$$\begin{bmatrix}1 & 0 & 0\end{bmatrix}\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}\begin{bmatrix}0\\1\\0\end{bmatrix}$$

Q 28

$$S_n = \begin{bmatrix} \langle 1z|S_n|1z\rangle & \langle 1z|S_n|0z\rangle & \langle 1z|S_n|-1z\rangle \\ \langle 0z|S_n|1z\rangle & \langle 0z|S_n|0z\rangle & \langle 0z|S_n|-1z\rangle \\ \langle -1z|S_n|1z\rangle & \langle -1z|S_n|0z\rangle & \langle -1z|S_n|-1z\rangle \end{bmatrix} \Rightarrow \hbar\begin{bmatrix} n_z & n_- & 0 \\ n_+ & 0 & n_- \\ 0 & n_+ & -n_z \end{bmatrix}$$

$$S_z = \hbar\begin{bmatrix}1 & 0 & 0\\0 & 0 & 0\\0 & 0 & -1\end{bmatrix}$$

likewise for $n = x$: $S_x = \frac{\hbar}{\sqrt{2}}\begin{bmatrix}0 & 1 & 0\\1 & 0 & 1\\0 & 1 & 0\end{bmatrix}$

$[S_x, S_y] = i\hbar S_z$

$$S_y = \frac{\hbar}{\sqrt{2}}\begin{bmatrix}0 & -i & 0\\i & 0 & i\\0 & i & 0\end{bmatrix}$$

$$[S_x,S_y] = \frac{\hbar^2}{2}\begin{bmatrix}0&1&0\\1&0&1\\0&1&0\end{bmatrix}\begin{bmatrix}0&-i&0\\i&0&-i\\0&i&0\end{bmatrix} - \frac{\hbar^2}{2}\begin{bmatrix}0&-i&0\\i&0&-i\\0&i&0\end{bmatrix}\begin{bmatrix}0&1&0\\1&0&1\\0&1&0\end{bmatrix} = \frac{\hbar^2}{2}\begin{bmatrix}i&0&-i\\0&0&0\\i&0&-i\end{bmatrix} - \frac{\hbar^2}{2}\begin{bmatrix}-i&0&-i\\0&0&0\\i&0&i\end{bmatrix}$$

$$= \frac{\hbar^2}{2}\begin{bmatrix}2i&0&0\\0&0&0\\0&0&-2i\end{bmatrix} \Rightarrow i\hbar^2\begin{bmatrix}1&0&0\\0&0&0\\0&0&-1\end{bmatrix} = i\hbar S_z$$ ✓

Q29 Problem 26 yields $\langle J_n\rangle_{mz} = \hbar m n_z$; $\langle J_n^2\rangle_{mz} = (\hbar m n_z)^2 + \frac{\hbar^2}{2}[j(j+1)-m^2](1-n_z^2)$

a) ~~Since $\langle J_n\rangle_{mn'} = \hbar m n'$ $\langle J_n^2\rangle_{mn'} = \hbar^2 m^2 + \frac{\hbar^2}{2}(j[j+1]-m^2)(1-n'^2)$~~

$\langle J_n\rangle_{mn'} = \langle j,m;n'|\{\frac{1}{2}(n_+J_- + n_-J_+) + n_zJ_z\}|j,m;n'\rangle$

° when $J_{+/-}$ acts on $|j,m;n'\rangle$ it results in $m = \pm 1 + m \Rightarrow$

kets don't match $\Rightarrow \langle jm;n'|jmn'\rangle = 0$ for $m \neq m\pm 1$

$\langle J_n\rangle_{mn'} = \langle j,m;n'|\{n_zJ_z\}|j,m;n'\rangle \Rightarrow \boxed{\hbar m(n\cdot n')}$

$\langle J_n^2\rangle_{mn'} = (-1)^2\langle J_n^2\rangle_{-mn'} = \langle J_n^2\rangle_{-m,n'} = \langle j;m;n'|\{n_z^2J_z + \frac{1}{2}(1-n_z^2)(J^2-J_z^2)|\cdot\cdot\rangle$

$= \hbar^2 m^2(n\cdot n')^2 + \frac{\hbar^2}{2}[j(j+1)-m^2](1-(n\cdot n')^2)$

b) for $\langle S_n\rangle_{n'} = \hbar m(n\cdot n')$ since $m = \frac{1}{2}$ (spin) $\langle S_n\rangle_{n'} = \frac{\hbar}{2}(n\cdot n')$

