PROBLEM 1.1.* Simplify each of the following expressions and write the answer in both rectangular form and polar form. The first one is done for you. Summarize your answers in a table, like the one shown below, but also describe your approach and **show your work** in each case.

	RECTANGULAR	POLAR
(a) $z = j(2+2j)$	$-2+2j$	$2\sqrt{2}e^{j0.75\pi}$
(b) $z = (1+j)^3$	$-2 + 2j$	$2\sqrt{2}\, e^{(3\pi/4)j}$
(c) $z = e^{j12\pi}e^{j\pi/6} + e^{j15\pi}e^{-j\pi/6}$	$0 + j$	$e^{(\pi/2)j}$
(d) $z = \dfrac{3-\sqrt{3}j}{3+\sqrt{3}j}$	$\frac{1}{2} - \frac{\sqrt{3}}{2}j$	$e^{(-\pi/3)j}$
(e) $z = (1+j)^4 - (1-j)^4$	0	0
(f) $z = j\cos(\pi/3) - \sin(\pi/3)$	$-\frac{\sqrt{3}}{2} + \frac{1}{2}j$	$e^{(5\pi/6)j}$

$\frac{5\pi}{6}$

$$(1+j)^3 \Rightarrow \left(\sqrt{2}\, e^{\pi/4\, j}\right)^3 \Rightarrow 2\sqrt{2}\; e^{3\pi/4\, j}$$

$$\tan^{-1}(1) = \pi/4 \qquad x = 2\sqrt{2}\cos\left(\tfrac{3}{4}\pi\right) + 2\sqrt{2}\sin\left(\tfrac{3\pi}{4}\right)$$

$$e^{12\pi j}e^{\frac{\pi}{6}j} + e^{15\pi j}e^{-\pi/6 j} \Rightarrow e^{j\left(\frac{73\pi}{6}\right)} + e^{j\left(\frac{89}{6}\pi\right)}$$

$$= \cos\left(\tfrac{73\pi}{6}\right) + j\sin\left(\tfrac{73\pi}{6}\right) + \cos\left(\tfrac{89\pi}{6}\right) + j\sin\left(\tfrac{89}{6}\pi\right)$$

$$\frac{\sqrt{3}}{2} + j\frac{1}{2} + -\frac{\sqrt{3}}{2} + \frac{1}{2}j \Rightarrow 0 + j$$

$$= e^{\frac{1}{2}\pi}$$

$$\sqrt{9+3}$$

$$\operatorname{atan}\left(\tfrac{\sqrt{3}}{3}\right)$$

$$z = \frac{3-\sqrt{3}j}{3+\sqrt{3}j} \Rightarrow \frac{\cancel{\sqrt{12}}\, e^{-\frac{\pi}{6}j}}{\cancel{\sqrt{12}}\, e^{\pi/6 j}} \Rightarrow e^{\left(-\frac{\pi}{6}-\frac{\pi}{6}\right)j} \Rightarrow e^{-\frac{\pi}{3}j} \Rightarrow \cos\left(-\tfrac{\pi}{3}\right) + j\sin\left(-\tfrac{\pi}{3}\right)$$

$$\frac{1}{2} - \frac{\sqrt{3}}{2}j$$

$$z = (1+j)^4 - (1-j)^4$$

$$\left(\sqrt{2}e^{\pi/4}\right)^4 - \left(\sqrt{2}\, e^{-\pi/4 j}\right)^4$$

$$4e^{\pi j} - 4e^{-\pi j}$$

$$4\cos(\pi) + 4j\sin(\pi) - 4\cos(-\pi) - 4j\sin(-\pi)$$

$$-4 + 0 - (-4) - 0 \Rightarrow 0$$

$$j\cos(\pi/3) - \sin(\pi/3)$$

$$\frac{1}{2}j - \frac{\sqrt{3}}{2}$$

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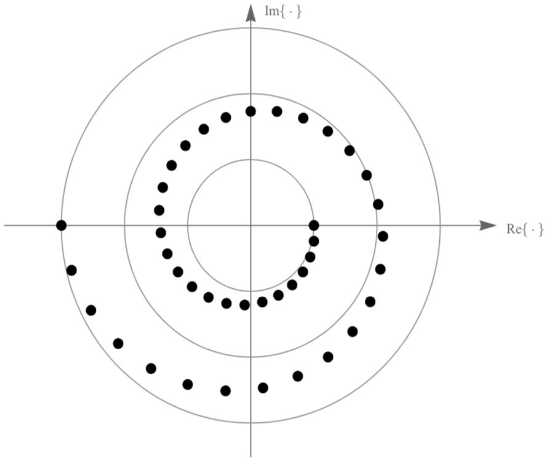
PROBLEM 1.2.* The figure below shows the locations in the complex plane of an unspecified complex number z when raised to the powers of $k \in \{0, 1, 2, 3, \ldots 40\}$. In other words, it show the locations in the complex plane of $z^0, z^1, z^2, z^3, z^4, z^5, \ldots$ and z^{40}:



Find z. Specify the anwer in polar form, $z = re^{j\varphi}$, with $r > 0$ and $\varphi \in (-\pi, \pi]$ to ensure that the answer is unique.

