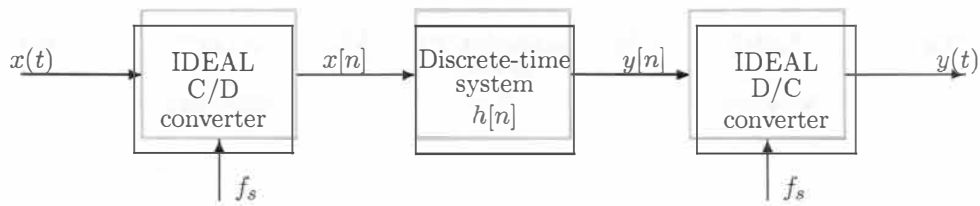


**PROBLEM sp-24-FINAL.1:**



This problem is related to the above block diagram of an ideal C-to-D converter, a filter, and an ideal D-to-C converter.

- (a) [2 points] If the output from the ideal C-to-D converter is  $x[n] = 3 \cos(0.5\pi n)$ , and the sampling rate is 2000 samples/sec, then determine one possible value of the input frequency of  $x(t)$ .

frequency

8000 Hz

4000 Hz

2000 Hz

1600 Hz

1200 Hz

1000 Hz

800 Hz

500 Hz

400 Hz

300 Hz

Pick a number from the list and enter its value in the answer box.

frequency =

- (b) [3 points] If the input is  $x(t) = 2 \cos(100\pi t - 0.1\pi) - 7 \cos(250\pi t + 0.2\pi)$ , and the system is given as  $h[n] = \delta[n] - \frac{\sin(0.4\pi n)}{\pi n}$ , find the smallest sampling frequency,  $f_s = f_{min}$ , such that  $y(t) = 0$  without aliasing.

Enter the value in the answer box.  $f_{min} =$

(c) [2 points] Suppose that

$$x(t) = \cos(400\pi t + \pi/2) + \cos(200\pi t + \pi/2).$$

Determine one value of  $f_s$  such that  $x[n] = 0$ .

frequency
8000 Hz
4000 Hz
2000 Hz
1600 Hz
1200 Hz
1000 Hz
800 Hz
500 Hz
400 Hz
300 Hz

Pick a number from the list and enter its value in the answer box.

$f_s =$

- (d) [5 points] Suppose that  $x(t) = \cos(500\pi t)$  and  $y(t) = 0.75 \cos(200\pi t)$ . Also, suppose that the discrete-time IIR system is given by the following difference equation:  $y[n] = a_1 y[n-1] + x[n] + b_2 x[n-2]$ . Determine a value for  $f_s$  that is larger than 200 Samples/s. The same  $f_s$  is used at both the Ideal C-to-D converter and the Ideal D-to-C converter.

frequency
750 Hz
700 Hz
650 Hz
600 Hz
550 Hz
500 Hz
450 Hz
400 Hz
350 Hz
300 Hz

[2 points] Pick a number from the list and enter its value in the answer

box.  $f_s =$

[3 points] Determine the value of  $a_1$  when the value for  $b_2$  is given as  $b_1 = 0.25$ .

$a_1 =$

- (e) [2 points] Suppose that the discrete-time LTI system is defined by the following difference equation:

$$y[n] = x[n] + b_1x[n-1] + x[n-2] + a_1y[n-1].$$

In this part, let  $a_1 = -0.3$ . The overall system can be used to null one continuous-time sinusoid. The frequency that is nulled is controlled by the value of  $b_1$ . If the sampling rate is  $f_s = 8000$  Hz, find the value of  $b_1$  so that the overall system nulls out a sinusoid at 60 Hz.

