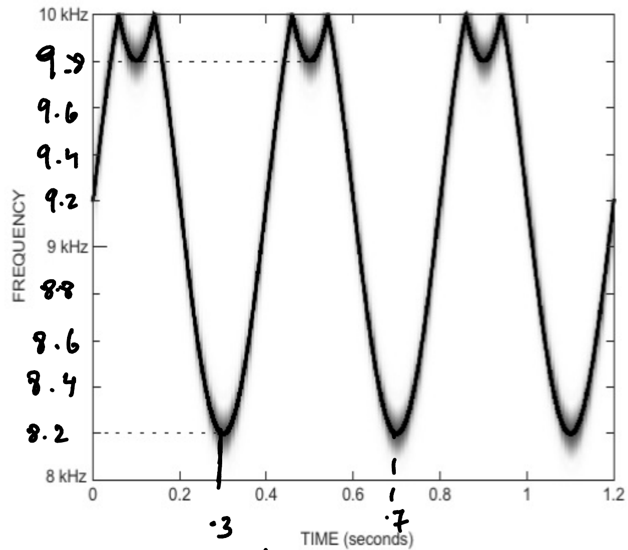
PROBLEM 6.1.* Suppose that running the following MATLAB code:

```
A =
B =
C =
fs =
dur =
t = 0:1/fs:dur;
x = cos(2*pi*B*t + A*cos(C*t));
spectrogram(x, hanning(300), [], 2^12, fs, 'yaxis');
```

leads to the following plot:

Peak @ 11.8 => $A = \frac{1000}{5\pi}$ 10.2

$f_i(t) = \frac{\partial \phi(t)}{\partial t}$



$\Delta t = T = .4$ sec

(a) Specify numerical values for the unspecified variables fs and dur.

(b) Specify a numerical value for C, with the constraint that $0 < C < 6\pi$.

(c) Specify one possible set of numerical values for the unspecified variables A and B, with the constraint that $0 < B < 10000$.

(d) Specify a second set of numerical values for the unspecified variables A and B, this time with the constraint that $10000 < B < 20000$.

a)

$f_s = 20$ kHz

since the plot (spectrogram) cuts off @ 10k Hz

dur = 1.2 seconds from graph

b)

$x = \cos(\psi(t))$

$\psi(t) = 2\pi B t + A\cos(Ct)$

$\frac{d\psi}{dt} = 2\pi B - AC\sin(Ct)$

$T = .4$

$f = \frac{1}{.4} = 2.5$ Hz

$\boxed{C = 5\pi}$

c) midline is 9200

$\Rightarrow B = \frac{9200}{2\pi} \Rightarrow \boxed{\frac{4600}{\pi}}$

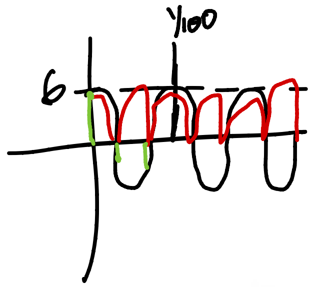
swing from 8.2 to 10.2 => 2000/2 is Amplitude

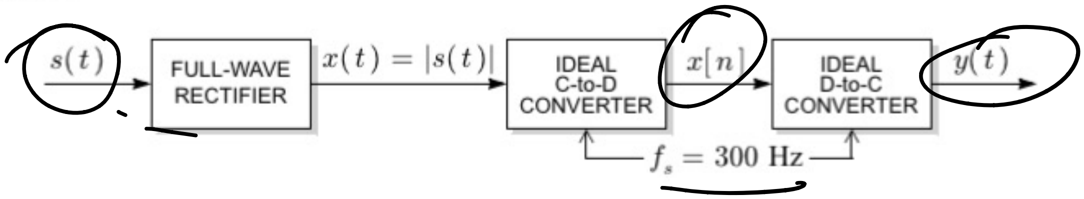
$A = \frac{1000}{5\pi} \Rightarrow \boxed{\frac{200}{\pi}}$

