The forestecology R package for fitting and assessing neighborhood models of the effect of interspecific competition on the growth of trees

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Abstract

- 1. Neighborhood competition models are powerful tools to measure the effect of interspecific competition. Statistical methods to ease the application of these models are currently lacking.
- 2. We present the forestecology package providing methods to i) specify neighborhood competition models, ii) evaluate the effect of competitor species identity using permutation tests, and iii) measure model performance using spatial cross-validation. Following Allen & Kim (2020), we implement a Bayesian linear regression neighborhood competition model.
- 3. We demonstrate the package's functionality using data from the Smithsonian Conservation Biology Institute's large forest dynamics plot, part of the ForestGEO global network of research sites. Given ForestGEO's data collection protocols and data formatting standards, the package was designed with cross-site compatibility in mind. We highlight the importance of spatial cross-validation when interpreting model results.
- 4. The package features i) tidyverse-like structure whereby verb-named functions

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can be modularly "piped" in sequence, ii) functions with standardized inputs/outputs
of simple features sf package class, and iii) an S3 object-oriented implementation of
the Bayesian linear regression model. These three facts allow for clear articulation of
all the steps in the sequence of analysis and easy wrangling and visualization of the
geospatial data. Furthermore, while the package only has Bayesian linear regression
implemented, the package was designed with extensibility to other methods in mind.

Keywords: forest ecology, interspecific competition, neighborhood competition, tree growth,
 R, ForestGEO, spatial cross-validation

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1 Introduction

Repeat-censused forest plots offer excellent opportunities to test neighborhood models of the effect of competition on the growth of trees (Canham et al. 2004). Neighborhood mod-33 els of competition have been used to: test whether the species identity of a competitor 34 matters [Uriarte et al. (2004); measure species-specific competition coefficients (Das 2012, 35 Tatsumi et al. (2016)); test competing models to see what structures competitive interac-36 tions, e.g. traits or phylogeny (Allen & Kim 2020, Uriarte et al. 2010); and inform selective 37 logging practices (Canham et al. 2006). Although these are well-described methods, few 38 methods are currently available for easy application. 39 We address this shortcoming with the forestecology R package providing methods 40 and data for forest ecology model fitting and assessment, available on CRAN (https:// 41 cran.r-project.org/package=forestecology) and on GitHub (https://github.com/ 42 rudeboybert/forestecology). The package is written to model stem diameter growth between two censuses based on neighborhood competition, largely following the methods in Allen & Kim (2020). 45 Let $i = 1, ..., n_j$ index all n_j trees of "focal" species j; let j = 1, ..., J index all J focal 46 species; and let k = 1, ..., K index all K "competitor" species. The average annual growth in diameter at breast height (DBH) y_{ij} (in centimeters/year) of the i^{th} tree of focal species j is modeled as

$$y_{ij} = \beta_{0,j} + \beta_{\text{dbh},j} \cdot \text{dbh}_{ij} + \sum_{k=1}^{K} \lambda_{jk} \cdot \text{BA}_{ijk} + \epsilon_{ij}$$
 (1)

where $\beta_{0,j}$ is the diameter-independent growth rate of species j; dbh_{ij} is the DBH of the focal tree at the earlier census and $\beta_{\text{dbh},j}$ the slope of that species's diameter-growth relationship; BA_{ijk} is the sum of the basal area of all trees of competitor species k and λ_{jk} quantifies the corresponding change in growth for individuals of group j from these competitors; and ϵ_{ij} is a random error term distributed Normal $(0, \sigma^2)$. Allen & Kim (2020)

- estimate all parameters via Bayesian linear regression, while exploiting Normal/Inverse
- 56 Gamma conjugacy to derive closed-form solutions to all posterior distributions¹. These
- 57 closed-form solutions are not as computationally expensive as approximations from Markov
- 58 Chain Monte Carlo algorithms.
- To evaluate whether competitor species identity matters, Allen & Kim (2020) run a per-
- 60 mutation test where a null hypothesis of no species grouping-specific effects of competition
- 61 is assumed, thus the species identity of all competitors can be permuted:

$$H_0: \lambda_{jk} = \lambda_j \text{ for all } k = 1, \dots, K$$
 (2)

vs. H_A : at least one λ_{jk} is different

- Furthermore, to account for the spatial autocorrelation in their estimates of out-of-
- sample model error, Allen & Kim (2020) use spatial cross-validation. Estimates of model
- error that do not account for this dependence tend to underestimate the true model error
- 65 (Roberts et al. 2017).
- The package is designed with "tidy" design principles in mind (Wickham et al. 2019).
- 67 Much like all tidyverse packages, forestecology has verb-named functions that can be
- 68 modularly composed using the pipe %>% operator to sequentially complete all necessary
- 69 analysis steps (Bache & Wickham 2020). Furthermore, the inputs and outputs of most
- 70 functions use the same "simple features for R" data structures from the sf package, a
- package for standardized and tidyverse-friendly wrangling and visualizing of spatial data
- 72 (Pebesma 2018).
- ⁷³ Currently the package only implements the Bayesian linear regression model detailed
- in Equation 1. As we demonstrate in Section 2.4 however, the fitting of this model is self-
- contained in a single function comp_bayes_lm() which returns an object of S3 class type

¹See S1 Appendix of Allen & Kim (2020), available at https://doi.org/10.1371/journal.pone. 0229930.s004

- comp_bayes_lm. This class has generic methods implemented to print, make predictions,
- and plot all results. Therefore the package can be modularly extended to fit other models as
- long as they are coded similarly to comp_bayes_lm() and have equivalent generic methods
- 79 implemented.

