VCF Math Specification v1.1
Project: VCF Research – Vector Cycle Framework (VCF)
Tagline: *Translating liquidity, macro, and behavioral momentum into a unified 3■D model of market equ

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1. Introduction

This document upgrades the previous v1.0 engine specification produced with Claude. It presents refined ma

The intent is to:

1. Formalize the **pillar■based geometric state variables** (Theta, Phi, and coherence).
2. Add **harmonic and wavelet■based features** that capture multi■scale behavior in both economic and m
3. Define **macro–market coupling metrics** (phase alignment / resonance).
4. Construct a **unified feature vector** and geometry space suitable for regime detection and backtesting.

This spec is written so that multiple AI systems (Claude, ChatGPT, etc.) and human collaborators can all imp

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2. Pillar Geometry (Phase I & II)

2.1 Data Normalization

For each metric \(x_i(t)\), define a normalized series \(z_i(t)\) via a standard z■score over a chosen sample w

\[
z_i(t) = \frac{x_i(t) - \mu_i}{\sigma_i},
\quad
\mu_i = \frac{1}{|\mathcal{T}|} \sum_{u \in \mathcal{T}} x_i(u),
\quad
\sigma_i^2 = \frac{1}{|\mathcal{T}|} \sum_{u \in \mathcal{T}} \left(x_i(u) - \mu_i\right)^2.
\]

Alternative normalization schemes (robust, rolling, logit, etc.) may be used, but all metrics in a given experim

2.2 Pillar Definitions

Partition the metric universe into disjoint (or tagged) sets:

- \(\mathcal{M}\): Macro metrics (GDP, CPI, PPI, unemployment, etc.)
- \(\mathcal{L}\): Liquidity / rates metrics (M2, 10■year yield, curve spreads, etc.)
- \(\mathcal{R}\): Risk / volatility metrics (VIX, MOVE, credit vol, etc.)
- \(\mathcal{E}\): Equity / return metrics (SPY, sectors, breadth, etc.)

For each date \(t\), define the **pillar scores** as simple averages of z■scores:

\[
M(t) = \frac{1}{n_M}\sum_{i \in \mathcal{M}} z_i(t), \quad
L(t) = \frac{1}{n_L}\sum_{i \in \mathcal{L}} z_i(t),
\]

\[
R(t) = \frac{1}{n_R}\sum_{i \in \mathcal{R}} z_i(t), \quad
E(t) = \frac{1}{n_E}\sum_{i \in \mathcal{E}} z_i(t),
\]

where $n_M = |\mathcal{M}|$, etc.

2.3 Geometric Angles and Coherence

Define two primary planes:

- **Macro–Liquidity plane**: coordinates $(L(t), M(t))$
- **Equity–Risk plane**: coordinates $(R(t), E(t))$

The **angles** (in degrees) are:

$$
\theta(t) = \arctan2\big(M(t),\,L(t)\big) \cdot \frac{180}{\pi},
$$

$$
\phi(t) = \arctan2\big(E(t),\,R(t)\big) \cdot \frac{180}{\pi}.
$$

The corresponding **coherence / radius** values are:

$$
C_{\theta}(t) = \sqrt{M(t)^2 + L(t)^2},
\quad
C_{\phi}(t) = \sqrt{E(t)^2 + R(t)^2}.
$$

These capture the magnitude of the combined signal in each plane. High coherence indicates strong, aligne

2.4 Example Regime Bands

For illustration, one can define economic cycle bands on $\theta(t)$:

- $\theta < 0^\circ$: Contraction / recessionary geometry
- $0^\circ \le \theta < 30^\circ$: Early expansion
- $30^\circ \le \theta < 60^\circ$: Late expansion
- $\theta \ge 60^\circ$: Overheated / late-cycle

Similarly, one can define risk regimes on $\phi(t)$:

- $\phi < 45^\circ$: Risk-off
- $45^\circ \le \phi < 70^\circ$: Neutral / mixed
- $\phi \ge 70^\circ$: Risk-on / stretched

The exact thresholds can be calibrated empirically, but the mathematical structure remains as above.

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3. Harmonic & Wavelet Analysis (Phase II)

3.1 Motivation

Standard geometry (Theta, Phi, coherence) captures the **instantaneous configuration** of macro and mark

To address this, we introduce **harmonic and wavelet features** that summarize how power is distributed ac

3.2 Detrending and Windowing

Given a scalar time series $x(t)$ (monthly or quarterly data), we first define a locally detrended series:

- Choose a local mean window length k (e.g., 60–120 months).
- Compute the rolling mean:

$$
\mu_k(t) = \frac{1}{k} \sum_{j=0}^{k-1} x(t-j).
$$

- Detrend:

$$
y(t) = x(t) - \mu_k(t).
$$

For wavelet or Fourier analysis, we work on a moving window of fixed length W (e.g., 120 months):

$$
y_{t-W+1:t} = [y(t-W+1),\,\dots,\,y(t)].
$$

3.3 Continuous Wavelet Transform (CWT) – Core Definition

The **continuous wavelet transform** (CWT) of a function $x(t)$ at scale $a > 0$ and translation \(b \in \mat

$$
W_x(a,b) = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{\infty} x(t)\,\overline{\psi\!\left(\frac{t-b}{a}\right)}\,dt,
$$

where $\psi(t)$ is the **mother wavelet** and the overline denotes complex conjugation.

