PROBLEM 9.1.* Sketch the frequency response $H(e^{j\hat{\omega}})$ corresponding to each impulse response.

(Hint: all frequency responses are real-valued. Use Table 7.1 along with the *linearity* and *modulation* DTFT properties from Table 7-2).

Label all important heights and frequencies.

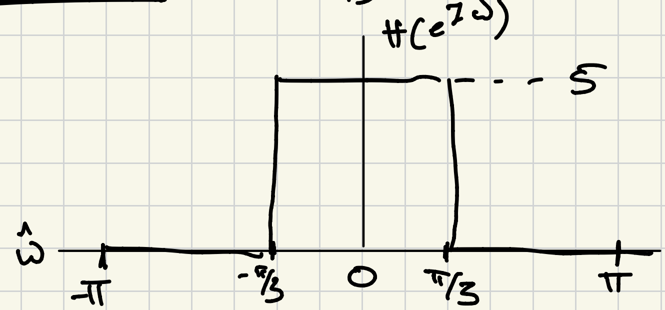
(a) $h[n] = 3\dfrac{\sin(\pi n/3)}{0.6\pi n}$.

(b) $h[n] = \delta[n] - \cos(\pi n)\dfrac{\sin(\pi n/3)}{\pi n}$.

(c) $h[n] = 4\delta[n] + \dfrac{\sin(0.2\pi n)}{0.25\pi n} - \dfrac{\sin(0.6\pi n)}{0.25\pi n} + 2\cos(0.6\pi n)\dfrac{\sin(0.2\pi n)}{0.25\pi n} + 2\cos(0.9\pi n)\dfrac{\sin(0.1\pi n)}{0.25\pi n}$.

a) $h[n] = 3\dfrac{\sin(\frac{\pi}{3}n)}{(.6)\pi n} = 5\cdot\dfrac{\sin(\pi/3\ n)}{\pi n}$ $\quad \hat{\omega}_b = \pi/3$

$H(e^{j\omega}) = 5\left[u(\omega+\pi/3) - u(\omega-\pi/3)\right]$

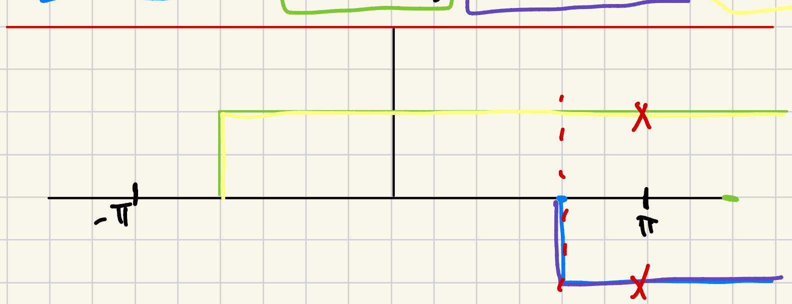


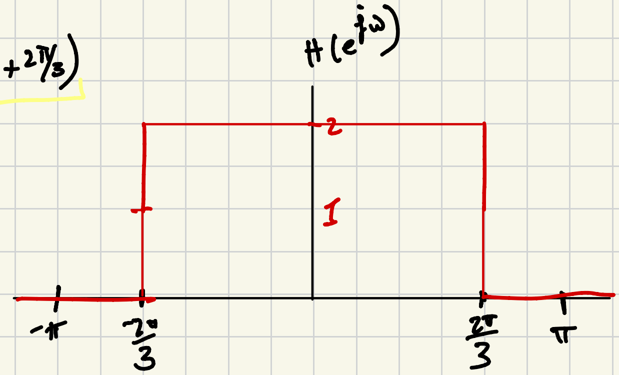
b) $h[n] = \delta[n] - \cos(\pi n)\dfrac{\sin(\frac{\pi}{3}n)}{\pi n}$ Let $x[n] = \dfrac{\sin(\frac{\pi}{3}n)}{\pi n} \xRightarrow{DTFT} X(e^{j\omega}) = u(\omega+\frac{\pi}{3}) - u(\omega-\pi/3)$

$h[n] \Rightarrow \delta[n] - \cos(\pi n)\,x[n] \xRightarrow{DTFT} H(e^{j\omega}) = 1 - \left(\frac{1}{2}X(e^{j(\omega-\pi)}) + \frac{1}{2}X(e^{j(\omega+\pi)})\right)$