2 forestecology workflow: a case study

- We present a case-study of forestecology's functionality on data from the Smithsonian
- 82 Conservation Biology Institute (SCBI) large forest dynamics plot in Front Royal, VA,
- USA, part of the ForestGEO global network of research sites (Bourg et al. 2013, Anderson-
- ⁸⁴ Teixeira et al. (2015)). The 25.6 ha (640 x 400 m) plot is located at the intersection of three
- of the major physiographic provinces of the eastern US—the Blue Ridge, Ridge and Valley,
- and Piedmont provinces—and is adjacent to the northern end of Shenandoah National
- 87 Park.

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- The package has the following goals: to evaluate i) the effect of competitor species
- 39 identity using permutation tests and ii) model performance using spatial cross-validation.
- We outline the four-step basic analysis sequence:
- 1. Compute the growth of stems based on two censuses.
- 92 2. Add spatial information:
- 1. Define a buffer region of trees.
 - 2. Add spatial cross-validation block information.
- 3. Identify all focal trees and their competitors.
- 4. Apply model, which includes:
- 97 1. Fit model.
 - 2. Compute predicted values.
- 3. Visualize posterior distributions.

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We start by loading all packages.

```
library(tidyverse)
library(lubridate)
library(sf)
library(patchwork)
library(forestecology)
library(blockCV)

# Resolve conflicting functions
filter <- dplyr::filter
select <- dplyr::select</pre>
```

2.1 Step 1: Compute the growth of trees based on census data

We first compute the growth of trees using data from two censuses. compute_growth() 102 computes the average annual growth based on census data that roughly follows ForestGEO 103 standards. Despite such standards, minor variations will still exist between sites, thereby 104 necessitating some data wrangling. For example, the SCBI site records all DBH values in 105 millimeters (Bourg et al. 2013), whereas the Michigan Big Woods site used in Allen & Kim 106 (2020) records them in centimeters (Allen et al. 2020). 107 We load both 2008 and 2014 SCBI census .csv files as they existed on GitHub on 108 2020/11/20 and perform minor data wrangling (Gonzalez-Akre et al. 2020). We then only 109 consider a 9 ha subsection of the 25.6 ha of the site to speed up computation for this 110

example: gx from 0-300 instead of 0-400 and gy from 300-600 instead of 0-640.

```
census_2013_scbi <- read_csv("scbi.stem2.csv") %>%
 select(stemID, sp, date = ExactDate, gx, gy, dbh, codes, status) %>%
 mutate(
    # Convert date from character to date
   date = mdy(date),
   # Convert dbh to be in cm
   dbh = as.numeric(dbh)/10
 ) %>%
 filter(gx < 300, between(gy, 300, 600))
census_2018_scbi <- read_csv("scbi.stem3.csv") %>%
 select(stemID, sp, date = ExactDate, gx, gy, dbh, codes, status) %>%
 mutate(
   date = mdy(date),
   dbh = as.numeric(dbh)/10
 ) %>%
 filter(gx < 300, between(gy, 300, 600))
```

These two data frames are then used as inputs to compute_growth(), along with id
specifying the variable that uniquely identifies each tree-stem. We also discard all resprouts
with code == R in the later census, since we are only interested in the growth of surviving,
and not resprouted, stems.

```
growth_scbi <-
compute_growth(
  census_1 = census_2013_scbi,
  census_2 = census_2018_scbi %>% filter(!str_detect(codes, "R")),
  id = "stemID"
```

```
growth_scbi %>%
  select(stemID, sp, dbh1, dbh2, growth, geometry)
## Simple feature collection with 7954 features and 5 fields
## geometry type: POINT
## dimension:
                  XY
## bbox:
                  xmin: 0.2 ymin: 300 xmax: 300 ymax: 600
## CRS:
                  NA
## # A tibble: 7,954 x 6
     stemID sp
                  dbh1 dbh2 growth
##
                                      geometry
      <dbl> <fct> <dbl> <dbl> <dbl> <dbl>
                                       <POINT>
## 1
         4 nysy 13.6 14.2 0.103 (14.2 428)
         5 havi 8.8 9.6 0.150 (9.4 436)
## 2
        6 havi 3.25 4 0.140 (1.3 434)
## 3
        77 qual 65.2 66 0.141 (34.7 307)
        79 tiam 47.7 46.8 -0.161
                                    (40 381)
## 5
## # ... with 7,949 more rows
```

The output growth_scbi is a data frame of class sf that includes among other variables
the species variable sp converted to a factor, the average annual growth in DBH (cm ·
y-1) for all stems that were alive at both time points, and the sf package's encoding of
geolocations of geometry type <POINT>. Given that growth_scbi is of class sf, it can be
easily plotted in ggplot2 using geom_sf() as seen in Figure 1.

```
ggplot() +
  geom_sf(data = growth_scbi %>% sample_n(500), aes(size = growth)) +
  scale_size_binned(limits = c(0.1, 1)) +
  labs(size = expression(paste(Growth, " (cm ",y^{-1},')')) )
```

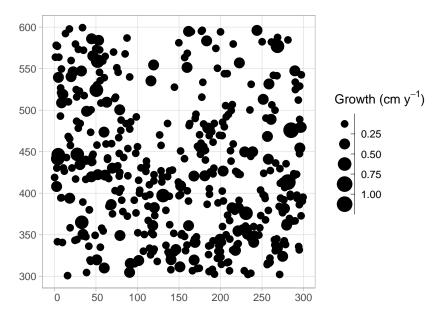


Figure 1: Step 1 - Compute growth of trees based on census data. A map of the growth of a random sample of 500 trees from a 9 ha subsection of the Smithsonian Conservation Biology Institute (SCBI) forest plot.