$b_1 =$

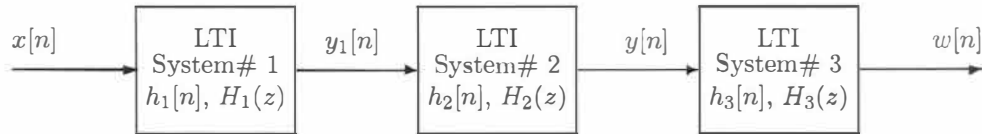
- (f) [2 points] Suppose that the discrete-time LTI system is defined by the following difference equation:

$$y[n] = x[n] + b_1x[n-1] + x[n-2] + a_1y[n-1].$$

In this part, let  $a_1 = 0$  and  $b_1 = 1$ . If the input  $x(t) = 1$  for all  $t$ , what is  $y(t)$  for all  $t$ ?

$y(t) =$

**PROBLEM sp-24-FINAL.2:**



The above block diagram depicts a cascade connection of three LTI systems. Suppose that the system function for system # 1 is given as  $H_1(z) = 2 - 2z^{-5}$ . Also, suppose that system # 2 is an FIR filter described by the following difference equation:  $y[n] = 3y_1[n] + 3y_1[n - 5]$ .

- (a) **[2 points]** Make a pole-zero plot for the first system, system # 1. Account for all poles and zeros.

- (b) **[2 points]** If we would like to replace the first two systems, # 1 and # 2, with a single overall system that has an impulse response  $h[n]$ , determine  $h[n]$  as a sum of scaled and shifted delta functions.

$h[n] =$

- (c) [3 points] What is  $y[n]$  when  $x[n] = \cos(\pi n + \pi/5)$ .

$$y[n] =$$

- (d) [2 points] Suppose that  $y[n] = 6\delta[n - 2] - 6\delta[n - 12]$  is the output of system# 2 when  $x[n] = \delta[n - B]$ , where  $B$  is an integer. Determine the value of  $B$ .

$$B =$$

- (e) [4 points] We would like to design system# 3 such that it inverts the cascade of the first two systems,  $H_1(z)$  and  $H_2(z)$ . Suppose that system# 3 can be defined by the following difference equation:  $w[n] = a_1w[n - 1] + a_{10}w[n - 10] + b_0y[n] + b_{10}y[n - 10]$ .

Determine the values of  $a_1$ ,  $a_{10}$ ,  $b_0$ , and  $b_{10}$ .

$$a_1 =$$

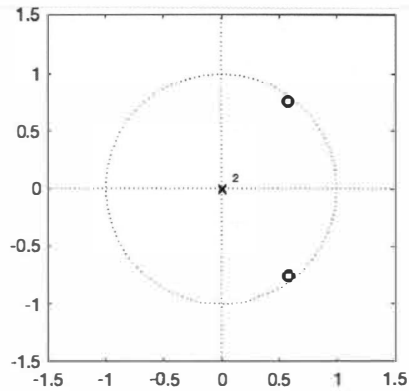
$$a_{10} =$$

$$b_0 =$$

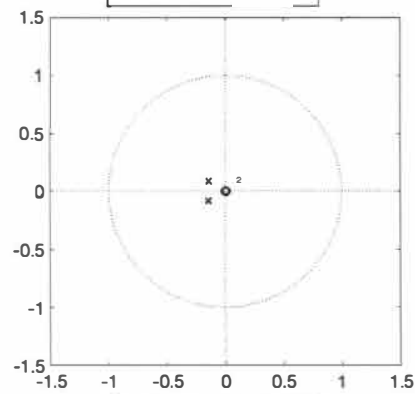
$$b_{10} =$$

### PROBLEM sp-24-FINAL.3:

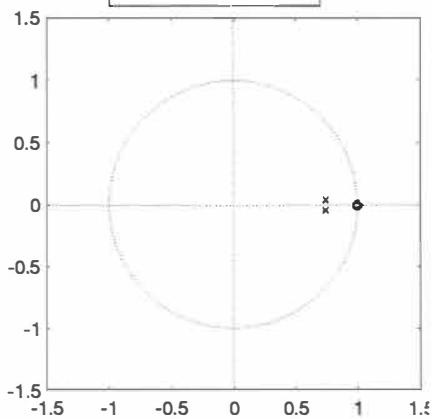
- (a) [2 points Each] Below are the pole-zero plots of the system functions,  $H(z)$ , of several discrete-time systems. Also, there are plots of impulse responses,  $h[n]$ , on the next page. For each pole-zero plot, enter the letters of the matching impulse response. There are more impulse response plots than pole-zero plots.



