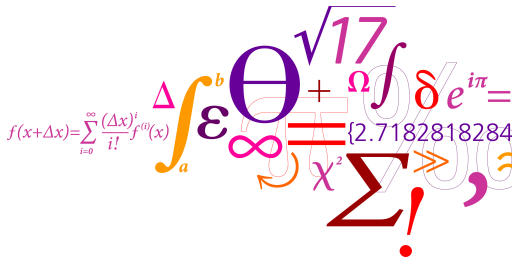


# 02224 Real-time systems

Timed Games, Controller Synthesis and UPPAAL-TIGA

Michael R. Hansen



**DTU Compute**

Department of Applied Mathematics and Computer Science

# Model Checking vs Controllor Synthesis

## Model Checking ( $M \models \phi$ )

- Given system model  $M$  and
- a property  $\phi$ ,

check (automatically) whether the behaviours of  $M$  satisfy  $\phi$ .

## Controller Synthesis

- Given environment model  $S$ ,
- a model  $C$  describing the moves the controller *can* do,
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- Controller continuously observes the system and can perform two actions
  - wait (that is, delay)
  - take a controllable move (thereby preventing delay)
- A 2-player game:  
Controller moves against uncontrollable environment moves
- Reachability games:  $\text{control} : A \langle \rangle \text{Win}$ , where  $\text{Win}$  is a state formula describing the winning states.
- Safety games:  $\text{control} : A[] \text{not Lose}$  where  $\text{Lose}$  is a state formula describing the losing states of the controller.
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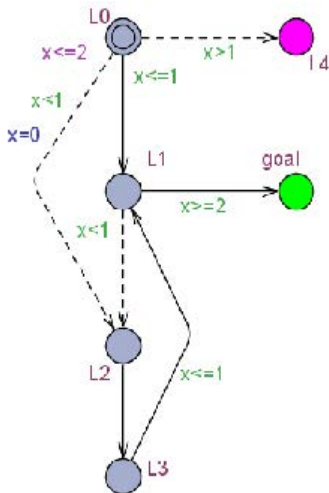
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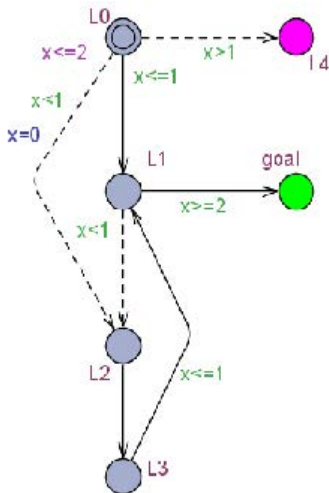
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- Solid transitions are controlled
- dashed are uncontrolled (environment) transitions
- Priority to environment transitions
- Reachability objective:  
*control :  $A \langle \rangle \text{goal}$*
- Control can, in a state, choose to wait (delay) or choose to take a controllable transition
- Reachability and Safety Games are decidable. Memoryless and region-based strategies are sufficient. [AMS98]

Figure: From Uppaal-TIGA tutorial

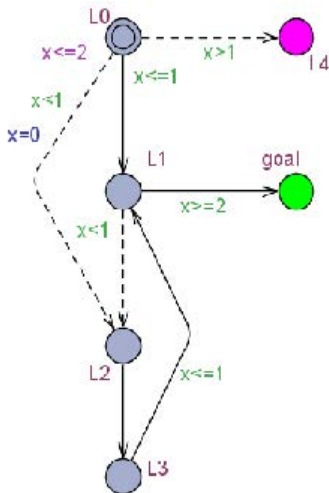
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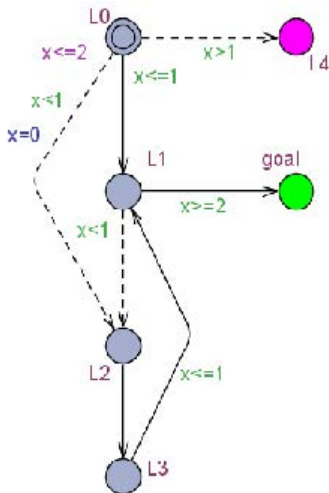
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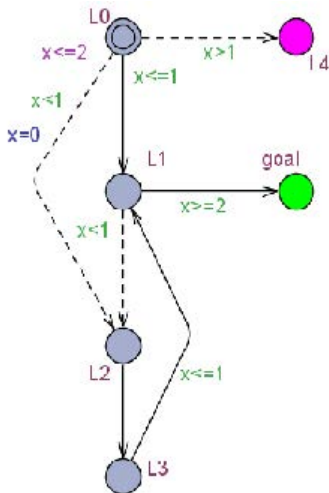
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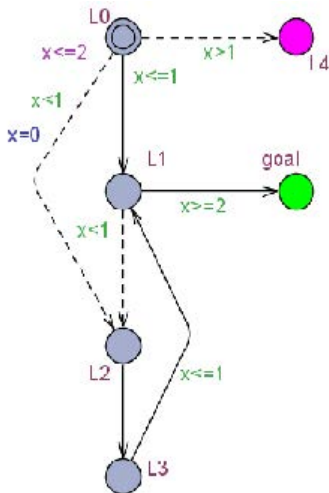
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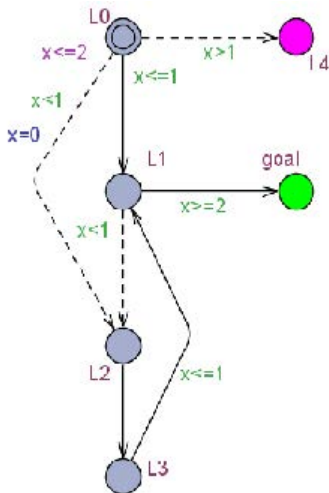


