

Lecture 3

How to minimize cost

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Acknowledgement

- Andrew Ng's ML class
 - <https://class.coursera.org/ml-003/lecture>
 - <http://www.holehouse.org/mlclass/> (note)
- Convolutional Neural Networks for Visual Recognition.
 - <http://cs231n.github.io/>
- Tensorflow
 - <https://www.tensorflow.org>
 - <https://github.com/aymericdamien/TensorFlow-Examples>

Hypothesis and Cost

$$H(x) = Wx + b$$

$$cost(W, b) = \frac{1}{m} \sum_{i=1}^m (H(x^{(i)}) - y^{(i)})^2$$

Simplified hypothesis

$$H(x) = Wx$$

$$\text{cost}(W) = \frac{1}{m} \sum_{i=1}^m (Wx^{(i)} - y^{(i)})^2$$

What $cost(W)$ looks like?

$$cost(W) = \frac{1}{m} \sum_{i=1}^m (W x^{(i)} - y^{(i)})^2$$

| x | Y |
|---|---|
| 1 | 1 |
| 2 | 2 |
| 3 | 3 |

- $W=I, cost(W)=?$

What $\text{cost}(W)$ looks like?

$$\text{cost}(W) = \frac{1}{m} \sum_{i=1}^m (W x^{(i)} - y^{(i)})^2$$

| x | Y |
|---|---|
| 1 | 1 |
| 2 | 2 |
| 3 | 3 |

- $W=1, \text{cost}(W)=0$

$$\frac{1}{3}((1 * 1 - 1)^2 + (1 * 2 - 2)^2 + (1 * 3 - 3)^2)$$

- $W=0, \text{cost}(W)=4.67$

$$\frac{1}{3}((0 * 1 - 1)^2 + (0 * 2 - 2)^2 + (0 * 3 - 3)^2)$$

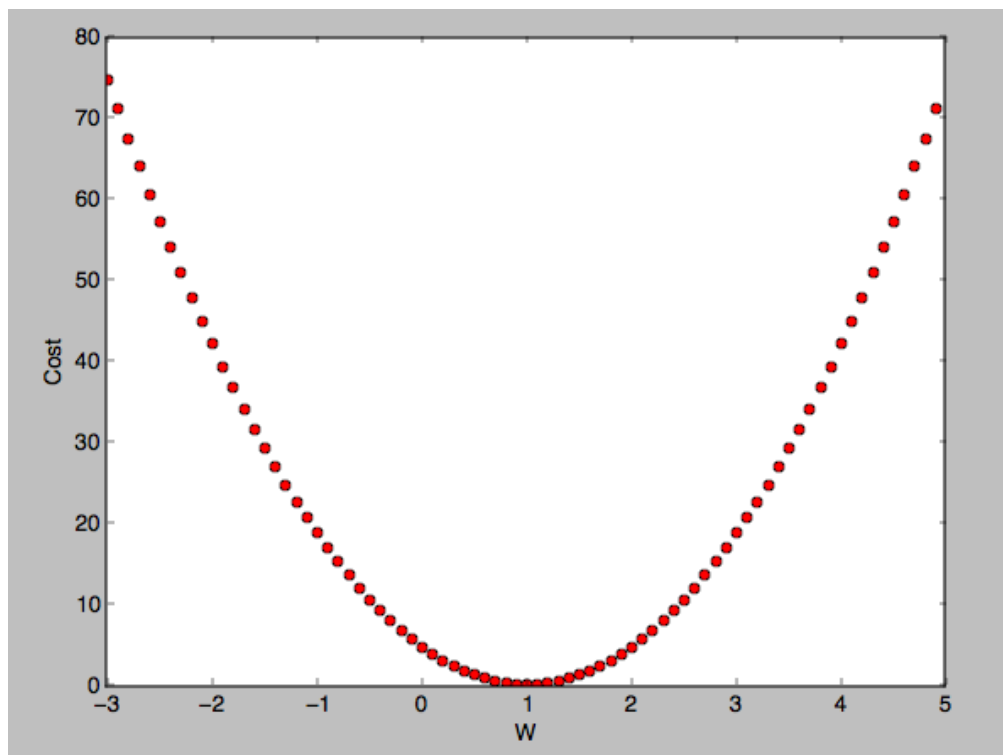
- $W=2, \text{cost}(W)=?$

What $\text{cost}(W)$ looks like?