2.2 Step 2: Add spatial information

We then add spatial information to growth_scbi. We first add a "buffer region" to the periphery of the study region. Since some of our model's explanatory variables are cumulative, we must ensure that all trees being modeled are not biased to have different neighbor structures. This is of concern for trees at the boundary of the study region who will not have all their neighbors included in the census stems. To account for such edge effects, only trees that are not part of this buffer region, i.e. are part of the interior of the study region, will have their growth modeled (Waller & Gotway 2004).

Our model of interspecific competition relies on a spatial definition of who competitor trees are: all trees within a distance comp_dist of a focal tree. Here we set comp_dist to 7.5m, a value informed by other studies (Canham et al. 2004, Uriarte et al. (2004), Canham et al. (2006)), but the package could also be used to compare multiple distances and see which is best supported (see Appendix 2 9). We use comp_dist and a manually constructed

sf representation of the study region's boundary as inputs to add_buffer_variable() to

add a buffer boolean variable to growth_scbi. All trees with buffer equal to FALSE will

be our focal trees whose growth will be modeled, whereas those with TRUE will only act as

competitor trees.

```
# Define competitive distance range
comp_dist <- 7.5

# Manually construct study region boundary
study_region_scbi <- tibble(
    x = c(0, 300, 300, 0, 0),
    y = c(300, 300, 600, 600, 300)
) %>%
    sf_polygon()

growth_scbi <- growth_scbi %>%
    add_buffer_variable(size = comp_dist, region = study_region_scbi)
```

The second element of spatial information we add are blocks corresponding to folds of a spatial cross-validation algorithm. Conventional cross-validation algorithms assign individual observations to folds by randomly resampling them all while assuming they are statistically independent. In the case of forest census data however, observations exhibit spatial autocorrelation. We therefore incorporate this dependence into the cross-validation algorithm by resampling spatial blocks of trees (Roberts et al. 2017, Pohjankukka et al. (2017)).

We first manually define an sf object defining four folds that partition the study region.

We then use the output of the spatialBlock() function from the blockCV package to

- associate each tree in growth_scbi to the correct foldID (Valavi et al. 2019). This foldID
 variable will be used in Section 2.6.
- Figure 2 illustrates the net effect of adding these two elements of spatial information to growth_scbi.

```
# Manually define spatial blocks to act as folds
n_fold \leftarrow 4
fold1 <- cbind(c(0, 150, 150, 0), c(300, 300, 450, 450))
fold2 \leftarrow cbind(c(150, 300, 300, 150), c(300, 300, 450, 450))
fold3 <- cbind(c(0, 150, 150, 0), c(450, 450, 600, 600))
fold4 \leftarrow cbind(c(150, 300, 300, 150), c(450, 450, 600, 600))
blocks_scbi <- bind_rows(</pre>
  sf_polygon(fold1), sf_polygon(fold2), sf_polygon(fold3),
  sf_polygon(fold4)
) %>%
  mutate(folds = c(1:n_fold) %>% factor())
# Associate each observation to a fold
spatial_block_scbi <-
  spatialBlock(speciesData = growth_scbi, k = n_fold,
                selection = "systematic", blocks = blocks_scbi,
                showBlocks = FALSE, verbose = FALSE)
growth_scbi <- growth_scbi %>%
  mutate(foldID = spatial_block_scbi$foldID %>% factor())
```

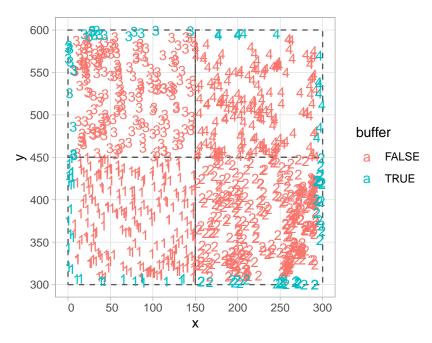


Figure 2: Step 2 - Add spatial information. A buffer region and spatial cross-validation blocks 1 through 4. The location of each tree is marked with its fold number where the folds are delineated with solid lines. The color of each digit indicates whether the tree is part of the buffer region (thus will only be considered as a competitor tree) or is part of the interior of the study region (thus is a focal tree whose growth is of modeled interest).

2.3 Step 3: Identify all focal and corresponding competitor trees

We then identify all focal trees and their corresponding competitor trees. More specifically, identify all trees that are not part of the buffer region, have a valid growth measurement, and have at least one neighbor within 7.5m. We do this using create_focal_vs_comp(), which takes the previously detailed comp_dist and id arguments as well as the sf representation of the spatial cross-validation blocks and returns a new data frame focal_vs_comp_scbi.

```
focal_vs_comp_scbi <- growth_scbi %>%
  create_focal_vs_comp(comp_dist, blocks = blocks_scbi, id = "stemID")
focal_vs_comp_scbi %>%
  select(focal_ID, focal_sp, geometry, growth, comp)
## # A tibble: 6,296 x 5
##
     focal_ID focal_sp geometry growth comp
##
        <dbl> <fct>
                          <POINT>
                                  <dbl> t>
## 1
                      (14.2 428) 0.103 <tibble [20 x 4]>
            4 nysy
## 2
            5 havi
                       (9.4 436) 0.150 <tibble [32 x 4]>
                         (40 381) -0.161 <tibble [20 x 4]>
## 3
           79 tiam
## 4
                       (38.7 422) 0.253 <tibble [12 x 4]>
           80 caca
## 5
           96 libe
                         (60 310) 0.262 <tibble [14 x 4]>
## # ... with 6,291 more rows
```

