Lecture 3.2: Sampling + Introduction to Bayesian Statistics

2014/02/12

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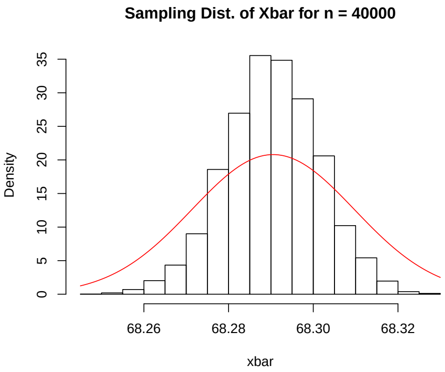

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i.e. sampling with replacement vs sampling without replacement.

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Say $N = 10000$.

- For $n = 100$

$$\sqrt{\frac{10000-100}{10000-1}} = 0.99$$

- For $n = 9000$

$$\sqrt{\frac{10000-9000}{10000-1}} = 0.32$$

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Capping n to be less than 10% of the population is a rule of thumb for keeping the correction factor “close” to 1.

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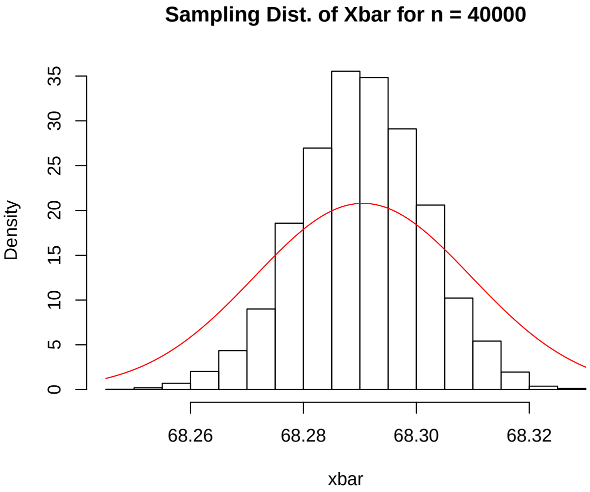

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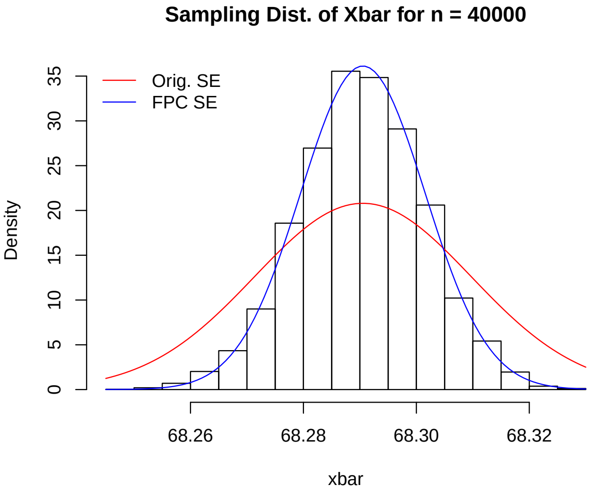

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- Conceptual: If we sample everybody in our study population, then we don't need to use statistics because we know exactly what the true μ is.
- Mathematical: If $n = N$, then $FPC = \sqrt{\frac{N-n}{N-1}} = 0$, hence $SE_{\overline{X}} = 0$. i.e. there is no variability in our sampling procedure. If we repeat the procedure many times, we get the exact same value every time: the true μ.

Bayesian Statistics

Switching Gears: Thomas Bayes

Philosophical Schools of Thought (My Understanding)

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- Subjectivism: the philosophical tenet that "our own mental activity is the only unquestionable fact of our experience"

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- Positivist View - Frequentist Probability: it defines an event's probability as the limit of its relative frequency in a large observable number of trials.
- Subjectivist View - Bayesian Probability: a subjective status by regarding it as a measure of the degree of belief of the individual assessing the uncertainty of a particular situation.

