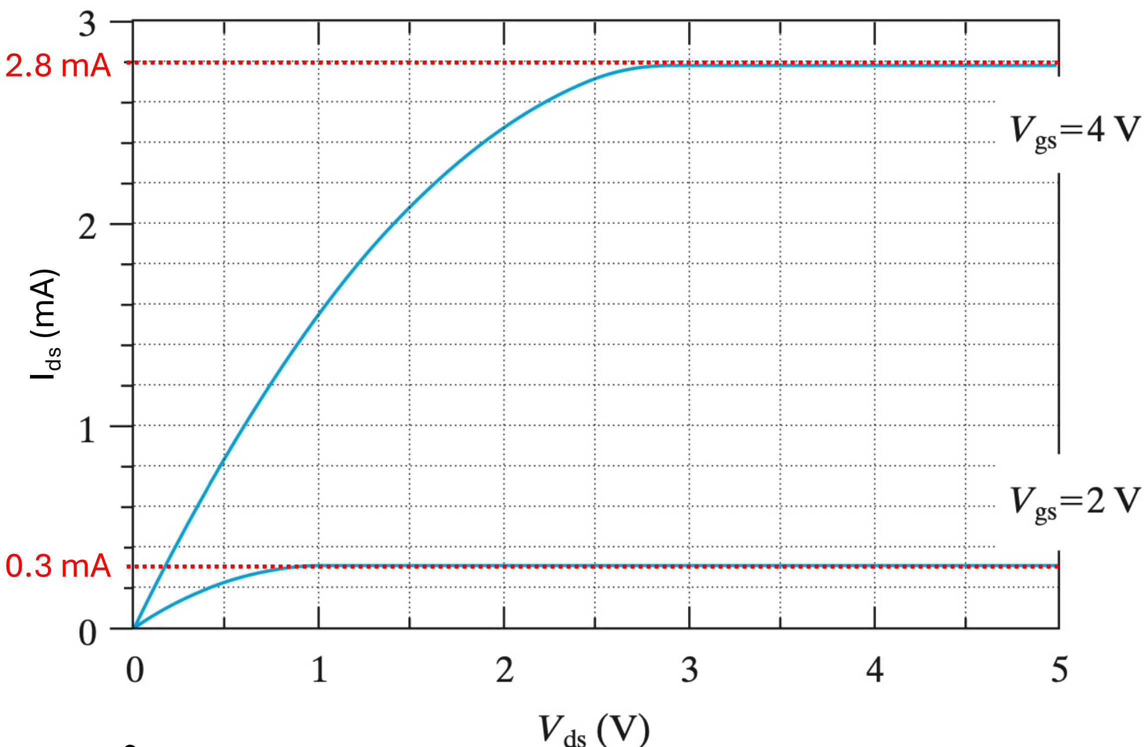
1. (20 points) The output characteristics of a MOSFET are shown below.
   a. (8 points) From the I-V characteristics below, determine the threshold voltage of the device by comparing the saturation current for the two operating conditions.
   b. (7 points) Find the Beta-factor (β) of the device.
   c. (5 points) Determine the drain current of the device when V_{ds} = 2V and V_{gs} = 3.5V.



a) I_{DS} for Saturation is

$$I_{DS} = \frac{\beta}{2}(V_{gs} - V_{tn})^2$$

$$\frac{I_{DS1}}{I_{DS2}} = \frac{\frac{\cancel{\beta}}{\cancel{2}}(V_{gs_1} - V_t)^2}{\frac{\cancel{\beta}}{\cancel{2}}(V_{gs_2} - V_t)^2} \Rightarrow \frac{2.8\,mA}{.3\,mA} = \frac{(4 - V_t)^2}{(2 - V_t)^2}$$

$$\sqrt{9.\overline{33}} = \frac{4 - V_t}{2 - V_t} \Rightarrow 3.055 = \frac{4 - V_t}{2 - V_t}$$

