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Tuesday, October 24, 2023 18:35

1. Let U be a subspace in inner product space V.
   a. Show that $U^{\perp} \cap U = \{\vec{0}\}$
   b. Show that $\left(U^{\perp}\right)^{\perp} = U$. It is true if U is just a subset?

2. In class we saw what the matrix for orthogonal projection onto a subspace U looks like in $\mathbb{R}^n$. In general let V be a finite-dimensional vector space and we say $P \in \mathscr{L}(V)$ is orthgonoal projection map onto subspace U if whenever $v \in V$ is decomposed as $u + w$, where $u \in U$ and $w \in U^{\perp}$, $Pv = u$. With this in mind, show that if $P \in \mathscr{L}(V)$ is an orthogonal projection map,
   a. $P^2 = P$
   b. $Pu = u$ if $u \in U$
   c. Im$P = U$
   d. $Pw = \vec{0}$ if $w \in U^{\perp}$
   e. Ker$P = U^{\perp}$
   f. $v - Pv \in U^{\perp}$

3. Let $u \in \mathbb{R}^n$ be a unit vector.
   a. Consider the linear transformation associated with matrix $I_n - uu^T$. What does it do to the vectors in $\mathbb{R}^n$.
   b. What about the matrix $2uu^T - I_n$

4. Let $A \in M_{m \times n}(\mathbb{R})$.
   a. Show that $(Col(A))^{\perp} = Nul\left(A^T\right)$
   b. Show that $\left(Col\left(A^T\right)\right)^{\perp} = Nul(A)$
   c. Let $A = \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{pmatrix}$. Determine a basis for $Nul\left(A^T\right)$. Draw a sketch to illustrate the result in (a) in this case.
   d. If A is symmetric, that is $A = A^T$, show that $Ax = b$ has a solution iff b is orthogonal to $Nul(A)$.

.,I gave the definition of orthogonal complement in class a
they should know without proving them.

This is the definition that we will use for orthogonal proje
considering the decomposition in the definition. Please als
them understand. I have not shown that the orthogonal p
next time.

I am shooting for projection onto $u^{\perp}$ and a reflection along u

I mentioned this result and also mentioned that it was mostly
mentioned that these are sometimes called the 4 fundament
though the two identifies are essentially the same by replacin
one both of them are in the codomain and the second pair ar
sum. You can also go over when A is symmetric, domain and
talk about the orthogonality of Nul(A) and Col(A).