PROBLEM 2.1.* Let $x(t)$ be the following sinsoid plus a constant:

$$x(t) = 10 + 9\cos(4\pi t - 0.4\pi).$$

$2\pi f_0 (t - t_0) = 4\pi t - .4\pi$

$2\pi f_0 t - 2\pi f_0 t_0 = 4\pi t - .4\pi$

$f_0 = 2 \quad T = \frac{1}{2} = .5 \quad t_0 = \frac{1}{10}$

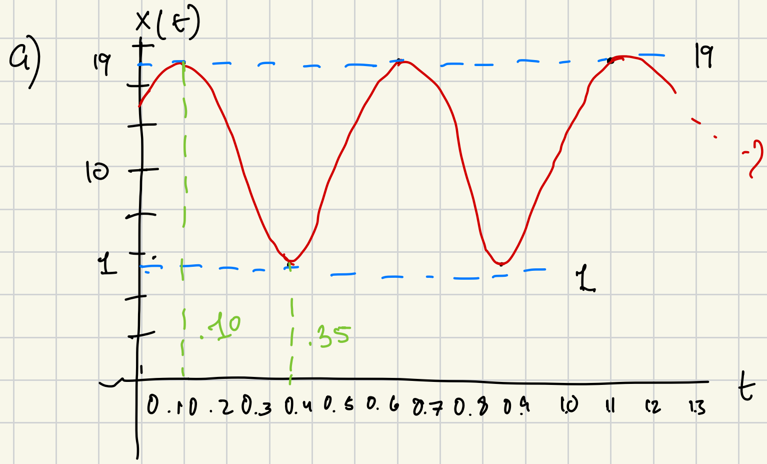
(a) Sketch $x(t)$ versus time t for time in the range $0 < t < 2$, *carefully* labeling important values for the time axis (like the times of the peaks) and important values for the vertical axis (like the value at the peaks and at the valleys).

(b) What is the time of the peak that happens closest to time zero?

(c) Evaluate the following integral:

$$\int_0^7 x(t)dt.$$

a)



b) $\boxed{t = .1 \text{ seconds}}$

c) $\int_0^7 x(t)\,dt \Rightarrow \int_0^7 10 + 9\cos(4\pi t - .4\pi)\,dt \Rightarrow 10\int_0^7 dt + 9\int_0^7 \cos(4\pi t - .4\pi)\,dt$

$70 + \frac{9}{4\pi}\int_0^7 \cos(u)\,du \qquad u = 4\pi t - .4\pi \qquad du = 4\pi \qquad \Rightarrow 70 + \frac{9}{4\pi}\left[\sin(4\pi t - .4\pi)\right]_0^7$

$70 + \frac{9}{4\pi}(1. \ldots E{-15}) = \boxed{70}$

PROBLEM 2.2.* Find a pair of sinusoids $s_1(t)$ and $s_2(t)$ that satisfy both of the following for all t:

let $s_1 = r_1 e^{j\phi_1}$

$s_2 = r_2 e^{j\theta_2}$

$$\sqrt{3}\,s_1(t) + s_2(t) = \cos(2048\pi t),$$

$$\sqrt{3}\,s_1(t) - s_2(t) = \sin(2048\pi t).$$

$\frac{e^{j\theta_1} + e^{-j\theta_1}}{2} = \cos(\theta_1)$

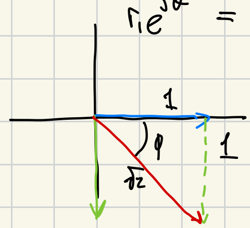
$\frac{e^{j\theta_1} - e^{-j\theta_1}}{2j} = \sin(\theta_1)$

Write your answers in standard form.

$2\sqrt{3}\,s_1(t) = \cos(2048\pi t) + \cancel{\sin(2048\pi t)} + \cos(2048\pi t - \pi/2)$

$r_1 e^{j\theta} = 1e^{j(0)} + 1e^{-j(\pi/2)}$

$\theta = -\pi/4$



$2\sqrt{3}\,r_1 = \sqrt{2}$

$r_1 = \frac{\sqrt{2}}{2\sqrt{3}} \Rightarrow \frac{\sqrt{6}}{6}$

$s_1(t) = \frac{\sqrt{6}}{6} e^{j(-\pi/4)}$

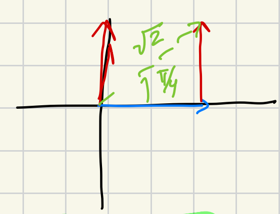
$2s_2(t) = \cos(2048\pi t) - \sin(2048\pi t)$