D) $A = -\frac{200}{\pi}$

$B = \frac{10.8 \text{ kHz}}{2\pi} = \frac{10800}{2\pi} = \boxed{\frac{5400}{\pi}}$

PROBLEM 6.2.* A full-wave rectifier is a device whose output is the absolute value of its input. Suppose that a rectified version of $s(t) = 6\cos(200\pi t)$ is input to the cascade of an ideal C-D converter and an ideal D-C converter, both with sampling rate $f_s = 300$ Hz, as shown below:





(a) Find an expression for the output $y(t)$, simplified as much as possible.

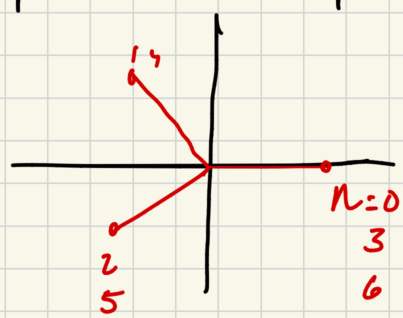
(*Hint 1*: This problem is easiest to solve in the time domain: first determine the values $x[0]$, $x[1]$, $x[2]$, etc. before considering what $y(t)$ might look like. *Hint 2*: The solution to HW5 Prob. 5.2(d) may be useful.)

(b) Sketch $x(t) = |s(t)|$ and $y(t)$ on the same scale for time in the range $0 < t < 0.03$. Confirm that the two curves intersect at the sampling times. Label the intersection of the two curves at time 0 by $x[0]$, the intersection at time $1/f_s$ by $x[1]$, the intersection at time $2/f_s$ by $x[2]$, and similarly label the intersection of the two curves at time n/f_s by $x[n]$ for all $n \in \{0, 1, \ldots 9\}$.

a) $|6\cos(2\pi(100)t)| \rightarrow x[n] = \left|6\cos\left(2\pi\left(\frac{100}{300}\right)n\right)\right| \Rightarrow \left|6\cos\left(\frac{2\pi}{3}n\right)\right|$

