$$y' = (x^3 - \sqrt{11x})^{\arcsin(x^2-1)} \cdot \frac{2x}{\sqrt{1-(x^2-1)^2}} \cdot \ln(x^3 - \sqrt{11x}) + \arcsin(x^2-1) \cdot$$

$$\cdot \frac{1}{x^3 - \sqrt{11x}} \cdot \left(3x^2 - \frac{\sqrt{11}}{2\sqrt{x}}\right)$$

12.10.2020

Интегралы и диф. ур.

$$\sin(3xy - 7y) + \frac{x^2+3xy}{y^2} = 2x + xy$$

$$\left(\sin(3xy-7y) + \frac{x^2+3xy}{y^2}\right)'_x = (2x+xy)'_x$$

$$(\sin(3xy-7y))' + \left(\frac{x^2+3xy}{y^2}\right)' = (2x)' + (xy)'$$

$$\cos(3xy-7y) \cdot (3xy-7y)'_x + \frac{(x^2+3xy)'_x \cdot y^2 - (x^2+3xy) \cdot (y^2)'_x}{(y^2)^2} = 2 \cdot (x)'_x +$$

$$+ (x)'_x y + x \cdot y'_x$$

$$\cos(3xy-7y) \cdot (3y + 3xy' - 7y') + \frac{(2x + 3y + 3xy')y^2 - (x^2+3xy) \cdot 2y \cdot y'}{y^4}$$

$$= 2 + y + xy'$$

$$\cos(3xy-7y)(3x-7) \cdot y' + \frac{3xy^2 - 2yx^2 - 6xy^2}{y^4} \cdot y' - xy' = 2 + y - \cos$$

$$- \cos(3xy - 7y) \cdot 3y - \frac{(2x+3y)y^2}{y^4}$$

$$y' = \frac{2 + y - 3y \cdot \cos(3xy-7y) - \frac{2x+3y}{y^2}}{(3x-7) \cdot \cos(3xy-7y) - \frac{3xy+2x^2}{y^3} - x}$$

$$\frac{1}{x} : \frac{1}{x^2} = \frac{1}{x} \cdot \frac{x^2}{1} = \cancel{x}$$

$dy = f'(x)dx$

$d^2y = f''(x)d^2x$

$d^3y = f'''(x)d^3x$

④ $dy - ?$ $\quad y = \dfrac{(x^2+x+1)\cdot 7^x}{(x^3-5)\ln x}$

$y' = \cancel{\left((x^2+x+1)\cdot 7^x\right)' - \left((x^3-5)\ln x\right)}$

$$y' = \frac{\left((x^2+x+1)7^x\right)'\cdot(x^3-5)\ln x - (x^2+x+1)7^x\cdot\left((x^3-5)\ln x\right)'}{\left((x^3-5)\ln x\right)^2} =$$

$$= \frac{\left((2x+1)7^x + (x^2+x+1)7^x\ln 7\right)\cdot(x^3-5)\ln x - 7^x(x^2+x+1)\cdot\left(3x^2\ln x + (x^3-5)\cdot\frac{1}{x}\right)}{(x^3-5)^2\ln^2 x}dx$$

⑤ dy, d^2y, d^3y

1) $y = \sqrt[4]{x}\ln x$

$$y' = \frac{1}{4}\cdot x^{\frac{1}{4}-1}\cdot\ln x + \sqrt[4]{x}\cdot\frac{1}{x} = \left(\frac{\ln x}{4\sqrt[4]{x^3}} + \frac{1}{\sqrt[4]{x^3}}\right)dx =$$

$$= \frac{\ln x + 4}{4\sqrt[4]{x^3}}\ \cancel{dx}; \quad dy = \frac{\ln x + 4}{4\sqrt[4]{x^3}}dx$$

$$y'' = \left(\frac{4+\ln x}{4\sqrt[4]{x^3}}\right)' = \frac{1/x\cdot 4\cdot\sqrt[4]{x^3} - (4+\ln x)\cdot 4\cdot\frac{3}{4}\cdot x^{3/4-1}}{16(x^{3/4})^2} = \frac{\frac{4\sqrt[4]{x^3}}{x} - \frac{3(4\dots}{4\dots}}{16\cdot x^{3/2}}$$

