Question 2

Monday, October 2, 2023 10:10 AM

2. Find a basis and then the dimension of the following subspace:

$$\text{span}\{2+x^2-2x^3, 1-2x+x^2-x^3, 5+2x+2x^2-5x^3, 3+6x-3x^3\}$$

$$\left[\begin{array}{cccc|c} 2 & 0 & 1 & -2 & 0 \\ 1 & -2 & 1 & -1 & 0 \\ 5 & 2 & 2 & -5 & 0 \\ 3 & 6 & 0 & -3 & 0 \end{array}\right]$$ switch R_2 & R_1

Redo

$$\left[\begin{array}{cccc|c} 1 & -2 & 1 & -1 & 0 \\ 2 & 0 & 1 & -2 & 0 \\ 5 & 2 & 2 & -5 & 0 \\ 3 & 6 & 0 & -3 & 0 \end{array}\right]$$

$$\begin{bmatrix} 3 \\ 6 \\ 0 \\ -3 \end{bmatrix} = 3\begin{bmatrix} 2 \\ 0 \\ 1 \\ -2 \end{bmatrix} - 3\begin{bmatrix} 1 \\ -2 \\ 1 \\ -1 \end{bmatrix}$$

Elim last row

$$\left[\begin{array}{cccc|c} 1 & -2 & 1 & -1 & 0 \\ 2 & 0 & 1 & -2 & 0 \\ 5 & 2 & 2 & -5 & 0 \end{array}\right]$$

But

$$\begin{bmatrix} 5 \\ 2 \\ 2 \\ -5 \end{bmatrix} = 3\begin{bmatrix} 2 \\ 0 \\ 1 \\ -2 \end{bmatrix} - \begin{bmatrix} 1 \\ -2 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 6-1 \\ 0+2 \\ 3-1 \\ -6+1 \end{bmatrix}$$

Thus eliminate last Row

$$\left[\begin{array}{cccc|c} 1 & -2 & 1 & -1 & 0 \\ 2 & 0 & 1 & -2 & 0 \end{array}\right]$$

Thus

$\{-2x+x^2-x^3,\ 2+x^2-2x^3\}$ forms a Basis and has dim = 2