MATH-1564-K Linear Algebra with Abstract Vector Spaces

Definitions

Notations

- $f : \mathcal{D} \to \mathcal{C}$
- $x \mapsto y$
- $f \circ g \circ h$
- $x \sim y$
- $S/\sim$
- $\{c_i\}_{i=1}^n$
- $\text{span}(v_1, \ldots, v_n)$
- $A \times B$
- $U + W$
- $x + U$
- $M_{m\times n}(\mathbb{R})$
- $M_n(\mathbb{R})$
- $\mathbb{R}_n[x], \mathcal{P}_n(\mathbb{R})$
- $\mathbb{R}[x], \mathcal{P}(\mathbb{R})$
- $[v]_\mathcal{B}$, $\text{Rep}_\mathcal{B}(v)$
- $V \cong W$
- $\text{Col}(A)$
- $\text{Row}(A)$
- $\text{Nul}(A)$
- $\text{rank}(A)$
- A^T
- A^{-1}
- I_n
- $GL_n(\mathbb{R})$
- $\text{id} : S \to S$
- $\mathcal{L}(V, W)$
- $\mathcal{L}(V), \mathcal{L}(V, V)$
- $\mathcal{L}(V, \mathbb{R})$
- $\text{Im}T$
- $\text{Ker}T$
- $\text{dim}U$
- $[T]_{\mathcal{B}\to\mathcal{C}}, [T]_\mathcal{C}^\mathcal{B}$
- $[T]_\mathcal{B}$
- $\langle \cdot, \cdot \rangle, \langle v_1, v_2 \rangle$
- $\| \cdot \|, \|x\|$
- $\text{Proj}_u(v)$
- T^*
- $A \succeq 0$
- $A \succ 0$
- $\sqrt{A}$
- SBWOC
- TFAE
- iff

1. A *binary relation* is
2. An *equivalance relation* is
3. Given $\sim$ an equivalence relation on set S, the *equivalence class* with representative $x \in S$ is
4. Given $\sim$ an equivalence relation on set S, $S/\sim$ is
5. A *linear system of equations* is
6. List all three *elementary row operations.*
7. Matrix is in ref if ; in rref if
8. Describe *Gauss-Jordan elimination.*
9. Let $u, v \in \mathbb{R}^n$. Their addition is defined to be
10. Let $u \in \mathbb{R}^n$ and $k \in \mathbb{R}$. ku is defined to be
11. Let $u, v \in \mathbb{R}^n$. Their *dot product* $u \cdot v$ is
12. Let $A \in M_{m\times n}(\mathbb{R})$ and $v \in \mathbb{R}^n$. Av is defined to be

13. Set V with two operations $+, \cdot$ over field $\mathbb{F}$ is a *vector space* over $\mathbb{F}$ if

14. Subset U in vector space V is a *subspace* if

15. A *linear combination* of vectors $v_1, \ldots, v_n$ is of the form

16. A *homogenous system* is

17. The *span* of $v_1, \ldots, v_n$ is the set

18. Vectors $v_1, \ldots, v_n$ are *linearly independent* if

19. They are *linearly dependent* if

20. A vector space V is *finite-dimensional* if

21. An *ordered basis* of a vector space V is

22. Let $\mathcal{B}$ be an ordered basis for finite dimensional vector space V. For vector $v \in V$, its *coordinate vector* w.r.t. basis $\mathcal{B}$ is

23. The *standard basis* for $\mathbb{R}^n$ is

24. The *dimension* of vector space V is

25. Let $A \in M_{m\times n}(\mathbb{R})$. Define $\mathrm{Col}(A)$, $\mathrm{Nul}(A)$, $\mathrm{Row}(A)$.

26. Let $A \in M_{m\times n}(\mathbb{R})$ and $B \in M_{n\times p}(\mathbb{R})$. Their product AB is

27. Let $A, B \in M_{m\times n}(\mathbb{R})$. Their sum $A + B$ is

28. Let $A \in M_{m\times n}(\mathbb{R})$ and $k \in \mathbb{R}$ then kA is

29. Let $A \in M_{m\times n}(\mathbb{R})$. Its *transpose*, A^T, is

30. Function $f : \mathcal{D} \to \mathcal{C}$ is injective/1-to-1 if, surjective/onto if and bijetive/1-to-1 correspondence if

31. Function T from vector space V to vector space W, both over $\mathbb{R}$, is a $\mathbb{R}$-*linear map* if

32. Let $A \in M_{m\times n}(\mathbb{R})$ then we can define a *linear map associated with* A by

33. A vector space *isomorphism* is

34. Two vector spaces are said to be *isomorphic* if

35. $\mathcal{L}(V, W)$ is used to denote

36. The *identity map* on set S is

37. We say linear map $T : V \to W$ is *invertible* if

38. *Row rank* of matrix A is

39. *Column rank* of matrix A is

40. *Rank* of matrix A is

41. Let $T \in \mathcal{L}(V, W)$. *Image* of T is and *Kernel* of T is

42. Let $T \in \mathcal{L}(V, W)$. $\text{rank}T$ is defined to be

43. $E \in M_n(\mathbb{R})$ is an *elementary matrix* if

44. Matrix $A \in M_n(\mathbb{R})$ is *invertible* if

45. Let V be a vector space over $\mathbb{R}$ with basis $B = \langle b_1, \ldots, b_n \rangle$ and W be a vector space over $\mathbb{R}$ with basis $C = \langle c_1, \ldots, c_n \rangle$. Then the *matrix associated with* T with respect to basis B and C, denoted by $[T]_{B \to C}$ is

