RUDRA GOEL

Maj: Computer engineering

Date: 12/9/2024

GTID: 903 897740

ECE 2040 Final exam submission

Problem 1. Mark the following statements as either **True** or **False**

(a) **True** or **False**: An LC circuit (with a DC source) is an oscillatory circuit.

(b) **True** or **False**: A circuit response may grow, rather than decay exponentially with time.

(c) **True** or **False**: The time constant of an RC circuit is $\tau = \frac{1}{RC}$ while the time constant of an RL circuit is $\tau = \frac{L}{R}$.

$\tau = RC \Rightarrow t/\tau$

(d) **True** or **False**: Kirchhoff's current and voltage laws hold in a frequency domain representation of an RLC circuit with a DC source.

$V = I \cdot Z_{RCL}$ Z_R, Z_C, Z_L

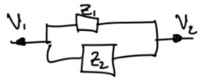
(e) **True** or **False**: The decay constant α in an **overdamped** system can also be referred to as a **damping factor** or a **damping coefficient**.

(f) **True** or **False**: The overall gain of two contiguous connected or concatenated summing amplifiers is equal to product of gain of each amplifier.

(g) **True** or **False**: A noninverting amplifier is an op amp circuit producing an output voltage that is a negative, scaled replica of the input voltage.

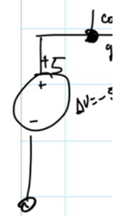
→ Retain Sign

(h) **True** or **False**: Impedances in parallel share a common voltage.



(i) **True** or **False**: An attenuator is a passive device that affect it input power such that the output power is less than the input power. If the aforestated fact is true, then this type of passive device can be fabricated from a network of resistors.

(j) **True** or **False**: Two voltage sources/supplies have +5 V, and −5 V terminals only. If these devices are connected such that the −5 V terminal from device 1 is connected to the +5 V terminal from device 2 then the device combination can be considered as a three terminal device with the *common ground* being the joint terminals.



(k) **True** or **False**: The capacitor does not allow sudden changes in voltage.

(l) **True** or **False**: The voltage applied to a 212 mH inductor in a circuit can be denoted as $v(t) = 15e^{-5t}$ V. The current in the circuit is $11.27e^{-10t}$ A.

$V = L\frac{di}{dt} \Rightarrow \int di = \frac{1}{L}\int v\,dt \Rightarrow I = \frac{1}{L}\int_0^t 15e^{-5t}$

$u = -5t$
$du = -5$

$\frac{-3}{L} e^{-5t}\Big]_0^t \Rightarrow$

(m) **True** or **False**: An inductor works as an open circuit if the circuit is power by a DC source.

(n) **True** or **False**: A capacitor can be used in place of a resistor in a circuit with a high frequency input AC source.

$Z_R \neq Z_C \Rightarrow R \neq \frac{-j}{\omega C}$

(o) **True** or **False**: The insulating medium between the two plates of capacitor is know as a capacitive medium.

"Dielectric"

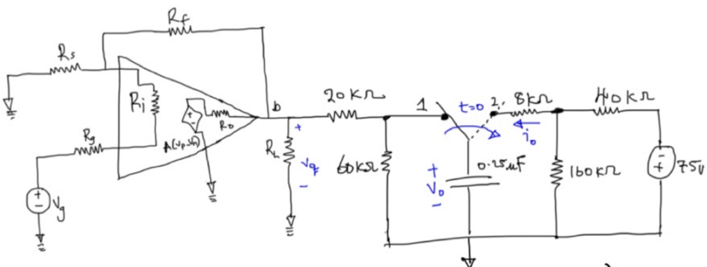
(p) When the input voltage [redacted] amplifier behaves as

(p) When the input voltage difference is small in magnitude, the operational amplifier behaves as iii
- (i) Non-linear device
- (ii) Complex device
- (iii) Linear device
- (iv) Bipolar device

(q) The condition for a Non-inverting amplifying circuit to function in linear region operation is i
- (i) $(R_s + R_f)/R_s < |V_{CC}/V_g|$
- (ii) $(R_s + R_f)/R_s \neq |V_{CC}/V_g|$
- (iii) $(R_s + R_f)/R_s > |V_{CC}/V_g|$
- (iv) $(R_s + R_f)/R_s = |V_{CC}/V_g|$

$$\underbrace{\left(\frac{R_s + R_f}{R_s}\right) V_g}_{\text{output voltage}} < |V_{cc}|$$

(r) All the rules and laws of D.C. circuit also apply to A.C. circuit containing
- (i) Capacitance only
- (ii) Inductance only
- (iii) Resistance only
- (iv) all the above



a) $(.25E{-}6)\left(\frac{d}{dt}\left[-60+90e^{-100t}\right]\right)$

$(.25e{-}6)(-9000e^{-100t}) = (-2.25e{-}3)e^{-100t}$

assuming in mA

$i = C\frac{dv}{dt}$

$\int dv = \int \frac{1}{C} i\,dt$

Fig. 1. $v = \frac{1}{C}\int_0^t i\,dt$

(s) The circuit shown in Fig. 1 stems from the concatenation of a realistic (non-ideal) non-inverting operational amplifier model and an RC circuit. The switch in the circuit has remained in position 1 for a long time. At $t = 0$, the switch moves to the position 2. If $R_s = 1000\ \Omega$, $R_f = 3000\ \Omega$, $R_g = 1000\ \Omega$, $R_i = 75\ \text{M}\Omega$, $R_o = 50\ \Omega$, $A = 10^5$ and $V_g = 10$ V. If

$u = -100t$, $du = -100$

$V_0(0) = 30$

(a) $V_0(t) = -60 + 90e^{-100t}$ for $t \geq 0$

(b) $i_0(t) = -2.25e^{-100t}$ for $t \geq 0^+$

b) $v = \frac{1}{.25e{-}6}\int_0^t (-2.25e{-}3)e^{-100t}\,dt$ $\quad \frac{1(-2.25e{-}3)}{(.25e{-}6)(-100)}\left(e^{-100t} - 1\right)$ $\Rightarrow V_0(t) = 90e^{-100t} - 60$

$90e^{-100t} - 90 + 30$

Check the consistency of the solutions (for $V_0(t)$ and $i_0(t)$) by deriving $V_0(t)$ from $i_0(t)$.

