② $\int x^2 \sin(x^3+1)\,dx = \Big[\, t = x^3+1 \Rightarrow dt = d(x^3+1) = (x^3+1)'_x\,dx = 3x^2\,dx$

$\Rightarrow x^2\,dx = \frac{1}{3}\,dt \,\Big] = \int \sin t \cdot \frac{1}{3}\,dt = \frac{1}{3}\int \sin t\,dt =$

$= -\frac{1}{3}\cos t + C = -\frac{1}{3}\cos(x^3+1) + C$

③ $\int \frac{x\,dx}{x^2+1} = \Big[\, t = x^2+1 \Rightarrow dt = d(x^2+1) = (x^2+1)'_x\,dx = 2x\,dx \Rightarrow x\,dx = \frac{1}{2}dt \,\Big] =$

$= \int \frac{1/2\,dt}{t} = \frac{1}{2}\int \frac{dt}{t} = \frac{1}{2}\ln|t| + C = \frac{1}{2}\ln|x^2+1| + C$

30.11.20

8.2.10 1) $\int \frac{x - \sin\frac{1}{x}}{x^2}\,dx = \int \left(\frac{x}{x^2} - \frac{\sin\frac{1}{x}}{x^2}\right)dx = \int \frac{dx}{x} - \int \frac{\sin\frac{1}{x}}{x^2}\,dx =$

$= \Big[\, t = \frac{1}{x} \Rightarrow dt = d\left(\frac{1}{x}\right) = \left(\frac{1}{x}\right)'_x dx = -\frac{dx}{x^2} \Rightarrow \frac{dx}{x^2} = -dt \,\Big] = \int \frac{dx}{x} -$

$- \int \sin t\,(-dt) = \int \frac{dx}{x} + \int \sin t\,dt = \ln|x| - \cos t + C = \ln|x| - \cos\frac{1}{x} + C$

2) $\int \frac{5x-1}{\sqrt{4-x^2}}\,dx = \int \left(\frac{5x}{\sqrt{4-x^2}} - \frac{1}{\sqrt{4-x^2}}\right)dx = 5\int \frac{x\,dx}{\sqrt{4-x^2}} - \int \frac{dx}{\sqrt{4-x^2}} =$

$= \Big[\, t = 4-x^2 \Rightarrow dt = d(4-x^2) = (4-x^2)'_x\,dx = -2x\,dx \Rightarrow x\,dx = -\frac{1}{2}dt \,\Big] =$

$= 5\int \frac{-\frac{1}{2}dt}{\sqrt{t}} - \int \frac{dx}{\sqrt{4-x^2}} = -\frac{1}{2}\cdot 5\int \frac{dt}{\sqrt{t}} - \int \frac{dx}{\sqrt{2^2-x^2}} = -\frac{5}{2}\cdot 2\sqrt{t} -$

$- \arcsin\frac{x}{2} + C = -5\sqrt{4-x^2} - \arcsin\frac{x}{2} + C$

8.2.15

1) $\int \frac{\sqrt{1-x^2}}{x^2}\,dx = \Big[\, x=\psi(t),\; x=\sin t \Rightarrow 1-x^2 = 1-\sin^2 t = \cos^2 t \Rightarrow \sqrt{\cos^2 t} = \cos t$

$x=\sin t \Rightarrow dx = d(\sin t) = (\sin t)'_t\,dt = \cos t\,dt \,\Big] = \int \frac{\sqrt{\cos^2 t}\,\cos t\,dt}{\sin^2 t} =$

$= \int \frac{\cos^2 t\,dt}{\sin^2 t} = \int \frac{1-\sin^2 t}{\sin^2 t}\,dt = \int \frac{1}{\sin^2 t}\,dt - \int \frac{\sin^2 t}{\sin^2 t}\,dt$

$= \int \frac{dt}{\sin^2 t} - \int dt = -\mathrm{ctg}\,t - t + C = [\, x=\sin t \Rightarrow t=\arcsin x \,] =$

$= -\mathrm{ctg}(\arcsin x) - \arcsin x + C = -\frac{\sqrt{1-x^2}}{x} - \arcsin x + C$

2) $\int \frac{dx}{\sqrt{x}\,(1+\sqrt{x})} = \Big[\, x=\psi(t),\; x=t^2 \Rightarrow \sqrt{x}=t;\; dx = d(t^2) = (t^2)'_t\,dt = 2t\,dt,\; t=\sqrt{x} \,\Big]$

$= \int \frac{2t\,dt}{\sqrt{t^2}\,(1+\sqrt{t^2})} = \int \frac{2t\,dt}{t(1+t)} = \int \frac{2\,dt}{1+t} = 2\int \frac{dt}{1+t} = [\, dt = d(t+1) \,] =$