More details regarding the figure:

- The axes are not labeled.
- The large gray circles are centered at the origin and are concentric with radii r_1, $2r_1$, and $3r_1$, for some unspecified r_1.
- The innermost point intersects both the real axis and the inner concentric circle, while the outermost point intersects both the real axis and the outer circle.

$z^0 = r_1 \Rightarrow (r e^{j0\pi})^0 \Rightarrow r^0 e^0 \Rightarrow r^0 = 1 = r_1$

$z^{40} = -3r_1 \Rightarrow (re^{j\theta})^{40} \Rightarrow r^{40} e^{j40\theta} = -3r_1$

$r^{40} = 3$

$e^{j40\theta} = -1$

$\cos(40\theta) + j\sin(40\theta) = -1$

$40\theta = 3\pi$

$\boxed{\theta = \frac{3\pi}{40}}$

$\sqrt[40]{3} = \frac{1255}{1221}$

$$\boxed{z = \frac{1255}{1221} e^{j\left(\frac{3\pi}{40}\right)}}$$

Made with Goodnotes

PROBLEM 1.3.* The equation $|z-6-8j| = 10$ specifies a shape in the complex plane.

$z = 6+8j \Rightarrow 10e^{.9273j}$

(a) Describe the shape, in words.

(b) Draw a picture of the shape in the complex plane. Carefully label the scale of both axes.

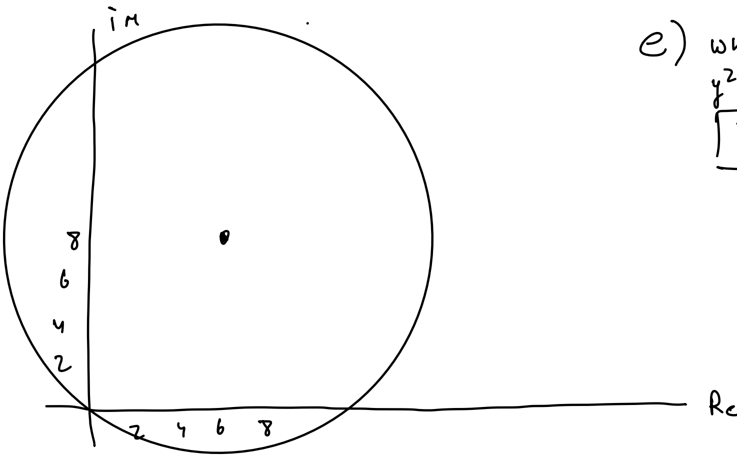
(c) Of all of the values of z that satisfy this equation, which has the largest real part?

(d) Specify all of the real values of z that satisfy this equation, if any.

(e) Specify all of the imaginary values of z that satisfy this equation, if any.

a) This is a circle centered @ $10e^{.9273j}$ w/ radius 10

b)



e) when $x = 0$

$y^2 - 16y = 0$

$\boxed{\mathrm{Im}\{z\} = 0 + 16}$

c) $z = Re^{.9273j}$

$\mathrm{Re}\{z\} = r\cos(.9273j)$

This is the center

largest Re{z} will be center + radius

$\Rightarrow$ z w/ largest Real is

$\boxed{z = 10e^{.9273j} + 10}$

D) $z = x+yi \Rightarrow |x+yi-6-8j| = 10 \Rightarrow |(x-6)+(y-8)j| = 10$

$\Rightarrow \left(\sqrt{(x-6)^2+(y-8)^2}\right)^2 = (10) \Rightarrow (x-6)^2 + (y-8)^2 = 10^2$

all Real of z is when $y=0$

$\Rightarrow x^2 - 12x + 36 + 54 \Rightarrow x^2 - 12x + 0 \Rightarrow \boxed{\mathrm{Re}\{z\} = 0 + 12}$

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PROBLEM 1.4.* Consider the complex number $z = e^{j0.4\pi}$.

(a) Sketch the locations of z, z^2, z^3, z^4, and z^5 in the complex plane. Include the unit circle in the sketch for reference.

