Question 3

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3. Consider $v \in \mathbb{R}^n$ and subspace $U \subseteq \mathbb{R}^n$. We know that we can write v as a sum of $v_1 \in U$ and $v_2 \in U^\perp$. Show that this decomposition is unique.

If we say $V = V_1 + V_2$, and $V_1 \in \bar{U}$, Then $V_1 = \text{proj}_{\bar{U}} \vec{V}$

$\Rightarrow V_1 = P\vec{V}$ for $P = [\text{proj}_{\bar{U}}(\cdot)]$

assume $\exists$ another Decomposition of $V = \tilde{V}_1 + \tilde{V}_2$

$\vec{V} = PV + V_2$. But $\tilde{V}_1$ is also $\text{proj}_{\bar{U}} \vec{V} \Rightarrow PV$

$\vec{V} = PV + \tilde{V}_2$ Thus $V_1 = \tilde{V}_1$

and By $\cancel{PV} + V_2 = \cancel{PV} + \tilde{V}_2 \therefore V_2 = \tilde{V}_2$