$= 2\int \frac{d(t+1)}{(t+1)} = \left[\, \int \frac{dx}{x} = \ln|x| + C \,\right] = 2\cdot \ln|1+t| + C =$

$= 2\ln|\sqrt{x}+1| + C = 2\ln(\sqrt{x}+1) + C$

$\int u v'\,dx = uv - \int v u'\,dx$

8.2.20

1) $\int x\cdot e^x\,dx = [\, u=x,\; v=e^x \,] = xe^x - \int e^x\,dx =$

$= xe^x - e^x + C = e^x(x-1) + C$

2) $\int \ln x \, dx = \int 1 \cdot \ln x \, dx = \left[\begin{array}{l} u = \ln x \Rightarrow u' = (\ln x)'_x = 1/x \\ v' = 1 \Rightarrow v = \int 1 dx = x \end{array} \right] =$

$= \ln x \cdot x - \int x \cdot \frac{1}{x} \, dx = x \ln x - \int 1 dx = x \ln x - x + C$

3) $\int x^2 \cos x \, dx = \left[\begin{array}{l} u = x^2 \Rightarrow u' = 2x \\ v' = \cos x \Rightarrow v = \sin x \end{array} \right] = x^2 \sin x - 2 \int x \sin x \, dx = \left[\begin{array}{l} u = x \Rightarrow u' = 1 \\ v' = \sin x \Rightarrow \dots \end{array} \right.$

$= x^2 \sin x - 2(x \cdot (-\cos x) - \int (-\cos x) \cdot 1 dx = x^2 \sin x - 2(-x \cos x + \int \cos x \, dx)$

$= x^2 \sin x - 2(-x \cos x + \sin x) + C = x^2 \sin x + 2x \cos x - 2 \sin x + C$

8.2.27 $\int e^x \cos x \, dx = \left[\begin{array}{l} u = e^x \Rightarrow u' = e^x \\ v' = \cos x \Rightarrow v = \sin x \end{array} \right] = e^x \sin x - \int e^x \sin x \, dx =$

$= \left[\begin{array}{l} u = e^x \Rightarrow u' = (e^x)' = e^x \\ v' = \sin x \Rightarrow v = -\cos x \end{array} \right] = e^x \sin x - \left(e^x (-\cos x) - \int (-\cos x) e^x \, dx \right) =$

$= e^x \sin x - (-e^x \cos x + \int e^x \cos x \, dx) = e^x \sin x + e^x \cos x - \int e^x \cos x \, dx$

Т.е. $\int e^x \cos x \, dx = e^x \sin x + e^x \cos x - \int e^x \cos x \, dx + C$

$2 \int e^x \cos x \, dx = e^x (\sin x + \cos x) + C$

$\int e^x \cos x \, dx = \frac{1}{2} e^x (\sin x + \cos x) + C$

8.2.30

(1) $\int \operatorname{arctg} x \, dx = \int 1 \cdot \operatorname{arctg} x \, dx = \begin{bmatrix} u = \operatorname{arctg} x \Rightarrow u' = \frac{1}{1+x^2} \\ v' = 1 \Rightarrow v = x \end{bmatrix} =$

$= x \operatorname{arctg} x - \int \left(x \cdot \frac{1}{1+x^2} \right) dx = x \operatorname{arctg} x - \int \frac{x \, dx}{1+x^2} = \begin{bmatrix} t = 1+x^2 = x = \sqrt{t-1} \\ dx = d\sqrt{t-1} = \frac{1}{2\sqrt{t-1}} \end{bmatrix}$

$= x \operatorname{arctg} x - \int \frac{\sqrt{t-1} \cdot \frac{1}{2\sqrt{t-1}}}{t} dt = x \operatorname{arctg} x - \int \frac{1}{2} \cdot \frac{dt}{t} =$

$= x \operatorname{arctg} x - \frac{1}{2} \int \frac{dt}{t} = x \operatorname{arctg} x - \frac{1}{2} \ln|t| + C =$

$= x \operatorname{arctg} x - \frac{1}{2} \ln(1+x^2) + C$

(2) $\int \sin\sqrt{x} \, dx = [\, x = t^2 \Rightarrow dx = d(t^2) = 2t\,dt \,] = \int 2t \sin t \, dt =$

$= \begin{bmatrix} v' = \sin t \Rightarrow v = -\cos t \\ u = 2t \Rightarrow u' = 2 \end{bmatrix} = -2t\cos t - \int -2\cos t \, dt =$

$= -2t\cos t + 2\int \cos t \, dt = -2t\cos t + 2\sin t + C = -2\sqrt{x}\cos\sqrt{x} + 2\sin\sqrt{x} + C$