The resulting focal_vs_comp_scbi has 6296 rows, representing the subset of the 7954 trees in growth_scbi that will be considered as focal trees. The variables focal_ID and focal_sp relate to tree-stem identification and species information. Most notably however is the variable comp, which contains information on all competitor trees saved in tidyr package list-column format (Wickham 2020). To inspect this information, we flatten the comp list-column for the tree with focal_ID 4 in the first row, here a tibble [20 × 4], into regular columns using unnest() from the tidyr package.

```
focal_vs_comp_scbi %>%
  filter(focal_ID == 4) %>%
  select(focal_ID, dbh, comp) %>%
  unnest(cols = "comp")

## # A tibble: 20 x 6

## focal_ID dbh comp_ID dist comp_sp comp_basal_area
```

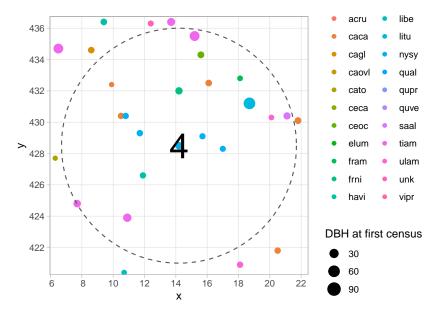


Figure 3: Step 3 - Identify all focal and corresponding competitor trees. The dashed circle extends 7.5m away from the focal tree 4 while all 20 competitor trees are within this circle.

##	<dbl> <dbl></dbl></dbl>	<db1> <d< th=""><th>lbl> <fct></fct></th><th><db1></db1></th><th></th></d<></db1>	lbl> <fct></fct>	<db1></db1>	
## 1	4 13.6	1836 7	7.48 tiam	0.0176	
## 2	4 13.6	1847 2	2.81 nysy	0.00332	
## 3	4 13.6	1848 1	.62 nysy	0.00396	
## 4	4 13.6	1849 2	2.62 nysy	0.00535	
## 5	4 13.6	1850 2	2.98 havi	0.00472	
## #	. with 15 mor	e rows			

We observe 4 variables describing 20 competitor trees: the unique tree-stem ID, the distance to the focal tree (all ≤ 7.5 m), the species, and the basal area (in m²) calculated as $\frac{\pi \times (\text{DBH/2})^2}{10000}$ for the DBH in cm from the earlier census. Saving competitor information in list-column format minimizes redundancy since we do not need to repeat information on the focal tree 20 times. We visualize the spatial distribution of these trees in Figure 3.

69 2.4 Step 4: Fit model

Lastly, we fit the competition Bayesian linear regression model for tree growth outlined in
Equation 1 using comp_bayes_lm(). This function has an option to specify prior distributions of all parameters, chosen here to be the defaults detailed in ?comp_bayes_lm.

```
comp_bayes_lm_scbi <- focal_vs_comp_scbi %>%
comp_bayes_lm(prior_param = NULL)
```

The resultingcomp_bayes_lm_scbi is an object of S3 class type comp_bayes_lm containing the posterior values of all parameters. Furthermore, this class includes generics for
three methods. First, the generic for print() displays the names of all prior and posterior
parameters and the model formula:

```
comp_bayes_lm_scbi
## Bayesian linear regression model parameters with a multivariate Normal
## likelihood. See ?comp_bayes_lm for details:
##
     parameter_type
##
                              prior posterior
## 1 Inverse-Gamma on sigma^2 a_0
                                     a\_star
## 2 Inverse-Gamma on sigma^2 b_0
                                     b_star
## 3 Multivariate t on beta
                              mu_0
                                    mu\_star
## 4 Multivariate t on beta
                               V_{-}O
                                     V_star
##
## Model formula:
## growth \sim sp + dbh + dbh * sp + acne * sp + acru * sp + amar * sp + astr
## * sp + caca * sp + caco * sp + cade * sp + caql * sp + caovl * sp + cato
## * sp + ceca * sp + ceoc * sp + chvi * sp + cofl * sp + crpr * sp + crsp
## * sp + divi * sp + elum * sp + faqr * sp + fram * sp + frni * sp + frpe
```

```
## * sp + havi * sp + ilve * sp + juci * sp + juni * sp + libe * sp + litu
## * sp + nysy * sp + pist * sp + pivi * sp + ploc * sp + prav * sp + prse
## * sp + qual * sp + quco * sp + qufa * sp + qumi * sp + qupr * sp + quru
## * sp + quve * sp + rops * sp + saal * sp + saca * sp + tiam * sp + ulam
## * sp + ulru * sp + unk * sp + vipr * sp
```

Next, the generic for predict() takes the posterior parameter values in comp_bayes_lm_scbi and a newdata data frame, and outputs a vector growth_hat of predicted DBH values $\widehat{y_{ij}}$ computed from the posterior predictive distribution.

```
focal_vs_comp_scbi <- focal_vs_comp_scbi %>%
mutate(growth_hat = predict(comp_bayes_lm_scbi, newdata = focal_vs_comp_scbi))
```

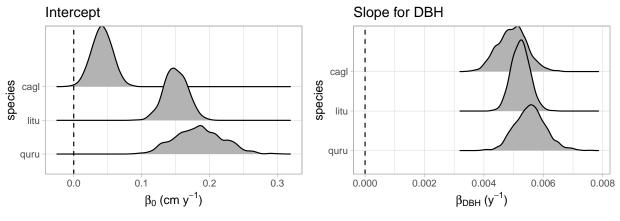
```
focal_vs_comp_scbi %>%
  select(focal_ID, focal_sp, dbh, growth, growth_hat)
## # A tibble: 6,296 x 5
    focal_ID focal_sp dbh growth growth_hat
        <dbl> <fct>
##
                       <dbl> <dbl>
                                         <db1>
## 1
                       13.6
                              0.103
                                        0.0809
            4 nysy
## 2
            5 havi
                      8.8
                              0.150
                                        0.112
## 3
           79 tiam
                      47.7 -0.161
                                        0.229
## 4
           80 caca
                        5.15 0.253
                                        0.121
## 5
           96 libe
                        2.3
                              0.262
                                        0.142
## # ... with 6,291 more rows
```

