$|\varphi_n\rangle = \begin{pmatrix} \cdot \\ \cdot \end{pmatrix}$

2×1 $\quad 1\times2 \quad \langle\varphi_n| = (\quad)$

14 if $A = \begin{pmatrix} 0 & i \\ 0 & 0 \end{pmatrix}$ $\quad A = \sum |\varphi_n\rangle\langle\varphi_n|$ where $\hat{A}^+$ is conjugate - Transpose

a) $A \neq A^+$ and $[A, A^+] \neq \hat{0}$

$A^+ = \begin{pmatrix} 0 & 0 \\ -i & 0 \end{pmatrix}$ That is $\langle\psi|A|\psi\rangle \neq \langle A^+\psi|\psi\rangle$

Since $\begin{pmatrix} 0 & i \\ 0 & 0 \end{pmatrix} \neq \begin{pmatrix} 0 & 0 \\ -i & 0 \end{pmatrix}$ Then $A \neq A^+$

$[A, A^+] \neq 0 \quad \therefore \quad \underset{A}{\begin{pmatrix} 0 & i \\ 0 & 0 \end{pmatrix}} \underset{B}{\begin{pmatrix} 0 & 0 \\ -i & 0 \end{pmatrix}} - \underset{B}{\begin{pmatrix} 0 & 0 \\ -i & 0 \end{pmatrix}} \underset{A}{\begin{pmatrix} 0 & i \\ 0 & 0 \end{pmatrix}} = \begin{pmatrix} 0-i^2 & 0 \\ 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & -i^2 \end{pmatrix}$

$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \neq \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow \neq 0$

b) Solve eigenvalue problem $A|\phi\rangle = a|\phi\rangle$, $|\phi\rangle \neq |null\rangle$

$A|\phi\rangle - a|\phi\rangle = 0 \Rightarrow A|\phi\rangle - a\mathbb{1}|\phi\rangle = 0 \quad (A - a\mathbb{1})|\phi\rangle = 0$

$\det(A - a\mathbb{1}) = 0$

$\det\left(\begin{pmatrix} 0 & i \\ 0 & 0 \end{pmatrix} - \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix}\right) \Rightarrow \det\begin{pmatrix} -a & i \\ 0 & -a \end{pmatrix} = 0$

$a^2 - i = 0 \Rightarrow a^2 = i \Rightarrow a = \sqrt{i}$

No vectors $|\phi\rangle$ span $\mathcal{H}_a$ the eigenspace, a subspace of $\mathcal{H}$ formed by eigenvalue $a = \sqrt{i}$

15 $\hat{A}$ is linear operator acting on $\mathcal{H}$ with $\dim(\mathcal{H})=2$

What is $\det(\hat{A})$ in terms of $tr(A)$ and $tr(\hat{A}^2)$?

$$\hat{A} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \qquad \det(A) = A_{11}A_{22} - A_{12}A_{21}$$

$$tr(A) = A_{11} + A_{22}$$

$$\hat{A}^2 = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} A_{11}A_{11} + A_{12}A_{21} & A_{11}A_{12} + A_{12}A_{22} \\ A_{21}A_{11} + A_{22}A_{21} & A_{21}A_{12} + A_{22}A_{22} \end{pmatrix}$$

$$tr(A^2) = A_{11}A_{11} + A_{12}A_{21} + A_{21}A_{12} + A_{22}A_{22}$$

$$(tr(A))^2 = \cancel{A_{11}A_{11}} + 2A_{11}A_{22} + \cancel{A_{22}A_{22}}$$

$$-\ tr(A^2) = \cancel{A_{11}A_{11}} + 2A_{12}A_{21} + \cancel{A_{22}A_{22}}$$

$$0 \quad (2A_{11}A_{22} - 2A_{12}A_{21})\tfrac{1}{2} = A_{11}A_{22} - A_{12}A_{21} = \det(\hat{A})$$

$$\therefore \det(\hat{A}) = \tfrac{1}{2}\left((tr(\hat{A}))^2 - tr(\hat{A}^2)\right)$$

