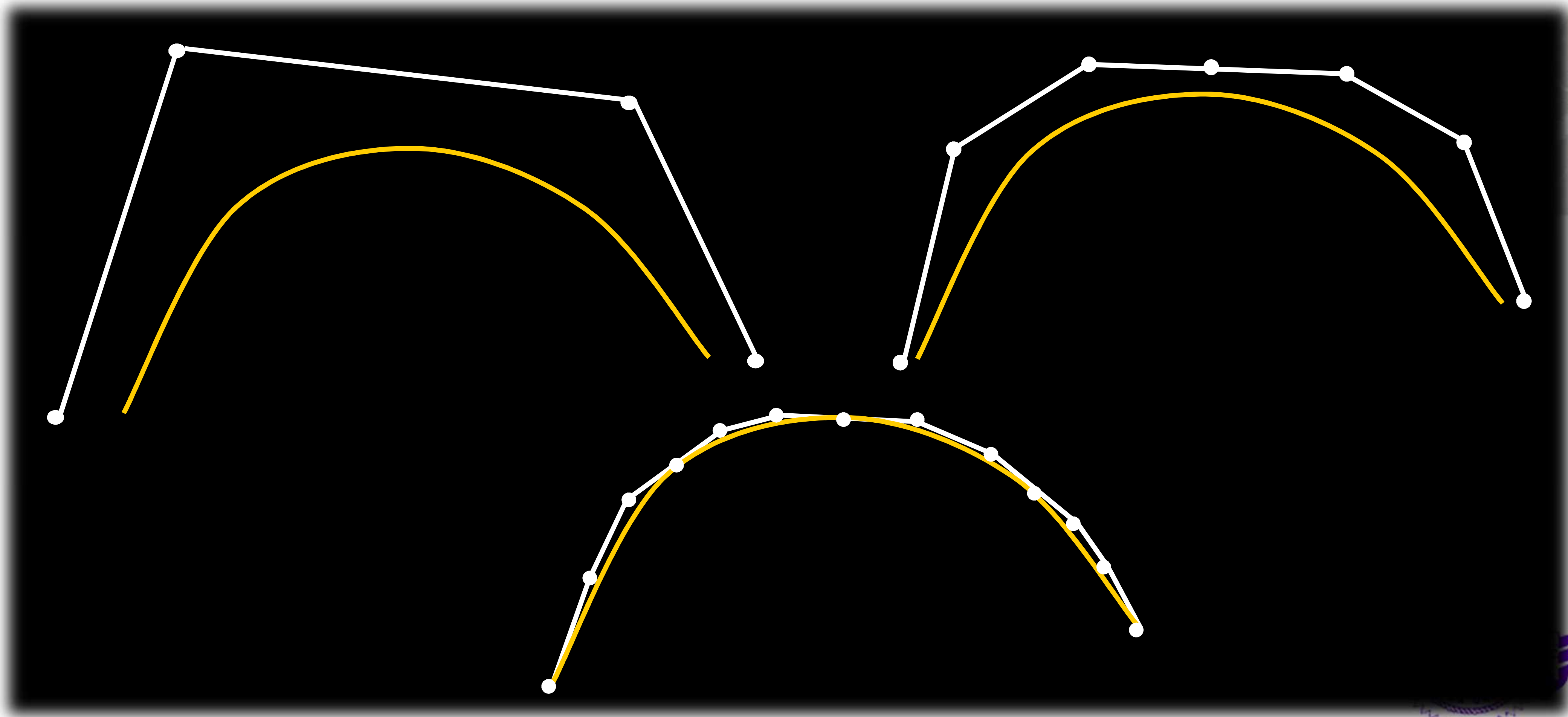
- **Level of details**



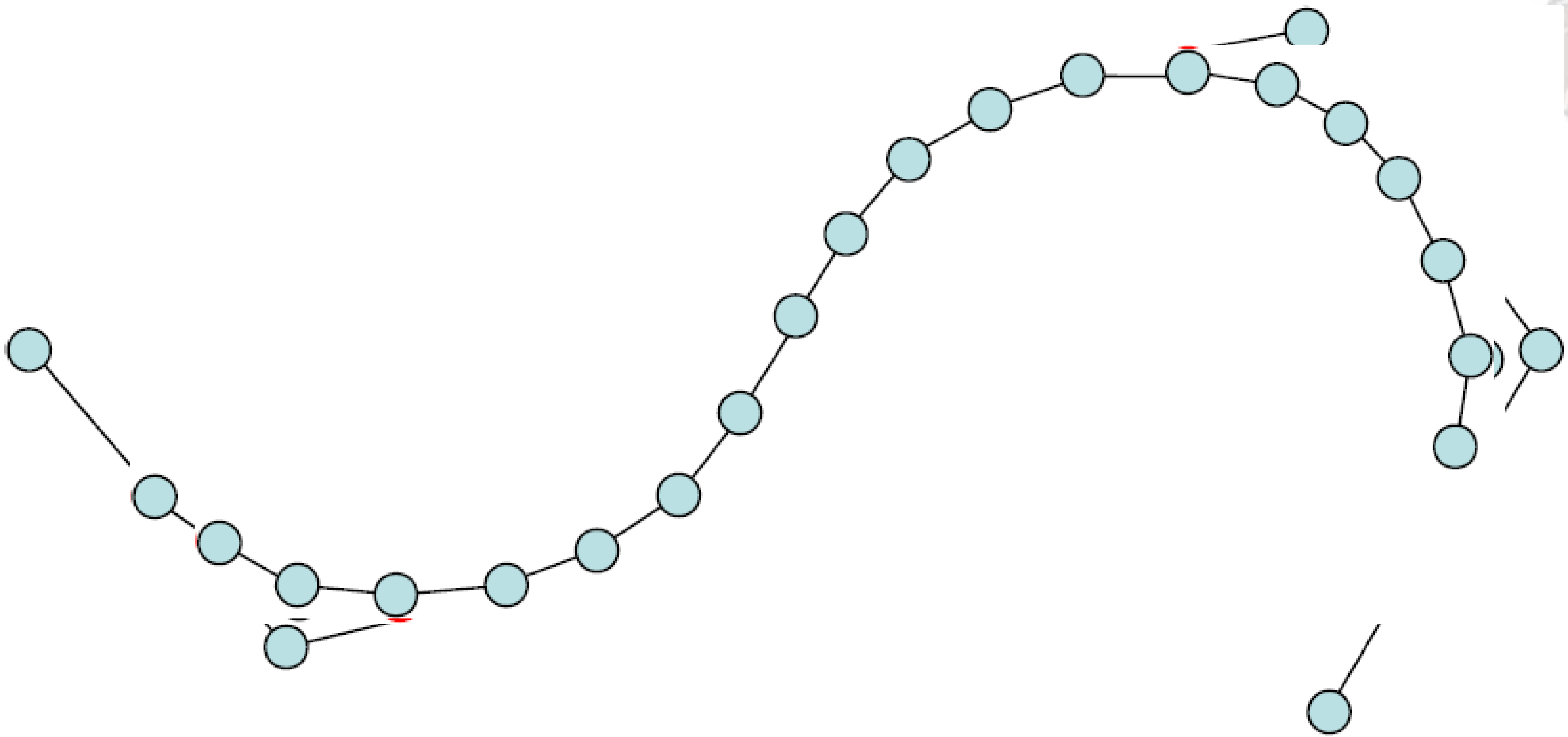


Approximation

