T3 a) $A = |\alpha\rangle\langle\alpha| - i\sqrt{2}\,|\alpha\rangle\langle\beta| + i\sqrt{2}\,|\beta\rangle\langle\alpha|$

Since $|\alpha\rangle$ and $|\beta\rangle$ form orthonormal Basis for the

$|\alpha\rangle = \begin{bmatrix}1\\0\end{bmatrix}$ and $|\beta\rangle = \begin{bmatrix}0\\1\end{bmatrix}$

$\begin{bmatrix}1\\0\end{bmatrix}[1\ 0] - i\sqrt{2}\begin{bmatrix}1\\0\end{bmatrix}[0\ 1] + i\sqrt{2}\begin{bmatrix}0\\1\end{bmatrix}[1\ 0]$

$\begin{bmatrix}1&0\\0&0\end{bmatrix} - -i\sqrt{2}\begin{bmatrix}0&1\\0&0\end{bmatrix} + i\sqrt{2}\begin{bmatrix}0&0\\1&0\end{bmatrix} = \begin{bmatrix}1&-i\sqrt{2}\\i\sqrt{2}&0\end{bmatrix}$

arbitrary $|\psi\rangle$

$A|\psi\rangle = \lambda|\psi\rangle$ where λ is eigenvector

$A|\psi\rangle - \lambda|\psi\rangle = 0$

$(A - \lambda I_n)|\psi\rangle = 0$ $\quad \det|A - \lambda I_n| = 0$

$\det\left(\begin{bmatrix}1-\lambda & -i\sqrt{2}\\ i\sqrt{2} & -\lambda\end{bmatrix}\right) = 0$ $\quad (-\lambda + \lambda^2) - (-i\sqrt{2})(i\sqrt{2}) = 0$ $\quad (\lambda - 2)(\lambda + 1) = 0$

$\lambda^2 - \lambda - (2) = 0$ $\quad \lambda^2 - \lambda - 2 = 0$ $\quad \boxed{\lambda = +2, -1}$

b) $\langle A\rangle_\psi = \langle\psi|A|\psi\rangle$

$[c^*, c^*]\begin{bmatrix}1 & -i\sqrt{2}\\ i\sqrt{2} & 0\end{bmatrix}\begin{bmatrix}c\\c\end{bmatrix} = [c^*\ c^*]\begin{bmatrix}c - i\sqrt{2}c\\ i\sqrt{2}c\end{bmatrix} = c^*(c - i\sqrt{2}c) + c^*(i\sqrt{2}c)$

But since $\langle\psi|\psi\rangle = 1$, Then $[c^*, c^*]\begin{bmatrix}c\\c\end{bmatrix} = 2c^*c = 2c^2 = 1$

$c = 1/\sqrt{2}$

$\Rightarrow \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}} - i\right) + \frac{1}{\sqrt{2}}(i)$

$\frac{1}{2} - \frac{i}{\sqrt{2}} + \frac{i}{\sqrt{2}} = \boxed{1/2} = \langle A\rangle_\psi$

c) $\text{Prob}(A|\psi\rangle = 2) + \text{Prob}(A|\psi\rangle = -1) = 1$

$+2\,\text{Prob}(A|\psi\rangle = 2) - 1\,\text{Prob}(A|\psi\rangle = -1) = 1/2$

$$\left[\begin{array}{cc|c}1 & 1 & 1\\ 2 & -1 & 1/2\end{array}\right]$$

$\text{Prob}(A|\psi\rangle = 2) = 1/2$

$\text{Prob}(A|\psi\rangle = -1) = 1/2$

T4 $|\psi\rangle = \frac{1}{\sqrt{2}}\{|+1\rangle - i|-1\rangle\}$ $|\pm 1\rangle$ eigenvectors of S_z eigenvalue $\pm\hbar$

~~$\Delta S_n = \sqrt{\langle S_n^2\rangle - \langle S_n\rangle^2}$ $\langle S_n\rangle = \pm\hbar n_z$~~

