MODULE 2 - PART C

ELECTRICAL CHARACTERISTICS OF A MOSFET

- In the last part (Part B) of Module 2, we have looked at some qualitative understanding of the current - voltage characteristics of MOSFETS. (I-V characteristics)
- Here, we will start by modeling the I-V relationships of the MOSFET

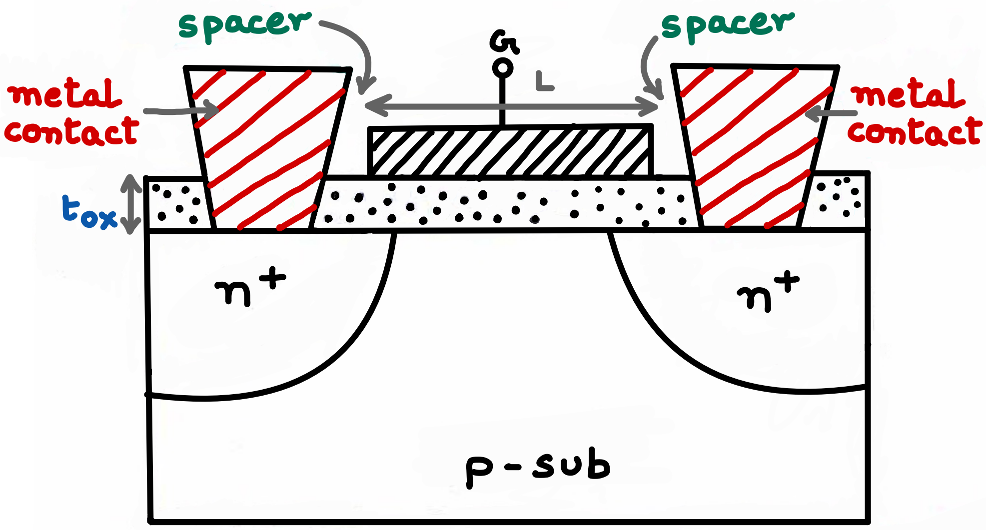
MODELING MOS I-V characteristics

- There are 2 approaches of deriving these relationships. First approach does not require Calculus and the second approach requires Calculus and is more accurate. We will look at both.

- APPROACH 1 :-

Let us start with the 1st approach

→ Looking at the side view of the MOSFET, we can see :-



S+D metal contacts (are made of Tungsten (W) based material)

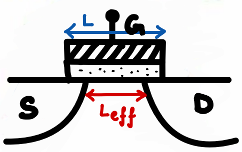
t_{ox} → thickness of the oxide layer (SiO_2, HfO_2)

spacers → ensure that the S+G+D contacts remain isolated

There is usually an overlap region betn S+G + D+G. This is the reason why the drawn length $L_{drawn} > L_{eff}$ (effective length)

Why does this overlap region occur?

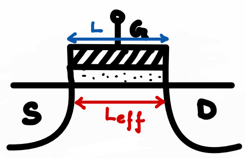
We use a SELF-ALIGNED process so that the S+D are aligned with the G edge, but after



doping, when the transistor is annealed (heated) at high temperatures, the n+ material diffuses under the gate and creates the overlap.

Here, in our approach 1, we are going to assume that,

L or $L_{drawn} = L_{eff}$



Let us now look at some of the main Parameters which we will use here.

PARAMETERS:

- $I_{DS} = \dfrac{\text{Charge in channel}}{\text{transit time}}$

since, current = Charge per unit time.

- τ = transit time
- Q = charge in motion or transit

(thus we can only have current in the inversion region i.e. $V_{GS} > V_{th}$)

- μ (mu) → mobility of e^-s in the case of nMOSes (of h^+s → holes in the case of PMOSes)
- C_g → gate capacitance (which we will discuss later)
- E → electric field (again we will discuss this later)
- ϵ (epsilon) → permittivity of the gate dielectric