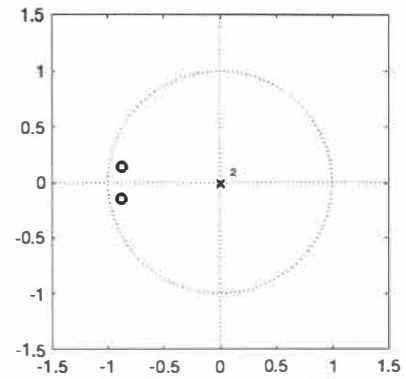
ANS =



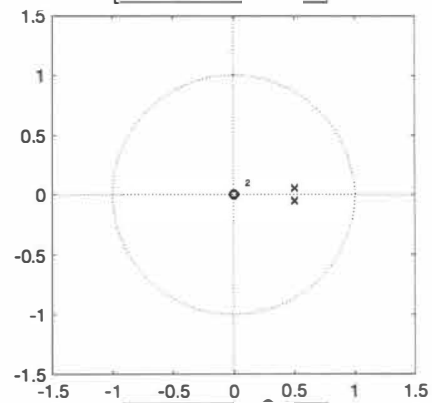
ANS =



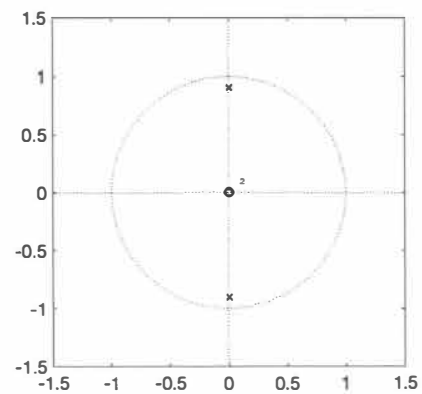
ANS =



ANS =

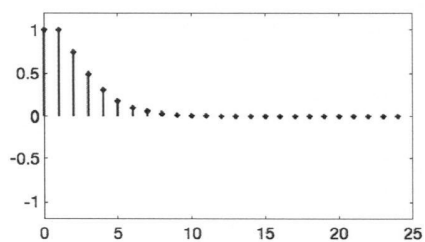


ANS =

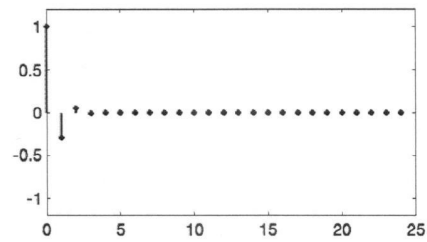


ANS =

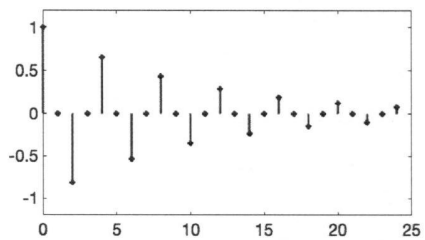




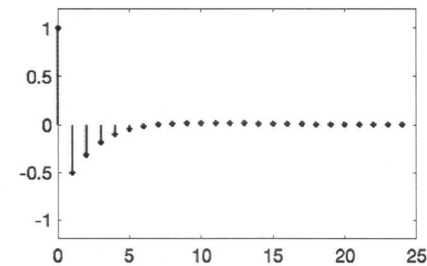
A



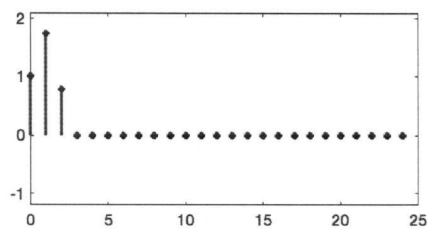
B



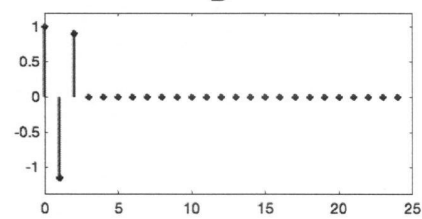
C



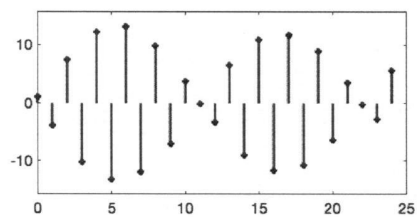
D



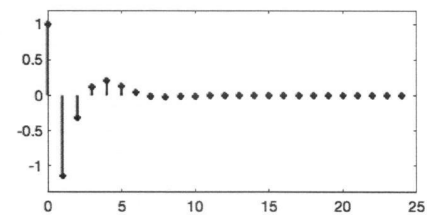
E



F



