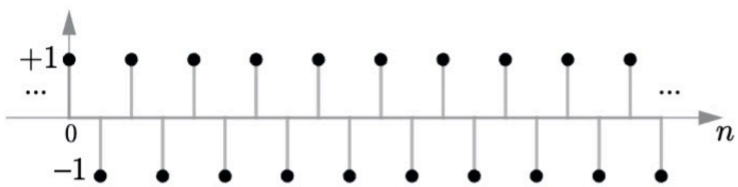
PROBLEM 5.1.* Download the WAV file `mystery.wav` from the canvas homework page. Import it into MATLAB and listen to it using the `audioread` and `soundsc` commands. Hidden in this audio file is a message. What is the message?
Specify the message as well as the MATLAB code you used to decode the message.

```
[xx, fs] = audioread("mystery.wav");

spectrogram(xx, 256, [], [], fs, 'yaxis');
```

PROBLEM 5.2.* Write each of the following as a discrete-time sinusoid in standard form $A\cos(\hat{\omega}n + \varphi)$, with $A \geq 0$, $0 \leq \hat{\omega} \leq \pi$, and $-\pi < \varphi \leq \pi$:

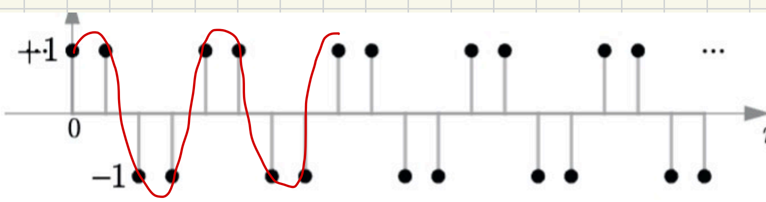
(a) $x_a[n] = \ldots 1, -1, 1, -1, \ldots$



if $x_a[n] = A\cos(\hat{\omega}n + \phi)$ $A = 1$, $\hat{\omega} = \pi$, $\phi = 0$

$x_a[n] = \cos(\pi n)$

(b) $x_b[n] = \ldots 1, 1, -1, -1, 1, 1, -1, -1, \ldots$



$x_b[n] = A\cos(\hat{\omega}n + \phi)$ period is 4 samples $= f_0 = 1/4$

$\hat{\omega} = \frac{2\pi}{4} = \pi/2$

$n=0 \Rightarrow A\cos(\phi) = 1 \longrightarrow \Rightarrow \cos(\phi) = 1/A$

$n=1 \Rightarrow A\cos(\pi/2 + \phi) = 1 \Rightarrow \cancel{\cos(\pi/2)\cos(\phi)} - \sin(\pi/2)\sin(\phi) = 1/A \Rightarrow \sin(\phi) = -1/A$

$n=2 \Rightarrow A\cos(\pi + \phi) = -1 \Rightarrow \cos(\pi)\cos(\phi) - \cancel{\sin(\pi)\sin(\phi)} = -1/A \Rightarrow \cos(\phi) = +1/A$

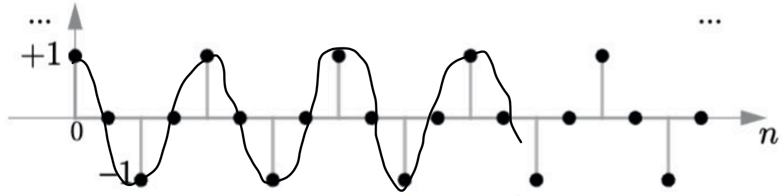
$n=3 \Rightarrow A\cos(3\pi/2 + \phi) = -1 \Rightarrow \cancel{\cos(3\pi/2)\cos(\phi)} - \sin(3\pi/2)\sin(\phi) = -1/A \Rightarrow \sin(\phi) = -1/A$

$\cos(-\pi/4) = \sqrt{2}/2$
$\sin(-\pi/4) = -\sqrt{2}/2$ $\Rightarrow$ $\boxed{\phi = -\pi/4}$ $\boxed{A = 2/\sqrt{2}}$

$\frac{\sqrt{2}}{2} = \frac{1}{A}$

$x[n] = \frac{2}{\sqrt{2}}\cos\left(\frac{\pi}{2}n - \pi/4\right)$

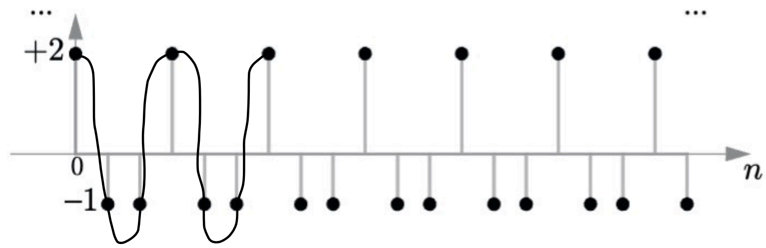
(c) $x_c[n] = \ldots 1, 0, -1, 0, 1, 0, -1, 0, \ldots$



