10/16

Friday, October 13, 2023 11:23

1. Let $T \in \mathscr{L}(V, W)$, $S \in \mathscr{L}(W, U)$. Let $\mathscr{B}$, $\mathscr{C}$, $\mathscr{D}$ be ordered basis for V, W, U, all finite-dimentional. Show that $[S \circ T]_{\mathscr{B} \to \mathscr{D}} = [S]_{\mathscr{C} \to \mathscr{D}} [T]_{\mathscr{B} \to \mathscr{C}}$
2. Let A, B be matrices with appropriate sizes. Show that
   a. Col(AB) $\subseteq$ Col(A)
   b. Ker(B) $\subseteq$ Ker(AB)
3. Let $A \in M_n(\mathbb{R})$. Show that if $Ax = y$ implies $x = y$ then $A = I_n$.
4. Some properties on inverses
   a. Show that $\left(A^{-1}\right)^{-1} = A$ if A is invertible.
   b. Let A, B be invertible squared matrices. Is AB in vertible?
5. True/False
   a. If A^2 is invertible then so is A.
   b. If $A^2 = I_n$, then $A = I_n$ or $A = -I_n$
   c. If matrix $\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$ is invertible then the submatrix $\begin{pmatrix} a & b \\ d & e \end{pmatrix}$ is also invertible.
   d. Any invertible matrix is a product of some elementary matrices

We used these two results in class but I can tell people were not very convinced. I want them to show this in studio.