- $W=1, \text{cost}(W)=0$
- $W=0, \text{cost}(W)=4.67$
- $W=2, \text{cost}(W)=4.67$

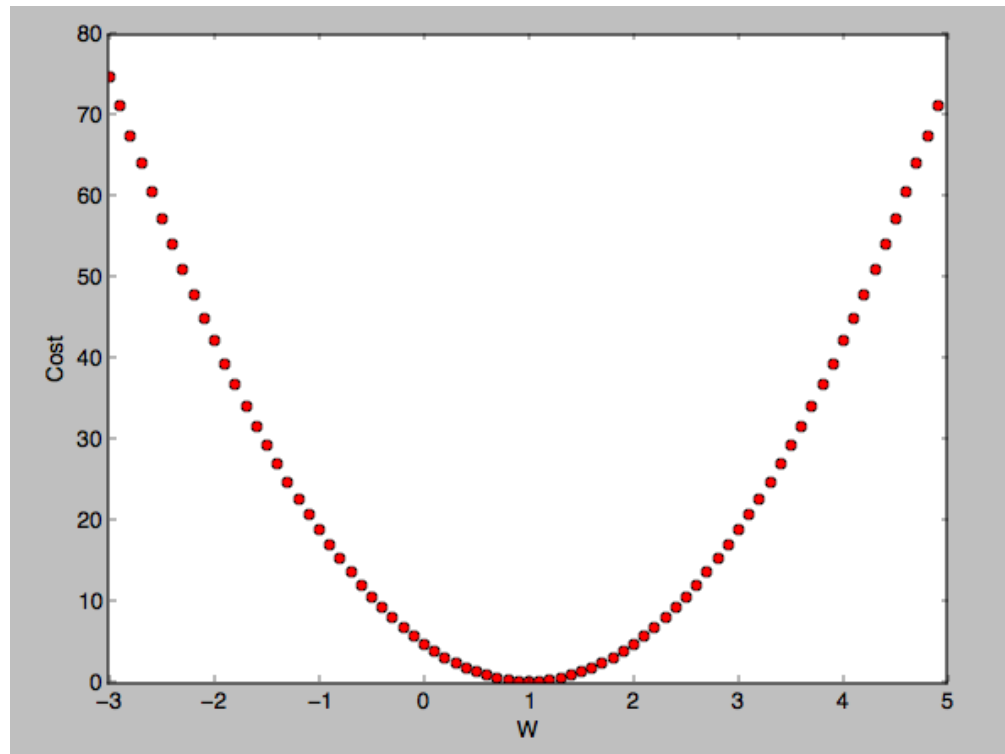
What $\text{cost}(W)$ looks like?

$$\text{cost}(W) = \frac{1}{m} \sum_{i=1}^m (W x^{(i)} - y^{(i)})^2$$



How to minimize cost?

