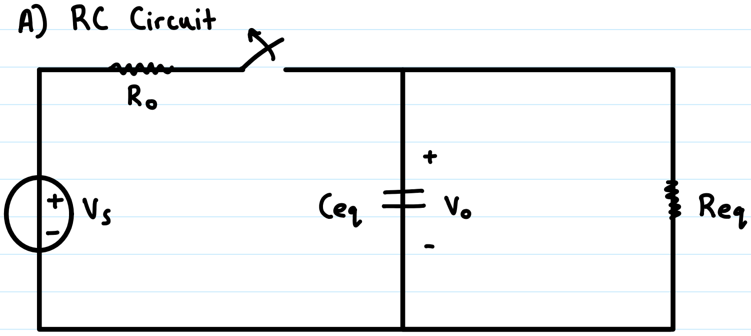
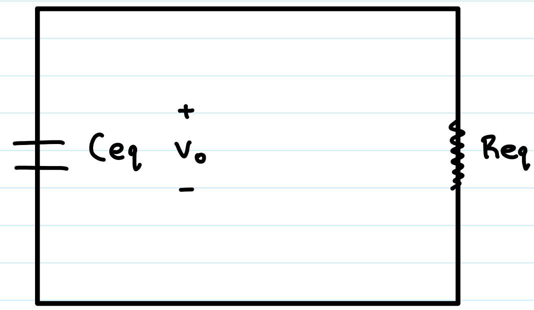
10/24/2024

Thursday, October 24, 2024 8:02 AM

A) RC Circuit

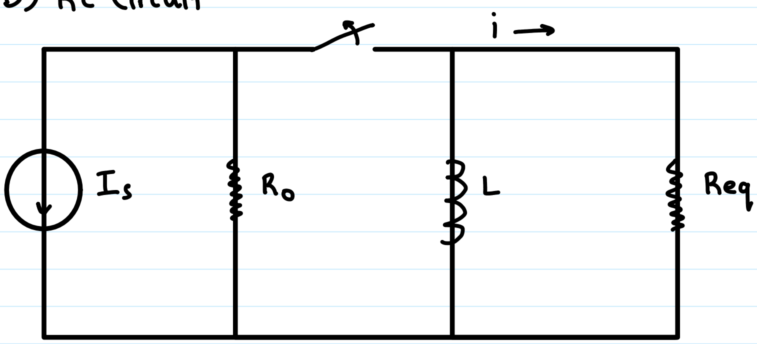


reduction →

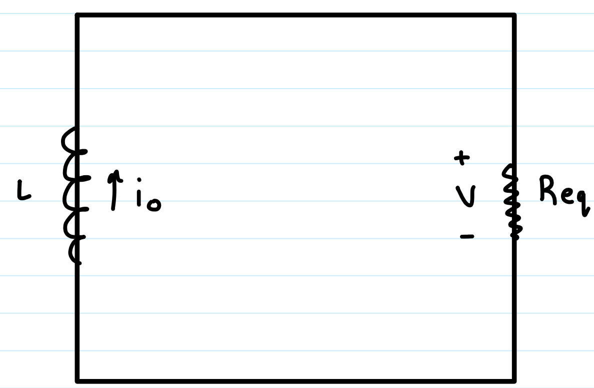


Prior to $t = 0$, switch is closed.
Prior to time $= 0$ $(t = 0^-)$, switch is closed.
Prior to time ≥ 0 $(t = 0^+)$, switch is open.