$H(e^{j\omega}) = 1 - \left[\frac{1}{2}u(\overset{\omega-\pi}{\omega_1} + \pi/3) - \frac{1}{2}u(\omega_1 - \pi/3) + \frac{1}{2}u(\overset{\omega+\pi}{\omega_2} + \pi/3) - \frac{1}{2}u(\omega_2 - \pi/3)\right]$

$H(e^{j\omega}) = 1 - \frac{1}{2}u(\omega - \frac{2\pi}{3}) + \frac{1}{2}(\omega - \overset{+2\pi/3}{\cancel{\frac{4\pi}{3}}}) \overset{-2\pi/3}{-\frac{1}{2}u(\omega + 4\pi/3)} + \frac{1}{2}u(\omega + 2\pi/3)$





Made with Goodnotes

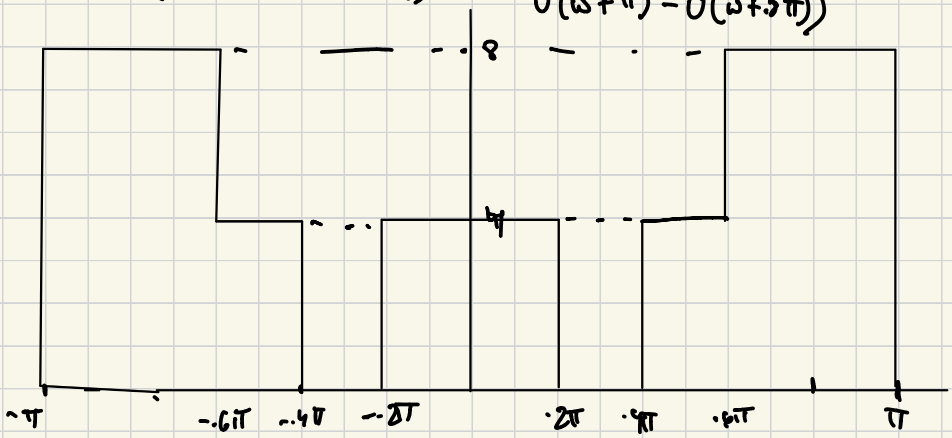
C) $h[n] = 4\delta[n] + \frac{1}{4}\frac{\sin(.2\pi n)}{\pi n} - \frac{1}{4}\frac{\sin(.6\pi n)}{\pi n} + 2\cos(.6\pi n)\left(\frac{1}{4}\right)\frac{\sin(.2\pi n)}{\pi n}$

$+ 2\cos(.9\pi n)\left(\frac{1}{4}\right)\frac{\sin(.1\pi n)}{\pi n}$

$U(\omega + .1\pi n) -$
$U(\omega - .1\pi n)$

$$H(e^{j\omega}) = 4 + \frac{1}{4}\Big(U(\omega+.2\pi) - U(\omega-.2\pi)\Big) - \frac{1}{4}\Big(U(\omega+.6\pi) - U(\omega-.6\pi)\Big)$$

$$+\frac{1}{4}\Big(U(\omega-.4\pi) - U(\omega-.8\pi) + U(\omega+.8\pi) - U(\omega+.4\pi)\Big) - \frac{1}{4}\Big(U(\omega-.8\pi) - U(\omega-\pi) + U(\omega+\pi) - U(\omega+.8\pi)\Big)$$



PROBLEM 9.2.* Determine the discrete-time sequence $x[n]$ that corresponds to each of the following discrete-time Fourier transforms. Answer in the form of a carefully labeled stem plot. (*Hints:* Although you could use the inverse-DTFT integral, it is not necessary. Use the tables instead. Euler might be useful for part (b) and part (c).)

(a) $X(e^{j\hat{\omega}}) = 32 + 16e^{-2j\hat{\omega}}$.

(b) $X(e^{j\hat{\omega}}) = 4\cos^2(2\hat{\omega})$.

(c) $X(e^{j\hat{\omega}}) = 6je^{-6j\hat{\omega}}\sin(2\hat{\omega})$.

