Question 8

Sunday, October 15, 2023 10:42 PM

8. For any two matrices $A \in M_{m\times n}(\mathbb{R})$ and $B \in M_{n\times k}(\mathbb{R})$ which satisfy $AB = 0$ prove that $\operatorname{rank}(B) + \operatorname{rank}(A) \leq n$.

if $AB=0$, then $(AB)_{ij} = \sum_{p=1}^{n} A_{ip}B_{pj} = 0$ arbitrary $i \leq m$, $j \leq k$

This implies that $B \in \text{null}(A)$, AKA $\ker(A)$

By Rank - nullity Theorem

$$\text{col}(B) \subseteq \text{null}(A)$$

$$\underbrace{\dim(\text{col}(B))}_{\text{Rank } B} \leq \dim(\text{null}(A))$$

$$\text{Rank } B + \text{Rank } A \leq \underbrace{\dim(\text{null } A) + \text{Rank}(A)}_{} = n$$

$\Rightarrow$ Rank B + Rank A $\leq n$