$x[n] = \{ \underline{|6|}, |-3|, |-3|, |6|, |-3|, \ldots \}$

$x[n] = \{ \underline{6}, 3, 3, 6, 3, \ldots \}$ $\omega = 3$



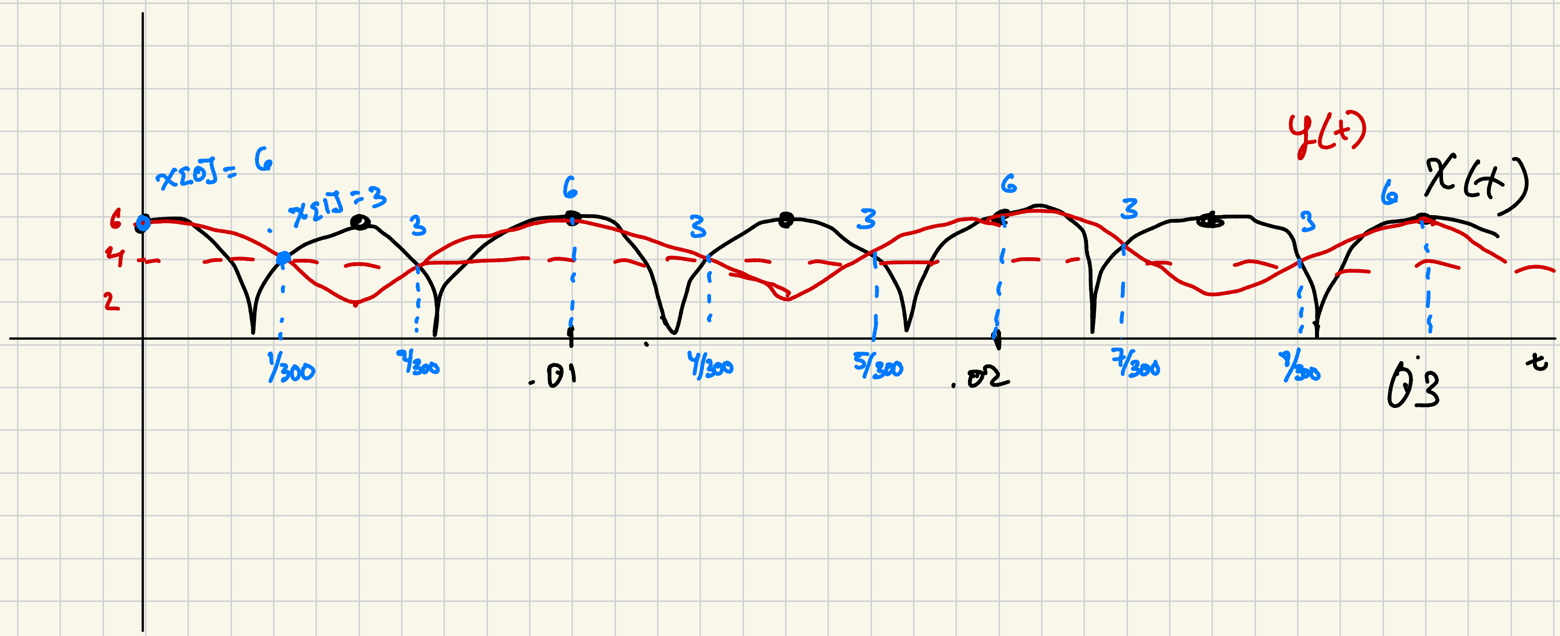
$x[n] = |6\cos(\frac{2\pi}{3}n)|$ $n = t f_s = 300t$

$$\boxed{y(t) = 4 + 2\cos(200\pi t)}$$

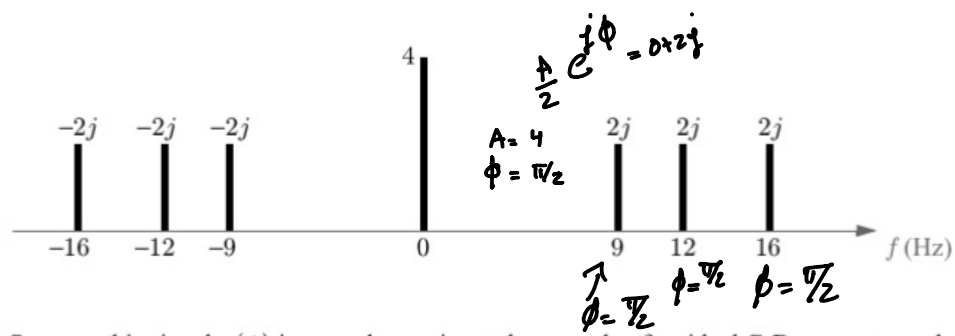
$\cos(\frac{2\pi}{3}(300)t)$

$\rightarrow \cos(200\pi t)$

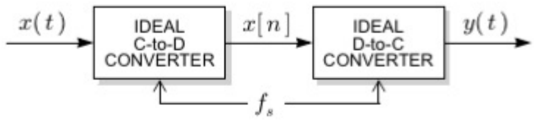
b



PROBLEM 6.3.* Consider a signal $x(t)$ whose spectrum is shown below:



Suppose this signal $x(t)$ is passed as an input the cascade of an ideal C-D converter and an ideal D-C converter, both with the same unspecified sampling rate f_s, as shown below:



(a) If $f_s = 40$ Hz, is the output $y(t)$ periodic?
If *YES*, specify its fundamental frequency f_0. If *NO*, explain why not.

(b) If $f_s = 1.1$ Hz, is the output $y(t)$ periodic?
If *YES*, specify its fundamental frequency f_0. If *NO*, explain why not.

(c) Find the largest sampling rate f_s for which the output $y(t)$ is periodic with fundamental frequency $f_0 = 3$ Hz.

