ECE 3030: Physical Foundations of Computer Engineering
Fall 2021
Homework 5—Total points 100
Due on Thursday 10/29/2021 at 11.59am. In case of a late submission, you will be penalized by 50 points for each day after the submission deadline has passed. You will receive no score if you submit after the solution has been posted.

Q1 Band diagrams of a MOS capacitor for three different values of the gate voltage V_G are shown in figure 1. V_t is its threshold voltage. All the variables shown in figure 1 have their usual meanings. [30 pts]

[Q1.1] Based on the information provided in figure 1, find out whether V_G larger or smaller than V_t for each of the three cases. Briefly explain your answer. [10 pts]

[Q1.2] Find out the values of electron density and hole density at x=0 and x=60 nm for each of the three cases. Assume that N_C=N_V=10^{25} /m^3, kT=25 meV, E_G= 1.1 eV (all variables have their usual meaning). [20 pts]

Solution to Q1:
If $E_{FS} - E_i(x = 0) > \phi_B$, $V_G > V_t$.
If $E_{FS} - E_i(x = 0) = \phi_B$, $V_G = V_t$.
If $E_{FS} - E_i(x = 0) < \phi_B$, $V_G < V_t$.

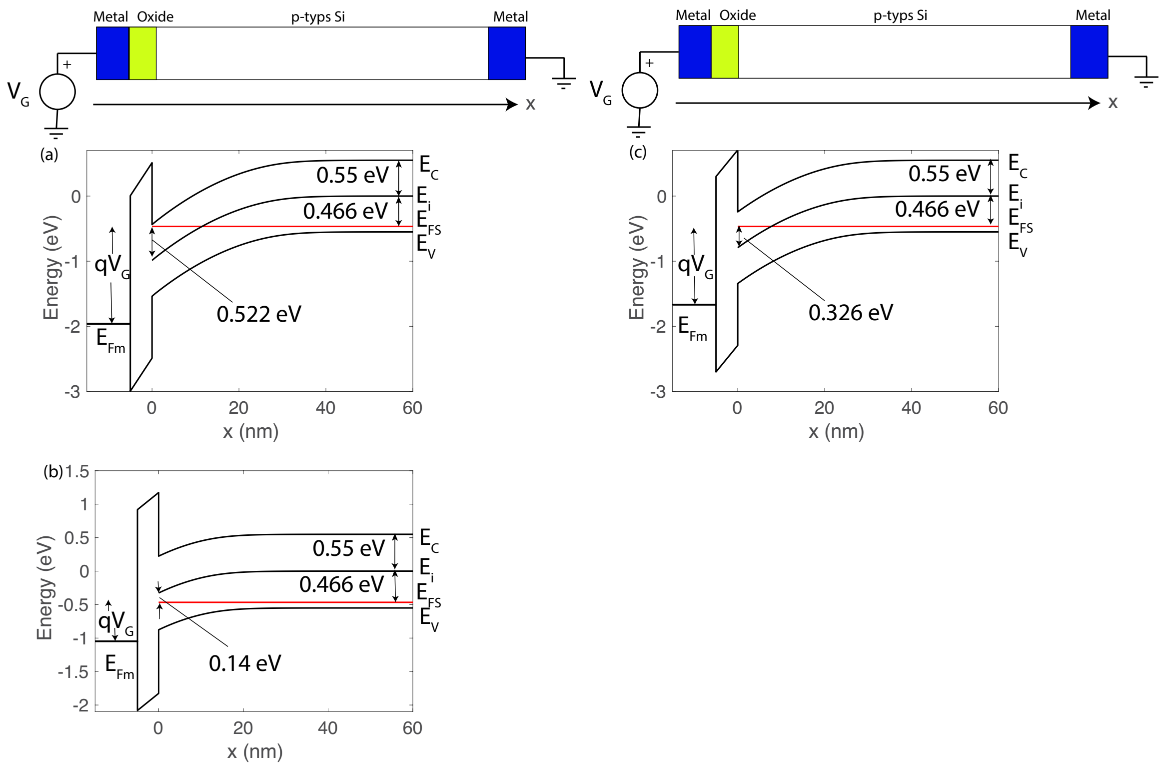

Figure 1: Band diagrams of a MOS capacitor at different gate voltages.

