Test 2

Problem t3

Observable A is represented by the operator

$$\hat{A} = |\alpha\rangle\langle\alpha| - i\sqrt{2}\,|\alpha\rangle\langle\beta| + i\sqrt{2}\,|\beta\rangle\langle\alpha|,$$

where vectors $|\alpha\rangle$ and $|\beta\rangle$ form an orthonormal basis for a two-dimensional Hilbert space.

(a) What are the possible outcomes of measurement of A?

(b) Find the expectation value of A in the state $|\psi\rangle \propto |\alpha\rangle + |\beta\rangle$, $\langle\psi|\psi\rangle = 1$.

(c) For the state $|\psi\rangle$ of part (b), compute the probabilities of each of the possible outcomes found in part (a).

Problem t4

Spin 2 particles are in the state $|\psi\rangle = \dfrac{1}{\sqrt{2}}\Big\{|+1\rangle - i\,|-1\rangle\Big\}$, where $|\pm1\rangle$ are eigenvectors of $\hat{S}_{\mathbf{z}}$ with eigenvalues $\pm\hbar$.

Evaluate the uncertainty of $S_{\mathbf{n}} = \mathbf{n}\cdot\mathbf{S}$ in this state (here $\mathbf{n}$ is a unit dimensionless vector).
For which $\mathbf{n}$ the uncertainty attains the smallest possible value? What is this value?
Feel free to use the relations

$$\langle S_{\mathbf{n}}\rangle_{\pm1} = \langle\pm1|\hat{S}_{\mathbf{n}}|\pm1\rangle = \pm\hbar n_{\mathbf{z}}, \quad \langle S_{\mathbf{n}}^2\rangle_{\pm1} = \langle\pm1|\hat{S}_{\mathbf{n}}^2|\pm1\rangle = (\hbar^2/2)\big(5 - 3n_{\mathbf{z}}^2\big)$$

that can be obtained by setting $j = 2$ and $m = \pm1$ in the expressions derived in Problem 26.