(t) Lauren Yao and Koby Dunn have been best friends since middle school. However, a recent argument about a circuit problem has put a strain on their friendship. The bone of contention is as follows:

Resistor

An A.C. voltage impressed across ~~a pure resistance of 3.5 Ohms in parallel with~~ a pure inductance

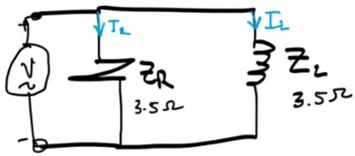
Resistor

An A.C. voltage impressed across a pure resistance of 3.5 Ohms in parallel with a pure inductance of impedance of 3.5 Ohms. The weight of the current (i.e., value of current is one component relative to the other) in the aforementioned passive components is in question.

If the current through the pure resistance and inductance are denoted as I_R and I_L, respectively.

(i) **Lauren** claims that $I_R > I_L$
(ii) **Koby** claims that $I_R < I_L$
(iii) **Roger** (paper vendor down their street) claims that $I_R = I_L$
(iv) **Mr Johnshon** (their neighbor) claims that none of the above answers are correct.

Help settle the dispute by stating the right answer.



Let the voltage across the pure Resistor be

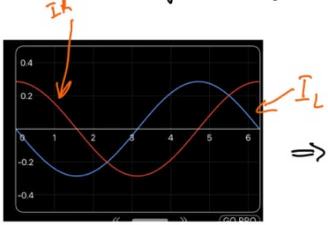
$V_R = A\cos(\omega t)$ where A is the Amplitude of sinusoid

$V = IR \Rightarrow \quad I_R = \frac{A}{Z_R}\cos(\omega t) \Rightarrow \frac{1}{3.5} A\cos(\omega t)$

(i) The current across an inductor w/ A AC current is 90° out-of-phase from its voltage.

V_L is V_R because of parallel $\Rightarrow V_L = A\cos(\omega t)$

$V = IR \Rightarrow \quad I_L = \frac{A}{3.5}\cos(\omega t + 90)$



* 90° offset

$\Rightarrow$ IV Mr. Johnson is Right since both currents are greater than the other @ different times. Precisely, they are greater / less than one another ½ the time.

Problem 2a. The parameters for the circuit shown in Fig. 2 are $R_a = 100\,\text{K}\Omega$, $R_1 = 500\,\text{K}\Omega$, $C_1 = 0.1\,\mu\text{F}$, $R_b = 25\,\text{K}\Omega$, $R_2 = 100\,\text{K}\Omega$ and $C_2 = 1\,\mu\text{F}$. The power supply voltage for each op amp is ± 6V. The

Ac coup

Ca

AC coupling

Caps -

inductor

Problem 2a. The parameters for the circuit shown in Fig. 2 are $R_a = 100\,\mathrm{K\Omega}$, $R_1 = 500\,\mathrm{K\Omega}$, $C_1 = 0.1\,\mu\mathrm{F}$, $R_b = 25\,\mathrm{K\Omega}$, $R_2 = 100\,\mathrm{K\Omega}$ and $C_2 = 1\,\mu\mathrm{F}$. The power supply voltage for each op amp is $\pm 6\mathrm{V}$. The signal voltage (V_g) for the cascaded integrating amplifiers jumps from 0 to 250 mV at $t = 0$. No energy is stored in the feedback capacitors at the instant the signal is applied.

(i) Derive an algebraic expression for the output voltage V_0 as a function of the input voltage V_g.
(ii) Derive the characteristic equation and solve for its roots.
(iii) Find the numerical expression of the differential equation for V_0.
(iv) Find $V_0(t)$ for $t \geq 0$.
(v) Find the numerical expression of the differential equation for V_{01}.
(vi) Find $V_{01}(t)$ for $t \geq 0$. $i = C\frac{dV}{dt}$

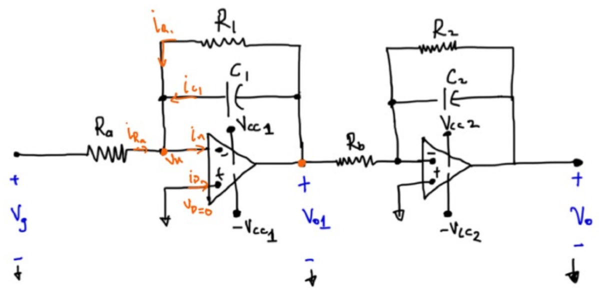


Fig. 2. Cascaded Integrating Amplifiers with RC Feedback Link.

KCL @ V_n

$i_g - i_n - i_{c_1} - i_{R_1} = 0$

$\frac{V_g - V_n}{R_a} - 0 - \frac{(V_n - V_{01})}{1/j\omega C_1} - \frac{(V_n - V_{01})}{R_1} = 0$

$V_n = V_p = 0$

$\frac{V_g}{R_a} = \frac{-V_{01}}{1/j\omega C_1} - \frac{V_{01}}{R_1}$ ☆

$= -\left(j\omega C_1 + \frac{1}{R_1}\right) V_{01}$

$\frac{V_g}{R_a} = -\left(\frac{j\omega C_1 R_1 + 1}{R_1}\right) V_{01}$

$\left(-\frac{R_1}{R_a(j\omega C_1 R_1 + 1)}\right) V_g = V_{01}$ ☆

Do same for V_0

$V_0 = H(S)\,V_g$

$V_g = \begin{cases} 0 & t < 0 \\ 250\text{ mV} & t \geq 0 \end{cases} = U(t)$

$V_0 = H(s)\,\mathcal{L}\{U(t)\}$

$V_0 \Rightarrow \frac{.25}{s} H(S)$ expand

(iii) $V_0(S) = \left(\frac{.25 R_1 R_2}{R_a R_b (s)(sC_1R_1 + 1)(sC_2R_2 + 1)}\right)$ in Laplace Domain