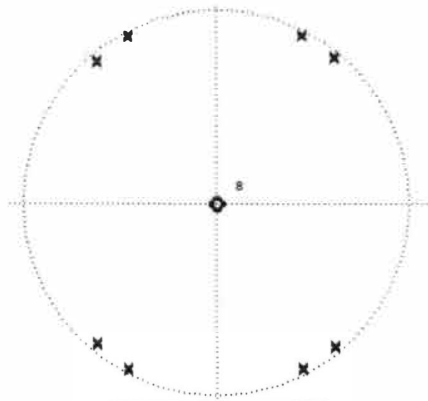
G



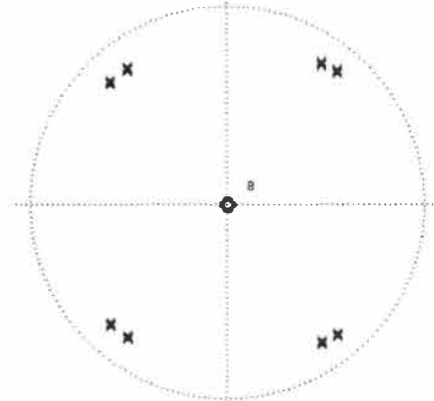
H

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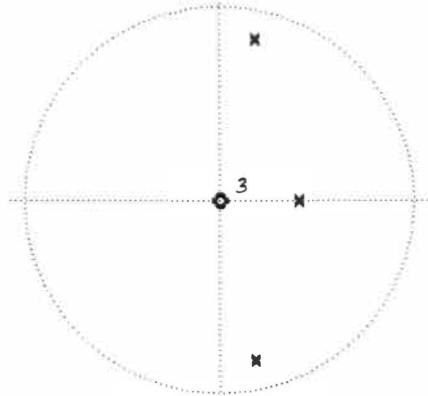
- (b) [2 points Each] Below are the pole-zero plots of the system functions,  $H(z)$ , of several discrete-time systems. Also, there are plots of frequency responses,  $H(e^{j\omega})$ , on the next page. For each pole-zero plot, enter the letters of the matching frequency response. There are more frequency response plots than pole-zero plots.