$\hat{\omega} = 2\pi\left(\frac{1}{4}\right) \qquad \cos\left(\frac{\pi}{2}n + \phi\right) \quad A = 1$

$x_c[n] = \cos\left(\frac{\pi}{2}n\right)$

(d) $x_d[n] = \ldots 2, -1, -1, 2, -1, -1, \ldots$



$\hat{\omega} = 2\pi\left(\frac{1}{3}\right) \Rightarrow x[n] = 2\cos\left(2\pi\left(\frac{1}{3}\right)n\right)$

$$\left.\begin{array}{l} n=0 \Rightarrow 2\cdot\cos(0+\phi) = 2 \\ n=1 \Rightarrow 2\cdot\cos\left(\frac{2\pi}{3}+\phi\right) = -1 \\ n=2 \Rightarrow 2\cdot\cos\left(\frac{4\pi}{3}+\phi\right) = -1 \end{array}\right\} \phi = 0$$

$x[n] = 2\cdot\cos\left(\frac{2\pi}{3}n\right)$

PROBLEM 5.3.* Write each of the following as a discrete-time sinusoid in standard form $A\cos(\hat{\omega}n + \varphi)$, with $A \geq 0$, $0 \leq \hat{\omega} \leq \pi$, and $-\pi < \varphi \leq \pi$.

$\hat{\omega} = \frac{M}{N_0}(2\pi)$

(a) $x_a[n] = \text{Re}\{\sqrt{2}\, e^{j(0.2\pi n + \pi/3)}\}$.

(b) $x_b[n] = -\sin(9.3\pi(n-4) + 0.3\pi)$.

(c) $x_c[n] = \sqrt{2} + \cos(2026\pi n) + \sqrt{2}\sin(2026(2\pi)n)$.

(d) $x_d[n] = \cos(7n) - \sin(7n) + \sqrt{2}\cos(7n + 0.25\pi)$

a) $x[n] = \sqrt{2}\cos(.2\pi n + \frac{\pi}{3})$

b) $x[n] = -\sin(9.3\pi n - 4(9.3\pi) + .3\pi)$

$= \sin(-9.3\pi n + 36.9\pi) \Rightarrow \cos(-9.3\pi + 36.4\pi)$

$\hat{\omega} = -9.3\pi + 2\pi l \; (l = 5)$

$= .7\pi$

$\phi = 36.4\pi - 2\pi l \; (l = 18)$

$\phi = .4\pi$

$x[n] = \cos(.7\pi n + .4\pi)$

c) $x[n] = \sqrt{2} + \underbrace{\cos(2026\pi n)}_{\forall n \in \mathbb{N},\ \text{is } 1} + \sqrt{2}\sin(2026(2\pi)n)$

$\Rightarrow (1 + \sqrt{2}) + \sqrt{2}\cos(2026(2\pi)n - \pi/2)$

$\sqrt{2}(\cancel{\cos(2026(2\pi)n)\cos(-\pi/2)} + \cancel{\sin(2026(2\pi)n)\sin(\pi/2)})$

0 0

$\boxed{x[n] \Rightarrow 1 + \sqrt{2}}$

D) $x[n] = \cos(7n) - \sin(7n) + \sqrt{2}\cos(7n + .25\pi)$

$\hat{\omega} = 7$ $\quad -\cos(7n - \pi/2)$

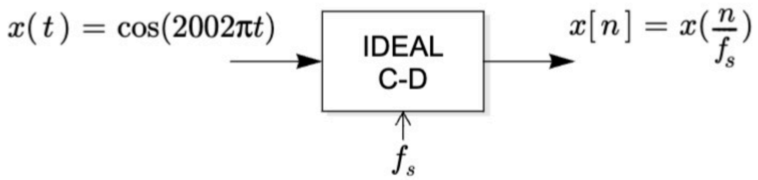
$e^{j(0)} + (-1)e^{j(-\pi/2)} + \sqrt{2}\, e^{j(.25\pi)} \Rightarrow 2.8284\cos(7n + \pi/4)$

$\hookrightarrow e^{j(\pi/2)}$

$7 - 2\pi l \; (l = 1) = .7168 = \hat{\omega}$

$x[n] \approx \underbrace{2.8284}_{(2\sqrt{2})} \cos(.7168n + \pi/4)$

PROBLEM 5.4.* A continuous-time sinusoid $x(t) = \cos(2002\pi t)$ is sampled with sampling rate f_s, resulting in a discrete-time sequence:



(a) Find the fundamental period N_0 for $x[n]$ when the sampling rate is $f_s = 3003$ Hz.

(b) Find the fundamental period N_0 for $x[n]$ when the sampling rate is $f_s = 1092$ Hz.

(c) Find the fundamental period N_0 for $x[n]$ when the sampling rate is $f_s = 507006.5$ Hz.