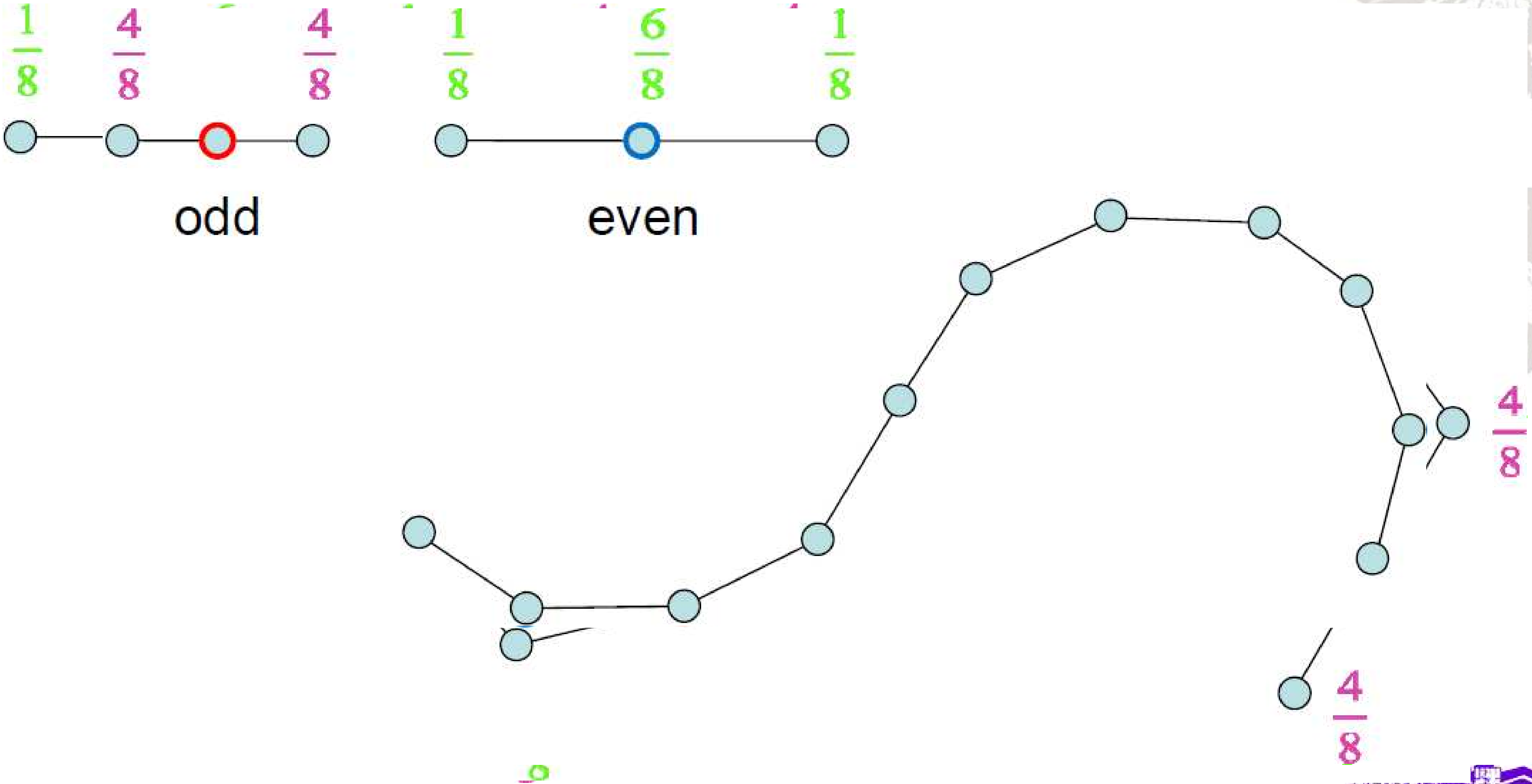
- Generated points at level j+1 are local averages of points at level j