16 a) observable A corresponds to operator (Hermitian)

$$A = \begin{pmatrix} 1 & e^{-i\frac{\pi}{3}} \\ e^{i\pi/3} & -1 \end{pmatrix} \qquad A^{+} = \begin{pmatrix} 1 & e^{i\pi/3} \\ e^{-i\pi/3} & -1 \end{pmatrix}$$

$A|\psi\rangle = a|\psi\rangle$ s.t. $\{a\}$ → complete set of measurements

$A|\psi\rangle = a\mathbb{1}|\psi\rangle$ on A: eigenvalues

$(A - a\mathbb{1})|\psi\rangle = 0$ Thus $\det(A - a\mathbb{1}) = 0$

$$\begin{pmatrix} 1 & e^{-i\pi/3} \\ e^{i\pi/3} & -1 \end{pmatrix} - \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} \Rightarrow \det\left[\begin{pmatrix} 1-a & e^{-i\pi/3} \\ e^{i\frac{\pi}{3}} & -1-a \end{pmatrix}\right] = 0$$

$$(1-a)(-1-a) - e^{-i\frac{\pi}{3}}e^{i\frac{\pi}{3}} = 0$$

$$-1 + a - a + a^2 - e^{i\pi/3 - i\pi/3} = 0$$

$$a^2 - 1 - e^0 = 0 \qquad a^2 - 2 = 0 \qquad \boxed{a = \sqrt{2} \ ; \ a = \pm\sqrt{2}}$$

b) find $|\psi\rangle$ s.t. a is largest

$A|\psi\rangle = a|\psi\rangle$

$A|\psi\rangle = \sqrt{2}\,|\psi\rangle$

$$\left[\begin{array}{cc|c} 1-a & e^{-i\pi/3} & 0 \\ e^{i\pi/3} & -1-a & 0 \end{array}\right] \Rightarrow (A - a\mathbb{1})\begin{pmatrix} |\psi_1\rangle \\ |\psi_2\rangle \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

for $a = \sqrt{2}$

$$\left[\begin{array}{cc|c} 1-\sqrt{2} & e^{-i\pi/3} & 0 \\ e^{i\pi/3} & -1-\sqrt{2} & 0 \end{array}\right]$$

$0 = (1-\sqrt{2})|\psi_1\rangle + e^{-i\pi/3}|\psi_2\rangle$

$0 = e^{i\pi/3}|\psi_1\rangle + (-1-\sqrt{2})|\psi_2\rangle$

$$\frac{|\psi_2\rangle}{|\psi_1\rangle} = -\frac{1-\sqrt{2}}{e^{-i\pi/3}} = (1-\sqrt{2})e^{i\pi/3} \qquad \frac{\psi_2}{\psi_1} = -\frac{e^{i\pi/3}}{-1-\sqrt{2}} \propto \frac{1}{1+\sqrt{2}}(e^{i\pi/3})$$

Thus $|\psi\rangle = c\begin{pmatrix} e^{i\pi/3} \\ 1-\sqrt{2} \end{pmatrix}$

16b $1-\sqrt{2}|\psi_1\rangle + e^{-i\frac{\pi}{3}}|\psi_2\rangle = e^{i\pi/3}|\psi_1\rangle + (-1-\sqrt{2})|\psi_2\rangle$

$\cancel{|\psi_1\rangle = \frac{e^{i\pi/3}}{1-\sqrt{2}}|\psi_1\rangle + \frac{(-1-\sqrt{2}-e^{-i\pi/3})}{1-\sqrt{2}}|\psi_2\rangle}$

$(1-\sqrt{2}-e^{i\frac{\pi}{3}})|\psi_1\rangle = (-1-\sqrt{2}-e^{-i\pi/3})|\psi_2\rangle$

$|\psi\rangle = (-1+\sqrt{2}+e^{i\pi/3})|\psi_1\rangle + (-1-\sqrt{2}-e^{-i\pi/3})|\psi_2\rangle$

16c $\hat{B} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ evaluate $\langle B \rangle$ on state $|\psi\rangle$

$\frac{1}{2\sqrt{2}}$ α $2\sqrt{2}$

$|\psi\rangle = C\begin{pmatrix} e^{-i\pi/3} \\ 1-\sqrt{2} \end{pmatrix}$ $\qquad \langle B \rangle = \langle \psi | B | \psi \rangle$