(d) $X(e^{j\hat{\omega}}) = \dfrac{\sin(3.5\hat{\omega})}{\sin(0.5\hat{\omega})}$. $\quad \dfrac{\sin(7(.5\omega))}{\sin(.5\omega)}$

(e) $X(e^{j\hat{\omega}}) = \dfrac{1-e^{-j5\hat{\omega}}}{1-e^{-j\hat{\omega}}}$.

a) $x[n] = 32\delta[n] + 16\delta[n-2]$

b) $X(e^{j\omega}) = 4\cos^2(2\omega) \Rightarrow 4\left(\dfrac{e^{j2\omega}+e^{-j2\omega}}{2}\right)^2 \Rightarrow \dfrac{\cancel{4}\,(e^{j2\omega}+e^{-j2\omega})^2}{\cancel{4}}$

$e^{j2\omega}e^{j2\omega} + \cancel{e^{-j2\omega}e^{j2\omega}} + \cancel{e^{j2\omega}e^{-j(2\omega)}} + e^{-j2\omega}e^{-j2\omega} + 2$ $\quad e^0 \to 1$, $e^0 \to 1$

$X(e^{j\omega}) = e^{j4\omega} + e^{-j4\omega} \xrightarrow{\text{IDTFT}} x[n] = \delta[n-4] + \delta[n+4] + 2\delta[n]$

c) $X(e^{j\omega}) = 6je^{-j6\omega}\sin(2\omega) \to 6\cancel{j}(e^{-j6\omega})\left(\dfrac{e^{j2\omega}+e^{-j2\omega}}{2\cancel{j}}\right) \to 3e^{-4\omega j} + 3e^{-8\omega j}$

$x[n] = 3\delta[n-4] + 3\delta[n-8]$

D) $X(e^{j\omega}) = \underbrace{\dfrac{\sin(7(\frac{1}{2}\omega))}{\sin(\frac{1}{2}\omega)}\, e^{-j\omega(7-1)/2}}_{\text{let } Y(e^{j\omega})}\; e^{j\omega(7-1)/2}$

$e^{j3\omega} \Rightarrow n_d = -3$

$y[n] = u[n] - u[n-7]$

$x[n] = y[n+3] = u[n+3] - u[n-4]$

E) $x(e^{j\omega}) = \dfrac{1-e^{-j5\omega}}{1-e^{-j\omega}} = \dfrac{1}{1-e^{-j\omega}} - \dfrac{e^{-j5\omega}}{1-e^{-j\omega}}$

$x[n] = u[n] - u[n-5]$

PROBLEM 9.3.* Evaluate the following convolutions.
Express your answer $y[n]$ as a function of n. Simplify as much as possible.

(a) $y_a[n] = \delta[n-2] * \delta[n-3]$.

(b) $y_b[n] = \cos(0.55\pi n) * \dfrac{\sin(0.4\pi n)}{\pi n}$.

(c) $y_c[n] = \left(\dfrac{\sin(0.7\pi n)}{\pi n} - \dfrac{\sin(0.4\pi n)}{\pi n}\right) * \left(\dfrac{\sin(0.3\pi n)}{\pi n} - \dfrac{\sin(0.2\pi n)}{\pi n}\right)$.

(d) $y_d[n] = \left(\dfrac{\sin(0.7\pi n)}{\pi n} - \dfrac{\sin(0.4\pi n)}{\pi n}\right) * \left(\dfrac{\sin(2\pi n/3)}{\pi n} - \dfrac{\sin(\pi n/3)}{\pi n}\right)$.

(e) $y_e[n] = \left(\delta[n] - \dfrac{\sin(0.4\pi n)}{\pi n}\right) * \dfrac{\sin(0.9\pi n)}{\pi n}$.

a) $y_a[n] \Rightarrow e^{-j\omega 2}\, e^{-j(3)\omega} \Rightarrow e^{-j5\omega} \longrightarrow y_a[n] = \delta[n-5]$

