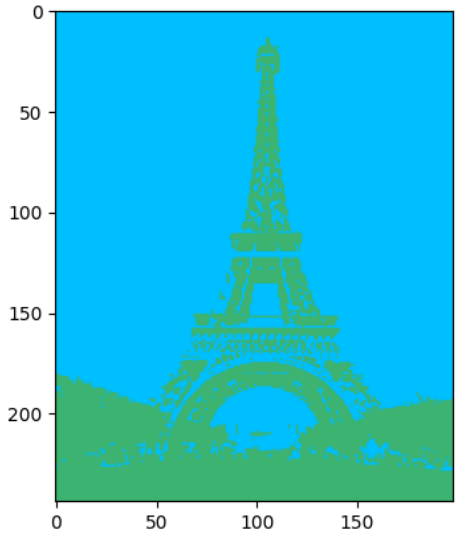
1.

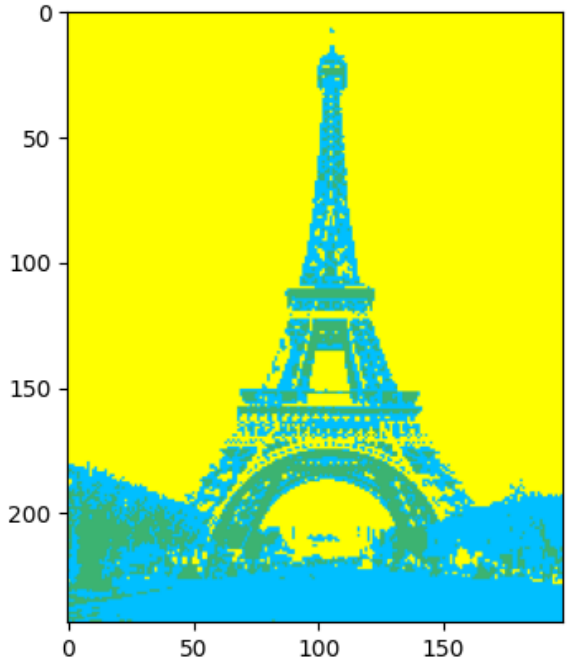
K = 2

LSE = `13847.297136161471`

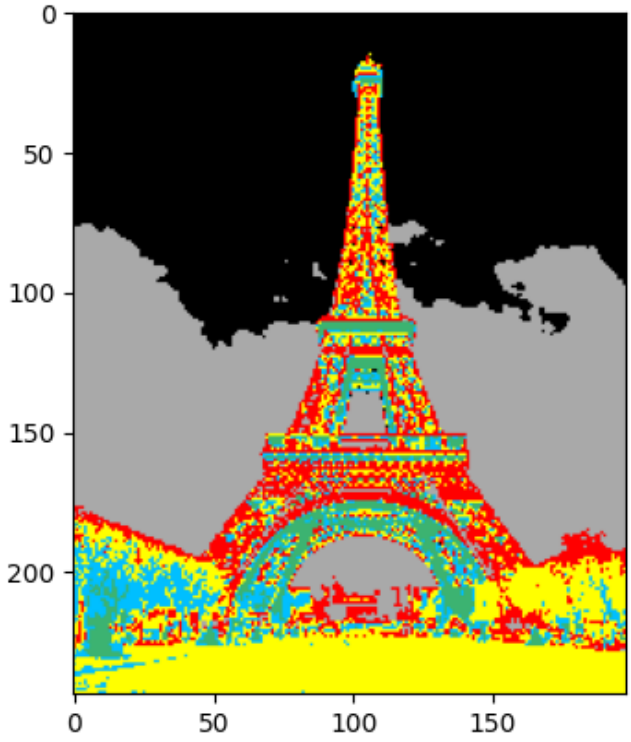


K = 3

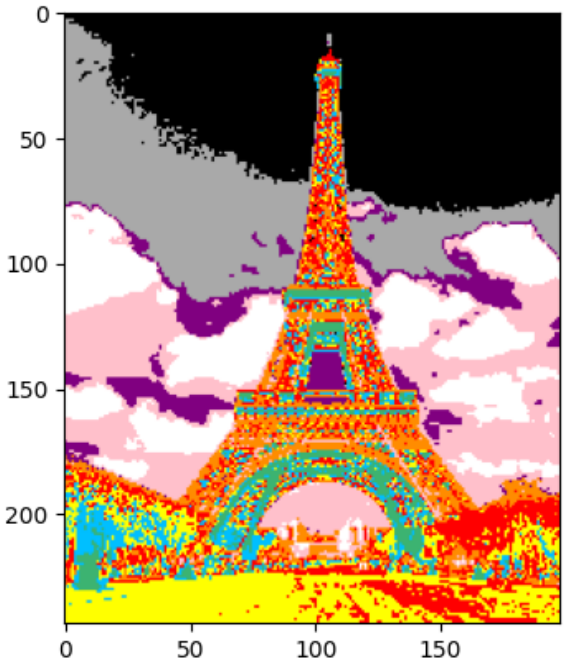
LSE = `13089.734490850286`



K = 6
LSE = 4709.379195482143

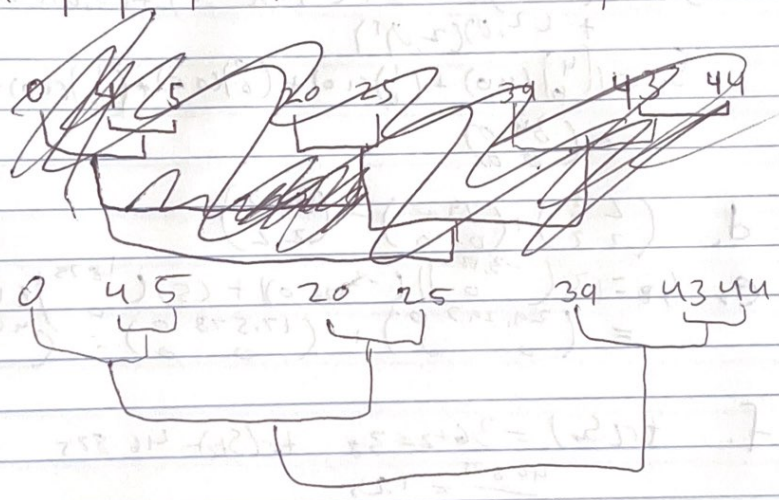


K = 10
LSE = 3252.0361424950825



2.

	0	4	5	20	25	39	43	44
0	0	4	5	20	25	39	43	44
4		0	1	16	21	35	39	40
5			0	15	20	34	38	39
20				0	5	19	23	24
25					0	14	18	19
39						0	4	5
43							0	1
44								0



The two top level clusters are [0, 4, 5, 20, 25] and [34, 43, 44]

3. a. $m_1 = \frac{1}{3}((1,1)+(2,2)+(3,3)) = (2,2)$

$m_2 = \frac{1}{5}((5,2)+(6,2)+(7,2)+(8,2)+(9,2)) = (7,2)$

b. $m = \frac{3(2,2)+5(7,2)}{3+5} = \frac{(6,6)+(35,10)}{8} = (5.125, 2)$

c. $S_1 = ((1,1)-(2,2))((1,1)-(2,2))^T + ((2,2)-(2,2))((2,4)-(2,2))^T + ((3,3)-(2,2))((3,3)-(2,2))^T$

$S_1 = \begin{pmatrix}1\\1\end{pmatrix}(1\ 1) + \begin{pmatrix}0\\0\end{pmatrix}(0\ 0) + \begin{pmatrix}1\\1\end{pmatrix}(1\ 1) = \begin{pmatrix}2&2\\2&2\end{pmatrix}$