$\langle \psi | = (e^{+i\frac{\pi}{3}}, 1-\sqrt{2}) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} e^{-i\pi/3} \\ 1-\sqrt{2} \end{pmatrix} \Rightarrow (e^{+i\pi/3}, 1-\sqrt{2}) \begin{pmatrix} 1-\sqrt{2} \\ e^{-i\pi/3} \end{pmatrix}$

$e^{+i\pi/3}(1-\sqrt{2}) + (1-\sqrt{2})e^{-i\pi/3} = (1-\sqrt{2})(e^{i\pi/3} + e^{-i\pi/3})$

$\cos(\frac{\pi}{3}) + \cancel{i\sin(\frac{\pi}{3})} + \cos(-\frac{\pi}{3}) + i\sin(-\frac{\pi}{3})$

$(1-\sqrt{2})(2\cos(\frac{\pi}{3})) =$

$(1-\sqrt{2})(1) = \boxed{1-\sqrt{2}}$

17a A is hermitian operator on $\mathcal{H}$ where $\dim \mathcal{H} = 2$

what are eigenvalues of $\hat{A}$ via $\mathrm{tr}\, A$ and $\mathrm{tr}(A^2)$

$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = a \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$ $\qquad (A - a\mathbb{1})|\psi\rangle = 0$

$\det \begin{pmatrix} A_{11}-a & A_{12} \\ A_{21} & A_{22}-a \end{pmatrix} = 0$

$(A_{11}-a)(A_{22}-a) - A_{12}A_{21} = 0$

$A_{11}A_{22} - aA_{22} - aA_{11} + a^2 - A_{12}A_{21} = 0 \Rightarrow a^2 - aA_{22} - aA_{11} = A_{12}A_{21} - A_{11}A_{22}$

$\cancel{a(a - A_{22} - A_{11}) = A_{12}A_{21} - A_{11}A_{22}}$

$\cancel{a = \left[\frac{A_{12}A_{21} - A_{11}A_{22}}{a - A_{22} - A_{11}}\right]}$ $\qquad 1a^2 - (A_{22}+A_{11})a - (A_{12}A_{21} - A_{11}A_{22}) = 0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ $\Big|_{a,\hat{A}} \Rightarrow a = \frac{(A_{22}+A_{11}) \pm \sqrt{(A_{22}+A_{11})^2 - 4(A_{11}A_{22} - A_{12}A_{21})}}{2}$

$\frac{1}{2}\mathrm{tr}(A) \pm \frac{1}{2}((\mathrm{tr}(A))^2 - 4(\det(A)))^{1/2}$

$a = \frac{1}{2}\mathrm{tr}(A) \pm \frac{1}{2}\sqrt{(\mathrm{tr}(A))^2 - 2((\mathrm{tr}(A))^2 - \mathrm{tr}(A^2))}$

$a = \frac{1}{2}\mathrm{tr}(A) \pm \frac{1}{2}\sqrt{2\,\mathrm{tr}(A^2) - (\mathrm{tr}(A))^2}$ ✓

17B) $\langle \psi | \psi \rangle = \alpha \in \mathbb{C}$

$$a = \frac{1}{2}tr(A) \pm \frac{1}{2}\sqrt{2tr(A^2) - (tr(A))^2} \quad , \quad tr(|\phi\rangle\langle\psi|) = \langle \psi | \phi \rangle$$

$A|x\rangle = \lambda|x\rangle$ some arbitrary $|x\rangle \in \mathcal{H}$

$$tr(A) \Rightarrow tr(|\phi\rangle\langle\psi| + |\psi\rangle\langle\phi|) = \langle\psi|\phi\rangle + \langle\phi|\psi\rangle \boxed{= \alpha^* + \alpha}$$

$$tr(A^2) \Rightarrow tr((|\phi\rangle\langle\psi| + |\psi\rangle\langle\phi|)(|\phi\rangle\langle\psi| + |\psi\rangle\langle\phi|))$$