$$\text{cost}(W) = \frac{1}{m} \sum_{i=1}^m (W x^{(i)} - y^{(i)})^2$$

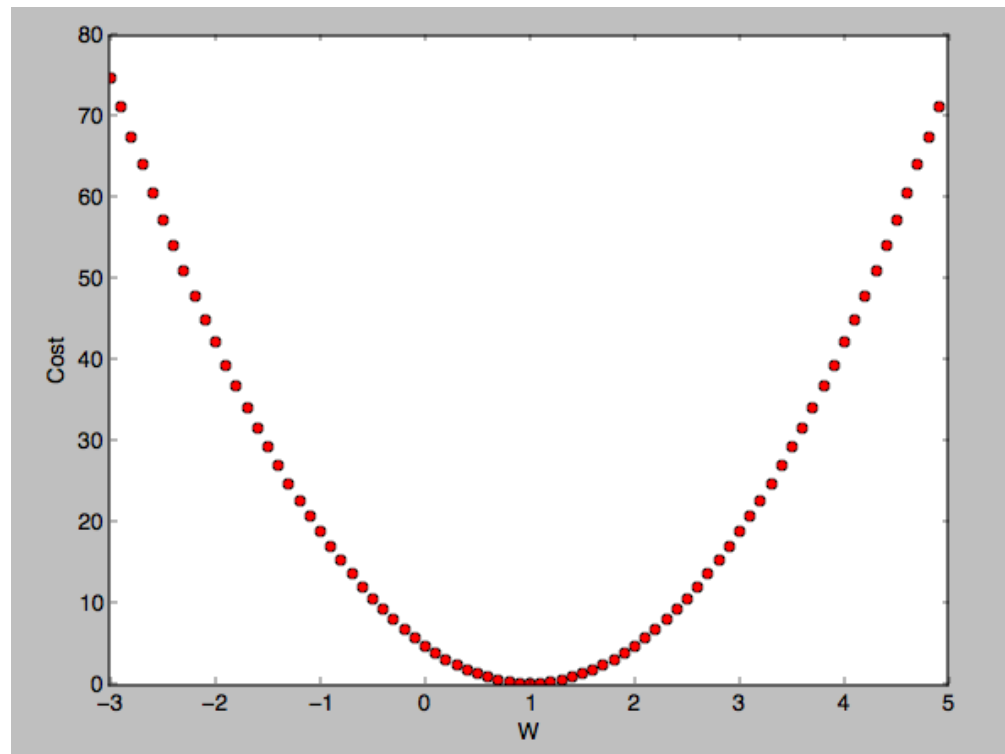


Gradient descent algorithm

- Minimize cost function
- Gradient descent is used many minimization problems
- For a given cost function, $cost(W, b)$, it will find W, b to minimize cost
- It can be applied to more general function: $cost(w_1, w_2, \dots)$

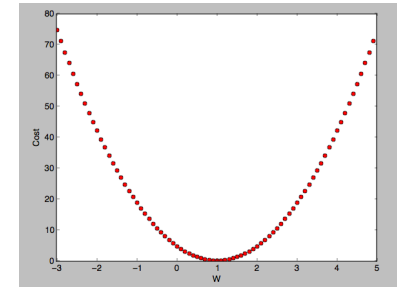
How it works?

How would you find the lowest point?



How it works?

- Start with initial guesses
 - Start at 0,0 (or any other value)
 - Keeping changing W and b a little bit to try and reduce $\text{cost}(W, b)$
- Each time you change the parameters, you select the gradient which reduces $\text{cost}(W, b)$ the most possible
- Repeat
- Do so until you converge to a local minimum
- Has an interesting property
 - Where you start can determine which minimum you end up



Formal definition

$$\text{cost}(W) = \frac{1}{m} \sum_{i=1}^m (W x^{(i)} - y^{(i)})^2$$



$$\text{cost}(W) = \frac{1}{2m} \sum_{i=1}^m (W x^{(i)} - y^{(i)})^2$$

Formal definition

$$cost(W) = \frac{1}{2m} \sum_{i=1}^m (W x^{(i)} - y^{(i)})^2$$

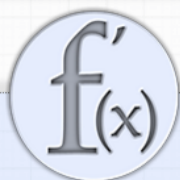
$$W := W - \text{👉} \frac{\partial}{\partial W} cost(W)$$

Formal definition

$$W := W - \alpha \frac{\partial}{\partial W} \frac{1}{2m} \sum_{i=1}^m (W x^{(i)} - y^{(i)})^2$$

$$W := W - \alpha \frac{1}{2m} \sum_{i=1}^m 2(W x^{(i)} - y^{(i)}) x^{(i)}$$

$$W := W - \alpha \frac{1}{m} \sum_{i=1}^m (W x^{(i)} - y^{(i)}) x^{(i)}$$



Derivative Calculator

Calculate derivatives online
— with steps and graphing!

