

Question 1

Sunday, November 12, 2023

3:11 PM

1. Compute the determinants of the following matrices:

$$\begin{pmatrix} 2 & 6 & 16 \\ -3 & -6 & 18 \\ 5 & 12 & 35 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ -1 & 5 & 2 \\ 3 & 2 & 0 \end{pmatrix}, \begin{pmatrix} 4 & 0 & 1 \\ -2 & 2 & -1 \\ 0 & 4 & -3 \end{pmatrix}, \begin{pmatrix} 4 & -4 & 2 & 1 \\ 1 & 2 & 0 & 3 \\ 2 & 0 & 3 & 4 \\ 0 & -1 & 2 & 1 \end{pmatrix}$$

$$\begin{array}{ccc} 2 & 6 & 16 \\ -3 & -6 & 18 \\ 5 & 12 & 35 \end{array}$$

$$-420 + 540 + -576 + 486 - 432 + 630 = \boxed{222}$$

$$\begin{array}{ccc} 1 & 2 & 3 \\ -1 & 5 & 2 \\ 3 & 2 & 0 \end{array}$$

$$0 + 12 - 6 - 45 - 4 - 0 = \boxed{-43}$$

$$\begin{array}{ccc} 4 & 0 & 1 \\ -2 & 2 & -1 \\ 0 & 4 & -3 \end{array}$$

$$-24 + 0 + -8 - 0 + 16 - 0 = \boxed{-16}$$

$$\begin{pmatrix} 4 & -4 & 2 & 1 \\ 1 & 2 & 0 & 3 \\ 2 & 0 & 3 & 4 \\ 0 & -1 & 2 & 1 \end{pmatrix} \rightarrow \begin{array}{cccc} 1 & 2 & 0 & 3 \\ 4 & -4 & 2 & 1 \\ 2 & 0 & 3 & 4 \\ 0 & -1 & 2 & 1 \end{array} \quad (-1)$$

$$\rightarrow \begin{array}{cccc} 1 & 2 & 0 & 3 \\ 0 & -12 & 2 & -11 \\ & -4 & 2 & -2 \end{array} \rightarrow \begin{array}{cccc} 1 & 2 & 0 & 3 \\ 0 & -12 & 2 & -11 \\ 0 & -4 & 2 & -2 \end{array}$$

$$1 + \frac{11}{12} = \frac{23}{12}$$

$$\begin{matrix} 0 & -1 & 2 & 1 \end{matrix}$$

$$\begin{matrix} 0 & 0 & \frac{1}{6} & \frac{23}{12} \end{matrix}$$

$$\begin{array}{cccc} 1 & 2 & 0 & 3 \\ 0 & -12 & 2 & -11 \\ 6 & 6 & 7 & 5 \\ 0 & 0 & 22 & 23 \end{array} \xrightarrow{(36)} \left| \begin{array}{cccc} 1 & 2 & 0 & 3 \\ 0 & -12 & 2 & -11 \\ 0 & 0 & 7 & 5 \\ 0 & 0 & 0 & \frac{5}{7} \end{array} \right| \begin{array}{l} 22 - \frac{22}{7}(7) \\ 23 - \frac{22}{7}(5) \end{array}$$

$$\rightarrow 1 \cdot -12 \cdot 7 \cdot \frac{5}{7} = -612\left(-\frac{1}{7}\right)\left(\frac{1}{36}\right)$$

$$\boxed{= 17}$$