b) $\cos(.55n\pi)$

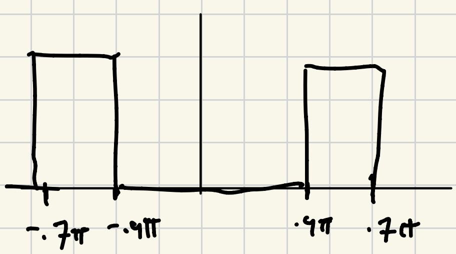
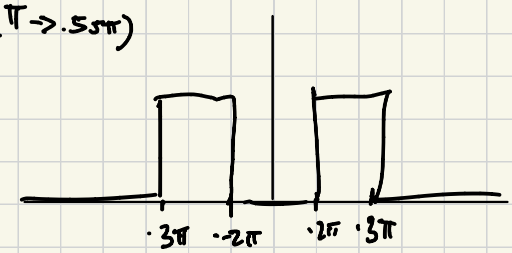
$\dfrac{e^{j.55\pi n}}{2} + \dfrac{e^{-j.55\pi n}}{2} \rightarrow \left[\tfrac{1}{2}\delta[\omega+.55\pi] + \tfrac{1}{2}\delta[\omega-.55\pi]\right]\left(u[\omega+.4\pi) - u[\omega-.4\pi]\right)$

$\underbrace{\tfrac{1}{2}\delta(\omega+.55\pi)u(\omega+.4\pi)}_{0} + \underbrace{\tfrac{1}{2}\delta[\omega-.55\pi)u(\omega+.4\pi)}_{0} - \underbrace{\tfrac{1}{2}u(\omega-.4\pi)\delta(\omega+.55\pi)}_{0} - \tfrac{1}{2}\delta[\omega-.55\pi)u(n-.4\pi)$

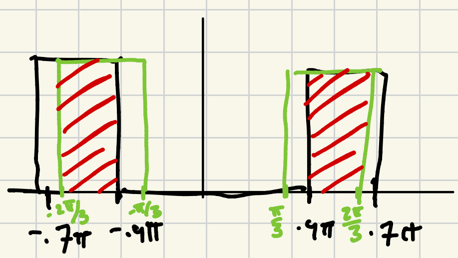
$-\tfrac{1}{2}\delta[\omega-.55\pi)$

$$y_b[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\Omega})\, e^{j\omega n}\, d\omega \Rightarrow \underset{\uparrow\ [\pi \to .55\pi)}{0} + 0 + 0 = \boxed{0}$$

No overlaping Regions

c)  $\times$  $\Rightarrow$

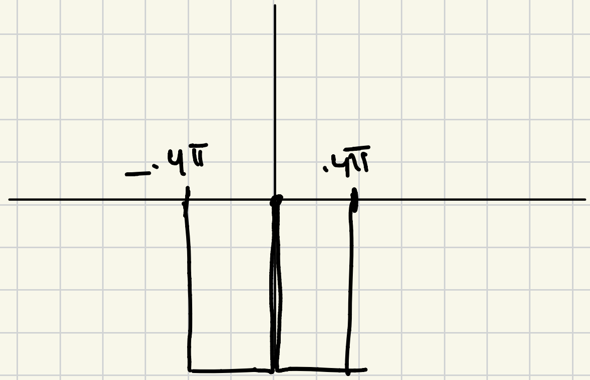
$\boxed{y_c[n] = 0}$

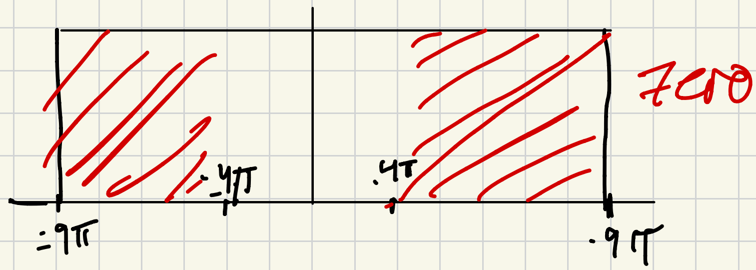
D) 

$\left[u[\omega + \tfrac{2\pi}{3}] - u[\omega - \tfrac{2\pi}{3}]\right] -$
$\left[u[\omega + .4\pi] - u[\omega - .4\pi]\right]$

$\Rightarrow \boxed{y_D[n] = \dfrac{\sin(\frac{2\pi}{3}n)}{\pi n} - \dfrac{\sin(.4\pi n)}{\pi n}}$

e)