$$= \frac{\frac{4}{\sqrt[4]{x}} - \frac{3(4+\ln x)}{\sqrt[4]{x}}}{16\cdot\sqrt{x^3}} = \frac{4-12-3\ln x}{16\cdot x^{1/4}\cdot x^{3/2}} = \frac{-8-3\ln x}{16\cdot\sqrt[4]{x^7}}$$

$$d^2y = \frac{-8-3\ln x}{16\sqrt[4]{x^7}}d^2x$$

$$y''' = \frac{-8-3\ln x}{16\sqrt[4]{x^7}} = \frac{-3\cdot\frac{1}{x}\cdot 16\, x^{\frac{7}{4}} - (-8-3\ln x)\, 16\cdot\frac{7}{4}\cdot x^{\frac{7}{4}-1}}{256(x^{\frac{7}{4}})^2} =$$

$$= \frac{-48\cdot x^{3/4} + (8+3\ln x)\,4\cdot 7\cdot x^{3/4}}{256\cdot x^{7/2}} = \frac{x^{3/4}\cdot 4\cdot(-12+(8+3\ln x)\cdot 7)}{256\cdot x^{7/2}} =$$

$$= \frac{(44+21\ln x)}{64\cdot\sqrt[4]{x^{11}}}$$

$$d^3y = \frac{44+21\ln x}{64\cdot\sqrt[4]{x^{11}}}\,dx^3$$

⑤ 2) $dy,\ d^2y,\ d^3y$ $\quad y = tg^2(x^5)$

$$y' = (tg^2(x^5))' = 2tg(x^5)\cdot(tg(x^5))' = 2tg(x^5)\cdot\frac{1}{\cos^2(x^5)}\cdot(x^5)' =$$

$$= \frac{10\cdot x^4\cdot tg(x^5)}{\cos^2(x^5)}\ ;\quad dy = \frac{10x^4\, tg(x^5)}{\cos^2(x^5)}\,dx$$

$$\left(\frac{10x^4\cdot tg(x^5)}{\cos^2(x^5)}\right)' = \frac{(10x^4\cdot tg(x^5))'\cdot\cos^2(x^5) - 10x^4\cdot tg(x^5)(\cos^2(x^5))'}{\cos^2(x^5)} =$$

$$= \frac{\cos^2(x^5)\cdot((10x^4)'\cdot tg(x^5) + 10x^4(tg(x^5))')}{\cos^4(x^5)} -$$

$$- \frac{10x^4\cdot tg(x^5)\cdot(-2)\sin(x^5)\cdot 5x^4}{\cos^4(x^5)} = \frac{40x^3\cdot\cos(x^5)\cdot\sin(x^5) +}{\cos^4(x^5)} -$$

$$- \frac{-100x^8\cdot\sin(x^5)\cdot tg(x^5)}{\cos^4(x^5)} = \frac{100x^8\cdot\sin x^5\cdot tg(x^5)}{\cos^4(x^5)} + \frac{50x^8}{\cos^4(x^5)} + \frac{40x^3\, tg(x^5)}{\cos^2(x^5)}$$

$$d^2y = \left(\frac{100x^8\cdot\sin(x^5)\cdot tg(x^5)}{\cos^4(x^5)} + \frac{50x^8}{\cos^4(x^5)} + \frac{40x^3\, tg(x^5)}{\cos^2(x^5)}\right)dx^2$$

$$\left(\frac{100x^8\sin^2(x^5)}{\cos^5(x^5)} + \frac{50x^8}{\cos^4(x^5)} + \frac{40x^3\, tg(x^5)}{\cos^2(x^5)}\right)' =$$

$$= \frac{(100x^8\cdot\sin^2(x^5))'\cdot\cos^5(x^5) - 100x^8\cdot\sin^2(x^5)\cdot(\cos^5(x^5))'}{\cos^{10}(x^5)} + \frac{(50x^8)'\cos^4(x^5)}{\cos^8(x^5)} -$$

$$- \frac{50x^8\cdot(\cos^4(x^5))'}{\cos^8(x^5)} + \frac{(40x^3\, tg(x^5))'\cos^2(x^5) - 40x^3\, tg(x^5)(\cos^2(x^5))'}{\cos^4(x^5)} =$$