(d) Find the largest sampling rate f_s for which the output $y(t)$ is periodic with fundamental frequency $f_0 = 9$ Hz.

(e) Find the largest sampling rate f_s for which the output $y(t)$ is periodic with fundamental frequency $f_0 = 12$ Hz.

a) Periodicity is Guaranteed if $\hat{\omega}_0 = 2\pi\left(\frac{f_0}{f_s}\right)$ can be Reduced and is Rational, and the Signal $x(t)$ is itself Periodic. $x(t)$ has fundamental frequency of 1 Hz $\Rightarrow$ $y(t)$ is also Periodic since the Sampling Rate will result in Perfect Reconstruction of $x(t)$.

b) $f_{0_x} = 9 \Rightarrow f_{1y} = \min_l |9 - 1.1l|$ $\quad l = 8 \Rightarrow f_{1y} = .2 \Rightarrow y(t) = \cos(2\pi(.2)t + \phi_1)$

$= 12 \Rightarrow f_{1y} = \min_l |12 - 1.1l|$ $\quad l = 11 \Rightarrow f_{1y} = .1 \Rightarrow y(t) = \cos(2\pi(.1)t + \phi_2)$

$16 \Rightarrow f_{1y} = \min_l |16 - 1.1l|$ $\quad l = 15 = f_{1y} = .5 \Rightarrow y(t) = \cos(2\pi(.5)t + \phi_3)$

Yes with $\boxed{f_0 = .1 \text{ Hz}}$

c)

Since 9 and 12 Hz are already fundamental to 3 — we need 16 Hz to fold to a multiple of 3 Hz or 0. if we sample @ 32 Hz, then with the 90° deg phase shift, the signal will be sampled every time $\cos(2\pi(16)t + \pi/2)$ crosses the x axis resulting in samples of only zero.
$\Rightarrow$ $y(t)$ only preserves 9 Hz and 12 Hz, both multiples of 3.

D) Periodic for $f_0 = 9$ Hz

$\Rightarrow$ want 12 Hz signal to cancel 16 Hz aliased signal.

$24 < f_s < 32 \Rightarrow \min_l |16 - f_s l| = 12$

$|16 - f_s| = 12 \Rightarrow f_s = \cancel{4}$ or 28

if $f_s = 28$ Hz $\Rightarrow$ for $f_1 = 16$ Hz, the output is

$4\cos(2\pi(16-28)t + \pi/2) \Rightarrow 4\cos(2\pi(-12)t + \pi/2) \Rightarrow$

$4\cos(2\pi(12) - \pi/2)$ which is 180° out of phase with original $4\cos(2\pi(12) + \pi/2)$ signal $\Rightarrow$ only 9 Hz signal remains

$\Rightarrow$ fundamental frequency of $y(t)$ is 9 Hz.

E) Same as 6.2D but get 16 to fold to 9 Hz

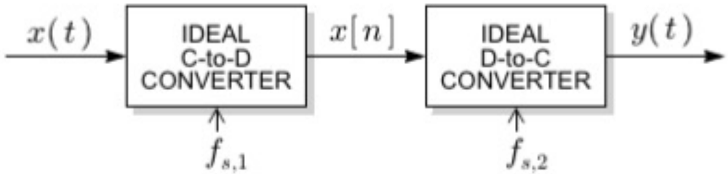
$\min_l |16 - f_s l| = 9$

$|16 - f_s| = 9 \Rightarrow f_s = 25$ or $\cancel{7\text{ Hz}}$

$4\cos(2\pi(16-25)t + \pi/2)$
$= 4\cos(-2\pi(9) + \pi/2)$
$= 4\cos(2\pi(9) - \pi/2)$

only $4\cos(2\pi(12) + \pi/2) = y(t)$ remains $\Rightarrow f_s = 25$

PROBLEM 6.4.* Consider the following cascade of an ideal C-D converter with sampling rate $f_{s,1}$ and an ideal D-C converter with sampling-rate parameter $f_{s,2}$:



While it is common for the two parameters to be identical ($f_{s,2} = f_{s,1}$), here we consider the case where they are different.

