Question 2

Sunday, December 3, 2023 4:12 PM

2. Orthogonally diagonalize the following matrices

$$A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}$$

$$\det(A - \lambda I) = P(\lambda)$$

$$\begin{array}{ccccc} -\lambda & 1 & 1 & -\lambda & 1 \\ 1 & -\lambda & 1 & 1 & -\lambda \\ 1 & 1 & -\lambda & 1 & 1 \\ - & - & -\ + & + & + \end{array}$$

$$-\lambda^3 + 1 + 1 + \lambda + \lambda + \lambda$$

$$-\lambda^3 + 3\lambda + 2 = P(\lambda)$$

$$\lambda(-\lambda^2 + 3) + 2 = P(\lambda)$$

$$\lambda = -1, \ 2$$

$$E_{\lambda=1} = \text{null}\begin{bmatrix} +1 & 1 & 1 \\ 1 & +1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \text{Span}\left\{ \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \right\}$$

$$E_{\lambda=2} = \text{null}\begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix} = \text{Span}\left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$$

$$A = \underbrace{\begin{bmatrix} -1/\sqrt{2} & -1/\sqrt{2} & 1/\sqrt{3} \\ 1/\sqrt{2} & 0 & 1/\sqrt{3} \\ 0 & 1/\sqrt{2} & 1/\sqrt{3} \end{bmatrix}}_{Q} \underbrace{\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}}_{D} \underbrace{\begin{bmatrix} -1/\sqrt{2} & -1/\sqrt{2} & 1/\sqrt{3} \\ 1/\sqrt{2} & 0 & 1/\sqrt{3} \\ 0 & 1/\sqrt{2} & 1/\sqrt{3} \end{bmatrix}^{-1}}_{Q^{-1}}$$

$$B = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$$

$$(3-\lambda)(3-\lambda) - 4 = P(\lambda)$$

$$9 - 6\lambda + \lambda^2 - 4 = P(\lambda)$$

$$\lambda^2 - 6\lambda + 5 \Rightarrow (\lambda - 5)(\lambda - 1) \quad \lambda = 5 \text{ \& } 1$$

$$E_5 = \text{null}\begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} = \text{Span}\left\{\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right\}$$

$$E_1 = \text{null}\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} = \text{Span}\left\{\begin{pmatrix} 1 \\ -1 \end{pmatrix}\right\}$$

$$B = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}\begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}^{-1}$$