PROB. 11.1.* Consider an LTI filter with impulse response $h[n] = \sqrt{2}\,\delta[n] + 0.5\delta[n-3]$.

(a) Find the system function $H(z)$ for this system.

(b) Sketch the pole-zero plot for $H(z)$.

(c) With the help of MATLAB, sketch its magnitude response $|H(e^{j\hat{\omega}})|$ versus $\hat{\omega}$. Make a connection between the locations of the zeros in the pole-zero plot and the frequencies for which $|H(e^{j\hat{\omega}})|$ is small.

a) $h[n] = [\sqrt{2},\ 0,\ 0,\ 1/2]$

$$\sqrt{2} + \frac{1}{2}z^{-3} = H(z)$$

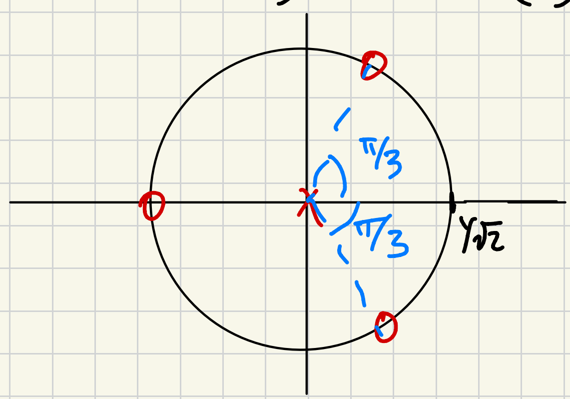
b) $$H(z)\left(\frac{z^3}{z^3}\right) = \frac{\sqrt{2}z^3 + \frac{1}{2}}{z^3} \Rightarrow \frac{\sqrt{2}\left(z^3 + \frac{1}{2\sqrt{2}}\right)}{z^3} \Rightarrow \frac{\sqrt{2}\left(z^3 + \frac{\sqrt{2}}{4}\right)}{z^3}$$

$$z = \left(re^{j\theta}\right)^3 = -\frac{\sqrt{2}}{4} \Rightarrow r^3 e^{j3\theta} = -\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) \Rightarrow re^{j\theta} = \left(\frac{-1}{2}\right)^{1/3}\left(\frac{\sqrt{2}}{2}\right)^{1/3}$$

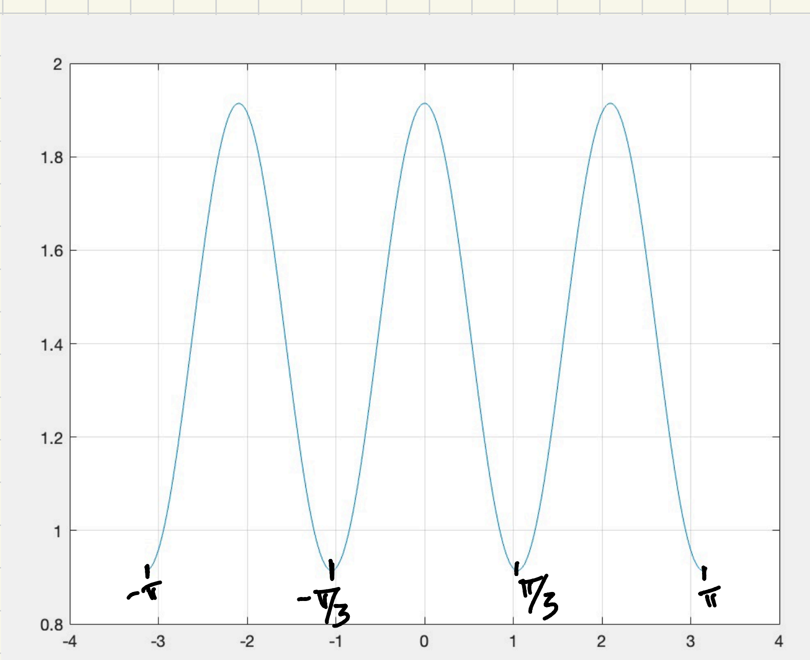
$$z = -\frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}(2)} \pm \frac{\sqrt{3}}{2\sqrt{2}}i$$

$$\frac{1}{\sqrt{2}}\left(\frac{1}{2} \pm \frac{\sqrt{3}}{2}i\right)$$

$$\theta = \pi/3$$



c)



Not zero @ locations but close since zeros occur @ $r = 1$ but $\frac{1}{\sqrt{2}} = r < 1 \Rightarrow$ close to zero

Made with Goodnotes

PROB. 11.2.* Consider an FIR system whose system function is $H(z) = 1 - z^{-2}$.

