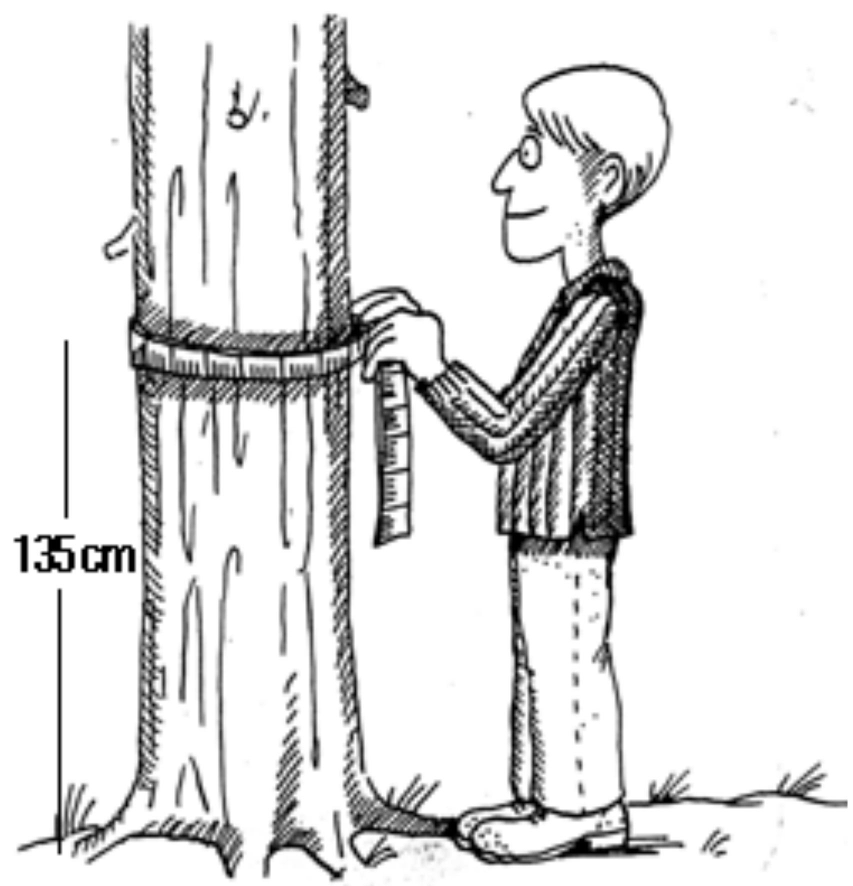
Fusing disparate measurement data for forecasting the growth of trees via Hidden Markov Models

Prof. Albert Y. Kim
UMass Amherst Statistics Seminar Series
Friday, January 22, 2021

Context

Diameter at Breast Height (dbh)

After species & location, one of the most informative variables about a tree is dbh



Smithsonian Conservation Biology Institute

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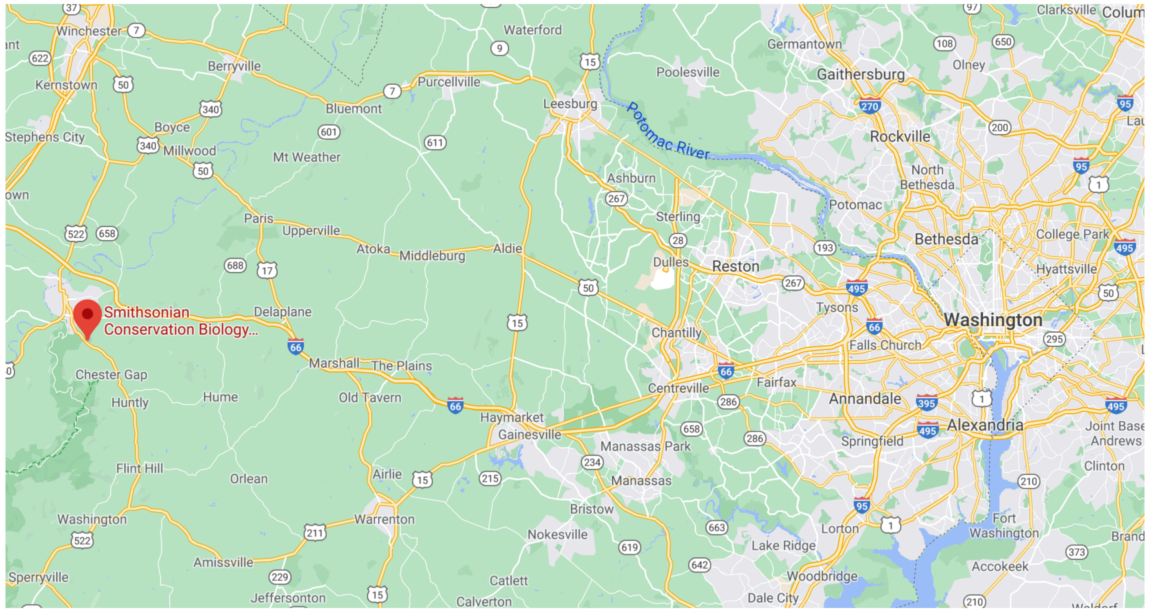

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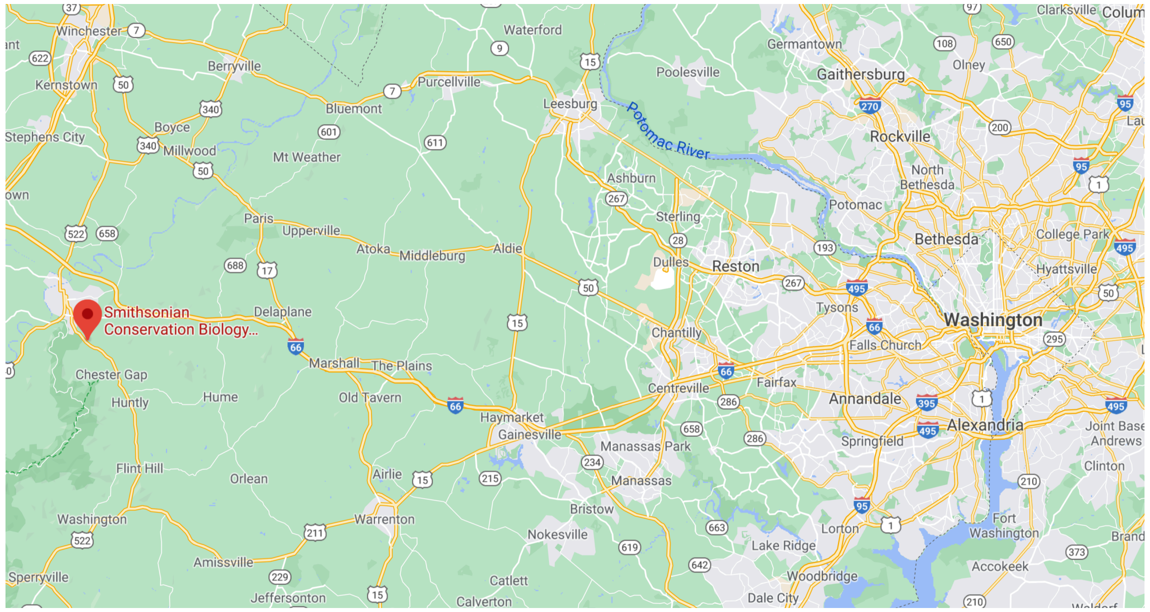

25.6 ha = 35.85 soccer fields

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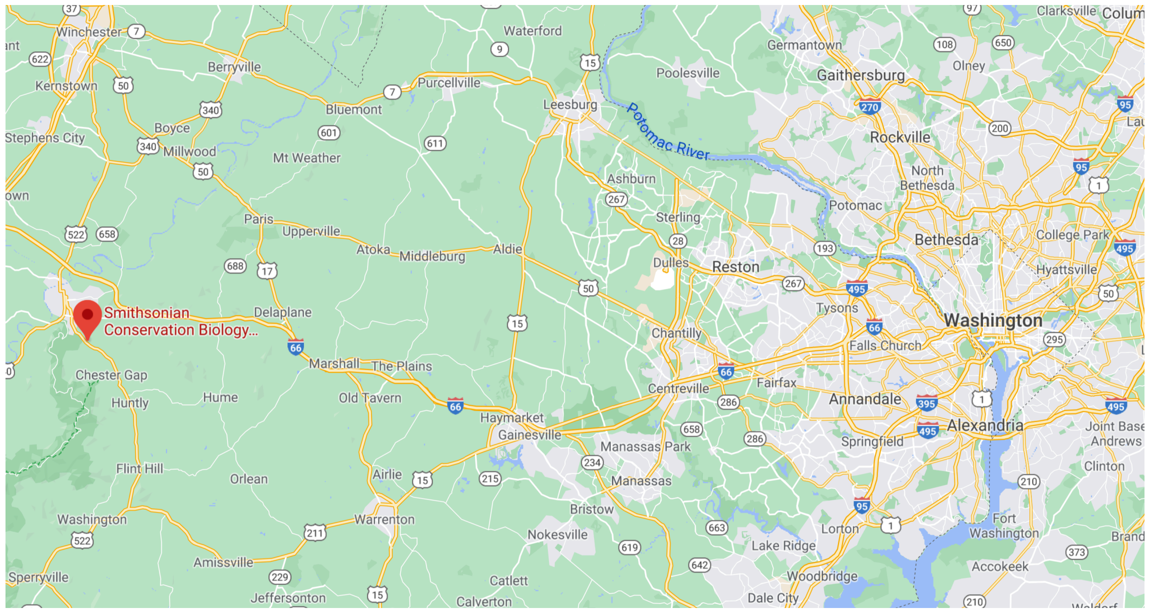

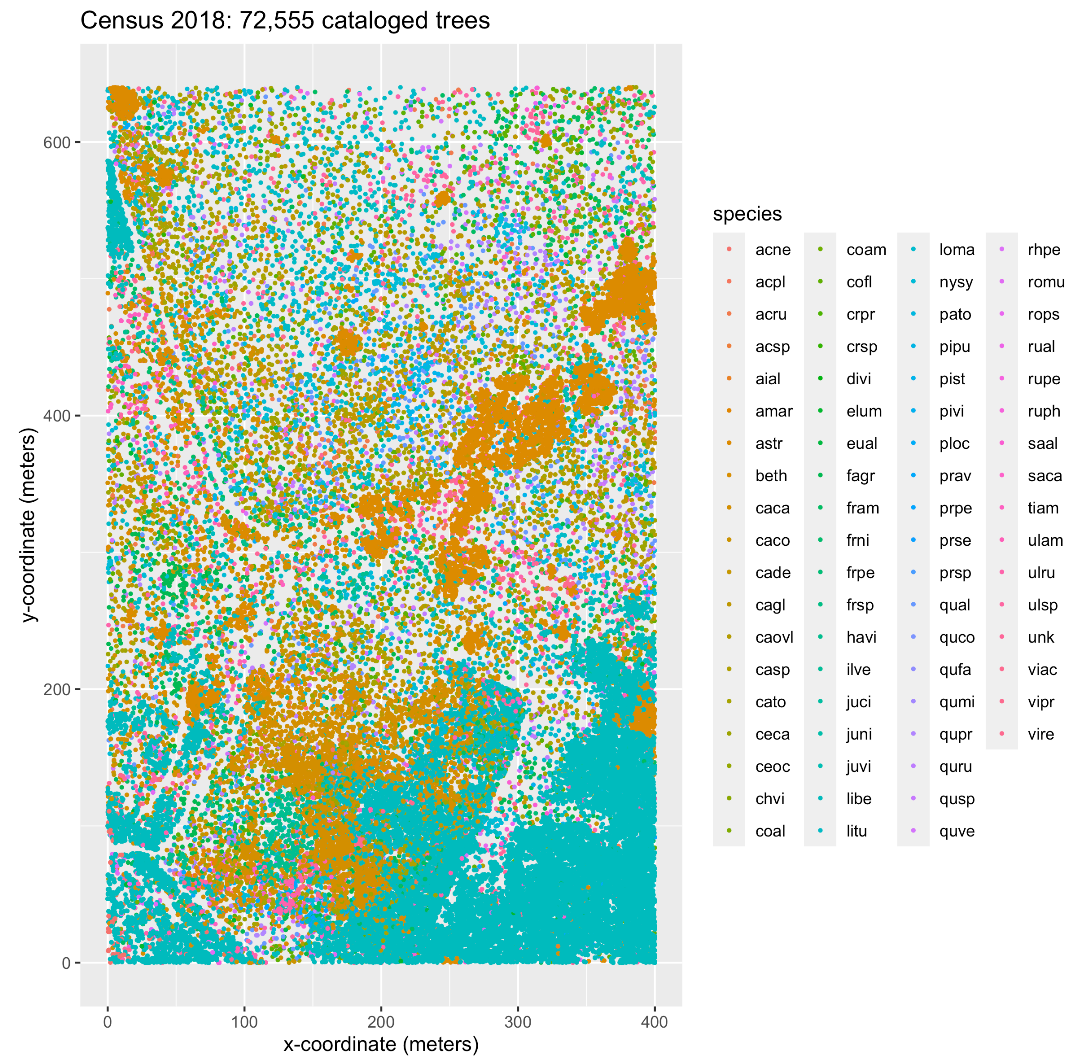

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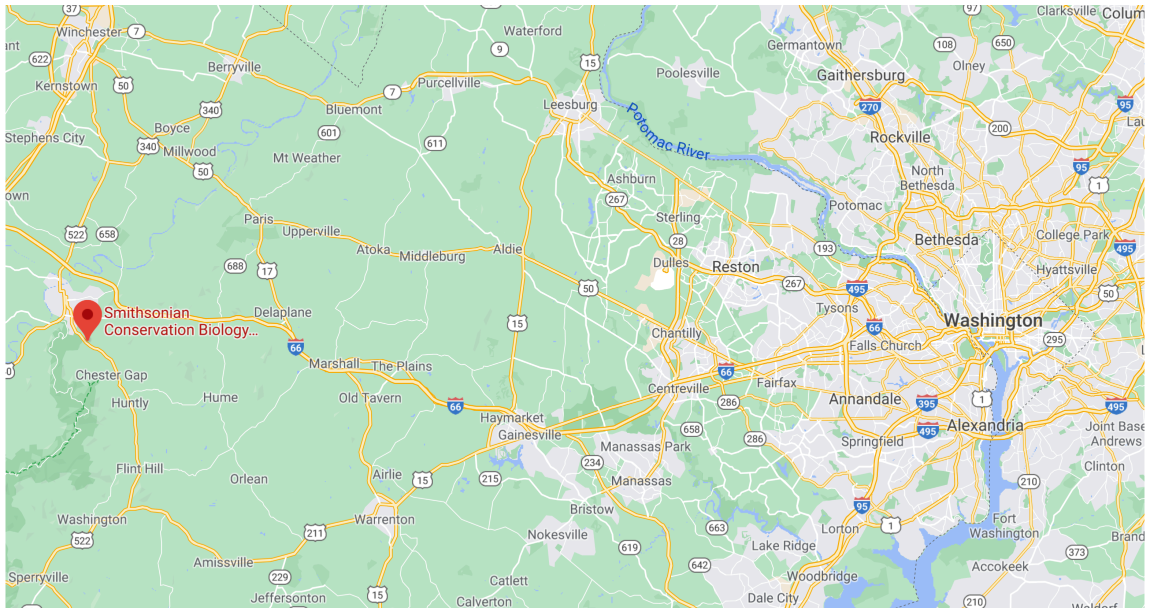

