Question 7

Monday, October 23, 2023 10:53 PM

7. Let $M \in M_n(\mathbb{R})$. Characterize M such that $< \cdot, \cdot >_M : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ defined by $< x, y >_M = (Mx)^T(My)$ is an inner product. Justify your answer.

if $\langle \cdot , \cdot \rangle_M : \mathbb{R}^n \times \mathbb{R}^n \longrightarrow \mathbb{R}$

$\langle x, y \rangle \longmapsto (Mx)^T(My)$

Then it holds for positive Definitness

$(Mx)^T(Mx) > 0$

$x^T M^T M x > 0$ and thus

if $Mx = \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix} \Rightarrow a_1^2 + \dots + a_n^2 = 0$ iff $Mx = 0$

in order for $M\vec{x} = \vec{0}$, x must contain trivial solution $(\vec{x} = \vec{0})$. for $M(x) = \vec{0}$ to hold for $x = \vec{0}$, Then $\exists\ A = M^{-1}$ where

$AMx = A\vec{0} \Rightarrow x = \vec{0}$

Thus M is <u>invertible</u>.