Question 4

Sunday, November 19, 2023 6:50 PM

4. Consider the matrix E from Q1

$$E = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & -1 \end{pmatrix}$$

(a) Find the eigenvalues of E^2. Is E^2 diagonalizable?

$E = \begin{bmatrix} 1 & 1/6 & 1 \\ 0 & -1/3 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1/6 & 1 \\ 0 & -1/3 & 1 \\ 0 & 1 & 0 \end{bmatrix}^{-1}$

$P \quad D \quad P^{-1}$

$E^2 = (PDP^{-1})(PDP^{-1}) \Rightarrow PDP^{-1}PDP^{-1} = PD^2P^{-1}$

$D^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

Diagonizable ✓ , e-vals: 1, 4

(b) Find the eigenvalues of E^{10}. Is E^{10} diagonalizable?

yes E^{10} Diagonizable

e-vals = $1, 2^{10} \Rightarrow 1, 1024$

(c) Find the eigenvalues of $E^3 - 5E^2 + 2E + 3I$. Is $E^3 - 5E^2 + 2E + 3I$ diagonalizable?

$\begin{bmatrix} 1 & & \\ & -1 & \\ & & 8 \end{bmatrix} - \begin{bmatrix} 5 & & \\ & 5 & \\ & & 20 \end{bmatrix} + \begin{bmatrix} 2 & & \\ & -2 & \\ & & 4 \end{bmatrix} + \begin{bmatrix} 3 & & \\ & 3 & \\ & & 3 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & -5 \end{bmatrix} \rightarrow PD^3P^{-1} - 5PD^2P^{-1} + 2PDP^{-1} + 3I_n$

$P(D^3 - 5D^2 + 2D + 3I_n)P^{-1}$

Matrix e-vals: 1, -5 $\Rightarrow$ Diagonizable ✓

(d) Is E invertible? If so, find the eigenvalues of E^{-1}. Is E^{-1} diagonalizable?

$E = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & -1 \end{pmatrix}$ $\det E = -2 \neq 0 \therefore$ E is invertible

If $E\vec{v} = \lambda\vec{v}$

$[E = PDP^{-1}]^{-1} \Rightarrow E^{-1} = P^{-1^{-1}} D^{-1} P^{-1} \Rightarrow E^{-1} = PD^{-1}P^{-1}$

Diagonizable

$D^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1/2 \end{bmatrix}$ e-vals: $1, -1, 1/2$

$E = \begin{bmatrix} 1 & 1/6 & 1 \\ 0 & -1/3 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1/6 & 1 \\ 0 & -1/3 & 1 \\ 0 & 1 & 0 \end{bmatrix}^{-1}$

$P \quad D \quad P^{-1}$

(e) Compute E^5.

$E^5 = PD^5P^{-1} \Rightarrow \begin{bmatrix} 1 & 1/6 & 1 \\ 0 & -1/3 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 32 \end{bmatrix} \begin{bmatrix} 1 & 1/6 & 1 \\ 0 & -1/3 & 1 \\ 0 & 1 & 0 \end{bmatrix}^{-1}$

$= \begin{bmatrix} 1 & 31 & 10 \\ 0 & 32 & 11 \\ 0 & 0 & -1 \end{bmatrix}$