→ take the inverse Laplace of $V_0(S)$ to determine V_0 in time-Domain

$\mathcal{L}^{-1}\{V_0(S)\} = \mathcal{L}^{-1}\left\{\frac{.25 R_1 R_2}{R_a R_b (s)(sC_1R_1 + 1)(sC_2R_2 + 1)}\right\}$

$V_0(t) = \frac{.25 R_1 R_2}{R_a R_B} \mathcal{L}^{-1}\left\{\frac{1}{(S)(sC_1R_1 + 1)(sC_2R_2 + 1)}\right\}$

$\frac{1}{S(sC_1R_1+1)(sC_2R_2+1)} = \frac{A}{S} + \frac{B}{sC_1R_1 + 1} + \frac{D}{sC_2R_2 + 1}$

$$\frac{V_g}{R_a} = -\left(\frac{j\omega C_1 R_1 + 1}{R_1}\right) V_{o1}$$

$$\left(-\frac{R_1}{R_a(j\omega C_1 R_1 + 1)}\right) V_g = V_{o1}$$

Do same for V_o

$$V_o = -\left(\frac{R_2}{R_b(j\omega C_2 R_2 + 1)}\right) V_{o1}$$

i) $$V_o = \left(\frac{R_1 R_2}{R_a R_b (j\omega C_1 R_1 + 1)(j\omega C_2 R_2 + 1)}\right) V_g$$

Characteristic equation for V_o

Let $K = \frac{R_1 R_2}{R_a R_b} V_g$

$$\Rightarrow V_o = \frac{K}{(j\omega C_1 R_1 + 1)(j\omega C_2 R_2 + 1)}$$

Convert to S Domain

$$V_o(s) = \frac{K}{(sC_1 R_1 + 1)(sC_2 R_2 + 1)}$$

Characteristic equation:

$$s^2 C_1 C_2 R_1 R_2 + sC_1 R_1 + sC_2 R_2 + 1$$

(ii) $$s^2 C_1 C_2 R_1 R_2 + s(C_1 R_1 + C_2 R_2) + 1 = 0$$

$$s^2 + s\left(\frac{C_1 R_1 + C_2 R_2}{C_1 R_1 C_2 R_2}\right) + \frac{1}{C_1 R_1 C_2 R_2} = 0$$

$$s = \frac{-\frac{C_1 R_1 + C_2 R_2}{C_1 R_1 C_2 R_2} \pm \sqrt{\left(\frac{C_1 R_1 + C_2 R_2}{C_1 R_1 C_2 R_2}\right)^2 - \frac{4}{C_1 R_1 C_2 R_2}}}{2}$$

$$s = -15 \pm \frac{1}{2}\sqrt{30^2 - 800}$$

$$-15 \pm \frac{1}{2}(10) \Rightarrow \boxed{\begin{matrix} s_1 = -10 \\ s_2 = -20 \end{matrix}}$$

$$V_{o1}(s) = H(s)\, \mathcal{L}\{v_g(t)\}$$

$$V_{o1}(s) = \left(\frac{-R_1}{R_a(sC_1 R_1 + 1)}\right)\left(\frac{.25}{s}\right)$$

to determine V_o in time - domain

$$\mathcal{L}^{-1}\{V_o(s)\} = \mathcal{L}^{-1}\left\{\frac{.25 R_1 R_2}{R_a R_b (s)(sC_1 R_1 + 1)(sC_2 R_2 + 1)}\right\}$$

$$V_o(t) = \frac{.25 R_1 R_2}{R_a R_B}\mathcal{L}^{-1}\left\{\frac{1}{(s)(sC_1 R_1 + 1)(sC_2 R_2 + 1)}\right\}$$

$$\frac{1}{s(sC_1 R_1 + 1)(sC_2 R_2 + 1)} = \frac{A}{s} + \frac{B}{sC_1 R_1 + 1} + \frac{D}{sC_2 R_2 + 1}$$

$$\Rightarrow \frac{A}{s} + \frac{B}{.05s + 1} + \frac{D}{.1s + 1}$$

via Partial fraction Decomposition

$$\Rightarrow \mathcal{L}^{-1}\left\{\frac{1}{s} + \frac{1}{s+20} + \frac{-2}{s+10}\right\}$$

$$V_o(t) = 5\left[1 + e^{-20t} - 2e^{-10t}\right]$$

IV) $$\boxed{V_o(t) = 5 + 5e^{-20t} - 10e^{-10t}}$$

Characteristic equation:

$$s^2 C_1C_2R_1R_2 + sC_1R_1 + sC_2R_2 + 1$$

(ii) $$s^2 C_1C_2R_1R_2 + s(C_1R_1 + C_2R_2) + 1 = 0$$

$$s^2 + s\left(\frac{C_1R_1 + C_2R_2}{C_1R_1C_2R_2}\right) + \frac{1}{C_1R_1C_2R_2} = 0$$

$$s = \frac{-\frac{C_1R_1 + C_2R_2}{C_1R_1C_2R_2} \pm \sqrt{\left(\frac{C_1R_1 + C_2R_2}{C_1R_1C_2R_2}\right)^2 - \frac{4}{C_1R_1C_2R_2}}}{2}$$

$$s = -15 \pm \frac{1}{2}\sqrt{30^2 - 800}$$

$$-15 \pm \frac{1}{2}(10) \Rightarrow \boxed{\begin{aligned} s_1 &= -10 \\ s_2 &= -20 \end{aligned}}$$

$$V_{o1}(s) = \underset{\downarrow}{H(s)}\, \mathcal{L}\{v_g(t)\}$$

$$V_{o1}(s) = \left(\frac{-R_1}{R_a(sC_1R_1 + 1)}\right)\left(\frac{.25}{s}\right)$$