ⓐ $\Phi_B = 0.466\,eV$ $\qquad E_{FS} - E_i(x=0) = 0.522\,eV > \Phi_B$

$\cancel{E_i(x=0) - E_{FS} = 0.522\,eV > \phi_B}$

$\boxed{V_G > V_t}$

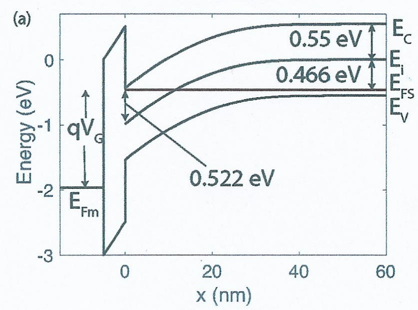

Figure 3: Band diagrams of a MOS capacitor at different gate voltages.

$\underline{x=0}$ $\quad E_c(x=0) - E_{FS} = 0.55 - 0.522 = 0.028\,eV$

$E_{FS} - E_V(x=0) = 0.55 + 0.522 = 1.072\,eV$

$$n_{(x=0)} = N_c\, e^{-\frac{E_c(x=0) - E_{FS}}{KT}} = 10^{25}\, e^{-\frac{.028}{.025}} = 3.26 \times 10^{24}\ m^{-3}$$

$$p(x=0) = N_v\, e^{-\frac{E_{FS} - E_V(x=0)}{KT}} = 10^{25}\, e^{-\frac{1.072}{.025}} = 2.38 \times 10^{6}\ m^{-3}$$

Alternatively, $N^* = N_c e^{-E_g/2KT} = 10^{25}\, e^{-1.1/2\times .025} = 2.79 \times 10^{15}\ m^{-3}$

$$n(x=0) = N^*\, e^{-\frac{E_i(x=0) - E_{FS}}{KT}} = 2.79\times10^{15}\, e^{-\frac{-.522}{.025}} = 3.26 \times 10^{24}\ m^{-3}$$

$$p(x=0) = N^*\, e^{\frac{E_i(x=0) - E_{FS}}{KT}} = 2.79\times10^{15}\, e^{\frac{-.522}{.025}} = 2.38 \times 10^{6}\ m^{-3}$$

$x = 60\,nm$ $\quad E_c(x=60\,nm) - E_{FS} = .55 + .466 = 1.016\,eV$

$E_{FS} - E_V(x=60\,nm) = 0.55 - .466 = 0.084\,eV$

$$n(x=60\,nm) = N_c\, e^{-\frac{E_c(x=60\,nm) - E_{FS}}{KT}} = 10^{25}\, e^{-\frac{1.016}{.025}} = 2.24 \times 10^{7}\ m^{-3}$$

$$p(x=60\,nm) = N_V\, e^{-\frac{E_{FS} - E_V(x=60\,nm)}{KT}} = 10^{25}\, e^{-\frac{.084}{.025}} = 3.47 \times 10^{23}\ m^{-3}$$

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(b) $\Phi_B = 0.466\,eV$

$E_{FS} - E_i(x=0) = -0.14 < \Phi_B$

$\underline{V_G < V_t}$

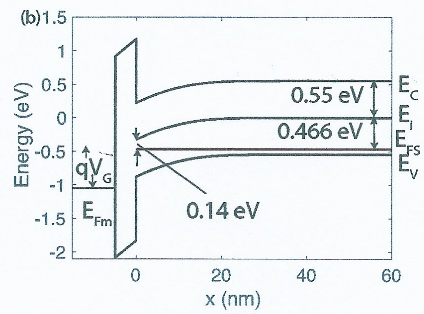

Figure 4: Band diagrams of a MOS capacitor at different gate voltages.

$x=0 \Rightarrow E_c(x=0) - E_{FS} = 0.55eV + 0.14eV = 0.69eV$

$E_{FS} - E_V(x=0) = 0.55eV - 0.14eV = 0.41eV$

$$n(x=0) = N_c e^{-\frac{E_c(x=0)-E_{FS}}{kT}} = 10^{25} e^{-\frac{0.69}{.025}} = 1.03 \times 10^{13}\ m^{-3}$$

$$p(x=0) = N_v e^{-\frac{E_F - E_V(x=0)}{kT}} = 10^{25} e^{-.41/.025} = 7.54 \times 10^{17}\ m^{-3}$$

alternatively:

$$n(x=0) = N^* e^{-\frac{E_i(x=0)-E_{FS}}{kT}} = 2.79 \times 10^{15} e^{-\frac{.14}{.025}} = 1.03 \times 10^{13}\ m^{-3}$$

$$p(x=0) = N^* e^{\frac{E_i(x=0)-E_{FS}}{kT}} = 2.79 \times 10^{15} e^{.14/.025} = 7.54 \times 10^{13}\ m^{-3}$$