- The parameter a controls **dilation** (scale): small a → high frequency / short cycles; large a → lo
- The parameter b controls **translation** in time.

The **wavelet power spectrum** at (a,b) is:

$$
P_x(a,b) = \frac{1}{a} \big| W_x(a,b) \big|^2.
$$

In practice, implementations often use the **Morlet wavelet**, a complex exponential modulated by a Gaussi

3.4 Scale–Frequency Mapping

For a given wavelet family, there is a mapping from **scale** a to **equivalent Fourier frequency** f (or

$$
f(a) \approx \frac{f_c}{a \cdot \Delta t}, \quad T(a) \approx \frac{1}{f(a)},
$$

where f_c is the center frequency of the mother wavelet and Δt is the sampling period. Many librar

This allows us to define bands in terms of **periods in months** (e.g., 6–24 months, 24–96 months) and then

3.5 Wavelet Harmonic Features (Short vs Long Cycles)

For each metric, on each window, we define **short■cycle** and **long■cycle** power:

- Let $\mathcal{S}$ be the set of scales corresponding to short periods (e.g., 6–24 months).
- Let $\mathcal{L}$ be the set of scales corresponding to long periods (e.g., 24–96 months).

Then, the integrated power in each band is:

\[
P_{\text{short}}(t) = \sum_{a \in \mathcal{S}} P_x(a, b_t),
\quad
P_{\text{long}}(t) = \sum_{a \in \mathcal{L}} P_x(a, b_t),
\]

where \(b_t\) is the time index corresponding to the end of the window.

Define the **harmonic ratio**:

\[
H_{\text{ratio}}(t) = \frac{P_{\text{short}}(t)}{P_{\text{long}}(t) + \varepsilon},
\]

with a small \(\varepsilon > 0\) to ensure numerical stability. A value \(H_{\text{ratio}}(t) > 1\) indicates that sh

Define the **dominant period** as the period associated with the scale (or frequency) of maximum power:

\[
a^*(t) = \arg\max_{a} P_x(a, b_t),
\quad
T_{\text{dom}}(t) = T(a^*(t)).
\]

These functions \(H_{\text{ratio}}(t)\) and \(T_{\text{dom}}(t)\) are computed separately for each metric.

3.6 Discrete Wavelet Transform and MODWT (Alternative Implementation)

As an alternative or complement to the CWT, one can use the **discrete wavelet transform (DWT)** or, bette

- The DWT yields a multiresolution decomposition into levels \(j = 1, 2, \dots, J\), each associated with a dya
- The MODWT is a non■decimated (redundant) version of the DWT that is **shift■invariant** and can handl

In this context, at each level \(j\), we obtain detail coefficients \(d_{j,t}\). The variance of these coefficients ov

The core ideas (short vs long power, dominant scale/level, etc.) remain the same, whether implemented via

3.7 Pillar■Level Harmonic Aggregation

For a pillar (e.g., Equity), with metric set \(\mathcal{E}\):

\[
H^{(E)}_{\text{ratio}}(t) = \frac{1}{n_E} \sum_{i \in \mathcal{E}} H_{\text{ratio},i}(t),
\]

\[
T^{(E)}_{\text{dom}}(t) = \frac{1}{n_E} \sum_{i \in \mathcal{E}} T_{\text{dom},i}(t).
\]

Analogous aggregates can be defined for Macro, Liquidity, and Risk pillars. These become additional state v

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4. Macro–Market Coupling & Resonance (Phase III)

4.1 Phase Alignment via Wavelets or Fourier

To study coupling between macro and markets, we consider **phase alignment** at specific frequencies or s

Using either CWT or DFT on a window, we obtain complex coefficients for each frequency (or scale) index \(

$$
\hat{y}_M(\omega_j) = A_M(\omega_j)\,e^{i \phi_M(\omega_j)},
\quad
\hat{y}_E(\omega_j) = A_E(\omega_j)\,e^{i \phi_E(\omega_j)}.
$$

The **phase difference** is:

$$
\Delta \phi(\omega_j, t) = \phi_E(\omega_j) - \phi_M(\omega_j),
$$

wrapped into $(-\pi, \pi]$ as needed.

Let $\omega_M^*(t)$ be the frequency (or scale) at which macro power is maximal in the window. Define the

$$
\text{Resonance}_{M,E}(t) = \cos\big(\Delta \phi(\omega_M^*(t), t)\big).
$$

Interpretation:

- $\text{Resonance} \approx +1$: Macro and equity cycles are **in phase** (moving together).
- $\text{Resonance} \approx -1$: **Anti-phase**, one is high when the other is low.
- $\text{Resonance} \approx 0$: Quadrature / disordered phase (misaligned).