(v) $$V_{o1}(s) = -\frac{.25R_1}{R_a}\left(\frac{1}{sC_1R_1 + 1}\right)$$

$$\frac{A}{s} + \frac{B}{.05s + 1} = \frac{1}{s(s(.05) + 1)}$$

$$A(s(.05) + 1) + B(s) = 1 \quad \text{for } s = 0,\ A = 1$$

$$s = -20$$
$$B = -1/20$$

$$\mathcal{L}^{-1}\{V_{o1}(s)\} = -\frac{.25R_1}{R_a}\mathcal{L}^{-1}\left\{\frac{1}{(sC_1R_1 + 1)s}\right\}$$

$$V_{o1}(t) = -\frac{.25R_1}{R_a}\left[\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - \frac{1}{20}\mathcal{L}^{-1}\left\{\frac{1}{.05s + 1}\right\}\right]$$

$$-\frac{.25R_1}{R_a}\left[1 - \mathcal{L}^{-1}\left\{\frac{1}{s + 20}\right\}\right]$$

$$-\frac{.25R_1}{R_a}\left[1 - e^{-20t}\right] \Rightarrow \quad 1.25e^{-20t} - 1.25 = V_{o1}(t) \quad \text{(VI)}$$

Problem 2b. Repeat Problem 2a with the feedback resistors R_1 and R_2 removed.

KCl @ V_n is Reduced

$$\frac{V_g}{R_a} = \frac{-V_{o1}}{1/sC_1}$$

$$\frac{V_g}{R_a} = -V_{o1}(sC_1)$$

$$\Rightarrow V_{o1} = \frac{-1}{R_a(sC_1)} V_g$$

$$V_o = \frac{-1}{R_b(sC_2)} V_{o1} \Rightarrow$$

$$V_o = \left(\frac{1}{R_a(sC_1)}\right)\left(\frac{1}{R_b(sC_2)}\right) V_g$$

(i) $$\boxed{V_o = \frac{1}{R_aR_b s^2 C_1C_2} V_g}$$

Transfer Function: let $k = \frac{1}{R_aR_b C_1C_2}$

$$H(s) = \frac{k}{s^2}$$

(ii) $$s^2 = 0$$

$$s = 0$$

$$V_o = H(s)\, V_g(s) \qquad V_g(s) = \mathcal{L}\{V_g(t)\}$$

$$V_o(s) = .25\, H(s)\left(\frac{1}{s}\right) \qquad \Rightarrow \frac{.25}{s}$$

(iii) $$V_o(s) = \frac{.25}{R_aR_b\, s^3 C_1C_2}$$

$$\mathcal{L}^{-1}\{V_o(s)\} = \frac{.25}{R_aR_b C_1C_2}\left(\frac{1}{2}\right)\mathcal{L}^{-1}\left\{\frac{1(2)}{s^3}\right\}$$

(IV) $$\boxed{V_o(t) = \begin{cases} 500t^2 & 0 \le t \le .1095 \\ 6 & t \ge .1095 \end{cases}}$$

(V) $$V_{o1} = \frac{-1}{R_aC_1 s}\left(\frac{.25}{s}\right) \Rightarrow -\left(\frac{.25}{R_aC_1}\right)\frac{1}{s^2} = V_o(s)$$

(VI) $$V_{o1}(t) = \frac{.25}{.01}\, t \Rightarrow -25t \Rightarrow \boxed{\begin{cases} -25t & 0 \le t \le .24 \\ -6 & t \ge .24 \end{cases}}$$

Problem 3. The n-type metal oxide semiconductor (MOSFET) transistor (see Fig. 3) is a three-terminal component that serves as a building block for almost all digital electronic devices.

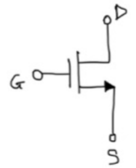


Fig. 3. MOSFET (NMOS) with terminals labeled as Gate (G), Source (S) and Drain (D)

The n-type MOSFET (NMOS) has two operation modes namely (i) triode mode, and (ii) saturation mode i.e., active mode. A large scale equivalent circuit model for the NMOS in the saturation mode is shown in Fig. 4.

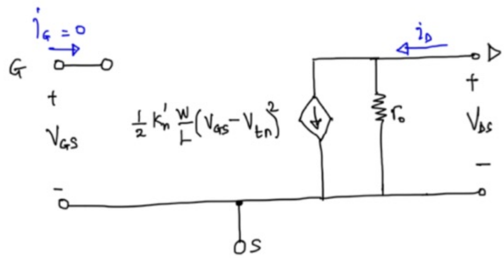


Fig. 4. Large-signal equivalent circuit model of the n-channel MOSFET in saturation.

In the circuit above, $r_0 \approx \left[\frac{di_D}{dv_{DS}}\right]^{-1}_{V_{GS}=\text{constant}}$ while the drain current $i_D = \frac{1}{2}k_n' \frac{W}{L}(V_{GS} - V_{tn})^2$.

There are rules guiding the operation of an NMOS transistor.

(i) **RULE** 1**:** The NMOS will operate in the saturation (active) mode if the drain voltage (V_D) is greater than the difference between the gate voltage (V_G) and the threshold voltage (V_{tn}) i.e. $V_D > V_G - V_{tn}$.

(ii) **RULE** 2**:** The source voltage (V_S) must not exceed the gate voltage (V_G). A situation where $V_S > V_G$ implies that the transistor is inactive i.e., cut off.