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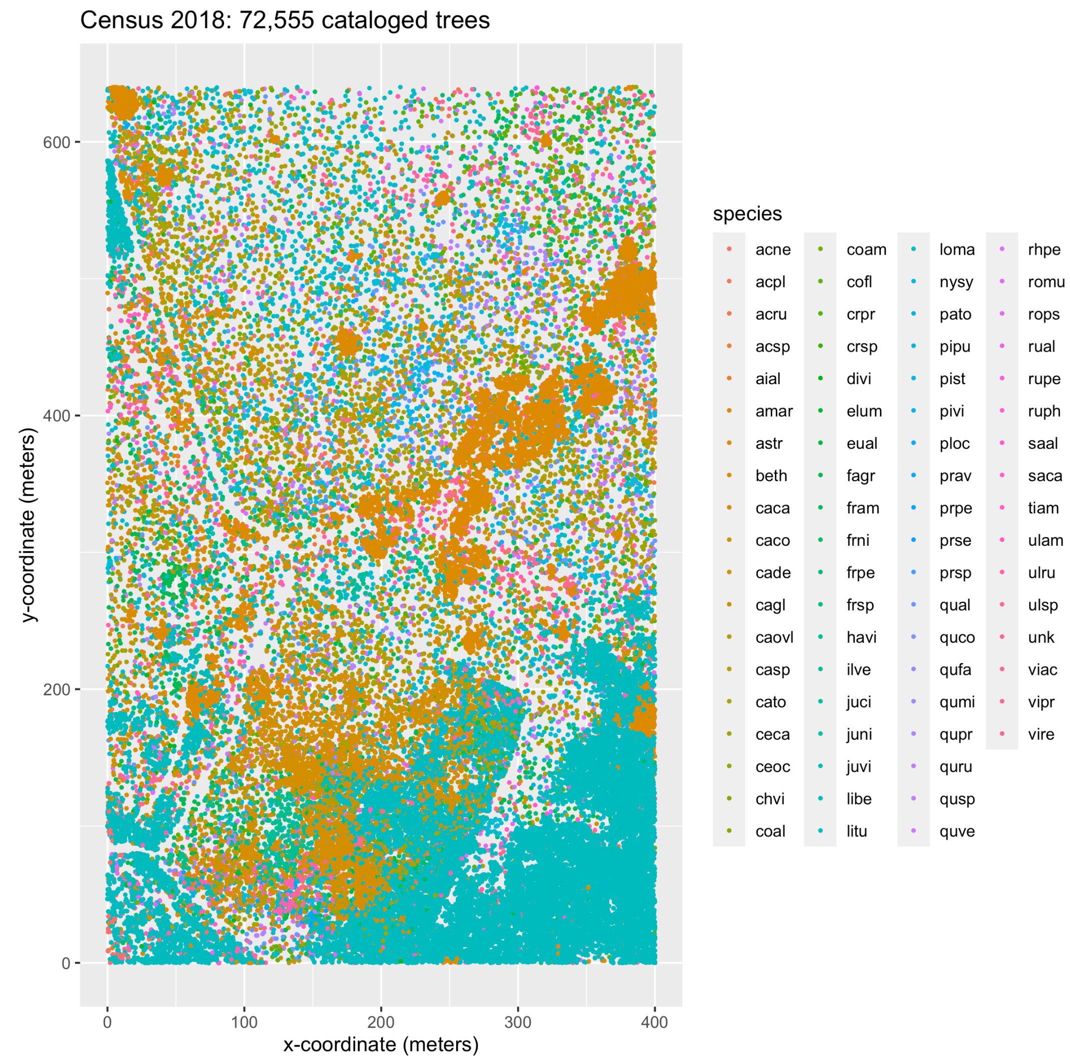
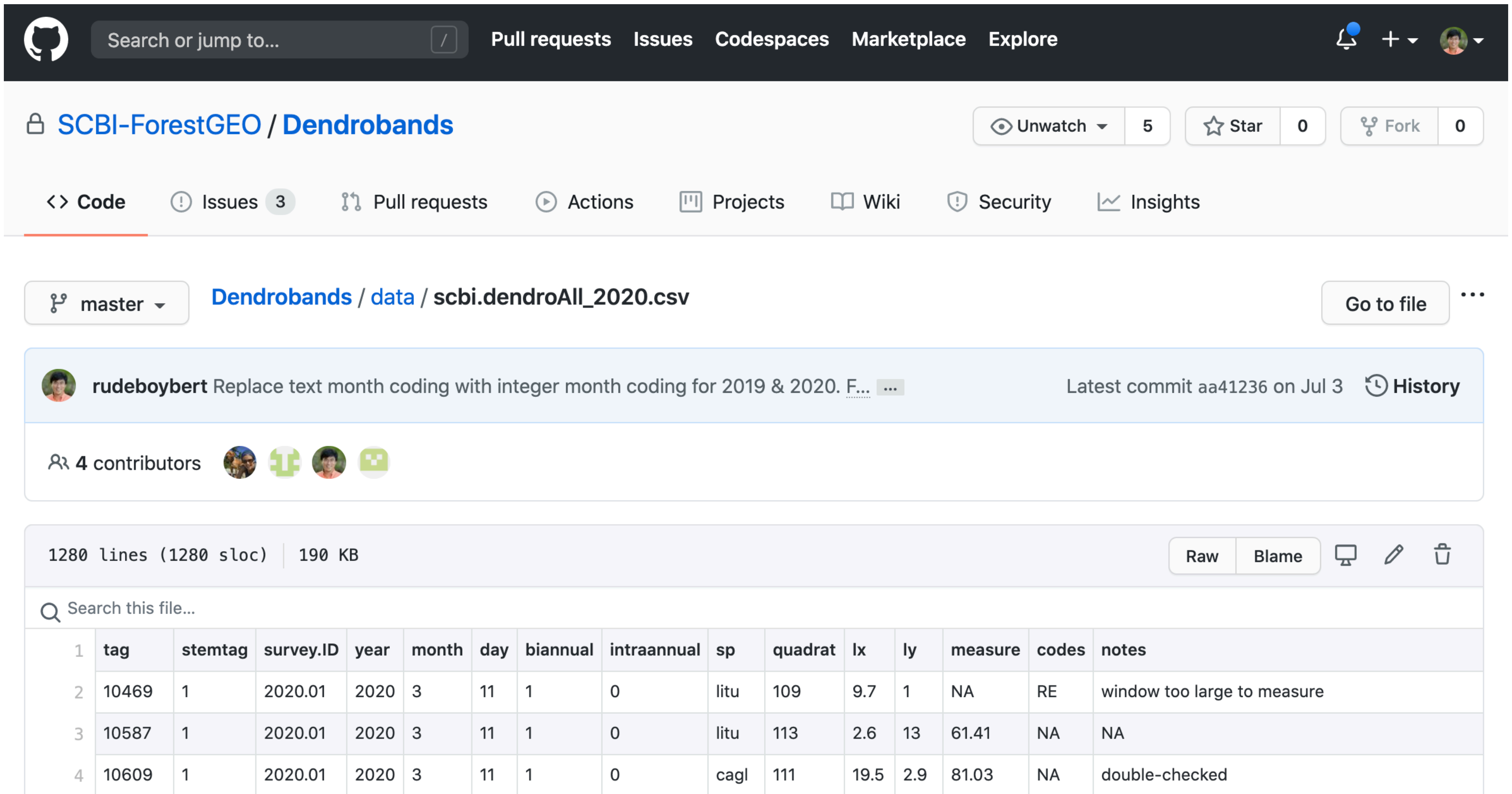


dbh >10mm are tagged

Data on GitHub



	tag	stemtag	survey.ID	year	month	day	biannual	intraannual	sp	quadrat	lx	ly	measure	codes	notes
2	10469	1	2020.01	2020	3	11	1	0	litu	109	9.7	1	NA	RE	window too large to measure
3	10587	1	2020.01	2020	3	11	1	0	litu	113	2.6	13	61.41	NA	NA
4	10609	1	2020.01	2020	3	11	1	0	cagl	111	19.5	2.9	81.03	NA	double-checked

Equipment to measure doh

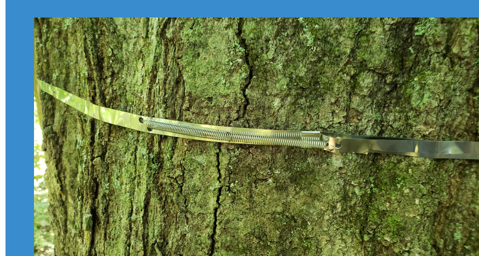
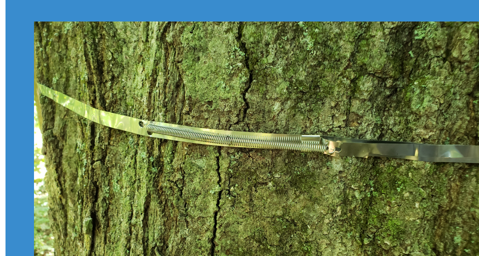
1. Measuring tape. Call this “census” data

2. Tree coring + dendrochronology. Call this “core” data

Equipment to measure doh

3. Dendrobands + Calipers:
Call this “dendro” data

Comparison Chart

	Data source	Measurement	Cost	Sources of Error?
	**Census via tape**	Diameter	Cheap	Large variation in dbh 📏 technique
	**Tree coring**	Ring width increment	Expensive	Standardized, cores are dried, no bark effects
	**Biannual dendroband (start & end of year)**	Increment (from baseline)	High setup, rapid follow-up	Climate induced variation in bark & device (-'ve growth)
	**Intraannual dendroband (every 2 weeks)**	"	"	"

Goal 🎯🎯🎯

Can we fuse these disparate data sources into a single model to forecast the growth of trees?

Model

Hidden Markov Models

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- Hidden: “Data fusion” via latent variables
  In our case: “true” dbh

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  - Propagate when forecasting

Minimum Viable Product

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- $\epsilon \sim$ Normal $\left(0, \sigma_\epsilon^2\right)$

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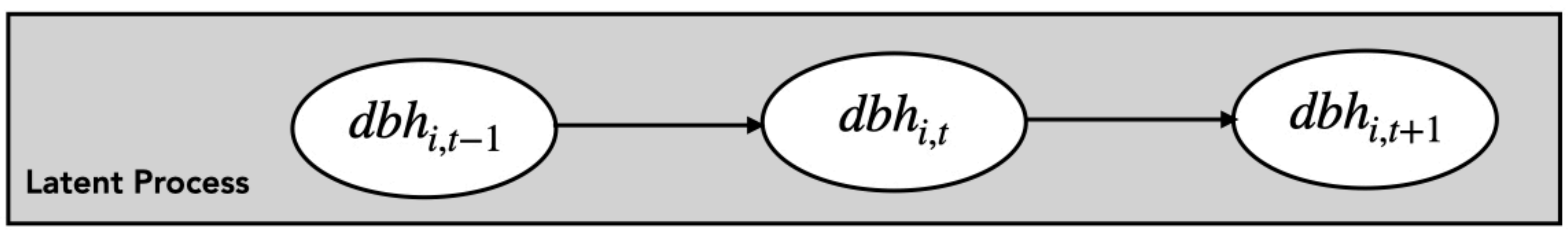