Also check the [Integral Calculator!](#)
[Ableitungsrechner](#) auf Deutsch



Hello there!

Was this calculator helpful to you? Then I would highly appreciate **small donations** via PayPal:



... or use [this link](#) for shopping on Amazon, without affecting your order.

Thank you!

Calculate the Derivative of ...

(x-a)^2

Go!

This will be calculated:

$$\frac{d}{dx} \left[(xa - y)^2 \right]$$

Not what you mean? *Use parentheses!* Set differentiation variable and order in "Options".

[About](#) [Help](#) [Examples](#) [Options](#)

The Derivative Calculator lets you calculate derivatives of functions online — for free!

Our calculator allows you to check your solutions to calculus exercises. It helps you practice by showing you the full working (step by step differentiation).

The Derivative Calculator supports computing first, second, ..., fifth derivatives as well as differentiating functions with many variables (partial derivatives), implicit differentiation and calculating roots/zeros. Interactive graphs/plots help visualize and better understand the functions.

For more about how to use the Derivative Calculator, go to "Help" or take a look at the examples.

And now: Happy differentiating!

Recommend this Website

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Result

Done! See the result further below.

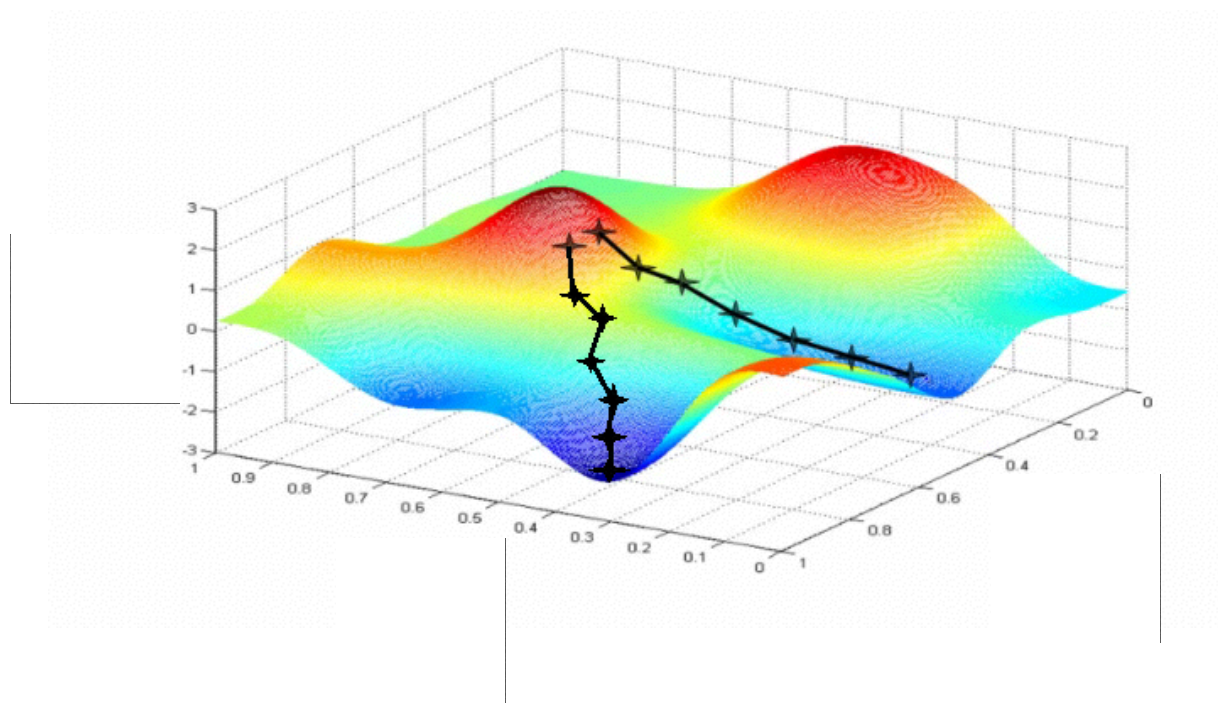
In order to not miss anything, please scroll all the way down.

