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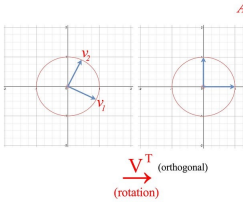
Thursday, November 30, 2023 23:31

1. Determine the singular values of matrix algebraically and geometrically.
$\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$
2. Determine a SVD of
$\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$

   Sketch how the unit circle is transformed under V^T, ΣV^T and $U\Sigma V^T$
3. If a square matrix is symmetric, what can you say about its eigenvalues and its singular values?
4. What can you say about the singular values of an orthogonal matrix? Explain this both geometrically and algebraically.
5. Consider an SVD of A given by $A = U\Sigma V^T$. What are the eigenvalues of AA^T? Can you give an orthonormal eigenbasis of AA^T? What conclusion can you draw on the relation between the eigenvalues of A^TA and AA^T ?
6. True or False
   a. If the col vectors of A, which is a 2×2 matrix, are orthogonal, then the singular values of A are the norms of its column vectors.
   b. Let $A, B \in M_2(\mathbb{R})$. Then AB, BA have the same singular values.
   c. If $A \in M_2(\mathbb{R})$ has singular values $1, 8$, then there exists a unit vector w such that $||Aw|| = 5$.



Please draw something like this vectors on the unit circle will be sigma_1 and. Use this to explair

Matrix is a scaling by factor $\sqrt{a^2+b^2}$, therefore $\sigma_1 =$

Non-zero eigenvalues of A

A. T
B. F consider $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$
C. T