Approximation

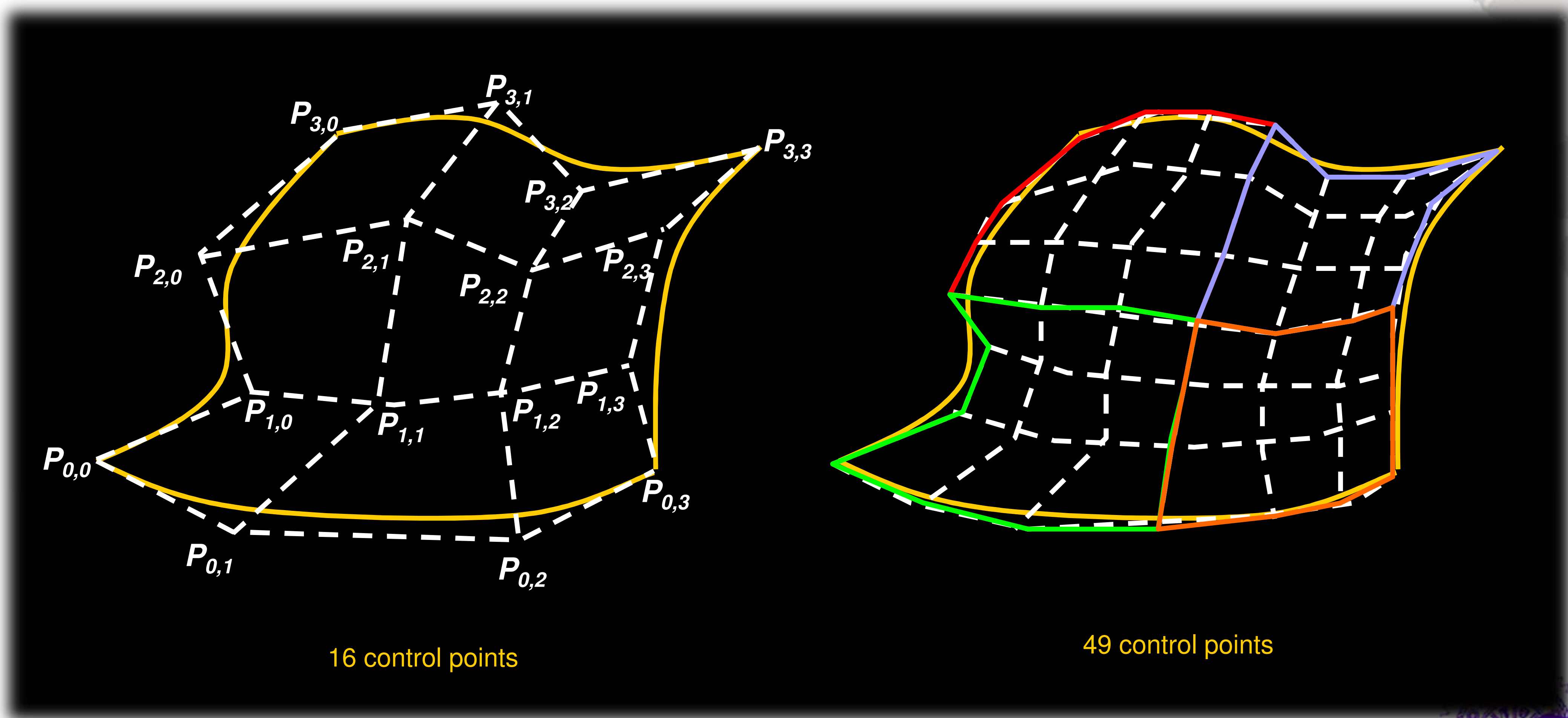


Approximation



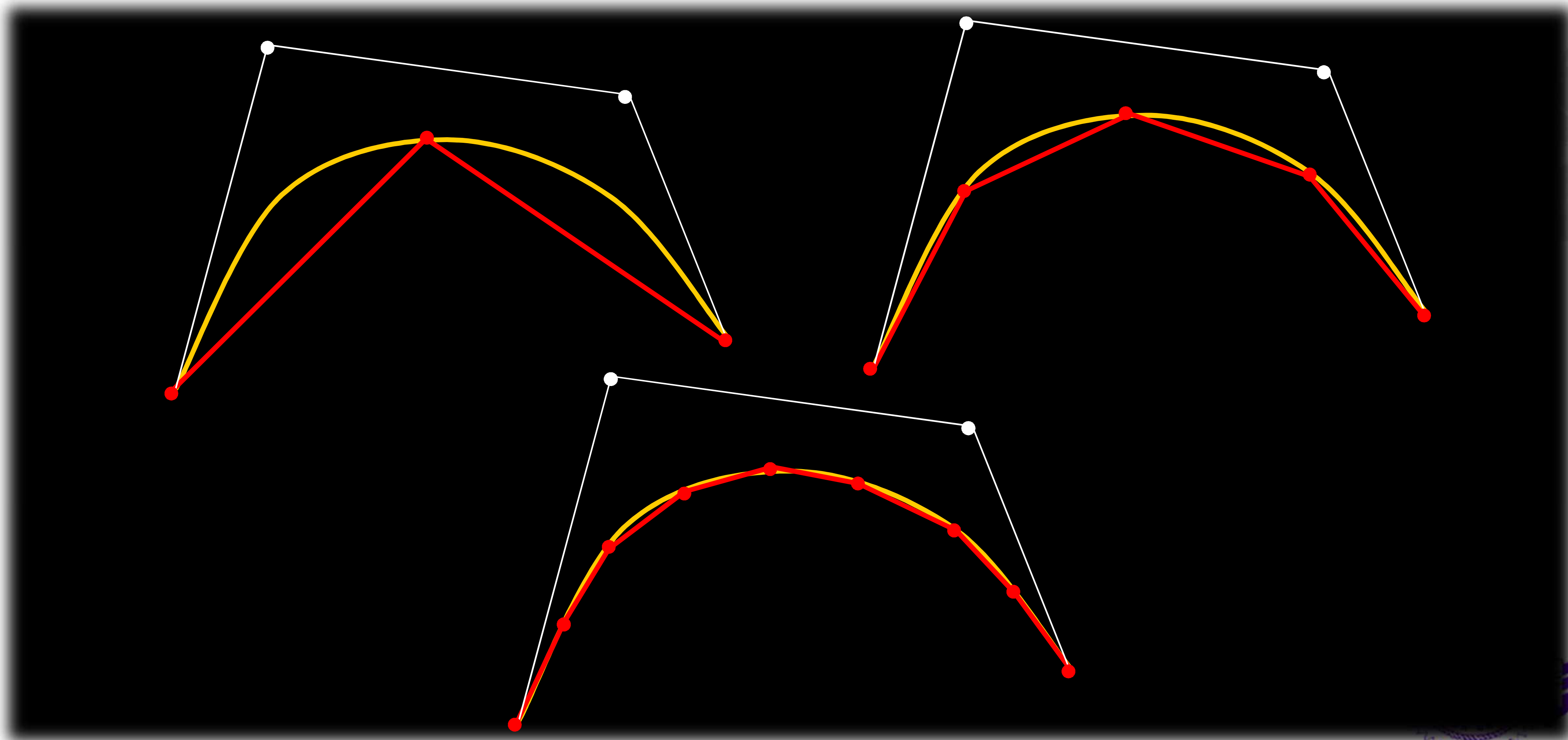
Bézier Surface

- Using control net to approximate the surface