Gradient descent algorithm

$$W := W - \alpha \frac{1}{m} \sum_{i=1}^m (Wx^{(i)} - y^{(i)})x^{(i)}$$



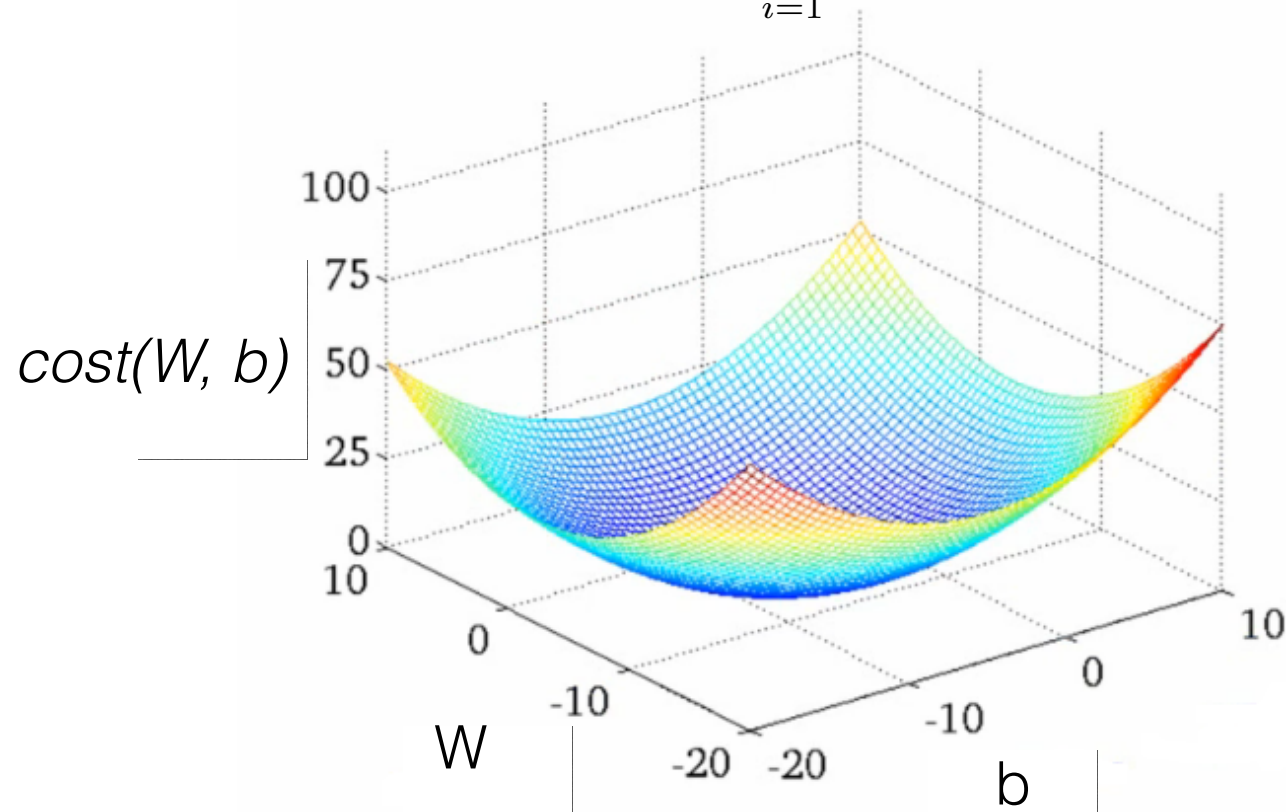
Convex function



www.holehouse.org/mlclass/

Convex function

$$cost(W, b) = \frac{1}{m} \sum_{i=1}^m (H(x^{(i)}) - y^{(i)})^2$$



Next
Multivariable logistic
regression