46. Let $T \in \mathcal{L}(V, V)$ and let B, C be two different bases for V. We have the identity $[T]_{B \to B} =$

47. Let B, C be two bases for vector space V. Then the *change of bases matrix* from B to C is given by

48. Let V be a vector space over $\mathbb{R}$. Function $\langle \cdot, \cdot \rangle : V \times V \to \mathbb{R}$ is an *inner product* if

49. Let V be a vector space over $\mathbb{R}$. Function $\| \cdot \| : V \to \mathbb{R}$ is a *norm* if

50. Given an inner product $\langle \cdot, \cdot \rangle$, the *norm* induced by it is given by

51. Two vectors in inner product space V are said to be orthogonal if

52. Consider vector v and non-zero vector u in inner product space V. The *orthogonal projection* of v onto the direction of u is given by

53. Vectors $\{v_1, \cdots, v_n\}$ in inner product space are said to form an *orthonormal set* if

54. An *orthonormal basis* for vector space V is

55. Describe *Gram-Schmidt process*.

56. Let U be a subspace with an orthonormal basis $\{q_1, \ldots, q_m\}$ in vector space V. Then for any vector $v \in V$, *the orthogonal projection* of v onto subspace U is given by

57. Let S be a subspace in inner product space V. The orthogonal complement, $S^\perp$, is defined to be

58. Let U be a subspace in inner product space V and consider the decomposition of vector $v \in V$ as $v_U + v_{U^\perp}$. We say $P \in \mathcal{L}(V)$ is a orthogonal projection map if

59. Let $P \in M_n\mathbb{R}$. P is an orthogonal projection matrix iff

60. Let $T \in \mathcal{L}$. Adjoint of T, denoted by T^*, is

61. Let $u, v \in \mathbb{R}^n$. The outer product of u, v is

62. Let $A \in M_{m \times n}(\mathbb{R})$ with linear independent columns. We say A admits a QR factorization if

63. We say x^* is a least-square solution to system $Ax = b$ if

64. We say $T \in \mathcal{L}(\mathbb{R})$ is an orthogonal transformation if
65. Matrix $Q \in M_n(\mathbb{R})$ is an orthogonal matrix if
66. We say map $f : V^n \to \mathbb{R}$ is a multi-linear map if
67. Let $V = \mathbb{R}^n$. Map $\det : V^n \to \mathbb{R}$ is a $n \times n$ determinant if
68. Describe Sarrus rule to find determinant of a 3×3 matrix.
69. Describe Laplace/cofactor expansion to find determinant of an $n \times n$ matrix.
70. Let $z = a + bi \in \mathbb{C}$. The real part of z is ; the imaginary part is; its complex conjugate is given by.
71. Let $z = a + bi \in \mathbb{C}$. Its polar form is, where its argument is given by , and its modulus is.
72. Matrix $A \in M_2(\mathbb{R})$ is called a scaling-rotation matrix if it is of the form
73. We say matrix $D \in M_n(\mathbb{R})$ is diagonal if
74. We say matrix $D \in M_n(\mathbb{R})$ is diagonal if
75. Let $A, B \in M_n(\mathbb{R})$. We say A is similar to B if
76. Let $T \in \mathcal{L}(\mathbb{R})$. We say T is diagonalizable if
77. Let $A \in M_n(\mathbb{R})$. We say A is diagonalizable if
78. Let $T \in \mathcal{L}(\mathbb{R})$. We say (λ, v) is an eigenpair of T if
79. Let $A \in M_n(\mathbb{R})$. We say (λ, v) is an eigenpair of A if
80. Let $A \in M_n(\mathbb{R})$ and λ an eigenvalue of A. The eigenspace corresponding to λ is
81. The polynomial in variable λ is the characteristic polynomial of $A \in M_n(\mathbb{R})$ if
82. The algebraic multiplicity of eigenvalue λ of matrix A is
83. The geometric multiplicity of eigenvalue λ of matrix A is
84. Matrix $A \in M_n(\mathbb{R})$ is symmetric if
85. $f : \mathbb{R}^n \to \mathbb{R}$ is a quadratic form if
86. Let $q(\vec{x}) = \vec{x}^T A \vec{x}$ be a quadratic form with A a real symmetric matrix. We say A is positive definite if ; positive semi-definite if .
87. Matrix $A \in M_n(\mathbb{R})$ is nilpotent if