(d) Answer *TRUE* or *FALSE*, and explain your reasoning:
In order for $x[n]$ to be periodic in this example, the sampling rate must be *rational*.

(e) Find the smallest integer-valued sampling rate f_s for which $x[n]$ is periodic with fundamental period $N_0 = 8$.

(f) Specify three distinct integer-valued sampling rates $f_s > 0$ for which $x[n]$ is periodic with fundamental period $N_0 = 60$. (Hint: There are eight.)

a) $\omega = 2\pi \frac{f_0}{f_s} = \frac{1001}{3003}(2\pi) = \frac{1}{3}(2\pi) \Rightarrow N_0 = 3$ Samples

b) $2\pi\left(\frac{1001}{1092}\right) \Rightarrow N_0 = 12$ Samples

$\frac{11}{12}$

c) $\hat{\omega} = 2\pi\left(\frac{1001}{507006.5}\right) \Rightarrow$

$\frac{2002}{1014013} \Rightarrow \frac{2}{1013} \Rightarrow$

$\boxed{N_0 = 1013 \text{ Samples}}$

D) if f_s were Not to Be Rational (irrational)
Then The term $\hat{\omega} = 2\pi \frac{f_0}{f_s}$ would Result in
M/N Being irrational. Specifically, N_0 would Not Be Rational
$\Rightarrow$ Signal is Non Periodic $\Rightarrow$ f_s MUST Be Rational (True)

E) $\hat{\omega} = 2\pi\left(\frac{M}{N_0}\right) \Rightarrow \cancel{2\pi}\left(\frac{M}{8}\right) = \cancel{2\pi}\left(\frac{1001}{f_s}\right) \Rightarrow f_s = \frac{8008}{M} \Rightarrow \frac{1001}{8008}$ can Be $\frac{1}{8}$

$\Rightarrow \boxed{f_s = 8 \text{ Hz}}$

PROBLEM 5.5.* In Prob. 5.4 the C-D input is specified, and you are asked about the output. Here we consider the reverse: the *output* is specified, and you are asked about the input.

Suppose that a continuous-time signal $x(t)$ is sampled at a sampling rate of 1024 samples/sec, resulting in the discrete-time sinusoidal signal $x[n]$ indicated below:

$f_s = 1024$

$$x(t) \longrightarrow \boxed{\text{IDEAL C-D}} \longrightarrow x[n] = 512\cos(0.125\pi n - 0.75\pi)$$

$\frac{1}{8}\pi n$

$f_s = 1024$ Hz

Knowing the C-D output $x[n]$ does not uniquely determine the C-D input $x(t)$; there are many continuous-time signals $x(t)$ that when sampled would produce this $x[n]$.
If we add a constraint that the input $x(t)$ is a sinusoid whose frequency is less than 2 kHz, then there are only four possible inputs. Name all four. In other words, specify four different continuous-time sinsusoidal signals (all in standard form)

$$x_1(t) = A_1\cos(2\pi f_1 t + \varphi_1),$$
$$x_2(t) = A_2\cos(2\pi f_2 t + \varphi_2),$$
$$x_3(t) = A_3\cos(2\pi f_3 t + \varphi_3),$$
$$x_4(t) = A_4\cos(2\pi f_4 t + \varphi_4)$$

that could have produced this particular $x[n]$, subject to the constraint that $0 < f_i < 2$ kHz in all four cases.

① $\frac{\cancel{2\pi} f_1}{f_s} = 2\pi \frac{M}{N_0} = \hat{\omega} \Rightarrow \cancel{2\pi}\left(\frac{1}{16}\right)$

$\frac{f_1}{1024} = \frac{1}{16} \Rightarrow f_1 = 64$ Hz

$x(t) = 512\cos(2\pi[64)t - .75\pi)$

② $\cancel{2\pi}\frac{f_1}{1024} = \hat{\omega} + 2\pi \Rightarrow 2.125\pi \Rightarrow \cancel{2\pi}\left(\frac{17}{16}\right) \Rightarrow f_2 = 1088$ Hz

$x(t) = 512\cos(2\pi(1088)t - .75\pi)$

③ $\cancel{2\pi}\frac{f_1}{1024} = \hat{\omega} + 4\pi \Rightarrow 4.125\pi = \cancel{2\pi}\left(\frac{33}{16}\right)$ X2

$\Rightarrow \hat{\omega} - 2\pi = -1.875\pi \Rightarrow \cancel{2\pi}\left(\frac{15}{16}\right) \Rightarrow -960$ $x(t) = 512\cos(2\pi(960)t + .75\pi)$

④ $2\pi\frac{f_1}{1024} = \hat{\omega} - 4\pi = -3.875\pi \Rightarrow 2\pi\left(-\frac{31}{16}\right) = f_1 = -1984$ $x(t) = 512\cos(2\pi(1984)t + .75\pi)$