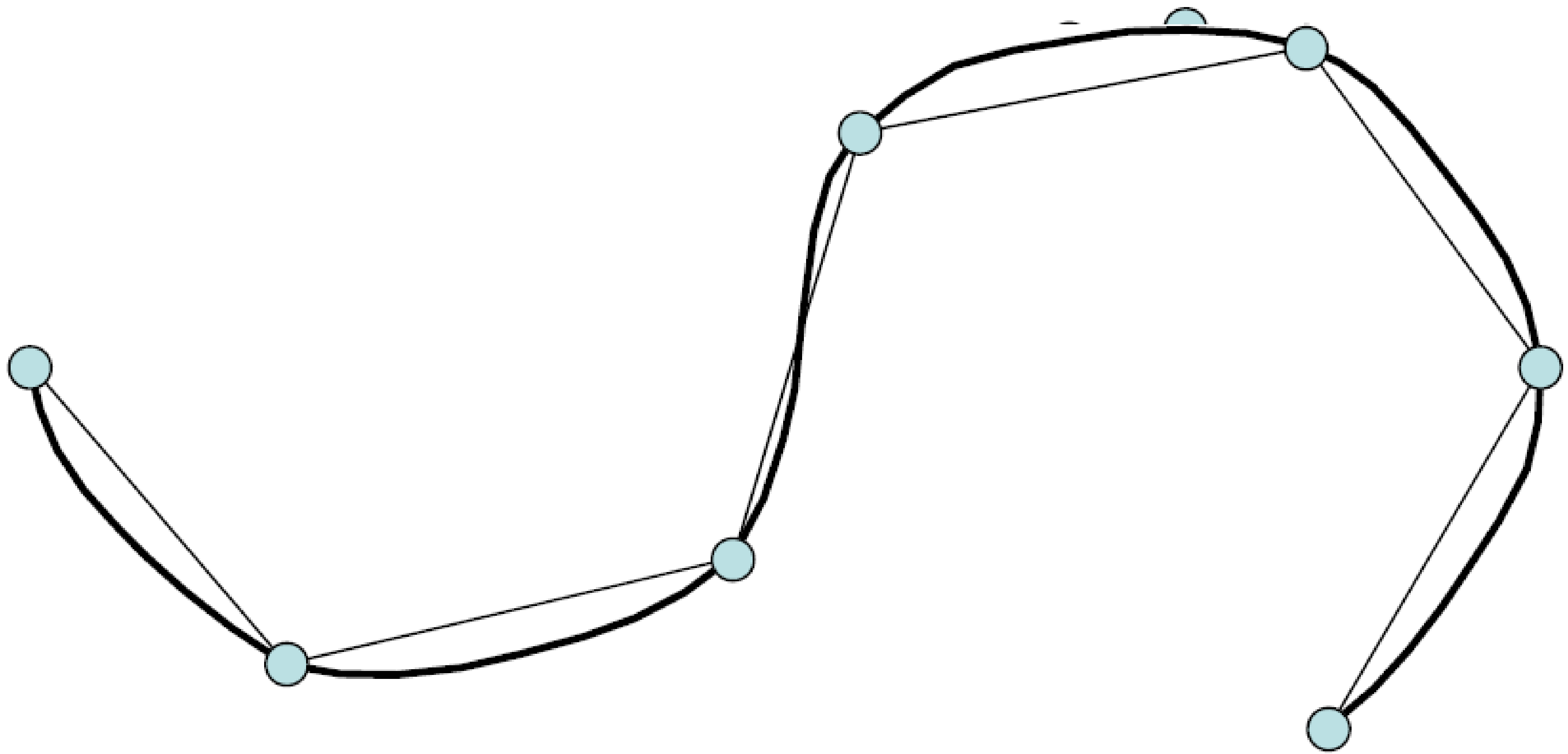


Interpolation

- Generated points are exactly the points on the curve/surface

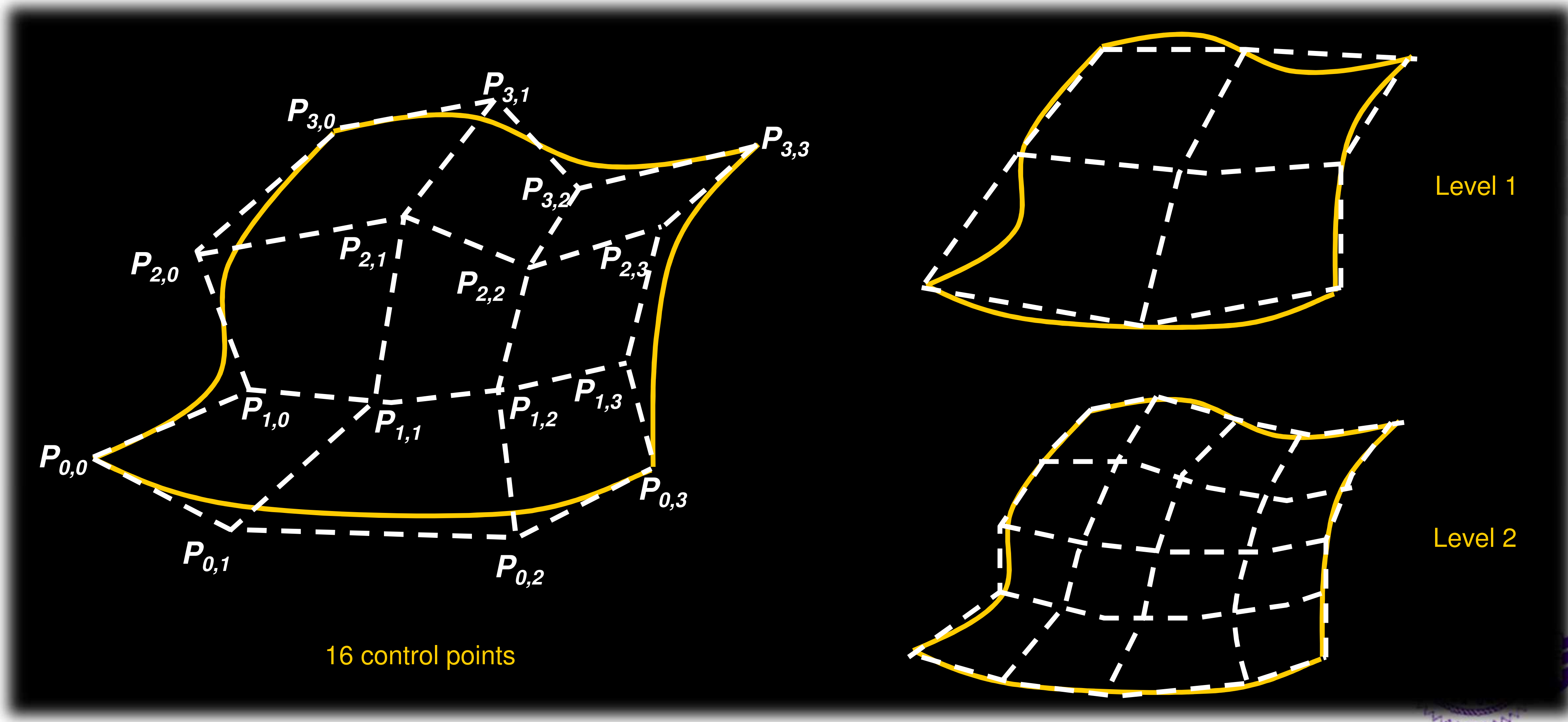


Interpolation