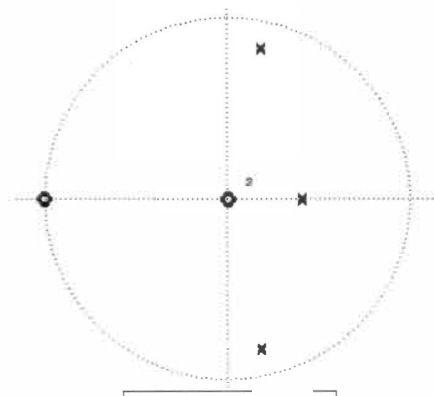
ANS =



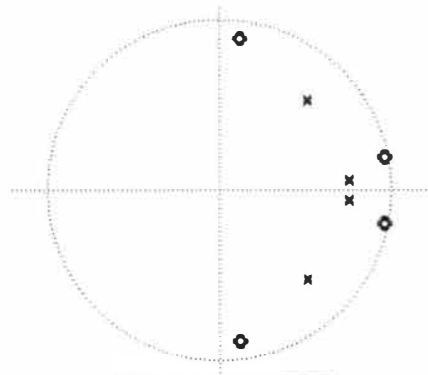
ANS =



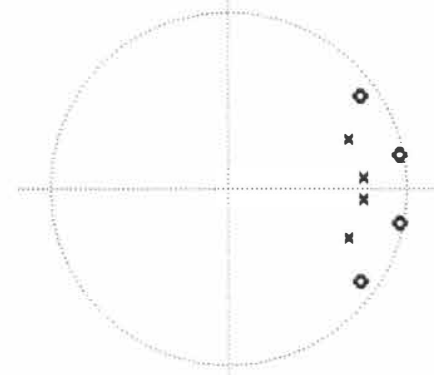
ANS =



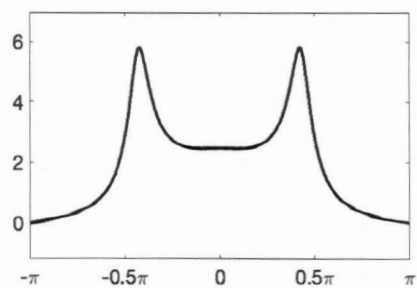
ANS =



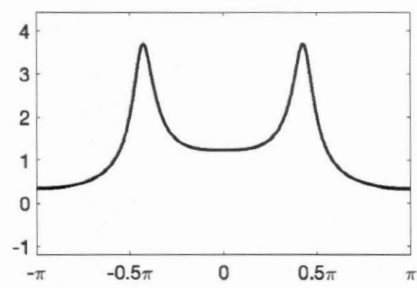
ANS =



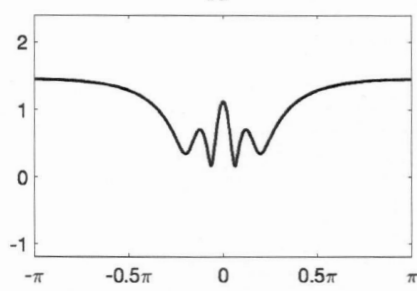
ANS =



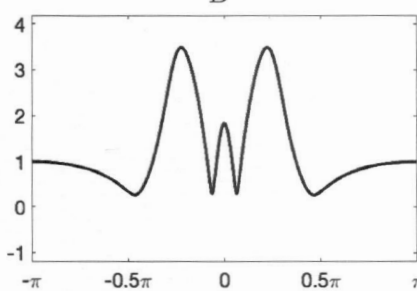
A



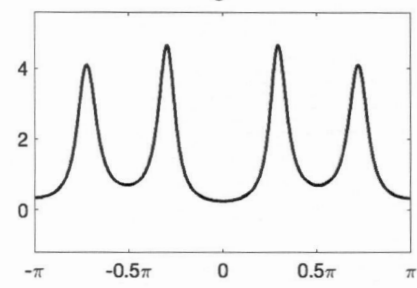
B



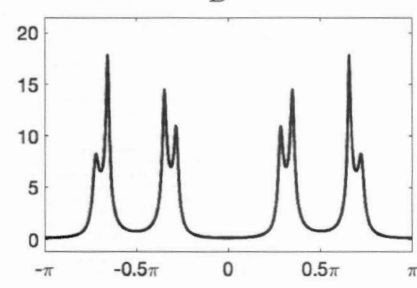
C



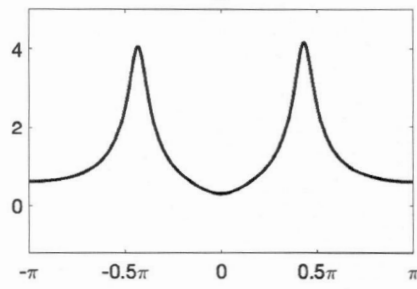
D



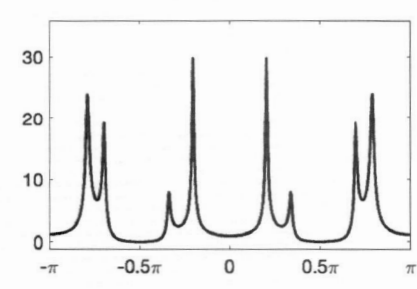
E



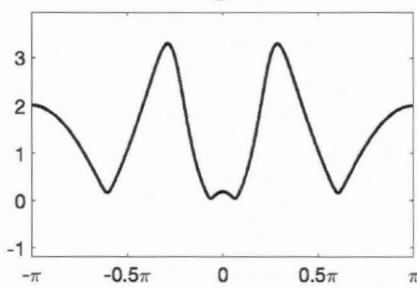
F



G



H



I

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**PROBLEM sp-24-FINAL.4:**

[3 points Each] Pick the correct frequency response characteristic and enter the number in the answer box. Each frequency response is used only once. There are more items on the right-hand list than on the left-hand list.

(a)  $h[n] = \left(-\frac{1}{2}\right)^n u[n]$

(b)  $H(z) = 1 + z^{-1} + z^{-2} + z^{-3} + z^{-4}$

(c) `yn = filter([1,1],1,xn)`

(d)  $h[n] = \sum_{k=0}^3 \delta[n-k]$

(e)  $y[n] = x[n-1] + 2x[n-3] + x[n-5]$

(f)  $y[n] = \frac{1}{2}y[n-1] + x[n]$

1.  $H(e^{j\hat{\omega}}) = \frac{1}{1-0.5e^{-j\hat{\omega}}}$

2.  $H(e^{j\hat{\omega}}) = \frac{1}{1+0.5e^{-j\hat{\omega}}}$

3.  $H(e^{j\hat{\omega}}) = 2e^{-j2\hat{\omega}}(0.5 + \cos(\hat{\omega}) + \cos(2\hat{\omega}))$

4.  $H(e^{j\hat{\omega}}) = \frac{1}{1-e^{-j0.5\hat{\omega}}}$

5.  $|H(e^{j\hat{\omega}})| = 2 \cos(\hat{\omega}/2)$

6.  $H(e^{j\hat{\omega}}) = \frac{\sin(2\hat{\omega})}{\sin(0.5\hat{\omega})} e^{-j1.5\hat{\omega}}$