$\langle S_n^2\rangle_{n'} = \frac{\hbar^2}{4}(n\cdot n')^2 + \frac{\hbar^2}{2}\left(\frac{1}{2}\left(\frac{3}{2}\right) - \frac{1}{4}\right)(1-(n\cdot n')^2)$

$= \frac{\hbar^2}{4}(n\cdot n')^2 + \frac{\hbar^2}{4} - \frac{\hbar^2}{4}(n\cdot n')^2 = \hbar^2/4 = (\hbar/2)^2$

c) $\langle S_n\rangle_{n'} = \hbar m(n\cdot n') = \boxed{\hbar(n\cdot n')}$ for $m = 1$ (spin 1)

$\langle S_n^2\rangle_{n'} = (-1)^2\langle S_n^2\rangle_{-1,n} = \hbar^2(n\cdot n')^2 + \frac{\hbar^2}{2}(1(1+1)-1)(1-(n\cdot n')^2)$

$= \hbar^2(n\cdot n')^2 + \frac{\hbar^2}{2} - \frac{\hbar^2}{2}(n\cdot n')^2 \Rightarrow \boxed{\frac{\hbar^2}{2}(1+(n\cdot n')^2)}$

Q30 $H = \omega n \cdot \hat{S}$ $\quad n = \frac{1}{\sqrt{2}}(x+z)$

@ $t=0$ $\quad |\psi(0)\rangle = \frac{1}{\sqrt{2}}(|+1\rangle - |-1\rangle)$

$P(t) = |\langle\psi(0)|\psi(t)\rangle|^2 \qquad \psi(t) = \hat{T}(t,0)|\psi(0)\rangle = 1 - \frac{it}{\hbar}\hat{H} - \frac{t^2}{2\hbar^2}\hat{H}^2 + \dots |\psi(0)\rangle$

$P(t) = |1 - \frac{it}{\hbar}\langle H\rangle - \frac{t^2}{2\hbar^2}\langle H^2\rangle \dots|^2 \qquad \langle H\rangle = \langle \omega(\frac{1}{\sqrt{2}}(x+z))\cdot S\rangle$

$\Rightarrow P(t) = 1 - \left(\frac{t\,\Delta H}{\hbar}\right)^2 \quad \Delta H = \sqrt{\langle H^2\rangle - \langle H\rangle^2} \qquad \langle H\rangle = \omega\langle S_n\rangle \Rightarrow \langle H\rangle = \frac{\omega}{\sqrt{2}}\langle S_x + 0\rangle$

$= \omega\hbar/\sqrt{2}\,(n_x)$

$\langle H^2\rangle = \omega^2\langle S_n^2\rangle = \frac{\omega^2\hbar}{2}(1 + n_x^2)$

$\sqrt{\frac{\omega^2}{2}\hbar^2 + \frac{\omega^2}{2}\hbar^2 n_x^2 - \frac{\omega^2\hbar^2}{2}n_x^2} = \sqrt{\omega^2\hbar^2/2} = \frac{\omega\hbar}{\sqrt{2}}$

$P(t) = 1 - \frac{t^2\omega^2\hbar^2}{2\hbar} \Rightarrow \boxed{1 - \frac{t^2\omega^2\hbar}{2}}$

Q31 $H = \begin{pmatrix} 1 & i\sqrt{3} \\ -i\sqrt{3} & -1 \end{pmatrix} \qquad (1-\lambda)(-1-\lambda) - (-i\sqrt{3})(i\sqrt{3}) = P(\lambda)$

a) $-1 + \lambda - \lambda + \lambda^2 - (-i^2(3)) = P(\lambda)$

$\lambda^2 - 1 - 3 = P(\lambda) \qquad \lambda = \pm 2$

$\hat{A}|\pm n\rangle = \pm 2|\pm n\rangle \qquad \sum e^{-it\lambda/\hbar}|\lambda\rangle\langle\lambda|$

$|+n\rangle = \begin{pmatrix} 1-2 & i\sqrt{3} \\ -i\sqrt{3} & -1-2 \end{pmatrix} = \left(\begin{array}{cc|c} -1 & i\sqrt{3} & 0 \\ -i\sqrt{3} & -3 & 0 \end{array}\right) = \begin{pmatrix} i\sqrt{3} \\ 1 \end{pmatrix}(-i\sqrt{3} \;\; 1) = \begin{bmatrix} 3 & i\sqrt{3} \\ -i\sqrt{3} & 1 \end{bmatrix}$

