Question 3

Sunday, November 19, 2023 5:19 PM

3. Consider the following matrices.

$$A = \begin{pmatrix} 8 & 3 & -3 \\ -6 & -1 & 3 \\ 12 & 6 & -4 \end{pmatrix} \quad B = \begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \quad C = \begin{pmatrix} 4 & 0 & 3 \\ 0 & 5 & 0 \\ 3 & 0 & -4 \end{pmatrix}$$

$$E = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & -1 \end{pmatrix} \quad F = \begin{pmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix} \quad G = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 5 \\ 0 & 0 & 0 & 3 \end{pmatrix}$$

For each of these matrices solve the following:

(a) Find all of the eigenvalues of the matrix, determine the geometric multiplicity and the algebraic multiplicity of each eigenvalue.

(b) Determine if the matrix is diagonalizable. If it is then find a diagonal matrix D and an invertible matrix P so that the matrix is equal to PDP^{-1}.

a) $\begin{bmatrix} 8 & 3 & -3 \\ -6 & -1 & 3 \\ 12 & 6 & -4 \end{bmatrix}$ $\det\left(\begin{bmatrix} 8-\lambda & 3 & -3 \\ 6 & -1-\lambda & 3 \\ 12 & 6 & -4-\lambda \end{bmatrix}\right) = P_A(\lambda)$

$P_A(\lambda) = -\lambda^3 + 3\lambda^2 - 4$

$(\lambda+1)(\lambda-2)^2 = P_a(\lambda)$

B) yes

$A = \begin{bmatrix} -1 & 1 & 1 \\ 2 & 0 & -1 \\ 0 & 2 & 2 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 1 & 1 \\ 2 & 0 & -1 \\ 0 & 2 & 2 \end{bmatrix}^{-1}$

for $\lambda = -1$, AM $= 1$, GM $= 1$

$E_{\lambda=1} = \text{null}\begin{bmatrix} 9 & 3 & -3 \\ -6 & 0 & 3 \\ 12 & 6 & -3 \end{bmatrix} = \text{span}\left\{\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}\right\}$

for $\lambda = 2$, AM $= 2$, GM $= 2$

$E_{\lambda 2} = \text{null}\begin{bmatrix} 6 & 3 & -3 \\ -6 & -3 & 3 \\ 12 & 6 & -6 \end{bmatrix} = \text{span}\left\{\begin{pmatrix} -1/2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1/2 \\ 0 \\ 1 \end{pmatrix}\right\}$

$6x_1 = 3x_3 - 3x_2$

$x_1 = \frac{1}{2}x_3 - \frac{1}{2}x_2$

$x_2 = x_2$

$x_3 = x_3$

$\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$ $P_B(\lambda) = -\lambda^3 - 3\lambda^2 + 9\lambda + 27$

$(\lambda+3)^2(\lambda-3)$

for $\lambda=-3$, $AM=2$, $GM=2$

$$E_{\lambda=-3} = \text{null}\left\{\begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}\right\} = \text{span}\left\{\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}\right\}$$

for $\lambda=3$, $AM=1$, $GM=1$

$$E_{\lambda=3} = \text{null}\begin{bmatrix} -4 & 2 & 2 \\ 2 & -4 & 2 \\ 2 & 2 & -4 \end{bmatrix} = \text{span}\left\{\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}\right\}$$

B) yes

$$B = \begin{bmatrix} -1 & -1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} -1 & -1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}^{-1}$$

$$C = \begin{pmatrix} 4 & 0 & 3 \\ 0 & 5 & 0 \\ 3 & 0 & -4 \end{pmatrix}$$

$$P_C(\lambda) = (5-\lambda)\begin{vmatrix} 4-\lambda & 3 \\ 3 & -4-\lambda \end{vmatrix} = (5-\lambda)\left[(4-\lambda)(-4-\lambda)-9\right]$$

$$-16+\lambda^2-9$$

$$\lambda^2-25$$

$$(5-\lambda)(\lambda-5)(\lambda+5)=0$$

$$(\lambda-5)^2(\lambda+5)$$

B) $$C = \begin{bmatrix} 3 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & -3 \end{bmatrix}\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & -5 \end{bmatrix}\begin{bmatrix} 3 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & -3 \end{bmatrix}^{-1}$$

for $\lambda=5$, $AM=2$, $GM=2$

$$E_{\lambda=5} = \text{null}\left\{\begin{matrix} -1 & 0 & 3 \\ 0 & 0 & 0 \\ 3 & 0 & -9 \end{matrix}\right\} = \text{span}\left\{\begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\right\}$$

for $\lambda=-5$, $AM=1$, $GM=1$

$$E_{\lambda=-5} = \text{null}\begin{bmatrix} 9 & 0 & 3 \\ 0 & 10 & 0 \\ 3 & 0 & 1 \end{bmatrix} = \text{span}\left\{\begin{pmatrix} 1 \\ 0 \\ -3 \end{pmatrix}\right\}$$

$$E = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & -1 \end{pmatrix}$$

B) $$E = \begin{bmatrix} 1 & 1/6 & 1 \\ 0 & -1/3 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} 1 & 1/6 & 1 \\ 0 & -1/3 & 1 \\ 0 & 1 & 0 \end{bmatrix}^{-1}$$

$$P_E(\lambda) = (\lambda-2)(\lambda-1)(\lambda+1)$$

for $\lambda=2$ $AM=1$, $GM=1$

for $\lambda=1$ $AM=1$ $GM=1$

for $\lambda=-1$ $Am=1$, $Gm=1$

$$F = \begin{pmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix}$$

B) Not <u>Diagonalizable</u>

$P_F(\lambda) = (\lambda-2)^3(\lambda-3)$

for $\lambda=2$, $Am=3$, $Gm=1$

$$E_{\lambda=2} = \text{null}\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \text{Span}\left\{\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}\right\}$$

for $\lambda=3$ $Am=1$, $Gm=1$

$$E_{\lambda=3} = \text{null}\begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \text{Span}\left\{\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}\right\}$$

$$G = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 5 \\ 0 & 0 & 0 & 3 \end{pmatrix}$$

B) $G = \begin{bmatrix} 0 & 0 & -i & i \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & -i \end{bmatrix}\begin{bmatrix} 0 & 0 & -i & i \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}^{-1}$

$P_g(\lambda) = (\lambda-2)(\lambda-3)(\lambda^2+1)$

for $\lambda=2$, $Am=1$, $Gm=1$

$$\text{null}\begin{bmatrix} -2 & 1 & 0 & 0 \\ -1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \text{Span}\left\{\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}\right\}$$

for $\lambda=3$, $Am=1$, $Gm=1$

null$[-3 \;\; 1 \;\; 0 \;\; 0$

$$\mathrm{Nul}\begin{bmatrix} -1 & -3 & 0 & 0 \\ 0 & 0 & -1 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \mathrm{Span}\left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

for $\lambda = i$, $AM = 1$, $GM = 1$

for $\lambda = -i$, $AM = 1$, $GM = 1$