7.  $\angle H(e^{j\hat{\omega}}) = -3\hat{\omega}$

8.  $|H(e^{j\hat{\omega}})| = 0.5 \cos(2\hat{\omega})$

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**PROBLEM sp-24-FINAL.5:**

The parts in this problem are independent from each other.

(a) [6 points] Two periodic sequences are given as follows:

$n$	...	0	1	2	3	4	5	6	7	8	9	10	11	12	...
$x_1[n]$	...	1	0.5	-0.5	-1	-0.5	0.5	1	0.5	-.5	-1	-0.5	0.5	1	...
$x_2[n]$	...	0	2	0	-2	0	2	0	-2	0	2	0	-2	0	...

Note that the two signals have different periodicity. Suppose that we add the two signals to have a new periodic signal  $x[n] = x_1[n] + x_2[n]$ . For the sequence  $x[n], n = 0, \dots, 11$ , determine the 12-point DFT sequence and write the values in the following table.

*Hint: Write  $x_1[n]$  and  $x_2[n]$  as simple sinusoids.*

$k$	0	1	2	3	4	5
$X[k]$						
$k$	6	7	8	9	10	11
$X[k]$						



- (b) [2 points] Suppose the DFT,  $X[k]$ , of a sequence  $x[n]$  shown below, is real. That is,  $X^*[k] = X[k]$  for  $k = 0, \dots, 7$ . Can the unknown values of  $x[n]$  be determined? If yes, give the missing values. If no, then justify your answer.

$$\{1, 7, ?, ?, 1, 6, 5, 7\}$$

circle one **YES** (give the missing values below) or **NO** (justify)

- (c) [1 point Each] Shown below are different outcomes that result from executing the following MATLABcode:

```
stem(abs(fft(ones(1,L),N)));
```

s Match each plot with the corresponding values for the variables  $N$  and  $L$  by writing a letter (A through I) in each answer box.



