26.10.20

11.3.18 $\quad 1{,}07^{3{,}97}$

$f(x;y) = x^y$, $x = 1{,}07$, $y = 3{,}97$

$x_0 = 1 \qquad \Delta x = x - x_0 = 1{,}07 - 1 = 0{,}07$

$y = 4 \qquad \Delta y = y - y_0 = 3{,}97 - 4 = -0{,}03$

$f(1;4) = 1^4 = 1$

$f(x;y) = f(x_0 + \Delta x; y_0 + \Delta y) \approx f(x_0;y_0) + f'_x(x_0;y_0)\Delta x + f'_y(x_0;y_0)\Delta y$

1) $f(x_0;y_0) = f(1;4) = 1^4 = 1$

2) $f'_x(x_0;y_0) = (x^y)'_x \Big|_{(x_0;y_0)} = y \cdot x^{y-1} \Big| = 4 \cdot 1^3 = 4$

3) $f'_y(x_0;y_0) = (x^y)'_y \Big|_{(x_0;y_0)} = x^y \cdot \ln x \Big|_{(x_0;y_0)} = 1^4 \cdot \ln 1 = 0$

4) $f(1{,}07; 3{,}97) \approx 1 + 4 \cdot 0{,}07 + 0 \cdot (-0{,}03) = 1 + 0{,}28 = 1{,}28$

$1{,}07^{3{,}97} \approx 1{,}28$

11.3.19 $\quad 1{,}04^{2{,}03}$

$f(x;y) = x^y$

$x_0 = 1 \qquad \qquad \Delta x = 0{,}04$

$\Rightarrow$

$y_0 = 2 \qquad \qquad \Delta y = 0{,}03$

11.3.21.

$\sin 28^\circ \cos 61^\circ$

$f(x;y) = \sin(x)\cos(y)$

$x = 28 \quad x_0 = 30 \quad \Delta x = -2$

$y = 61 \quad y_0 = 60 \quad \Delta y = 1$

$f(x_0;y_0) = \sin 30 \cos 60 = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

$f'_x(x_0;y_0) = (\sin(x)\cos(y))'_x = \cos(x)\cos(y)\big|_{(x_0;y_0)} = \cos 30 \cos 60 =$

$f'_y(x_0;y_0) = (\sin(x)\cos(y))'_y = -\sin(x)\sin(y)\big|_{(x_0;y_0)} = -\sin 30 \sin 60 = \frac{\sqrt{3}}{4}$

$f(x;y) \approx f(x_0;y_0) + f'_x(x_0;y_0)\Delta x + f'_y(x_0;y_0)\Delta y = \frac{1}{4} + \frac{\sqrt{3}}{4}(-2) + \frac{\sqrt{3}}{4} \cdot 1$

$= \frac{1}{4} - \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{4} = \frac{\sqrt{3} + 1 - 2\sqrt{3}}{4} = \frac{1 - \sqrt{3}}{4}$

11.3.22.

$\sqrt{(\sin^2 1{,}55 + 8 \cdot e^{0{,}015})^5}$

$f(x;y) = \sqrt{(\sin^2 x + 8e^y)^5} = (\sin^2 x + 8e^y)^{5/2}$

1) $f'_x(x;y) = ((\sin^2 x + 8e^y)^{\frac{5}{2}})'_x = \frac{5}{2} \cdot ((\sin^2 x + 8e^y)^{\frac{3}{2}}) \cdot (2\sin x \cos x) =$

$\frac{5}{2} \cdot 2\sin x \cos x (\sin^2 x + 8e^y)^{3/2} = \frac{5}{2} \sin(2x)(\sin^2 x + 8e^y)^{3/2}$

2) $f'_y(x;y) = ((\sin^2 x + 8e^y)^{5/2})'_y = \frac{5}{2}((\sin^2 x + 8e^y)^{3/2}) \cdot (0 + 8e^y) =$

$\frac{5}{2} \cdot 8e^y(\sin^2 x + 8e^y)^{3/2} = 20e^y(\sin^2 x + 8e^y)^{3/2}$

3) $x = 1{,}55 \qquad x_0 = \frac{\pi}{2} \approx \frac{3{,}1415}{2} \approx 1{,}571 \qquad \Delta x = 1{,}55 - 1{,}571 = -0{,}021$

$y = 0{,}015 \qquad y_0 = 0 \qquad \Delta y = y - y_0 = 0{,}015 - 0 = 0{,}015$

4) $f(x_0; y_0) = f\left(\frac{\pi}{2}; 0\right) = (\sin^2\frac{\pi}{2} + 8e^0)^{5/2} = (1^2 + 8\cdot 1)^{5/2} = 9^{5/2} = 3^5 = 243$

5) $f'_x(x_0, y_0) = \left(\frac{5}{2}\sin(2x)(\sin^2 x + 8e^y)^{3/2}\right)\Big|_{(\frac{\pi}{2}; 0)} = \left(\frac{5}{2}\sin(2\cdot\frac{\pi}{2})(\sin^2\frac{\pi}{2} + 8e^0)\right)^{3/2} =$

$= \frac{5}{2}\cdot 0\,(1 + 8\cdot 1)^{3/2} = 0$

6) $f'_y(x_0, y_0) = \left(20e^y(\sin^2 x + 8e^y)^{3/2}\right)\Big|_{(\frac{\pi}{2}; 0)} = 20\cdot e^0(\sin^2\frac{\pi}{2} + 8e^0)^{3/2} = 20(1+8)^{3/2} =$

$= 20\cdot 9^{3/2} = 20\cdot 27 = 540$

7) $f(x; y) \approx f(x_0; y_0) + f'_x(x_0; y_0)\Delta x + f'_y(x_0; y_0)\Delta y = 243 + 0\cdot(-0{,}021) + 540\cdot 0{,}015$

$= 243 + 540\cdot 0{,}015 = 243 + 8{,}1 = 251{,}1$

$\sqrt{(\sin^2(1{,}55) + 8e^{0{,}015})^5} \approx 251{,}1$

$f(x; y; z) \approx f(x_0; y_0, z_0) + f'_x(x_0; y_0; z_0)\Delta x + f'_y(x_0; y_0, z_0)\Delta y + f'_z(x_0; y_0; z_0)\Delta z$