$(z^3_y)' + (3x^2y)'_y + (xz)'_y + (y^2z^2)'_y + y'_y - (2x)'_y = 0 \quad | \, (\ )'_y$

$3z^2 z'_y + 3x^2 + xz'_y + 2yz^2 + 2y^2 z z'_y + 1 = 0$

$z'_y(3z^2 + x + 2y^2 z) = -1 - 3x^2 - 2yz^2 \Rightarrow z'_y = \frac{-1 - 3x^2 - 2yz^2}{3z^2 + x + 2y^2 z}$

11.4.34. $x + y + z = e^{-(x+y+z)}$, $z'_x - ?$, $z'_y - ?$

$F'_x(x;y;z) = (x + y + z - e^{-(x+y+z)})'_x = 1 + 0 + 0 - e^{-(x+y+z)} \cdot (-x-y-z)'_x =$

$= 1 - e^{-(x+y+z)} \cdot (-1-0-0) = 1 + e^{-(x+y+z)}$

$F'_y(x;y;z) = (x + y + z - e^{-(x+y+z)})'_y = 1 - e^{-(x+y+z)} \cdot (-1) = 1 + e^{-(x+y+z)}$

$F'_z(x;y;z) = (x + y + z - e^{-(x+y+z)})'_z = 1 + e^{-(x+y+z)}$

$\frac{\partial z}{\partial x} = -\frac{1 + e^{-(x+y+z)}}{1 + e^{-(x+y+z)}} = -1dx \quad \frac{\partial z}{\partial y} = -\frac{1 + e^{-(x+y+z)}}{1 + e^{-(x+y+z)}} = -1dy$

$dz = -1dx - 1dy = -dx - dy$

11.5.1. $z = x^3 - x^2y - y^3$

1. $\frac{\partial z}{\partial x} = 3x^2 - 2xy; \quad \frac{\partial z}{\partial y} = -x^2 - 3y^2$

2. $\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x}(3x^2 - 2xy) = 6x - 2y;$

$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y}(3x^2 - 2xy) = -2x;$

$$\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial}{\partial x}(-x^2 - 3y^2) = -2x;$$

$$\frac{\partial^2 z}{\partial y^2} = \frac{\partial}{\partial y}(x^2 - 3y^2) = -6y;$$

(3) $$\frac{\partial^3 z}{\partial x^3} = \frac{\partial}{\partial x}\left(\frac{\partial^2 z}{\partial x^2}\right) = \frac{\partial}{\partial x}(6x - 2y) = 6$$

$$\frac{\partial^3 z}{\partial x^2 \partial y} = \frac{\partial}{\partial y}(6x - 2y) = -2$$

$$\frac{\partial^3 z}{\partial x \partial y^2} = \frac{\partial}{\partial y}(-2x) = 0$$

$$\frac{\partial^3 z}{\partial y^3} = \frac{\partial}{\partial y}(-6y) = -6$$

Все послед. производные будут равны нулю

11.5.2. $z = e^{xy^3}$; $\frac{\partial^4 z}{\partial x^4}$; $\frac{\partial^4 z}{\partial x^3 \partial y}$; $\frac{\partial^4 z}{\partial x^2 \partial y^2}$

(1) $$\frac{\partial z}{\partial x} = (e^{xy^3})'_x = e^{xy^3} \cdot (xy^3)'_x = e^{xy^3} y^3$$

$$\frac{\partial^2 z}{\partial x^2} = (e^{xy^3} y^3)'_x = y^3 \cdot y^3 \cdot e^{xy^3} = y^6 e^{xy^3}$$

$$\frac{\partial^3 z}{\partial x^3} = (y^6 e^{xy^3})'_x = y^9 e^{xy^3}$$

$$\frac{\partial^4 z}{\partial x^4} = (y^9 e^{xy^3})'_x = y^{12} e^{xy^3}$$

(2) $$\frac{\partial^4 z}{\partial x^3 \partial y} = \frac{\partial}{\partial y}\left(\frac{\partial^3 z}{\partial x^3}\right) = (y^9 e^{xy^3})'_y = (y^9)'_y e^{xy^3} + y^9 (e^{xy^3})'_y =$$

$$= 9y^8 e^{xy^3} + y^9 e^{xy^3} \cdot 3xy^2 = 9y^8 e^{xy^3} + y^{11} 3x e^{xy^3}$$

