23.11.20

8.2.15 $\int \frac{dx}{\sqrt{16-9x^2}}$

1 способ: $\int \frac{dx}{\sqrt{4^2-(3x)^2}} = \left[\int \frac{dx}{\sqrt{a^2-x^2}},\ y = 3x \Rightarrow x = \frac{y}{3} = dx = d\left(\frac{y}{3}\right) = \right.$

$\left. = \left(\frac{y}{3}\right)' dy = \frac{1}{3} y' dy = \frac{1}{3} dy \right] = \int \frac{1/3\, dy}{\sqrt{4^2-y^2}} = \frac{1}{3} \int \frac{dy}{\sqrt{4^2-y^2}} = \frac{1}{3} \cdot \left(\arcsin \frac{y}{4} + C\right) =$

$= \frac{1}{3}\left(\arcsin \frac{3x}{4} + C\right) = \frac{1}{3} \arcsin \frac{3x}{4} + C$

2 способ:

$\int \frac{dx}{\sqrt{16-9x^2}} = \int \frac{dx}{\sqrt{4^2-(3x)^2}} = \left[\int f(ax+b)\,dx = \frac{1}{a} F(ax+b) + C,\ a \neq 0; \right.$

$\left. f(ax+b) = \frac{1}{\sqrt{4^2-(3x)^2}} \right] = \frac{1}{3} \cdot \arcsin\left(\frac{3x}{4}\right) + C$

8.2.22 1) $\int \sin^2 x\, dx = \left[\sin^2 x = \frac{1-\cos 2x}{2} \right] = \int \frac{1-\cos 2x}{2}\, dx =$

$= \int \left(\frac{1}{2} - \frac{1}{2}\cos 2x\right) dx = \int \frac{1}{2}\, dx - \int \frac{1}{2} \cos 2x\, dx = \frac{1}{2} \int dx - \frac{1}{2} \int \cos 2x\, dx =$

$= \left[\int \cos x\, dx = \sin x + C;\ f(ax+b)\,dx = \frac{1}{a} F(ax+b) + C;\ ax+b = 2x+0 \right] =$

$= \frac{1}{2} x - \frac{1}{2} \cdot \frac{1}{2} \cdot \sin(2x) + C = \frac{1}{2} x - \frac{1}{4} \sin 2x + C$

2) $\int \frac{x^2}{x^2+1}\, dx = \int \frac{x^2+1-1}{x^2+1}\, dx = \int \left(\frac{x^2+1}{x^2+1} - \frac{1}{x^2+1} \right) dx = \int \left(1 - \frac{1}{x^2+1}\right) dx =$

$= \int 1\, dx - \int \frac{dx}{x^2+1^2} = x - \frac{1}{1} \operatorname{arctg} \frac{x}{1} + C = x - \operatorname{arctg} x + C$

Основные методы интегрирования

① Метод подстановки (замена переменной)

$\int f(\varphi(x)) \cdot \varphi'(x)\,dx$, $\varphi'(x)$ и $f(x)$ - непрерывны на интервале

замена $t = \varphi(x) \Rightarrow \int f(\varphi(x)) \cdot \varphi'(x)\,dx = \int f(t)\,dt$

② Метод подстановки v2.0

$\int f(x)\,dx$ замена $x = \psi(t) \Rightarrow \int f(x)\,dx = \int f(\psi(t)) \cdot \psi'(t)\,dt$

③ Интегрирование по частям (метод стрелок)

$u(x)$, $v(x)$ - непрерывны на интервале, $\exists\, u'(x)$, $v'(x)$

$$\int u v'\,dx = uv - \int v u'\,dx$$

$$\int u\,dv = uv - \int v\,du$$

$$\int \underset{\downarrow \atop F(x)}{f(x)} \cdot \underset{\downarrow \atop g'(x)}{g(x)}\,dx = F(x)\,g(x) - \int F(x)\,g'(x)\,dx$$

Решение задач:

821. ① $\int (7x-1)^{23}\,dx = \left[t = 7x-1 \Rightarrow dt = d(7x-1) = (7x-1)'_x\,dx = 7\,dx \Rightarrow dx = \frac{1}{7}\,dt \right] = \int t^{23}\,\frac{1}{7}\,dt = \frac{1}{7}\int t^{23}\,dt = \left[\int x^2\,dx\right] = \frac{1}{7} \cdot \frac{t^{24}}{24} + C =$

$= \frac{t^{24}}{168} + C = \frac{(7x-1)^{24}}{168} + C$

② $\int x^2 \sin(x^3+1)\,dx = \Big[\, t = x^3+1 \Rightarrow dt = d(x^3+1) = (x^3+1)'_x\,dx = 3x^2\,dx$

$\Rightarrow \cancel{dx = \frac{dt}{3x^2}}\; x^2\,dx = \frac{1}{3}\,dt \,\Big] = \int \sin t \cdot \frac{1}{3}\,dt = \frac{1}{3}\int \sin t\,dt =$

$= -\frac{1}{3}\cos t + C = -\frac{1}{3}\cos(x^3+1) + C$

③ $\int \frac{x\,dx}{x^2+1} = \Big[\, t = x^2+1 \Rightarrow dt = d(x^2+1)' = (x^2+1)'_x\,dx = 2x\,dx \Rightarrow x\,dx = \frac{1}{2}dt \,\Big]$

$= \int \frac{1/2\,dt}{t} = \frac{1}{2}\int \frac{dt}{t} = \frac{1}{2}\ln|t| + C = \frac{1}{2}\ln|x^2+1| + C$