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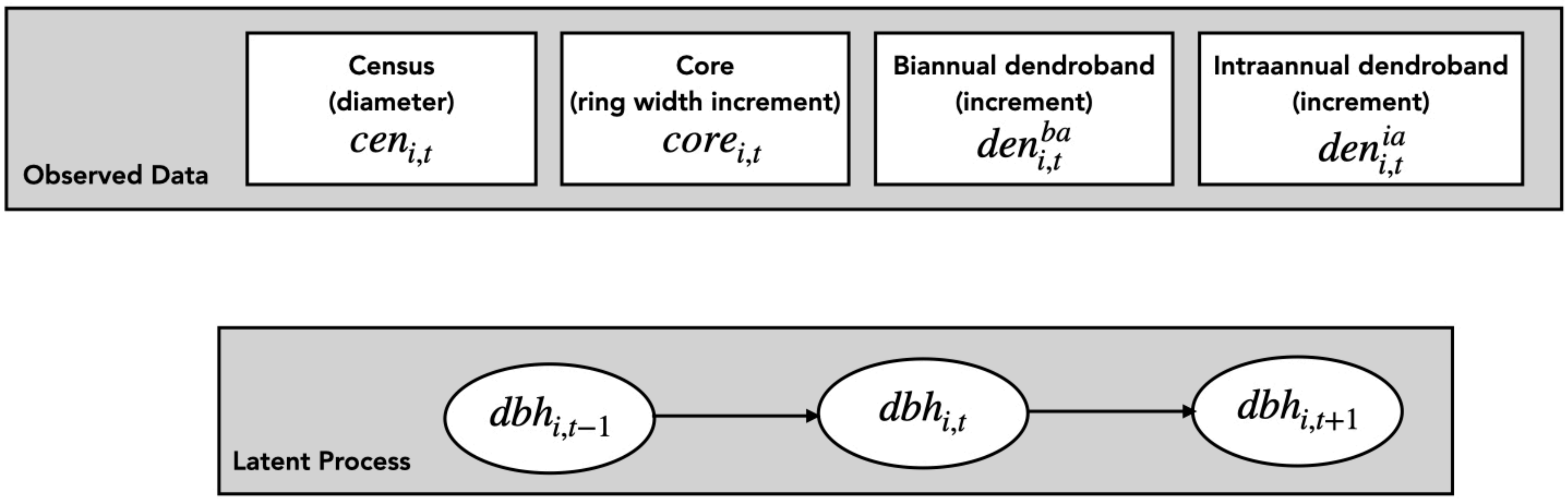
Census
(diameter)
$cen_{i,t}$
Core
(ring width increment)
$core_{i,t}$
Biannual dendroband
(increment)
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Observed Data
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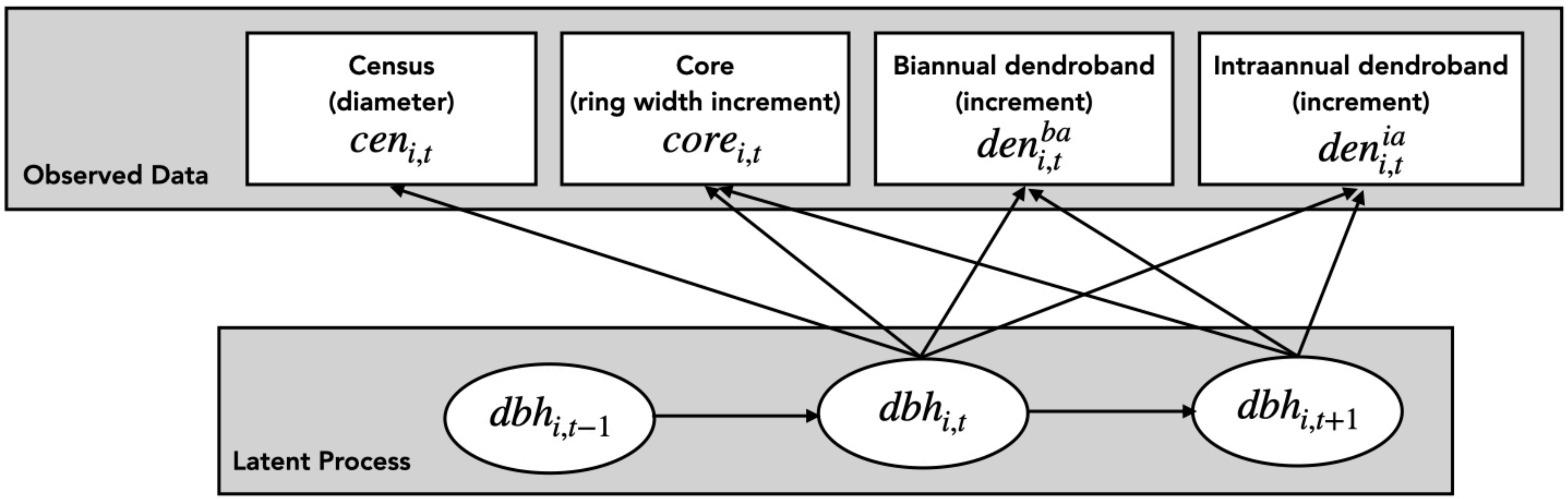
Latent Process

Model



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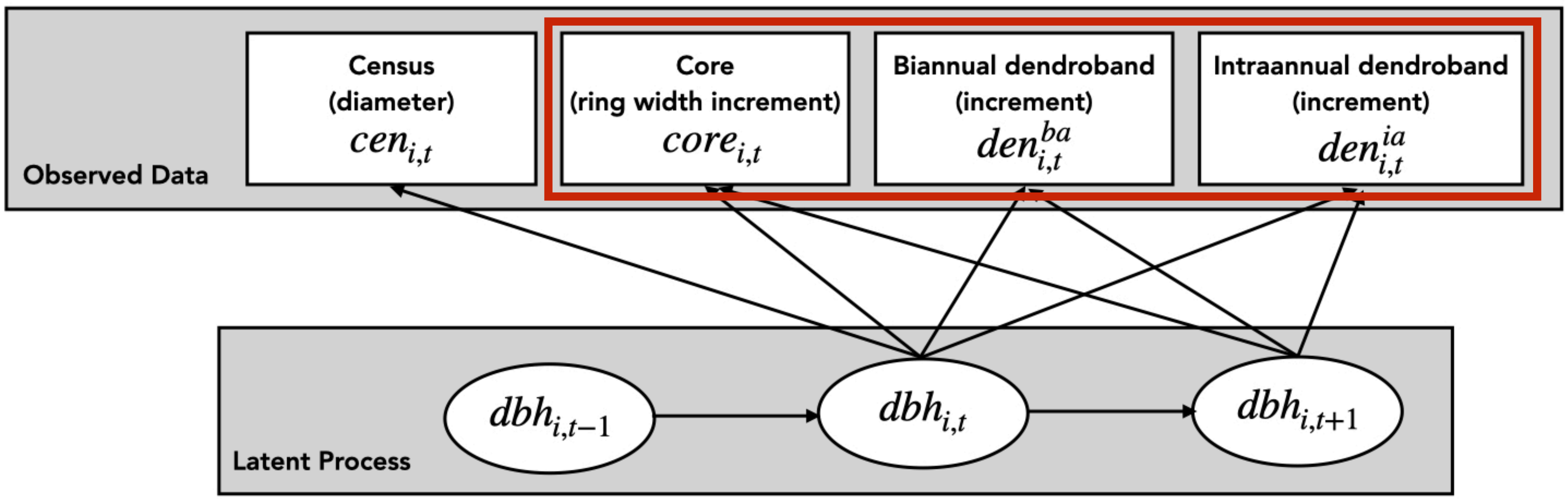

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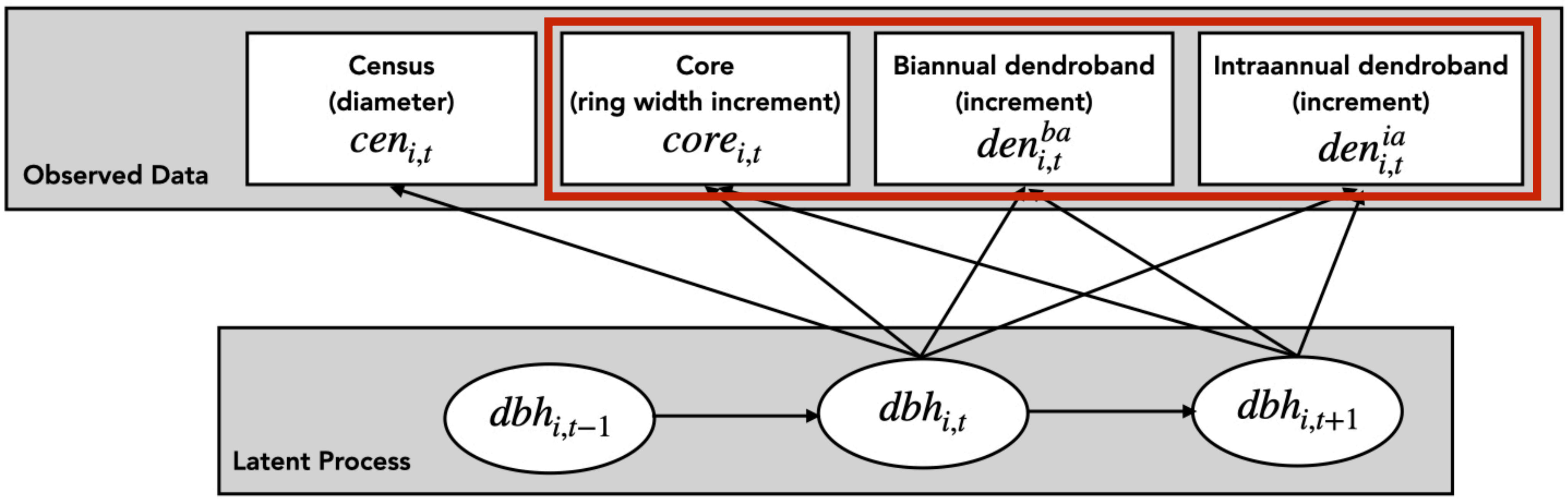
Increments = $dbh_{i,t-1} - dbh_{i,t}$



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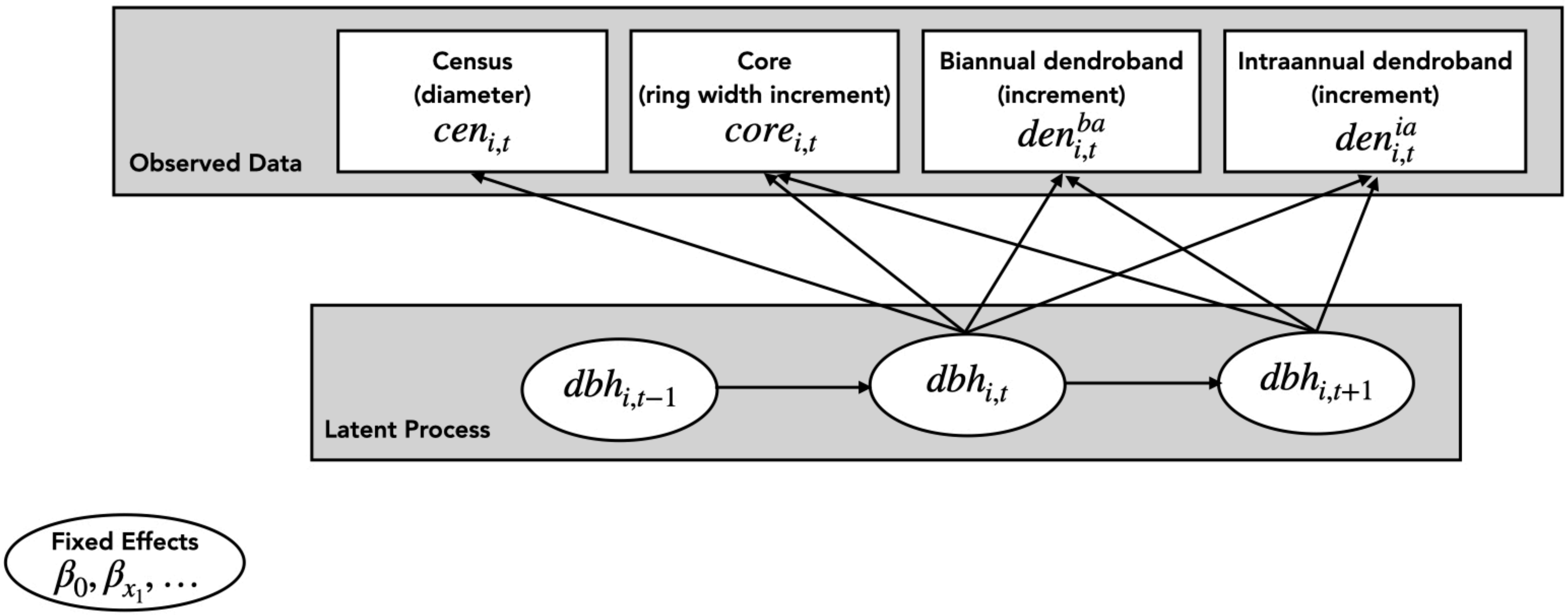