We can now compare the observed and predicted growths to compute the root mean squared error (RMSE) of our model:

```
model_rmse <- focal_vs_comp_scbi %>%
    rmse(truth = growth, estimate = growth_hat) %>%
    pull(.estimate)
model_rmse
## [1] 0.128
```

Lastly, the generic for ggplot2::autoplot() allows us to visualize all posterior distributions, as seen in Figure 4. Setting type to "intercepts" and "dbh_slopes" returns species-specific posterior distributions for $\beta_{0,j}$ and $\beta_{dbh,j}$ respectively, while setting type = "competition" returns competition coefficients $\lambda_{j,k}$.

For many users the visualizations of $\lambda_{j,k}$ will be of particular interest as they provide insight into species-specific competitive interactions, where negative values indicate a competitor species which slows the growth of a focal species. Here, for example, we see that tulip poplars (litu) have a strong negative effect on the growth of conspecifics but relatively



Competitor species in rows, focal species in columns Ex: Top row, second column: competitive effect of cagl on litu

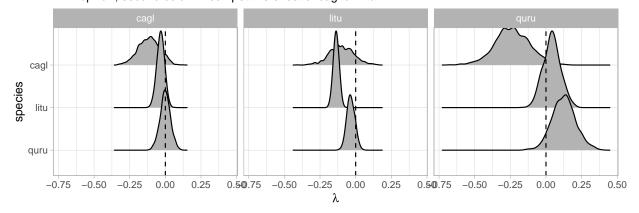


Figure 4: Step 4 - Fit model. Posterior distributions of all parameters. For compactness we include only three species.

lesser effect on pignut hickory (cagl) and red oak (quru) neighbors.

Currently the forestecology package can only fit the competition Bayesian linear regression model in Equation 1. However, it can be extended to any model as long as it is implemented in a function similar to comp_bayes_lm().

2.5 Evaluate the effect of competitor species identity using permutation tests

To evaluate the effect of competitor species identity, we use the above four steps along with 196 the permutation test in Equation 2. Under a null hypothesis where competitor species 197 identity does not matter, we can permute the competitor species identities within each 198 focal tree, compute the RMSE test statistic, repeat this process several times to construct 199 a null distribution, and compare it to the observed RMSE to assess significance. Going 200 back to our example in Section 2.3 of focal tree with focal_ID 4 and its 20 competitors, 201 the permutation test only randomly resamples the comp_sp variable without replacement, 202 leaving all other variables intact. This resampling is nested within each focal tree in order 203 to preserve neighborhood structure. We perform this permutation test once again using comp_bayes_lm() but by setting run_shuffle = TRUE.

```
model_rmse_shuffle <- focal_vs_comp_scbi %>%
    rmse(truth = growth, estimate = growth_hat_shuffle) %>%
    pull(.estimate)
model_rmse_shuffle
## [1] 0.131
```

The resulting permutation test RMSE of 0.131 is larger than the earlier RMSE of 0.128, suggesting that models that do incorporate competitor species identity better fit the data.

2.6 Evaluate model performance using spatial cross-validation

To evaluate model performance, we use spatial cross-validation. The model fit in Section 2.4 uses the same data to both fit and assess model performance. Given the spatial-autocorrelation of our data, this can potentially lead to overfit models (Roberts et al. 2017). To mitigate this risk, we use the spatial cross-validation blocking scheme encoded in the foldID variable from Section 2.2 and visualized in Figure 2.

At each iteration of the cross-validation, one fold acts as the test set and the remaining
three act as the training set. We fit the model to all focal trees in the training set, apply
the model to all focal trees in the test set, compute predicted values, and compute the
RMSE. Furthermore, to maintain spatial independence between the test and training sets,
a "fold buffer" that extends 7.5m outwards from the boundary of the test set is considered;
all trees within this "fold buffer" are excluded from the training set (see Figure 5).

This process is repeated for each of the four folds acting as the test set, then the four RMSE's are averaged to provide a single estimate of model error. This algorithm is implemented in run_cv(), which acts as a wrapper function to both comp_bayes_lm() that fits the model and predict() that returns predicted values.

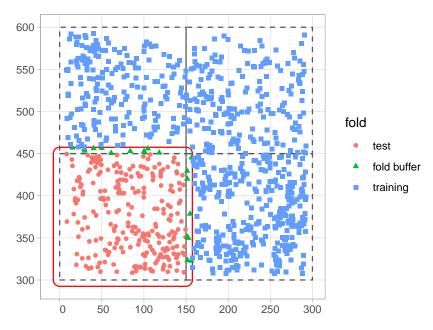


Figure 5: Schematic of spatial cross-validation. Using the k=1 fold (bottom-left) as the test set, k=2 through 4 as the training set, along with a "fold buffer" extending outwards from the test set to maintain spatial independence between it and the training set.

```
focal_vs_comp_scbi <- focal_vs_comp_scbi %>%
    run_cv(comp_dist = comp_dist, blocks = blocks_scbi)
```

```
model_rmse_cv <- focal_vs_comp_scbi %>%
    rmse(truth = growth, estimate = growth_hat) %>%
    pull(.estimate)
model_rmse_cv
## [1] 0.14
```

The resulting RMSE of 0.14 computed using cross-validation is larger than the earlier RMSE of 0.128, suggesting that models that do not account for spatial autocorrelation generate model error estimates that are overly optimistic, i.e. RMSE values that are too low.

