1.1.

$$\frac{x^{n+2}}{x^{n-2}} = x^{n+2-n+2} = x^4$$

1.2.

$$x^{-1} \cdot 8 = 2 \quad / : 8$$

$$x^{-1} = \frac{1}{4}$$

$$\frac{1}{x} = \frac{1}{4}$$

$$x = 4$$

1.3.

$$a = 5$$

$$b = 10$$

$$\left(a^b\right)^0 = \left(5^{10}\right)^0 = 5^0 = 1$$

1.4.

$$\frac{\sqrt{4x}}{\sqrt{x}} = \frac{2\cancel{\sqrt{x}}}{\cancel{\sqrt{x}}} = 2$$

1.5

$$x^2 + (x+1)^2 = (x+2)^2$$

$$x^2 + x^2 + 2x + 1 = x^2 + 4x + 4$$

$$x^2 - 2x - 3 = 0$$

$$x_{1,2} = \frac{2 \pm \sqrt{4+12}}{2} = \frac{2 \pm 4}{2} = \begin{cases} x_1 = 3 \\ x_2 = -1 \end{cases}$$

1.6.

$$2^x > 1024$$

$$2^x > 2^{10}$$

$$x > 10$$

2.1.

if $C=0$ then $F=32$

if $C=100$ then $F=212$

$32 = a + b \cdot 0$

$32 = a$

$212 = a + b \cdot 100$

$212 = 32 + b \cdot 100$

$180 = b \cdot 100$

$1{,}8 = b$

$F = C$

$C = 32 + 1{,}8\, C$

$-32 = 0{,}8\, C$

$\underline{\underline{-40}} = C$

2.2

$f(x) = 5x + 4$

$f(3) = 5 \cdot 3 + 4 = 19$

2.3

$x^2 - 4x + 3 = 0$

$x_{1,2} = \frac{4 \pm \sqrt{16-12}}{2} = \frac{4 \pm 2}{2} = \begin{cases} x_1 = 3 \\ x_2 = 1 \end{cases}$

2.4

$10 \cdot 1{,}02^{90} = 59{,}4313$

2.5

$e^{\ln 5} = 5$

3.1.

$$\sum_{i=1}^{\infty} \frac{12}{6^i} \qquad a_n = 12 \cdot \frac{1}{6^i} \qquad a = 12 \qquad b = \frac{1}{6}$$

$$\sum_{i=1}^{\infty} \frac{12}{6^i} = \frac{12 \cdot \frac{1}{6}}{1 - \frac{1}{6}} = \frac{2}{\frac{5}{6}} = \frac{6 \cdot 2}{5} = \frac{12}{5}$$

3.2

$$\lim_{x \to 1} \frac{6^{1-x}}{x} = \lim_{x \to 1} \frac{6^{1-1}}{1} = \lim_{x \to 1} \frac{6^0}{1} = 1$$

3.3.

$f(x) = x^5 - 8$

$x = -3$

$f'(x) = 5x^4$

$f'(-3) = 5 \cdot (-3)^4 = 5 \cdot 81 = 405$

3.4

$$\frac{d}{dx} \frac{x^3 + 2x - 1}{x - 2} = \frac{(3x^2 + 2)(x - 2) - (x^3 + 2x - 1) \cdot 1}{(x-2)^2} =$$

$$\boxed{\left(\frac{f}{g}\right)' = \frac{f'g - f \cdot g'}{g^2}}$$

$$= \frac{3x^3 - 6x^2 + 2x - 4 - x^3 - 2x + 1}{(x-2)^2} =$$

$$= \frac{2x^3 - 6x^2 - 3}{(x-2)^2}$$

3.5

$$\frac{d^2}{dx^2} 4x^4 + 4x^2$$

$d_1 = 16x^3 + 8x$

$d_2 = 48x^2 + 8$

3.6

$$\frac{d}{dx}\ \frac{\ln x}{e^x} = \frac{\frac{1}{x}\cdot e^x - \ln x \cdot e^x}{(e^x)^2} = \frac{\frac{1}{x} - \ln x}{e^x}$$

3.7

$f(x) = 3x^2 - 5x + 2$

$f'(x) = 6x - 5$

$f''(x) = 6 \implies f'' > 0 \implies$ convex

$f'(x) = 0$

$6x - 5 = 0$

$x = \frac{5}{6}$ stationary point , $\implies$ local minimum

	$-\infty < x < \frac{5}{6}$	$x = \frac{5}{6}$	$\frac{5}{6} < x < +\infty$
$f(x)$	$\searrow$	local min	$\nearrow$
$f'(x)$	$-$	0	$+$
$f''(x)$	$+$	$+$	$+$

3.8.

