Question 2

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2. In class, we have shown that if two finite-dimensional vector spaces are of the same dimension, then they are isomorphic. Show the converse is also true.

if $q \Rightarrow p$

Converse: if $p \Rightarrow Q$

if 2 V.S. have same Dimension, Then isomorphic

if 2 V.S. are isomorphic, Then same Dimension

<u>Proof</u>|

Consider $\bar{V}$ with Basis $\{v_1, \dots, v_n\}$ and

$\bar{\omega}$ ...... $\{w_1, \dots, w_n\}$

if $\bar{V}$ and $\bar{\omega}$ are isomorphic, $\exists$ a

linear map $T \in \mathcal{L}(\bar{V}, \bar{\omega})$, an isomorphism

s.t. $T(V_i) = w_i$ $\forall i$ $\{1, \dots, N\}$ dim of(V).

as such, if T is an isomorphism for $\bar{V} \to \bar{\omega}$

Then T is Bijective meaning There is a

one-to-one Relation between $\bar{V}$ and $\bar{\omega}$.

as such since $T(v_i) = \omega_i$ $\forall i \in \{1, \dots, n\}$, Then

$\dim(\bar{V}) = \dim(\bar{\omega})$.

T is a bijective map, Thus $\ker(T) = \{\vec{0}\}$

and By Rank-nullity

$$\dim(\ker(T)) + \dim(\operatorname{im}(T)) = \dim V$$

$$0 + \dim(W) = \underline{\dim(V)}$$