Будницкий Н.В.

Интегрирование. Часть 1.

8.2.29. $\int \frac{dx}{x^2\sqrt{x}} = \int \frac{dx}{x^{5/2}} = \int x^{-\frac{5}{2}} dx = \frac{x^{-5/2+1}}{-5/2+1} + C = \frac{x^{-3/2}}{-3/2} + C$

$= -\frac{2}{3} x^{-3/2} + C = -\frac{2}{3\sqrt{x^3}} + C$

Проверка: $\left(-\frac{2}{3\sqrt{x^3}}\right)' = -\frac{2}{3}\cdot\left(\frac{1}{\sqrt{x^3}}\right)' = -\frac{2}{3}\cdot\left(x^{-3/2}\right)' = -\frac{2}{3}\cdot\left(-\frac{3}{2}\right)\cdot x^{-5/2}$

$= \frac{1}{x^{5/2}} = \frac{1}{x^2\sqrt{x}}$

8.2.30. $\int \frac{dx}{x^2+3} = \int \frac{dx}{x^2+\sqrt{3}^2} = \frac{1}{\sqrt{3}} \operatorname{arctg} \frac{x}{\sqrt{3}} + C$

Проверка: $\left(\frac{1}{\sqrt{3}} \cdot \operatorname{arctg} \frac{x}{\sqrt{3}}\right)' = \frac{1}{\sqrt{3}} \cdot \left(\operatorname{arctg} \frac{x}{\sqrt{3}}\right)' = \frac{1}{\sqrt{3}} \cdot \left(\left(\frac{x}{\sqrt{3}}\right)' \cdot \frac{1}{1+\left(\frac{x}{\sqrt{3}}\right)^2}\right) =$

$= \frac{1}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}} \cdot \frac{1}{\frac{3+x^2}{3}} = \frac{1}{3} \cdot \frac{3}{x^2+3} = \frac{1}{x^2+3}$

8.2.32. $\int \frac{1}{5^x} dx = \int 5^{-x} dx = -\frac{5^{-x}}{\ln 5} + C = -\frac{1}{5^x \ln 5} + C$

Проверка: $\left(-\frac{5^{-x}}{\ln 5}\right)' = \frac{5^{-x} \ln 5}{\ln 5} = 5^{-x} = \frac{1}{5^x}$

8.2.37. $\int \frac{dx}{\sqrt{4-x^2}} = \int \frac{dx}{\sqrt{2^2-x^2}} = \arcsin \frac{x}{2} + C$ Проверка: $\frac{1}{2} \cdot \frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}} =$

$= \frac{1}{2\sqrt{1-\frac{x^2}{4}}} = \frac{1}{\sqrt{4\left(1-\frac{x^2}{4}\right)}} = \frac{1}{\sqrt{4-x^2}}$

8.1.33. $\int \frac{dx}{\sqrt{x^2-1}} = \int \frac{dx}{\sqrt{-1+x^2}} = \int \frac{dx}{\sqrt{x^2+(-1)}} =$

$= \ln\left| x + \sqrt{x^2-1} \right| + C$

8.1.34. $\int \frac{dx}{x^2-25} = \int \frac{dx}{x^2-5^2} = \frac{1}{2 \cdot 5} \ln\left| \frac{x-5}{x+5} \right| + C =$

$= \frac{1}{10} \ln\left| \frac{x-5}{x+5} \right| + C$

8.1.35. $\int \left(x + \frac{2}{x}\right)^2 dx = \int x^2 dx + \int \frac{1}{x} dx + \int \frac{4}{x^2} dx =$

$= \frac{x^3}{3} + \ln x + \frac{1}{4} \cdot \frac{x^{-1}}{-1} + C = \frac{1}{3} x^3 + \ln x - \frac{1}{4x} + C$

8.1.36. $\int \frac{dx}{4x^2+1} = \int \frac{1}{4} \frac{dx}{x^2+1/4} = \frac{1}{4} \int \frac{dx}{x^2+(1/2)^2} = \frac{1}{4} \cdot 2 \operatorname{arctg} 2x + C =$

$= \frac{1}{2} \operatorname{arctg} 2x + C$

8.1.37. $\int \left(7^x - \frac{8}{x} + 4\cos x\right) dx = \int 7^x dx - \int \frac{8}{x} dx + \int 4\cos x \, dx =$

