

## C-8.1

In order to find the  $k$ th smallest key in the union of the keys from  $S$  and  $T$ , we can do a “double” binary search. In other words, we will begin by examining the  $k/2$ th element in the sequence  $S$ . Next, we will find the largest element in  $T$  that is less than  $S[k/2]$  by binary search. Then, we will add the indices of the elements we were examining in  $S$  and  $T$ . If the sum equals  $k$ , then the max of the two elements is our result. If the sum is greater than  $k$ , then we will do a binary search to the right (upwards) in  $S$ . If the sum is less than  $k$ , then we will do a binary search to the left (downwards) in  $S$ . This is followed once again by searching in  $T$  for largest element less than the current element in  $S$ , etc. This method does a binary search on  $S$  which requires  $O(\log n)$  “probes.” However, for each probe of the search, it does a binary search on  $T$  which takes  $O(\log n)$  time. Thus, the entire method takes  $O(\log^2 n)$  time.