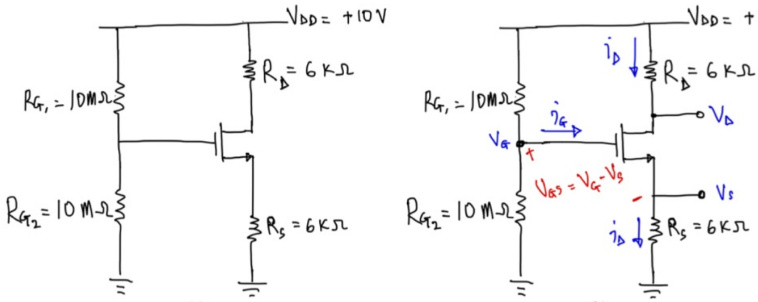
Using the rules stated above, analyze the active circuit shown in Fig. 5(b) – annotated version of Fig. 5(a). Find the voltages (V_S, V_D, V_{GS}) at all nodes and the current i_D. Let $V_{tn} = 1$ V, $i_G = 0$ A, and $k_n'(W/L) = 1\,\text{mA/V}^2$.

There are rules guiding the operation of an NMOS transistor.

(i) **RULE 1:** The NMOS will operate in the saturation (active) mode if the drain voltage (V_D) is greater than the difference between the gate voltage (V_G) and the threshold voltage (V_{tn}) i.e. $V_D > V_G - V_{tn}$.

(ii) **RULE 2:** The source voltage (V_S) must not exceed the gate voltage (V_G). A situation where $V_S > V_G$ implies that the transistor is inactive i.e., cut off.

Using the rules stated above, analyze the active circuit shown in Fig. 5(b) – annotated version of Fig. 5(a). Find the voltages (V_S, V_D, V_{GS}) at all nodes and the current i_D. Let $V_{tn} = 1$ V, $i_G = 0$ A, and $k_n'(W/L) = 1\,\text{mA/V}^2$.



(a) (b)

Fig. 5. (a) Original circuit (b) circuit with additional details

$$V_S = i_D \cdot R_S \Rightarrow i_D(6k) = i_D \cdot 6 \quad \text{as mA}$$

$V_g = 5\text{V}$ Due to R_{g1} & R_{g2} Being Same => voltage divider directly in half

$$V_{gs} = 5 - i_D R_S \Rightarrow 5 - i_D(6)$$

$$i_D = \frac{1}{2} k'\left(\frac{W}{L}\right)(V_{gs} - V_{Tn})^2$$

$$\Rightarrow \frac{1}{2}(1)(5 - i_D(6) - 1)^2$$

$$i_D = \frac{1}{2}(1)(4 - 6i_D)^2$$

Home Insert Draw View

$$i_D = \frac{1}{2} k' \left(\frac{W}{L}\right)\left(V_{gs} - V_{Tn}\right)^2$$

$$\Rightarrow \frac{1}{2}(1)\left(5 - i_D(6) - 1\right)^2$$

$$i_D = \frac{1}{2}(1)\left(4 - 6 i_D\right)^2$$

$$i_D = \frac{1}{2}\left[16 - 48 i_D + 36 i_D^2\right)$$

$$i_D = 18 i_D^2 - 24 i_D + 8$$

$$18 i_D^2 - 25 i_D + 8 = 0$$

$$i_{D_1} = .89 \text{ mA}$$

$$i_{D_2} = .5 \text{ mA}$$

$$V_s = i_D \cdot R_s$$

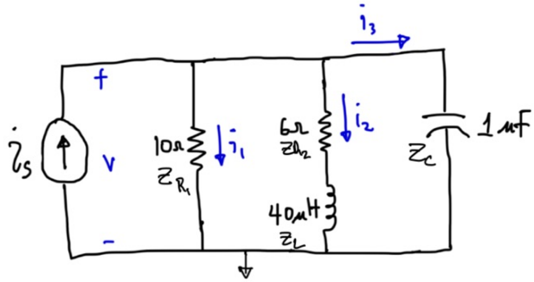
$$= (.89e{-3})(6E3) \Rightarrow 5.34V > 5$$

$$\Rightarrow V_s > V_g$$ which is Not allowed via Rule 2

$$\boxed{\begin{aligned} i_D &= .5 \text{ mA} \\ V_s &= 3 \text{ V} \\ V_g &= 5 \text{ V} \\ V_{gs} &= 2 \text{ V} \\ V_D &= 10 - V_s = 7 \text{ V} \end{aligned}}$$

Made with Goodnotes

Problem 4a. The sinusoidal current source in the circuit shown in Fig. 6 produces the current $i_s = 8\cos 200000t$ A.
(a) Construct the frequency-domain equivalent circuit.
(b) Find the steady-state expression for $v, i_1, i_2,$ and i_3.



$Z_C = \dfrac{1}{s(1e\text{-}6)}$ $\quad Z_L = s(40e\text{-}6)$ $\quad Z_{R_1} = 10\Omega$

$Z_{R_2} = 6\Omega$

$s = j\omega$

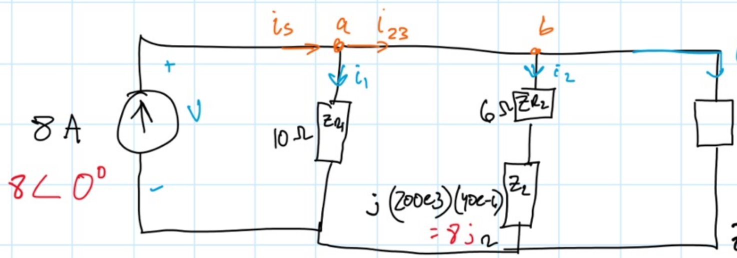
convert $I = 8\cos(200{,}000\,t)$ to frequency Domain

$\underbrace{Ae^{j\phi}}_{\text{phasor form}}e^{j\omega t}$

$\phi = 0$

$I = 8e^{j(0)} \Rightarrow 8\,A$

a)



