07.12.20

8.1.2. $\int x^{10}\,dx = \frac{x^{11}}{11} + C$

8.1.3. $\int \frac{dx}{x^7} = \frac{x^{-6}}{-6} + C = -\frac{1}{6x^6} + C$

8.1.4. $\int \sqrt[4]{x}\,dx = \int x^{1/4}\,dx = \frac{x^{5/4}}{5/4} + C = \frac{4\sqrt[4]{x^5}}{5} + C$

8.1.5. $\int \frac{dx}{x^2+9} = \int \frac{dx}{x^2+3^2} = \frac{1}{3}\,\mathrm{arctg}\,\frac{x}{3} + C$

8.1.6 $\int \frac{dx}{x^2-\frac{1}{2}} = \int \frac{dx}{x^2-\sqrt{\frac{1}{2}}^2} = \left[\frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C\right] = \frac{1}{2\sqrt{1/2}}\ln\left|\frac{x-1/2}{x+1/2}\right| + C =$

$= \frac{1}{\sqrt{2}}\ln\left|\frac{x-\sqrt{1/2}}{x+\sqrt{1/2}}\right| + C$

8.1.7. $\int \frac{dx}{\sqrt{x^2+3}} = \left[\ln\left|x+\sqrt{x^2+a}\right| + C\right] = \ln\left|x+\sqrt{x^2+3}\right| + C$

8.1.9 $\int \frac{x^4+x^2-6x}{x^3}\,dx = \int\left(\frac{x^4}{x^3}+\frac{x^2}{x^3}-\frac{6x}{x^3}\right)dx = \int\left(x+\frac{1}{x}-\frac{6}{x^2}\right)dx$

$= \int x\,dx + \int\frac{1}{x}\,dx - 6\int\frac{1}{x^2}\,dx = \frac{x^2}{2} + \ln|x| + 6\,\frac{1}{x} + C$

8.1.10 $\int\left(\frac{5}{x} - \frac{10}{\sqrt[4]{x^3}} - \frac{3}{x^2+7}\right)dx = 5\int\frac{dx}{x} - 10\int\frac{dx}{\sqrt[4]{x^3}} - 3\int\frac{dx}{x^2+7} =$

$= 5\int\frac{dx}{x} - 10\int x^{-3/4}\,dx - 3\int\frac{dx}{x^2+\sqrt{7}^2} = 5\ln|x| - 10\,\frac{x^{1/4}}{1/4} - 3\,\frac{1}{\sqrt{7}}\,\mathrm{arctg}\,\frac{x}{\sqrt{7}} + C$

$= 5\ln|x| - 40x^{1/4} - \frac{3}{\sqrt{7}}\,\mathrm{arctg}\,\frac{x}{\sqrt{7}} + C$

8.1.11 $\int \sqrt{x}\,(x^2+1)\,dx = \int (x^2\sqrt{x} + \sqrt{x})\,dx = \int (x^2 \cdot x^{1/2} + x^{1/2})\,dx =$

$= \int (x^{5/2} + x^{1/2})\,dx = \int x^{5/2}\,dx + \int x^{1/2}\,dx = \frac{x^{7/2}}{7/2} + \frac{x^{3/2}}{3/2} + C =$

$= \frac{2x^3\sqrt{x}}{7} + \frac{2x\sqrt{x}}{3} + C$

8.1.12 $\int \frac{3+\sqrt{4-x^2}}{\sqrt{4-x^2}}\,dx = \int \left(\frac{3}{\sqrt{4-x^2}} + 1\right) dx = \int \frac{3}{\sqrt{4-x^2}}\,dx + \int dx =$

$= 3\int \frac{dx}{\sqrt{4-x^2}} + \int 1\,dx = 3 \arcsin \frac{x}{2} + x + C$

8.1.13 $\int \frac{(x^3+2)^2}{\sqrt{x}}\,dx = \int \frac{x^6+4x^3+4}{\sqrt{x}}\,dx = \int \frac{x^6}{x^{1/2}}\,dx + 4\int \frac{x^3}{x^{1/2}}\,dx + 4\int \frac{1}{x^{1/2}}\,dx$

$= \int x^{11/2}\,dx + 4\int x^{5/2}\,dx + 4\int x^{-1/2}\,dx = \frac{x^{13/2}}{13/2} + 4\,\frac{x^{7/2}}{7/2} + 4\,\frac{x^{1/2}}{1/2} + C =$

$= \frac{2x^6\sqrt{x}}{13} + \frac{8x^3\sqrt{x}}{7} + 8\sqrt{x} + C$

8.1.14 $\int \left(4\sin x + 8x^3 - \frac{12}{\cos^2 x}\right) dx = 4\int \sin x\,dx + 8\int x^3\,dx - 12\int \frac{1}{\cos^2 x}\,dx$

$= -4\cos x + 8\,\frac{x^4}{4} - 12\,\mathrm{tg}\,x + C = -4\cos x + 2x^4 - 12\,\mathrm{tg}\,x + C$

8.1.16 $\int \cos(2x)\,dx = \left[\begin{array}{ll} dx = \frac{1}{2}\,2dx = \frac{1}{2}\,d(2x) & ax = 2x + 0 \\ \int f(ax+b)\,dx = \frac{1}{a}F(ax+b) + C,\ a \neq 0 & \end{array}\right] =$

