MATH-1564-K Linear Algebra with Abstract Vector Spaces

Definitions

Notations

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 $\bullet \ x \mapsto y$ $\bullet \ f \circ g \circ h$

• $x \sim y$

• S/ ~

 $\bullet \ \{c_i\}_{i=1}^n$

• $\operatorname{span}(v_1,\ldots,v_n)$

 \bullet $A \times B$

 \bullet U+W

 \bullet x + U

• $M_{m \times n}(\mathbb{R})$

• $M_n(\mathbb{R})$

• $\mathbb{R}_n[x], \mathcal{P}_n(\mathbb{R})$

• $\mathbb{R}[x], \mathcal{P}(\mathbb{R})$

• $[v]_{\mathcal{B}}$, $\operatorname{Rep}_{\mathcal{B}}(v)$

 $\bullet \ V \cong W$

• Col(A)

• Row(A)

• Nul(A)

• rank(A)

 \bullet A^T

 $\bullet \ A^{-1}$

• *I*_n

• $GL_n(\mathbb{R})$

• $id: S \to S$

• $\mathcal{L}(V,W)$

• $\mathcal{L}(V), \mathcal{L}(V, V)$

• $\mathcal{L}(V,\mathbb{R})$

 \bullet ImT

 \bullet KerT

• $\dim U$

• $[T]_{\mathcal{B} \to \mathcal{C}}, [T]_{\mathcal{C}}^{\mathcal{B}}$

• $[T]_{\mathcal{B}}$

 $\bullet <\cdot,\cdot>,< v_1,v_2>$

 $\bullet \ \|\cdot\|,\|x\|$

• $\operatorname{Proj}_u(v)$

• SBWOC

• TFAE

iff

1. A binary relation is

2. An equivalence relation is

3. Given \sim an equivalence relation on set S, the equivalence class with representative $x \in S$ is

4. Given \sim an equivalence relation on set $S, S/\sim$ is

5. A linear system of equations is

6. List all three elementary row operations.

7. Matrix is in ref if; in rref if

8. Describe Gauss-Jordan elimination.

9. Let $u, v \in \mathbb{R}^n$. Their addition is defined to be

10. Let $u \in \mathbb{R}^n$ and $k \in \mathbb{R}$. ku is defined to be

11. Let $u, v \in \mathbb{R}^n$. Their dot product $u \cdot v$ is

12. Let $A \in M_{m \times n}(\mathbb{R})$ and $v \in \mathbb{R}^n$. Av is defined to be

13. Set V with two operations +, over field \mathbb{F} is a vector space over \mathbb{F} if

- 14. Subset U in vector space V is a *subspace* if
- 15. A linear combination of vectors v_1, \ldots, v_n is of the form
- 16. A homogenous system is
- 17. The span of v_1, \ldots, v_n is the set
- 18. Vectors v_1, \ldots, v_n are linearly independent if
- 19. They are linearly dependent if
- 20. A vector space V is finite-dimensional if
- 21. An ordered basis of a vector space V is
- 22. Let \mathcal{B} be an ordered basis for finite dimensional vector space V. For vector $v \in V$, its coordinate vector w.r.t. basis \mathcal{B} is
- 23. The standard basis for \mathbb{R}^n is
- 24. The dimension of vector space V is
- 25. Let $A \in M_{m \times n}(\mathbb{R})$. Define Col(A), Nul(A), Row(A).
- 26. Let $A \in M_{m \times n}(\mathbb{R})$ and $B \in M_{n \times p}(\mathbb{R})$. Their product AB is
- 27. Let $A, B \in M_{m \times n}(\mathbb{R})$. Their sum A + B is
- 28. Let $A \in M_{m \times n}(\mathbb{R})$ and $k \in \mathbb{R}$ then kA is
- 29. Let $A \in M_{m \times n}(\mathbb{R})$. Its transpose, A^T , is
- 30. Function $f: \mathcal{D} \to \mathcal{C}$ is injective/1-to-1 if, surjective/onto if and bijetive/1-to-1 correspondence if
- 31. Function T from vector space V to vector space W, both over \mathbb{R} , is a \mathbb{R} -linear map if
- 32. Let $A \in M_{m \times n}(\mathbb{R})$ then we can define a linear map associated with A by
- 33. A vector space isomorphism is
- 34. Two vector spaces are said to be isomorphic if
- 35. $\mathcal{L}(V,W)$ is used to denote
- 36. The *identity map* on set S is
- 37. We say linear map $T: V \to W$ is invertible if
- 38. $Row\ rank\ of\ matrix\ A$ is
- 39. $Column\ rank\ of\ matrix\ A$ is
- 40. Rank of matrix A is

- 41. Let $T \in \mathcal{L}(V, W)$. Image of T is and Kernel of T is
- 42. Let $T \in \mathcal{L}(V, W)$. rank T is defined to be
- 43. $E \in M_n(\mathbb{R})$ is an elementary matrix if
- 44. Matrix $A \in M_n(\mathbb{R})$ is invertible if
- 45. Let V be a vector space over \mathbb{R} with basis $B = \langle b_1, \ldots, b_n \rangle$ and W be a vector space over \mathbb{R} with basis $C = \langle c_1, \ldots, c_n \rangle$. Then the matrix associated with T with respect to basis B and C, denoted by $[T]_{B \to C}$ is
- 46. Let $T \in \mathcal{L}(V,V)$ and let B,C be two different bases for V. We have the identity $[T]_{B\to B} =$
- 47. Let B, C be two bases for vector space V. Then the *change of bases matrix* from B to C is given by
- 48. Let V be a vector space over \mathbb{R} . Function $\langle \cdot, \cdot \rangle : V \times V \to \mathbb{R}$ is an inner product if
- 49. Let V be a vector space over \mathbb{R} . Function $\|\cdot\|: V \to \mathbb{R}$ is a norm if
- 50. Given an inner product $\langle \cdot, \cdot \rangle$, the norm induced by it is given by
- 51. Two vectors in inner product space V are said to be orthogonal if
- 52. Consider vector v and non-zero vector u in inner product space V. The orthogonal projection of v onto the direction of u is given by
- 53. Vectors $\{v_1, \dots, v_n\}$ in inner product space are said to form an orthonormal set if
- 54. An orthonormal basis for vector space V is
- 55. Describe Gram-Schmidt process.
- 56. Let U be a subspace with an orthonormal basis $\{q_1, \ldots, q_m\}$ in vector space V. Then for any vector $v \in V$, the orthogonal projection of v onto subspace U is given by