1)

n^{th} gen	$n+1$ gen
W	W/x
L	L/x
t_{ox}	t_{ox}/x
V_{DD}	V_{DD}/x
V_{th}	V_{th}/x

$$f = \frac{1}{t_{delay}} = \frac{I_{ON}}{C_L V_{DD}} \quad , \quad P_{Total} = \underset{\text{Power per transistor}}{P} \cdot \underset{\text{\# of transistor}}{N}$$

$$f_n = \frac{I_{ON,n}}{C_{L,n} \cdot V_{DD,n}}$$

$$f_{n+1} = \frac{I_{ON,n+1}}{C_{L,n+1} V_{DD,n+1}} = \frac{\frac{I_{ON,n}}{x}}{\frac{C_{L,n}}{x} \cdot \frac{V_{DD,n}}{x}} = x \frac{I_{ON,n}}{C_{L,n} \cdot V_{DD,n}}$$

$$= x \cdot f_n$$

$$P_{Total,n} = P_n \cdot N_n = C_{L,n} \cdot V_{DD,n}^2 \cdot f_n \cdot \frac{A_{chip}}{W_n \cdot L_n}$$

$$P_{Total,n+1} = C_{L,n+1} \cdot V_{DD,n+1}^2 \cdot f_{n+1} \cdot \frac{A_{chip}}{W_{n+1} \cdot L_{n+1}}$$

$$= \frac{C_{L,n}}{\cancel{x}} \cdot \left(\frac{V_{DD,n}}{\cancel{x}}\right)^2 \cdot f_n \cdot x \cdot \frac{A_{chip}}{\frac{W_n}{\cancel{x}} \cdot \frac{L_n}{\cancel{x}}}$$

$$= C_{L,n} \cdot V_{DD,n}^2 \cdot f_n \cdot \frac{A_{chip}}{W_n \cdot L_n}$$

$$= P_{Total,n}$$

Frquency increased by a factor of x, but the total power consumption of the chip and the chip area is still the same across generations, which means the chip power density remains the same throughout generations.

2)

$$P_{Total,n} = P_{Total,n+1}$$

$$P_n N_n = P_{n+1} \cdot N_{n+1}$$

$$\cancel{C_{L,n}} \cancel{V_{DD,n}}^2 \cancel{f_n} \cdot \frac{\cancel{A_{chip}}}{\cancel{W_n} \cdot \cancel{L_n}} = \frac{\cancel{C_{L,n}}}{x} \cancel{V_{DD,n}}^2 \cancel{f_n} \cdot k \cdot \frac{\cancel{A_{chip}}}{\frac{\cancel{W_n}}{x} \cdot \frac{\cancel{L_n}}{x}}$$

$$1 = \frac{1}{x} \cdot x^2 k = xk$$

$$k = \frac{1}{x}$$

The frequency is scaled by a factor of 1/x

3) There will always be a small current flowing from the drain to source, regardless of the input voltage. Therefore, the inverter will always dissipate power even if the input voltage is 0V.

4) $t_1 = 20ns \quad , \quad P_1 = 1.3W$

$$t_{20} = \frac{t_1}{x^{20-1}} = \frac{20ns}{2^{19}} = 3.815 \times 10^{-14} s$$

$$P_{20} = \frac{P_1}{x^{2(20-1)}} = \frac{1.3W}{2^{38}} = 4.729 \times 10^{-12} W$$