$x = 60nm$: same as case (a)

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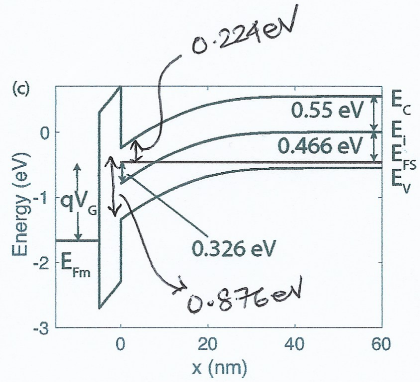

Figure 5: Band diagrams of a MOS capacitor at different gate voltages.

(c) $\phi_B = 0.446\text{ eV}$ ~~[illegible]~~

$E_{FS} - E_i(x=0) = 0.326\text{ eV} < \phi_B \quad V_G < V_t$

$x = 0\text{ nm}$: $E_c(x=0) - E_{FS} = 0.55 - 0.326 = 0.224\text{ eV}$

$E_{FS} - E_V(x=0) = 0.55 + 0.326 = 0.876\text{ eV}$

$$n_{(x=0)} = N_c e^{-\frac{E_c(x=0) - E_{FS}}{KT}} = 10^{25} e^{-\frac{.225}{.025}} = \underline{1.23 \times 10^{21}\text{ m}^{-3}}$$

$$p(x=0) = N_v e^{-\frac{E_{FS} - E_V(x=0)}{KT}} = 10^{25} e^{-\frac{.876}{.025}} = \underline{6.05 \times 10^{9}}\text{ m}^{-3}$$

alternatively:

~~$x = 60\text{ nm}: E_c$~~

$$n(x=0) = N^* e^{-\frac{E_i(x=0) - E_{FS}}{KT}} = 2.79 \times 10^{15} e^{-\frac{-.326}{.025}} = 1.28 \times 10^{21}\text{ m}^{-3}$$

$$p(x=0) = N^* e^{\frac{E_i(x=0) - E_{FS}}{KT}} = 2.79 \times 10^{15} e^{+\frac{-.326}{.025}} = \underline{6.06 \times 10^{9}\text{ m}^{-3}}$$

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$x = 60\text{ nm}$ same as case (a) & (b)

Q2 In class, we have derived an expression for the threshold voltage V_t in an n-type MOSFET. How does V_t change if the following parameters are increased. [25 pts]

Q2.1 Acceptor doping density, N_a.

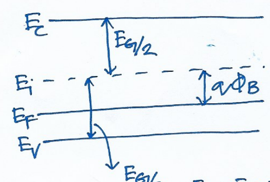
Q2.2 Oxide thickness, t_{ox}.

Q2.1 Dielectric constant of oxide, ϵ_{ox}.

Solution to Q2:

Q1: $V_t = 2\phi_B + \dfrac{\sqrt{4\varepsilon_0\varepsilon_{si}\, q N_a \phi_B}}{C_{ox}}$

$\cancel{C_{ox} = \varepsilon_0\varepsilon_{ox}}$ $\quad C_{ox} = \dfrac{\varepsilon_0\varepsilon_{ox}}{t_{ox}}$



$$q\phi_B = E_i - E_F = E_i - E_V - (E_F - E_V)$$
$$= \frac{E_G}{2} + kT\ln\frac{N_a}{N_V}$$
$$\Rightarrow \phi_B = \frac{E_G}{2q} + \frac{kT}{q}\ln\frac{N_a}{N_V}$$

$$p = N_a = N_V e^{-\left(\frac{E_F - E_V}{kT}\right)}$$
$$\Rightarrow -(E_F - E_V) = kT\ln\frac{N_a}{N_V}$$

Q1.1: $N_a\uparrow$ $\quad \phi_B\uparrow$ $\quad V_t = 2\phi_B + \dfrac{\sqrt{4\varepsilon_0\varepsilon_{si}\, q N_a \phi_B}}{C_{ox}}$ $\uparrow$

$\boxed{V_t \text{ increases}}$ if N_a increases.

