Midterm 2

Tuesday, November 19, 2024 12:47 PM

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MID-TERM EXAM 2 Fall 2024

ECE 2040

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MAJOR: Computer Engineering

INSTRUCTIONS

This is an open book exam. You are allowed to use your calculators and MATLAB.

This is a take-home exam (due date: November 23, 2024, by 11:59 PM).

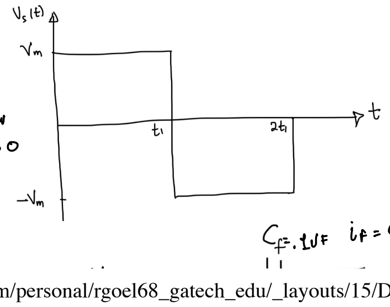
Upload a copy of your completed exam to canvas.

LATE SUBMISSIONS ARE NOT ALLOWED.

Please note that solutions similar to those of other students in the class will NOT be graded.

Problem 1 (20 points). Assume that the numerical values for the signal voltage in Fig. 1 are $V_m = 50$ mV and $t_1 = 1$s. This signal voltage is applied to the integrating-amplifier circuit shown in Fig. 2. The circuit parameters of the amplifier are $R_s = 100K\Omega$, $C_f = 0.1\mu$F, and $V_{cc} = 6$ V. The initial voltage on the capacitor is zero.
(a) Calculate $V_o(t)$.
(b) Plot $V_o(t)$ versus t.

$$V_s(t) = \begin{cases} 50e\text{-}3 & 0 \le t \le 1 \\ -50e3 & 1 \le t \le 2 \end{cases}$$



for integrator amplifier circuits $i_s = i_f \Rightarrow 0$ "tied to ground"

$C_f = .1uF$ $\quad i_f = C_f \frac{dV_o}{dt} = C_f \frac{\partial(0 - V_o)}{\partial t}$

$= -C_f \frac{\partial V_o}{}$

Fig. 1.

$$i_s = \frac{V_s - 0}{R_s} = \frac{V_s}{R_s}$$

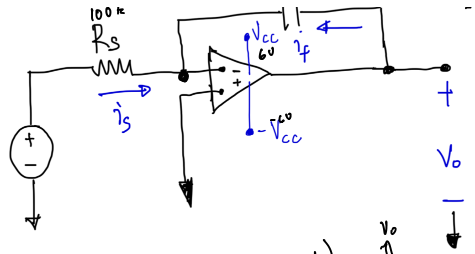


Fig. 2.

$$i_s = i_f$$

$$\Rightarrow \frac{V_s}{R_s} = -C_f \frac{\partial V_o}{\partial t}$$

$$\int_0^t \frac{V_s}{R_s} = -\int_0^t C_f \frac{\partial V_o}{\partial t}$$

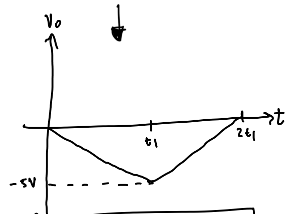
$$\Rightarrow -\frac{1}{C_f R_s} \int_0^t V_s = \int_0^t \frac{\partial V_o}{\partial t}$$

a) $V_o(t) = -\frac{1}{C_f R_s} \int_0^t V_s \, \partial t \Rightarrow$

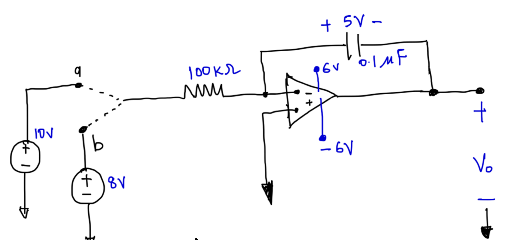
$$\frac{1}{C_f R_s} = 100$$

$$\begin{cases} \frac{-1}{C_f R_s}(50e{-3})\, t & 0 \le t \le 1 \\ \frac{1}{C_f R_s}(50e{-3}) + & 1 \le t \le 2 \end{cases} \quad \boxed{\begin{cases} -5t & 0 \le t \le t_1 \\ 5t - 10 & t_1 \le t \le t_2 \end{cases}}$$

b)



Problem 2 (20 points). At the instant the switch makes contact with the terminal **a** in Fig. 3, the voltage on the 0.1μF capacitor is 5 V. The switch remains at terminals **a** for 9 ms and then moves instantaneously to terminal **b**. How many milliseconds after making contact with the terminal **a** does the operational amplifier saturate?



$$V_o(t) = \frac{-1}{C_f R_s} \int_0^t V_{in} \, \partial t \Rightarrow \frac{-1}{C_f R_s} \int 10 \, \partial t$$

Initial voltage on capacitor

Fig. 3.