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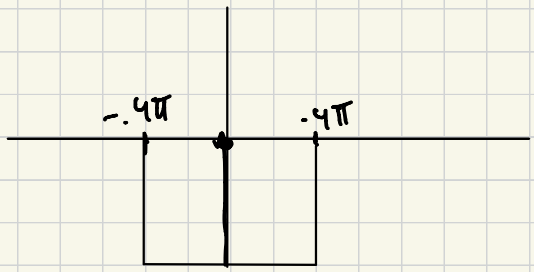
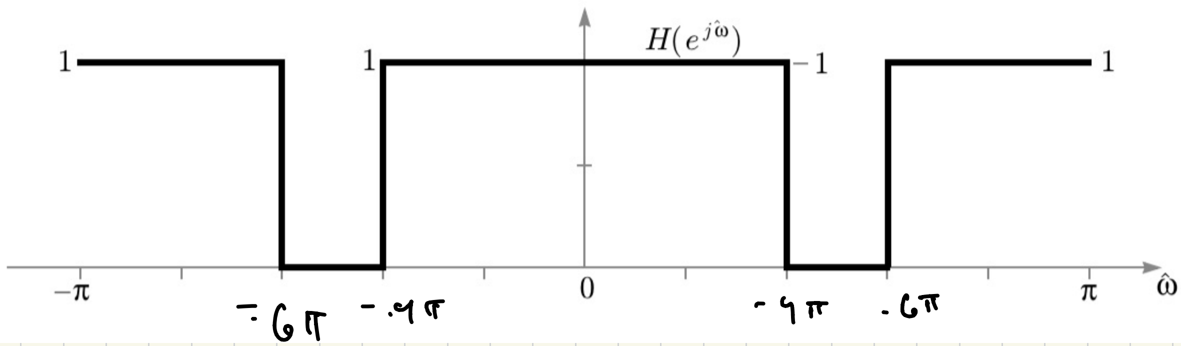


Same thing

$$\boxed{y_e[n] = \frac{\sin(.9\pi n)}{\pi n} - \frac{\sin(.4\pi n)}{\pi n}}$$

PROBLEM 9.4.* Shown below is the frequency response $H(e^{j\hat{\omega}})$ of a *band-stop (or notch) filter* that rejects sinusoids whose frequencies are in the "band" between 0.4π and 0.6π, and passes everything else:



(a) If the filter input is the "...+ – + –..." sequence $x[n] = (-1)^n$, what is the output $y[n]$?

(b) If the filter input is the sinusoid $x[n] = \cos(0.5\pi n)$, what is the output $y[n]$?

In the remainder of the problem we explore four different ways to write the impulse response.

(c) The impulse response of this filter can be written as:

$$h[n] = A\frac{\sin(\hat{\omega}_1 n)}{\pi n} + B\cos(\pi n)\frac{\sin(\hat{\omega}_2 n)}{\pi n}.$$

Find numeric values for the constants A, B, $\hat{\omega}_1$ and $\hat{\omega}_2$.

a) $x[n] = (-1)^n \Rightarrow \cos(\pi n) = \omega = \pi \Rightarrow$ accepted

$$\Rightarrow \boxed{y[n] = \cos(\pi n)}$$

B) $y[n] = 0$

c) $h[n] = A\dfrac{\sin(\omega_1 n)}{\pi n} + \dfrac{B}{2}e^{j\pi n}\dfrac{\sin(\omega_2 n)}{\pi n} + \dfrac{B}{2}e^{-j\pi n}\dfrac{\sin(\omega_2 n)}{\pi n}$

$$\begin{cases} A & |\omega| < \omega_1 \\ 0 & \omega_1 < |\omega| < \pi \end{cases} + \begin{cases} B/2 & |\omega| < \omega_2 - \pi \\ 0 & \omega_2 - \pi < |\omega| < \pi \end{cases} + \begin{cases} B/2 & |\omega| < \omega_2 + \pi \\ 0 & \omega_2 + \pi < |\omega| < \pi \end{cases}$$

$\omega_1 = .4\pi$ $\quad$ $\omega_2 = .4\pi$

$A = 1$ $\quad$ $B = 2$

(d) The impulse response of this filter (also) can be written as:

$$h[n] = \delta[n] - A\cos(\hat{\omega}_1 n)\frac{\sin(\hat{\omega}_2 n)}{\pi n}.$$

Find numeric values for the constants A, $\hat{\omega}_1$, and $\hat{\omega}_2$.

