Question 4

Sunday, November 12, 2023 4:59 PM

4. (a) Compute the determinant of the following $n \times n$ matrix:

$$\begin{pmatrix} 4 & 1 & 1 & \dots & 1 \\ 1 & 4 & 1 & \dots & 1 \\ 1 & 1 & 4 & \dots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \dots & 4 \end{pmatrix}$$

for $n = 2$ $\Delta = 15$

$n = 3$ $\Delta = 54$

$n = 4$ $\Delta = 189$

$n = 5$ $\Delta = 648$

$$4, \underset{n=2}{15/4}, \underset{n=3}{18/5}, \underset{n=4}{21/6}, \underset{n=5}{24/7}$$

$$4 \cdot \left(\prod_{p=2}^{n} \frac{3(p+3)}{p+2} \right)$$

(b) For the matrix,

$$A = \begin{pmatrix} 1 & 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 1 & 1 \\ 1 & 0 & 0 & 0 & \dots & 0 & 1 \end{pmatrix}$$

Prove that $|A| = 1 + (-1)^{(n+1)}$. (Note the 1 on the left lowest corner)

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For This matrix to Be Reduced to Echelon form, we first Subtract The 1^{st} Row from The last. This turns the A_{n2} entry to -1. Then Add The Second Row to The last Row to turn $A_{n2}=0$. But this Results in $A_{n3} = 1$. Perform This process iteratively until The A_{nn-1} Term is Non-zero. Note, if n is odd, This term will Be -1 Since You Subtracted The 1^{st} Row and every other Subsequent Rows from The last to turn 1 to Zero. likewise if n is even.

Performing a Subtraction or addition to The n^{th} Row when $A_{nn-1} = \pm 1$ will Result in $A_{nn} = 0, 2$. At This point Since A is in echelon form, The total product Of The main diagonals is determinant A_{nn} making $|A| = 0$ or 2. 0 if n is even and 2 if n is odd $\Rightarrow |A| = 1 + (-1)^{n+1}$