$$6.11 - 3.055 V_t = 4 - V_t$$

$$2.055 V_t = 2.11$$

$$\boxed{V_t = 1.0267\,V}$$

Made with Goodnotes

b) $I_{DS} = \frac{\beta}{2}(V_{gs} - V_{tn})^2$

$I_{DS} = .0028 = \frac{\beta}{2}(4 - 1.0267)^2$

$\Rightarrow$

$\boxed{\beta = .000633484 9 \ A/V^2}$

c) $V_{DS} = 2V \quad V_{gs} = 3.5V$

$3.5 - V_{tn} = 2.47$

$V_{DS} < V_{gs} - V_{tn}$

$\Rightarrow$ linear Region

$I_{DS} = \beta\left[(V_{gs} - V_t) - \frac{V_{DS}}{2}\right]V_{DS}$

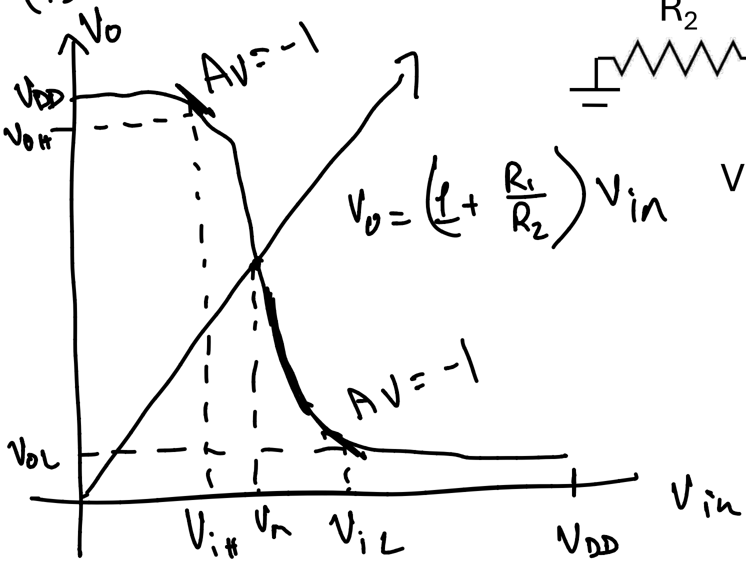
$\beta\left(2.47 - \frac{1}{2}\right)(2)$

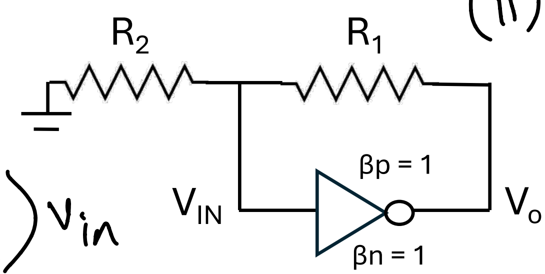
$\boxed{= .0025A = I_{DS}}$

Symmetric

2. (a) (10 points) Consider the inverter circuit shown below. Assume that Vtn = |Vtp|.
   i. (2 points) Draw the VTC of the inverter.
   ii. (2 points) Write down the relationship between V_{IN} and V_O using voltage division.
   iii. (3 points) On the same graphs as (i) now sketch the relationship between V_{IN} and V_O through the resistor network.
   iv. (3 points) From these two plots show the final voltage where the output, V_O settles down (equilibrium point). Assume that the value of R_2 is very high and it does not disrupt the normal operation of the inverter.

(i)





(ii)

$$V_{in} = \frac{R_2}{R_1 + R_2} V_O$$

$$\Rightarrow V_{in} = \frac{1}{1 + \frac{R_1}{R_2}} V_O$$

$$V_O = \left(1 + \frac{R_1}{R_2}\right) V_O$$

(IV) if $R_2 >> R_1 \Rightarrow V_O = (1)\, V_{in}$

$$V_M = \frac{V_{DD} - \cancel{|V_{tp}|} + \cancel{V_{tn}}}{2} \qquad \text{since } \sqrt{\frac{\beta_n}{\beta_p}} = 1$$

$$V_M = \frac{1}{2} V_{DD}$$

$\Rightarrow$ equilibrium point is intersection of V_O w.r.t. V_{in} and VTC of inverter

$$\Rightarrow \boxed{V_O = \frac{1}{2} V_{DD}}$$

(b) (10 points) In part (a), if R1 is a variable resistor and its value increases, how does the final value of V_O change? What is V_O for the two limiting cases: (i) when R1 is 0, and (ii) when R1 is ∞

Given that $V_O = \left(1 + \frac{R_1}{R_2}\right) V_{in}$ By voltage Division

V_O will increase as R_1 increases.

