C-6.6

We will show this using induction. For $i(T)=0$, then $e(T)=2i(T)+1=1$. This is obviously true. For $i(T)=1$, then $e(T)=2i(T)+1=2+1=3$. Again, this is obviously true from our problem definition. Now let us assume that the $e(T)$ equation holds true for $k'<k$, i.e., for any $i(T)=k'<k$, $e(T)=2i(T)+1$.

Now consider $i(T)=k$. Then, $e(T)=2(k-1)+1+(3-1)$. That is, the number of external nodes is equal to the number of external nodes for a tree with $k-1$ internal nodes plus 3 (we added an internal node which must have 3 children) minus 1 (in creating the new internal node, we made an external node into an internal node). Thus, $e(T)=2k-2+3=2k+1$. This is what we needed to show.