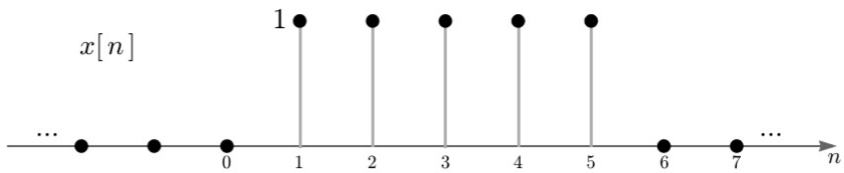
PROBLEM 7.1.* The following difference equation defines an LTI discrete-time system:

$$y[n] = x[n] + 2x[n-1] + x[n-2].$$

(a) Sketch a stem plot of the impulse response $h[n]$. Label each nonzero value.

(b) Find the filter coefficients $\{b_k\}$ in the FIR representation $y[n] = \sum_{k=0}^{M} b_k x[n-k]$.

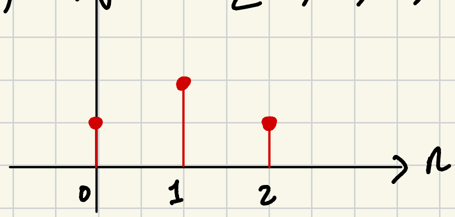
(c) Sketch a stem plot of the output $y[n]$ when the input is $x[n] = u[n-1] - u[n-6]$, the rectangular sequence whose stem plot is shown below. Use convolution.



(d) Sketch a stem plot of the output $y[n]$ when the input is the plus-minus sequence (a maximum-frequency sinusoid) $x[n] = (-1)^n = \cos(\pi n)$.

(You don't need Chap. 6 to solve this problem, you can stick to the time domain and use the tools of Chap. 5.)

a) $h[n] = \{1, 2, 1, 0, 0, \ldots\}$



b) $b_k \in \{1, 2, 1\}$

c) $x[n] = \{\underline{0}, 1, 1, 1, 1, 1, 0, 0, \ldots\}$

$y[n] = h[n] * x[n]$

=> 0 1 1 1 1 1 0 0

1 2 1 → 0

1 2 1 → 1

1 2 1 → 3

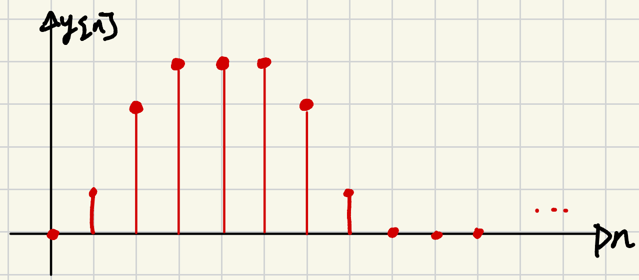
1 2 1 → 4

1 2 1 → 4

1 2 1 → 4

1 2 1 → 3

1 2 1 → 1



D) $x[n] = \cos(\pi n) = \{1, -1, 1, -1, 1, -1, \ldots\}$

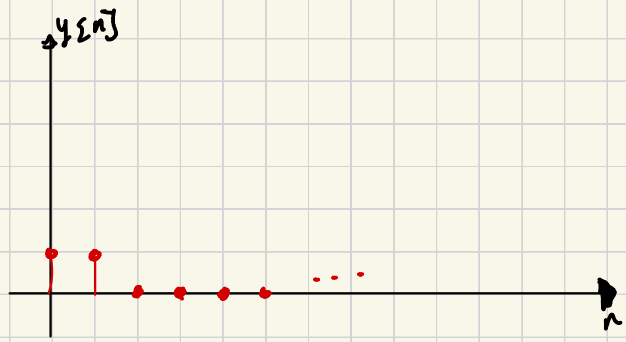
1 2 1 → 1 $n=0$

1 2 1 → 1 $n=1$

1 2 1 → 0 $n=2$

1 2 1 → 0 $n=3$

⋮



PROBLEM 7.2.* Suppose we want to convolve the following two signals:

$x[n] = \{2, 1, 3, -1, -2, 1, 2, -2, 1\}$

$$x[n] = 2\delta[n] + \delta[n-1] + 3\delta[n-2] - \delta[n-3] - 2\delta[n-4] + \delta[n-5] + 2\delta[n-6] - 2\delta[n-7] + \delta[n-8],$$

$$h[n] = 2\delta[n] + \delta[n-1]. \quad h[n] = \{2, 1\}$$

There are many ways to implement convolution, but for this problem you must implement it using a *convolution table*. Even then you have an extra degree of freedom that follows from the fact that convolution is commutative: You can view either of the above two signals as the filter input, and the other as the filter impulse response, it doesn't matter which is which, the filter output will be the same. The point of this problem is to implement it both ways, to confirm that they yield the same answer, and to observe that one way is easier than the other.