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- Bayesian Statistics: the true θ is a distribution of values that reflects our belief in the plausibility of different values.

Specific Example

Concrete example: the probability of flipping heads

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1. A prior distribution $\Pr(\theta)$. It reflects our prior belief about θ.
2. The likelihood function $\Pr(X|\theta)\,(= L(\theta|X))$. This is the mechanism that generates the data.
3. A posterior distribution $\Pr(\theta|X)$. We update our belief about θ after observing data X.

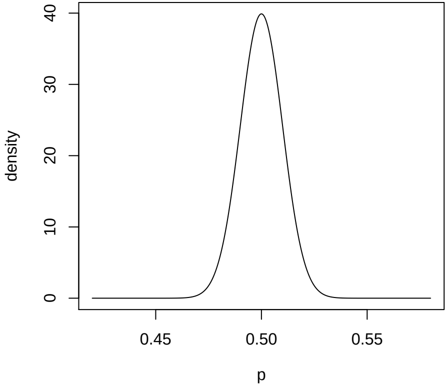
$$\Pr(\theta|X) = \frac{\Pr(X|\theta) \cdot \Pr(\theta)}{\Pr(X)}$$

The Issue: The Bayesian Procedure

Where do you come up with $\Pr(\theta)$? It's completely subjective! You decide!

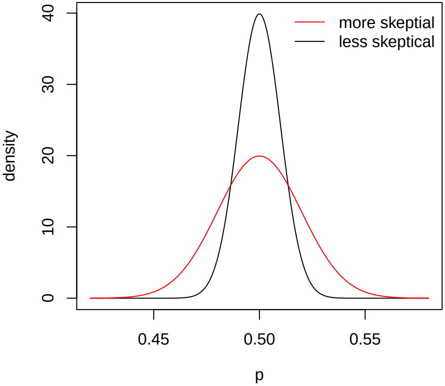
Prior Distribution

This distribution can reflect someone's prior belief of p.



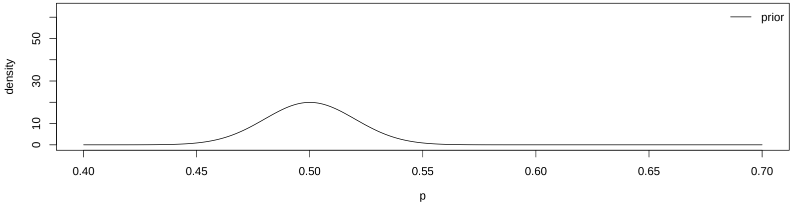
Prior Distribution

Say someone is more skeptical that $p = 0.5$, we can lower it.



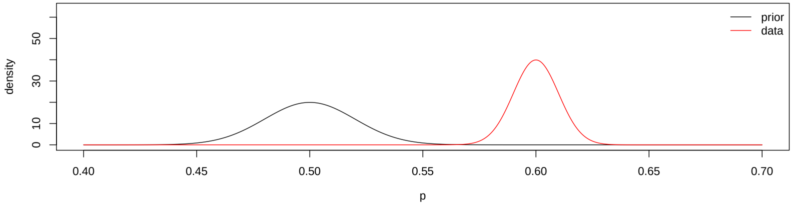
The Bayesian Procedure

Say we have the following prior belief centered at $p = 0.5$



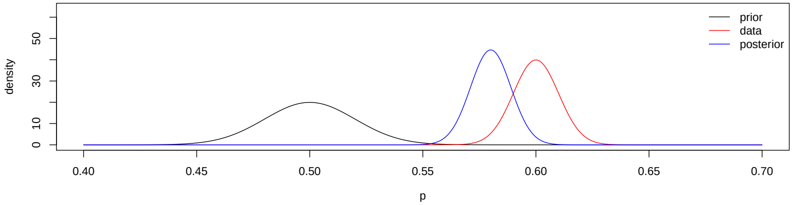
The Bayesian Procedure

Say we collect data, represented by the red line, suggesting $p = 0.6$



The Bayesian Procedure

We then update our belief, as reflected in the posterior distribution!