3 Importance of spatial cross-validation

run_cv() also accepts the run_shuffle argument in order to permute competitor species 229 identity as described in Section 2.5. Figure 6 compares model performance for 49 permuta-230 tions of competitor species and RMSE calculations, both with and without cross-validation. 231 Without cross-validation, competitor species identity does matter as the observed RMSE 232 was significantly lower than the permutation null distribution of RMSE. However, once we 233 incorporate spatial cross-validation, this improvement disappears. These results suggest 234 that in this 9 has subplot of the SCBI plot, competitive interactions do not depend on the 235 identity of the competitor, which is the opposite of what has been observed in other loca-236 tions (Allen & Kim 2020, Uriarte et al. (2004)). This provides a striking example of the 237 importance of cross-validation, as without it the over-fit model gives rise to an incorrect 238 conclusion. 239

²⁴⁰ 4 Conclusion and future work

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The forestecology package provides an accessible way to fit and test models of neighborhood competition. The package follows the tidy data design principles, leverages the sf package for spatial data, and S3 open-oriented model implementation structure. We hope that the package will increase the use of neighborhood competition models to better understand what structures plant competition.

While the package is designed with ForestGEO plot data in mind, we envision that it can be modified to work on any single large, mapped forest plot in which at least two measurements of each individual have been taken. Furthermore, we hope that future versions of the package will be flexible to other plot layouts, for example inventory plot-structure with many spatially separated plots like the US Forest Service Forest Inventory and Analysis plots (Smith 2002).

We also hope to extend the forestecology package's functionality to account for a

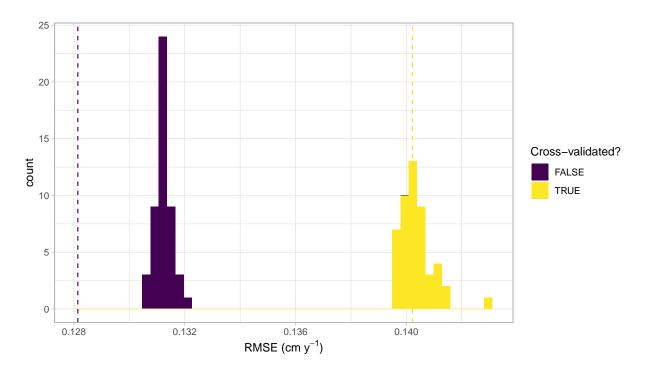


Figure 6: Comparison of root mean squared error of models for standard, permuted, and spatially cross-validated error estimates. The dotted lines show observed RMSE while the histograms show the null distribution of RMSE for 49 permutations under the null hypothesis of no competitor species identity effects. The colors indicate whether spatial cross-validation was used or not.

larger variety of models for tree growth. One clear future direction would be to allow competition based on species trait values rather than species identity. There is evidence that traits predict competitive outcomes (Kunstler et al. 2012, Lasky et al. (2014), Uriarte et al. (2010)). Thus an extension of the model would allow λ values from Equation 1 to be a function of the traits of competing species.

Lastly, the forestecology current uses the blockCV package behind the scenes to cre-

ate the spatial blocks acting as folds for our spatial cross-validation algorithm detailed in Sections 2.2 and 2.6 (Valavi et al. 2019). This back-end functionality could be substituted with the spatialsample package for spatial resampling infrastructure; a tidymodels package under active development as of 2021 (Silge 2021, Kuhn & Wickham (2020)).

5 Acknowledgments

We thank Sophie Li for their feedback on the package interface. The authors declare no conflicts of interest.

₂₆₆ 6 Author's contributions

AYK and DNA conceived the ideas and coded a draft of the package. AYK wrote an initial manuscript draft. SPC rewrote much of the package's code to align with R and "tidy" best practices (Wickham et al. 2019). All authors contributed to subsequent drafts and gave final approval for manuscript.

7 Data accessibility

We intend to archive all data and source code for this manuscript on GitHub at https://
github.com/rudeboybert/forestecology and on Zenodo upon acceptance. The example
Smithsonian Conservation Biology Institute census data are available on GitHub at https:

```
//github.com/SCBI-ForestGEO/SCBI-ForestGEO-Data/tree/master/tree_main_census/
data/census-csv-files and are archived on Zenodo at https://doi.org/10.5281/zenodo.
277 2649301 (Gonzalez-Akre et al. 2020).
```