(a) Implement the convolution using a "tall, narrow" table that has lots of rows: the zero-th row represents $x[0]h[n]$, the next row represents $x[1]h[n-1]$, the next $x[2]h[n-2]$, and in general the k-th row (for $k \in \{0, 1, 2, ...\}$) represents $x[k]h[n-k]$.
The convolution is found by adding all of the rows.

(b) Implement the convolution the other way, using a "short, fat" table with only two main rows: the zero-th row represents $h[0]x[n]$, the next row represents $h[1]x[n-1]$. The convolution is found by adding these two rows.

(c) Verify that the answers in (a) and (b) are identical.

(d) In your opinion, which was easier to implement, (a) or (b)?

a)

$x[n]$ \ $h[n]$	2	1
2	4	2
1	2	1
3	6	3
-1	-2	-1
-2	-4	-2
1	2	1
2	4	2
-2	-4	-2
1	2	1

$y[n]: \{4, 4, 7, 1, -5, 0, 5, -2, 0, 1\}$

c) Signals match ✓

D) B was easier to implement

⇒ Column vector signal is 2nd in conv eqn

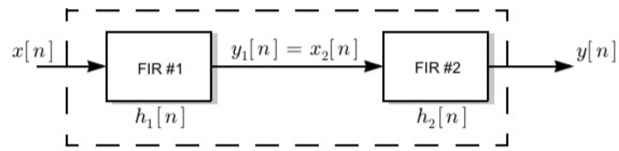
$$y[n] = \sum_k x[k]\,h[n-k]$$

b)

$h[n]$ \ $x[n]$	2	1	3	-1	-2	1	2	-2	1
2	4	2	6	-2	-4	2	4	-4	2
1	2	1	3	-1	-2	1	2	-2	1

$h[n] = \{4, 4, 7, 1, -5, 0, 5, -2, 0, 1\}$

PROBLEM 7.3.* The figure below depicts a *cascade* connection of two FIR filters, in which the output $y_1[n]$ of the first filter is the input $x_2[n]$ to the second filter, and the output of the second filter is the *overall* output $y[n]$:



The "overall" filter formed by this cascade has input $x[n]$ and output $y[n]$, as depicted by the dashed box.

(a) Find the overall difference equation (relating $y[n]$ to $x[n]$) when:

- the first filter is a two-point averager
- the second filter is a first difference filter

(b) Find the overall step response (i.e., find $y[n]$ when $x[n] = u[n]$) when:

- the first filter is a five-point averager
- the second filter is a first difference filter

(c) Find the second impulse response $h_2[n]$ when the difference equations defining the first filter and the overall filter are:

$$y_1[n] = 2x[n] + 2x[n-1] - x[n-2],$$
$$y[n] = 4x[n] + 10x[n-3] - x[n-4] - x[n-5].$$

Specify your answer in two forms: (i) as a stem plot of $h_2[n]$ vs n; and (ii) by listing numerical values for $h_2[0]$, $h_2[1]$, $h_2[2]$, $h_2[3]$, and $h_2[4]$.

a) $y_1[n] = \frac{1}{2}(x[n] + x[n-1]) \Rightarrow h_1[n] = \{\frac{1}{2}, \frac{1}{2}\}$

$y_2[n] = x[n] - x[n-1] \rightarrow h_2[n] = \{1, -1\})$

$h[n] = h_1[n] * h_2[n] = \{1/2, 0, -1/2\}$

$y[n] = \frac{1}{2}x[n] - 1/2\, x[n-2]$

b) $h_1[n] = \{1/5, 1/5, 1/5, 1/5, 1/5\}$ $\quad h_2[n] = \{1, -1\}$

$h[n] = \{1/5, 0, 0, 0, 0, -1/5\}$

1

1/5
1/5
1/5
1/5
1/5
0

$S[n] = \{\underbrace{1/5, 1/5, 1/5, 1/5, 1/5}, 0, 0, ..\}$

$S[n] = \begin{cases} .2 & \text{for } 0 \le n \le 4 \\ 0 & \text{for } n \ge 5 \end{cases}$

