09.11.20

11.4.20 $\quad 2x^2-3y^2+5xy-y^3x+x^5=37$

$(x_0;y_0)=(2;-3)$

1) $y=y(x)$ - ф-ия в окрестности точки $x=2$, неявная, задана ур-ем?

2) если для п 1) „да", тогда $y'(x)$, $y'(x_0=2)$

Решение:

1) $f(x;y)=2x^2-3y^2+5xy-y^3x+x^5-37$

$f(x_0;y_0)=f(2;-3)=2\cdot 2^2-3\cdot(-3)^2+5\cdot 2\cdot(-3)-(-3)^3\cdot 2+2^5-37=$

$=8-27-30+54+32-37=8-8=0$

$f'_x=(2x^2-3y^2+5xy-y^3x+x^5-37)'_x=4x-y^3+5x^4+5y$

$f'_x(x_0;y_0)=f'_x(2;-3)=(4x+5y-y^3+5x^4)\Big|_{(2;-3)}=$

$=4\cdot 2+5\cdot(-3)-(-3)^3+5\cdot 2^4=8-15-(-27)+5\cdot 16=100$

$f'_y=(2x^2-3y^2+5xy-y^3x+x^5-37)'_y=-6y+5x-3y^2x$

$f'_y(x_0;y_0)=f'_y(2;-3)=-6y+5x-3y^2x\Big|_{(2;-3)}=-6(-3)+5\cdot 2-3(-3)^2\cdot 2$

$=18+10-54=-26$

т.к. $f'_y(x_0;y_0)=-26\neq 0 \Rightarrow \exists$ неявная ф-ия $y=y(x)$, дифф.

в некоторой окрестности точки $x_0=2$. Тогда $y'(x)=-\frac{F'_x}{F'_y}=$

$=-\frac{4x+5y-y^3+5x^4}{-6y+5x-3y^2x}=\frac{4x+5y-y^3+5x^4}{6y-5x+3y^2x}$

$y'(x_0) = -\dfrac{F'_x(x_0;y_0)}{F'_y(x_0;y_0)} = -\dfrac{100}{-26} = \dfrac{50}{13}$

11.4.21. $\ell: -8x^2+xy^2+2x^3y-7=0$; $\ell \cap x=1$ = в нескольких точках

1) $F(x;y) = -8x^2+xy^2+2x^3y-7$

дано: $x=1$, тогда подставим в $F(x;y)$

$F(1;y) = -8\cdot 1^2 + 1\cdot y^2 + 2\cdot 1^3 y - 7 = y^2+2y-15$

То есть $y^2+2y-15=0$

$\left.\begin{array}{l} y_1+y_2=-2 \\ y_1 y_2 = -15 \end{array}\right| \Rightarrow \begin{array}{l} y_1=-5 \\ y_2=3 \end{array}$

Тогда ур-ие $F(x;y)=0$ определяет в окрестности $x=1$ две функции

Проверим: $\dfrac{\partial F}{\partial y} = (-8x^2+xy^2+2x^3y-7)'_y = 0 + x\cdot 2y + 2x^3\cdot 1 - 0 = 2xy+2x^3$

Имеем две точки: $(1;3)$ и $(1;-5)$. Подставим в $\dfrac{\partial F}{\partial y}$

$\dfrac{\partial F}{\partial y}(1;3) = (2xy+2x^3)\big|_{(1;3)} = 2\cdot 1\cdot 3 + 2\cdot 1^3 = 8$

$\dfrac{\partial F}{\partial y}(1;-5) = (2xy+2x^3)\big|_{(1;-5)} = 2\cdot 1\cdot(-5) + 2\cdot 1^3 = -8$

Т.к. $\dfrac{\partial F}{\partial y}(1;3) \neq \dfrac{\partial F}{\partial y}(1;-5)$, то $F(x;y)$ опред. 2 однознач. ф-ции

$y=y_1(x)$ и $y=y_2(x)$

$y_1(1)=3 \quad y_2(1)=-5$

2) $\frac{\partial F}{\partial x} = F'_x(x;y) = (-8x^2 + xy^2 + 2x^3y - 7)'_x = -8\cdot 2x + y^2\cdot 1 + 2\cdot 3x^2y - 0 =$