The same construction can be applied to other pairs (Macro vs Risk, Liquidity vs Equity, etc.).

4.2 Additional Coupling Measures (Optional)

Other coupling metrics that can be added later include:

- **Wavelet coherence**, a normalized cross-spectrum in time–frequency space.
- **Phase-locking value (PLV)** across windows.
- **Directional measures**, e.g., lag-structure between macro and market cycles.

For now, the Resonance Index defined above is sufficient for a first Phase III implementation.

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5. Unified State Vector & Regime Geometry (Phase III)

5.1 Construction of Unified Feature Vector

The goal in Phase III is to define a **single, agnostic state vector** that combines pillar geometry, harmonic I

For each time t, define:

$$
\mathbf{X}(t) = \big[
M(t),\,L(t),\,R(t),\,E(t),\,
C_{\theta}(t),\,C_{\phi}(t),\,
H^{(M)}_{\text{ratio}}(t),\,H^{(L)}_{\text{ratio}}(t),\,H^{(E)}_{\text{ratio}}(t),\,
T^{(M)}_{\text{dom}}(t),\,T^{(E)}_{\text{dom}}(t),\,
\text{Resonance}_{M,E}(t), \dots
\big]^{\top},
$$

where “$\dots$” can be extended to include additional features (e.g., Risk pillar harmonic stats, other resona

5.2 Dimensionality Reduction to VCF Geometry

To obtain a low-dimensional **geometry** suitable for visualization and regime clustering, apply principal co

- Let $W \in \mathbb{R}^{K \times d}$ be the matrix of the first d principal components (eigenvectors of th
- Let $\bar{\mathbf{X}}$ denote the mean of the feature vector over time.

Then:

$$
\mathbf{Y}(t) = W^{\top} \left(\mathbf{X}(t) - \bar{\mathbf{X}}\right),
$$

where $\mathbf{Y}(t) \in \mathbb{R}^d$ are the **VCF coordinates**. For example, $d = 3$ for a 3-D geom

Define:

$$
Y_1(t), Y_2(t), Y_3(t)
$$

as the components of $\mathbf{Y}(t)$. Then introduce VCF-space angles and radius:

$$
\Theta_{\text{VCF}}(t) = \arctan2\big(Y_2(t), Y_1(t)\big) \cdot \frac{180}{\pi},
$$

$$
\Phi_{\text{VCF}}(t) = \arctan2\big(Y_3(t), Y_1(t)\big) \cdot \frac{180}{\pi},
$$

$$
R_{\text{VCF}}(t) = \sqrt{Y_1(t)^2 + Y_2(t)^2 + Y_3(t)^2}.
$$

These are the **fully agnostic geometric coordinates** of the system, combining economic and market inforr

5.3 Regime Detection in Geometry Space

Collect all $\mathbf{Y}(t)$ into a matrix $Y \in \mathbb{R}^{T \times d}$. Apply a clustering method such as

- Choose a number of clusters K_{regime}.
- Fit the model to $\{\mathbf{Y}(t)\}$.
- Assign each time t to a regime:

$$
\text{Regime}(t) = \arg\min_{k \in \{1,\dots,K_{\text{regime}}\}} \left\|\mathbf{Y}(t) - \mu_k\right\|^2,
$$

where μ_k are the cluster centroids.

These empirically discovered regimes live in VCF geometry space and are **agnostic** as to whether signal

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6. Implementation Notes

6.1 Practical Choices

- **Sampling frequency**: monthly data is preferred for macro-heavy panels; higher-frequency implementa
- **Wavelet family**: Morlet or similar complex wavelet, which balances time and frequency localization, is re
- **Boundary conditions**: reflection or periodic extension should be used consistently in MODWT/DWT impl
- **Window sizes**:
  - Local detrending k: typically 5–10 years (60–120 months).
  - Spectral window W: often aligned with k or slightly shorter.

6.2 Data Pipeline Integration

1. Build normalized panel of all metrics.
2. Compute pillar scores and basic geometry (Section 2).
3. For each metric, compute wavelet/MODWT features on a rolling window (Section 3).
4. Aggregate harmonic features at the pillar level.
5. Compute macro–market resonance indices (Section 4).
6. Assemble the unified feature vector $\mathbf{X}(t)$ (Section 5.1).
7. Apply PCA (or similar) to derive VCF coordinates $\mathbf{Y}(t)$ (Section 5.2).
8. Cluster in geometry space to obtain regimes (Section 5.3).

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7. Next Steps for the Study

- Finalize **Phase I** (economic geometry only) with charts and historical narrative.
- Build **Phase II** market-sector and harmonic panel (using CWT or MODWT).
- Implement **Phase III** unified geometry + regime classification as per this spec.
- Layer on portfolio backtests using static, tilt-based, and geometry-driven allocation rules.
- Prepare academic-style documentation with methodology, results, and robustness checks.

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8. References (Conceptual & Technical)

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