$f(x,y) = x^2 + y^3$

$f(2,3) = 2^2 + 3^3 = 4 + 27 = 31$

3.9.

$f(x,y) = \ln(x-y) \implies x - y > 0$

$x > y$

3.10.

$$\frac{\partial}{\partial x} x^5 + xy^3 = 5x^4 + y^3$$

3.11.

$$f(x,y) = x^2y^2 + 10$$

$$f'x = 2xy^2 \qquad f'y = 2yx^2$$

$$f''x - 2y^2 \qquad f''y = 2x^2$$

$$f'x = 2xy^2 = 0 \qquad 2xy^2 = 2yx^2$$

$$f'y = 2yx^2 = 0 \qquad x = y$$

$$2x \cdot x^2 = 0$$

$$x = 0$$

$$y = 0 \quad \text{local minimum}$$

3.12.

$$\cancel{\max}\ f(x) = x^2y^2 \rightarrow \max\ \cancel{st}\ \text{s.t.} \quad x + y = 10 \qquad x + y - 10 = 0$$

$$\mathcal{L} = x^2y^2 - \lambda(x+y-10)$$

$$\left.\begin{aligned} \frac{\partial \mathcal{L}}{\partial x} &= 2xy^2 - \lambda = 0 \\ \frac{\partial \mathcal{L}}{\partial y} &= 2yx^2 - \lambda = 0 \end{aligned}\right\} \quad \begin{aligned} 2xy^2 - \lambda &= 2yx^2 - \lambda \\ y &= x \end{aligned}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = x + y - 10 = 0 \implies 2x - 10 = 0$$

$$x = 5$$

$$y = 5$$

4.1.

A · B

		1	1	7
		2	8	2
2	6	14	50	26
5	1	7	13	37
1	9	19	73	25

4.2

B · A

			2	2
			4	6
			1	3
1	9	1	39	59
2	1	2	10	16

4.3

$$\begin{bmatrix} 7{,}1 & 9{,}1 & 4{,}7 \\ 2 & 7{,}8 & 1{,}1 \\ 4 & 4{,}44 & 0 \end{bmatrix}^T = \begin{bmatrix} 7{,}1 & 2 & 4 \\ 9{,}1 & 7{,}8 & 4{,}44 \\ 4{,}7 & 1{,}1 & 0 \end{bmatrix}$$

4.4

$$\begin{bmatrix} 1 & 9 \\ 2 & 8 \end{bmatrix} \quad det = 1 \cdot 8 - 9 \cdot 2 = -10$$

5.1.

d1		1	2	3	4	5	6
d2	1	11	21	31	41	51	61
	2	12	22	32	42	52	62
	3	13	23	33	43	53	63
	4	14	24	34	44	54	64
	5	15	25	35	45	55	65
	6	16	26	36	46	56	66

sample space

5.2

Drug user	Drug test +	−
Yes (1%)	99%	1%
No (99%)	0,5%	99,5%

$$P = 1\% \cdot 99\% + 99\% \cdot 0{,}5\% = \underline{\underline{1{,}485\%}}$$

5.3

Drug user	Drug test +	−		+	−
Yes (1%)	$1 \cdot 0{,}99$	$1 \cdot 0{,}01$	=	0,99%	0,01%
No (99%)	~~99 · 0,05~~ $99 \cdot 0{,}005$	~~99 · 0,95~~ $99 \cdot 0{,}995$		0,495%	~~0,98 505%~~ 0,98 505

$$\frac{0{,}99\%}{0{,}99\% + 0{,}495\%} = \frac{2}{3}$$