²⁷⁸ 8 Appendix 1: Replicate RMSE comparison

This code replicates Figure 6: A comparison of root mean squared error of models for standard, permuted, and spatial cross-validated error estimates.

```
library(tidyverse)
library(lubridate)
library(here)
library(sf)
library(viridis)
library(forestecology)
library(blockCV)
library(tictoc)
# Compute growth of trees based on census data -----
census_2013_scbi <- here("paper/scbi.stem2.csv") %>%
  read_csv() %>%
  select(stemID, sp, date = ExactDate, gx, gy, dbh, codes, status) %>%
 mutate(
    date = mdy(date),
    dbh = as.numeric(dbh)/10
  ) %>%
```

```
filter(gx < 300, between(gy, 300, 600))
census_2018_scbi <- here("paper/scbi.stem3.csv") %>%
 read_csv() %>%
  select(stemID, sp, date = ExactDate, gx, gy, dbh, codes, status) %>%
 mutate(
   date = mdy(date),
   dbh = as.numeric(dbh)/10
  ) %>%
 filter(gx < 300, between(gy, 300, 600))
growth_scbi <-
  compute_growth(
   census_1 = census_2013_scbi,
   census_2 = census_2018_scbi %>% filter(!str_detect(codes, "R")),
   id = "stemID"
  )
# Add spatial information -----
# Define buffer region using competitive distance range
comp_dist <- 7.5
study_region_scbi <- tibble(</pre>
 x = c(0, 300, 300, 0, 0),
  y = c(300, 300, 600, 600, 300)
```

```
) %>%
  sf_polygon()
growth_scbi <- growth_scbi %>%
  add_buffer_variable(size = comp_dist, region = study_region_scbi)
# Manually define spatial blocks to act as folds
fold1 <- rbind(c(0, 300), c(150, 300), c(150, 450), c(0, 450))
fold2 <- rbind(c(150, 300), c(300, 300), c(300, 450), c(150, 450))
fold3 <- rbind(c(0, 450), c(150, 450), c(150, 600), c(0, 600))
fold4 \leftarrow rbind(c(150, 450), c(300, 450), c(300, 600), c(150, 600))
n_fold \leftarrow 4
blocks_scbi <- bind_rows(</pre>
  sf_polygon(fold1), sf_polygon(fold2),
  sf_polygon(fold3), sf_polygon(fold4)
) %>%
  mutate(folds = c(1:n_fold) %>% factor())
# Associate each observation to a fold
SpatialBlock_scbi <- spatialBlock(</pre>
  speciesData = growth_scbi, k = n_fold, selection = "systematic",
 blocks = blocks_scbi, showBlocks = FALSE, verbose = FALSE
)
growth_scbi <- growth_scbi %>%
```

```
mutate(foldID = SpatialBlock_scbi$foldID %>% factor())
# Compute focal versus competitor tree information ------
focal_vs_comp_scbi <- growth_scbi %>%
  create_focal_vs_comp(comp_dist, blocks = blocks_scbi, id = "stemID")
# Fit model and make predictions ------
# Number of permutation shuffles:
num\_shuffle \leftarrow 49
# Save results here
run_time <- 0
observed_RMSE <- 0
observed_RMSE_CV <- 0
shuffle_RMSE <- vector("list", 1)</pre>
shuffle_RMSE_CV <- vector("list", 1)</pre>
filename <- here("paper/simulation_results/") %>%
 str_c("2021-03-03_scbi_", num_shuffle, "_shuffles")
# Run all simulations
# 0. Setup simulation for this species type ----
# Start clock
tic()
```

```
# 1. Compute observed test statistic: RMSE with no cross-validation ----
# Fit model (compute posterior parameters)
comp_bayes_lm_scbi <- focal_vs_comp_scbi %>%
  comp_bayes_lm(prior_param = NULL, run_shuffle = FALSE)
# Make predictions and compute RMSE
observed_RMSE <- focal_vs_comp_scbi %>%
 mutate(growth_hat =
          predict(comp_bayes_lm_scbi, focal_vs_comp_scbi)) %>%
  rmse(truth = growth, estimate = growth_hat) %>%
 pull(.estimate)
# 2. Compute observed test statistic: RMSE with cross-validation ------
observed_RMSE_CV <- focal_vs_comp_scbi %>%
  run_cv(comp_dist = comp_dist, blocks = blocks_scbi) %>%
  rmse(truth = growth, estimate = growth_hat) %>%
 pull(.estimate)
# 3. Permutation distribution: RMSE with no cross-validation ---
# Compute num_shuffle permutation test statistics
shuffle_RMSE <- numeric(length = num_shuffle)</pre>
for(j in 1:num_shuffle){
  # Fit model (compute posterior parameters) with shuffling
  comp_bayes_lm_scbi <- focal_vs_comp_scbi %>%
    comp_bayes_lm(prior_param = NULL, run_shuffle = TRUE)
```

```
# Make predictions and compute RMSE
  shuffle_RMSE[j] <- focal_vs_comp_scbi %>%
   mutate(growth_hat =
             predict(comp_bayes_lm_scbi, focal_vs_comp_scbi)) %>%
   rmse(truth = growth, estimate = growth_hat) %>%
   pull(.estimate)
}
# 4. Permutation distribution: RMSE with cross-validation -----
\# Compute num_shuffle permutation test statistics
shuffle_RMSE_CV <- numeric(length = num_shuffle)</pre>
# Compute num_shuffle permutation test statistics
for(j in 1:num_shuffle){
  # Compute and save RMSE
  shuffle_RMSE_CV[j] <- focal_vs_comp_scbi %>%
   run_cv(comp_dist = comp_dist, blocks = blocks_scbi,
           run_shuffle = TRUE) %>%
    rmse(truth = growth, estimate = growth_hat) %>%
   pull(.estimate)
  # Status update
  str_c("Shuffle with permutation ", j, " at ", Sys.time()) %>% print()
}
# 5. Save results ----
```

```
clock <- toc(quiet = TRUE)</pre>
run_time <- clock$toc - clock$tic</pre>
model_comp_tbl <- tibble(</pre>
  run_time = run_time,
  observed_RMSE = observed_RMSE,
  observed_RMSE_CV = observed_RMSE_CV,
  shuffle_RMSE = shuffle_RMSE,
  shuffle_RMSE_CV = shuffle_RMSE_CV,
)
save(model_comp_tbl, file = filename %>% str_c(".RData"))
# Visualize results -----
model_comp <- bind_rows(</pre>
 model_comp_tbl %>%
    select(run_time, observed = observed_RMSE,
           shuffle = shuffle_RMSE) %>%
    mutate(CV = FALSE),
  model_comp_tbl %>%
    select(run_time, observed = observed_RMSE_CV,
           shuffle = shuffle_RMSE_CV) %>%
    mutate(CV = TRUE)
) %>%
  gather(type, RMSE, -c(run_time, CV))
```

```
model_comp_observed <- model_comp %>%
  filter(type == "observed") %>%
  unnest(cols = c(RMSE))
model_comp_shuffle <- model_comp %>%
  filter(type == "shuffle") %>%
  unnest(cols = c(RMSE))
cv_plot <- ggplot() +</pre>
  geom_vline(data = model_comp_observed,
             aes(xintercept = RMSE, col = CV),
             linetype = "dashed", show.legend = F) +
  geom_histogram(data = model_comp_shuffle,
                 aes(x = RMSE, fill = CV), bins = 50) +
  labs(
    fill = "Cross-validated?",
    x = expression(paste("RMSE (cm ", y^{-1}, ")"))
  ) +
  scale_color_viridis(discrete = TRUE, option = "D") +
  scale_fill_viridis(discrete = TRUE) +
  theme_light()
cv_plot
filename %>%
  str_c(".pdf") %>%
 ggsave(plot = cv_plot, width = 16/2, height = 9/2)
```