Q1.2: $t_{ox}\uparrow$ $\quad C_{ox}\downarrow$ $\quad V_t\uparrow$

Q1.3: $\varepsilon_{ox}\uparrow$ $\quad C_{ox}\uparrow$ $\quad V_t\downarrow$

Q3 Consider an n-type MOSFET with N_A=7×10^{18} m^3. The gate length of the MOSFET L=2 μm, width W=12 μm and the oxide thickness t_{ox}= 8 nm. Take N_C=N_V=10^{25} m^{-3}, E_G=1.12 eV, n_i=1.5×10^{16} m^{-3}, kT=0.026 eV, vacuum permittivity $\varepsilon_\circ$=8.854×10^{-12} F/m, dielectric constant of oxide ε_{ox}=4, dielectric constant of silicon ε_{Si}=12, electron mobility μ_n=230×10^{-4} m^2/Vs, hole mobility μ_p=83 ×10^{-4} m^2/Vs.

Calculate $\phi_B = |E_F - E_i|/q$, the oxide capacitance C_{ox}, the maximum depletion width W_{max} and the threshold voltage V_t. [25 pts]

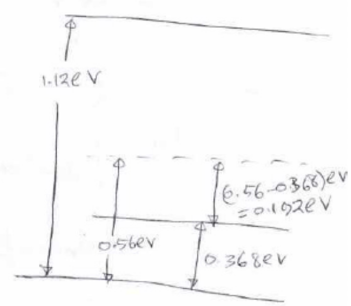
Solution to Q3:

$$p = N_A = N_V e^{\frac{E_V - E_F}{k_B T}}$$

$$\Rightarrow E_V - E_F = k_B T \ln \frac{N_A}{N_V}$$

$$= 0.026 \ln \frac{7\times10^{18}}{10^{25}}$$

$$= -0.368\,eV$$



(a) $q\psi_B = |E_i - E_F| = (0.55 - 0.368)eV = 0.192\,eV$

(b) $C_{ox} = \dfrac{\varepsilon_0 \varepsilon_{ox}}{t_{ox}} = \dfrac{8.854\times10^{-12}\,F/m \times 4}{8\times10^{-9}\,m}$

$= 4.427\times10^{-3}\,F/m^2$

(c) $W_{max} = \sqrt{\dfrac{2\varepsilon_0\varepsilon_{si}(2\psi_B)}{q N_A}} = \sqrt{\dfrac{2\times12\times8.854\times10^{-12}\times2\times0.192}{1.6\times10^{-19}\times7\times10^{18}}}$

$= 8.536\,\mu m$

$$V_t = \frac{\sqrt{4 \varepsilon_0 \varepsilon_{si} q N_A \varphi_B}}{C_{ox}} + 2\varphi_B$$

$$= \frac{\sqrt{4 \times 8.854 \times 10^{-12} \times 12 \times 1.6 \times 10^{-19} \times 7 \times 10^{18} \times 0.192}}{4.927 \times 10^{-3}}$$

$$+ 2 \times 0.193$$

$$= 0.386\ V$$

Q4 Draw the band diagram for n-type semiconductor (V_g is gate voltage and V_{th} is threshold voltage). [20 pts]

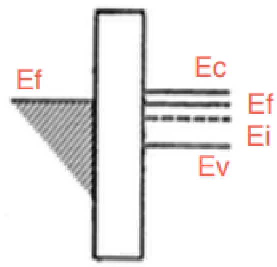
Q4.1 when $V_g = 0$

Q4.2 when $V_g > V_{th}$

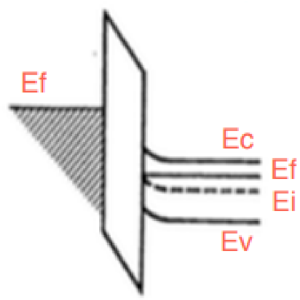
Q4.3 when $V_g \leq V_{th}$

Solution to Q4:
n-type semiconductor look the same as for p-type with reverse the unequal signs in the voltage conditions. $V_g > V_{th}$ is depletion for n-type semiconductor $V_g < V_{th}$ is inversion for n-type semiconductor.

Q4.1



Q4.2



Q4.3