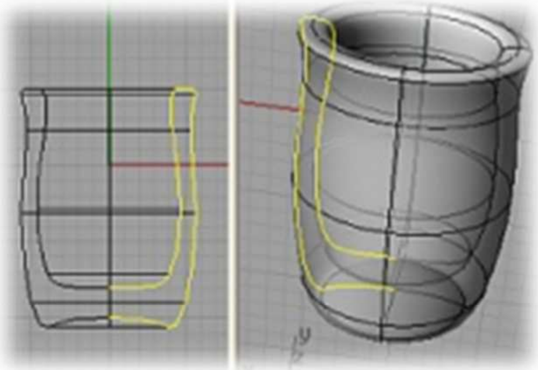
Bézier Surface

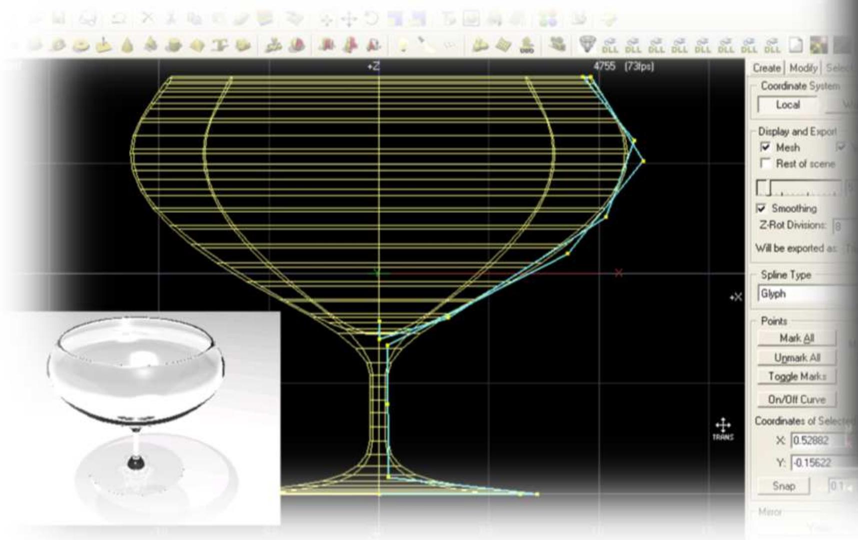
- Depends on the subdivision level to generate intermediate points on the surface