9 Appendix 2: Compare different competitive distances

For all the above analyses we took the cut off distance (comp_dist) for two stems to compete as 7.5 m. This distance has been estimated in previous neighborhood competition studies in forests (Canham et al. 2004, Uriarte et al. (2004), Canham et al. (2006)). We used 7.5 m in Allen & Kim (2020) as an average of the values estimated in other studies. But our package can be used to find which distance is best supported by the data. Here we provide an example using another section of the SCBI plot to provide an additional example of the CV block layout. To speed computation we do not consider species differences in competitive effects and treat all species as the same.

```
census_2013_scbi <- read_csv("scbi.stem2.csv") %>%
  select(stemID, sp, date = ExactDate, gx, gy, dbh, codes, status) %>%
  mutate(
    date = mdy(date),
    dbh = as.numeric(dbh)/10
) %>%
  filter(gx < 400, gy < 400)

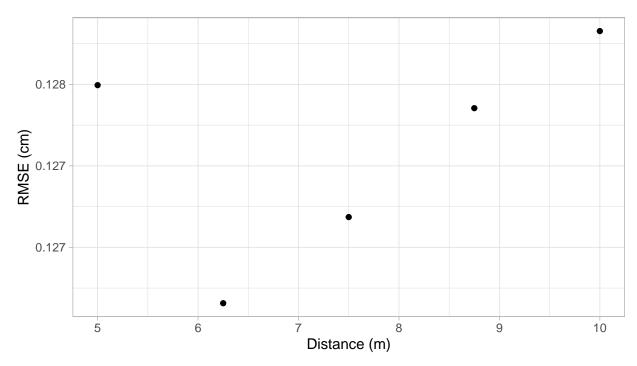
census_2018_scbi <- read_csv("scbi.stem3.csv") %>%
  select(stemID, sp, date = ExactDate, gx, gy, dbh, codes, status) %>%
  mutate(
    date = mdy(date),
    dbh = as.numeric(dbh)/10
) %>%
  filter(gx < 400, gy < 400)</pre>
```

```
growth_scbi <-</pre>
  compute_growth(
    census_1 = census_2013_scbi,
    census_2 = census_2018_scbi %>% filter(!str_detect(codes, "R")),
    id = "stemID"
  ) %>%
  # make all species the same, needs to be factor with at least two levels
  mutate(sp = factor('A', levels = c('A', 'B')) )
study_region_scbi <- tibble(</pre>
 x = c(0, 400, 400, 0, 0),
 y = c(0, 0, 400, 400, 0)
) %>%
  sf_polygon()
n_fold \leftarrow 4
fold1 <- cbind(c(0, 200, 200, 0), c(0, 0, 200, 200))
fold2 \leftarrow cbind(c(200, 400, 400, 200), c(0, 0, 200, 200))
fold3 <- cbind(c(0, 200, 200, 0), c(200, 200, 400, 400))
fold4 \leftarrow cbind(c(200, 400, 400, 200), c(200, 200, 400, 400))
blocks_scbi <- bind_rows(</pre>
  sf_polygon(fold1), sf_polygon(fold2), sf_polygon(fold3),
 sf_polygon(fold4)
) %>%
 mutate(folds = c(1:n_fold) %>% factor())
```

```
# Associate each observation to a fold
spatial_block_scbi <-</pre>
  spatialBlock(speciesData = growth_scbi, k = n_fold,
               selection = "systematic", blocks = blocks_scbi,
               showBlocks = FALSE, verbose = FALSE)
growth_scbi <- growth_scbi %>%
  mutate(foldID = spatial_block_scbi$foldID %>% factor())
mult_dist_comp <- tibble(</pre>
  dist = c(5, 6.25, 7.5, 8.75, 10),
 rmse = 0
)
for (i in 1:length(mult_dist_comp$dist))
{
  comp_dist <- mult_dist_comp$dist[i]</pre>
  growth_scbi <- growth_scbi %>%
    add_buffer_variable(size = comp_dist, region = study_region_scbi)
  focal_vs_comp_scbi <- growth_scbi %>%
    create_focal_vs_comp(comp_dist = comp_dist, blocks = blocks_scbi, id = "stemID") %>
    run_cv(comp_dist = comp_dist, blocks = blocks_scbi)
  mult_dist_comp$rmse[i] <- focal_vs_comp_scbi %>%
```

```
rmse(truth = growth, estimate = growth_hat) %>%
  pull(.estimate)
}

mult_dist_comp %>%
  ggplot(aes(dist,rmse)) +
  geom_point() +
  xlab('Distance (m)') +
  ylab('RMSE (cm)')
```



Here we see that a cut off distance of approximately 6 m minimizes RMSE.

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