# Homework 1

## Problem 1

Starting with the spin 1/2 relation $\langle \mathbf{a}\cdot\mathbf{S}\rangle_{\mathbf{n}} = (\hbar/2)(\mathbf{a}\cdot\mathbf{n})$, where $\mathbf{a}$ is an arbitrary reference vector and $\langle\cdots\rangle_{\mathbf{n}}$ denotes the expectation value evaluated for the ensemble specified by the condition $\mathrm{Prob}_{\mathbf{n}}(S_{\mathbf{n}}=\hbar/2)=1$ [see, e.g., Eq. (1.18a) and (1.20) in the Lecture Notes],

(a) compute the uncertainty $\Delta_{\mathbf{n}}(\mathbf{a}\cdot\mathbf{S})$.

(b) show that $\mathrm{Prob}_{\mathbf{n}}(S_{\mathbf{n}'}=\hbar/2) = \dfrac{1}{2}(1+\mathbf{n}\cdot\mathbf{n}')$.

## Problem 2

Is it possible to prepare a pure ensemble of spin 1/2 particles for which

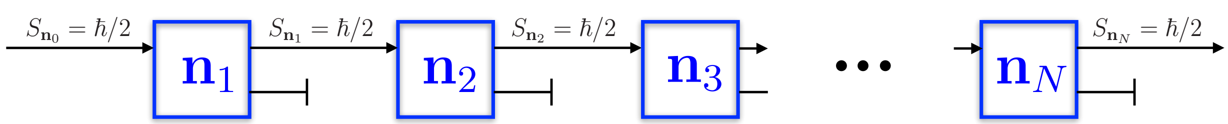
(a) $\langle S_{\mathbf{x}}\rangle = \langle S_{\mathbf{y}}\rangle = \langle S_{\mathbf{z}}\rangle = 0\,?$

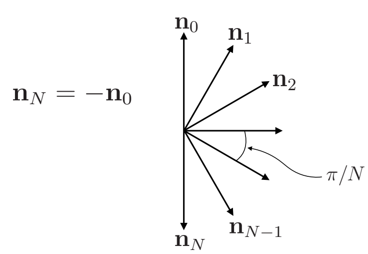
(b) $\langle S_{\mathbf{x}}\rangle + \langle S_{\mathbf{y}}\rangle + \langle S_{\mathbf{z}}\rangle = 0\,?$

(If your answer is *yes*, provide an example of an ensemble with the claimed property. If your answer is *no*, explain why not.)

## Problem 3

A filtered beam of spin 1/2 particles with $S_{\mathbf{n}_0}=\hbar/2$ is sent through $N$ consecutive Stern-Gerlach filters oriented in the directions of the unit vectors $\mathbf{n}_1, \mathbf{n}_2, \ldots, \mathbf{n}_N = -\mathbf{n}_0$ such that the angle between $\mathbf{n}_i$ and $\mathbf{n}_{i+1}$ is $\pi/N$.




Let $\mathcal{P}(N)$ be the probability that particles in the initial beam successfully pass all $N$ filters.

(a) Evaluate $\mathcal{P}(N)$ for $N=2$ and $N=3$.

(b) Find $\mathcal{P}(\infty)=\lim_{N\to\infty}\mathcal{P}(N)$ and evaluate the leading term in the expansion of $\mathcal{P}(\infty)-\mathcal{P}(N)$ in $1/N \ll 1$.
*Suggestion*: expand $\ln\mathcal{P}(N)$ to the lowest non-vanishing order in $1/N$.

## Problem 4

Measurements on spin 1/2 particles in a pure quantum state specified by the Bloch vector $\mathbf{n}$ have found

$$\langle S_{\mathbf{n}_1}\rangle_{\mathbf{n}} = \langle S_{\mathbf{n}_2}\rangle_{\mathbf{n}} = \frac{\hbar\alpha}{2}, \quad |\alpha|\le 1,$$

where the unit vectors $\mathbf{n}_{1,2}$ satisfy $\mathbf{n}_1\cdot\mathbf{n}_2=\alpha^2$. Express the Bloch vector $\mathbf{n}$ via $\mathbf{n}_1$, $\mathbf{n}_2$, and $\alpha$. Make sure that the expression you found has correct $\alpha\to 0$ and $\alpha\to 1$ limits.
*Suggestion*: write $\mathbf{n}$ as a linear combination of the three mutually orthogonal vectors $\mathbf{n}_1+\mathbf{n}_2$, $\mathbf{n}_1-\mathbf{n}_2$, and $\mathbf{n}_1\times\mathbf{n}_2$.