(a) Find the difference equation relating the filter output $y[n]$ to the input $x[n]$.

(b) Find the filter impulse response $h[n]$.

(c) Find the filter frequency response $H(e^{j\hat{\omega}})$.

(d) Find the dc gain of the filter.

(e) Sketch the magnitude response $|H(e^{j\hat{\omega}})|$. (*Hint:* It should look like a rectified sinusoid.)

(f) Sketch the pole-zero plot.

a) $H(z) = 1 - z^{-2} \Rightarrow h[n] = [1 \quad 0 \quad -1]$

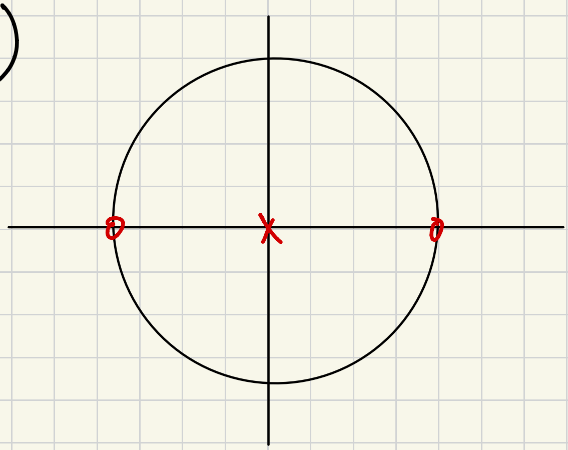
$y[n] = x[n] - x[n-2]$

b) $h[n] = \delta[n] - \delta[n-2]$

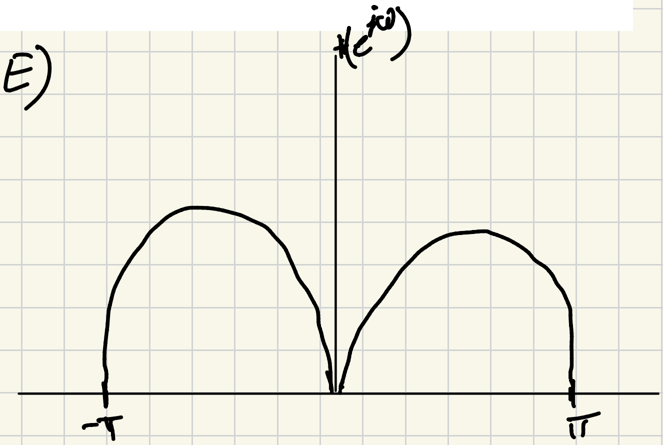
c) $H(e^{j\omega}) = 1 - e^{-2j\omega}$

D) $H(e^{j\omega})\Big|_{\omega=0} = 0$

f)



E)



$H(e^{j\omega}) = e^{-j\omega}(e^{j\omega} - e^{-j\omega})$

$e^{-j\omega}(2i\sin(\omega))$

$e^{-j\omega}(2i\sin(\omega))$

$H(z) = \dfrac{z^2 - 1}{z^2}$

Made with Goodnotes

PROB. 11.3.* A first-order LTI system whose system function is of the form $H(z) = 1 - e^{j\hat{\omega}_0}z^{-1}$ completely nulls out a complex exponential input signal of the form $e^{j\hat{\omega}_0 n}$.
The concept of *cascading* systems can be used to null out multiple complex exponential signals. For example, the system function $(1 - e^{j0.3\pi}z^{-1})(1 - e^{-j0.3\pi}z^{-1})$ nulls both $e^{j0.3\pi n}$ and $e^{-j0.3\pi n}$, and because of the Euler relation, it also nulls the real sinusoid of the form $A\cos(0.3\pi n + \varphi)$, regardless of the sinusoid amplitude or phase.