$\epsilon = \epsilon_0 \epsilon_r$ where,

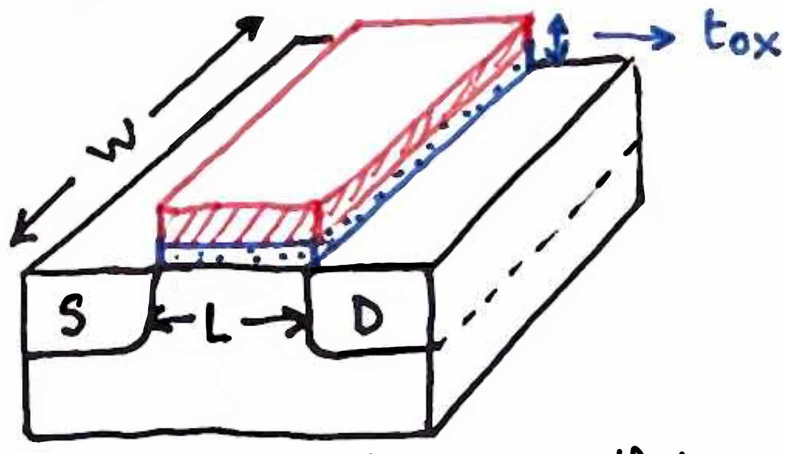
$\epsilon_0 \rightarrow$ permittivity of free space

$\epsilon_r \rightarrow$ permittivity of the material w.r.t. free space ; $\epsilon_r = 1$ for free space

So, we see that the current is Charge / time and hence we have to first find the charge & the time. Thus, the first question we will try to answer is :-

WHAT IS THE CHARGE IN THE CHANNEL ?

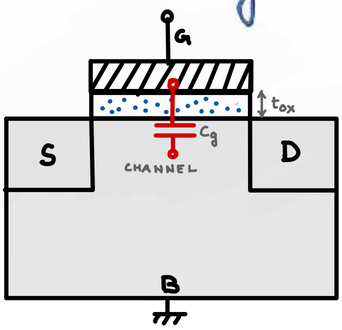
To calculate the charge, first let us redraw the side view of our MOSFET, this time in 3-D (so, going into our paper or screen)



$L \rightarrow$ channel length ; $t_{ox} \rightarrow$ thickness of the oxide layer

$W \rightarrow$ channel width

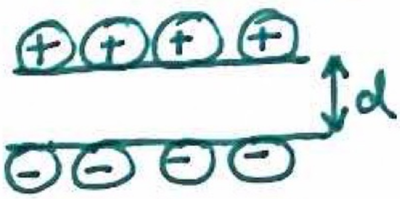
• Firstly, we see that, there is an oxide betⁿ the semiconductor + the Gate (high-K metal used now ; poly Si was used earlier). Hence we will have a gate capacitor. Let us call this C_g.



$\therefore$ C_g is the Capacitance between the Gate and the Channel.

Now, looking at the 3-D view, we can easily tell that this capacitor is infact a parallel plate capacitor, in the inversion region.

We know for a parallel plate capacitor, we have 2 plates separated by a dielectric as below:-



Here, the capacitance is given by :- $C = \dfrac{\epsilon A}{d}$

where A is the area of the plates + d is the separation distance as shown.

$\epsilon = \epsilon_0 \epsilon_r$ or $\epsilon_0 K$ where,

$\epsilon_0 \rightarrow 8.854 \times 10^{-12}$ F/m (Farad / meter) permittivity of free space.

ϵ_r or $K \rightarrow$ relative permittivity of the dielectric material

In our scenario, for the MOSFET,

$$C_g = \frac{\epsilon A}{d}$$

and the area A of the plates is given by L × W i.e the channel length × width and the separation distance is tox.

$$\therefore \boxed{C_g = \epsilon_{ox} \frac{WL}{t_{ox}}}$$

where, $\epsilon_{ox} \rightarrow$ permittivity of the oxide.

We also write,

$$\boxed{C_g = C_{ox} WL}$$

where, $C_{ox} = \dfrac{\epsilon_{ox}}{t_{ox}}$

and is actually the gate capacitance per unit area

$\therefore$ Now to answer our original question $\rightarrow$ What is the charge in the channel?

We know that the charge Q is given by

$$Q = CV$$

Now we already have an estimate of C.

How do we find V?

Before we answer this let us see what exactly is this V?