$V_o(9) = -1000(.009) + 5$

$= -4$ volts

① $\frac{-10}{.01}(t) + C$

$V_o(t) = -1000t + 5 = -6$

$-1000t = 11$

$t = \frac{11}{1000} = .011$

$t = 11$ ms

Does not saturate @ 9 ms → saturates @ 11 ms

When switch to B

② $V_o(t) = \frac{1}{C_f R_s} \int_0^t 8 \, \partial t$

$V_o = \frac{-8}{.01} t - 4$

$+6 + 4 = -800t \Rightarrow t = 2.5\text{ms} + 9\text{ ms} = \boxed{11.5 \text{ ms}}$ till saturation

Problem 3 (20 points). A sequential switching circuit is shown in Fig. 4. The two switches shown in the circuit have been closed for a long time. At $t = 0$, switch 1 is opened. Then 35 ms later, switch 2 is opened. Assume that $i_a = i_o$, $R_1 = 10\,\Omega$, $R_2 = 10\,\Omega$, $R = 10\,\Omega$, $V_{cc} = 15$ V, $R_{load} = 40\,\Omega$. Note that Q_1 is an *npn* bipolar junction transistor.

① Solve for V_o

$-\frac{V_a}{R_L} = \frac{V_o}{R_L} - \frac{V_{cc}}{R}$ $\quad V_a = -\frac{V_o R}{R_L} + V_{cc}$

(a) Find $i_L(t)$ for $0 \le t \le 35$ ms.
(b) Find $i_L(t)$ for $t \ge 35$ ms.
(c) What percentage of the initial energy stored in the 150 mH inductor is dissipated in the $18\,\Omega$ resistor?

① $i_a = i_o$

$i_o = \frac{V_o}{R_{Load}} = i_a$

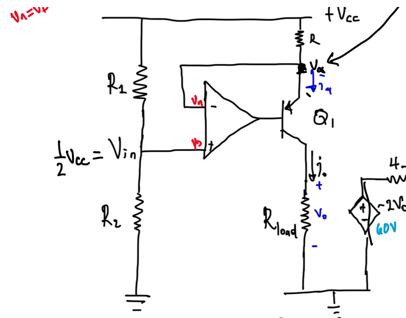
$i_a = \frac{V_{cc} - V_a}{R}$

$\frac{V_{cc} - V_a}{R} = \frac{V_o}{R_{Load}}$

$\frac{-V_o R}{R_{Load}} + V_{cc} = V_a$

⇒ since $V_n = V_p$

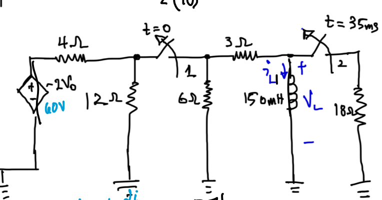
$V_p = \frac{1}{2} V_{cc} = V_{cc} - \frac{V_o R}{R_{Load}}$