$$③\ \frac{\partial^3 z}{\partial x^2 \partial y} = \frac{\partial}{\partial y}\left(\frac{\partial^2 z}{\partial x^2}\right) = \frac{\partial}{\partial y}\left(y^6 e^{xy^3}\right) = (y^6)'_y e^{xy^3} + y^6 (e^{xy^3})'_y = 6y^5 e^{xy^3} + y^6 e^{xy^3} \cdot 3xy^2 = 6y^5 e^{xy^3} + 3xy^8 e^{xy^3}$$

$$④\ \frac{\partial^4 z}{\partial x^2 \partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial^3 z}{\partial x^2 \partial y}\right) = \frac{\partial}{\partial y}\left(6y^5 e^{xy^3} + 3xy^8 e^{xy^3}\right) =$$

$$(6y^5 e^{xy^3})'_y + (3xy^8 e^{xy^3})'_y = (6y^5)'_y e^{xy^3} + 6y^5 (e^{xy^3})'_y +$$

$$(3xy^8)'_y e^{xy^3} + 3xy^8 \cdot (e^{xy^3})'_y = 30y^4 e^{xy^3} + 18xy^7 e^{xy^3} +$$

$$+ 24xy^7 e^{xy^3} + 9x^2 y^{10} = 30y^4 e^{xy^3} + 42xy^7 e^{xy^3} + 9x^2 y^{10} =$$

$$= 3y^4 e^{xy^3}(10 + 14xy^3 + 3x^2y^6)$$

11.5.3. $d^2 z - ?$ $z = \operatorname{arctg} \frac{y}{x}$

$$①\ dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy$$

$$\frac{\partial z}{\partial x} = \left(\operatorname{arctg} \frac{y}{x}\right)'_x = \frac{(y/x)'_x}{1 + (y/x)^2} = \frac{y(1/x)'_x}{1 + \frac{y^2}{x^2}} = \frac{-y/x^2}{1 + y^2/x^2} = -\frac{y}{x^2+y^2}$$

$$\frac{\partial z}{\partial y} = \left(\operatorname{arctg} \frac{y}{x}\right)'_y = \frac{(y/x)'_y}{1 + y^2/x^2} = \frac{1/x}{\frac{x^2+y^2}{x^2}} = \frac{1}{x} \cdot \frac{x^2}{x^2+y^2} = \frac{x}{x^2+y^2}$$

$$dz = -\frac{y}{x^2+y^2} dx + \frac{x}{x^2+y^2} dy$$

② $\dfrac{\partial^2 z}{\partial x^2} = \dfrac{\partial}{\partial x}\left(-\dfrac{y}{x^2+y^2}\right) = -\left(\dfrac{(y')_x(x^2+y^2)+y(x^2+y^2)'_x}{(x^2+y^2)^2}\right) =$

$= +\dfrac{2xy}{(x^2+y^2)^2}$

$\dfrac{\partial^2 z}{\partial x\partial y} = \dfrac{\partial}{\partial y}\left(-\dfrac{y}{x^2+y^2}\right) = -\left(\dfrac{(y')_y(x^2+y^2)-y(x^2+y^2)'_y}{(x^2+y^2)^2}\right) =$

$= -\dfrac{x^2+y^2-2y^2}{(x^2+y^2)^2} = \dfrac{y^2-x^2}{(x^2+y^2)^2}$

$\dfrac{\partial^2 z}{\partial y^2} = \dfrac{\partial}{\partial y}\left(\dfrac{x}{x^2+y^2}\right) = \left(\dfrac{x'_y(x^2+y^2)-x(x^2+y^2)'_y}{(x^2+y^2)^2}\right) =$

$= -\dfrac{2xy}{(x^2+y^2)^2}$

$d^2 z = \dfrac{\partial^2 z}{\partial x^2}dx^2 + 2\dfrac{\partial^2 z}{\partial x\partial y}dxdy + \dfrac{\partial^2 z}{\partial y^2}dy^2 =$

$= \dfrac{2xy\,dx^2}{(x^2+y^2)^2} + 2\dfrac{y^2-x^2}{(x^2+y^2)^2}dxdy - \dfrac{2xy\,dy^2}{(x^2+y^2)^2} =$

$= \dfrac{2}{(x^2+y^2)^2}\left(xy\,dx^2 + (y^2-x^2)dxdy - xy\,dy^2\right)$

pun(6, 5) -> mpy(6. owr// ...)

