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Aran, please define these terms.

Consider function f from domain D to codomain C. We say f is **injective**, or **one-to-one**, if for all x, y in D, $f(x) = f(y)$ implies $x = y$.

We say f is **surjective,** or **onto,** if $\forall\, y \in C$, there exists some $x \in D$ such that $f(x) = y$. In other words, f is sujrective if range $f = C$.

A function that is both injective and surjective is **bijective,** or a **one-to-one correspondence**.

1. For each of the following function, determine if it is injective, surjective, and bijective with the above definitions.
   a. $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = 3x + 5.$
   b. $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = |2x - 4|.$
   c. $f : \mathbb{R}^2 \to \mathbb{R}$ given by $f(x, y) = x^2 + 2y.$
   d. $f : \mathbb{R}^2 \to \mathbb{R}^2$ given by $f(x, y) = (2x - y, x + y).$

2. Let A, B be two finite sets such that $|A| = n \geq 1,\ |B| = m \geq 1$. Consider function $f : A \to B$. Prove or disprove:
   a. If f is injective, $n \leq m$.
   b. If f is surjective, $n \geq m$.
   c. If $n = m$, then f is injective iff f is surjective.

3. Consider three functions $f : A \to B,\ g : B \to C,\ h : C \to D$. Show that $(h \circ g) \circ f = h \circ (g \circ f)$. In other words, function compositions are associative.

4. Consider three functions $f : A \to B,\ g : B \to C$. Prove the following.
   a. If g, f are injective then so is $g \circ f$
   b. If $g \circ f$ are injective then so is f
   c. If g, f are surjective then so is $g \circ f$
   d. If $g \circ f$ are surjective then so is g