$e^{0j} - e^{-\pi/2 j}$

$2s_2(t) = e^{0j} + e^{-3\pi/2 j} \qquad \theta = \pi/4$

$2r_1 = \sqrt{2} \Rightarrow R_1 = \sqrt{2}/2$

$s_2(t) = \frac{\sqrt{2}}{2} e^{j(\pi/4)}$



$s_1(t) = \frac{\sqrt{6}}{6}\cos\left(2048\pi t - \frac{\pi}{4}\right)$

$s_2(t) = \frac{\sqrt{2}}{2}\cos(2048\pi t + \pi/4)$

PROBLEM 2.3.* Suppose that the following equation is true for all t:

$$\sin(2026\pi t + 2026.5\pi) + \cos(2\pi f_0 t - \frac{\pi}{6}) + \cos(2\pi f_0(t - t_0)) = A\cos(2\pi f_0 t).$$

(a) Find $f_0 > 0$.

(b) There are multiple possibilities for the value of t_0; find the *smallest* $t_0 > 0$.

(c) Find $A > 0$.

$2026\pi - 2026\pi = \phi = 0$

a) $\cos(2026\pi t + \cancel{2026.5\pi - \frac{\pi}{2}}) + \cos(2\pi f_0 t - \pi/6) + \cos(2\pi f_0(t - t_0)) = A\cos(2\pi f_0 t)$

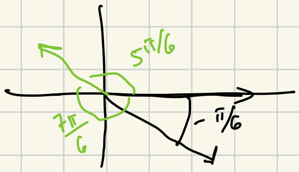
$f_0 = 1013\text{ Hz}$

$e^{j0} + e^{j(-\pi/6)} + e^{-2026\pi t_0} = Ae^{j(0)}$ $2026\pi t - 2026\pi t_0$ $\phi = 0$

$Ae^{j(0)}$

$A + 0j$

b)



$-2026\pi t_0 = \frac{5\pi}{6}$ X

$-2026\pi t_0 = \frac{-7\pi}{6} \Rightarrow \boxed{t_0 = \frac{7}{12156}}$

c) $\overbrace{1}^{e^{0j}} + \overbrace{\cancel{\frac{\sqrt{3}}{2}} - \cancel{\frac{1}{2}j}}^{e^{-\pi/6 j}} + \overbrace{-\cancel{\frac{\sqrt{3}}{2}} + \cancel{\frac{1}{2}j}}^{e^{-7\pi/6 j}} = A \Rightarrow \boxed{A = 1}$

PROBLEM 2.4.* Define $x(t)$ as the following sum of sinusoids:

$\pi t - \pi t_0$ $\pi t - \pi t_0 - .1\pi$ ϕ $\Rightarrow \phi = -\pi(t_0 + .1)$

$$x(t) = \cos(\pi t) + \cos(\pi t - 0.1\pi) + \cos(\pi t - 0.2\pi) + \cos(\pi t - 0.3\pi) + \cos(\pi t - 0.4\pi).$$

(a) Find the smallest positive value for the delay $t_0 \geq 0$ so that the delayed signal can be written as:

$X(t) = 1 + e^{-.1\pi} + e^{-.2\pi} + e^{-.3\pi} + e^{-.4\pi}$

$$x(t - t_0) = A\sin(\pi t)$$

$\Rightarrow Ae^{-\pi/2} = A\cos(\pi t - \pi/2$

for some unspecified positive constant A.

(b) Find the positive constant $A > 0$ in part (a).

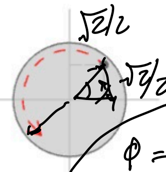
a) $Ae^{-\pi/2 j} = e^{-j\pi t_0} + e^{-j\pi(t_0 + .1)} + e^{-j\pi(t_0 + .2)} + e^{-j\pi(t_0 + .3)} + e^{j\pi(t_0 + .4)}$

$-\frac{5}{2} - \cancel{\frac{1}{2}} = -t_0 - t_0 - .1 - t_0 - .2 - t_0 - .3 - t_0 - .4$

$\Rightarrow -5t_0 - 1 = \frac{-5}{2} \Rightarrow -5t_0 = -\frac{3}{2} \Rightarrow t_0 = 3/10$

b) $4.5201 = A$

PROBLEM 2.5.* The magnetic disk in a hard drive spins in a counterclockwise direction at a rate of 7200 rpm (rotations per minute). The disk has a defect that appears as a large black dot that is a distance 4 (in unspecified units) from the center of the disk. Let $x(t)$ and $y(t)$ denote the horizontal and vertical coordinates, respectively, of the black dot at time t.



