# Part I

## 1)

1F-5

b) $A^2 = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ $\quad A^3 = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$ $\quad A^n = \begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix}$

1F-8

a) $\begin{pmatrix} 2 & -1 & 1 \\ 3 & 0 & 1 \\ 1 & 4 & -1 \end{pmatrix}$

1F-9

a)

1G-3

$A = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & 1 \\ -1 & -1 & 2 \end{pmatrix}$ $\quad M = \begin{pmatrix} 3 & -1 & 1 \\ 1 & 3 & 2 \\ -2 & -1 & 1 \end{pmatrix}$ $\quad |A| = 5$

$M^T = \begin{pmatrix} 3 & 1 & -2 \\ -1 & 3 & -1 \\ 1 & 2 & 1 \end{pmatrix}$ $\quad A^{-1} = \frac{1}{5} M^T = \begin{pmatrix} 3/5 & 1/5 & -2/5 \\ -1/5 & 3/5 & -1/5 \\ 1/5 & 2/5 & 1/5 \end{pmatrix}$

$$Ax = b \iff x = A^{-1}b = \begin{pmatrix} 3/5 & 1/5 & -2/5 \\ -1/5 & 3/5 & -1/5 \\ 1/5 & 2/5 & 1/5 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$$

1G-4.

$$\begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & 1 \\ -1 & -1 & 2 \end{pmatrix} x = y \qquad A = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & 1 \\ -1 & -1 & 2 \end{pmatrix} \qquad M = \begin{pmatrix} 3 & -1 & 1 \\ 1 & 3 & 2 \\ -2 & -1 & 1 \end{pmatrix}$$

$$|A| = 5 \quad A^{-1} = \begin{pmatrix} 3/5 & 1/5 & -2/5 \\ -1/5 & 3/5 & -1/5 \\ 1/5 & 3/5 & 1/5 \end{pmatrix}$$

$$x = \begin{pmatrix} 3/5 & 1/5 & -2/5 \\ -1/5 & 3/5 & -1/5 \\ 1/5 & 3/5 & 1/5 \end{pmatrix} y$$

$$x_1 = \frac{3}{5}y_1 + \frac{1}{5}y_2 - \frac{2}{5}y_3$$
$$x_2 = \frac{-1}{5}y_1 + \frac{3}{5}y_2 - \frac{1}{5}y_3$$
$$x_3 = \frac{1}{5}y_1 + \frac{3}{5}y_2 + \frac{1}{5}y_3$$

1G-5

$$(B^{-1}A^{-1})(AB) = B^{-1}(A^{-1}A)B = B^{-1}B = I$$

Thus, $(AB)^{-1} = B^{-1}A^{-1}$

$\boxed{2}$

1H-3

a) $\begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & 1 \\ -1 & c & 2 \end{pmatrix} x = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ $\qquad 2 - c + 5 + 2c + 1 = 0$

$c + 8 = 0 \qquad c = -8$

b) $\begin{pmatrix} 2-c & 1 \\ 0 & -1-c \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$(2-c)(-1-c) = 0 \qquad c = -1$ or $c = 2$

c) $\begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & 1 \\ -1 & -8 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$\langle 1, -1, 1 \rangle \times \langle 2, 1, 1 \rangle = \langle -2, 1, 3 \rangle$

$(-2, 1, 3)$

1E-1

a) $N = \langle 1, 2, -2 \rangle \qquad \langle 1, 2, -2 \rangle \cdot \langle x-2, y, z+1 \rangle = 0$

$x - 2 + 2y - 2z - 2 = x + 2y - 2z - 4 = 0$

c) $\overrightarrow{PQ} = \langle 1, -1, 1 \rangle \quad \overrightarrow{PR} = \langle -2, 3, 1 \rangle$

$\begin{pmatrix} i & j & k \\ 1 & -1 & 1 \\ -2 & 3 & 1 \end{pmatrix} \quad \overrightarrow{PQ} \times \overrightarrow{PR} = \langle -4, -3, 1 \rangle$

$\langle 1, 0, 1 \rangle \qquad \langle -4, -3, 1 \rangle \cdot \langle x-1, y, z-1 \rangle$

$= -4(x-1) - 3y + z - 1$

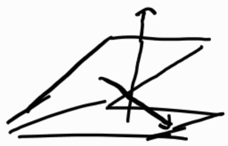
$= -4x - 3y + z + 3 = 0$

$4x + 3y - z - 3 = 0$

1E-2

$2x - y + z = 3 \quad x + y + 2z = 1$

$N_1 = \langle 2, -1, 1 \rangle \quad N_2 = \langle 1, 1, 2 \rangle$



$2 - 1 + 2 = 3$

$\sqrt{4+2} \quad \cos(\theta) \frac{3}{6} = \frac{1}{2}$

$\theta = \pi/3$

1E-6

$D = \frac{|d|}{\sqrt{a^2+b^2+c^2}}$ Normal vector $N$ of $ax+by+cz=d$

$= \langle a, b, c \rangle$

unit vector of $N$, $n = \frac{1}{\sqrt{a^2+b^2+c^2}} \langle a, b, c \rangle$

$|\overrightarrow{OX}| \cdot n = \langle x, y, z \rangle \langle a, b, c \rangle \frac{1}{\sqrt{a^2+b^2+c^2}}$

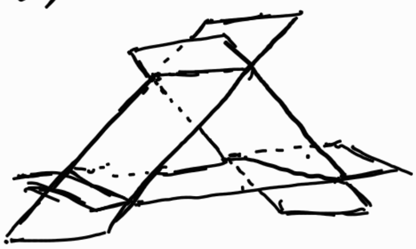
$= |\overrightarrow{OX}| \cos(\theta)$

$= \frac{ax+by+cz}{\sqrt{a^2+b^2+c^2}} = \frac{|d|}{\sqrt{a^2+b^2+c^2}} = D$

Part II

Problem 1.

a)



b)

Problem 2.

$P_1 = M_1 + 2M_2 + 3M_3 \qquad P_2 = M_1 + 3M_2 + 5M_3 \qquad P_3 = 3M_1 + 5M_5 + 8M_3$

$137M_1 \quad 279M_2 \quad 448M_3$

a) $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 3 & 5 & 8 \end{pmatrix} \begin{pmatrix} M_1 \\ M_2 \\ M_3 \end{pmatrix} = \begin{pmatrix} P_1 \\ P_2 \\ P_3 \end{pmatrix} \qquad A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 3 & 5 & 8 \end{pmatrix}$

b) $\begin{pmatrix} -1 & 7 & -4 \\ -1 & -1 & 1 \\ 1 & -2 & 1 \end{pmatrix} \qquad \begin{pmatrix} -1 & -1 & 1 \\ 7 & -1 & -2 \\ -4 & 1 & 1 \end{pmatrix} \qquad |A| = -1 + 14 - 12$
$= 1$

$A^{-1} = \begin{pmatrix} -1 & -1 & 1 \\ 7 & -1 & -2 \\ -4 & 1 & 1 \end{pmatrix}$