Made with Goodnotes

C) $h_1[n] = \{\underline{2}, 2, -1\}$ $\quad h[n] = \{\underline{4}, 0, 0, 10, -1, -1\}$

$h_2[n] = ?$

The convolved signal is dimension $N + M - 1$ where N is samples in $h_1[n]$ and M is samples in h_2

$\Rightarrow 6 = 3 + M - 1 \Rightarrow M = 4 \Rightarrow h_2$ has 4 elements

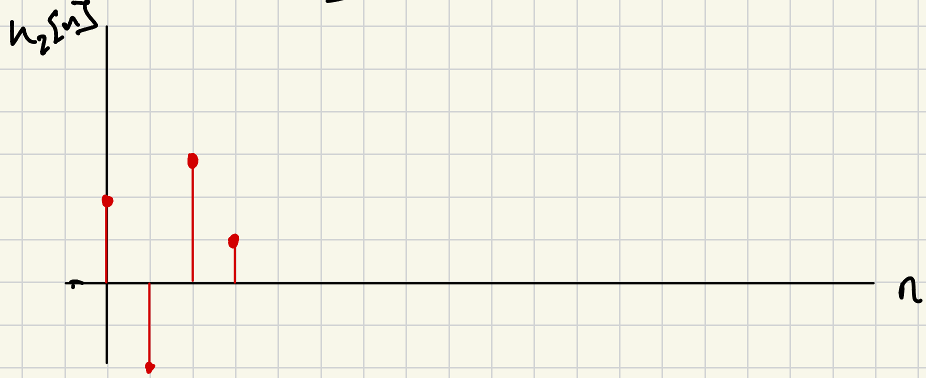
$h[n] = h_1 * h_2 \Rightarrow \sum_k h_1[k]\, h_2[n-k]$

	a	b	c	d
2	2a	2b	2c	2d
2	2a	2b	2c	2d
-1	-a	-b	-c	-d

$$2a = 4$$
$$2a + 2b = 0$$
$$-a + 2b + 2c = 0$$
$$-b + 2c + 2d = 10$$
$$-c + 2d = -1$$
$$-d = -1$$

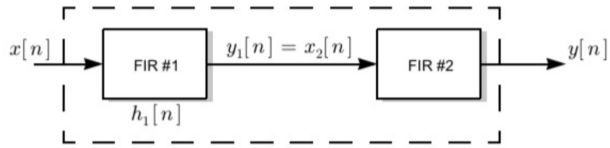
$$\begin{array}{ccccc} a & b & c & d & \\ \end{array}$$
$$\left[\begin{array}{cccc|c} 2 & 0 & 0 & 0 & 4 \\ 2 & 2 & 0 & 0 & 0 \\ -1 & 2 & 2 & 0 & 0 \\ 0 & -1 & 2 & 2 & 10 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & -1 \end{array}\right]$$

$h_2[n] = \{\underline{2}, -2, 3, 1\}$



PROBLEM 7.4.* Consider the following serial cascade of two FIR filters, where

- the first filter has impulse response $h_1[n] = 6\delta[n] + 12\delta[n-1]$;
- the second filter is a six-point averager:



(a) Find numerical values for the following overall outputs:

$$y[0],\ y[1],\ \ldots\ y[8]$$

when the overall input signal is $x[n] = 2\delta[n] - \delta[n-1]$.

(b) Find numerical values for the following overall inputs:

$$x[0]$$
$$x[1]$$
$$x[2]$$

when the overall output signal is $y[n] = 2\delta[n] + 4\delta[n-1] - 2\delta[n-6] - 4\delta[n-7]$.