(a) Design the coefficients $\{b_k\}$ of an FIR filter of minimal order that nulls out the signal:

$$x_a[n] = \sqrt{2}\cos(0.2\pi n + 0.3\pi).$$

(b) Design the coefficients $\{b_k\}$ of an FIR filter of minimal order that nulls out the signal:

$$x_b[n] = \sqrt{2}\sum_{k=0}^{4}\cos(0.2\pi k)\cos(0.2\pi kn + 0.3\pi).$$

(c) Sketch the pole-zero plot of the system function for the filter of part (b).

a) to null out freq $\omega_0 = .2\pi n$

we design the system function as:

$$H(z) = (1 - e^{j(.2\pi)}z^{-1})(1 - e^{j(-.2\pi)}z^{-1})$$

$$1 - e^{-j.2\pi}z^{-1} - e^{+j.2\pi}z^{-1} + e^{j(.2\pi - .2\pi)}z^{-2}$$

$$1 - (e^{-j.2\pi} + e^{j.2\pi})z^{-1} + z^{-2}$$

$$1 - 2\cos(.2\pi)z^{-1} + z^{-2}$$

$$b_k = \{1, -2\cos(.2\pi), 1\}$$

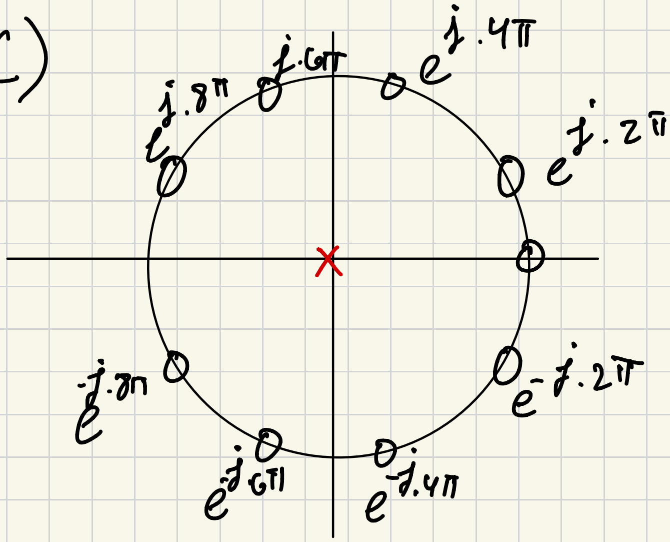
b) $\omega = 0, .2\pi, .4\pi, .6\pi, .8\pi$ to Null

$$(1 - 2\cos(.\pi)z^{-1} + z^{-2})(1 - 2\cos(.2\pi)z^{-1} + z^{-2})(1 - z^{-1})$$

or convolve each individual filter

$$\Rightarrow b_k = \{ \underset{b_0}{1}, \underset{b_1}{-1}, \underset{b_2}{1}, \underset{b_3}{-1}, \underset{b_4}{1}, \underset{b_5}{-1}, \underset{b_6}{1}, \underset{b_7}{-1}, \underset{b_8}{1}, \underset{b_9}{-1} \}$$

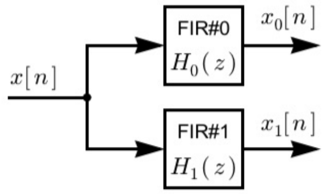
c)



PROB. 11.4.* Let $x[n]$ be an unspecified periodic sequence with fundamental period $N_0 = 3$. Applying the Euler relation to its DFS representation, such a signal can be written as a constant plus a sinusoid at the fundamental frequency $\frac{2\pi}{N_0} = \frac{2\pi}{3}$, according to:

$$x[n] = \underbrace{B}_{x_0[n]} + \underbrace{A\cos(2\pi n/3 + \varphi)}_{x_1[n]},$$

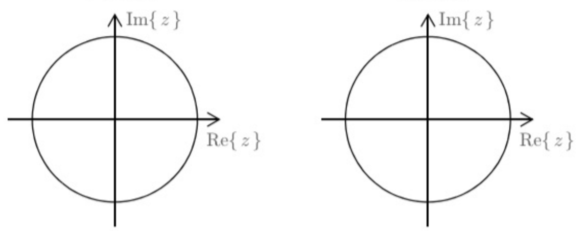
where $x_0[n]$ is the constant component, and $x_1[n]$ is the sinusoidal component. We can separate $x[n]$ into these components using a bank of FIR filters, as shown below:



(a) Design the coefficients $\{b_0, b_1, b_2\}$ of FIR#0 so that its output is $x_0[n]$.

(b) Design the coefficients $\{b_0, b_1, b_2\}$ of FIR#1 so that its output is $x_1[n]$.