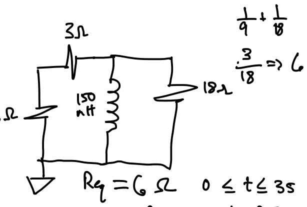


$\frac{1}{2}V_{cc} = \frac{V_0 R}{R_{Load}}$

$\Rightarrow \frac{V_{cc} R_{Load}}{2R} = V_0$

$\Rightarrow V_0 = \frac{(15)(40)}{2(10)} = \boxed{30V}$





$\frac{1}{9} + \frac{1}{18}$

$\frac{3}{18} \Rightarrow 6\Omega$

$R_{eq} = 6\Omega \quad 0 \le t \le 35$

$R_{eq} = 9 \quad = t \ge 35$

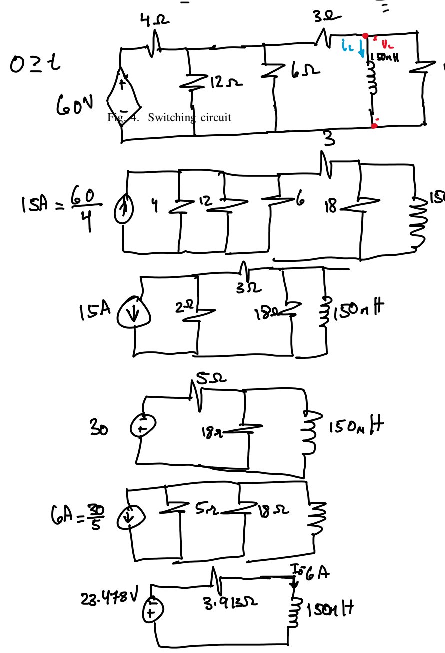


Fig. 4. Switching circuit

$V = L\frac{di}{dt}$

$I_0 = \frac{60}{5.71}$ for $t \le 0$

$i_L(t) = I_0 e^{-\frac{R}{L}t}$

from $0 \le t \le 35ms$

a) $i_L(t) = 6\, e^{-\frac{6}{150e-3}t} \Rightarrow \boxed{6e^{-40t}}$

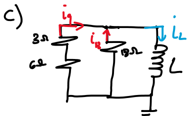
b) I_0 @ $t = 35$ ms

$i_L(35ms) = 6\, e^{-40(35e-3)} \approx 1.47958A$

$\boxed{i_L(t) = -1.47958\, e^{-60(t-0.035)}}$

$E_0 = \frac{1}{2}(150e-3)(6)^2 = 2.7j$

c)



$i_9 = 2\, i_{18}$

$i_L = i_9 + i_{18} \Rightarrow 3\, i_{18}$

$i_{18}(t) = \frac{1}{3} i_L(t) = 2\, e^{-40t}$

$P = I^2 R \Rightarrow (2\, e^{-40t})^2 (18\Omega)$

$(18\Omega)\int_0^{35} 4\, e^{-80t}\, dt \Rightarrow .84527j$

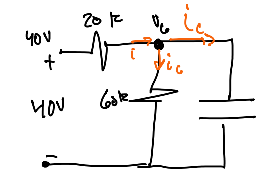
$\frac{P_{18}}{E_0} = \frac{.84527}{2.7} = \boxed{31.306\%}$

Problem 4 (20 points). The circuit shown in Fig. 5 stems from the concatenation of a realistic (non-ideal) non-inverting operational amplifier model and an RC circuit. The switch in the circuit has remained in position 1 for a long time. At $t = 0$, the switch moves to the position 2. If $R_s = 1000\,\Omega$, $R_f = 3000\,\Omega$, $R_g = 1000\,\Omega$, $R_i = 75\,\text{M}\Omega$, $R_o = 50\,\Omega$, $A = 10^5$ and $V_g = 10\text{V}$, find the following:
(a) $V_0(t)$ for $t \ge 0$
(b) $i_0(t)$ for $t \ge 0^+$

assuming it Behaves like an ideal op amp.

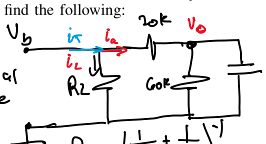
$V_b = \left(1 + \frac{R_f}{R_s}\right)V_g$

$= 4\, V_g = 40\,V$



$\left(\frac{20k}{20k+60k}\right)40 = \frac{1}{4}(40) = 30V$ @ V_b across 20k

initial state



KCL @ $i_{20k} + i_{60k} - i_c = 0$

$R_{eq} = \left(\frac{1}{R_L} + \frac{1}{80}\right)^{-1}$

$V_0 = \left(\frac{20k}{20k+60k}\right)V_b$

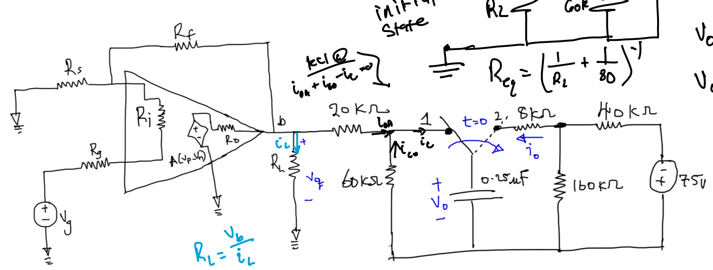
$V_0 = \frac{V_b}{4}$

$R_{eq} = \left(\frac{80}{80R_L} + \frac{R_L}{80R_L}\right)^{-1}$

$\left(\frac{80+R_L}{80R_L}\right)^{-1}$

$\boxed{R_{eq} = \frac{80R_L}{80+R_L}}$

$V_c = V_{cc} - (V_{cc} - V_0)e^{-t/\tau}$



$R_L = \frac{V_b}{i_L}$

Fig. 5.

