Question 2

Sunday, October 22, 2023 7:20 PM

2. Consider the following ordered bases of $\mathbb{R}^3$:

$$\mathcal{B} =< \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} >$$

$$\mathcal{C} =< \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} >$$

$$\mathcal{E} =< \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} >$$

Find the following matrices of transition from basis to basis:

$$[id]_{\mathcal{B}\to\mathcal{E}}, [id]_{\mathcal{C}\to\mathcal{E}}, [id]_{\mathcal{E}\to\mathcal{B}}, [id]_{\mathcal{E}\to\mathcal{C}}, [id]_{\mathcal{B}\to\mathcal{C}}, [id]_{\mathcal{C}\to\mathcal{E}}.$$

$$[id]_{B\to E} = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \quad [id]_{C\to E} = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 0 \\ -1 & 0 & 3 \end{bmatrix}$$

$$[id]_{E\to B} = \begin{bmatrix} 1/2 & 1/2 & -1/2 \\ -1/2 & 1/2 & 1/2 \\ 1/2 & -1/2 & 1/2 \end{bmatrix}$$

$$[id]_{E\to C} = \begin{bmatrix} -3/8 & 3/4 & 1/8 \\ 3/4 & -1/2 & -1/4 \\ -1/8 & 1/4 & 3/8 \end{bmatrix}$$

$$[id]_{B\to C} = \begin{bmatrix} 3/8 & 7/8 & -1/4 \\ 1/4 & -3/4 & 1/2 \\ 1/8 & 5/8 & 1/4 \end{bmatrix}$$

$$[\mathrm{id}]_{C \to e} = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 0 \\ -1 & 0 & 3 \end{bmatrix}$$