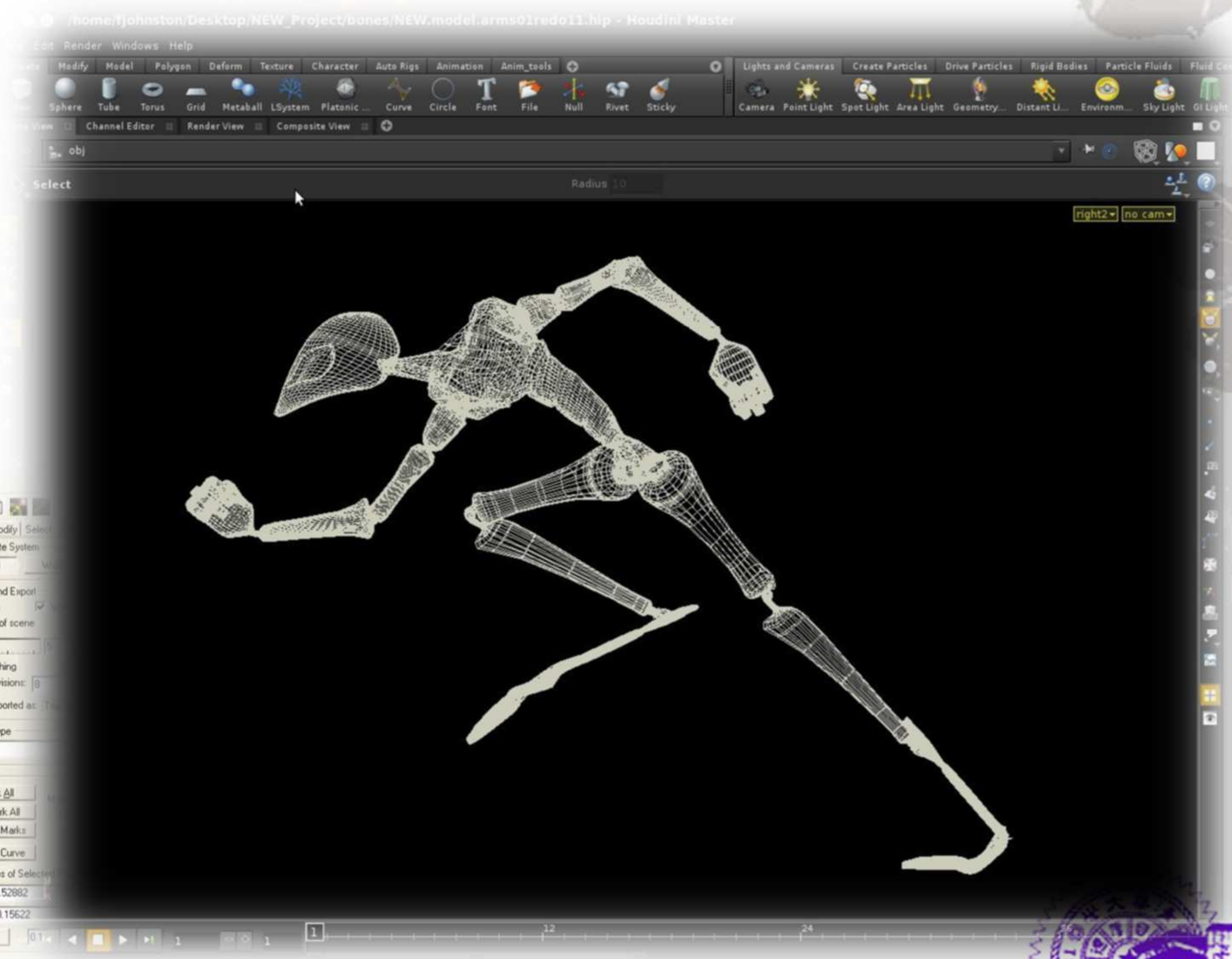


Modeling in Real

- Rotational Sweep

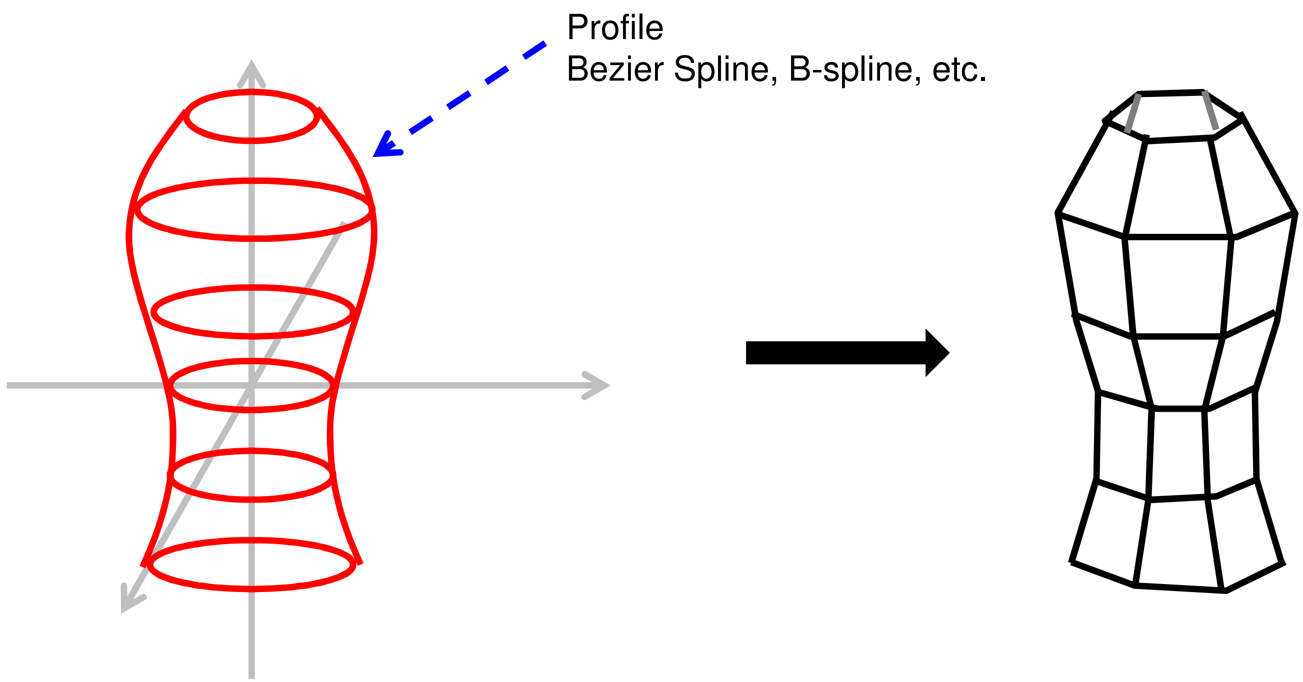






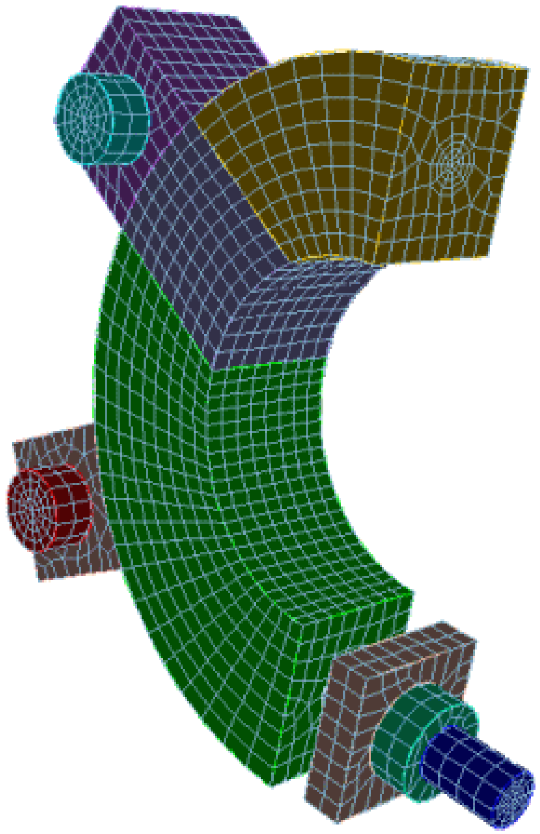