a) $\sum V = 0 \Rightarrow V_0 - 40k\left((.25e{-6})\frac{dV}{dt}\right) + 60 = 0$

$V_0 + 60 - .01\frac{dV}{dt} = 0$

$\frac{dV}{dt} - 100V_0 = 6000$

MOIF $e^{\int P\,dx} \Rightarrow e^{-100t}$

$\int e^{-100t}\frac{dV}{dt} - 100e^{-100t}V_0 = \int 6000e^{-100t}$

$u = -100t$

$du = -100$

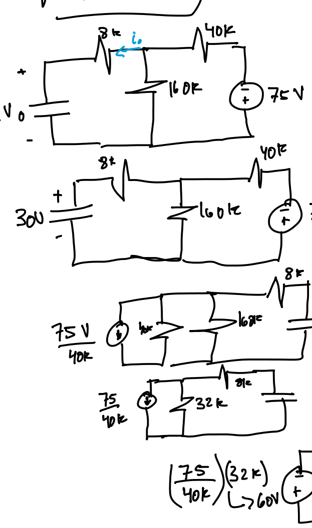
$\Rightarrow e^{-100t}V_0 = (6000)\left(\frac{-1}{100}\right)\int e^u\, du$

$e^{-100t}V_0 = -60\, e^{-100t} + C$

$V_0 = -60 + Ce^{100t}$

$30 = -60 + Ce^0 \Rightarrow C = 90$

$\boxed{V_0 = 90e^{-100t} - 60}$



$\left(\frac{75}{40k}\right)(32k) \to 60V$

$\tau = 40(.25e{-6})$

$\Rightarrow$

b) $i_0(t) = \frac{V_0(t)}{R_{th}} = \boxed{\frac{90\, e^{-100t} - 60}{40k}}$

Problem 5 **(20 points).** A custom circuit for strain measurement in solids can be found in Fig. 6. For this circuit, do the following:

(a) (**15 points**) Prove that the relationship between the input and output voltages can be expressed as:

$$V_0 = \frac{R_f\,(2\Delta R)}{R^2 - (2\Delta R)^2} V_{\text{ref}} \tag{1}$$

(b) (**5 points**) If $R = 120\,\Omega$ with $\delta = \frac{\Delta R}{R} = 0.01$, and the power supplies to the op-amp are ± 15 V with the reference voltage V_{ref} taken from the positive power supply. Calculate the value of R_f if the output voltage is 5 V.

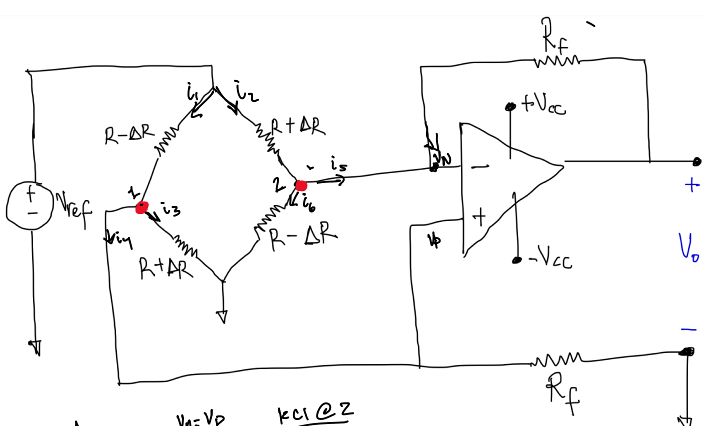


Fig. 6.