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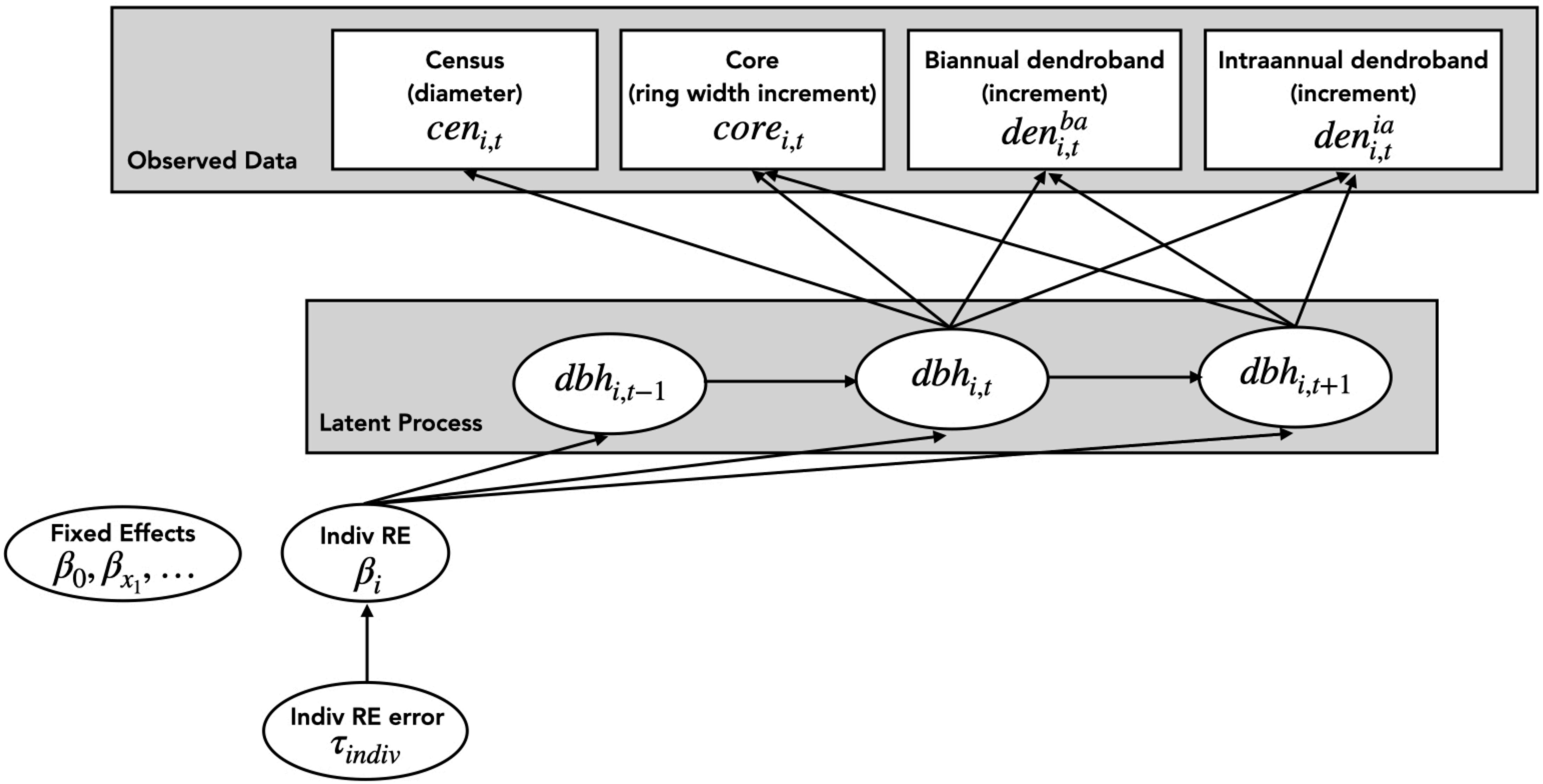
Latent Process
Fixed Effects
$\beta_0, \beta_{x_1}, \ldots$

Model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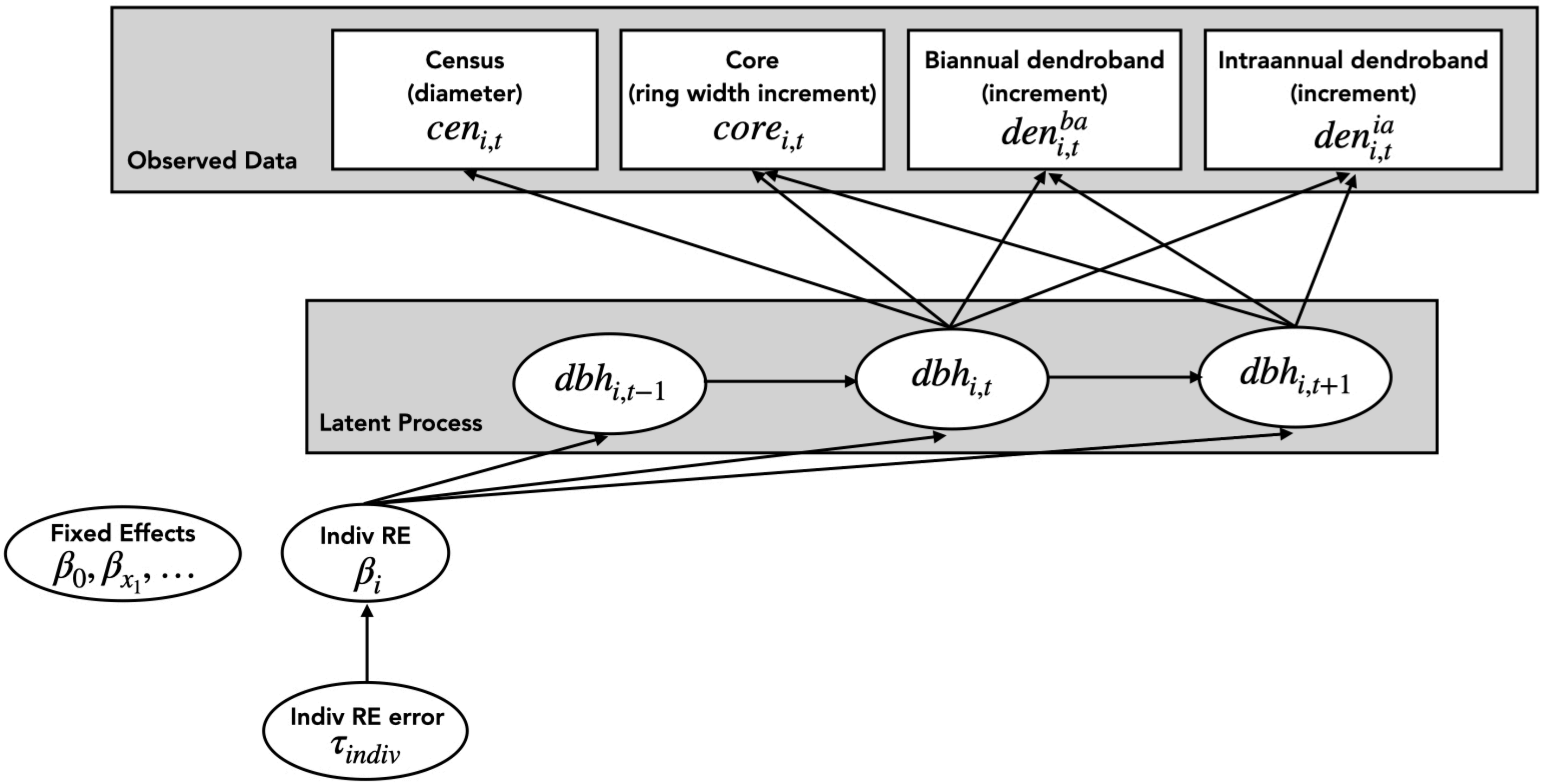
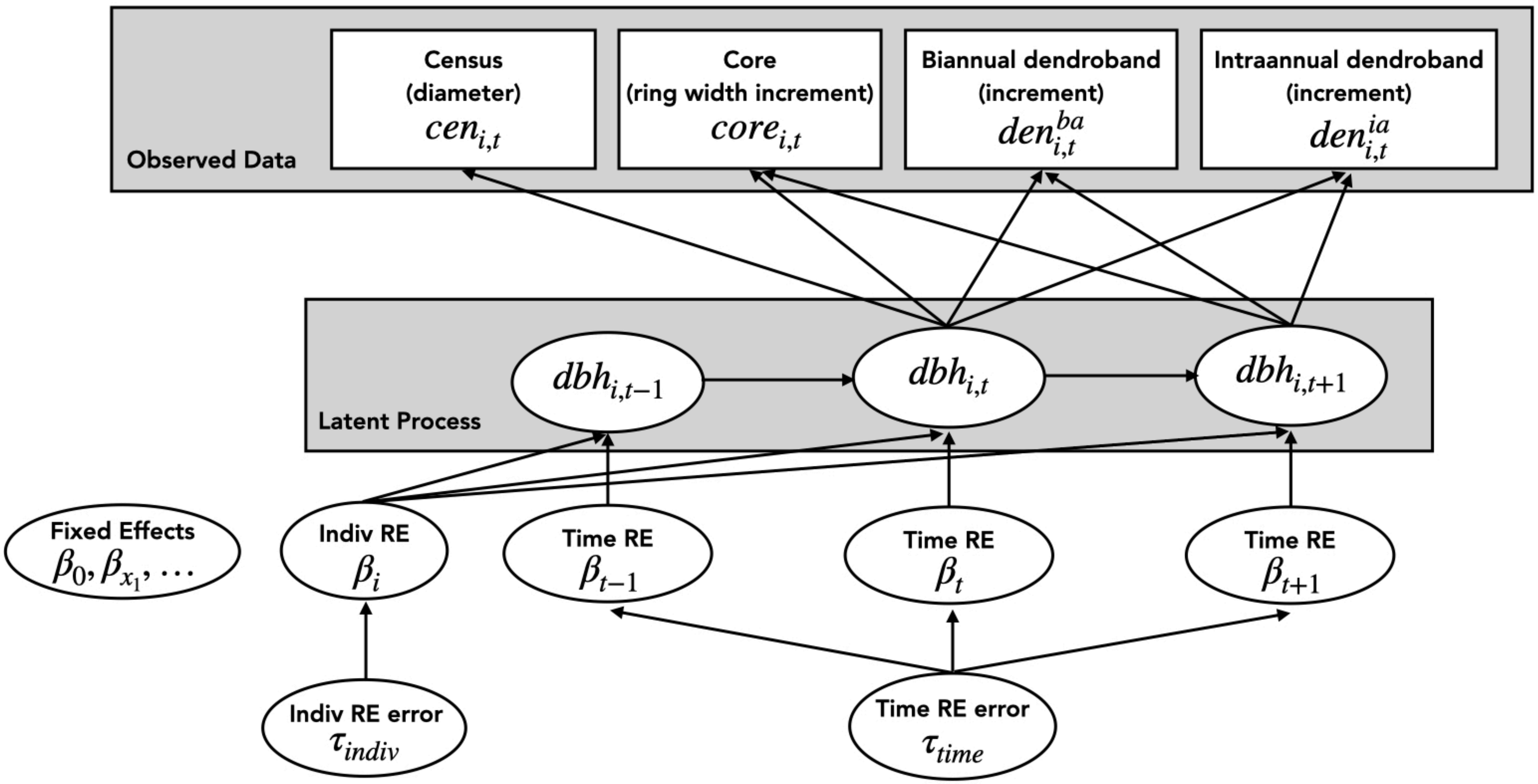
Indiv RE
β_i
Indiv RE error
τ_{indiv}

Model



FYI: Express variances via precision $\tau = \dfrac{1}{\sigma^2}$

Model



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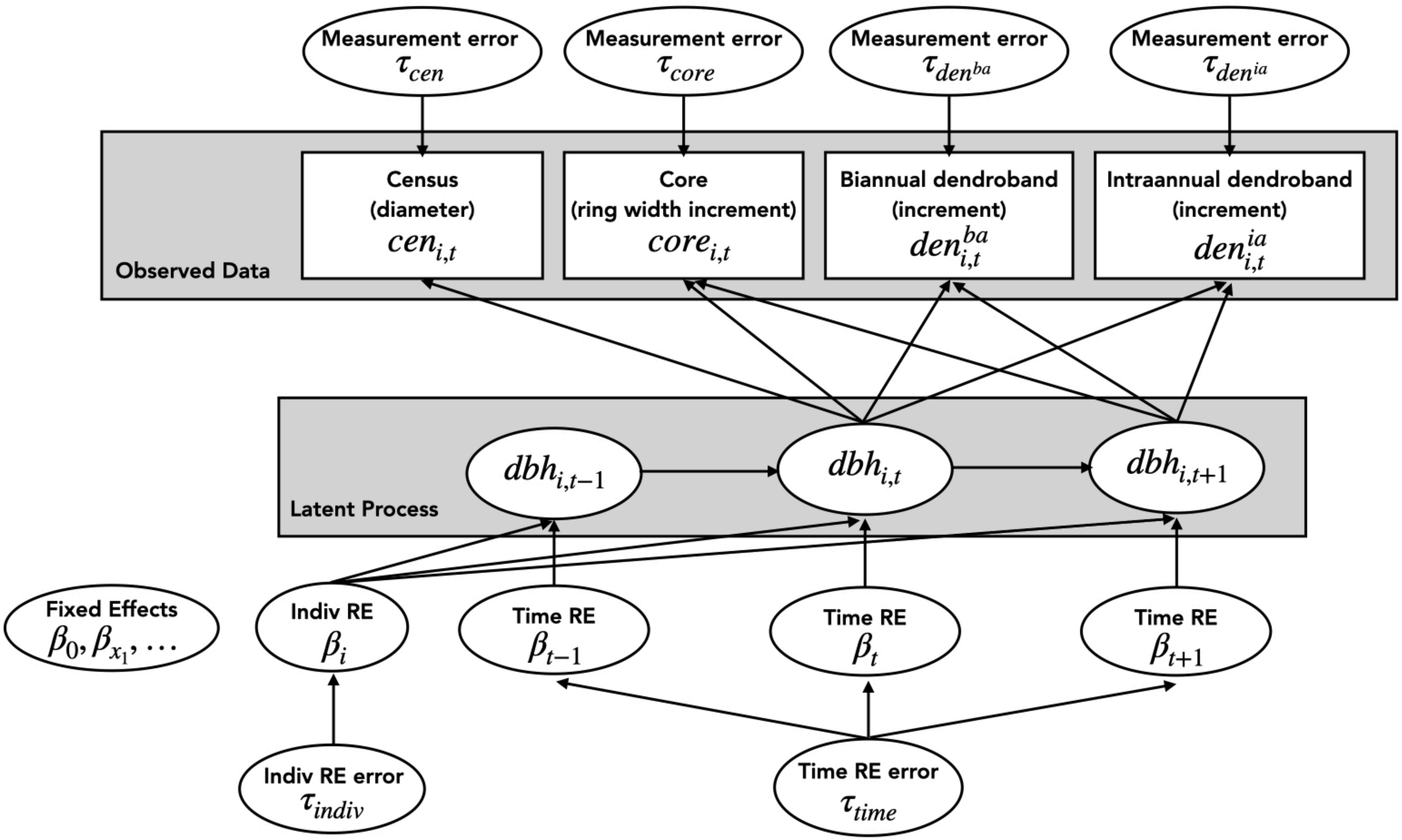
Indiv RE error τ_{indiv}
Time RE error τ_{time}