(e) The impulse response of this filter (also) can be written as:

$$h[n] = \delta[n] - A\frac{\sin(\hat{\omega}_1 n)}{\pi n} + A\frac{\sin(\hat{\omega}_2 n)}{\pi n}.$$

Find numeric values for the constants A, $\hat{\omega}_1$, and $\hat{\omega}_2$.

(f) The impulse response of this filter can (also) be written as:

$$h[n] = A\cos(\hat{\omega}_1 n)\frac{\sin(0.2\pi n)}{\pi n} + B\cos(\hat{\omega}_2 n)\frac{\sin(0.2\pi n)}{\pi n}.$$

Find numeric values for the constants A, B, $\hat{\omega}_1$ and $\hat{\omega}_2$.

D) $1 - \begin{cases} \frac{1}{2}A & |\omega| < \omega_2 + \omega_1 \\ 0 & \omega_2 + \omega_1 < |\omega| < \pi \end{cases} - \begin{cases} A/2 & |\omega| < \omega_2 - \omega_1 \\ 0 & \omega_2 - \omega_1 < |\omega| < \pi \end{cases}$

$A = 2$

$\omega_1 = .5\pi \quad \omega_2 = .1\pi$

E) $1 - \begin{cases} A & |\omega| < \omega_1 \\ 0 & \omega_1 < |\omega| < \pi \end{cases} + \begin{cases} A & |\omega| < \omega_2 \\ 0 & \omega_2 < |\omega| < \pi \end{cases}$

$A = 1 \quad \omega_1 = .6\pi$

$\omega_2 = .4\pi$

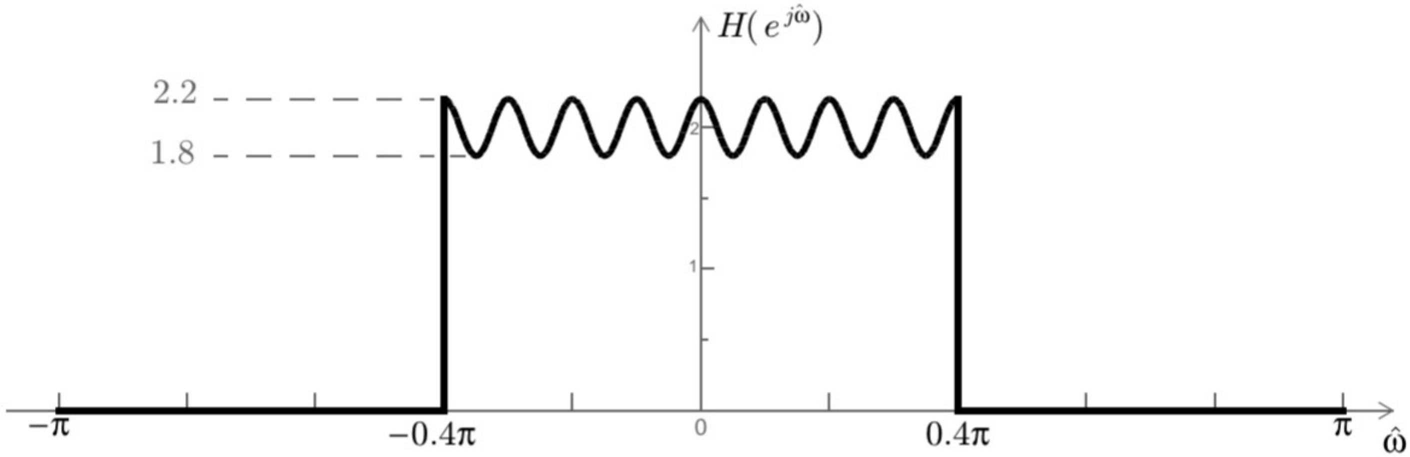
F) $\begin{cases} \frac{1}{2}A & |\omega| < .2\pi + \omega_1 \\ 0 & \omega_1 + .2\pi < |\omega| < \pi \end{cases} + \begin{cases} A/2 & |\omega| < .2\pi - \omega_1 \\ 0 & .2\pi - \omega_1 < |\omega| < \pi \end{cases} + \ldots \quad B$

$A = 2 \quad \omega_1 = .2\pi \quad \omega_2 = .8\pi$

$B = 2$

Made with Goodnotes

PROBLEM 9.5.* Consider the LTI filter defined by its frequency response $H(e^{j\hat{\omega}})$, which is the real-valued function of $\hat{\omega}$ shown below:



(This is just as it looks: $H(e^{j\hat{\omega}})$ oscillates sinusoidally between 1.8 and 2.2, undergoing precisely eight cycles as $\hat{\omega}$ ranges from -0.4π to 0.4π.)