$$= \frac{(800x^7\sin^2(x^5)+100x^8\cdot 2\sin(x^5)\cdot\cos(x^5)\cdot 5x^4)\cos^5(x^5)}{\cos^{10}(x^5)} -$$

$$- \frac{100x^8\sin^2(x^5)\cdot 5\cos^4(x^5)\cdot(-\sin(x^5))5x^4}{\cos^{10}(x^5)} + \frac{400x^7\cdot\cos^4(x^5)}{\cos^8(x^5)} -$$

$$- \frac{50x^8\cdot 4\cos^3(x^5)\cdot(-\sin(x^5))\cdot 5x^4}{\cos^8(x^5)} + \frac{\left(120x^2\, tg(x^5)+40x^3\cdot 5x^4\cdot\frac{1}{\cos^2(x^5)}\right)'\cos^2(x^5)}{\cos^4(x^5)}$$

$$- \frac{40x^3\, tg(x^5)\cdot 2\cos(x^5)\cdot 5x^4\cdot(-\sin(x^5))}{\cos^4(x^5)} = \Bigg(\frac{800x^7\sin^2(x^5)+1000x^{12}\cdot\sin(x^5)\cos(x^5)}{\cos^5(x^5)}$$

$$+ \frac{2500x^{16}\sin^2(x^5)\cdot\cos^4(x^5)\cdot\sin(x^5)}{\cos^{10}(x^5)} + \frac{400x^7}{\cos^4(x^5)} + \frac{1000x^{12}\sin(x^5)}{\cos^5(x^5)} +$$

$$+ \frac{120x^2\, tg(x^5) + \frac{200x^7}{\cos^2(x^5)}}{\cos^2(x^5)} + \frac{400x^7\, tg(x^5)\sin(x^5)}{\cos^3(x^5)}\Bigg) dx^3 = d^3y$$

6) 1) $\lim\limits_{x\to 0}\frac{\ln\sin(7x)}{\ln(5x)} = \left[\frac{\infty}{\infty}\right] = \lim\limits_{x\to 0}\frac{(\ln\sin(7x))'}{(\ln(5x))'} =$

$$= \lim_{x\to 0}\frac{\frac{7}{\sin 7x}\cdot\cos 7x}{x} = \lim_{x\to 0}\frac{7x\cos 7x}{\sin 7x} = \left[\frac{0}{0}\right] = \lim_{x\to 0}\frac{(7x\cos 7x)'}{(\sin 7x)'} =$$

$$= \lim_{x\to 0}\frac{7\cdot\cos x - 49x\sin 7x}{7\cos 7x} = \frac{7}{7} = 1$$

2) $\lim\limits_{x \to 0} \dfrac{x^5}{x^2 - \sin x} = \left[\dfrac{0}{0}\right] = \lim\limits_{x \to 0} \dfrac{(x^5)'}{(x^2 - \sin x)'} = \lim\limits_{x \to 0} \dfrac{5x^4}{2x - \cos x} = \dfrac{0}{1} = 0$

4) $\lim\limits_{x \to 1} \left(\dfrac{1}{1-x^3} - \dfrac{1}{1-x^2}\right) = [0-0] = \lim\limits_{x \to 1} \left(\dfrac{1}{(1-x)(1^2+x+x^2)} - \dfrac{1}{(1-x)(1+x)}\right)$

$= \lim\limits_{x \to 1} \left(\dfrac{1+x}{(1-x)(1+x)(1+x+x^2)} - \dfrac{1+x+x^2}{(1-x)(1+x)(1+x+x^2)}\right) = \lim\limits_{x \to 1} \dfrac{x^2}{(1-x)(1+x)(1+x+x^2)}$

$= \dfrac{1}{0} = \infty$

3) $\lim\limits_{x \to 0+0} (x \cdot \ln(x)) = [0 \cdot \infty] = \lim\limits_{x \to 0+0} \dfrac{\ln x}{\frac{1}{x}} = \left[\dfrac{\infty}{\infty}\right] =$

$= \lim\limits_{x \to 0+0} \dfrac{(\ln x)'}{(1/x)'} = \lim\limits_{x \to 0+0} \dfrac{1/x}{-1/x^2} = -\lim\limits_{x \to 0+0} \dfrac{1/x}{1/x^2} = -\lim\limits_{x \to 0+0} x = 0$