6) 2) $\lim\limits_{x\to0} \frac{x^5}{x^2-\sin x} = \left[\frac{0}{0}\right] = \lim\limits_{x\to0}\frac{(x^5)'}{(x^2-\sin x)'} = \lim\limits_{x\to0}\frac{5x^4}{2x-\cos x} = \frac{0}{1} = 0$

4) $\lim\limits_{x\to1}\left(\frac{1}{1-x^3}-\frac{1}{1-x^2}\right) = [0-0] = \lim\limits_{x\to1}\left(\frac{1}{(1-x)(1+x+x^2)} - \frac{1}{(1-x)(1+x)}\right)$

$= \lim\limits_{x\to1}\left(\frac{1+x}{(1-x)(1+x)(1+x+x^2)} - \frac{1+x+x^2}{(1-x)(1+x)(1+x+x^2)}\right) = \lim\limits_{x\to1}\frac{x^2}{(1-x)(1+x)(1+x+x^2)}$

$= \frac{1}{0} = \infty$

3) $\lim\limits_{x\to0+0}(x\cdot\ln(x)) = [0\cdot\infty] = \lim\limits_{x\to0+0}\frac{\ln x}{\frac{1}{x}} = \left[\frac{\infty}{\infty}\right] =$

$= \lim\limits_{x\to0+0}\frac{(\ln x)'}{(1/x)'} = \lim\limits_{x\to0+0}\frac{1/x}{-1/x^2} = -\lim\limits_{x\to0+0}\frac{1/x}{1/x^2} = -\lim\limits_{x\to0+0} x = 0$

19.10.2020

Частные приращения:

1) $\Delta_x z = f(x+\Delta x; y) - f(x,y)$

2) $\Delta_y z = f(x; y+\Delta y) - f(x,y)$

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Полное приращение:

$\Delta z = f(x+\Delta x, y+\Delta y) - f(x,y)$

$\Delta z \neq \Delta_x z + \Delta_y z$

Частная производная

$z'_x = \lim\limits_{\Delta x\to0}\frac{\Delta_x z}{\Delta x}$; $\frac{\partial z}{\partial x}$; $\frac{\partial f}{\partial x}(x,y)$; $\frac{\partial}{\partial x} z$; $\frac{\partial}{\partial x} f$; $\frac{\partial}{\partial x} f(x,y)$

$z'_y = \lim\limits_{\Delta y\to0}\frac{\Delta_y z}{\Delta y}$

11.3.1. $z = xy^2 - \frac{x}{y}$ $\Delta_x z, \Delta_y z, \Delta z$ - ?

$M_0(3; -2)$; $\Delta x = 0,1$; $\Delta y = -0,05$

1) $M_0(3; -2)$

$\exists\, x_0 = 3,\ y_0 = -2$ тогда:

$x = x_0 + \Delta x = 3 + 0,1 = 3,1$

$y = y_0 + \Delta x = -2 + (-0,05) = -2,05$

Тогда $M_1(3,1; -2,05)$

2) $z(M_0) = z(3; -2) = \left[z = xy^2 - \frac{x}{y} \right] = 3 \cdot (-2)^2 - \frac{3}{-2} = 3 \cdot 4 + \frac{3}{2} = 13,5$

$z(x_0 + \Delta x; y_0) = z(3,1; -2) = 3,1 \cdot (-2)^2 - \frac{3,1}{-2} = 3,1 \cdot 4 + \frac{3,1}{2} =$

$= 12,4 + 1,55 = 13,95$

$z(x_0; y_0 + \Delta y) = z(3; -2,05) = 3 \cdot (-2,05)^2 - \frac{3}{-2,05} = 3 \cdot 4,2025 + \frac{3}{2,05}$

$\approx 12,6075 + 1,4634 = 14,0709 \approx 14,07$

$z(M_1) = z(x; y) = z(3,1; -2,05) = 3,1 \cdot (-2,05)^2 - \frac{3,1}{-2,05} =$

$3,1 \cdot 4,2025 + \frac{3,1}{2,05} \approx 13,0278 + 1,5122 = 14,54$

3) $\Delta_x z = z(x_0 + \Delta x; y_0) - z(x_0; y_0) = 13,95 - 13,5 = 0,45$

$\Delta_y z = z(x_0; y_0 + \Delta y) - z(x_0; y_0) = 14,07 - 13,5 = 0,57$

4) $\Delta z = z(x_0 + \Delta x; y_0 + \Delta y) - z(x_0; y_0) = 14,54 - 13,5 = 1,04$

11.3.2.