(a) Find the filter output $y[n]$ in response to the filter input $x[n] = (\sqrt{2}\sin(0.25\pi n))^2$.

(b) Define $g[n]$ as the following sinc function:

$$g[n] = \frac{\sin(0.4\pi n)}{\pi n}.$$

The impulse response $h[n]$ of the above filter (i.e., the one defined by the above frequency-response plot) can be written in terms of this sinc function, according to:

$$h[n] = Ag[n] + Bg[n+D] + Cg[n-D].$$

Find numerical values for the unspecified constants A, B, C, and D.

a) $x[n] = 2\left(\frac{1}{2j}\left(e^{j\pi/4\, n} - e^{-j\pi/4\, n}\right)\right)^2$

$$-\frac{2}{4}\left(e^{j\pi/4\, n}e^{j\pi/4\, n} - \underbrace{\cancel{e^{-j\pi/4\, n}e^{j\pi/4\, n}}}_{-e^0} - \underbrace{\cancel{e^{j\pi/4\, n}e^{-j\pi/4\, n}}}_{-e^0} + e^{-j\pi/4\, n}e^{-j\pi/4\, n}\right)$$

$$-\frac{1}{2}\left(e^{j.5\pi n} + e^{-j.5\pi n} - 2\right)$$

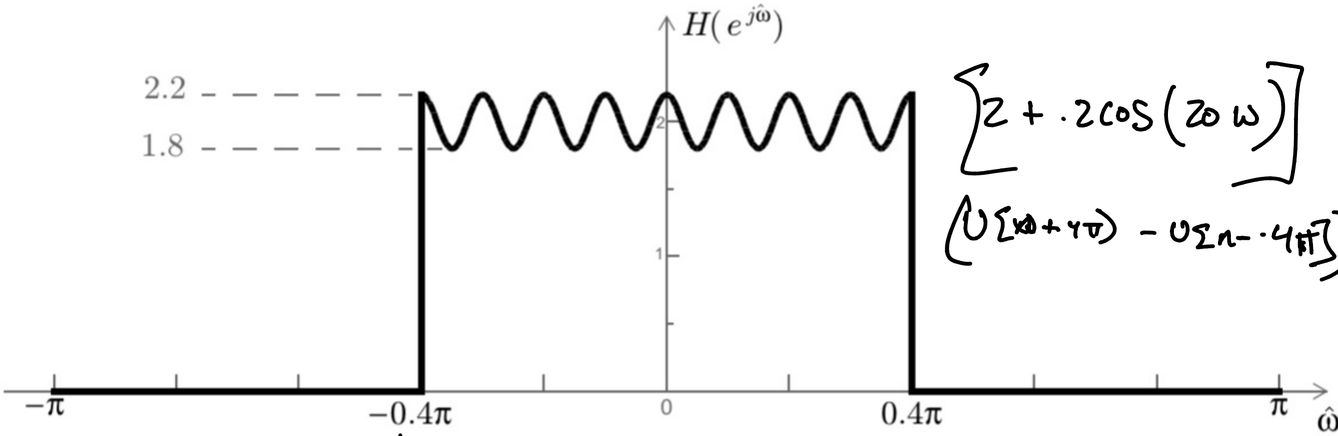
$1 - \cos(.5\pi n)$ $\quad \omega = \frac{1}{2}\pi \Rightarrow$ cos term attenuated

DC of fsq is $2.2 \Rightarrow X(e^{j\omega}) \cdot H(e^{j\omega}) = Y(e^{j\omega})$

$\boxed{y[n] = 2.2}$

Made with Goodnotes

PROBLEM 9.5.* Consider the LTI filter defined by its frequency response $H(e^{j\hat{\omega}})$, which is the real-valued function of $\hat{\omega}$ shown below:



$$\left[2 + .2\cos(20\omega)\right]$$
$$\left(U[\omega+.4\pi] - U[n-.4\pi]\right)$$

$$2\frac{\sin(.4\pi n)}{\pi n} + \frac{.2}{2}e^{j20\omega}U[\omega+.4\pi] - \frac{.2}{2}e^{j20\omega}U[n-.4\pi]$$