$S_2 = ((-2,0)(-2,0)^T + (-1,0)(-1,0)^T + (0,0)(0,0)^T + (1,0)(1,0)^T + (2,0)(2,0)^T)$

$S_2 = \begin{pmatrix}4\\0\end{pmatrix}(4\ 0) + \begin{pmatrix}1\\0\end{pmatrix}(1\ 0) + \begin{pmatrix}0\\0\end{pmatrix}(0\ 0) + \begin{pmatrix}1\\0\end{pmatrix}(1\ 0) + \begin{pmatrix}4\\0\end{pmatrix}(4\ 0)$

$= \begin{pmatrix}34&0\\0&0\end{pmatrix}$

d. $\begin{pmatrix}2&2\\2&2\end{pmatrix} + \begin{pmatrix}34&0\\0&0\end{pmatrix} = \begin{pmatrix}36&2\\2&2\end{pmatrix}$

e. $S_B = (3)\begin{pmatrix}-3.125\\0\end{pmatrix}(-3.125\ 0) + (5)\begin{pmatrix}1.875\\0\end{pmatrix}(1.875\ 0)$

$= \begin{pmatrix}29.297&0\\0&0\end{pmatrix} + \begin{pmatrix}17.578&0\\0&0\end{pmatrix} = \begin{pmatrix}46.875&0\\0&0\end{pmatrix}$

f. $tr(S_W) = 36+2 = 38 \quad tr(S_B) = 46.875$

$\frac{46.875}{38} = 1.235$

4. Confusion Matrix:

		TP	FN
		4	1
FP	1		TN 4

Accuracy = $4+4/10 = 0.8$

Precision = $4/(4+1) = 0.8$

Recall = $4/(4+1) = 0.8$

F1 score = $\frac{2*(0.8*0.8)}{(0.8)+(0.8)} = 0.8$

Specificity = $4/(4+1) = 0.8$