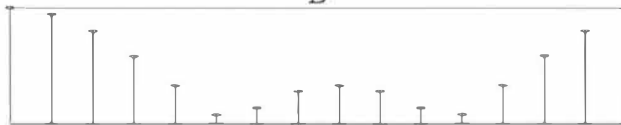
A

$L = 2, N = 16$  **ANS =**



B

$L = 2, N = 64$  **ANS =**



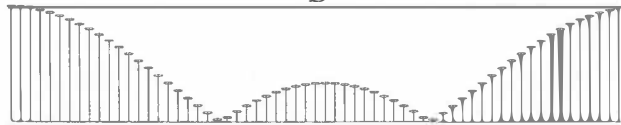
C

$L = 3, N = 16$  **ANS =**



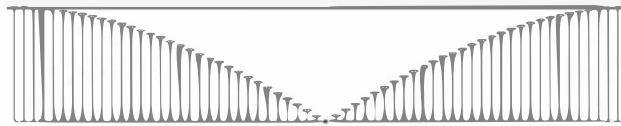
D

$L = 3, N = 64$  **ANS =**



E

$L = 8, N = 16$  **ANS =**



F

$L = 8, N = 64$  **ANS =**



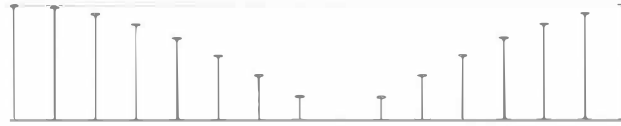
G

$L = 15, N = 16$  **ANS =**



H

$L = 15, N = 64$  **ANS =**



I

$L = 16, N = 16$  **ANS =**

**PROBLEM sp-24-FINAL.6:**

The parts in this problem are independent from each other.

- (a) [4 points] Simplify the following expression of  $x[n]$  with a single term.

$$x[n] = \sum_{k=-\infty}^{\infty} \left( \frac{\sin(0.7\pi k)}{\pi k} \right) \cdot \left( \frac{\sin(0.85\pi(n-k))}{\pi(n-k)} \right)$$

$x[n] =$

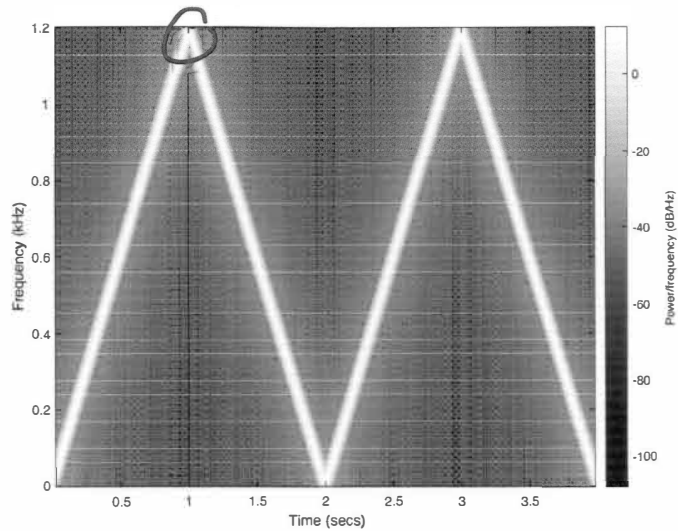
- (b) [4 points] Determine the fundamental frequency of the signal

$$x(t) = \cos(40\pi t) \cos(24\pi t) + \cos(60\pi t)$$

$f_0 =$  2 Hz

- (c) [4 points] Running the following MATLAB code produces the plot below, for a specific value of the parameter  $W$ .

```
fsamp = 2400;  
tmax = 4;  
tt = 0:(1/fsamp):tmax;  
xx = real((20+15*j)*exp(j*2*pi*W*(tt.^2)));  
spectrogram(xx,128,120,512,fsamp,'yaxis')
```



Note that the y-axis is in kHz, i.e., the highest frequency shown is 1200 Hz.

Determine the numerical value of the parameter  $W$  in the MATLAB code.

W =