11/6

Friday, November 3, 2023 10:33

1. Let $a, b, c, d \in \mathbb{R}^4$. What is det(d,b,3c,a) knowing that det(a,b,c,d) = 2?
2. Use definition of determinant to show that
   a. A matrix with two identical columns has determinant of 0.
   b. A matrix with a zero column has determinant of 0.
   c. A matrix with linearly dependent columns has determinant of 0.
   d. Determinant of a diagonal matrix is the product of its diagonal elements.
3. Use a series of column operations to show that matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ has determinant $ad - bc$.
4. Let E be an elementary matrix. Show that $E,\ E^T$ have the same determinant. (Hint: there are only three types of row operations. What do their transposes look like?)
5.
   a. From 2.c, what can you say about invertibility and determinant?
   b. With the help of 3, if matrix $\begin{pmatrix} 3 & 5 \\ 5 & 3 \end{pmatrix} - \lambda I_2$ is not invertible, what is the value of λ?

In class, we defined formally what determinant is that is
it is a map from $M\_n(\mathbb{R})$ to $\mathbb{R}$ such that
1. swapping two cols would reverse the sign
2. adding one scalar multiple of one vector to another will not change its value
3. scaling one col vector by a scalar will scale the determinant as well
4. det I_n = 1

I have not mentioned anything about transpose, row operations. So please don't use

Please cover this transpose result because I will need it on Tuesday.