Lecture 4.3: Poisson Distribution

2013/09/27

Previously... Variance of Linear Functions of Random Variables

Theorem

If X is a random variable and a and b are constants then

$$Var(aX + b) = a^2 Var(X)$$

i.e. the variance is invariant to translations on the real line, but not multiplication.

Previously... Moments

Definition

If k is a positive integer, the kth moment is defined to be

$$\mathbb{E}[X^k]$$

The kth central moment is defined to be

$$\mathbb{E}\left[(X-\mu)^k\right]$$

So the variance is the 2nd central moment. We'll revisit this later when we study moment generating functions.

Previously... Discrete Uniform Distribution

A discrete RV X has the discrete uniform distribution on the integers $1, 2, \ldots, N$ if it has a PMF of the form:

$$f(x) = \frac{1}{N} \text{ for } x = 1, 2, \ldots, N$$

Previously... Negative Binomial

Suppose that independent trials, each having probability p of success, are performed until a total of r success is accumulated. If we let X be the random number of trials required, then we say $X \sim$ Negative Binomial(r, p) with PMF

$$f(x) = \binom{x-1}{r-1} p^r (1-p)^{x-r}$$

Previously... Binomial vs Negative Binomial

Inverse of each other:

- Binomial: Fix the number of trials n, what is the random number of successes in those n trials?
  A Binomial(n, p) is n independent instances of a Bernoulli(p).
- Negative Binomial: Fix the number of successes r, what is the random number of trials until you achieve r successes.
  A Negative Binomial(r, p) is r independent instances of a Geometric(p).

Binomial vs Negative Binomial

In fact:

Goals for Today

- Poisson Distribution

Theorem: Back to Binomial Distribution

Poisson Distribution

Binomial/Poisson Link

Examples of Poisson Data

In practice, this distribution is often used to model counts for rare events where we have a measure of exposure and a rate. Examples:

- Car accidents at an intersection
  - n is the # of cars crossing the intersection
  - p is the probability of an accident (small)
- Nuclear physics: Radioactive decay
- Classic (von Bortkiewicz 1898), for the chance of a Prussian cavalryman being killed by the kick of a horse.

Example

Poisson PMF

Next Time

Continuous random variables:

- Instead of probability mass function being summed, probability density function being integrated
- Continuous Expectation/Variance