Question 9

Sunday, October 15, 2023   10:43 PM

9. The following claims are either **true or false**. Determine which case is it for each claim and prove your answer.

   (a) For any two $m \times n$ matrices A and B we have $\text{rank}(A + B) = \text{rank}(A) + \text{rank}(B)$.

   (b) For any two $m \times n$ matrices A and B we have $\text{rank}(A + B) \leq \text{rank}(A) + \text{rank}(B)$.

$3 \times 5 \qquad 3 \times 5$

a)

false

$$\text{let} \quad A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \quad \text{Rank}(A+B) = 2$$

$$\text{Rank}(A) + \text{Rank}(B) = 4 \neq 2$$

B) let A and B Be Represented as follows

$$A = \begin{pmatrix} 1 & & 1 \\ a_1 & \cdots & a_n \\ 1 & & 1 \end{pmatrix} \quad B = \begin{pmatrix} 1 & & 1 \\ b_1 & \cdots & b_n \\ 1 & & 1 \end{pmatrix} \quad A + B = \begin{pmatrix} 1 & & 1 \\ a_1 + b_1 & \cdots & a_n + b_n \\ 1 & & 1 \end{pmatrix}$$

where A is Spanned By $\{a_1, \dots, a_n\}$

where B is Spanned By $\{b_1, \dots, b_n\}$

and A + B is Spanned by $\{a_1, b_1, \dots, a_n, b_n\}$

but every Colum vector in A+B Can Be Rep. as a lin. Comb. of $\{a_1, \dots, a_n\}$ and/or $\{b_1, \dots, b_n\}$

Thus The Cols of A+B is Spanned By at

most 2n Basis vectors $\left(\text{Basis of } A = n + \text{Basis of } B = \atop n \right)$

Thus $\text{Rank}(A+B) \leq \text{Rank}(A) + \text{Rank}(B)$