Question 5

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5. Let $P \in \mathcal{L}(V)$ be an orthogonal projection map in inner product space V that projects vectors into subspace U. Show from first principle that $< x, Py >=< Px, y >=< Px, Py >$ for all $x, y, z \in V$.

if $\langle x, Py \rangle = \langle Px, y \rangle$

if we consider the spectral decompisition of vector any vector $\vec{v} \in \bar{V}$, then $\vec{v} = \vec{v}_U + \vec{v}_{U^\perp}$ $v_U \in U$ and $v_{U^\perp} \in U^\perp$. Then $x = x_U + x_{U^\perp}$

$$\langle x_U + x_{U^\perp}, y_U \rangle = \langle x_U, y_U + y_{U^\perp} \rangle$$

$$\langle x_U, y_U \rangle + \langle x_{U^\perp}, y_U \rangle = \langle x_U, y_U \rangle + \langle x_U, y_{U^\perp} \rangle$$

But since any vector in $U^\perp$ and U are made of basis vectors orthogonal to one another, any vector $v \in U^\perp$ and $v' \in U$ are orthogonal and their inner product is zero.

$$\langle x_U, y_U \rangle + \cancel{\langle x_{U^\perp}, y_U \rangle} = \langle x_U, y_U \rangle + \cancel{\langle x_U, y_{U^\perp} \rangle}$$

$$\langle x, Py \rangle = \langle x_U, y_U \rangle \Rightarrow \langle x, Py \rangle = \langle Px, Py \rangle$$