HW1

Thursday, August 29, 2024 8:24 AM

ECE 2040 Homework 1

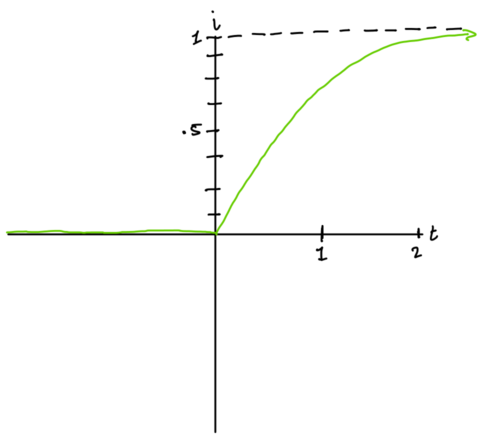
Due Date: September 6^{th}, 2024

Problem 1. Let $i(t) = (1 - e^{-2t})\,u(t)$ denote a time-varying current in a circuit. Assume $q(0) = 1$ C. Note that $u(t)$ is a step function with

$$u(t) = \begin{cases} 1, & t \geq 0 \text{ sec}, \\ 0, & t < 0 \text{ sec}, \end{cases}$$

(a) Sketch $i(t)$
(b) Find $q(t)$ for $t \geq 0$.

t	$i(t)$
0	0
1	$1-e^{-2} \approx .865$
2	$1-e^{-4} \approx .9816$
3	$1-e^{-6} \approx .9975$



$$i(t) = \frac{\partial Q}{\partial t}$$

$$\int_0^t \partial q = \int_0^t i(t)\, dt$$

$$q(t) = \int_0^t (1 - e^{-2t})\, u(t)\, \partial t$$

$$Q(t) - \int \partial t \;\; -\left(-\tfrac{1}{2}\right)\int_0^t -2e^{-2t}\, \partial t \qquad u = -2t \quad \partial u = -2\partial t$$

$$Q(t) = t \quad + \tfrac{1}{2}\int e^{u}\, \partial u$$

$$+ \tfrac{1}{2} e^{-2t}$$

$$q(t) = t + \tfrac{1}{2} e^{-2t} + c$$

$$q(0) = 0 + \tfrac{1}{2} e^{0} + c$$

$$1 = \tfrac{1}{2} + c$$

$$c = \tfrac{1}{2}$$

$$\boxed{Q(t) = t + \tfrac{1}{2} e^{-2t} + \tfrac{1}{2}}$$

Problem 2. *Ohm's Law* can be expressed as stated in equation (1) below

$$\mathbf{V} = \mathbf{I} \cdot \mathbf{R} \qquad (1)$$

where $\mathbf{V}$, $\mathbf{I}$ and $\mathbf{R}$ denote the voltage, current and resistance in a circuit, respectively.
(a) Prove that equation (1) is linear.
(b) State the conditions under which the linearity assumption of equation (1) would fail.

a) Ohm's law above is of the form

$$y = mx + b$$
$$V = IR + 0$$

if we Rearrange the terms s.t. current is isolated, we can test the equation being linear in an experiment more easily. For a fixed voltage and variable Resistance, we can measure the change in current via an ammeter.

$y = m\ x + b$

$I = V\left(\frac{1}{R}\right)$. for a fixed voltge, current is inversly proportional to Resistance, a linear Relationship.

B) The equation above would fail if The circuit Does not contain Ohmic materials in other words, extremely high temperature would cause This equation to turn non linear as the increasing heat would simultaneously change The Resistance properties and produce a Nonlinear Relationship between current voltage and Resistance.

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Problem 3. Let $i(t) = (1 - e^{-2t})\, u(t - \tau)$ denote a time-varying current in a circuit with a 100 kΩ resistor R_1. Note that $u(t)$ is a step function with

$$u(t) = \begin{cases} 1, & t \geq \tau \text{ sec}, \\ 0, & t < \tau \text{ sec}, \end{cases}$$

Using *Ohm's law*

(a) Assume $\tau = 0$, provide an expression for the voltage across the R_1.
(b) Assume $\tau = 1$, provide an expression for the voltage across the R_1.

a) $\tau = 0 \Rightarrow i(t) = (1 - e^{-2t})\, u(t)$

$$\Rightarrow v(t) = \begin{cases} 100{,}000(1 - e^{-2t}) & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$\Rightarrow v(t) = 100{,}000(1 - e^{-2t})$ since t must Be ≥ 0

b)

$$v(t) = \begin{cases} 100{,}000(1 - e^{-2t}) & t \geq 1 \\ 0 & t < 1 \end{cases}$$

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Problem 4. Let $i(t) = 5e^{-t} u(t - \tau)$ A, and $v(t) = -3e^{-2t} u(t - \tau)$ V . Note that $u(t)$ is a step function with

$(5e^{-3})(-3e^{-6})$

$$u(t) = \begin{cases} 1, & t \geq \tau \text{ sec}, \\ 0, & t < \tau \text{ sec}, \end{cases}$$

Using *Ohm's law*

(a) Assuming $\tau = 0$, what is the power supplied in the circuit as function of time.
(b) Assuming $\tau = 1$, what is the power absorbed in the circuit as function of time.
(c) Assuming $\tau = 0$, what is the energy absorbed at $t = 3$ s?
(d) Assuming $\tau = 1$, what is the energy supplied at $t = 3$ s?

$P = IV \Rightarrow A \cdot V \Rightarrow \frac{C}{S} V$

a) for $\tau = 0$ $\quad P(t) = i(t) \cdot v(t)$

$5e^{-t}(1) \cdot -3e^{-2t}(1)$

$$\boxed{P(t) \Rightarrow +15e^{-3t}\ W}$$

$15e^{-3t}$ watts of Power is supplied to The circuit

B) for $\tau = 0$ $\quad i(t) = 5e^{-t}\, u(t-1)$

$v(t) = -3e^{-2t}\, u(t-1)$

$$\boxed{P(t) = -15e^{-3t}\, u(t-1)\ W}$$

$15e^{-3t}\, u(t-1)$ watts of Power is absorbed By The circuit

written as a Piecewise function:

$$\boxed{P(t) = \begin{cases} -15e^{-3t} & t \geq 1 \\ 0 & t < 1 \end{cases}}$$

c) $-15e^{-3t}$ for $\tau = 0$

$P(t) = \frac{dE}{dt} \Rightarrow \int P(t)\,dt = \int dE$

$\Rightarrow E = \int P(t)\,dt \Rightarrow \int_0^3 -15e^{-3t}\,dt \quad u = -3t,\ du = -3dt$

$\int_0^3 5(-3)e^{u}\,du$

$\Rightarrow 5e^{-3t}\Big]_0^3 \Rightarrow \boxed{5e^{-9} + 5\ \text{J}}$

$\Rightarrow$ -4.9994 J absorbed by circuit

D) for $\tau = 1$, $t = 3$

$E = \int_0^3 P(t)\,dt \Rightarrow \underbrace{\int_0^1 P(t)\,dt}_{\Rightarrow 0} + \int_1^3 P(t)\,dt$

$\Rightarrow \int_1^3 -15e^{-3t}\,dt = 5e^{-3t}\Big]_1^3 \Rightarrow 5e^{-9} - 5e^{-3}$

$\boxed{5(e^{-9} - e^{-3})\ \text{J}}$

$\Rightarrow$ .2483 J supplied to the circuit