(c) Let $H_i(z)$ be the system function of the i-th FIR filter. Sketch the pole-zero plot for both of the system functions $H_0(z)$ and $H_1(z)$. Align them side-by-side like this:



(d) Find a *simplified* expression for the sum of the component system functions:

$$H(z) = H_0(z) + H_1(z),$$

where $H_i(z)$ is the system function of the i-th FIR filter. (Simplify the answer as much as possible.)

a) must null frequencies $\omega = \frac{2\pi}{3}$ and DC gain must be 1

$$\Rightarrow \left(1 - z^{-1}(e^{j2\pi/3})\right)\left(1 - z^{-1}(e^{-j2\pi/3})\right)$$

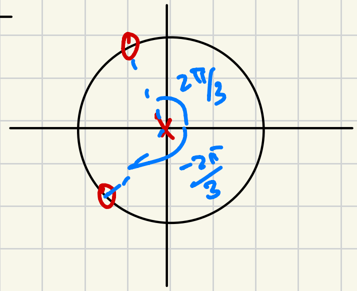
$$\Rightarrow \{1 \quad -2\cos(\tfrac{2\pi}{3}) \quad 1\} \Rightarrow b_k = \{1/3 \quad 1/3 \quad 1/3\}$$

b) $x_1[n] = x[n] - x_0[n] \Rightarrow H(z) = 1 - \frac{1}{3}(1 + z^{-1} + z^{-2}) \Rightarrow \frac{2}{3} - \frac{z^{-1}}{3} - \frac{z^{-2}}{3} \Rightarrow b_k = \{2/3, -1/3, -1/3\}$

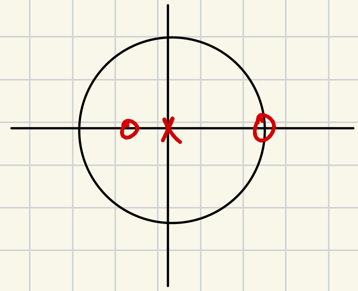
c) $H_0(z) = \frac{\left(\frac{1}{3}\right)(z - e^{j2\pi/3})(z - e^{-j2\pi/3})}{z^2}$

$H_1(z) = \frac{2z^2 + z + 1}{z^2} = \frac{(2z+1)(z-1)}{3z^2}$

FIR 0



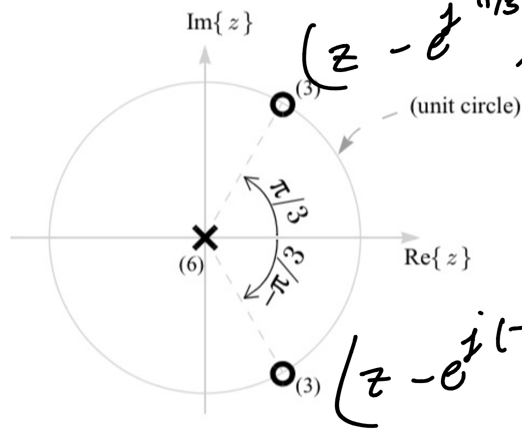
FiR 1



D) $\dfrac{z^2 + z + 1}{3z^2} + \dfrac{(z+1)(z-1)}{2z^2}$

$$\frac{z^2 \cancel{+z} \cancel{+1} + 2z^2 \cancel{-z} \cancel{-1}}{3z^2} \quad \frac{3z^2}{3z^2} = \boxed{1}$$

PROBLEM 11.5.* Consider an LTI filter whose system function $H(z)$ has the pole-zero plot shown below, with three zeros (multiplicity 3) on the unit circle with angle $\pi/3$, and similarly three zeros at the conjugate location, and six poles at the origin:



$(z - e^{j\pi/3})^3$

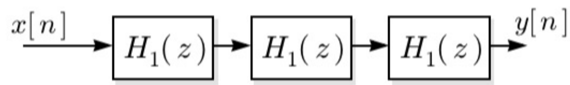
$(z - e^{j(-\pi/3)})^3$

Assume further that the evaluating $H(z)$ at $z = 1$ yields $H(1) = 1$.
(A pole-zero plot is invariant to an arbitrary scaling constant, some extra constraint like this is needed to uniquely define a filter.)

(a) Find the dc gain of the filter.

(b) Write the difference equation relating the filter output $y[n]$ to its input $x[n]$.