KCL @ 1

$-i_1 + i_3 + i_4 = 0$

$$\frac{-(V_{Ref} - V_p)}{R-\Delta R} + \frac{V_p}{R+\Delta R} + \frac{V_p}{R_f}$$

$V_n = V_p$

KCL @ 2

$-i_2 + i_5 + i_6 = 0$

$$\frac{-(V_{Ref} - V_n)}{R+\Delta R} + \frac{-(V_0 - V_n)}{R_f} + \frac{V_n}{R-\Delta R} = 0$$

$$R_f(R+\Delta R)\ \frac{V_p - V_{Ref}}{R-\Delta R} + \frac{V_p}{R+\Delta R}\,(R-\Delta R)R_f + \frac{V_p}{R_f}\,(R+\Delta R)(R-\Delta R)$$

$$\frac{(V_p - V_{Ref})(R+\Delta R)R_f + V_p(R-\Delta R)(R_f) + V_p(R+\Delta R)(R-\Delta R)}{(R-\Delta R)(R+\Delta R)R_f}$$

$$\frac{V_p(R+\Delta R)(R_f) - V_{ref}(R+\Delta R)(R_f) + V_p(R-\Delta R)(R_f) + V_p(R+\Delta R)(R-\Delta R)}{(R-\Delta R)(R+\Delta R)R_f}$$

$$V_p\left[(R+\Delta R)(R_f) + (R-\Delta R)(R_f) + (R+\Delta R)(R-\Delta R)\right] - V_{ref}(R+\Delta R)(R_f) = 0$$

(1) $$V_p = \frac{V_{ref}(R+\Delta R)(R_f)}{(R+\Delta R)(R_f) + (R-\Delta R)(R_f) + (R+\Delta R)(R-\Delta R)}$$

Z

$$(R_f)(R+\Delta R)\,\frac{V_n}{R-\Delta R} + \frac{V_n - V_{ref}}{R+\Delta R}\,(R-\Delta R)R_f + \frac{V_n - V_o}{R_f}\,(R+\Delta R)(R-\Delta R) = 0$$

$$V_n(R+\Delta R)R_f + (V_n - V_{ref})(R-\Delta R)R_f + (V_n - V_o)(R+\Delta R)(R-\Delta R) = 0$$

$$V_n(R+\Delta R)R_f + V_n(R-\Delta R)R_f - V_{ref}(R-\Delta R)R_f + V_n(R+\Delta R)(R-\Delta R) - V_o(R+\Delta R)(R-\Delta R) = 0$$

$$V_n\left[(R+\Delta R)R_f + (R-\Delta R)R_f + (R+\Delta R)(R-\Delta R)\right] = V_{ref}(R-\Delta R)R_f + V_o(R+\Delta R)(R-\Delta R)$$

(2) $$V_n = \frac{V_{ref}(R-\Delta R)R_f + V_o(R+\Delta R)(R-\Delta R)}{(R+\Delta R)R_f + (R-\Delta R)R_f + (R+\Delta R)(R-\Delta R)}$$ $V_P = V_n$

$$\frac{V_{ref}(R+\Delta R)(R_f)}{(R+\Delta R)(R_f) + (R-\Delta R)(R_f) + (R+\Delta R)(R-\Delta R)} = \frac{V_{ref}(R-\Delta R)R_f + V_o(R+\Delta R)(R-\Delta R)}{(R+\Delta R)R_f + (R-\Delta R)R_f + (R+\Delta R)(R-\Delta R)}$$

$$\frac{(V_{ref})(R+\Delta R)(R_f)\left[\cancel{(R+\Delta R)R_f + (R-\Delta R)R_f + (R+\Delta R)(R-\Delta R)}\right]}{\cancel{(R+\Delta R)R_f + (R-\Delta R)(R_f) + (R+\Delta R)(R-\Delta R)}} - V_{ref}(R-\Delta R)R_f = V_o(R+\Delta R)(R-\Delta R)$$

Bless

$$\frac{V_{ref}(R+\Delta R)(R_f) - V_{ref}(R-\Delta R)R_f}{(R+\Delta R)(R-\Delta R)} = V_o$$

* could not derive 2 in denominator

$$V_o = \frac{V_{ref}R_f\left[(R+\Delta R) - (R-\Delta R)\right]}{R^2 - \Delta R^2} \Rightarrow \boxed{\frac{V_{ref}R_f(2\Delta R)}{R^2 - \Delta R^2}}$$

b) $V_o = 5$ $\frac{\Delta R}{R} = .01$ $R = 120$, $\Delta R = 1.2$

$$\frac{(V_o)(R^2 - \Delta R^2)}{V_{ref}(2\Delta R)} = R_f$$

$$\frac{(5)(120^2 - 1.2^2)}{15(2.4)} = \boxed{R_f = 1,999.8\,\Omega}$$