~~$\langle S_n^2\rangle = \frac{\hbar^2}{2}(5-3n_z^2)$~~

~~$\sqrt{\frac{\hbar^2}{2}(5-3n_z^2) - (\pm\hbar n_z)^2}$~~

~~$\frac{\hbar^2}{2}5 - \frac{3}{2}\hbar^2 n_z^2 - \hbar^2 n_z^2 \Rightarrow \sqrt{\frac{5}{2}\hbar^2 - \frac{5}{2}\hbar^2 n_z^2}$~~

~~$\Rightarrow \boxed{\hbar\sqrt{\frac{5}{2}(1-n_z^2)}}$~~

~~For ΔS_n to Be smallest, we say $\Delta S_n = 0$~~

~~$\hbar\sqrt{5/2(1-n_z^2)} = 0$ iff $n_z^2 = 1 \Rightarrow n_z = \pm 1$~~

~~Thus vector $n = \langle c_1\begin{pmatrix}1\\0\\0\end{pmatrix}, c_2\begin{pmatrix}0\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\\pm 1\end{pmatrix}\rangle$ where c_1, c_2 are any coefficients.~~

$\langle\psi| = \frac{1}{\sqrt{2}}\{\langle +1| + i\langle -1|\}$ $|\psi\rangle = \frac{1}{\sqrt{2}}\{|+1\rangle - i|-1\rangle\}$

$\langle\psi|S_n^2|\psi\rangle = \frac{1}{2}\{\underbrace{\langle +1|S_n^2|+1\rangle}_{\frac{\hbar^2}{2}(5-3n_z^2)} + i\underbrace{\langle -1|S_n^2|+1\rangle}_{\approx i\langle -1|\beta i S_-^2|+1\rangle} - i\underbrace{\langle +1|S_n^2|-1\rangle}_{\approx -i\langle +1|\alpha i S_+^2|-1\rangle} + \underbrace{\langle -1|S_n^2|-1\rangle}_{\frac{\hbar^2}{2}(5-3n_z^2)}\}$

Then $\langle\psi|S_n^2|\psi\rangle = \frac{1}{2}\{\hbar^2(5-3n_z^2) - \beta\langle -1|S_-\underbrace{S_-|+1\rangle}_{\hbar\sqrt{j(j+1)-m(m-1)}|0\rangle} + \alpha\langle +1|S_+\underbrace{S_+|-1\rangle}_{\hbar\sqrt{j(j+1)-m(m+1)}|0\rangle}\}$ Consider proportionals αi

$(\ldots)^2|-1\rangle$ $(\ldots)^2|+1\rangle$

$= \frac{1}{2}\{\hbar^2(5-3n_z^2) - \hbar^2(j(j+1)-m(m-1))\beta + \hbar^2(j(j+1)-m(m+1))\alpha\}$ $j=2$ $m=\pm 1$

$\Rightarrow \frac{1}{2}\{\hbar^2(5-3n_z^2) + 4\hbar^2(\alpha-\beta)\} \Rightarrow \boxed{\frac{\hbar^2}{2}(5-3n_z^2) + 2\hbar^2(\alpha-\beta) = \langle\psi|S_n^2|\psi\rangle}$

$\langle\psi|S_n|\psi\rangle = \frac{1}{2}\{\langle +1|S_n|+1\rangle + i\underbrace{\langle -1|S_n|+1\rangle}_{} - i\underbrace{\langle +1|S_n|-1\rangle}_{|0\rangle} + \langle -1|S_n|-1\rangle\}$

$\hbar n_z + i\langle -1|\frac{1}{2}(n_+S_- + n_-S_+) + \underbrace{n_zS_z|+1\rangle}_{|+2\rangle \Rightarrow 0}\} = 0$ Both -1 & $+1$ don't matter

$\Rightarrow \langle\psi|S_n|\psi\rangle = 0$

$\Delta S_n = \sqrt{\langle S_n^2\rangle - \langle S_n\rangle^2} \Rightarrow \boxed{\hbar\sqrt{\frac{1}{2}(5-3n_z^2) + 2(\alpha-\beta)}}$ for some α, β imaginary proportionality consts.

When $n = \langle 0, 0, \pm\sqrt{5/3}\rangle$

The uncertainty is minimized to $\boxed{\hbar\sqrt{2(\alpha-\beta)}}$ where $S_n^2|-1\rangle \approx \alpha i S_+^2|-1\rangle$

$S_n^2|1\rangle \approx \beta i S_-^2|+1\rangle$