11.5.4. $d^2z - ?$ $\quad z = \frac{xy}{x-y}$ $\quad$ Док-ть: $z''_{x^2} + 2z''_{xy} + z''_{y} = \frac{2}{x-y}$

$$z'_x = \left(\frac{xy}{x-y}\right)'_x = \frac{(xy)'_x(x-y) - xy(x-y)'_x}{(x-y)^2} = \frac{y(x-y) - xy}{(x-y)^2} = -\frac{y^2}{(x-y)^2}$$

$$z'_y = \left(\frac{xy}{x-y}\right)'_y = \frac{(xy)'_y(x-y) - xy(x-y)'_y}{(x-y)^2} = \frac{x(x-y) + xy}{(x-y)^2} = \frac{x^2}{(x-y)^2}$$

$$z''_{x^2} = \left(-\frac{y^2}{(x-y)^2}\right)'_x = -\frac{(y^2)'(x-y)^2 - y^2((x-y)^2)'}{(x-y)^4} = \frac{2y^2(x-y)}{(x-y)^4} = \frac{2y^2}{(x-y)^3}$$

$$z''_{xy} = \left(-\frac{y^2}{(x-y)^2}\right)'_{y} = -\frac{(y^2)'_y(x-y)^2 - y^2((x-y)^2)'_y}{(x-y)^4} =$$

$$= -\frac{2y(x-y)^2 - 2y^2(2x-y)\cdot(-1)}{(x-y)^4} = -\frac{2y(x-y)(x-y+y)}{(x-y)^4} = -\frac{2xy}{(x-y)^3}$$

$$z''_{y^2} = \left(\frac{x^2}{(x-y)^2}\right)'_y = \frac{(x^2)'_y(x-y)^2 - x^2((x-y)^2)'_y}{(x-y)^4} = \frac{2x^2}{(x-y)^3}$$

$$d^2z = \frac{2(y^2dx^2 - 2xy\,dxdy + x^2dy^2)}{(x-y)^3}$$

$$z''_{x^2} + 2z''_{xy} + z''_{y^2} = \frac{2y^4 - 4xy + 2x^2}{(x-y)^3} = \frac{2(x-y)^2}{(x-y)^3} = \frac{2}{x-y}$$

11.5.5. $d^3 z - ?$ $z = \frac{xy}{x+y}$

(1) $z'_x = \left(\frac{xy}{x+y}\right)'_x = \frac{(xy)'_x (x+y) - xy(x+y)'_x}{(x+y)^2} =$

$= \frac{y(x+y) - xy}{(x+y)^2} = \frac{y^2 + xy - xy}{(x+y)^2} = \frac{y^2}{(x+y)^2}$

$z'_y = \left(\frac{xy}{(x+y)}\right)'_y = \frac{(xy)'_y (x+y) - xy(x+y)'_y}{(x+y)^2} = \frac{x(x+y) - xy}{(x+y)^2} = \frac{x^2}{(x+y)^2}$

(2) $z''_{x^2} = \left(\frac{y^2}{(x+y)^2}\right)'_x = \frac{(y^2)'_x (x+y)^2 - y^2((x+y)^2)'_x}{(x+y)^4} =$

$= \frac{-2y^2(x+y)}{(x+y)^4} = -\frac{2y^2}{(x+y)^3}$

$z''_{xy} = \left(\frac{y^2}{(x+y)^2}\right)'_y = \frac{(y^2)'_y (x+y)^2 - y^2((x+y)^2)'_y}{(x+y)^4} = \frac{2y(x+y)^2 - 2y^2(x+y)}{(x+y)^4}$

$= \frac{2y(x+y)((x+y) - y)}{(x+y)^4} = \frac{2xy}{(x+y)^3}$

$z''_{y^2} = \left(\frac{x^2}{(x+y)^2}\right)'_y = \frac{(x^2)'_y (x+y)^2 - x^2((x+y)^2)'_y}{(x+y)^4} =$

$= \frac{-2x^2(x+y)}{(x+y)^4} = -\frac{2x^2}{(x+y)^3}$

③ $d^2z = -\frac{2y^2}{(x+y)^3}dx^2 + 4\frac{xy}{(x+y)^3}dxdy - \frac{2x^2}{(x+y)^3}dy^2 =$

$= -2\frac{(ydx - xdy)^2}{(x+y)^3}$

④ $d^3z = z'''_{x^3}dx^3 + 3z'''_{x^2y}dx^2dy + 3z'''_{xy^2}dxdy^2 + z'''_{y^3}dy^3$

$d^3z = \frac{6}{(x+y)^4}\left(y^2dx^3 - (2xy - y^2)dx^2dy - (2xy - x^2)dxdy^2 + x^2dy^3\right)$