$Z_C = \dfrac{1}{j(200e3)(1e\text{-}6)}$

$= 5/j\,\Omega$

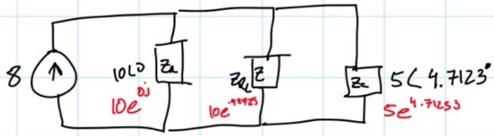
$Z_C = -5j\,\Omega$

$6 + 8j = Z_{RL}$

$\Rightarrow 10\angle .92729$

$6+8j = Z_{RL}$

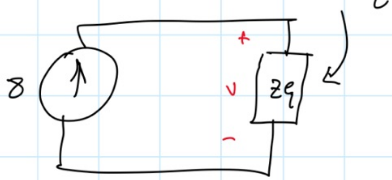
$\Rightarrow 10\angle .92729$



$$\left[\frac{1}{10} + \frac{1}{10}e^{-.92729j} + \frac{1}{5}e^{-4.7123j}\right]^{-1}$$

$$(10)\left[1 + e^{-.92729j} + 2e^{-4.7123j}\right]^{-1}$$

$$= 5e^{-.6435} = Z_{eq}$$



$i_1 = \frac{v}{Z_{R_1}}$

$i_2 = \frac{v}{(Z_{R_2}+Z_L)}$

$i_3 = \frac{v}{Z_C}$

$V = I \cdot Z_{eq} \Rightarrow 8 \cdot 5\, e^{-.6435j}$

$$\boxed{\begin{aligned} &V \Rightarrow 40e^{-.6435j}\ V \Rightarrow 40\cos(200{,}000t - 36.87)\ V \\ &i_1 = 4e^{-.6435j}\ A \Rightarrow 4\cos(200000t - 36.87)\ A \\ &i_2 = 4e^{-1.5708j}\ A \Rightarrow 4\cos(200{,}000t - 90)\ A \\ &i_3 = 8e^{.9273j}\ A \Rightarrow 8\cos(200{,}000t + 53.13)\ A \end{aligned}}$$

Problem 4b. For the circuit shown in Fig. 7, find:

(i) The value of R_L that results in maximum power being transferred to R_L

(ii) Calculate the maximum power that can be delivered to R_L

(iii) When R_L is adjusted for maximum power transfer, what percentage of the power delivered by the 360 V source reaches R_L?

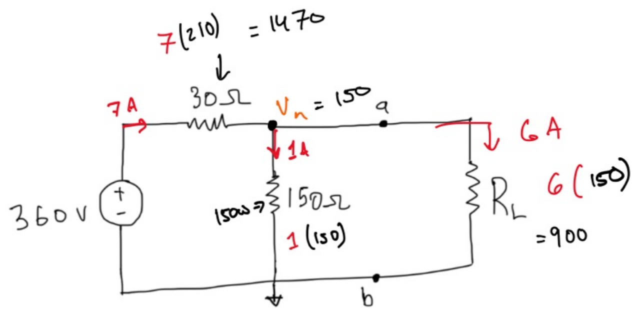


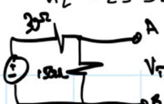
Fig. 7. RLC Network with sinusoidal current source

(i) Maximum Power Transfer Theorem States to deliver max power, $R_L = R_{Th}$

$\Rightarrow \boxed{R_L = 25\,\Omega}$

(ii) Given That $R_L = 25\,\Omega$

$V_{Th} \Rightarrow$



$\frac{150}{180}(360) = 300$

$P_{max} = \frac{\frac{1}{4}V_{Th}^2}{R_{Th}} = \boxed{900 \text{ Watts}}$

(iii)

$P_{30\Omega} = \frac{V^2}{R} = \frac{210^2}{30} = 1470$ watts

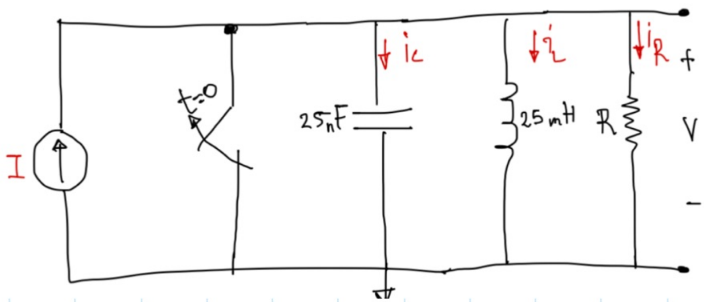
$P_{150\Omega} = \frac{V^2}{R} = \frac{150^2}{150} = 150$ watts

$P_{R_L} = \frac{150^2}{25} = 900$ watts

$\Rightarrow \boxed{35.71\%}$ of 360 V source

Problem 5a. The initial energy stored in the circuit in Fig. 8 is zero. At $t = 0$, a dc current source of 24 mA is applied to the circuit. The value of the resistor is $400\ \Omega$.

(a) What is the initial value of i_L?
(b) What is the initial value of di_L/dt?
(c) What are the roots of the characteristic equation?
(d) What is the numerical expression for $i_L(t)$ when $t \geq 0$?
(e) Find $i_L(t)$ for $t \geq 0$ if the resistor R in Fig. 8 is increased to $625\ \Omega$.
(f) Find $i_L(t)$ for $t \geq 0$ if the resistor R in Fig. 8 is increased to $500\ \Omega$.



a) Since there is no energy in circuit for $t<0$
i_L @ $t=0$ will be zero

$i_L(0)=0$

b) Given that there is no energy in the capacitor @ $t=0$
then voltage across it is zero.