(This is just as it looks: $H(e^{j\hat{\omega}})$ oscillates sinusoidally between 1.8 and 2.2, undergoing precisely eight cycles as $\hat{\omega}$ ranges from -0.4π to 0.4π.)

$$+\frac{.2}{2}e^{-j20\omega}U[\omega+.4\pi] - \frac{.2}{2}e^{-j20\omega}U[n-.4\pi]$$

(a) Find the filter output $y[n]$ in response to the filter input $x[n] = (\sqrt{2}\sin(0.25\pi n))^2$.

(b) Define $g[n]$ as the following sinc function:

$$g[n] = \frac{\sin(0.4\pi n)}{\pi n}.$$

$G(e^{j\omega}) = U[\omega+.4\pi] - U[\omega-.4\pi]$

The impulse response $h[n]$ of the above filter (i.e., the one defined by the above frequency-response plot) can be written in terms of this sinc function, according to:

$$h[n] = Ag[n] + Bg[n+D] + Cg[n-D].$$

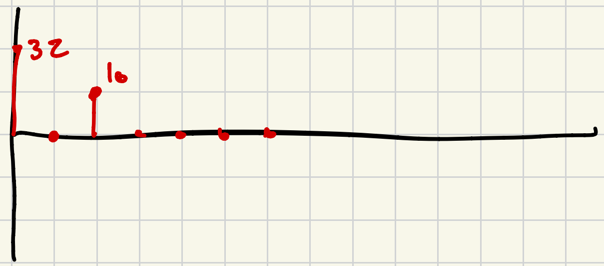
Find numerical values for the unspecified constants A, B, C, and D.

B) $A = 2$ $D = 20$

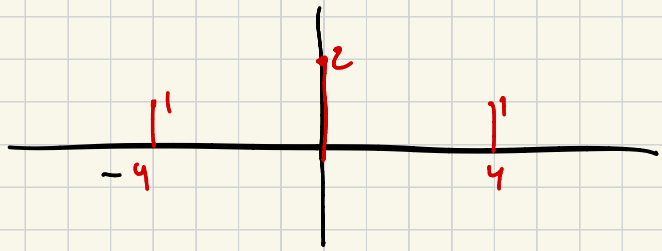
$B = .1$ $C = .1$

9.2 Stem Plots

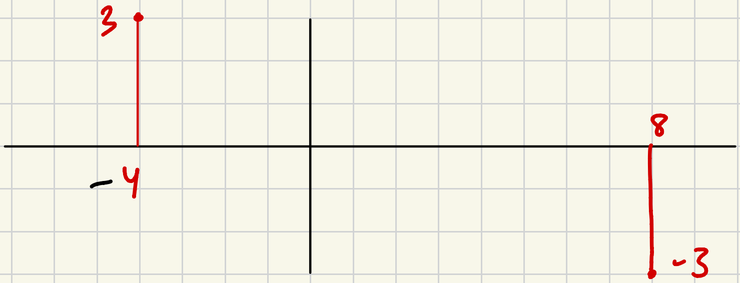
a).



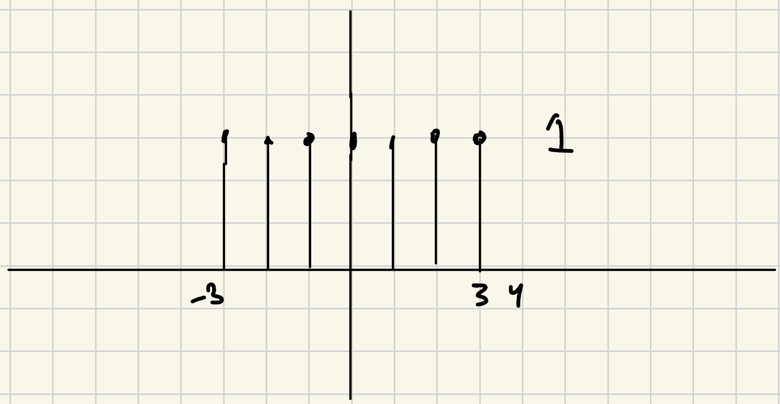
b)



c)



D)



E)