Modeling in Real

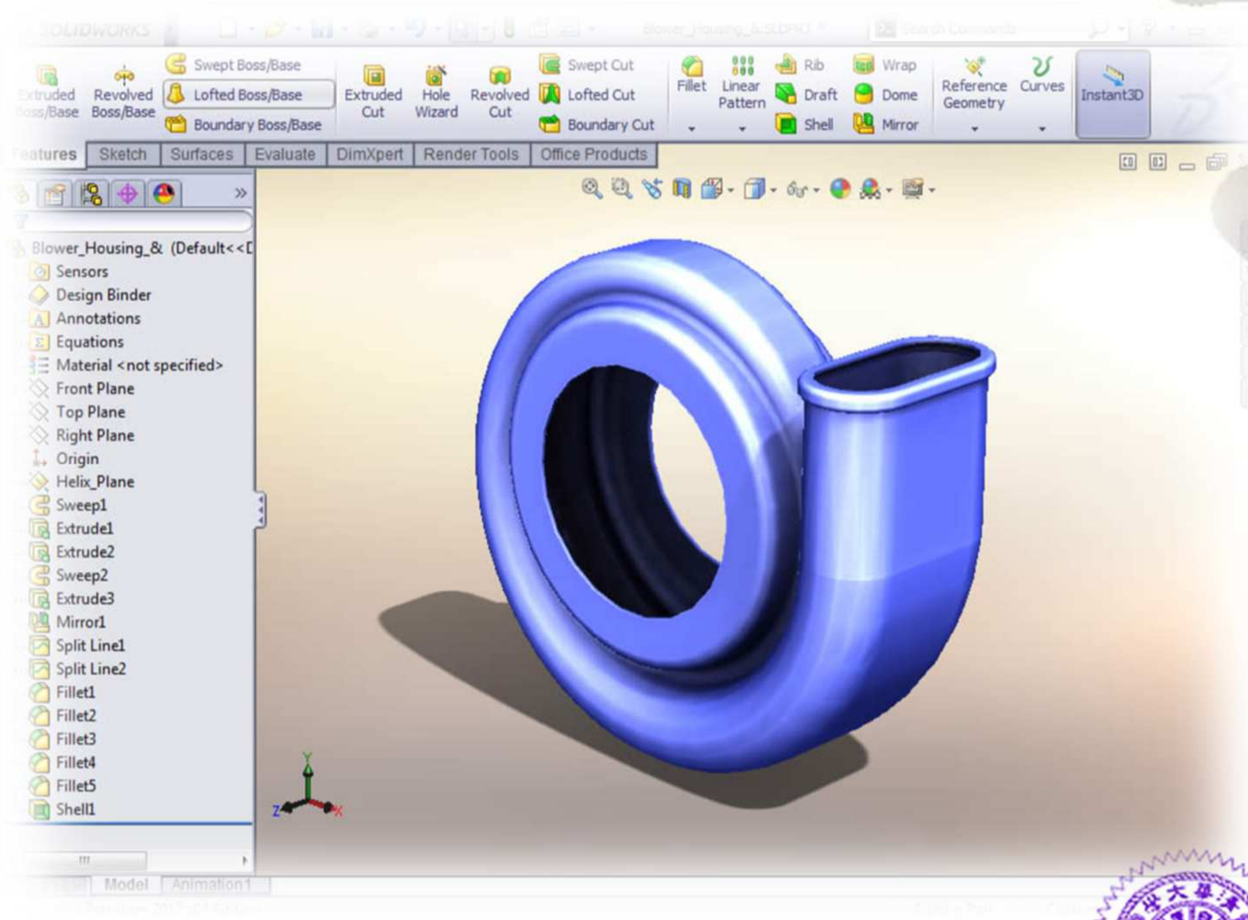
- Rotational Sweep
  - An object is defined by rotating a given profile (or area) along an axis