a) $h_1[n] = \{6, 12\}$ $\quad h_2[n] = \{1/6, 1/6, 1/6, 1/6, 1/6, 1/6\}$

$h[n] = \{1, 3, 3, 3, 3, 3, 2\}$

$x[n] = \{2, -1\}$ $\quad y[n] = x[n] * h[n]$

$y[n] = \{\underline{2}, 5, 3, 3, 3, 3, 1, -2, 0\}$

b) $y[n] = \{\underline{2}, 4, 0, 0, 0, 0, -2, -4\}$ $\qquad 4 = 6 - 2$

$0 = 6 - 6$

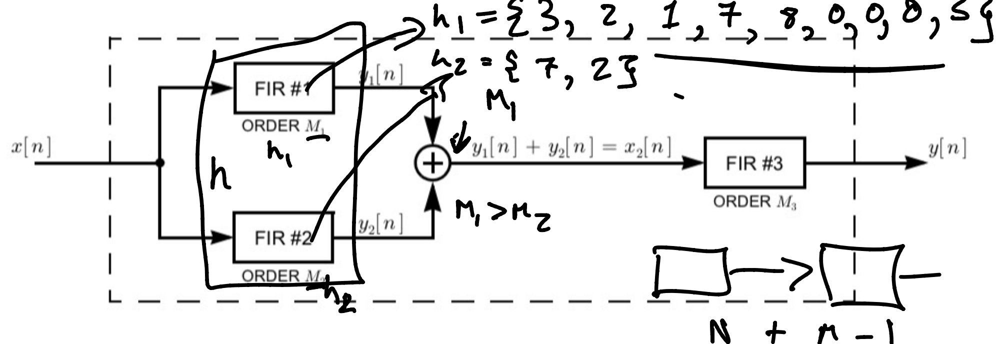
$x[0] = 2$

$8 = 7 + M - 1$

$x[1] = -2$

$x[2] = 0$

PROBLEM 7.5.* The figure below shows an overall filter constructed from three building-block filters using a combination of a parallel cascade and serial cascade:



The overall input $x[n]$ is fed as an input to both the first and second building-block filters, whose outputs are added before being fed to the last building block filter, yielding an overall output $y[n]$. Let M_i denote the order[1] of the i-th building-block filter, for $i \in \{1, 2, 3\}$, and let M denote the order of the *overall* filter (from $x[n]$ to $y[n]$), as indicated by the dashed box.

(a) Find M in the special case when all three building block filters are 8-point averagers.

(b) For the special case when $M_1 \geq M_2$ and $h_1[M_1] + h_2[M_1] \neq 0$:
Find an equation for the overall order M,
expressed as a function of the unspecified building-block orders $M_1 \geq M_2$, M_2, and M_3.

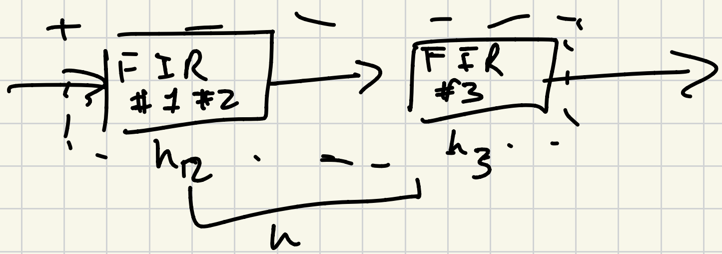
(c) How might the answer in part (b) change without the constraint $h_1[M_1] + h_2[M_1] \neq 0$?

(d) Find the overall impulse response $h[n]$ when:

- FIR#1 is a first difference filter
- FIR#2 is a two-point averaging filter
- FIR#3 is a ten-point averaging filter

a) When all Building Filters are 8-pt averagers

Since #1 and #2 are parallel summed =>



$h = h_{12} * h_3$

order of h is $N + M - 1$

$\Rightarrow 8 + 8 - 1 = \boxed{15}$

b) $M_1 + M_3 - 1$

c) without the constraint $h_1[M_1] + h_2[M_1] \neq 0$, meaning $M_1 = M_2$, Then the overall order is still $M_1 + M_3 - 1 = M$ since $M_1 = M_2$ in this case.

D) $h_1[n] = x[n] - x[n-1] \Rightarrow \{1, -1\}$

$h_2[n] = \frac{1}{2} x[n] + \frac{1}{2} x[n-1] \Rightarrow \{\frac{1}{2}, \frac{1}{2}\}$

$h_3[n] = \{\frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}\}$

$h_{12}[n] = \{\frac{3}{2}, -\frac{1}{2}\} * h_3[n]$

$h[n] = \{\frac{3}{20}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, -\frac{1}{20}\}$