$$\Rightarrow tr(\underset{①}{|\phi\rangle\langle\psi|\phi\rangle\langle\psi|} + \underset{②}{|\psi\rangle\langle\phi|\phi\rangle\langle\psi|} + \underset{③}{|\phi\rangle\langle\psi|\psi\rangle\langle\phi|} + \underset{④}{|\psi\rangle\langle\phi|\psi\rangle\langle\phi|})$$

$$\Rightarrow tr(|\phi\rangle\alpha^*\langle\psi| + |\psi\rangle\langle\psi| + |\phi\rangle\langle\phi| + |\psi\rangle\alpha\langle\phi|)$$

$$\Rightarrow \alpha^* tr(|\phi\rangle\langle\psi|) + tr(|\psi\rangle\langle\psi|) + tr(|\phi\rangle\langle\phi|) + \alpha tr(|\psi\rangle\langle\phi|)$$

$$\Rightarrow \alpha^*\langle\psi|\phi\rangle + 1 + 1 + \alpha\langle\phi|\psi\rangle$$

$$\Rightarrow \alpha^*\langle\psi|\phi\rangle + \alpha\langle\phi|\psi\rangle + 2 \Rightarrow \boxed{(\alpha^*)^2 + \alpha^2 + 2}$$

$$(tr(A))^2 = (\langle\psi|\phi\rangle + \langle\phi|\psi\rangle)(\langle\psi|\phi\rangle + \langle\phi|\psi\rangle)$$

$$\Rightarrow (\alpha^* + \alpha)(\alpha^* + \alpha) \Rightarrow \boxed{(\alpha^*)^2 + 2\alpha\alpha^* + \alpha^2}$$

$$a = \frac{1}{2}(\alpha^* + \alpha) \pm \frac{1}{2}\sqrt{2(\alpha^*)^2 + 2\alpha^2 + 4 - (\alpha^{*2} + 2\alpha\alpha^* + \alpha^2)}$$

$$a = \frac{1}{2}(\alpha^* + \alpha) \pm \frac{1}{2}\sqrt{(\alpha^*)^2 + \alpha^2 - 2\alpha\alpha^* + 4}$$

17C let $\alpha = a + bi$ and $\alpha^* = a - bi$ where eigenvalue $a \neq a$ (alpha)

$$a = \frac{1}{2}(a - bi + a + bi) \pm \frac{1}{2}\sqrt{(a-bi)(a-bi) + (a+bi)(a+bi) - 2(a+bi)(a-bi) + 4}$$

$$a = \frac{1}{2}(2a) \pm \frac{1}{2}\sqrt{a^2 - 2abi + b^2i^2 + a^2 + 2abi + b^2i^2 - 2(a^2 - b^2i^2) + 4}$$

$$a = a \pm \frac{1}{2}\sqrt{2a^2 + 2b^2i^2 - 2a^2 + 2b^2i^2 + 4} \Rightarrow a = a \pm \frac{1}{2}\sqrt{-4b^2 + 4}$$

$$\Rightarrow a = a \pm \sqrt{1 - b^2} \in \mathbb{R}$$, eigenvalue is element of Reals

18 $\hat{S}_n = n \cdot \hat{S} = \begin{pmatrix} \langle +z|S_n|+z\rangle & \langle +z|S_n|-z\rangle \\ \langle -z|S_n|+z\rangle & \langle -z|S_n|-z\rangle \end{pmatrix} = \hbar/2 \begin{pmatrix} n_z & n_- \\ n_+ & -n_z \end{pmatrix}$

$n_\pm = n_x \pm i n_y$

$$S_n|\psi\rangle = \lambda|\psi\rangle \implies \det(\hat{S}_n - \lambda \mathbb{1}) = 0$$

$\det\left[\frac{\hbar}{2}\begin{pmatrix} n_z & n_- \\ n_+ & -n_z \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}\right] = 0 \implies \det\begin{bmatrix} \frac{\hbar}{2}n_z - \lambda & \frac{\hbar}{2}n_- \\ \frac{\hbar}{2}n_+ & -\frac{\hbar}{2}n_z - \lambda \end{bmatrix} = 0$

$(\frac{\hbar}{2}n_z - \lambda)(-\frac{\hbar}{2}n_z - \lambda) - (\frac{\hbar}{2}n_-)(\frac{\hbar}{2}n_+) = 0$

