

Strictly Local Languages

$\text{fac}_k(w)$ u is k -fac of w if $w = w_1 u w_2$

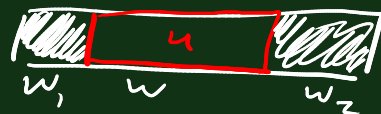
k -fac of w

G

G is a SL_k grammar if $G \subseteq \text{fac}_k(\Sigma^*)$

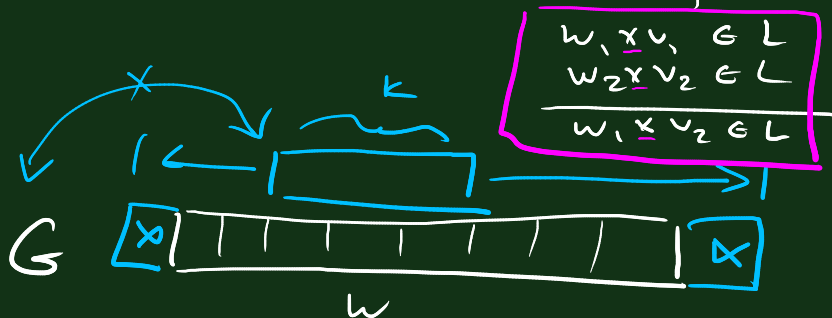
$w \models G$ iff $\text{fac}_k(w) \cap G = \emptyset$

$$L(G) = \{w \in \Sigma^* \mid w \models G\}$$



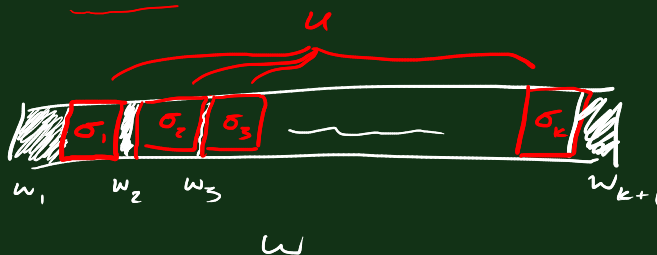
L is SL_k iff $L = L(G)$ for some SL_k grammar G

Thm (Rogers & Pullum 2011) L is SL_k iff $w_1, v_1, w_2, v_2 \in \Sigma^*$ and for any string x $|x| = k-1$, then



Strictly Piecewise (Rogers et al 2010)

subsequence $u = \sigma_1 \sigma_2 \dots \sigma_k \subseteq w$ iff $w = w_1 \sigma_1 w_2 \sigma_2 \dots w_k \sigma_k w_{k+1}$
 $u \in \Sigma^*$ $\sigma_i \in \Sigma$ $w_i \in \Sigma^*$



Z-subseq of abca

(ac)

$\lambda abca$
 $\overline{w_1} \overline{\sigma_1} \overline{w_2} \overline{\sigma_2} \overline{w_3}$

(ba)

$abca \lambda$
 $\overline{w_1} \overline{\sigma_1} \overline{w_2} \overline{\sigma_2} \overline{w_3}$

(ab)

$w_2 = \lambda$
 $\lambda abca$
 $\overline{w_1} \overline{\sigma_1} \overline{\sigma_2} \overline{w_3}$

$\text{subseq}_k(w) = \{u \mid u \subseteq w \text{ and } |u| = k\}$ if $|w| \geq k$
 $\{w\}$ if $|w| < k$

$$\text{subseq}_k(L) = \bigcup_{w \in L} \text{subseq}_k(w)$$

G is SP_k grammar iff $G \subseteq \text{subseq}_k(\Sigma^*)$

$w \models G$ iff $\text{subseq}_k(w) \cap G = \emptyset$

$$L(G) = \{w \mid w \models G\}$$

Long-distance interactions in phonology

Source /si-tʃi-z-a?/ → ʃi-tʃi-z-a? ^{*}ʃi-tʃi-z-a? _z
 /na-s-ɣatʃ/ → na-ʃ-ɣatʃ

~~ʃ~~ s...ʃ

✓ ʃ...s

$$L_s \subset \{\overbrace{s, s, a, t}^z\}^*$$

$$\underline{W} \stackrel{?}{\models} G_s$$

$$L_s = \{SataS, SataS, \dots\}$$

$$\overline{L_s} = \{sataS, \dots\}$$

$$G_s = \{s\}$$

$$\text{subseq}_z(SataS) = \{Sa, St, Sa, SS, at, aa, aS, ta, tS\}$$

$$\text{subseq}_z(\cancel{sateS}) = \{sa, st, se, s, at, aa, aS, \textcircled{t}\}$$

$$L \rightsquigarrow SP_k \quad \text{iff} \quad L = L(G) \quad \text{for some } SP_k \text{ grammar}$$

Thm (Royer et al 2010)

SP langs are exactly those closed under subsequence

$$L \rightsquigarrow SP \quad \text{iff} \quad \forall w \in L, \text{ if } u \sqsubseteq w, \text{ then } u \in L$$