$= \frac{1}{2}\sin(2x) + C$

8.1.12 $\int (9x+2)^{17} dx = \left[\begin{array}{l} \int x^k dx \\ \int f(ax+b)dx = \frac{1}{a} F(ax+b) + C \end{array} \right] = \frac{1}{9} \frac{(9x+2)^{17+1}}{17+1} + C$

$= \frac{(9x+2)^{18}}{162} + C$

8.1.18. $\int \frac{dx}{8x-1} = \frac{1}{8} \ln|8x-1| + C$

8.1.19. $\int 4^{3-5x} dx = \left[\begin{array}{l} \int a^x dx = \frac{a^x}{\ln a} + C \\ \int f(ax+b) = \frac{1}{a} F(ax+b) + C \end{array} \right] = -\frac{1}{5} \frac{4^{3-5x}}{\ln 4} + C =$

$= -\frac{4^{3-5x}}{5 \ln 4} + C$

8.1.20 $\int \sqrt{3x+4}\, dx = \left[\begin{array}{l} \int x^k dx = \frac{x^{k+1}}{k+1} \\ \int f(ax+b) = \frac{1}{a} F(ax+b) + C \end{array} \right] = \frac{1}{3} \frac{(3x+4)^{3/2}}{3/2} + C =$

$= \frac{2}{9} \sqrt{3x+4}^{3} + C$

8.1.21 $\int \frac{dx}{3x^2-25} = \int \frac{dx}{3(x^2 - \frac{25}{3})} = \int \frac{1}{3} \frac{dx}{x^2 - \frac{25}{3}} = \frac{1}{3} \int \frac{dx}{x^2 - (\frac{5}{\sqrt{3}})^2} =$

$= \frac{1}{3} \cdot \frac{1}{2 \cdot \frac{5}{\sqrt{3}}} \cdot \ln \left| \frac{x - 5/\sqrt{3}}{x + 5/\sqrt{3}} \right| + C = \frac{\sqrt{3}}{30} \ln \left| \frac{x\sqrt{3} - 5}{x\sqrt{3} + 5} \right| + C$

8.1.23. $\int \cos^2 x\, dx = \left[\cos^2 \alpha = \frac{1 + \cos 2\alpha}{2} \right] = \int \frac{1 + \cos(2x)}{2} dx = \frac{1}{2} \int dx + \frac{1}{2} \int \cos(2x) dx$

$= \frac{1}{2} x + \frac{1}{2} \frac{1}{2} \sin(2x) + C = \frac{1}{2} x + \frac{1}{4} \sin(2x) + C$

8.1.24 $\int \frac{x-2}{x+3} dx = \int \frac{x+3-3-2}{x+3} dx = \int \frac{x+3-5}{x+3} dx = \int \left(\frac{x+3}{x+3} - \frac{5}{x+3} \right) dx$

$= \int \left(1 - \frac{5}{x+3}\right) dx = \int dx - 5 \int \frac{1}{x+3} dx = x - 5 \frac{1}{1} \ln|x+3| + C$

8.1.25. $\int \frac{x^2dx}{x^2-9} = \int \frac{x^2-9+9}{x^2-9}dx = \int\left(\frac{x^2-9}{x^2-9} + \frac{9}{x^2-9}\right)dx = \int dx + \int \frac{9}{x^2-9}dx =$

$= x + 9\int \frac{1}{x^2-9}dx = x + 9 \cdot \frac{1}{2\cdot 3}\ln\left|\frac{x-3}{x+3}\right| + C = x + \frac{3}{2}\ln\left|\frac{x-3}{x+3}\right| + C$

8.1.26 $\int \frac{5+\sin^3 x}{\sin^2 x}dx = 5\int \frac{dx}{\sin^2 x} + \int \sin x\, dx = -5\,ctg\, x - \cos x + C$

8.2.2. $\int \sqrt{4x-5}\,dx = [t = 4x-5 \Rightarrow dt = d(4x-5) = 4dx \Rightarrow dx = \frac{1}{4}dt] =$

$= \int \sqrt{t}\,\frac{1}{4}dt = \frac{1}{4}\int\sqrt{t}\,dt = \frac{1}{4}\,\frac{t^{3/2}}{3/2} + C = \frac{1}{4}\cdot\frac{2t^{3/2}}{3} + C =$

$= \frac{1}{6}t^{3/2} + C = \frac{1}{6}(4x-5)^{3/2} + C$

8.2.3. $\int \frac{dx}{(3x+2)^4} = \int (3x+2)^{-4}dx = [t = 3x+2 \Rightarrow dt = d(3x+2) = 3dx \Rightarrow dx = \frac{1}{3}dt]$

$= \int t^{-4}\,\frac{1}{3}dt = \frac{1}{3}\int t^{-4}dt = \frac{1}{3}\,\frac{t^{-3}}{-3} + C = -\frac{1}{9t^3} + C = -\frac{1}{9\cdot(3x+2)^3} + C =$

8.2.4. $\int \sin^3 x \cos x\,dx = [t = \sin x \Rightarrow \sin^3 x = t,\ dt = d(\sin x) = \cos x\,dx] =$

$= \int t^3 dt = \frac{t^4}{4} + C = \frac{\sin^4 x}{4} + C$

8.2.5. $\int e^{x^3}x^2dx = [t = x^3 \Rightarrow dt = d(x^3) = 3x^2dx \Rightarrow x^2dx = \frac{1}{3}dt] = \int e^t\,\frac{1}{3}\,dt =$

$= \frac{1}{3}e^t + C = \frac{1}{3}e^{x^3} + C$