$z = x^2 y$; $M_0(1;2)$; $\Delta x = 0,1$; $\Delta y = -0,2$

1) $M_0(1;2) \Rightarrow x_0 = 1, y_0 = 2 \Rightarrow x = x_0 + \Delta x = 1 + 0,1 = 1,1$; $y = y_0 + \Delta y = 2 + (-0,2) = 1,8$

2) $z(x_0; y_0) = z(1;2) = 1^2 \cdot 2 = 2$

$z(x_0 + \Delta x; y_0) = z(1,1; 2) = (1,1)^2 \cdot 2 = 1,21 \cdot 2 = 2,42$

$z(x_0; y_0 + \Delta y) = z(1; 1,8) = 1^2 \cdot 1,8 = 1,8$

$z(x_0 + \Delta x; y_0 + \Delta y) = z(1,1; 1,8) = 1,1^2 \cdot 1,8 = 1,21 \cdot 1,8 = 2,178$

3) $\Delta_x z = z(x_0 + \Delta x; y_0) - z(x_0, y_0) = 2,42 - 2 = 0,42$

$\Delta_y z = z(x_0; y_0 + \Delta y) - z(x_0; y_0) = 1,8 - 2 = -0,2$

4) $\Delta z = z(x_0 + \Delta x; y_0 + \Delta y) - z(x_0; y_0) = 2,178 - 2 = 0,178$

Дифференциал функции

$dz = \underbrace{z'_x dx}_{d_x z} + \underbrace{z'_y dy}_{d_y z}$ $\qquad dx = \Delta x \qquad dy = \Delta y$

полный диф-ал $\qquad$ частные диф-ы

$z'_x = f'_x(x;y)$

$z'_y = f'_y(x;y)$

$f(x_0 + \Delta x; y_0 + \Delta y) \approx f(x_0; y_0) + f'_x(x_0; y_0) \cdot \Delta x + f'_y(x_0; y_0) \cdot \Delta y$

линеаризация функции $z = f(x;y)$ в окрестности точки $M_0(x_0; y_0)$

11.3.9

$z = \frac{x}{y^3} + \frac{y}{x^3} - \frac{1}{6x^2 y}$ $\qquad z'_x - ?$ $\quad z'_y - ?$

$z'_x = \left(\frac{x}{y^3} + \frac{y}{x^3} - \frac{1}{6x^2 y}\right)' = [y = const] = \frac{1}{y^3} \cdot (x)'_x + y \cdot (x^{-3})'_x -$

$\frac{1}{6y} \cdot (x^{-2})'_x = \frac{1}{y^3} + y(-3)x^{-4} - \frac{1}{6y} \cdot (-2) \cdot x^{-3} = \frac{1}{y^3} - \frac{3y}{x^4} + \frac{1}{3x^3 y}$

$$z'_y = \left(\frac{x}{y^3} + \frac{y}{x^3} - \frac{1}{6x^2y}\right)'_y = [x = const] = x \cdot (y^{-3})'_y + \frac{1}{x^3} \cdot (y)'_y - \frac{1}{6x^2} \cdot (y^{-1})'_y$$

$$= x \cdot (-3) y^{-4} + \frac{1}{x^3} - \frac{1}{6x^2} \cdot (-1) y^{-2} = -\frac{3x}{y^4} + \frac{1}{x^3} + \frac{1}{6x^2y^2}$$

11.3.10. $z = \dfrac{x^2 - 2xy}{y^2 + 2xy + 1}$ $\quad z'_x, z'_y - ?$

$$z'_x = \left(\frac{x^2 - 2xy}{y^2 + 2xy + 1}\right)'_x = \frac{(x^2 - 2xy)'_x (y^2 + 2xy + 1) - (x^2 - 2xy)(y^2 + 2xy + 1)'_x}{(y^2 + 2xy + 1)^2} =$$

$$= \frac{(2x - 2y)(y^2 + 2xy + 1) - (x^2 - 2xy)2y}{(y^2 + 2xy + 1)^2}$$

$$z'_y = \left(\frac{x^2 - 2xy}{y^2 + 2xy + 1}\right)'_y = \frac{(x^2 - 2xy)'_y (y^2 + 2xy + 1) - (x^2 - 2xy)(y^2 + 2xy + 1)'}{(y^2 + 2xy + 1)^2} =$$

$$= \frac{-2x(y^2 + 2xy + 1) - (x^2 - 2xy)(2y + 2x)}{(y^2 + 2xy + 1)^2}$$

11.3.16 $z = \cos \dfrac{x^2 + y^2}{x^3 + y^3}$ $\quad z'_x, z'_y, d_x z, d_y z, dz - ?$