Modeling in Real

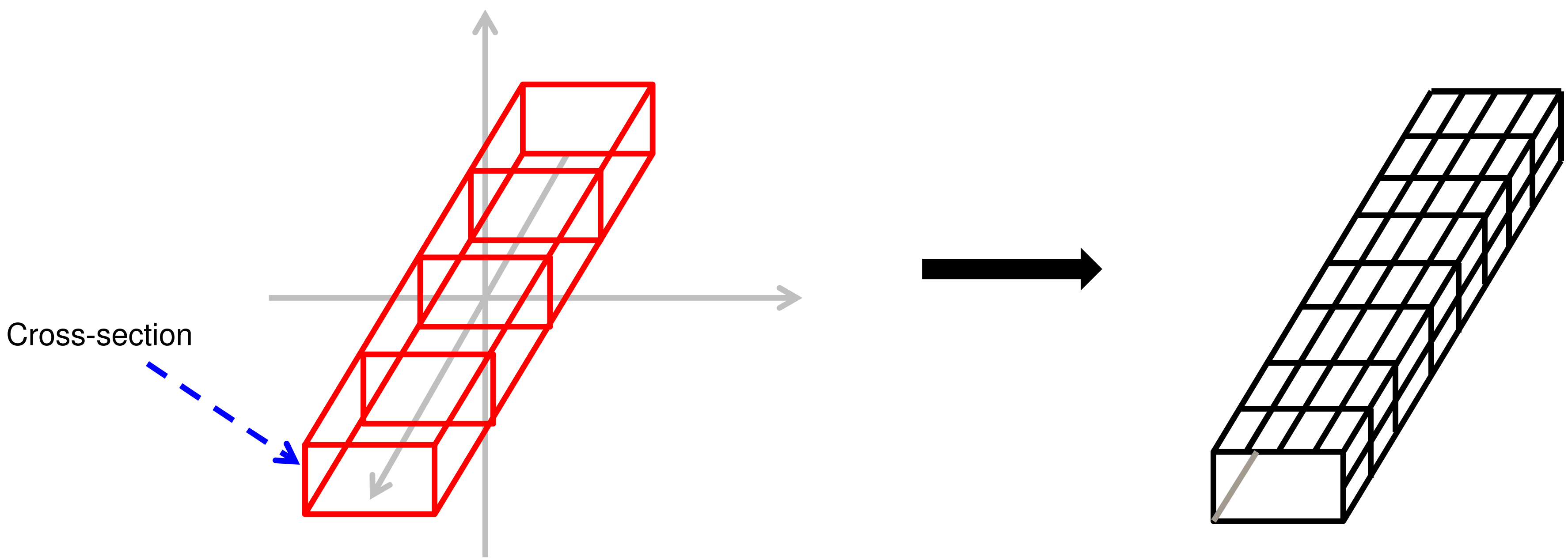
- Translational Sweep (Extrusion)





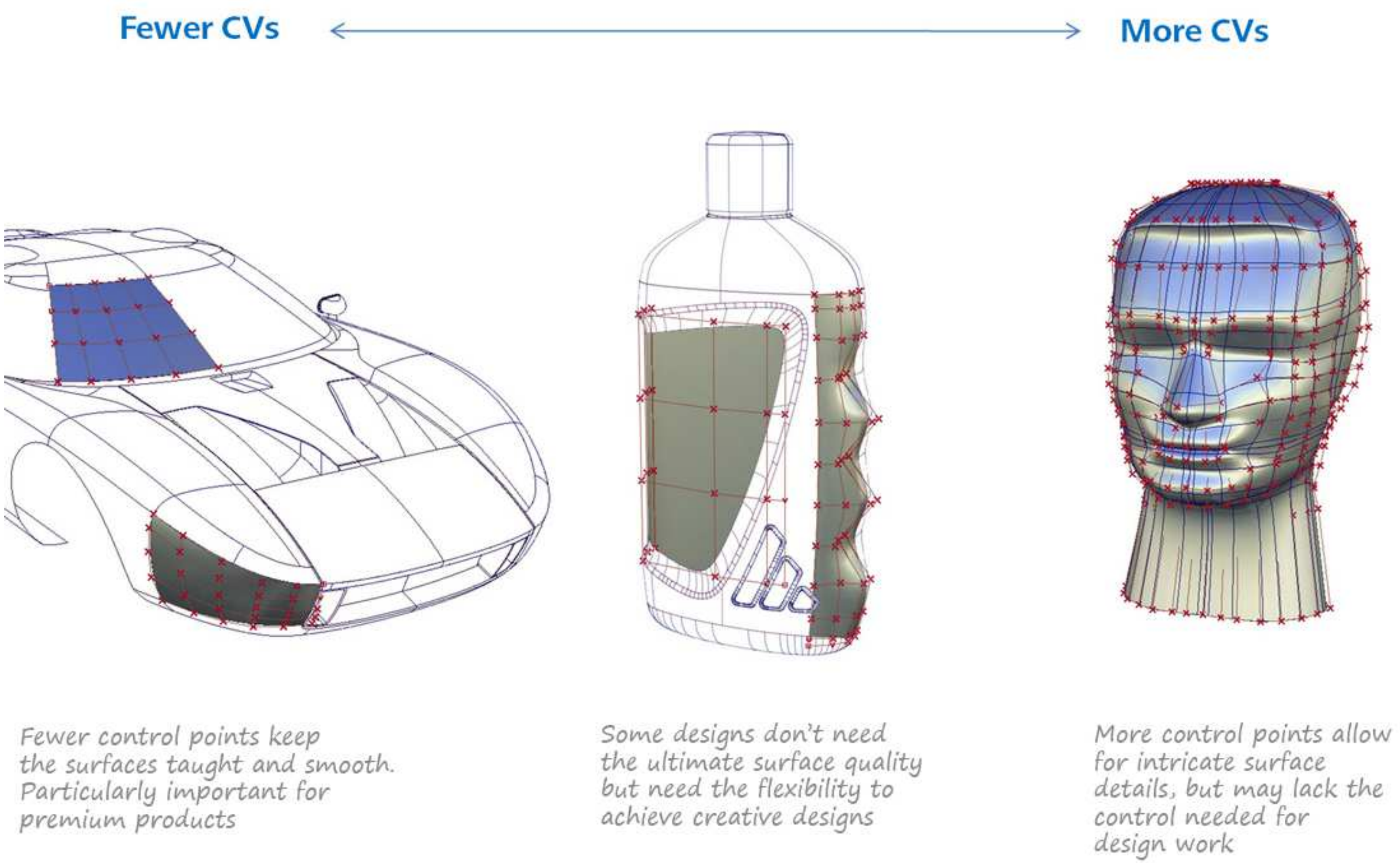
Modeling in Real

- **Translational Sweep**
  - An object is defined by translating a given profile (or cross-section) along an axis or a path



Modeling in Real

- **Freeform Modeling**



Modeling in Real

- **Freeform Modeling**
  - **Describe the skin of a 3D geometric element**
  - **Shape change by manipulating the control points**

Q&A