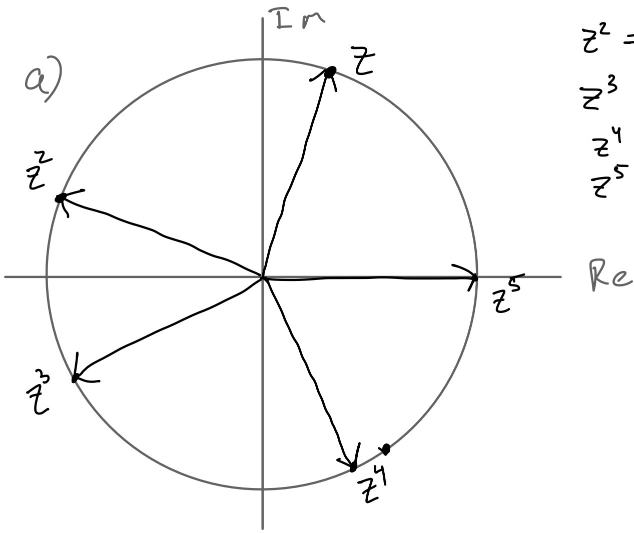
(b) Specify the three smallest positive integers n for which $z^n = 1$.

(c) Specify the three smallest positive integers N for which $\sum_{k=0}^{N-1} z^k = 1$.

(d) Specify the three smallest positive integers m for which $\sum_{k=0}^{4} z^{mk} = 0$.

(e) Specify the three smallest positive integers m for which $\sum_{k=0}^{4} z^{mk} \neq 0$.

a)



$z^2 = e^{j(.8)\pi}$

$z^3 = e^{j(1.2)\pi}$

$z^4 = e^{j(1.6)\pi}$

$z^5 = e^{j(2\pi)}$

$z^{0,2,4,6,8}$

$z^6 = e^{j(2.4\pi)}$

$z^7 = e^{j(2.8\pi)}$

$z^8 = e^{j(3.2\pi)}$

$z^9 = e^{j(3.6\pi)}$

0 4 8 12 16

$z^{0,3,6,9,12}$

z

b) $n \in \{5, 10, 15\}$

c) $N \in \{1, 6, 11\}$

D) $m \in \{1, 2, 3\}$

E) $m \in \{5, 10, 15\}$

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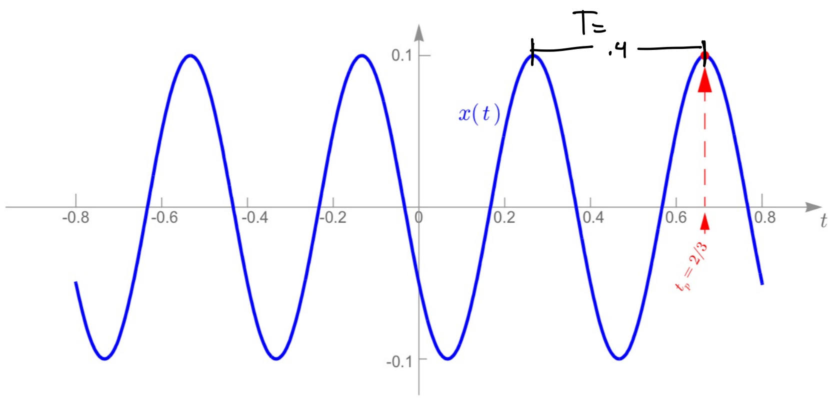
PROBLEM 1.5.* The waveform shown below can be represented by $x(t) = \mathrm{Re}\{Xe^{j\omega_0 t}\}$:



As indicated in the figure, it achieves a peak at time $t_p = 2/3$.

(a) Find $\omega_0 \geq 0$.

(b) Find the complex phasor X, expressed in polar form.

(c) The waveform can also be written in *standard form* as $x(t) = A\cos(2\pi f_0 t + \varphi)$. Find the amplitude A, the Hertzian frequency f_0 (in Hz), and the phase φ. (*Standard form* means that $A \geq 0$, $f_0 \geq 0$, and $-\pi < \varphi \leq \pi$, which makes the answers unique.)

(d) There are infinitely many values for t_0 for which the given signal can be written as:

$$x(t) = A\cos(2\pi f_0(t - t_0)).$$

Of these, specify the three that are *smallest*, when constrained to be positive ($t_0 > 0$).

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a) $T = \frac{2\pi}{\omega} \Rightarrow \omega_0 = \frac{2\pi}{T} \Rightarrow \frac{2\pi}{.4} = \boxed{5\pi \text{ rad/sec}}$

$X = Ae^{j(-2\pi f t_0)}$

B) $\omega_0 = 5\pi$ rad/sec $\quad x(t) = Xe^{j5\pi t}$ $\quad \boxed{X = .1e^{\frac{10\pi}{3}j}}$ where ϕ is Phase of $x(t)$

c) $A = 0.1$

$\phi = -2\pi f_0 t_p \Rightarrow -2\pi(2.5)\left(\frac{2}{3}\right) = -\frac{10\pi}{3}$ rad

$f = \frac{\omega}{2\pi} = \frac{5\pi}{2\pi} = 2.5$ Hz

D) $t_0 = \frac{2}{3} - \frac{2}{5} \Rightarrow \frac{10}{15} - \frac{6}{15} = \boxed{\frac{4}{15}, \frac{10}{15}, \frac{16}{15}}$

Made with Goodnotes