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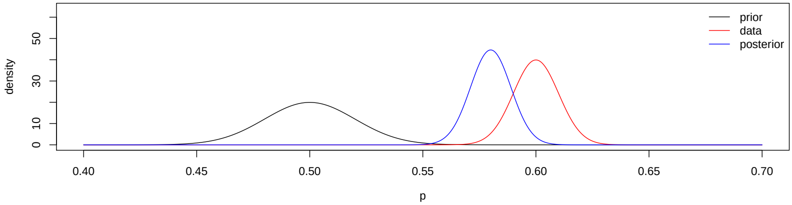
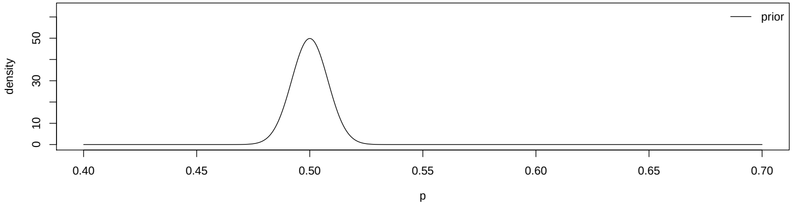
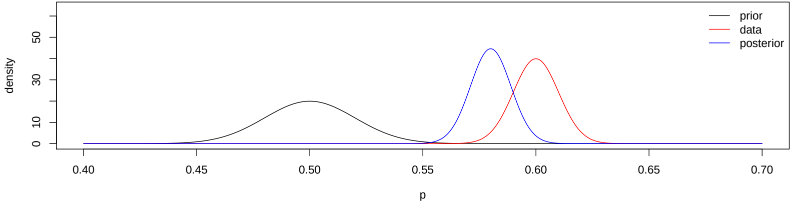
Now say we have a stronger prior belief that $p = 0.5$

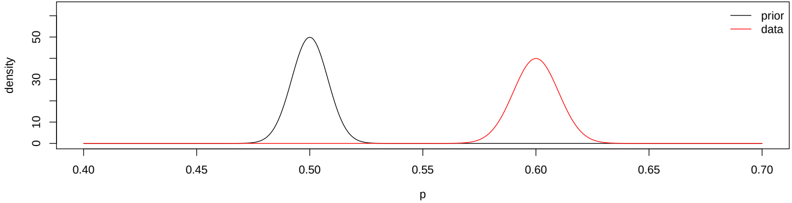




The Bayesian Procedure

Say we observed the same data (as represented in red).





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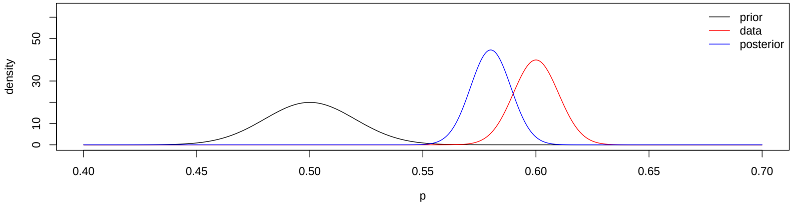
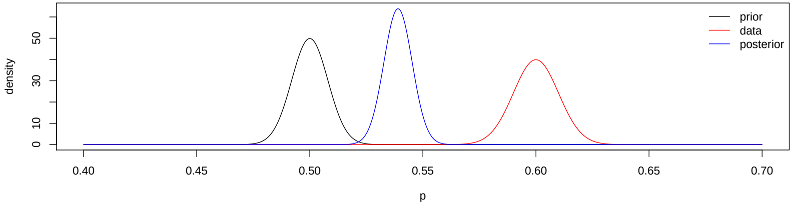
The posterior in this case is pulled left due to the sharper prior.





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The debate rages on...