

## C-11.2

The justification is very similar to the argument that the number of iterations in `KMPMatch()` is  $O(n)$ .

Define  $k = i - j$  for the sake of analysis. One of the following conditions occurs at each iteration of the loop:

- If  $P[i] = P[j]$ , then  $i$  increases by 1, and  $k$  does not change, since  $j$  also increases by 1.
- If  $P[i] \neq P[j]$  and  $j > 0$ , then  $i$  does not change and  $k$  increases by at least 1, since in this case  $k$  changes from  $i - j$  to  $i - f(j - 1)$ , which is an addition of  $j - f(j - 1)$ , which is positive because  $f(j - 1) < j$ .
- If  $P[i] \neq P[j]$  and  $j = 0$ , then  $i$  increases by 1 and  $k$  increases by 1, since  $j$  does not change.

As a result, the number of iterations is at most  $2m$ . Therefore, `KMPFailureFunction()` runs in  $O(m)$  time.