The Capacitance C_g is basically the capacitance bet^n the G & the channel. We have seen this earlier.

Now, V will also be the voltage bet^n the G & the channel. ∴ $V_{GCh} = V_G - V_{Ch}$

What is V_{ch}?

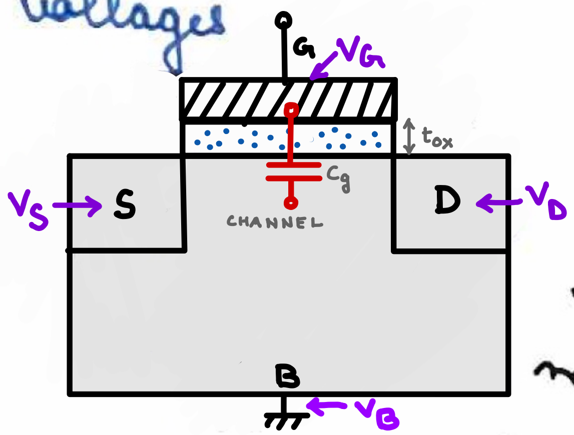
There is no clear answer to this question.

Firstly, we will look at the SIMPLE VERSION of the answer to this question.

This SIMPLE VERSION is actually what our FIRST APPROACH will be.

What is V_{GC} (APPROACH 1)?

Let us take a look at the channel more carefully now, and write down the different voltages

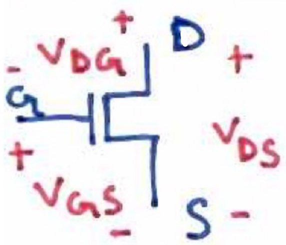


Now the channel voltage is not constant.

We will break this up into 2 parts.

On the source side, we can say that the channel voltage is $V_{GC_{Source}} = V_{GS}$

On the Drain side, let us see what we can write:-



We can write :-

$V_{DS} = V_{DG} + V_{GS}$

or, $V_{GS} = V_{DS} - V_{DG}$

or, $V_{GS} = V_{DS} + V_{GD}$ $(\because V_{DG} = -V_{GD})$

or, $V_{GD} = V_{GC_{Drain}} = V_{GS} - V_{DS}$

Assuming the voltage in the channel changes linearly, we can say that the CHANNEL VOLTAGE is the average of $V_{GC_{source}}$ and $V_{GC_{Drain}}$.

That is,

$$\boxed{V_{GC} = \frac{V_{GC_{source}} + V_{GC_{Drain}}}{2}}$$

$$\therefore V_{GC} = \frac{V_{GS} + (V_{GS} - V_{DS})}{2}$$

or, $\boxed{V_{GC} = V_{GS} - \frac{V_{DS}}{2}}$

Now we will calculate the channel charge.

$$Q_{channel} = CV$$

where, $C = C_{ox} WL$

Keep in mind that this happens in the inversion region where, $V_{GS} > V_{th}$

∴ The effective V_{GS} or the actual overdrive voltage is $V_{GS} - V_{th}$ (also sometimes called $V_{GST} = V_{GS} - V_t$ or V_{Gt}

Hence we can substitute this in place of V_{GS}.

∴ We have,

$$V_{GC} = V_{GS} - V_{th} - \frac{V_{DS}}{2}$$

or, $$\boxed{V_{GC} = V_{GS} - \frac{V_{DS}}{2} - V_{th}}$$

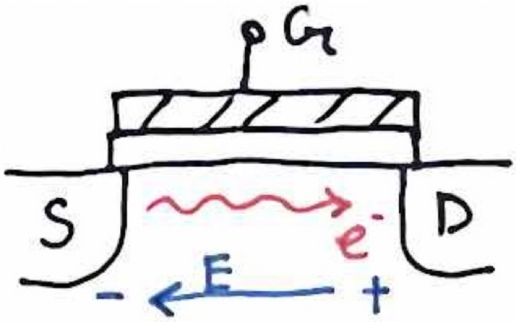
∴ $Q_{channel} = CV$ or,

$$\boxed{Q = C_{ox} WL \left(V_{GS} - \frac{V_{DS}}{2} - V_{th}\right)}$$

where, $C_{ox} = \frac{\epsilon_{ox}}{t_{ox}}$ cap/area

To find the current, we now need to determine the carrier velocity.

