T1a $|\langle S_x\rangle_n| > \Delta_n S_y$ $\quad \Delta_n S_y = \frac{\hbar}{2}|n \times y|$

$$\begin{bmatrix} i & j & k \\ n_1 & n_2 & n_3 \\ 0 & 1 & 0 \end{bmatrix} =$$

$$((n_2 \cdot 0) - (n_3))\,i +$$

$$0\,\hat{j} + (n_1 - 0)\,\hat{k}$$

$$|\langle S_x\rangle_n| > \Delta_n S_y = \frac{\hbar}{2}\sqrt{\vec{n}_3^{\,2} + \vec{n}_1^{\,2}}$$

$$\left|\frac{\hbar}{2}\left(\widehat{\langle 1,0,0\rangle} \cdot \langle n_1, n_2, n_3\rangle\right)\right|$$

$$\left|\cancel{\frac{\hbar}{2}}(\vec{n}_1)\right| > \cancel{\frac{\hbar}{2}}\sqrt{n_3^2 + n_1^2} \quad \rightarrow 0 > n_3^2 \; ??$$

$$|\vec{n}_1| > \sqrt{n_3^2 + n_1^2} \quad \cancel{|n_1|^2} > n_3^2 \cancel{+n_1^2}$$

Not possible since one unit vector n_1 cannot be greater than the whole magnitude of the entire Bloch vector.

T2b $|\langle S_x\rangle_n| \overset{?}{=} \Delta_n S_y$

$$|\vec{n}_1| \overset{?}{=} \sqrt{n_3^2 + n_1^2} \quad \text{where } n_3 = 0$$

$$|\vec{n}_1| = \sqrt{n_1^2} \Rightarrow |n_1| = n_1 \checkmark$$

Such bloch vector $\vec{n} = \langle \frac{1}{\sqrt{2}}, 0, 0\rangle$

T2 $|\psi_1\rangle = \cos\frac{\pi}{8}|+n\rangle + e^{i\pi/4}\sin(\pi/8)|-n\rangle$

$|\psi_2\rangle = e^{i\pi/4}\cos(\pi/8)|+n\rangle + \sin(\pi/8)|-n\rangle$

$\langle\psi_2|\psi_1\rangle = \left(e^{-i\pi/4}\cos(\pi/8)|+n\rangle + \sin(\pi/8)|-n\rangle\right)\left(\cos(\pi/8)|n\rangle + e^{i\pi/4}\sin\pi/8|-n\rangle\right)$

$= e^{-i\pi/4}\cos^2\frac{\pi}{8} + e^{i\pi/4}\sin^2\frac{\pi}{8})$

$e^{-i\pi/4}\left(\frac{1}{2}+\frac{1}{2}\cos\left(\frac{\pi}{4}\right)\right) + e^{i\pi/4}\left(\frac{1}{2}-\frac{1}{2}\cos\left(\frac{\pi}{4}\right)\right)$

$\frac{1}{2}\left(\underset{\cos\frac{\pi}{4}-i\sin\frac{\pi}{4}}{e^{-i\pi/4}}(1+\cos\pi/4) + \underset{\cos\frac{\pi}{4}+i\sin(\frac{\pi}{4})}{e^{i\pi/4}}(1-\cos\pi/4)\right)$

$\frac{1}{2}\left(\cos\frac{\pi}{4} \cancel{-i\sin\frac{\pi}{4}} \cancel{+\cos^2\frac{\pi}{4}} - i\sin\frac{\pi}{4}\cos\frac{\pi}{4} + \cos\frac{\pi}{4} \cancel{+i\sin\frac{\pi}{4}} \cancel{-\cos^2\frac{\pi}{4}} - i\sin\frac{\pi}{4}\cos\frac{\pi}{4}\right)$

$\frac{1}{2}\left(2\cos\frac{\pi}{4} - 2i\sin\frac{\pi}{4}\cos\frac{\pi}{4}\right) = \cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\cos\frac{\pi}{4}$

$\langle\psi_2|\psi_1\rangle = \frac{\sqrt{2}}{2}\left(1 - i\frac{\sqrt{2}}{2}\right);\ |\langle\psi_2|\psi_1\rangle|^2 = \left(\frac{\sqrt{2}}{2} - \frac{1}{2}i\right)\left(\frac{\sqrt{2}}{2} + \frac{1}{2}i\right) = \frac{1}{2} - \frac{1}{4}i^2 = \frac{1}{2}+\frac{1}{4}$

$\cos\theta = 3/4 \quad \theta = \cos^{-1}\left(\frac{3}{4}\right) = \alpha = .7227$ rad?

$\langle\psi_1|\psi_2\rangle = e^{i\pi/4}\cos^2\frac{\pi}{8} + e^{-i\pi/4}\sin^2\frac{\pi}{8}$

$\left(\cos\frac{\pi}{4} + i\sin\pi/4\right)\left(\frac{1}{2}+\frac{1}{2}\cos\left(\frac{\pi}{4}\right)\right) + \left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right)\left(\frac{1}{2}-\frac{1}{2}\cos\left(\frac{\pi}{4}\right)\right)$

should be simple angle

$\frac{1}{2}\left(\cos\frac{\pi}{4} \cancel{+i\sin\frac{\pi}{4}} \cancel{+\cos^2\frac{\pi}{4}} + i\sin\frac{\pi}{4}\cos\frac{\pi}{4} + \cos\frac{\pi}{4} \cancel{-i\sin\frac{\pi}{4}} \cancel{-\cos^2\frac{\pi}{4}} + i\sin\frac{\pi}{4}\cos\frac{\pi}{4}\right)$

$|\langle\psi_1|\psi_2\rangle|^2 \frac{1}{2}\left(2\cos\frac{\pi}{4} + 2i\sin\frac{\pi}{4}\cos\frac{\pi}{4}\right) \Rightarrow \left(\frac{\sqrt{2}}{2} + \frac{1}{2}i\right)\left(\frac{\sqrt{2}}{2} - \frac{1}{2}i\right) = \frac{1}{2} - \frac{1}{4}i^2 = \frac{3}{4}$

$= 3/4$

Verify $\langle\psi_1|\psi_1\rangle = 1 \Rightarrow \left(\cos\frac{\pi}{8}|n\rangle + e^{-i\pi/4}\sin\frac{\pi}{8}|-n\rangle\right)\left(\cos\frac{\pi}{8}|n\rangle + e^{i\pi/4}\sin\frac{\pi}{8}|-n\rangle\right)$

$= \cos^2\frac{\pi}{8} + e^0\sin\frac{\pi}{8} = 1$

$|\langle n_1|n_2\rangle|^2 = \frac{1}{2}(1 + \underbrace{n_1\cdot n_2}_{})$ $\quad \cos^{-1}\left(\frac{1}{2}\right) =$

$|\langle\psi_1|\psi_2\rangle|^2 = \frac{3}{4} = \frac{1}{2}(1 + \cos\alpha)$

$6/4 - 1 = \cos\alpha$

$\frac{6}{4} - \frac{4}{4} = \frac{2}{4} = \frac{1}{2} = \cos\alpha$

$\alpha = \cos^{-1}\left(\frac{1}{2}\right)$

$\boxed{\alpha = \pi/3}$