(i) When R_1 is 0, $V_O = V_{in}$

and the intersection of the curves in 2a(III) will be $\frac{1}{2}$ the supply voltages (V_m or tripping point of inverter)

(ii) When $R_1 \to \infty$ $\lim_{R_1 \to \infty} V_O = \infty$, But in our case, the slope will grow to ∞ or practically a vertical line. This means that V_O will be the supply voltage fully when R_1 is ∞ and will behave like a standard inverter.

3. (a) (12 points) Consider an NMOS transistor with $\beta_n = 0.2\ mA/V^2$ and $V_{tn} = 0.5V$ and the following terminal voltages. Identify the region of operation of the device in each of these configurations and determine the current through the device in these configurations:

   i. $V_G = 1.2$ V, $V_D = 0.8$ V, $V_S = 0.2$ V
   ii. $V_G = 0.5$ V, $V_D = 0.9$ V, $V_S = 0.1$ V
   iii. $V_G = 1.0$ V, $V_D = 0.4$ V, $V_S = 0.2$ V

(i) $V_g = 1.2V \quad V_D = .8V \quad V_s = .2V$

$V_{gS} = 1V \quad V_{DS} = .6V \quad V_{gt} = V_{gs} - V_{tn} = \frac{1}{2}$

$V_{gs} > V_{tn} \longrightarrow$ Device ON

$V_{DS} > V_{gS} - V_{tn} \longrightarrow$ Saturation Region

$I_{DS} = \frac{\beta}{2}(V_{gs} - V_{tn})^2 \Rightarrow \frac{.2}{2}\left(\frac{1}{2}\right)^2 = .1(.25)$

$I_{DS} = .025\,mA$

(ii) $V_g = .5 \quad V_D = .9 \quad V_s = .1$

$V_{gS} = .4V \quad V_{DS} = .8V$

$V_{gS} < V_{tn} \longrightarrow$ Device off $\Rightarrow$ Cut-off Region

$I_{DS} \simeq 0A$

(iii) $V_g = 1 \quad V_D = .4 \quad V_s = .2$

$V_{gS} = .8 \quad V_{DS} = .2 \quad V_{gt} = V_{gs} - V_{tn} = .8 - .5 = .3$

$V_{gS} > V_{tn} \longrightarrow$ on

$V_{DS} < V_{gt} \Rightarrow$ linear Region

$I_{DS} = .5[(.8 - .5) - .1](.2) = \boxed{.02\,mA}$

$$V_M = \frac{V_{DD} - |V_{tp}| + \sqrt{\beta_n/\beta_p}\,V_{tn}}{1 + \sqrt{\beta_n/\beta_p}}$$

(b) (8 points) A CMOS inverter is designed with NMOS and PMOS transistors where $V_{tn}=|V_{tp}|=0.6V$. Assume V_{DD} = 3.3V.
What is the inverter switching threshold (also called the trip point, V_M) voltage for the following three cases:

- βn= 0.250 mA/V^2, βp= 0.125 mA/V^2 2
- βn= 0.125 mA/V^2, βp= 0.250 mA/V^2 1/2
- βn= 0.250 mA/V^2, βp= 0.250 mA/V^2 1

Comment on how the inverter's V_M changes as the β-factor changes.

Case 1: $\sqrt{\beta_n/\beta_p} = \sqrt{\frac{.250}{.125}} = \sqrt{2}$

$$V_M = \frac{3.3 - .6 + \sqrt{2}(.6)}{1+\sqrt{2}} = \boxed{1.4698\ V}$$

Case 2: $\sqrt{\beta_n/\beta_p} = \sqrt{1/2} \Rightarrow \frac{\sqrt{2}}{2}$

$$V_M = \frac{3.3 - .6 + \frac{\sqrt{2}}{2}(.6)}{1+\sqrt{2}/2} = \boxed{1.8301\ V}$$

Case 3: $\sqrt{\frac{\beta_n}{\beta_p}} = 1$

$$V_M = \frac{3.3 - 0.6 - 0.6}{2} = \boxed{1.65\ V}$$

- When the Ratio of β_n/β_p is grater than 1, the tripping point is less than $V_{DD}/2$

- Whereas when the Ratio is less than 1, the tripping point is Grater than $V_{DD}/2$.

When β_n/β_p is Not 1, there is Bias in the inverter Characteristics.