$-\frac{\hbar^2}{4}n_z^2 + \lambda^2 - \frac{\hbar^2}{4}n_- n_+ = 0$

$-\frac{\hbar^2}{4}(n_z^2 + \lambda^2 + n_- n_+) = 0$

$-\hbar^2/4\,(n_z^2 + \lambda^2 + (n_x - i n_y)(n_x + i n_y)) = 0$

$-\hbar^2/4\,(n_z^2 + \lambda^2 + n_x^2 - i^2 n_y^2) = 0$

$-\frac{\hbar^2}{4}(n_z^2 + \lambda^2 + n_x^2 + n_y^2) = 0$

$\lambda^2 = \frac{\hbar^2}{4}(n_x^2 + n_y^2 + n_z^2) \implies \lambda^2 = \frac{\hbar^2}{4}$ $\boxed{\lambda = \pm\frac{\hbar}{2}}$

$\langle +z|+z\rangle = n_z$

$\langle +z|-z\rangle = n_x - i n_y$

$\langle -z|+z\rangle = n_x + i n_y$

$\langle -z|-z\rangle = -n_z$

for $\lambda = \hbar/2$

$\left[\begin{pmatrix} \frac{\hbar}{2}n_z & \frac{\hbar}{2}n_- \\ \frac{\hbar}{2}n_+ & -\frac{\hbar}{2}n_z \end{pmatrix} - \begin{pmatrix} \hbar/2 & 0 \\ 0 & \hbar/2 \end{pmatrix}\right]\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \implies \begin{bmatrix} \frac{\hbar}{2}n_z - \frac{\hbar}{2} & \frac{\hbar}{2}n_- \\ \frac{\hbar}{2}n_+ & -\frac{\hbar}{2}n_z - \frac{\hbar}{2} \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\frac{\hbar}{2}(n_z - 1)\alpha + \frac{\hbar}{2}n_- \beta = 0 \implies \frac{\hbar}{2}(\cos\theta - 1)\alpha + \frac{\hbar}{2}(n_x - i n_y)\beta$

$\frac{\hbar}{2}n_+\alpha \cancel{+_x} - \frac{\hbar}{2}(n_z + 1)\beta = 0$

$+\frac{\hbar}{2}(\sin\theta\cos\phi - i\sin\theta\sin\phi)\beta$

$\cancel{\cos\theta - 1}$

$\frac{\hbar}{2}\sin\theta(\cos\phi - i\sin\phi)\beta$

$-(1 - \cos(2\theta)) = 2\sin^2(\theta)$ ① $-\hbar\sin^2(\frac{\theta}{2})\alpha + \frac{\hbar}{2}\sin\theta\, e^{-i\phi}\beta = 0$

$\cos(2\theta) - 1 = -2\sin^2(\theta)$ ② $\frac{\hbar}{2}\sin\theta e^{i\phi}\alpha + \hbar\sin^2(\frac{\theta}{2})\beta = 0$

$\cos(\theta) - 1 = -2\sin^2(\frac{\theta}{2})$ $\to \alpha = \dfrac{-\frac{\hbar}{2}\sin\theta\, e^{-i\beta}\beta}{-\hbar\sin^2(\frac{\theta}{2})} = \dfrac{\hbar\sin\theta\, e^{-i\beta}}{1 - \cos(\theta)} b$

$\frac{\hbar}{2}(n_x + i n_y)\alpha + \frac{\hbar}{2}(\cos\theta + 1)\beta = 0$

$\frac{\hbar}{2}(\sin\theta\cos\phi + i\sin\theta\sin\phi)\alpha + \frac{\hbar}{2}(\cos\theta + 1)\beta = 0$ $\frac{\hbar}{2}\sin\theta e^{i\phi}\left(\frac{\sin\theta e^{-i\beta}\beta}{1 - \cos(\theta)}\right) + \hbar\sin^2(\frac{\theta}{2})\beta = 0$

$\hbar/2\sin\theta e^{i\phi}\alpha + \frac{\hbar}{2}(2\sin^2(\theta/2))\beta = 0$ $\dfrac{\frac{\hbar}{2}\sin^2\theta}{1 - \cos(\theta)}\beta + \hbar\sin^2(\frac{\theta}{2})\beta = 0$