(a) Suppose that $x(t)$ is the recording of a pop song that starts at time 0 and ends at time 240, a total of 4 minutes. A radio broadcaster wants to use the above-pictured system to shorten the song from 4 minutes to 3 minutes (180 seconds) to leave more time for commercials. If $f_{s,1} = 44.1$ kHz, find $f_{s,2}$ so that $y(t)$ starts at time 0 and ends at time 180.

(b) Using $f_{s,2} \neq f_{s,1}$ will not only change the duration of the song, it will also change the pitch. Suppose a segment of the song consists of a single tone at $f_0 = 800$ Hz, namely $x(t) = \cos(1600\pi t)$ for $t \in (1, 1.1)$; the corresponding tone for $y(t)$ will be at a different frequency $f_0{}' \neq 800$ Hz.

Find $f_0{}'$ when $f_{s,1} = 44.1$ kHz, and when $f_{s,2}$ is chosen according to part (a).

a) $x[n] = A\cos\left(2\pi\left(\frac{f_0}{f_s}\right)n + \phi\right)$ $\quad 0 < t < 240$

$n \in \{0 : 1/44.1\text{kHz} : 240\}$

$t = n f_s$ $\quad$ n has $(44.1E3)(240)$ Samples

$\boxed{f_{s2} = 58.8\text{ kHz}}$ $\quad 240(44.1e3) = 180 \cdot f_s$

b) $x(t) = \cos(2\pi(800)t)$ $\quad f_0 = 800$ $\quad \hat{\omega}_0 = \frac{8\cancel{00}}{441\cancel{00}}$

$x[n] = \cos\left(2\pi\left(\frac{8}{441}\right)n\right)$

$y(t) = x[n = t f_s] = \cos\left(2\pi\left(\frac{8}{441}\right)(t)(58.8e3)\right)$

$y(t) = \cos(2\pi(1066.7)t) \Rightarrow \boxed{f_1 = 1066.7\text{ Hz}}$

Made with Goodnotes

PROBLEM 6.5.* The following difference equation defines an LTI discrete-time system:

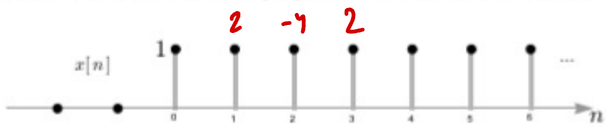
$$y[n] = 2x[n] - 4x[n-1] + 2x[n-2].$$

(a) Find the filter coefficients $\{b_k\}$ in the FIR representation $y[n] = \sum_{k=0}^{M} b_k x[n-k]$.

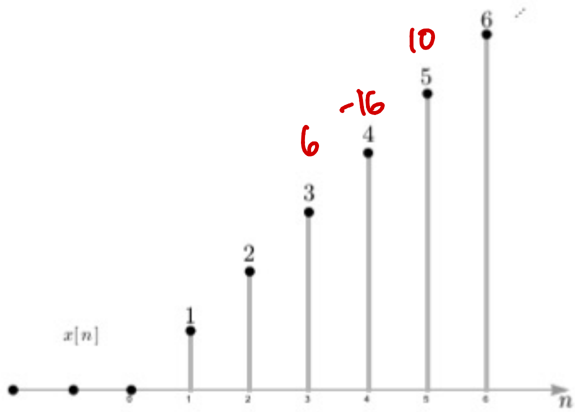
(b) Sketch a *stem plot* of the impulse response $h[n]$. Label each nonzero value.