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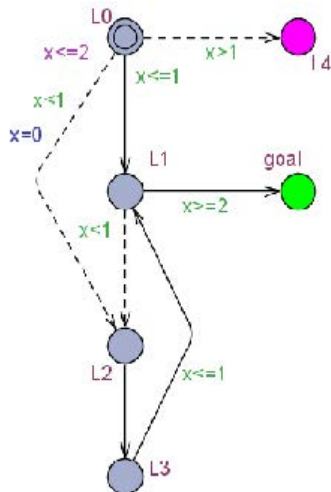
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# Backward computation of Winning States: An example



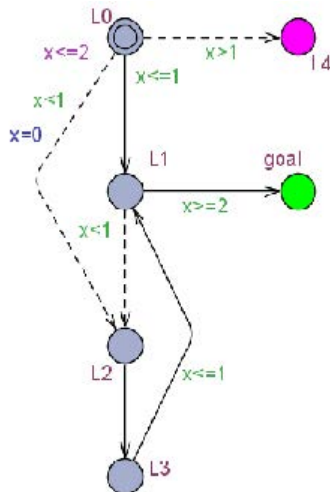
L0 : { }

L1 :  $\{x \mid 1 \leq x\}$

L2 : { }

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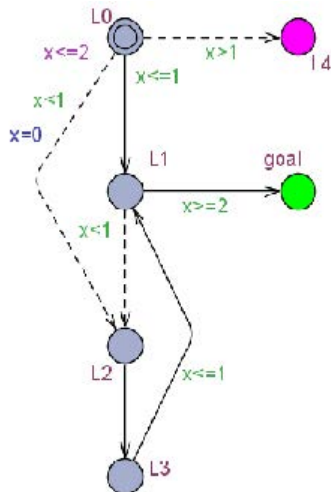
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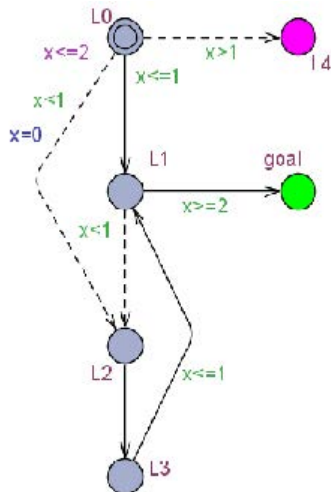
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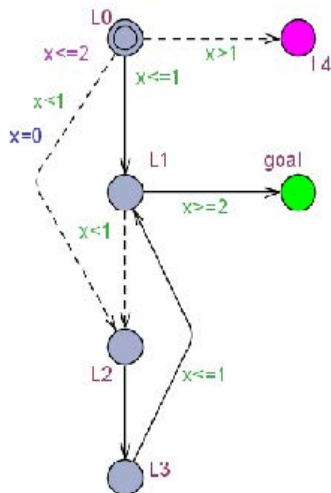
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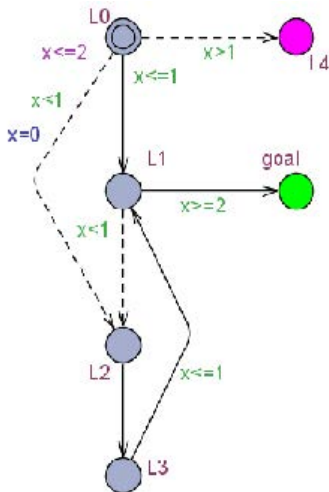
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# Computation of Winning Strategies: An example

Reachability Games `control A <> goal`: Take actions that lead to winning states. (Partition states to guarantee progress)



L0,  $x < 1$  : wait  
 L0,  $x = 1$  : goto L1

L1,  $x < 2$  : wait  
 L1,  $x \geq 2$  : goto goal

L2,  $x \leq 1$  : goto L3

L3,  $x < 1$  : Wait  
 L3,  $x = 1$  : goto L1

## Reachability properties:

- `control: A[p U q]`
- `control: A<> p`

## Safety properties:

- `control: A[p W q]`
- `control: A[] p`



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