Lecture 1.3: Permutations/Combinations + Hypergeometric + Birthday Problem

2013/09/06

Previously...

We introduced the Kolmogorov Axioms:

Let S be a sample space and let E be an event in S, then

1. $0 \leq \mathbb{P}(E)$ for every $E \in S$
2. $\mathbb{P}(S) = 1$
3. Countable Additivity: Let $E_1, E_2, \ldots$ be mutually exclusive[1] events, then

$$\mathbb{P}\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} \mathbb{P}\left(E_i\right)$$

[1] $E_i \cap E_j = \emptyset$ for any $i \neq j$. AKA disjoint

Previously...

Say we have an experiment where every one of N possible outcomes is likely. Let event E be some union of the outcomes. We introduced the following definition of probability:

Goals for today

- Review of counting methods: factorials, combinations, permutations
- Sampling with replacement vs without replacement
- Consider some discrete examples of probability

Factorials and Permutations

Say you have n distinct objects and we draw r of these objects without replacement. How many possible sequences can you draw? i.e. order matters

Combinations

Again, say you have n distinct objects and we draw r of these objects without replacement. How many possible samples can you draw where the order does not matter?

You probably already knew that this is simply "n choose r" $= C_r^n$, but let's tie it in with permutations. How many ways can we order r objects? $r!$

Comparison

$$\text{Permutations (order matters)} = P_r^n \quad =$$

$$\text{Combinations (order does not matter)} = C_r^n = \binom{n}{r} \quad =$$

The latter is also known as the binomial coefficient, which we will revisit later.

Urn Problem

Counting situation: say we have an urn with

- 20 balls in it: 12 of which are black, 8 of which are red
- We draw a sample of 10 balls without replacement
- Let $E = \{7 \text{ black balls \& 3 red ones are chosen}\}$

Using our definition of probability, find $\mathbb{P}(E)$

Hypergeometric Distribution

Birthday Problem

Say you have n people. Assume no February 29 birthdays and that all 365 possible birth dates are equally likely. We want to know the probability that at least two people have the same birthday.

1. GUESS how many people you need to have at least 90% probability of this being true.
2. Write down the formula for the probability for n people

Hints:

1. Use the definition of probability
2. Sometimes when $\mathbb{P}(E)$ is difficult to compute, use the fact that $\mathbb{P}(E) = 1 - \mathbb{P}(E^c)$
3. Start simple!

Birthday Problem

Birthday Problem

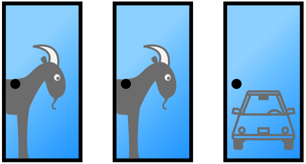
http://people.reed.edu/~albkim/MATH391/lectures/birthday.pdf

Next Time...

- We will discuss conditional probability: what is the probability of event B happening given that event A happened?
- One famous application of conditional probability is for the solution of the Monty Hall Problem, a classic problem in statistics, named after the host of the game show "Let's Make a Deal."

Monty Hall Problem

You are presented with 3 closed doors. Behind two of the doors are goats, and behind the third is a car. You pick a door, which remains closed. The host, who knows behind which of the doors lies the car, then opens one of other two doors to reveal a goat. He asks "Do you want to switch doors or not?"



What do you do? We will demonstrate the most effective strategy via conditional probability