(c) Let $H_1(z) = 1 + b_1 z^{-1} + b_2 z^{-2}$ be the system function of an order-two FIR filter. Find real values for the coefficients b_1 and b_2 so that the filter with system function $H(z)$ — having the above pole-zero plot — can be constructed by cascading three copies of the $H_1(z)$ filter in series, as illustrated below:



(d) Use MATLAB to generate a plot of the magnitude response $|H(e^{j\hat{\omega}})|$ versus $\hat{\omega}$.

(e) Looking at the plot from part (d), how are nulling frequencies of the frequency response (the values of $\hat{\omega}$ where the magnitude response is zero) related to the locations of the zeros in the given pole-zero plot?

a) Given that $H(z)\big|_{z=1} = 1$

we know $z = re^{j\omega} = 1 \Rightarrow \omega = 0$ meaning

DC gain is 1

b) $H(z) = \dfrac{(z-e^{j\pi/3})^3(z-e^{-j\pi/3})^3}{z^6} \Rightarrow \left[\dfrac{(z-e^{j\pi/3})(z-e^{-j\pi/3})}{z^2}\right]^3$

$b_k = [1 \ \ -2\cos(\pi/3) \ \ 1] * [1 \ \ -2\cos(\pi/3) \ \ 1] \ldots$ ↑ 3 cascaded filters

$[1 \ \ -1 \ \ 1] * [1 \ \ -1 \ \ 1] * [1 \ \ -1 \ \ 1] = [1 \ \ -3 \ \ 6 \ \ -7 \ \ 6 \ \ -3 \ \ 1]$

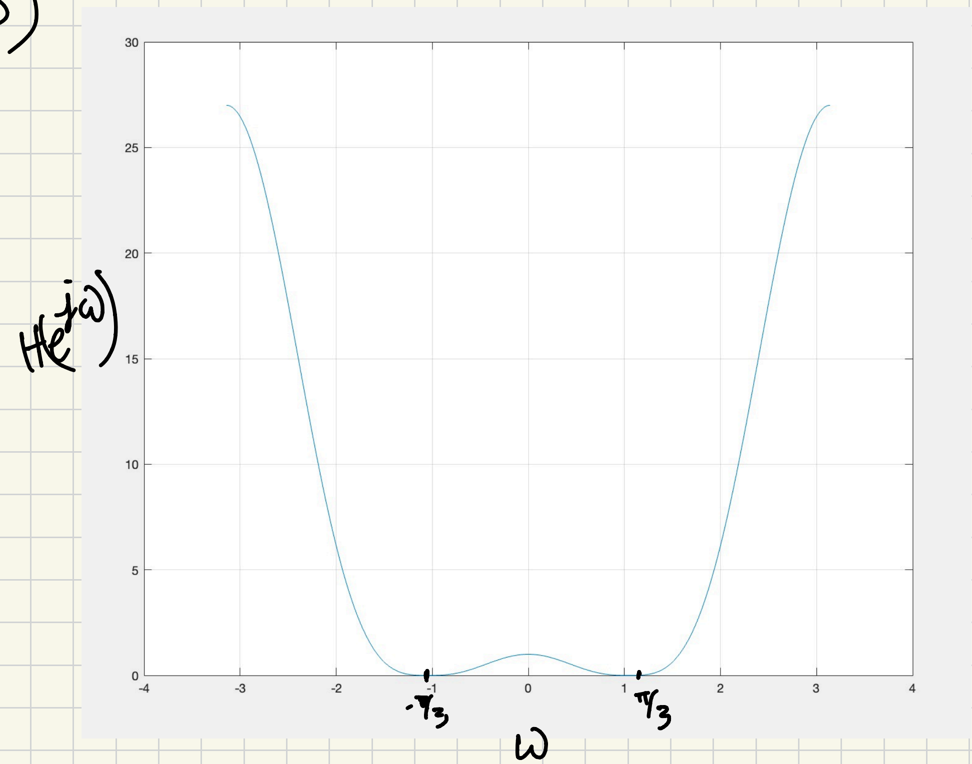
$y[n] = x[n] - 3x[n-1] + 6x[n-2] - 7x[n-3] + 6x[n-4] - 3x[n-5] + 1x[n-6]$

C) Solved in Part B

$b_1 = -1$

$b_2 = 1$

D)



E) The zeros of the Pole-zero Plot are @ $\pm^{j}\pi/3$ which is what $|H(e^{j\omega})|$ plot shows. furthermore, the Degree of each zero being 3 makes the frequencies in that Region more heavily attenuated.