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Tuesday, October 10, 2023 15:15

1. For each of the following matrices, interpret geometrically what T_A does.
   a. $A = \begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \end{pmatrix}$
   b. $A = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$
   c. $A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$
   d. $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & cos\theta & -sin\,\theta \\ 0 & sin\theta & cos\theta \end{pmatrix}$

2. Determine the matrix for each of the following transformations on $\mathbb{R}^2$
   a. Projection onto the line spanned by $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$
   b. Reflection along $y = x$ followed by refelction along x-axis.
3. Consider linear map $T : M_2\ (\mathbb{R}) \ \rightarrow \ M_2\ (\mathbb{R})$

   $$A \mapsto \begin{pmatrix} 2 & -4 \\ -1 & 2 \end{pmatrix} A$$

   Let $E = < \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} >$ be an ordered basis for $M_2\ (\mathbb{R})$ and let $B = \ < \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} >$
   a. Show B is basis for $M_2\ (\mathbb{R})$
   b. Find $[T]_{E\rightarrow E},\ [T]_{E\rightarrow B},\ [T]_{B\rightarrow E},\ [T]_{B\rightarrow B}$
   c. Use $[T]_{E\rightarrow E}$ to determine a basis and dimension of ImT and KerT. What results did you use here that guarantee your algorithm is correct? Explain.

Aran, the main result we covered in class is that for $T : V -$
$B = \ < b_1, \ldots,\ b_n >$ be an ordered basis for V and C
matrix representation of T is

$$[T]_{B\rightarrow C} = \begin{pmatrix} | & & | \\ [T\,(b_1)]_C & \cdots & [T\,(b_n)]_C \\ | & & | \end{pmatrix}$$

where $[T\,(b_i)]_C$ is basis vector $b\ \_\ i$'s image under T w

We can basically use the Nul[T] and Col[T] to find KerT and I
previously in studio. That is surjective maps preserve spanni
independence. And since the coordinate map $[\cdot]_{\mathscr{B}}$ is a bijec
matrix and the map very easily.