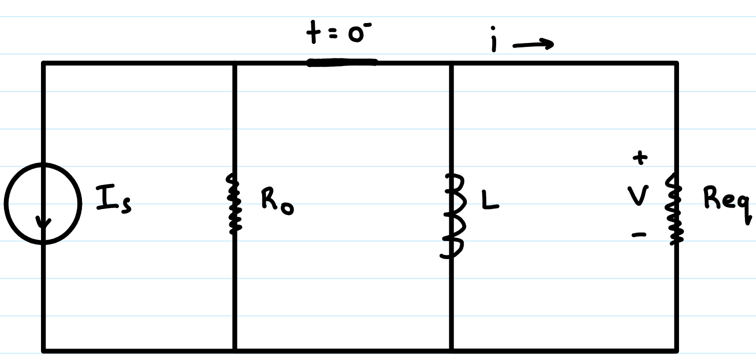
B) RL Circuit



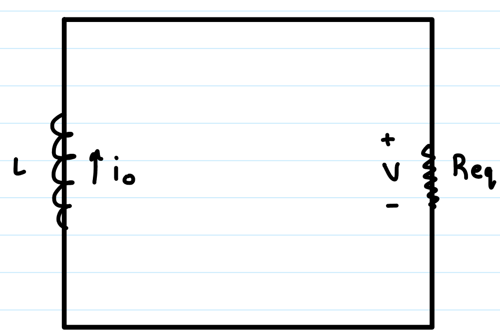
reduction →



1)
What is the natural response of circuit B at $t \geq 0$?



circuit reduction →

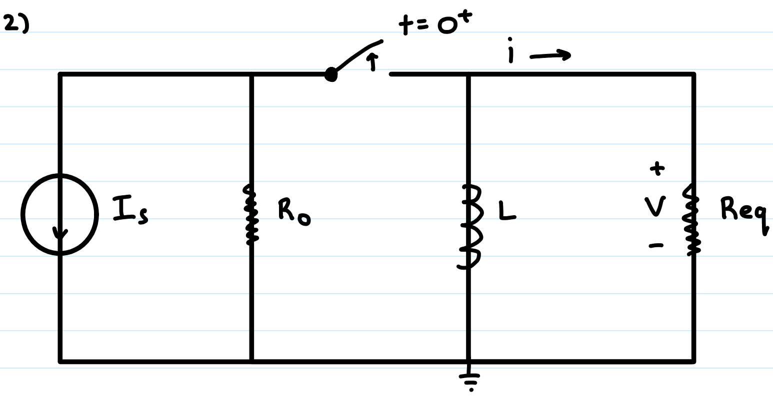


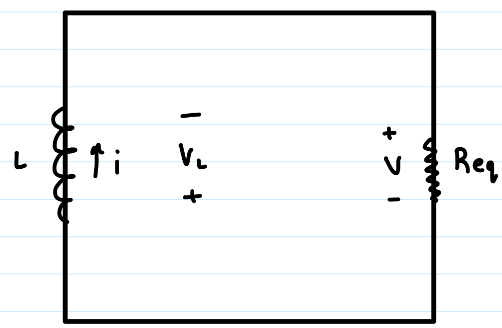
$V = L\,(di/dt)$

$V = 0$

$t = 0^-$, current through is I_s. ("Shunt") - small/little resistance

2)





KVL: $V_L + V = 0$

$L\,(di/dt) = V_L$

$V = Ri$

$\longrightarrow L\,(di/dt) + Ri = 0$ (differential equation).

$L\,(di/dt) + Ri = 0$

$$\frac{di}{dt} = \frac{-Ri}{L}$$

$$\frac{di}{i} = -\frac{R}{L}\,dt$$

$$\int_{i(t=0)}^{i(t)} \frac{1}{x}\,dx = \int -\frac{R}{L}\,dt$$

$$\ln \frac{i(t)}{i(0)} = -\frac{R}{L}t$$

$$e^{\ln\left(i(t)/i(0)\right)} = e^{-\frac{R}{L}t}$$

$$i(t) = i(0)\left(e^{-\frac{R}{L}t}\right)$$

$$i(0) = i(0^-) = i(0^+) = I_0$$

$$\boxed{i(t) = I_0\, e^{-\frac{R}{L}t}}$$

Ohm's Law: $V = IR$

$v(t) = I_0 R e^{-\frac{R}{L}t}$

Ohm's Law: $V = IR$

$V(t) = I_o R e^{-\frac{R}{L}t}$

$V(0^-) = 0$ (across inductor).

$V(0^+) = I_o R$.

*) Power dissipated in the resistor can be obtained from:

$P = Vi = i^2 R = \frac{V^2}{R}$

$P = I_o e^{-(R/L)t} I_o e^{-(R/L)t} R$

$P = I_o^2 e^{-2(R/L)t}, \ t \geq 0 \ (t = 0^+)$

Time constant (τ) ("tau")

