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Tuesday, September 26, 2023 17:23

1. Consider the following maps. Determine if there are linear maps. For those that are, determine if they are injective, surjective, or bijective.

   i. $T : M_2(\mathbb{R}) \to \mathbb{R}^2$ given by: $T\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a \\ d \end{pmatrix}$

   ii. $T : \mathbb{R}_2[x] \to M_2(\mathbb{R})$ given by: $T(a + bx + cx^2) = \begin{pmatrix} a-b & a+b \\ c-a & c+b \end{pmatrix}$.

   iii. $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by: $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+1 \\ y+1 \end{pmatrix}$.

   iv. $T : \mathbb{R}_n[x] \to \mathbb{R}_n[x]$ given by: $[Tp](x) = p'(x)$.

   v. $T : \mathbb{R}_2[x] \to \mathbb{R}_2[x]$ given by: $[Tp](x) = p^2(x)$.

2. Is there a linear transformation from $R^2 \to R^3$ such that

   $$T\begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad T\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}, \quad \text{and} \quad T\begin{pmatrix} 5 \\ 4 \end{pmatrix} = \begin{pmatrix} 5 \\ -1 \\ -1 \end{pmatrix}$$

3. Let $T : V \to W$ be a linear map from vector space V, W over R. Let $S \subset V$ be a collection of vectors $\{v_1, \ldots, v_n\}$.
   a. If v is a linear combination of $\{v_1, \ldots, v_n\}$, is $T(v)$ a linear combination of $\{T(v_1), \ldots, T(v_n)\}$?
   b. If $T(v)$ is a linear combination of $\{T(v_1), \ldots, T(v_n)\}$, , is v a linear combination of $\{v_1, \ldots, v_n\}$?
   c. Redo part (a), part (a) if T is injective.
   d. Redo part (a), part (a) if T is surjective.