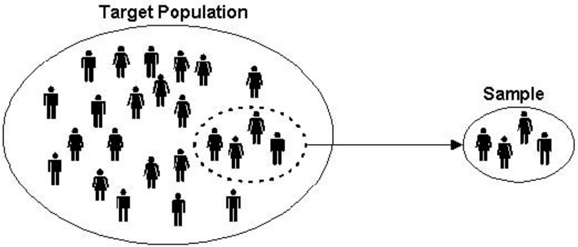
Populations and Samples (From MATH 141 2013/9/6)

We want to study and make statements about some aspect of a study/target population.

1. What proportion of Americans support a military intervention in Syria?
2. What proportion of televisions were tuned into the MTV Music awards last week?
3. What is average mercury content in swordfish in the Atlantic Ocean?

Populations and Samples

However, it is often really unrealistic/inconvenient/expensive to collect data for every case in the population. Therefore, we take a sample of cases.



If the sample is representative of the desired population then our results will generalize. This is called generalizability. If the sample is not representative, then the results will be biased.

Populations and Samples

We want the statistic[1] based on sample observations $X_1, \dots, X_n$, for example the sample mean

$$\overline{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$$

to generalize to the true population mean μ.

We view the as of yet still random $\overline{X}$ as an estimator of μ and the observed $\overline{x}$ as an estimate of μ.

[1] Any function of observable random variables that does not depend on any unknown parameters

Populations and Samples

How do we take a representative sample? In its simplest form, the way for your sample to be representative is to take a simple random sample from the population.

Back to 391: The set of random variables $X_1, \ldots, X_n$ is said to be a simple random sample of size n from a population with density function $f(x)$ if the joint pdf has the form

$$f(x_1, x_2, \ldots, x_n) = f(x_1)f(x_2)\ldots f(x_n) = \prod_{i=1}^{n} f(x_i)$$

i.e. the observations are independent & identically distributed IID.

Direction of the Class Now

We will study the properties of estimators like $\overline{X}$, for example:

- Does $\overline{X}$ "converge" to μ as $n \to \infty$? "Converge" in what sense? Weak Law of Large Numbers
- What is the expected value, variability, and distributional form of $\overline{X}$ as a function of sample size n? Central Limit Theorem
- We later study properties like unbiasedness, minimum variance'ness, sufficiency'ness, etc.