

## Lecture 14.1: Final Review

2013/12/09

# Condition Expectation + Variance

- ▶  $\mathbb{E}[Y|X = x]$
- ▶  $\text{Var}[Y|X = x]$

Know the proofs to both. Know how to apply these to hierarchical models.

# Multivariate Transformations

- ▶ The Jacobian
- ▶ Important: The support! Don't forget this!

# Multivariate Transformations

Using this to find the distribution of sums of random variables  
 $S = X_1 + X_2$ :

- ▶ Make use of a dummy variable
- ▶ Convolution formula

$$\begin{aligned}f_S(s) &= \int_{-\infty}^{\infty} f(t, s - t) dt \\&= \int_{-\infty}^{\infty} f_1(t) f_2(s - t) dt \text{ if independent} \\f_S(s) &= \sum_{\text{all } t} f(t, s - t) \\&= \sum_{\text{all } t} f_1(t) f_2(s - t) \text{ if independent}\end{aligned}$$

# Order Statistics

- ▶ Definition
- ▶ Joint PDF of  $X_{(1)}, \dots, X_{(n)}$
- ▶ PDF of  $X_{(j)}$
- ▶ Joint PDF of  $X_{(i)}$  and  $X_{(j)}$

# Estimators

For basic estimators:

- ▶  $\bar{X}$
- ▶  $S^2$

show properties like

- ▶ Expectation
- ▶ Variance

# Convergence

- ▶ Convergence in Probability
- ▶ Convergence in Distribution

Might have to show convergence along the lines of examples in lecture on 2013/11/22:

- ▶ Convergence in probability of the maximum of uniforms
- ▶ Convergence in distribution of  $n(1 - X_{(n)})$

# Weak Law of Large Numbers

- ▶ State Theorem
- ▶ Proof
- ▶ Interpretation



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# Central Limit Theorem

- ▶ State Theorem
- ▶ Proof: Convergence of MGF's
- ▶ Interpretation
- ▶ Applications (From 2013/12/2):
  - ▶ Approximation to the Binomial + Continuity Correction
  - ▶ Sums of IID random variables.

# Exponential Families

- ▶ Understand all the components
- ▶ What  $t_i(x)$  means
- ▶ Be able to re-express any family of distributions as an exponential family
- ▶ Theorem on expectation and variance of

$$\sum_{i=1}^k \frac{\partial w_i(\theta)}{\partial \theta_j} t_i(X)$$

# Exponential Families

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- ▶ Be able to re-express any family of distributions as an exponential family

# Characteristic Functions

# Markov Chains

- ▶ Definition
- ▶ Example done in class

# Wednesday's Lecture

- ▶ Any questions you may have
- ▶ Going to go over previous questions and emphasize important points to keep in mind for each one