4. (20 points) The output conductance of an NMOS represents the sensitivity of the drain current, I_{ds} to the drain-to-source voltage, V_{ds}. It is given by the following:

$$g_{ds} = \frac{\partial I_{ds}}{\partial V_{ds}}$$

a) Consider that the NMOS is operating in the linear region. Derive an expression for the output conductance.
b) If the Beta-factor is given as β = 0.2mA/V², determine the output conductance when V_{gs} = 3.3 V, V_T = 1.2 V and V_{ds} = 1.0 V.
c) As the V_{ds} increases, the NMOS approaches saturation. How does the output conductance change?
d) Derive an expression for (g_m/g_{ds}) of the transistor in the linear region in the condition when $V_{ds} << (V_{gs} - V_T)$. Here:

$$g_m = \frac{\partial I_{ds}}{\partial V_{gs}}$$

a) $g_{ds} = \frac{\partial I_{DS}}{\delta V_{DS}}$

$I_{DS} = \beta [V_{GS} - V_{tn} - \frac{V_{DS}}{2}) V_{DS}$

$\boxed{g_{ds} = \beta (V_{GS} - V_{tn}) - \frac{1}{4}\beta V_{DS}}$

b) $g_{ds} = .2 (3.3 - 1.2 - .25) \Rightarrow \boxed{.37 \frac{mA}{V}}$

c) as V_{DS} grows, g_{ds} deceases and eventually Reaches zero as $\frac{\partial I_{DS}}{\partial V_{DS}}$ is zero for a nMOS operating in saturation as $I_{DS} = \frac{\beta}{2}(V_{GS} - V_{tn})^2$, no V_{DS} to differentiate on.

D) $I_{DS} = \beta \left(V_{gs} - V_{tn} - \frac{V_{DS}}{2} \right) V_{DS}$

when $V_{DS} \ll V_{gs} - V_{tn}$ can be omitted

$$I_{DS} = \beta (V_{gs} - V_{tn}) V_{DS}$$

$$g_{ds} = \frac{\partial I_{DS}}{\partial V_{DS}} = \beta (V_{gs} - V_{tn})$$

$$g_m = \frac{\partial I_{DS}}{\partial V_{gs}} = \beta V_{DS}$$

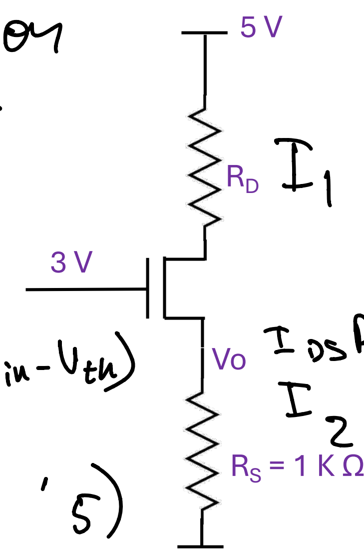
$$\frac{g_m}{g_{ds}} = \frac{\cancel{\beta} V_{DS}}{\cancel{\beta} (V_{gs} - V_{tn})} \Rightarrow \boxed{\frac{V_{DS}}{V_{gs} - V_{tn}}}$$

5. (20 points) In the circuit of a non-ideal resistive inverter shown below the NMOS transistor has the following parameters:

$\beta_n = 2m\ A/V^2$; $V_T = 1V$.

(i) (12 points) Determine the voltage V_O when the transistor operates in saturation
(ii) (8 points) Determine the maximum value of R_D needed to ensure that transistor operates in saturation.

(i) When in Saturation The Device is On $\Rightarrow I_1 = I_2$



I_1

$I_{DS} R_S$

I_2

$$V_0 = \frac{R_S}{R_D + R_S}(V_{in} - V_{tn}) \quad 5)$$

$$= \frac{1}{1 + \frac{R_D}{R_S}} \, 4$$

$$V_0 = \frac{4}{1 + R_D/1000}$$

$$I_{DS} = \beta(V_{gs} - V_{tn})^2$$

$$V_{gs} = 3 - I_{DS} R_S$$

$$I_{DS} = (2)(2 - I_{DS}(1000))^2$$

$$I_{DS} = 2 \quad 4 - 4000 I_{DS} + 1000^2 I_{DS}$$

$$V_0 = I_{DS}(1000) \quad \boxed{8\,V}$$

(ii) $V_{DS} > V_{gs} - V_{tn}$

$R_D =$

$1000\,\Omega$

$| \, ($