⇒ voltage across inductor is also zero

$$v = L\frac{di}{dt} \Rightarrow 0 = L\frac{di}{dt} \Rightarrow \boxed{\frac{di_L}{dt} = 0}$$

c)

$$\underbrace{\left[s^2 + \frac{s}{RC} + \frac{1}{LC}\right]}_{} = 0$$

↓
this is the characteristic Eqn

$$S_1 = \frac{-1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

$$S_2 = -\frac{1}{2RC} - \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

My Notebook @ Georgia Institute of Technology

Home Insert Draw View

c)

$$\underbrace{\left[s^2 + \frac{s}{RC} + \frac{1}{LC} \right]}_{\text{this is the characteristic Eqn}} = 0$$

$$S_1 = \frac{-1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

$$S_2 = -\frac{1}{2RC} - \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

$$S_1 = \frac{-1}{2(400)(25e-9)} - \sqrt{\left[\frac{1}{2(400)(25e-9)}\right]^2 - \frac{1}{(25e-3)(25e-9)}}$$

$$= -80,000$$

$$S_2 = -\frac{-1}{2(400)(25e-9)} + \sqrt{\left[\frac{1}{2(400)(25e-9)}\right]^2 - \frac{1}{(25e-3)(25e-9)}}$$

$$= -20,000$$

D) Since the Radian frequency squared is less than Damping frequency squared $\Rightarrow$ overdamped inductor

$$i_L = I_f + A_1 e^{S_1 t} + A_2 e^{S_2 t}$$

$$\frac{di_L}{dt} = A_1 S_1 e^{S_1 t} + A_2 S_2 e^{S_2 t}$$

Taken @ $t=0$

$$I_F + A_1 + A_2 = 0 \Rightarrow$$

$$A_1 S_1 + A_2 S_2 = 0 \Rightarrow$$

$$\left[\begin{array}{cc|c} -80e3 & -20e3 & 0 \\ 1 & 1 & -.024 \end{array}\right]$$

$$\Rightarrow -.032 + .008$$

$$\boxed{i_L = .024 - .032 e^{-20000t} + .008 e^{-80000t}}$$

Made with Goodnotes

e) if R is increased to 625 Ω

D) Since The Radian frequency squared is less Than Damping frequency squared ⇒ overdamped inductor

$$i_L = I_f + A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$\frac{di_L}{dt} = A_1 s_1 e^{s_1 t} + A_2 s_2 e^{s_2 t}$$

Taken @ $t=0$

$$I_F + A_1 + A_2 = 0 \Rightarrow \qquad \left[\begin{array}{cc|c} -80e3 & -20e3 & 0 \\ 1 & 1 & -.024 \end{array}\right]$$

$$A_1 S_1 + A_2 S_2 = 0 \Rightarrow$$

$$\Rightarrow -.032 \ \& \ .008$$

$$\boxed{i_L = .024 - .032\, e^{-20000t} + .008\, e^{-80000t}}$$

e) if R is increased to 625 Ω

$$s = \frac{1}{2RC} \pm \sqrt{[1/2RC]^2 - \frac{1}{LC}} \Rightarrow -3.2e4 \pm j\,2.4e4$$

$$\omega_o = \frac{1}{LC} \qquad \alpha = \frac{1}{2RC}$$

$$\omega_o = 1.6e9 \qquad \alpha = 32e3$$

$\omega_o^2 > \alpha^2 \Rightarrow$ roots turn complex & The System is underdamped

$$\Rightarrow \quad i_L = I_f + B_1 e^{s_1 t}\cos(\omega_d t) + B_2 e^{s_2 t}\sin(\omega_d t)$$

$$@\ t = 0 \Rightarrow 0 = I_f + B_1(1)(1) + \cancel{B_2(1)(0)} \to 0$$

$$\Rightarrow B_1 = -I_f \Rightarrow -.024$$

$$\frac{di_L}{dt} = B_1\left(\cancel{-\omega_d \sin(\omega_d t)e^{-\alpha t}} + -\alpha \underbrace{\cancel{e^{-\alpha t}}}_{1} \underbrace{\cancel{\cos(\omega_d t)}}_{1}\right) +$$

$$B_2\left(\omega_d \underbrace{\cancel{\cos(\omega_d t)}}_{1} \underbrace{e^{-\alpha t}}_{1} + \cancel{s_2 e^{\alpha t}\sin(\omega_d t)}\right)\Big|_{t=0}$$

$$\frac{di_L(0)}{dt} = -B_1 + \omega_d B_2 = 0$$

$$\frac{-.024(s_1)}{\omega_d} = B_2 = -.032$$

$$\boxed{i_L(t) = .024 - .024\, e^{-32e3t}\cos(24000t) - .032\, e^{-32e3t}\sin(24000t)}$$

$\omega_0 = 1.6e9 \qquad \alpha = 32e3$

$\omega_0^2 > \alpha^2 \Rightarrow$ Roots turn complex + The System is Underdamped

$\Rightarrow i_L = I_F + B_1 e^{S_1 t} \cos(\omega_d t) + B_2 e^{S_2 t} \sin(\omega_d t)$

@ $t = 0 \Rightarrow 0 = I_F + B_1(1)(1) + \cancel{B_2(1)(0)} = 0$

$\Rightarrow B_1 = -I_F \Rightarrow -.024$

$$\frac{di_L}{dt} = B_1\left(\cancel{-\omega_d \sin(\omega_d t) e^{-\alpha t}} + -\alpha \underbrace{e^{-\alpha t}}_{1} \underbrace{\cos(\omega_d t)}_{1}\right) +$$

$$B_2\left(\omega_d \underbrace{\cos(\omega_d t)}_{1} \underbrace{e^{-\alpha t}}_{1} + \cancel{S_2 e^{\alpha t} \sin(\omega_d t)}\right)\Big|_{t=0}$$

$$\frac{di_L(0)}{dt} = -B_1 + \omega_d B_2 = 0$$

$$\frac{-.024(S_1)}{\omega_d} = B_2 = -.032$$

$$\boxed{i_L(t) = .024 - .024 e^{-32e3t} \cos(24000t) - .032 e^{-32e3t} \sin(24000t)}$$

f) Given $R = 500$

$$S = \frac{1}{2RC} \pm \sqrt{\left[\frac{1}{2RC}\right]^2 - \frac{1}{LC}} \Rightarrow S = \frac{1}{2RC} = 40{,}000$$

