28.12.20

8.2.16. $\int \sqrt{9-x^2}\,dx = \left[\begin{matrix} \sqrt{1-x^2} \\ x=\sin t \Rightarrow 1-x^2 = 1-\sin^2 t = \cos^2 t \end{matrix} \right] =$

$= [\, x = 3\sin t \Rightarrow 9-x^2 = 9-(3\sin t)^2 = 9-9\sin^2 t = 9(1-\sin^2 t) = 9\cos^2 t,$

$dx = d(3\sin t) = (3\sin t)'_t\,dt = 3\cos t\,dt \,] = \int \sqrt{9-(3\sin t)^2}\;3\cos t\,dt =$

$= \int \sqrt{9\cos^2 t}\;3\cos t\,dt = \int 3\cos t\;3\cos t\,dt = 9\int \cos^2 t\,dt =$

$= 9\int \frac{1+\cos 2t}{2}\,dt = \frac{9}{2}\int (1+\cos 2t)\,dt = \frac{9}{2}\left(\int dt + \int \cos 2t\,dt\right) =$

$= \frac{9}{2}\left(t + \frac{1}{2}\sin 2t\right) + C = \frac{9}{2}\left(\arcsin\frac{x}{3} + \frac{1}{2}\sin\left(2\arcsin\frac{x}{3}\right)\right) + C =$

$= \cancel{\quad} =$

$= \frac{9}{2}\arcsin\frac{x}{3} + \frac{x}{2}\sqrt{9-x^2} + C$

8.2.17. $\int \frac{dx}{x\sqrt{x+1}} = \left[\begin{matrix} x+1 = t^2 \Rightarrow \\ \Rightarrow x = t^2-1 \Rightarrow dx = d(t^2-1) = 2t\,dt \end{matrix} \right] =$

$= \int \frac{2t\,dt}{(t^2-1)\sqrt{t^2}} = \int \frac{2t\,dt}{t(t^2-1)} = 2\int \frac{dt}{t^2-1} = 2\,\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| + C =$

$= \ln\left|\frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}\right| + C$

8.2.18. $\int x\sqrt{2-x}\,dx = \left[\begin{matrix} 2-x = t^2 \Rightarrow t = \sqrt{2-x} \\ x = 2-t^2 \Rightarrow dx = -2t\,dt \end{matrix} \right] = \int (2-t^2)\sqrt{t^2}\,(-2t\,dt) =$

$= \int (2-t^2)\,t\,(-2)t\,dt = \int (2t(-2)t - t^2 t(-2)t)\,dt = \int (-4t^2+2t^4)\,dt =$

$= -4\int t^2\,dt + 2\int t^4\,dt = -4\frac{t^3}{3} + 2\frac{t^5}{5} + C = \frac{2}{5}(\sqrt{2-x})^5 - \frac{4}{3}(\sqrt{2-x})^3 + C$

8.2.19

$$\int \frac{\sqrt{x}\,dx}{x+16} = \left[\begin{matrix} x = t^2 \Rightarrow t = \sqrt{x} \\ dx = 2t\,dt \end{matrix}\right] = \int \frac{\sqrt{t^2}\,2t\,dt}{t^2+16} = \int \frac{t\,2t\,dt}{t^2+16} = 2\int \frac{t^2\,dt}{t^2+16}$$

$$= 2\int \frac{(t^2+16)-16}{t^2+16}\,dt = 2\int \left(\frac{t^2+16}{t^2+16} - \frac{16}{t^2+16}\right)dt = 2\left(\int dt - 16\int \frac{dt}{t^2+16}\right) =$$

$$= 2\left(t - 16\,\frac{1}{4}\operatorname{arctg}\frac{t}{4}\right) + C = 2t - 8\operatorname{arctg}\frac{t}{4} + C = 2\sqrt{x} - 8\operatorname{arctg}\frac{\sqrt{x}}{4} + C$$

8.2.21 $\int x\sin x\,dx = \left[\begin{matrix} u = x \Rightarrow u' = x' = 1 \\ v' = \sin x \Rightarrow v = -\cos x \end{matrix}\right] = x(-\cos x) - \int(-\cos x)\cdot 1\,dx$

$= -x\cos x + \int \cos x\,dx = -x\cos x + \sin x + C$

18.12.2020

8.2.22 $\int (2x-1)\,e^{3x}\,dx = \left[\begin{matrix} u = 2x-1 \Rightarrow u' = 2 \\ v' = e^{3x} \Rightarrow v = \int e^{3x}\,dx \end{matrix}\right] = \left[\begin{matrix} \int e^{3x}\,dx;\ t = 3x \\ dt = d(3x) = 3\,dx \Rightarrow dx = \frac{1}{3}dt \end{matrix}\right]$

$\left[\int e^{t}\,\frac{1}{3}\,dt = \frac{1}{3}\int e^t\,dt = \frac{1}{3}e^t + C\right] = \ldots \frac{1}{\ldots}\,{}^{3x} \int \frac{\ldots}{\ldots}\,{}^{3x}$