Lecture 2.1: Intuition Behind Sampling Distributions

2014/01/29

Behavior of Statistics

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Two Important Conceptual Questions:

1. If we compute the sample mean $\overline{x}$ of these points, are we going to always get exactly 5?
2. Say we do this once and the sample mean is $\overline{x} = 5.025$. If we repeat this procedure
   (i.e. generate a new sample of 100 points and compute $\overline{x}$)
   are we going to get $\overline{x} = 5.025$ exactly?

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The sampling distribution describes the random behavior of these estimates.

Example: Sampling Distribution of the Sample Mean

Consider the histogram of 1000 instances of $\overline{x}$, where each $\overline{x}$ is computed from a sample of $n = 100$ Normal($\mu = 5, \sigma^2 = 4$) RV's.

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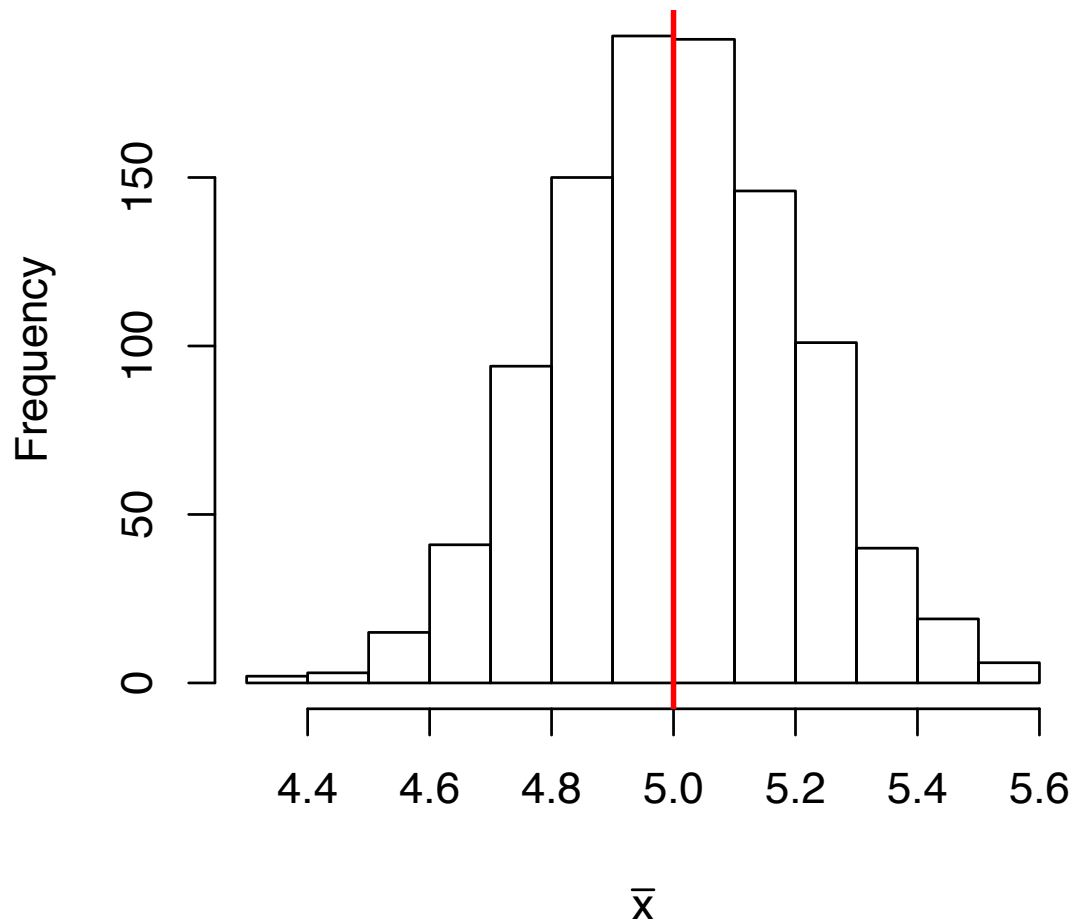

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Notice the $\sqrt{n}$ in the denominator: n increases, SE decreases!

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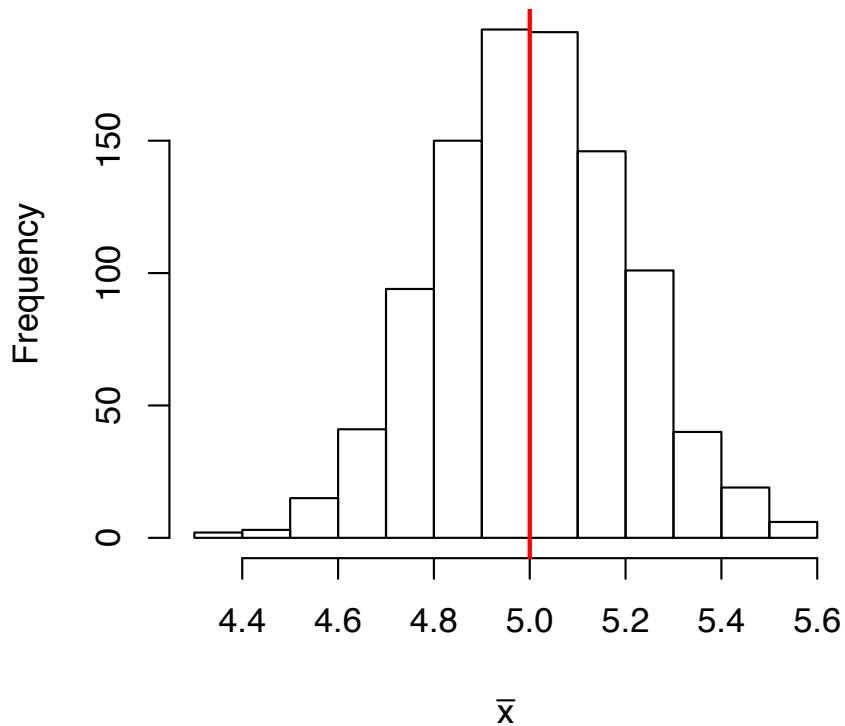
Recall $n = 100$ with $X_i \sim \text{Normal}(\mu = 5, \sigma^2 = 4)$. The standard error is the SD of the sampling distribution:

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{2}{\sqrt{100}} = \frac{2}{10} = 0.2$$



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