$|-n\rangle = \begin{pmatrix} 1+2 & i\sqrt{3} \\ -i\sqrt{3} & -1+2 \end{pmatrix} = \left(\begin{array}{cc|c} 3 & i\sqrt{3} & 0 \\ -i\sqrt{3} & 1 & 0 \end{array}\right) = \begin{pmatrix} 1 \\ i\sqrt{3} \end{pmatrix}(1 \;\; -i\sqrt{3}) = \begin{bmatrix} 1 & -i\sqrt{3} \\ i\sqrt{3} & 3 \end{bmatrix}$

$\hat{T}(t) = e^{-it(2)/\hbar}|n\rangle\langle n| + e^{it(2)/\hbar}|-n\rangle\langle n| = |n\rangle\langle n| + e^{4it/\hbar}|-n\rangle\langle -n|$

$= e^{-it(2)\hbar}\begin{bmatrix} 3 & i\sqrt{3} \\ -i\sqrt{3} & 1 \end{bmatrix} + e^{it(2)/\hbar}\begin{bmatrix} 1 & -i\sqrt{3} \\ i\sqrt{3} & 3 \end{bmatrix} = \begin{bmatrix} 3 & i\sqrt{3} \\ -i\sqrt{3} & 1 \end{bmatrix} + e^{4it/\hbar}\begin{bmatrix} 1 & -i\sqrt{3} \\ i\sqrt{3} & 3 \end{bmatrix}$

b) $A_t = T^\dagger(t) A T(t) \qquad T^\dagger = |n\rangle\langle n| + e^{-4it/\hbar}|-n\rangle\langle -n|$

$A_t = (|n\rangle\langle n| + e^{-4it/\hbar}|-n\rangle\langle -n|)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}(|n\rangle\langle n| + e^{4it/\hbar}|-n\rangle\langle -n|) = (|n\rangle\langle n| + e^{-4it/\hbar}|-n\rangle\langle -n|)\left[\begin{pmatrix} -i\sqrt{3} & 1 \\ 3 & i\sqrt{3} \end{pmatrix} + e^{4it/\hbar}\begin{pmatrix} i\sqrt{3} & 3 \\ 1 & -i\sqrt{3} \end{pmatrix}\right]$

$A_t = e^{-4it/\hbar}\begin{bmatrix} -4\sqrt{3}i & 4 \\ 12 & 4\sqrt{3}i \end{bmatrix} + e^{4it/\hbar}\begin{bmatrix} 4\sqrt{3}i & 12 \\ 4 & -4\sqrt{3}i \end{bmatrix} \Rightarrow \begin{bmatrix} -4i\sqrt{3} & 4 \\ 12 & 4i\sqrt{3} \end{bmatrix} + e^{8it/\hbar}\begin{bmatrix} 4i\sqrt{3} & 12 \\ 4 & -4i\sqrt{3} \end{bmatrix}$

c) @ $t=0$ $\quad |\psi(0)\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \langle A\rangle_t \Rightarrow \langle\psi(0)|A_t|\psi(0)\rangle = \langle\psi(0)|\frac{1}{\sqrt{2}}\begin{pmatrix} 4-4i\sqrt{3} \\ 12+4i\sqrt{3} \end{pmatrix} + \frac{e^{8it/\hbar}}{\sqrt{2}}\begin{pmatrix} 12+4i\sqrt{3} \\ 4-4i\sqrt{3} \end{pmatrix}$

$\frac{1}{2}\left[4-4i\sqrt{3} + 12+4i\sqrt{3} + e^{8it/\hbar}(12+4i\sqrt{3}+4-4i\sqrt{3})\right] = \frac{1}{2}(16 + 16e^{8it/\hbar})$

Q32 $H(t) = H_0 + V\delta(t/t_0)$

$$\delta\left(\frac{t}{t_0}\right) = \lim_{\tau \to +0} f_\tau(t)$$

$$H(t) = \hat{H}_0 + \hat{V}\left\{\lim_{\tau \to +0} f_\tau(t)\right\} = \hat{H}_0 + \frac{t_0}{\tau}$$

$$T(\tau - \tau) = e^{-i(\tau+\tau)H/\hbar} \Rightarrow e^{-i(2\tau)(H_0 + t_0/\tau)\frac{1}{\hbar}} \Rightarrow e^{-i(2\tau)H_0/\hbar} e^{-i(2\tau)t_0/\tau\hbar}$$

$$\Rightarrow \lim_{\tau \to +0} \underbrace{e^{-i(2\tau)H_0/\hbar}}_{e^0 \doteq 1} e^{-i2t_0/\hbar} = \boxed{e^{-i2t_0/\hbar}}$$