$= -16x + y^2 + 6x^2y$

$\frac{\partial F}{\partial x}(1;3) = (-16x + y^2 + 6x^2y)\Big|_{(1;3)} = -16\cdot 1 + 3^2 + 6\cdot 1^2\cdot 3 = -16 + 9 + 18 = 11$

$\frac{\partial F}{\partial x}(1;-5) = (-16x + y^2 + 6x^2y)\Big|_{(1;-5)} = -16\cdot 1 + (-5)^2 + 6\cdot 1^2\cdot(-5) = -21$

$y'_1(1) = -\frac{F'_x(1;3)}{F'_y(1;3)} = -\frac{11}{8}$ | $y'_2(1) = -\frac{F'_x(1;-5)}{F'_y(1;-5)} = -\frac{-21}{-8} = -\frac{21}{8}$

3) Касательная t_1 к $y_1(x)$ в т. $(1;3)$

$y'_1(x) = y'_1(1) = -\frac{11}{8} = tg\,\varphi$, где $\varphi = t\,^\wedge y_1$

t_1: $y - y_0 = k(x - x_0)$, где $k = tg\,\varphi$

Т.е. $t_1: y - 3 = -\frac{11}{8}(x-1) \quad |\cdot 8$

$8y - 24 = -11x + 11$

$8y - 24 + 11x - 11 = 0$

$8y + 11x - 35 = 0$

$11x + 8y - 35 = 0$

Касательная t_2 к $y_2(x)$ в т. $(1;-5)$

$y'_2(x) = y'_2(1) = -\frac{21}{8} = tg\,\alpha$, где $\alpha = t_2\,^\wedge y_2$

t_2: $y - y_0 = k_2(x - x_0)$, где $k_2 = tg\,\alpha$

Т.е. $t_2: y - (-5) = -\frac{21}{8}(x-1)$

$21x + 8y + 19 = 0$

11.4.33 $z = z(x;y): z^3+3x^2y+xz+y^2z^2+y-2x=0$

$\frac{\partial z}{\partial x}; \frac{\partial z}{\partial y}; dz = ?$

1 способ: Th 11.12: $\frac{\partial z}{\partial x} = -\frac{F'_x}{F'_z}$, $\frac{\partial z}{\partial y} = -\frac{F'_y}{F'_z}$

$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$

$F(x;y;z) = z^3+3x^2y+xz+y^2z^2+y-2x$

$F'_x(x;y;z) = (z^3+3x^2y+xz+y^2z^2+y-2x)'_x = 6xy+z-2$

$F'_y(x;y;z) = (z^3+3x^2y+xz+y^2z^2+y-2x)'_y = 3x^2+2yz^2+1$

$F'_z(x;y;z) = (z^3+3x^2y+xz+y^2z^2+y-2x)'_z = 3z^2+x+2y^2z$

$\frac{\partial z}{\partial x} = -\frac{6xy+z-2}{3z^2+x+2y^2z}$; $\frac{\partial z}{\partial y} = -\frac{3x^2+2yz^2+1}{3z^2+x+2y^2z}$

$dz = \frac{2-6xy-z}{3z^2+x+2y^2z}dx - \frac{3x^2+2yz^2+1}{3z^2+x+2y^2z}dy$

2 способ: $(z^3)'_x + (3x^2y)'_x + (xz)'_x + (y^2z^2) + (y)'_x - (2x)'_x = 0$

$z(x;y)$

$3x^2+3\cdot 2xy+(x'_x z + x\cdot z'_x) + y^2\cdot 2z\cdot z'_x + 0 - 2 = 0$

$3x^2+6xy+z+x\cdot z'_x+2y^2z\cdot z'_x - 2 = 0$

$z'_x(3z^2+x+2y^2z) = 2-6xy-z$

$z'_x = \frac{2-6xy-z}{3z^2+x+2y^2z}$

$$(z^3)'_y + (3x^2y)'_y + (xz)'_y + (y^2z^2)'_y + y'_y - (2x)'_y = 0 \quad | \; (\;)'_y$$

$$3z^2 z'_y + 3x^2 + xz'_y + 2yz^2 + 2y^2 z z'_y + 1 = 0$$

$$z'_y(3z^2 + x + 2y^2 z) = -1 - 3x^2 - 2yz^2 \Rightarrow z'_y = \frac{-1 - 3x^2 - 2yz^2}{3z^2 + x + 2y^2 z}$$