Lecture 4.2: Discrete Variance

2013/09/25

Previously... Expected Value

The expected value is a weighted average of all possible values x. This can be thought of as a measure of center.

If X is a discrete random variable with PMF $f(x)$, then the expected value of X is defined by

$$\mathbb{E}[X] = \sum_x x \cdot f(x)$$

This is also called the mean and expectation of X. Typically denoted by μ.

Previously... Linearity of Expectations

Theorem

If X is a (discrete) random variable with PMF $f(x)$, a and b are constants, and $g(x)$ and $h(x)$ are real-valued functions whose domains include the possible values of X, then

$$
\begin{aligned}
\mathbb{E}[a \cdot g(X) + b \cdot h(X)] &= a \cdot \mathbb{E}[g(X)] + b \cdot \mathbb{E}[h(X)] \\
&= a \cdot \sum_x g(x) \cdot f(x) + b \cdot \sum_x h(x) \cdot f(x)
\end{aligned}
$$

Previously... Variance

The variance σ^2 AKA Var(X) of a distribution is

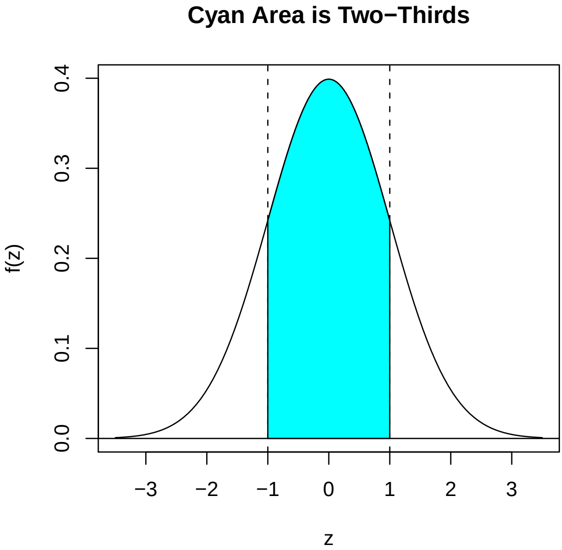
$$\mathbb{E}\left[(X-\mu)^2\right] = \sum_x (x-\mu)^2 \cdot f(x)$$

It is the expected squared deviation from the mean, and not absolute deviation (like in our example). i.e. not

$$\mathbb{E}\left[|X-\mu|\right] = \sum_x |x-\mu| \cdot f(x)$$

The standard deviation (SD) $\sigma = \sqrt{\sigma^2}$.

Ex: Standard Normal $\mu = 0$, $\sigma = 1$



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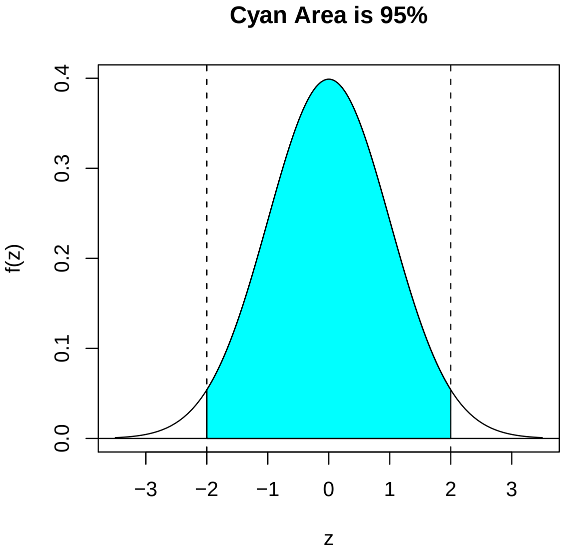

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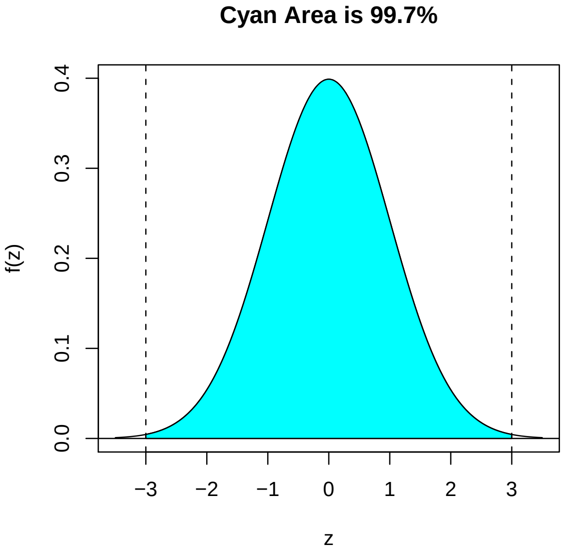

Previously... Alternate Expression of Variance

If X is a random variable, then

$$\begin{aligned} Var(X) &= \mathbb{E}[X^2] - (\mathbb{E}[X])^2 = \mathbb{E}[X^2] - \mu^2 \\ &= \sum_x x^2 \cdot f(x) - \left(\sum_x x \cdot f(x) \right)^2 \end{aligned}$$

Goals for Today

- More properties of Variance
- Define moments
- More distributions:
  - Discrete Uniform
  - Negative Binomial

Alternate Expression of Variance

Variance of Linear Functions of Random Variables

Moments

Discrete Uniform Distribution

Expectation of Discrete Uniform Distribution

Variance of Discrete Uniform Distribution

Negative Binomial

Say the Sharks & Jets are playing in a best-of-7 hockey playoff series. i.e. the first team to win $r = 4$ games wins. For each game, say $p = \mathbb{P}(\text{Sharks win}) = 0.6$ and the games are independent. What is the probability that the Sharks win the series in exactly $x = 6$ games?

Negative Binomial

Suppose that independent trials, each having probability p of success, are performed until a total of r success is accumulated. If we let X be the random number of trials required, then we say $X \sim \text{Negative Binomial}(r, p)$ with PMF

Binomial vs Negative Binomial

Next Time

- Poisson Random Variables