Model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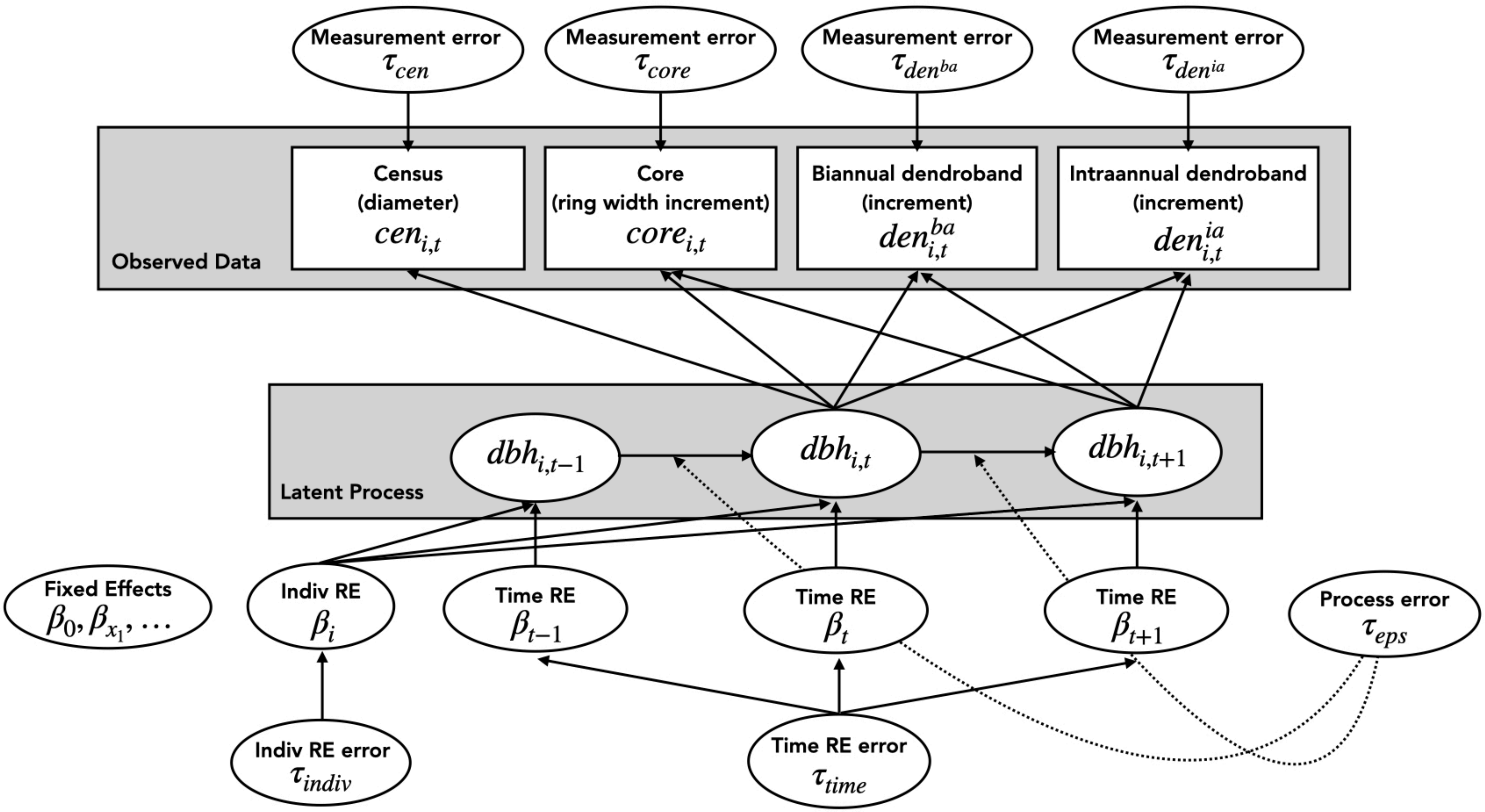
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Time RE β_t
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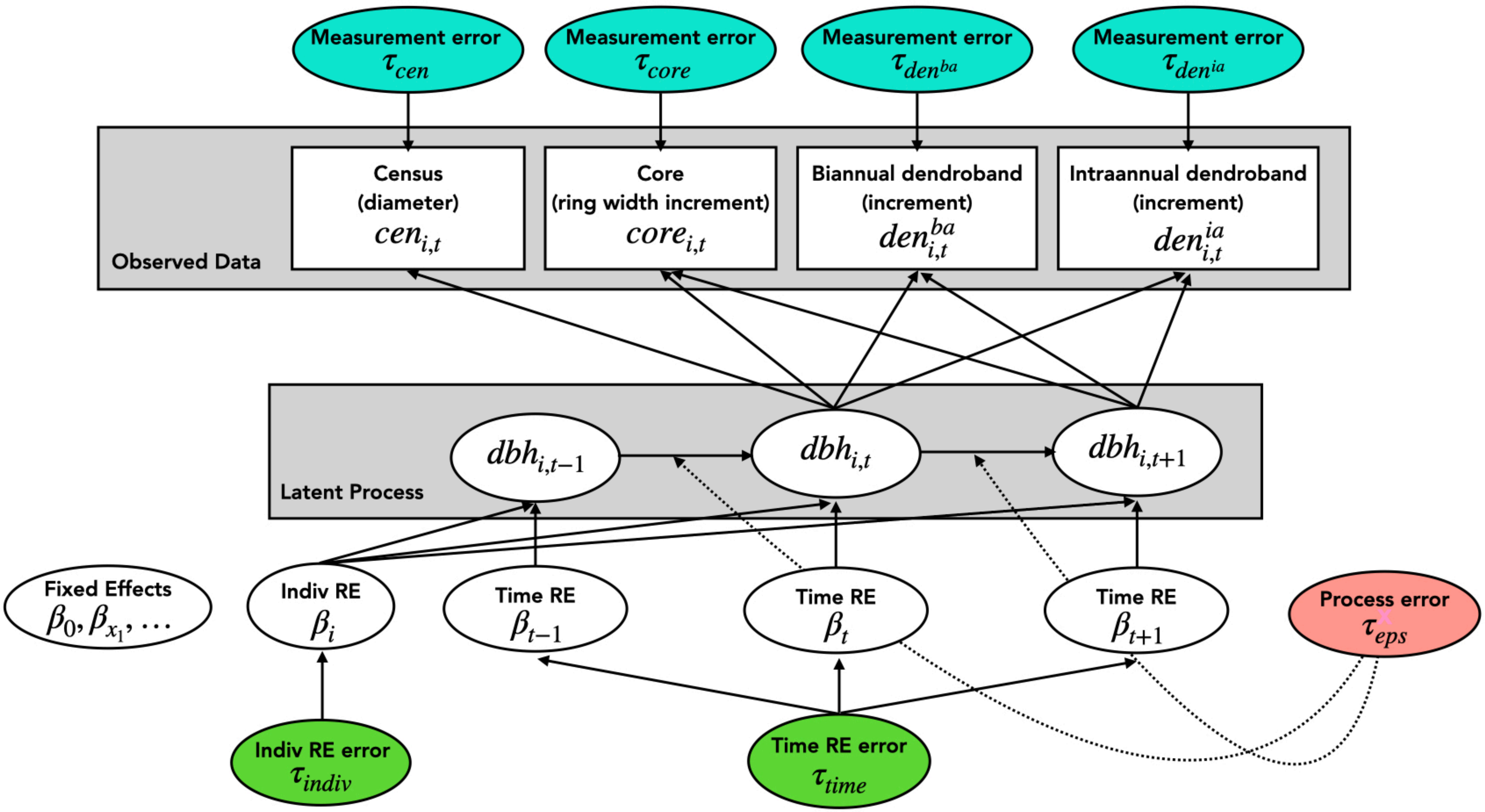
Process error τ_{eps}
Indiv RE error τ_{indiv}
Time RE error τ_{time}

Model



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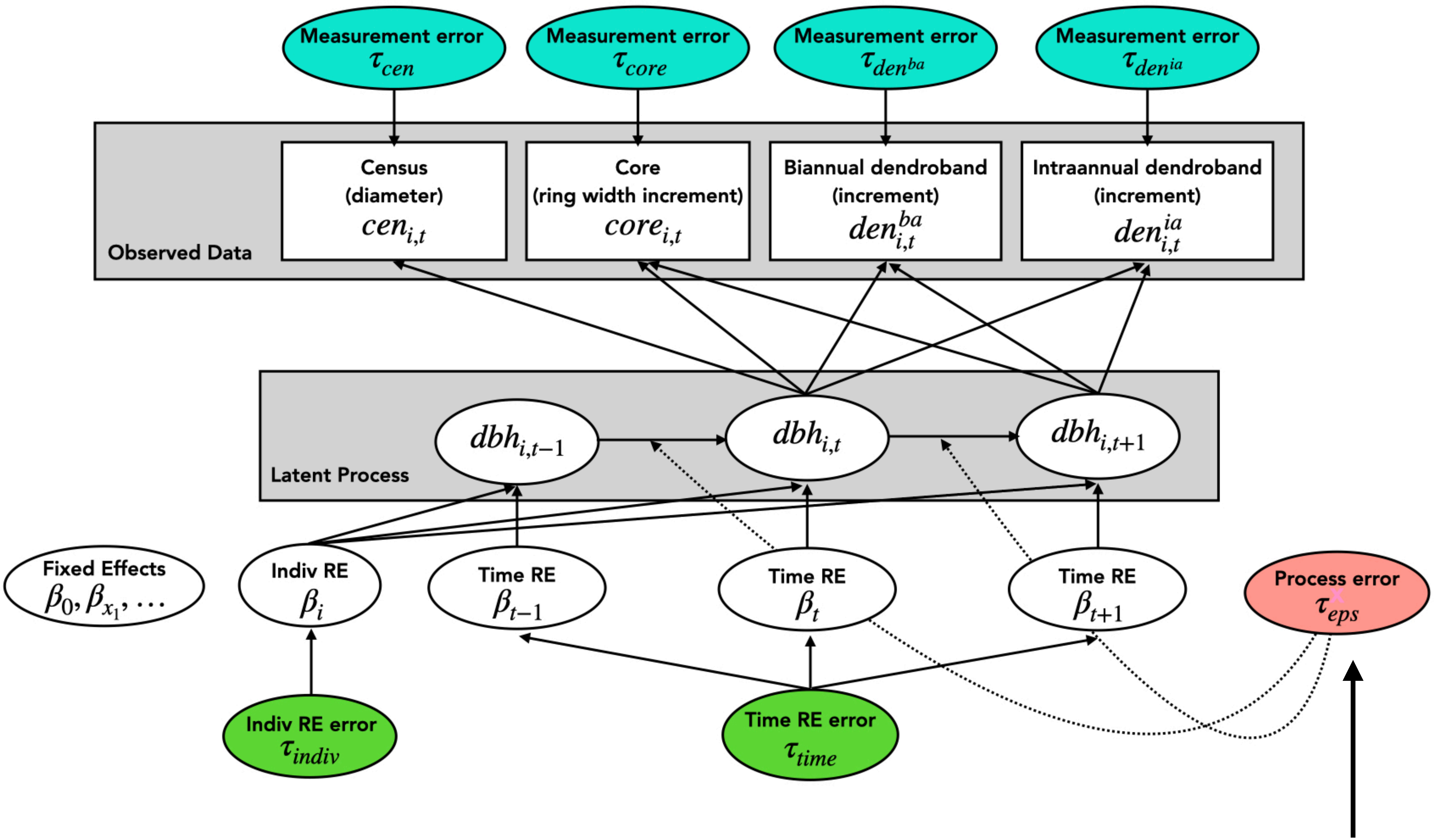
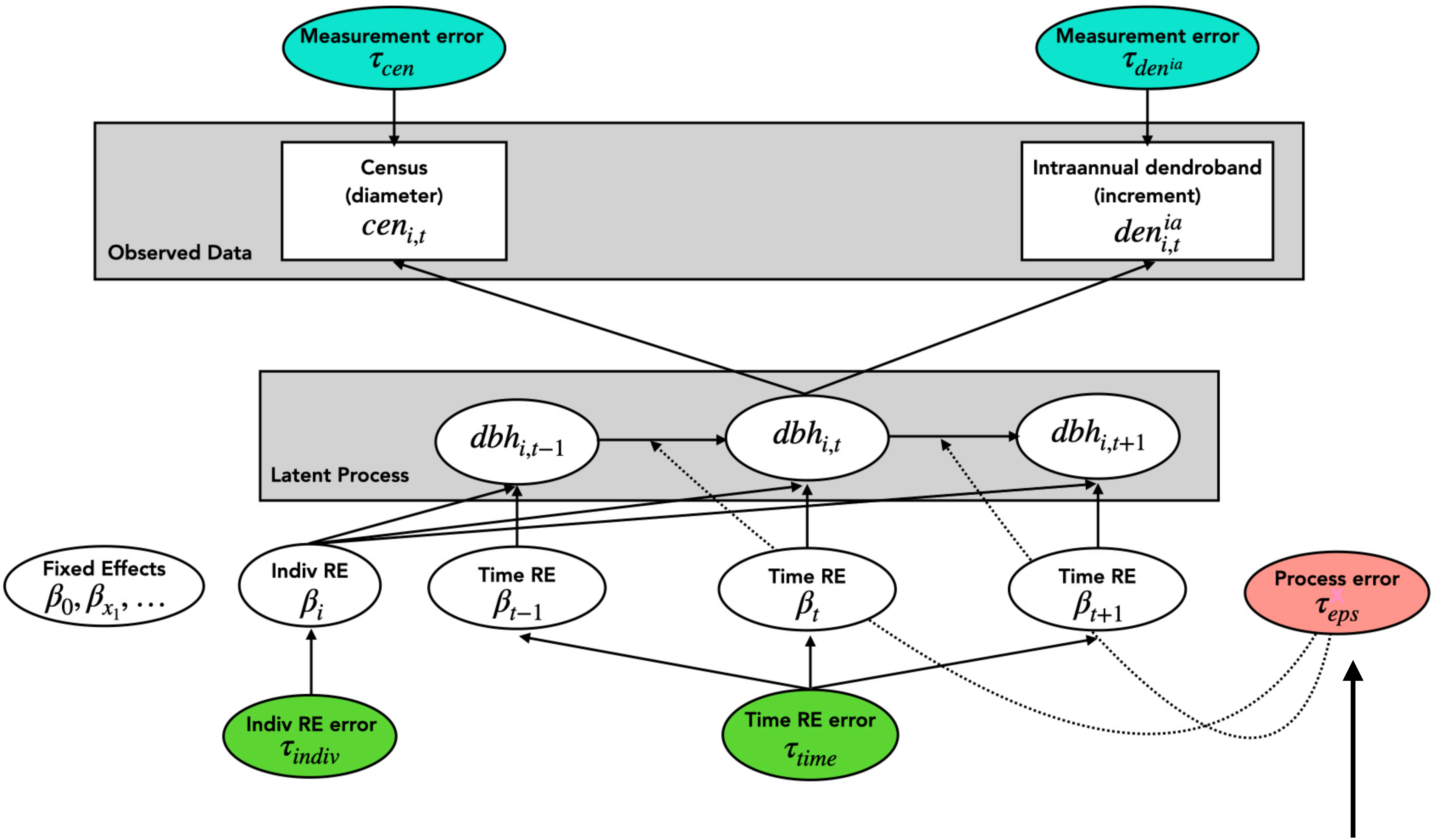