1) $z'_x = \left[\dfrac{\partial z}{\partial x}\right] = \left(\cos \dfrac{x^2 + y^2}{x^3 + y^3}\right)'_x = -\sin \dfrac{x^2 + y^2}{x^3 + y^3} \cdot \left(\dfrac{x^2 + y^2}{x^3 + y^3}\right)'_x = -\sin \dfrac{x^2 + y^2}{x^3 + y^3} \cdot$

$$\cdot \frac{(x^2 + y^2)'_x (x^3 + y^3) - (x^2 + y^2)(x^3 + y^3)'_x}{(x^3 + y^3)^2} = -\sin \frac{x^2 + y^2}{x^3 + y^3} \cdot$$

$$\cdot \frac{2x(x^3 + y^3) - 3x(x^2 + y^2)}{(x^3 + y^3)^2} = \frac{3x(x^2 + y^2) - 2x(x^3 + y^3)}{(x^3 + y^3)^2} \sin \frac{x^2 + y^2}{x^3 + y^3}$$

$$z'_y = \left[\frac{\partial z}{\partial y}\right] = \left(\cos \frac{x^2 + y^2}{x^3 + y^3}\right)' = \frac{3y^2(x^2 + y^2) - 2y(x^3 + y^3)}{(x^3 + y^3)^2} \sin \frac{x^2 + y^2}{x^3 + y^3}$$

2) $d_x z = z'_x dx = \frac{3x^2(x^2+y^2) - 2x(x^3+y^3)}{(x^3+y^3)^2} \sin \frac{x^2+y^2}{x^3+y^3} dx$

$d_y z = z'_y dy = \frac{3y^2(x^2+y^2) - 2y(x^3+y^3)}{(x^3+y^3)^2} \sin \frac{x^2+y^2}{x^3+y^3} dy$

3) $dz = d_x z + d_y z = \frac{3x^2(x^2+y^2) - 2x(x^3+y^3)}{(x^3+y^3)^2} \sin \frac{x^2+y^2}{x^3+y^3} dx +$

$+ \frac{3y^2(x^2+y^2) - 2y(x^3+y^3)}{(x^3+y^3)^2} \sin \frac{x^2+y^2}{x^3+y^3} dy =$

$= \frac{1}{(x^3+y^3)^2} \cdot \sin \frac{x^2+y^2}{x^3+y^3} \left((3x^2(x^2+y^2) - 2x(x^3+y^3)) dx + (3y^2(x^2+y^2) - 2y(x^3+y^3)) dy \right)$

11.3.17 $u = \frac{x}{\sqrt{y^2+z^2}}$; du - ?

$du = u'_x dx + u'_y dy + u'_z dz$

$u'_x = \left(\frac{x}{\sqrt{y^2+z^2}} \right)'_x = \frac{1}{\sqrt{y^2+z^2}}$

$u'_y = \left(\frac{x}{\sqrt{y^2+z^2}} \right)'_y = x \cdot \left((y^2+z^2)^{-\frac{1}{2}} \right)'_y = -\frac{1}{2} x (y^2+z^2)^{-3/2} \cdot 2y =$

$= \frac{-xy}{\sqrt{(y^2+z^2)^3}}$

$u'_z = \left(\frac{x}{\sqrt{y^2+z^2}} \right)'_z = \frac{-xz}{\sqrt{(y^2+z^2)^3}}$

$du = \frac{1}{\sqrt{y^2+z^2}} dx + \frac{-xy}{\sqrt{(y^2+z^2)^3}} dy + \frac{-xz}{\sqrt{(y^2+z^2)^3}} = \frac{dx}{\sqrt{y^2+z^2}} - \frac{xy\,dy + xz\,dz}{\sqrt{(y^2+z^2)^3}}$