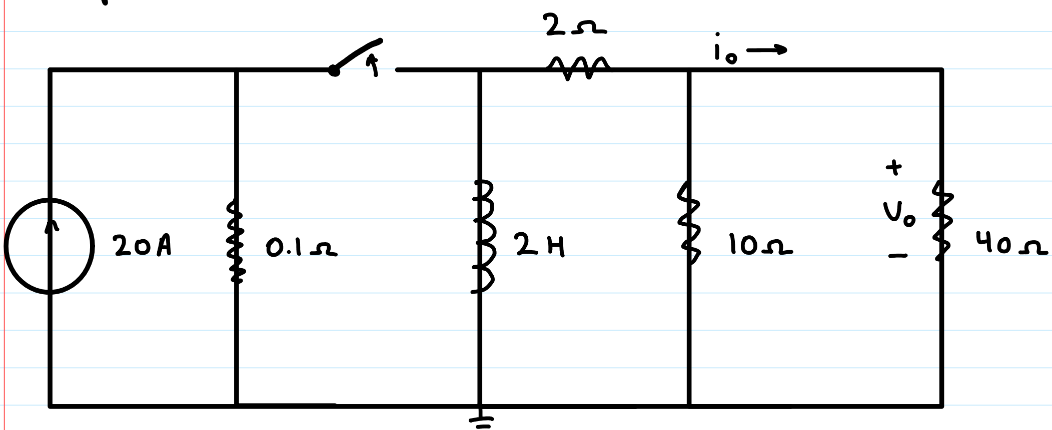
$\tau = \frac{L}{R}$ ("tau")

$i(t) = I_o e^{-t/\tau}$

$$w(t) = \int_0^t P(x)\,dx = \int_0^t I_o^2 R e^{-2(R/L)(x)}\,dx$$

$$= \frac{1}{2} L I_o^2 (1 - e^{-2(R/L)t}), \ t \geq 0, \ t(0^+).$$

Example: RL Circuit

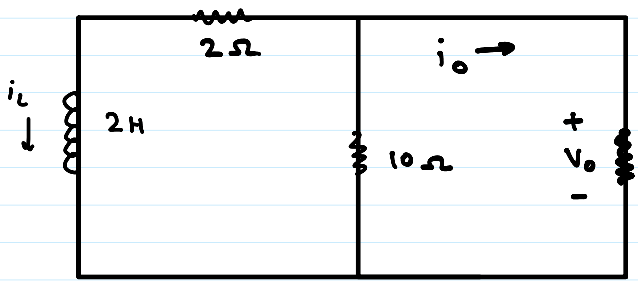


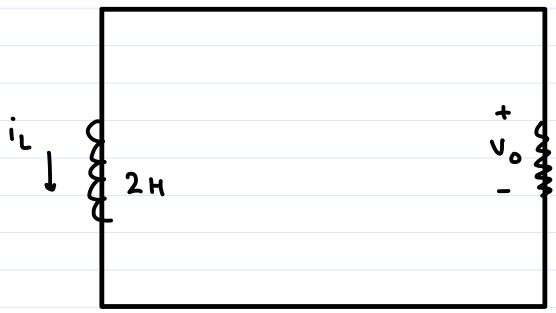
a) $i_L(t) = ?$ for $t \geq 0$.

$i_L(t) = 20\text{ A}$

$i_L(0^-) = i_L(0^+) = i_L(0)$

b) $i_0(t) = ?$ for $t \geq 0^+$





$\frac{1}{10} + \frac{1}{40} = \frac{1}{R_{eff}}$

$R_{eq} = 2 + (10 \parallel 40)$

$= 10\,\Omega$

Time constant:

$\tau = \frac{L}{R} = \frac{2}{10} = 0.2$ seconds.

Natural response:

$$\boxed{i(t) = 20e^{-5t}\ A, \quad t \geq 0^+}$$

$i_0 = -i_L\left(\frac{10}{40+10}\right)$

$$\boxed{i_0(t) = -4e^{-5t}\ A, \quad t \geq 0.}$$

$V_0(t) = 40(-4)\,e^{-5t}\ A$

$V_0(t) = -160e^{-5t}\ A$

$P_{10\Omega}(t) = \frac{V_0^2}{10}$

$= 2560\,e^{-10t}$ W, $t \geq 0^+$

$$\int^{w(t)} P_{10\Omega}(t)\,dt = 256\ J$$

$W_{Inductor} = \frac{1}{2} L I^2(0) = 400\ J$

$i_0 = -i_L\left(\frac{1}{5}\right)$

% energy dissipated by $10\,\Omega$ $(256/400) \sim 64\%$