Moral: only error this propagates across time in forecasts

Model as of 2021/1/22



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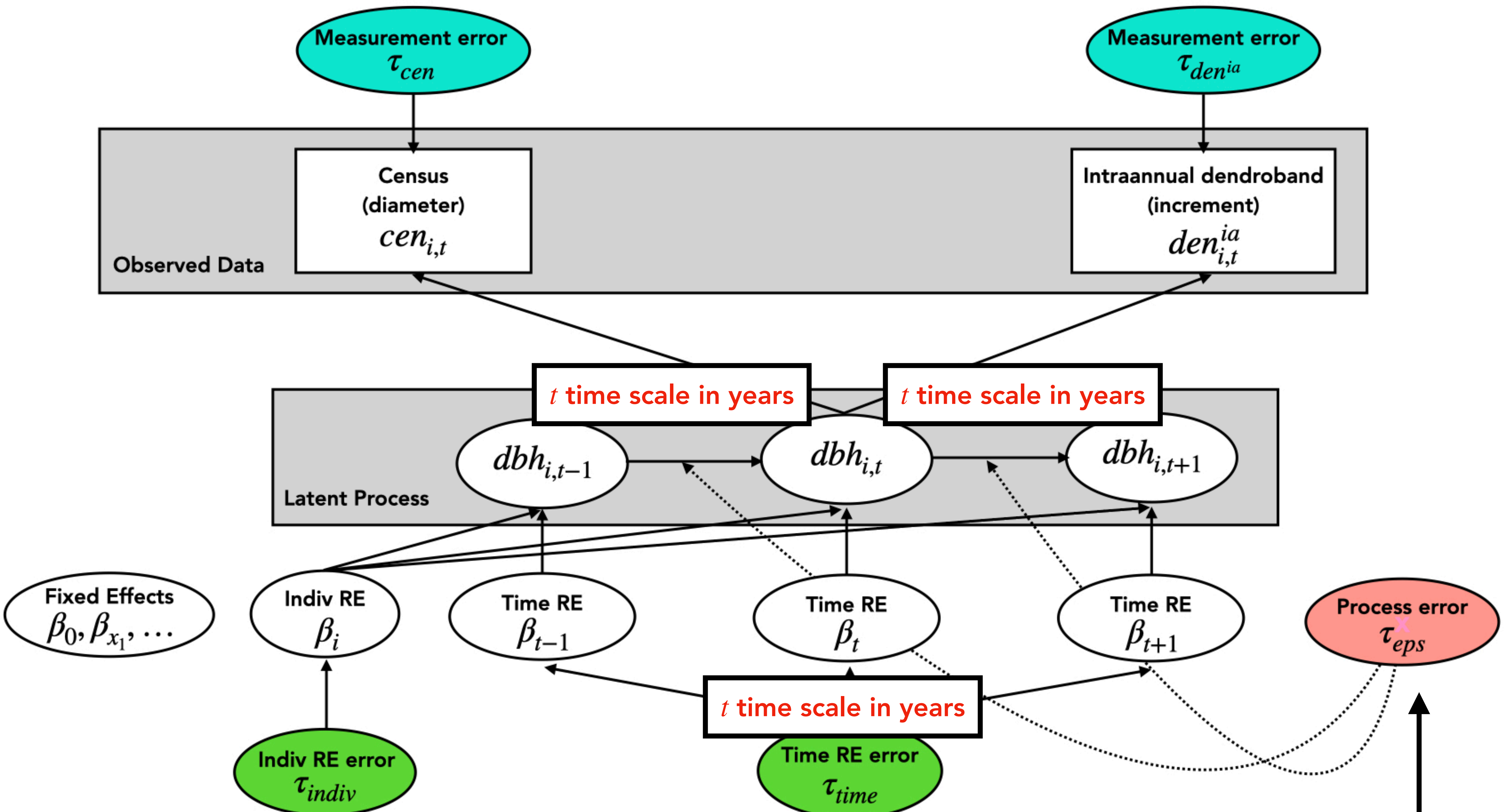

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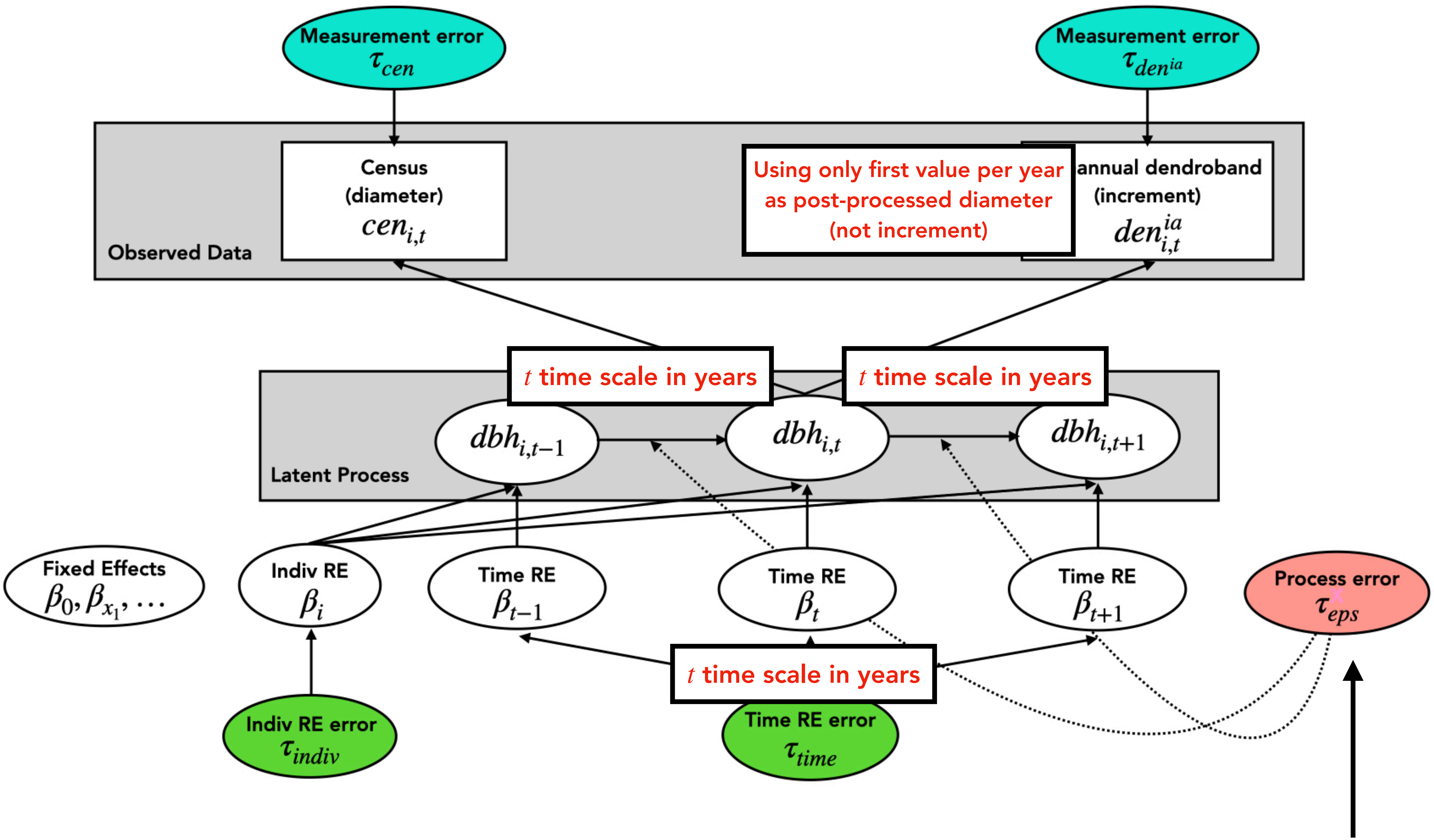

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Results

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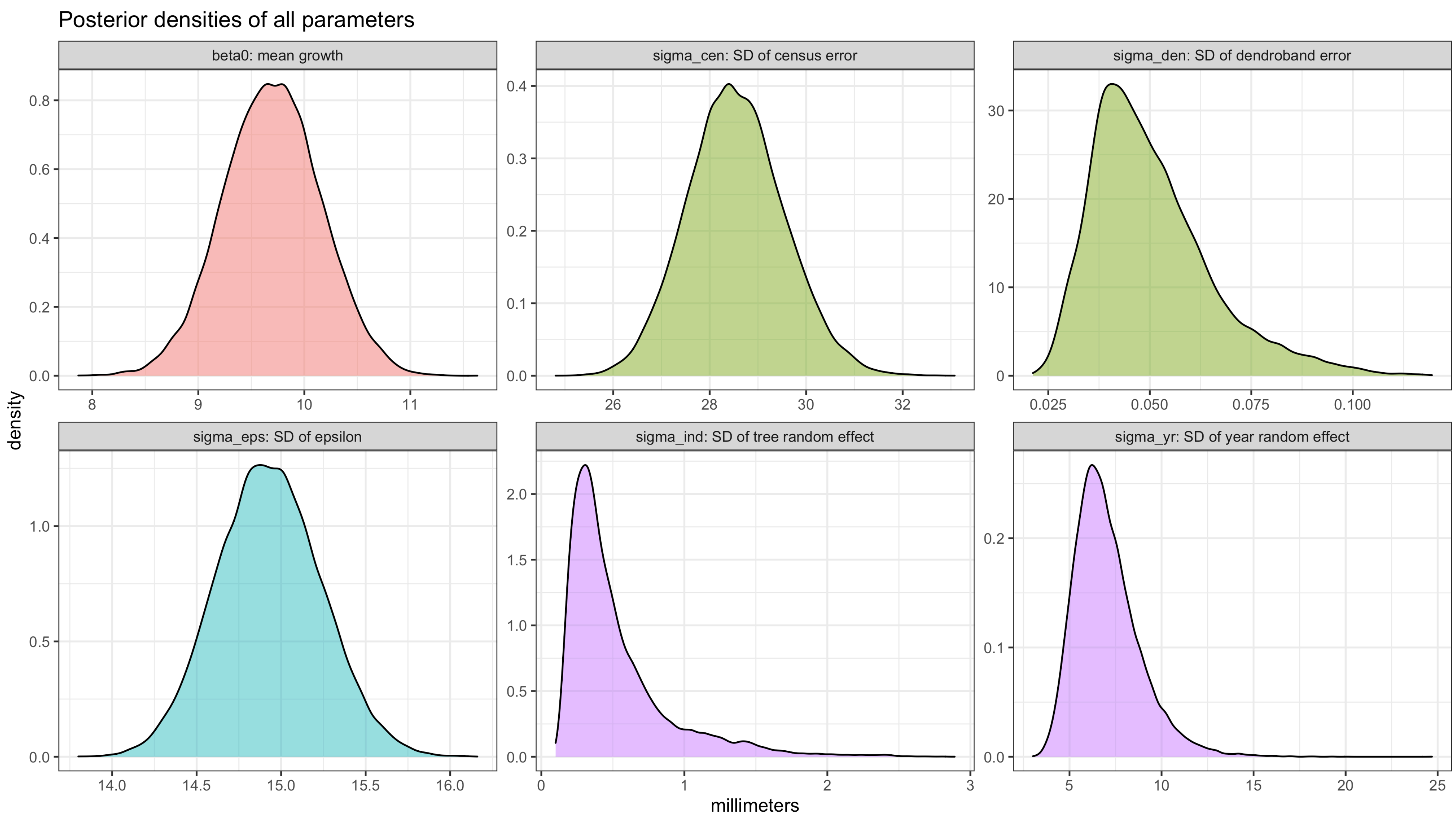
- Forecast into 2020 - 2022 by treating these years as missing values