① $-\hbar \sin^2(\frac{\theta}{2})\alpha + \frac{\hbar}{2}\sin\theta e^{-i\phi} = 0$

② $\frac{\hbar}{2}\sin\theta e^{i\phi}\alpha + \hbar\sin^2(\theta/2)B = 0$

$$\alpha = \frac{-\hbar/2\sin\theta e^{-i\phi}}{-\hbar\sin^2(\theta/2)}B \Rightarrow \frac{\sin\theta e^{-i\phi}}{2\sin^2(\frac{\theta}{2})}B$$

$$\frac{\hbar}{2}\sin\theta e^{i\phi}\left(\frac{\sin\theta e^{-i\phi}}{2\sin^2(\theta/2)}B\right) + \hbar\sin^2(\theta/2)B = 0$$

$$\frac{\hbar\sin^2\theta}{4\sin^2\frac{\theta}{2}}B + \hbar\sin^2(\frac{\theta}{2})B = 0$$

$$B = -\frac{\hbar\sin^2\theta}{4\sin^2\frac{\theta}{2}} - \hbar\sin^2(\frac{\theta}{2})$$

$$\left(\frac{-\hbar}{4}\right)\left(\frac{1}{2}\right) 1 - \cos$$

$$\begin{bmatrix} \frac{\hbar}{2}(\cos\theta - 1) & \frac{\hbar}{2}(\sin\theta(\cos\phi - i\sin\phi)) \\ \frac{\hbar}{2}\sin\theta(\cos\phi + i\sin\phi)) & -\frac{\hbar}{2}(\cos\theta + 1) \end{bmatrix} \begin{bmatrix} \alpha \\ B \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\frac{\hbar}{2}(\cos\theta - 1)\alpha + \frac{\hbar}{2}\sin\theta e^{-i\phi}B = 0 \Rightarrow (\cos\theta - 1)\alpha + \sin\theta e^{-i\phi}B = 0$

$\frac{\hbar}{2}\sin\theta e^{i\phi}\alpha - \frac{\hbar}{2}(\cos\theta + 1)B = 0 \Rightarrow \sin\theta e^{i\phi}\alpha - (\cos\theta + 1)B = 0$

from Eq 2: $B = \frac{\sin\theta e^{i\phi}}{\cos\theta + 1}\alpha$

$(\cos\theta - 1)\alpha + \sin\theta e^{-i\phi}\left(\frac{\sin\theta e^{i\phi}}{\cos\theta + 1}\right)\alpha = 0 \Rightarrow (\cos\theta - 1)\alpha + \frac{\sin^2\theta}{\cos\theta + 1}\alpha = 0$

$\alpha((\cos\theta + 1)(\cos\theta - 1) + \sin^2\theta) = 0 \Rightarrow \alpha(\cos^2\theta - 1 + \sin^2\theta) = 0 \quad \alpha(0) = 0$?...

$\frac{\hbar}{2}(n_z - 1)\alpha + \frac{\hbar}{2}B(n_x - in_y) = 0 \quad B = \frac{+\alpha(n_x + in_y)}{+(n_z + 1)}$

$\frac{\hbar}{2}\alpha(n_x + in_y) - \frac{\hbar}{2}B(n_z + 1) = 0$

$(n_z - 1)\alpha + \left(\frac{\alpha(n_x + in_y)}{n_z + 1}\right)(n_x - in_y) = 0$

$\alpha(n_z - 1) + \alpha(n_x^2 + n_y^2)/(n_z + 1) = 0$

18 Eigenvectors for $+\lambda = \hbar/2$ $|n\rangle = \cos\frac{\theta}{2}|+z\rangle + e^{i\phi}\sin(\frac{\theta}{2})|-z\rangle$

Answer → is a solution since $S_n|n\rangle = \lambda|n\rangle$

and for $\lambda = -\hbar/2$ $|-n\rangle = \sin\theta/2\,|+z\rangle - e^{+i\phi}\cos\frac{\theta}{2}|-z\rangle$

is sol since $S_n|-n\rangle = -\frac{\hbar}{2}|-n\rangle$