$\frac{7^x}{\ln 7} - 8 \ln x + 4 \sin x + C$

8.1.38 $\int \left(\frac{\sqrt{3}}{\cos^2 x} - \sqrt[3]{x} - \frac{2}{x^4}\right) dx = \int \frac{\sqrt{3}}{\cos^2 x} dx - \int \sqrt[3]{x} \, dx - \int \frac{2}{x^4} dx =$

$= \sqrt{3} \int \cos^{-2} x \, dx - \int x^{1/3} dx - 2\int x^{-4} dx = \sqrt{3} \operatorname{tg} x - \frac{x^{4/3}}{4/3} - 2 \cdot \frac{x^{-3}}{-3} + C$

$= \sqrt{3} \operatorname{tg} x - \frac{3x\sqrt[3]{x}}{4} + \frac{2}{3x^3} + C$

8.1.39 $\int \frac{\sqrt{x} - 3\sqrt[5]{x^2} + 1}{\sqrt[4]{x}} dx = \int \frac{\sqrt{x}\, dx}{\sqrt[4]{x}} - \int \frac{3\sqrt[5]{x^2}\, dx}{\sqrt[4]{x}} + \int \frac{1}{\sqrt[4]{x}} dx =$

$= \int \frac{x^{1/2}}{x^{1/4}} dx - 3\int \frac{x^{2/5}}{x^{1/4}} dx + \int x^{-1/4} dx = \int x^{1/4} dx - 3\int x^{3/20} dx$

$+ \int x^{-1/4} dx = \frac{x^{5/4}}{5/4} - 3 \cdot \frac{x^{23/20}}{23/20} + \frac{x^{3/4}}{3/4} + C = \frac{4x^{5/4}}{5} - \frac{60x^{23/20}}{23}$

$+ \frac{4x^{3/4}}{3} + C$

8.1.40 $\int (0{,}7x^{-0{,}1} + 0{,}2 \cdot (0{,}5)^x) dx = \int 0{,}7x^{-0{,}1} dx + \int 0{,}2 \cdot 0{,}5^x dx =$

$= 0{,}7\int x^{-0{,}1} dx + 0{,}2\int 0{,}5^x dx = 0{,}7 \cdot \frac{x^{0{,}9}}{0{,}9} + 0{,}2 \frac{0{,}5^x}{\ln 0{,}5} + C =$

$= \frac{7x^{0{,}9}}{9} + \frac{0{,}2 \cdot 0{,}5^x}{\ln 0{,}5} + C$

8.1.41 $\int (5\,\mathrm{sh}\,x - 7\,\mathrm{ch}\,x + 1) dx = 5\int \mathrm{sh}\,x\, dx - 7\int \mathrm{ch}\,x\, dx + \int 1 dx =$

$= 5\,\mathrm{ch}\,x - 7\,\mathrm{sh}\,x + x + C$

8.1.42 $\int (x^2 - 1)(\sqrt{x} + 4) dx = \int (x^2\sqrt{x} + 4x^2 - \sqrt{x} - 4) dx =$

$\int x^{5/2} dx + 4\int x^2 dx - \int \sqrt{x} dx - \int 4 dx = \frac{x^{7/2}}{7/2} + 4 \cdot \frac{x^3}{3} - \frac{x^{3/2}}{3/2} - 4x + C$

$= \frac{2x^3\sqrt{x}}{7} + \frac{4x^3}{3} - \frac{2x\sqrt{x}}{3} - 4x + C$

$$8.1.43 \int \frac{7-\sqrt{x^2+\pi}}{\sqrt{x^2+\pi}}\,dx = \int \frac{7\,dx}{\sqrt{x^2+\pi}} - \int dx = 7\ln\left|x+\sqrt{x^2+\pi}\right| - x + C$$

$$8.1.44 \int \left(\frac{\sqrt{x}-5}{x}\right)^3 dx = \int \frac{\sqrt{x^3} - 3\sqrt{x^2}\cdot 5 + 3\sqrt{x}\cdot 25 - 125}{x^3}\,dx =$$

