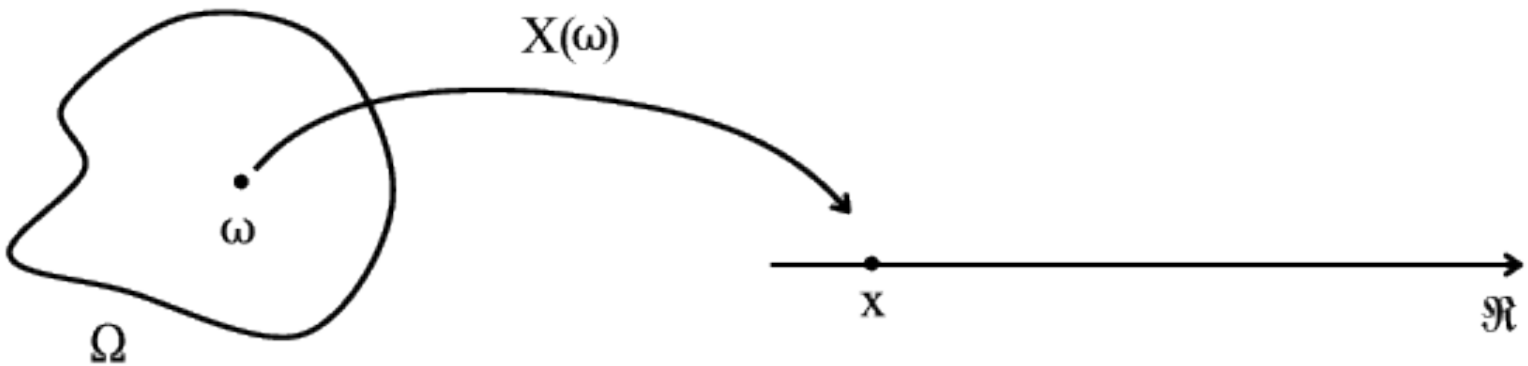
Lecture 3.3: Discrete Random Variables

2013/09/20

Previously... Random Variables

Definition

A function $X(\cdot)$ that maps the sample space S to the real line in such a way that, that for every $\omega \in S$, $X(\omega)$ is a real number, is called a random variable (RV for short).



Cumulative Distribution Function

Definition

The distribution function (AKA cumulative distribution function) of a random variable X is a function $F : \mathbb{R} \longrightarrow [0, 1]$ given by $F(x) = \mathbb{P}(X \leq x)$

A function $F(x)$ is a CDF for some random variable X if and only if it satisfies the following properties

- $\lim_{x \to -\infty} F(x) = 0$
- $\lim_{x \to \infty} F(x) = 1$
- $\lim_{h \to 0^+} F(x + h) = F(x)$ (right continuous)
- $a < b$ also implies $F(a) \leq F(b)$

Lemmas

Let $F(\cdot)$ be the distribution function of X. Then

- $\mathbb{P}(X > x) = 1 - F(x)$
- $\mathbb{P}(x < X \leq y) = F(y) - F(x)$
- $\mathbb{P}(X = x) = F(x) - \lim_{y \uparrow x} \mathrm{F(y)}$

Goals for Today

- Introduce Discrete Random Variables
- Discuss some (intuitive) properties
- Examples of Discrete Random Variables:
  - Bernoulli
  - Binomial
  - Geometric

Discrete Random Variables

Properties

Hypergeometric Distribution

Refresher:

- N total number of balls in the urn
- n number of balls drawn from the urn without replacement
- K number of "success" balls in the urn (in our case, black)
- X is the (random) number of "success" balls drawn

What is the probability that we draw k successes?

$$\mathbb{P}(X = k) = \frac{\binom{K}{k} \times \binom{N-K}{n-k}}{\binom{N}{n}}$$

Bernoulli Random Variable

Bernoulli Random Variable CDF

Binomial Random Variable

Say we roll a die $n = 10$ times. What is the probability of obtaining $k = 6$ 1's? Here, getting a 1 is a "success."

Geometric Random Variable

Say we are interested in the number of rolls it takes until we get a 1. What is the probability that it takes k rolls until we get a 1?

Next Time

- Expectation of a RV. i.e. the mean/average. Measure of central tendency.
- Variance and standard deviation of a RV. Measure of spread
- Properties