Posterior Distributions



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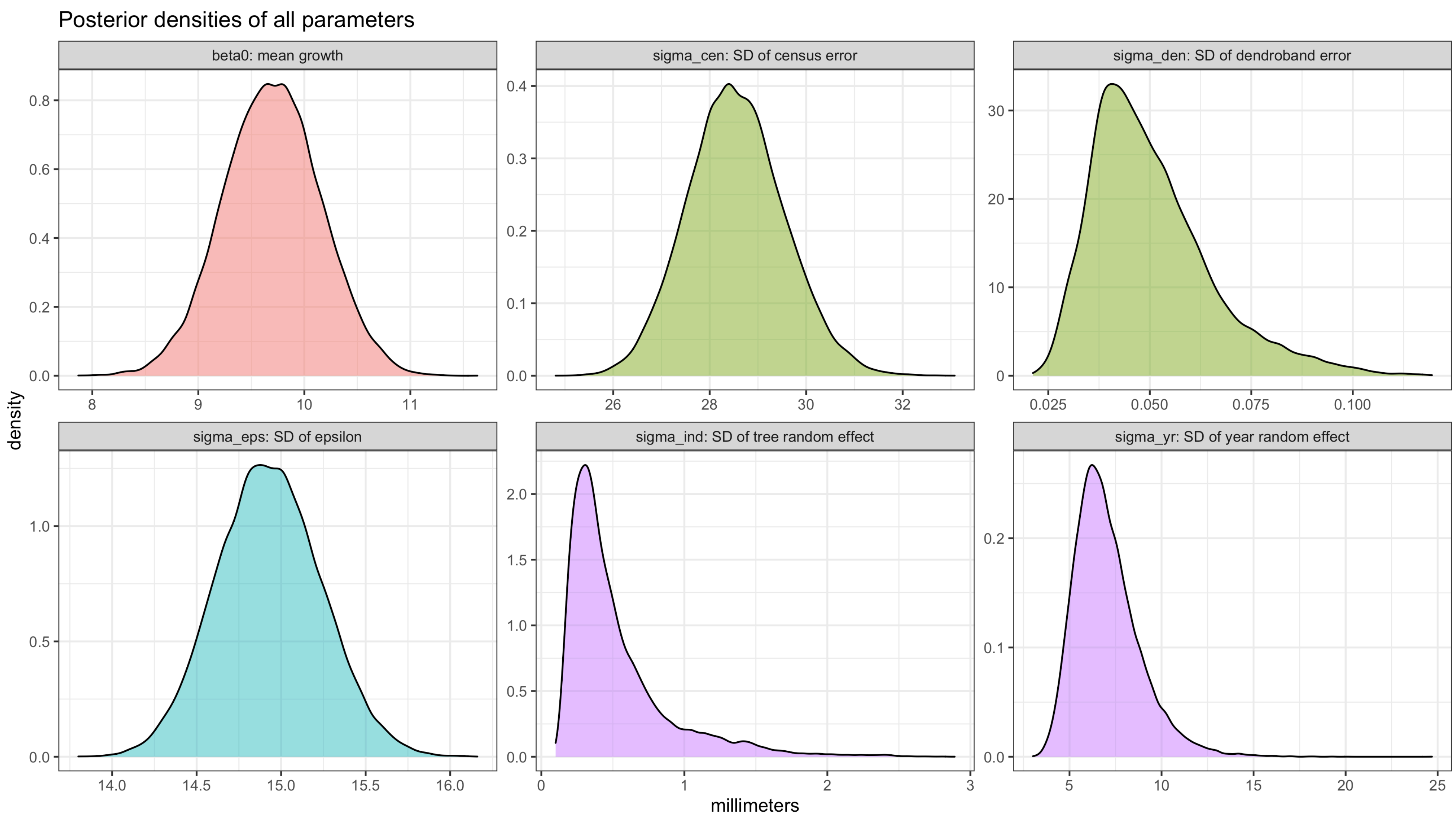

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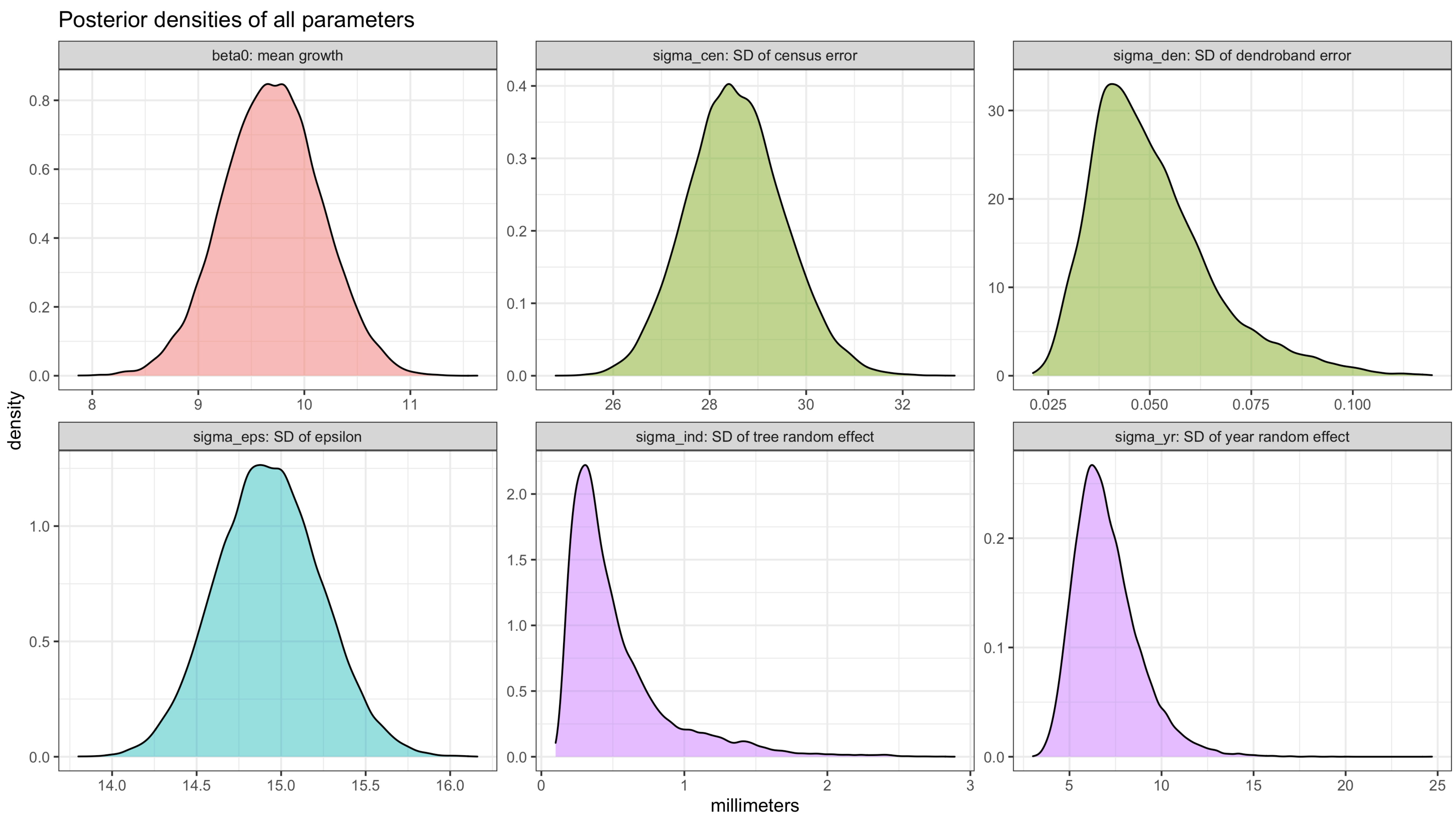

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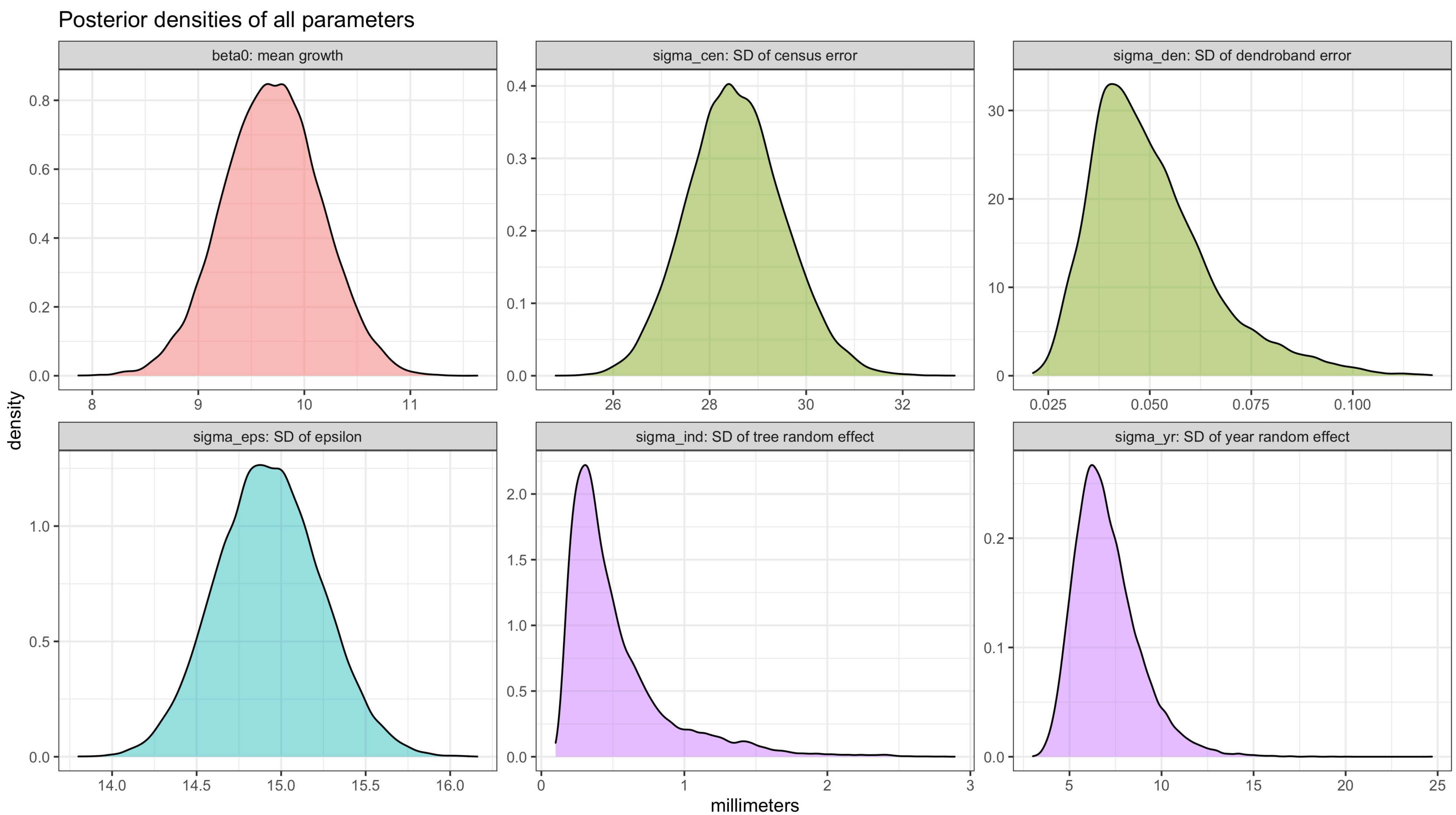

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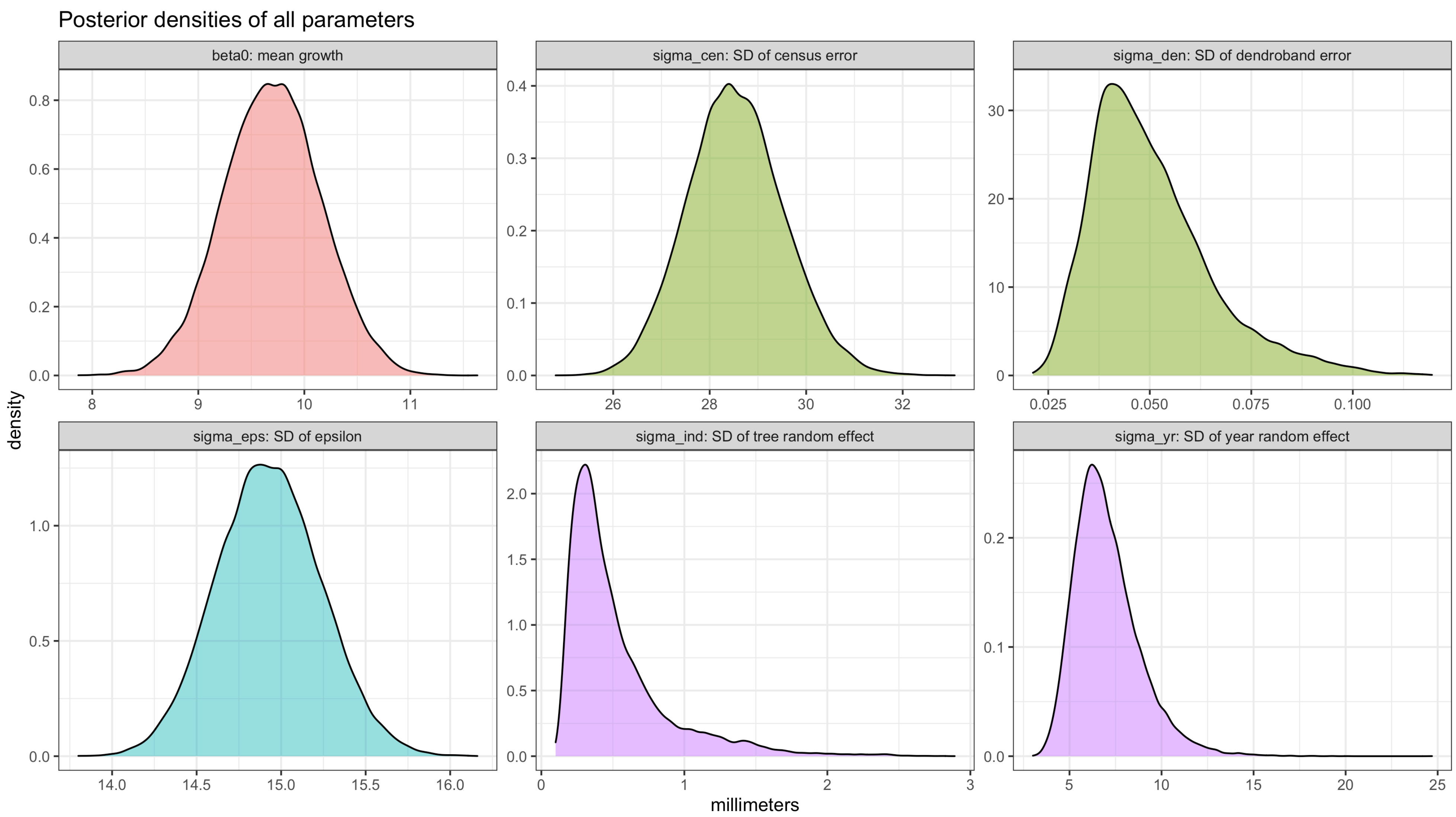