(c) Sketch a stem plot of the output $y[n]$ when the input is $x[n] = \delta[n-1] + \delta[n-2] + \delta[n-3]$, as sketched below:

(d) Sketch a stem plot of the output $y[n]$ when the input is the unit step, $x[n] = u[n]$:



(e) Sketch a stem plot of the output $y[n]$ when the input is the unit ramp, $x[n] = nu[n]$, as sketched below:

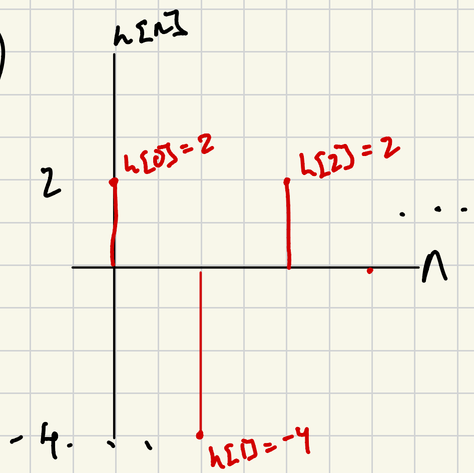


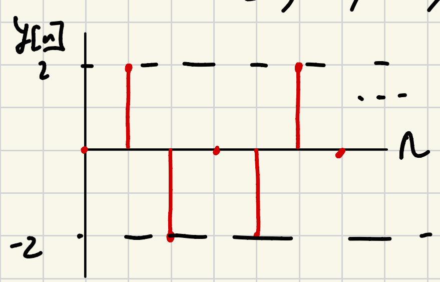
a) $b_n \in \{\underline{2}, -4, 2\}$

c) $x[n] = \{\underline{0}, 1, 1, 1\}$

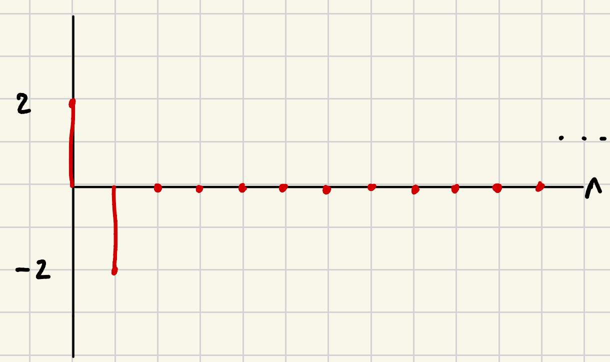
$y[n] = \{\underline{0}, 2, -2, 0, -2, 2\}$

b)

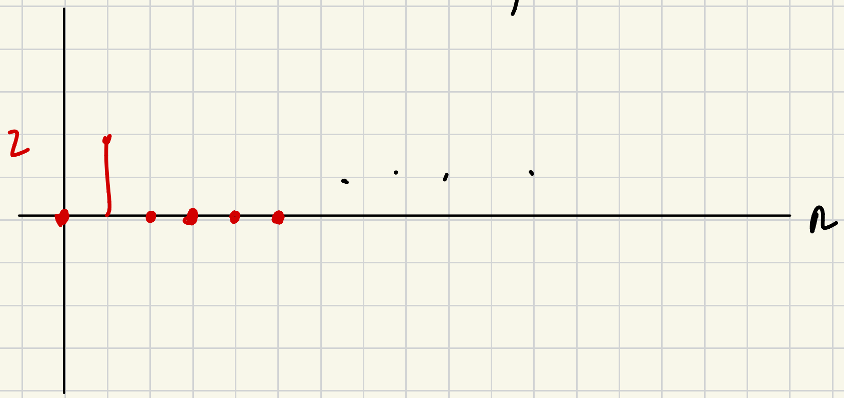




D) $x = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$



e) $x = \begin{cases} n, & n \geq 0 \\ 0, & n < 0 \end{cases}$



$y[n] = 2x[n] - 4x[n-1] + 2x[n-2]$

$x[n-1] = x[n-2] + 1$

$x[n] = x[n-2] + 2$

$\Rightarrow$

$\cancel{2x[n-2]} \cancel{+4} \cancel{-4x[n-2]} \cancel{-4} \cancel{+2x[n-2]}$

$= 0$ for $n \geq 2$