is such That $\omega_0^2 = \alpha^2$

Resulting in a Repeated Root

$\Rightarrow i_L = I_F + A_1 t e^{St} + A_2 e^{St}$

@ $t=0 \Rightarrow A_2 = -I_F = -.024$

$$\frac{di_L}{dt} = A_1\left(t(S)e^{St} + e^{St}\right) + A_2 S e^{St}$$

$$\frac{di_L(0)}{dt} = A_1 + A_2 S = 0$$

$A_1 = -A_2 S \Rightarrow .024(40000) = 960$

$$\boxed{i_L = .024 + 960 t e^{40000t} + -.024 e^{40000t}}$$

Made with Goodnotes

Problem 5b. Energy is stored in the circuit in Fig. 8 at the instant the dc current source is applied. If the resistance $R = 500\ \Omega$ and initial current in the inductor is 29 mA with the initial voltage across the capacitor being 50 V. Find

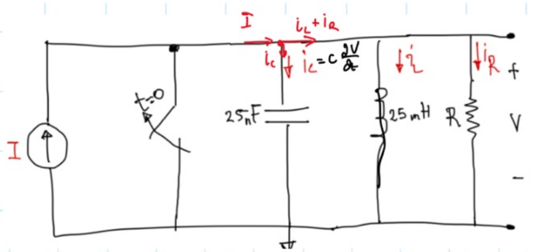
(i) $i_L(0)$
(ii) $di_L(0)/dt$
(iii) $i_L(t)$ for $t \geq 0$
(iv) $v(t)$ for $t \geq 0^+$.

$V_L = L \frac{\partial i}{\partial t}$

(i) $i_L(0) = \boxed{.029\ A}$

(ii) $V_L = (25e\text{-}3) \frac{\partial i_L(0)}{\partial t}$

$\Rightarrow \frac{50}{25e\text{-}3} = \boxed{2{,}000\ A/s}$



$I - i_c - (i_L + i_R) = 0$

$.024 - C\frac{\partial V}{\partial t} - i_L(t) - \frac{V(t)}{R} = 0$

Given That $V_L = L\frac{\partial i_L}{\partial t}$

$V_L = V_c = V_R = V$

$\frac{\partial V}{\partial t} = L\frac{\partial^2 i_L}{\partial t^2}$

$-CL\frac{\partial^2 i_L}{\partial t^2} - \frac{L}{R}\frac{\partial i_L}{\partial t} - i_L + .024 = 0$

$LC\frac{\partial^2 i_L}{\partial t^2} + \frac{L}{R}\frac{\partial i_L}{\partial t} + i_L = .024$

$\frac{\partial^2 i_L}{\partial t^2} + \frac{1}{RC}\frac{\partial i_L}{\partial t} + \frac{i_L}{LC} = \frac{.024}{LC}$

let $i_L = e^{\alpha t} \Rightarrow \frac{di_L}{dt} = \alpha e^{\alpha t} \ldots$

$\alpha^2 e^{\alpha t} + \frac{1}{RC}\alpha e^{\alpha t} + \frac{1}{LC}e^{\alpha t} = 0$ first solve the homogenous solution

$e^{\alpha t}(\alpha^2 + \frac{1}{RC}\alpha + \frac{1}{LC}) = 0$

$\alpha^2 + \frac{1}{RC}\alpha + \frac{1}{LC} = 0$

$\alpha = -\frac{1}{2RC} \pm \frac{1}{2}\sqrt{[1/RC]^2 - 4/LC}$

$= -40\,000 \pm 0$

general $i_{Lg}(t) = A_1 e^{-40000t}(t) + A_2 e^{-40000t} + i_{LP}$

$i_L(0) = .029 = A_2 + i_{LP}$ ← .024 $\quad A_2 = .005$

$\frac{\partial i_L(t)}{\partial t} = A_1\left(e^{-40000t} + t(-40000)e^{-40000t}\right) + A_2(-40000)e^{-40000t}$

let $i_L = e^{\alpha t} \Rightarrow \frac{di_L}{dt} = \alpha e^{\alpha t} \ldots$

$$\alpha^2 e^{\alpha t} + \frac{1}{RC}\alpha e^{\alpha t} + \frac{1}{LC} e^{\alpha t} = 0$$

first solve the homogenous solution

$$e^{\alpha t}\left(\alpha^2 + \frac{1}{RC}\alpha + \frac{1}{LC}\right) = 0$$

$$\alpha^2 + \frac{1}{RC}\alpha + \frac{1}{LC} = 0$$

$$\alpha = -\frac{1}{2RC} \pm \frac{1}{2}\sqrt{[1/RC]^2 - 4/LC}$$

$$= -40\,000 \pm 0$$

General $i_{Lg}(t) = A_1 e^{-40000t}(t) + A_2 e^{-40000t} + i_{LP}$

$$i_L(0) = .029 = A_2 + i_{LP} \quad .024 \qquad A_2 = .005$$

$$\frac{d\, i_L(t)}{dt} = A_1\left(e^{-40000t} + t(-40000)e^{-40000t}\right) + A_2(-40000)e^{-40000t}$$

$$\frac{d\, i_L(0)}{dt} = A_1 + A_2(-40000) = 2000$$

$$A_1 + -200 = 2000$$

$$A_1 = 2200$$

(iii) $\boxed{i_L(t) = .024 + 2200\, t\, e^{-40000t} + .005 e^{-40000t}}$

(IV) $L\frac{d i_L(t)}{dt} = V_L \Rightarrow$

$$V_L(t) = (25e{-3})\left(2200(-40000)t e^{-40000t} + 2200 e^{-40000t} - 200 e^{-40000t}\right)$$

$$\boxed{V(t) = (-2.2e6)\, t\, e^{-40000t} + 50 e^{-40000t}}$$