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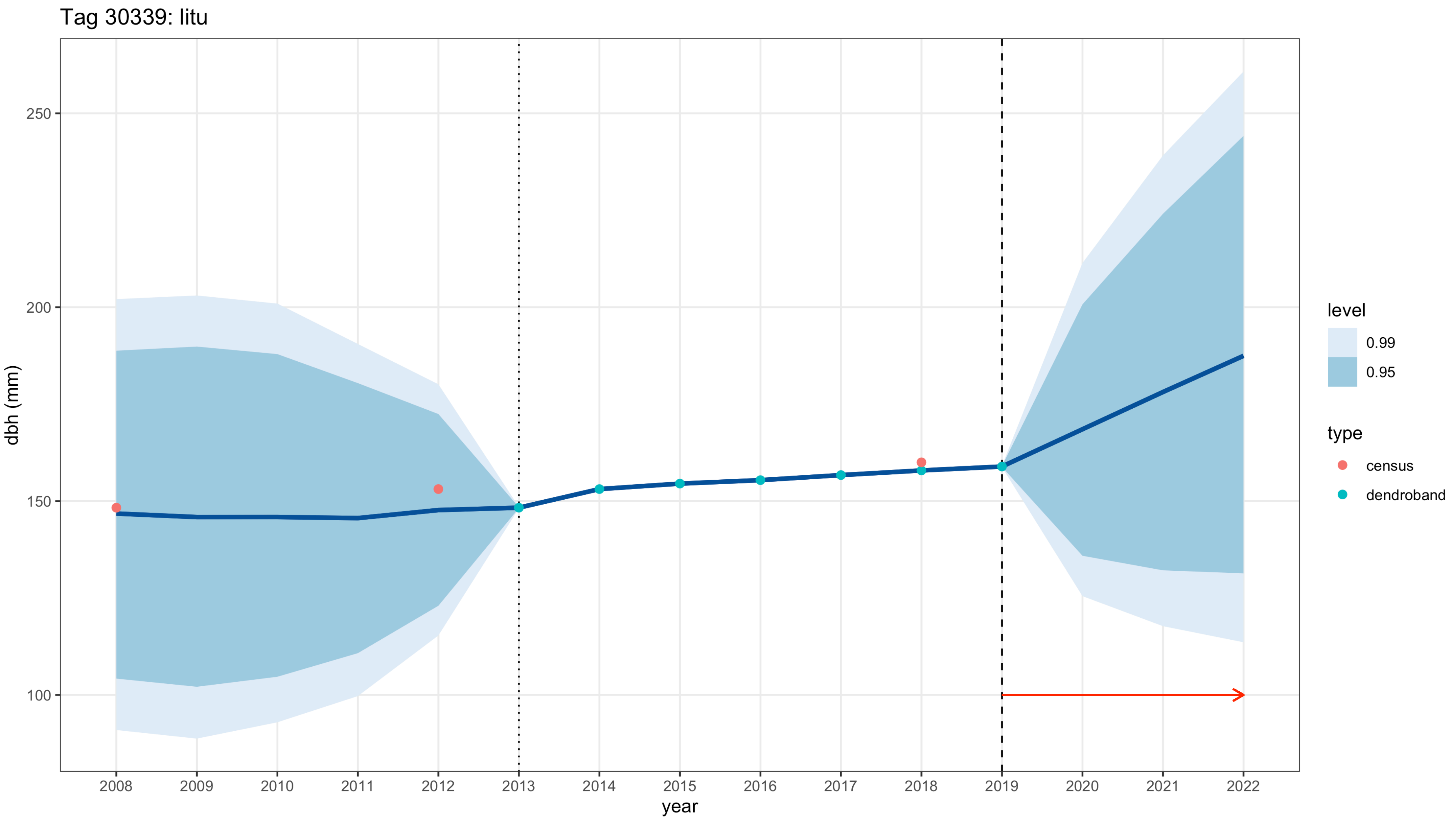
- σ_ϵ = remaining process error that propagates in forecasts across time

One particular tulip poplar

tag_stem	type	sp	`2007`	`2008`	`2009`	`2010`	`2011`	`2012`	`2013`	`2014`	`2015`	`2016`	`2017`	`2018`	`2019`
<chr>	<chr>	<chr>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>
30339_3	census	litu	NA	148.	NA	NA	NA	153.	NA	NA	NA	NA	NA	160	NA
30339_3	dendroband	litu	NA	NA	NA	NA	NA	NA	149.	155.	156.	157.	157.	159.	160.

One particular tulip poplar diameter

y = modeled true latent $dbh_{i,t}$



Dendroband installed in 2013

Future Work

TODO

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TODO

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  - Covariates: In particular species & starting diameter
- Choose appropriate time scale for t

Thanks!

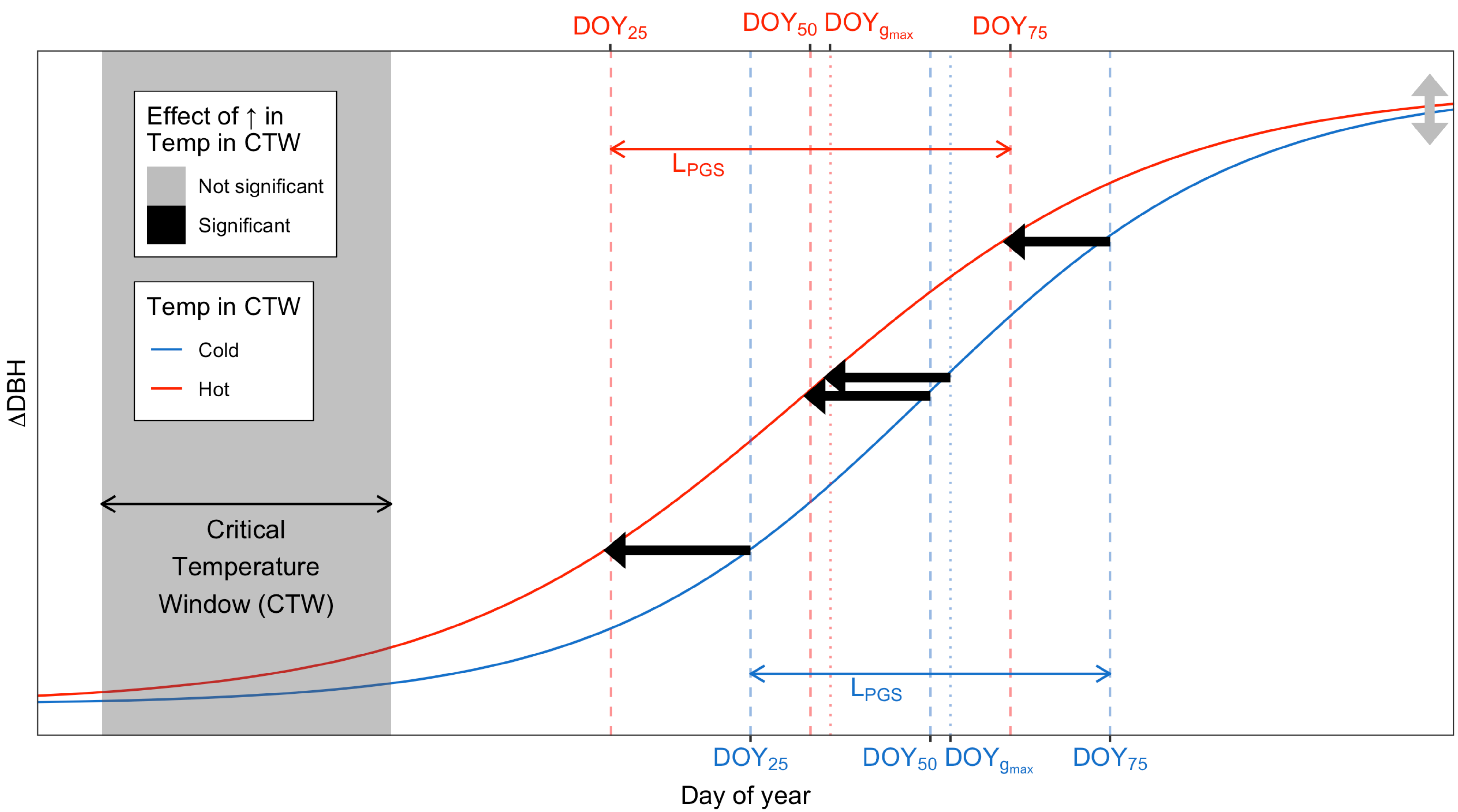
Slides on Twitter @rudeboybert

Nature doesn't always play nice

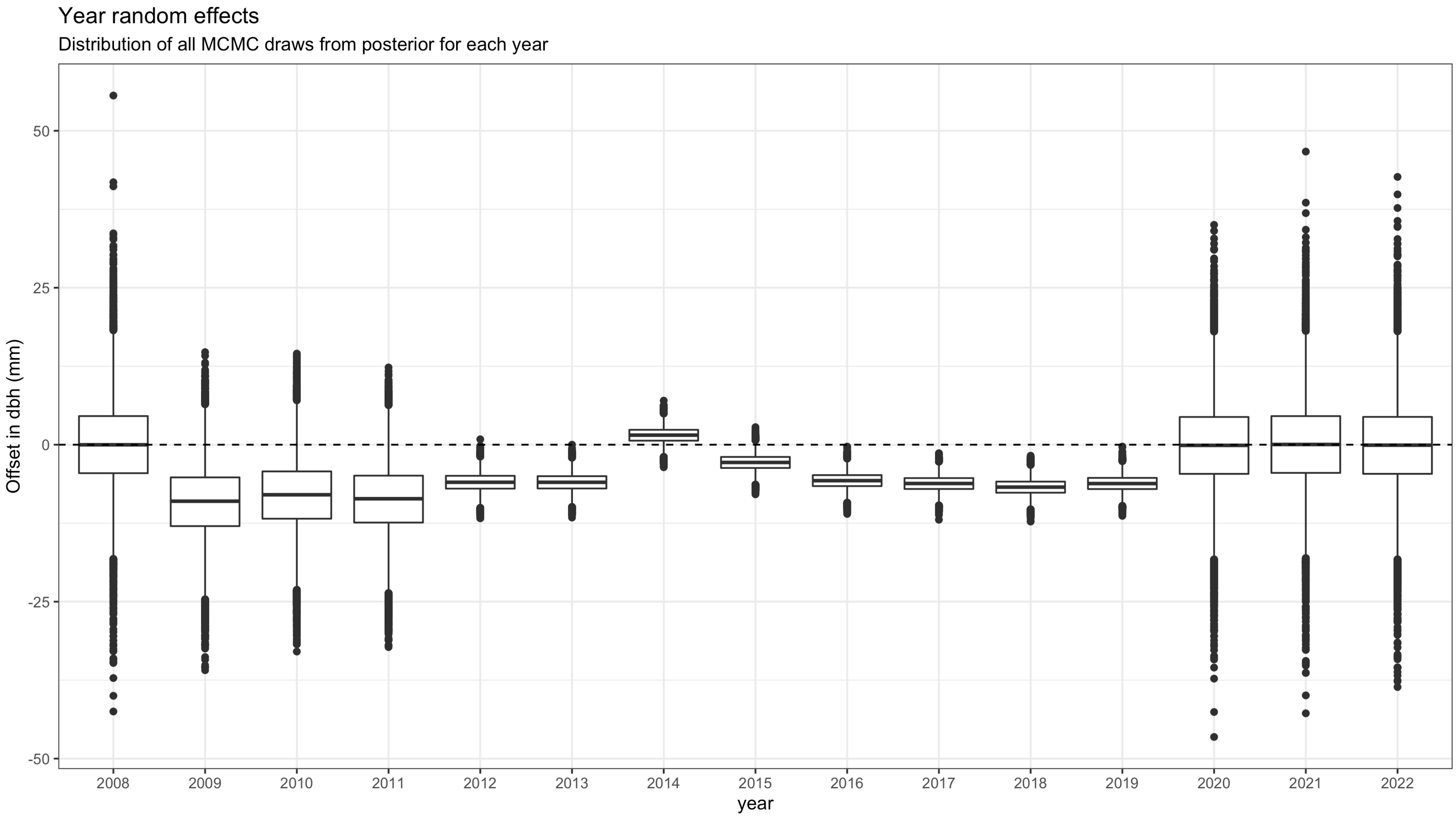
Nature doesn't always play nice

Thus could biannual dendroband data be more error prone than intraannual?

Intra-annual effect of climate



Year Random Effects



Individual Random Effects