$$= \int \frac{\sqrt{x}}{x^2}\,dx - 15\int \frac{1}{x^2}\,dx + 75\int \frac{\sqrt{x}}{x^3}\,dx - 125\int \frac{dx}{x^3} = \int x^{-3/2}\,dx - 15\int x^{-2}\,dx + 75\int x^{-5/2}\,dx -$$

$$-125\int x^{-3}\,dx = -\frac{2}{\sqrt{x}} + \frac{15}{x} - \frac{50}{x\sqrt{x}} + \frac{125}{2x^2} + C$$

$$8.1.45 \int \sin 7x\,dx = \left[7x = t \Rightarrow dt = (7x)'_x\,dx = 7dx \Rightarrow dx = \frac{1}{7}\,dt\right] =$$

$$= \int \sin t \cdot \frac{1}{7}\,dt = \frac{1}{7}\int \sin t\,dt = -\frac{1}{7}\cos t + C = -\frac{1}{7}\cos 7x + C$$

$$8.1.46 \int \sqrt[5]{2x-8}\,dx = \int (2x-8)^{1/5}\,dx = \left[\int f(2x+(-8)) = \frac{1}{2}F(2x-8)\right] =$$

$$= \frac{1}{2}\left(\frac{(2x-8)^{6/5}}{6/5}\right) + C = \frac{1}{2}\cdot\frac{5(2x-8)^{6/5}}{6} + C = \frac{5(2x-8)^{6/5}}{12} + C$$

$$8.1.47. \int (1-4x)^{2001}\,dx = -\frac{1}{4}\cdot\frac{(1-4x)^{2002}}{2002} + C$$

$$8.1.48. \int \frac{dx}{9x^2+7} = \int \frac{dx}{(3x)^2+(\sqrt{7}^2)} = \left[t = 3x \Rightarrow x = \frac{t}{3};\ dx = \left(\frac{t}{3}\right)' dt = \frac{1}{3}dt\right]$$

$$= \int \frac{1/3\,dt}{t^2+\sqrt{7}^2} = \frac{1}{3\sqrt{7}}\operatorname{arctg}\frac{t}{\sqrt{7}} + C = \frac{1}{3\sqrt{7}}\operatorname{arctg}\frac{3x}{\sqrt{7}} + C$$

$$8.1.49. \int \frac{dx}{(6x+11)^4} = \int (6x+11)^{-4}\,dx = \frac{1}{6}\cdot\frac{(6x+11)^{-3}}{-3} + C =$$

$$= -\frac{1}{18(6x+11)^3} + C$$

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8.1.50 $\int \frac{dx}{25x^2+1} = \int \frac{dx}{(5x)^2+1^2} = [\,t=5x \Rightarrow dx=\frac{1}{5}dt\,] =$

$= \int \frac{1/5\,dt}{t^2+1^2} = \frac{1}{5}\,\mathrm{arctg}\,t + C = \frac{1}{5}\,\mathrm{arctg}\,5x + C$

8.1.51 $\int 3^{2-11x}\,dx = -\frac{1}{11}\,\frac{3^{2-11x}}{\ln 3} + C = -\frac{3^{2-11x}}{11\ln 3} + C$

8.1.52 $\int \frac{dx}{\sqrt{4x^2-1}} = \int \frac{dx}{\sqrt{(2x)^2-1^2}} = \int \frac{dx}{\sqrt{(2x)^2+(-1)}} = [\,2x=t;\ x=\frac{t}{2} \Rightarrow dx=\frac{1}{2}dt\,]$

$= \int \frac{1/2\,dt}{\sqrt{t^2+(-1)}} = \frac{1}{2}\ln\left|t+\sqrt{t^2-1}\right| + C = \frac{1}{2}\ln\left|2x+\sqrt{4x^2-1}\right|$

8.1.53 $\int \sin^2 3x\,dx = \int \frac{1-\cos 6x}{2}\,dx = \int \frac{1}{2}dx - \int \frac{\cos 6x}{2}\,dx =$

$= \frac{1}{2}\int dx - \frac{1}{2}\int \cos(6x+0)\,dx = \frac{1}{2}x - \frac{1}{12}\sin 6x + C$

8.1.54 $\int \cos^2 8x\,dx = \int \frac{1+\cos 16x}{2} = \frac{1}{2}\int dx + \frac{1}{2}\int \cos 16x\,dx =$