$\phi = \frac{\pi}{4}$

$\frac{7200 \text{ rot}}{1 \text{ min}} \Big| \frac{1 \text{ min}}{60 \text{ Sec}}$

$\frac{7200}{60} = \frac{\text{rot}}{\text{Sec}} = \text{Hz}$

$120 \text{Hz} = f_0$

The figure shows a snapshot taken at time 0, at which time the horizontal and vertical coordinates of the black dot are identical, as illustrated in the figure, so that $x(0) = y(0)$.

$e^{j(0)}$ $\quad e^{j(\pi/2)}$ $\quad a^2+b^2 = 4$

(a) Write the sum $s(t) = x(t) + y(t)$ as a function of t as a single sinusoid, in standard form.

$\cos(240\pi t + \frac{\pi}{2})$

(b) Find the smallest positive t_0 for which the following equation is true for all time t:

$$x(t) + y(t - t_0) = 0.$$

$\cos(240\pi t - 240\pi t_0 + \frac{\pi}{2})$

$\sin(240\pi t - 240\pi t_0 + \pi/4)$

(c) Find the real positive constants β and A and f_0 for which for which the following equation is true for all t:

$$x(t) + \beta y(t) = A\cos(2\pi f_0 t - \frac{\pi}{12}).$$

$\frac{\sqrt{2}}{2} A = 4$

$A = \frac{8}{\sqrt{2}}$

$= 4\sqrt{2}$

$x\sqrt{2}$ $\quad x$ $\quad x$

$x\sqrt{2} = 4$

$\Rightarrow x = 4/\sqrt{2}$

$\frac{4\sqrt{2}}{2} = \boxed{2\sqrt{2}}$

$4 \cdot 2 = 8$

a) $S(t) = x(t) + y(t)$

$x(t) = A\cos(240\pi t + \pi/4)$

$y(t) = B\sin(240\pi t + \pi/4)$

$\sqrt{\frac{A^2}{2} + \frac{B^2}{2}} = 4 \Rightarrow \frac{A^2}{2} + \frac{B^2}{2} = 16$

$x(0) = y(0)$ $\qquad A^2 + B^2 = 32$

$\frac{A\sqrt{2}}{2} = \frac{B\sqrt{2}}{2} = A = B$ $\qquad 2A^2 = 32$

$A^2 = 16 \Rightarrow A = 4$

$x(t) = 4\cos(240\pi t + \pi/4) \rightarrow 4e^{j\pi/4}$

$y(t) = 4\sin(240\pi t + \pi/4) \rightarrow 4\cos(240\pi t + \pi/4 - \pi/2) \rightarrow 4e^{-j\pi/4}$

$S(t) = 4e^{j(\pi/4)} + 4e^{-j(\pi/4)} \Rightarrow 4(e^{j\pi/4} + e^{-j\pi/4})$

$= 4(2\cos(240\pi t + \pi/4)) \Rightarrow 8\cos(240t + \pi/4)$

b) $S(t) = 4\cos(240\pi t + \pi/4) + 4\cos(240\pi t - 240\pi t_0 - \pi/4)$

$\cancel{240\pi t} + \frac{\pi}{4} - \pi = \cancel{240\pi t} - 240\pi t_0 - \pi/4$

$\frac{\pi}{2} - \pi = -240\pi t_0$

$-\cancel{\pi}/2 = -240\cancel{\pi} t_0 \Rightarrow \boxed{\frac{1}{480} = t_0}$

c) $4\cos(240\pi t + \pi/4) + \beta 4\cos(240\pi t - \pi/4) = A\cos(240\pi t - \pi/12)$ $\qquad f_0 = 120$

$4e^{j\pi/4} + \beta 4 e^{j(-\pi/4)} = Ae^{j(-\pi/12)}$

$4\cos(\pi/4) + j4\sin(\pi/4) + \beta 4\cos(-\pi/4) + j\beta 4\sin(-\pi/4) = A\cos(-\pi/12) + Aj\sin(-\pi/12)$

$2\sqrt{2} + j2\sqrt{2} = -B2\sqrt{2} = jB2\sqrt{2} + A\cos(-\pi/12) + Aj\sin(-\pi/12)$

$A\cos(-\pi/12) - B2\sqrt{2} = 2\sqrt{2}$

$Aj\sin(-\pi/12) + Bj2\sqrt{2} = j2\sqrt{2}$

$\boxed{A = 8 \quad B = 1.7321}$