$= \frac{1}{2}x + \frac{1}{32}\cos 16x + C$

8.1.55 $\int \mathrm{tg}^2 x\,dx = \int \frac{\sin^2 x}{\cos^2 x}\,dx = \int \frac{1-\cos^2 x}{\cos^2 x}\,dx = \int \frac{1}{\cos^2 x}\,dx - \int dx =$

$= \mathrm{tg}\,x - x + C$

8.1.56 $\int \frac{4x+1}{x-5}dx = \int \frac{4x\,dx}{x-5} + \int \frac{dx}{x-5} = [\, t=(x-5) \Rightarrow dt=(x-5)' \Rightarrow$

$\Rightarrow dt = dx;\ x = t+5] = 4\int \frac{dt(t+5)}{t} + \int \frac{dt}{t} =$

$= 4\left(\int \frac{t\,dt}{t} + \int \frac{5\,dt}{t}\right) + \int \frac{dt}{t} = 4\int dt + 20\int \frac{dt}{t} + \int \frac{dt}{t} =$

$= 4x + 21 \ln|x-5| + C$

8.1.57. $\int (3\,tg\,x - 2\,ctg\,x)^2 dx = \int 9\,tg^2x\,dx - \int 12\,tg\,x\,ctg\,x\,dx + \int 4\,ctg^2x\,dx =$

$= 9\int tg^2x\,dx - 12\int dx + 4\int ctg^2x\,dx = 9\int \frac{\sin^2x}{\cos^2x}dx - 12\int dx +$

$+ 4\int \frac{\cos^2x}{\sin^2x}dx = 9\int \frac{1-\cos^2x}{\cos^2x}dx - 12\int dx + 4\int \frac{1-\sin^2x}{\sin^2x}dx =$

$= 9\int \frac{dx}{\cos^2x} - 9\int dx - 12\int dx + 4\int \frac{1}{\sin^2x} - 4\int dx =$

$= 9\,tg\,x - 4\,ctg\,x - 25x + C$

8.1.58. $\int \frac{4\sqrt{1-x^2} + 3x^2}{x^2-1}dx = \int \frac{4\sqrt{1-x^2}}{x^2-1}dx + \int \frac{3x^2}{x^2-1}dx =$

$= 4\int (1-x^2)^{-1/2}dx + 3\int \frac{x^2-1+1}{x^2-1}dx = -4\int \frac{1}{\sqrt{1-x^2}}dx + 3\int \frac{x^2-1}{x^2-1}dx +$

$+ 3\int \frac{1}{x^2-1} = -4\arcsin x + 3x + \frac{3}{2}\ln\left|\frac{x-1}{x+1}\right| + C$

8.1.59. $\int \frac{\cos 2x \, dx}{\sin^2 x \cos^2 x} = \int \frac{\cos^2 x - \sin^2 x}{\sin^2 x \cos^2 x} dx = \int \frac{dx}{\sin^2 x} - \int \frac{dx}{\cos^2 x} =$

$= -\mathrm{ctg}\, x - \mathrm{tg}\, x + C$

8.1.60. $\int \frac{\sin 2x \, dx}{\cos x} = \int \frac{2 \sin x \cos x \, dx}{\cos x} = 2\int \sin x \, dx = -2\cos x + C$

Будищий К.8. часть 2.

8.2.33. $\int \cos(6x+1)\, dx = \frac{1}{6} \sin(6x+1) + C$

8.2.34. $\int \frac{dx}{\sqrt[3]{(5x-2)^4}} = \int (5x-2)^{-4/3} dx = \frac{1}{5} \cdot \frac{(5x-2)^{-1/3}}{-1/3} + C =$

$= -\frac{3}{5\sqrt[3]{5x-2}} + C$

8.2.35. $\int \frac{\sqrt{\mathrm{tg}\, x}\, dx}{\cos^2 x} = \left[\mathrm{tg}\, x = t \Rightarrow dt = \frac{dx}{\cos^2 x} \Rightarrow dx = \cos^2 x \, dt \right] =$

$= \int \frac{\sqrt{t} \cos^2 x \, dt}{\cos^2 x} = \int \sqrt{t}\, dt = \frac{t^{3/2}}{3/2} + C = \frac{2t^{3/2}}{3} + C =$

$= \frac{2\sqrt{\mathrm{tg}^3 x}}{3} + C$