For our nMOS, the carriers are the electrons.



- Charge is carried by the e^-s & the e^-s are moving from S to D

We need to determine the velocity of these e^-s. (V_e) Now, we know,
$V_e \propto E$ where, E is the Electric Field from D to S. The e^-s are moving because of this Electric Field, because this acts as a force on these e^-s (Force will be Charge × Field Electric Field is Force per unit charge)

$\because V_e \propto E$ and hence

$\boxed{V_e = \mu E}$ where,

$\mu \rightarrow$ proportionality constant and it's known as the mobility of the carriers (e^-s)

In general for Silicon, electron mobility is 2 to 3 times higher than hole mobility, but for modern transistors, these 2 mobilities μ_p & μ_n are almost equal because of many advanced processing steps. Since we are dealing with long channel MOSFET the older notion that $\mu_n = 2/3 \mu_p$ holds true.

But in our course we will not go into the details of these advanced processing steps.

Now that we know what the velocity is, $v_e = \mu E$. We need to find E.

We know $E = \dfrac{F}{q}$ . . . (i)

Energy = Charge × Voltage

Also,

Energy $= \int_0^L F.dx = FL$ . . . (ii)

Say we have 2 points,

with potential V_1 & V_2

Potential V_1 is the work done (energy) required to bring a unit charge from ∞ to that point.

$\therefore$ $\text{Energy}_{V_1} = V_1 q$

for bringing q charge.

Also, similarly,

$\text{Energy}_{V_2} = V_2 q$

$\therefore$ $\Delta \text{Energy} = q(V_2 - V_1)$

or,

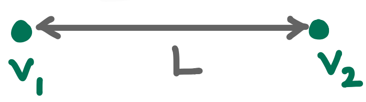
$q(V_2 - V_1) = FL$ from (ii)

$$\therefore \frac{F}{q} = \frac{V_2 - V_1}{L}$$

from (i), $\frac{F}{q} = E$ (electric field)

$$\therefore E = \frac{V_2 - V_1}{L} \text{ or, } \boxed{E = \frac{V_{21}}{L}}$$

where L is the distance betn V_1 & V_2.



∴ The electric field in the channel of length L betⁿ D & S is given by :-

$$E = \frac{V_D - V_S}{L} \quad \text{or} \quad \boxed{E = \frac{V_{DS}}{L}}$$

$$\therefore \boxed{v_e = \mu \frac{V_{DS}}{L}}$$

$\because v_e = \mu E$

From here, we will find the time it takes for carriers to transit the channel. (τ)

$$v_e = \frac{\text{LENGTH}}{\text{time}} = \frac{L}{\tau}$$

$$\therefore \tau = \frac{L}{v_e}$$

$$\text{or, } \boxed{\tau = \frac{L^2}{\mu V_{DS}}}$$

Now we will put everything together and find the current in the LINEAR REGION as below :-

$$I_{DS} = \frac{Q_{channel}}{\tau}$$

or, $$I_{DS} = \frac{C_{ox} WL \left(V_{GS} - \frac{V_{DS}}{2} - V_{th}\right)}{L^2 / \mu V_{DS}}$$

Rearranging, we have,

$$I_{DS} = \left(\frac{\epsilon_{ox} \mu}{t_{ox}}\right)\left(\frac{W}{L}\right)\left(V_G - \frac{V_{DS}}{2} - V_{th}\right) V_{DS}$$

where, $\epsilon_{ox} / t_{ox} = C_{ox}$

$$\boxed{I_{DS} = \beta \left(V_{Gt} - \frac{V_{DS}}{2}\right) V_{DS}}$$

where,

β (Beta factor) $= \frac{\epsilon_{ox} \mu}{t_{ox}} \cdot \frac{W}{L}$

V_{Gt} = overdrive $= V_G - V_{th}$

This equation for I_{DS} is true in the linear region because we had assumed that the voltage V_{DS} DECREASES LINEARLY from the Source to the Drain when we calculated V_{GC}.

From this equation that we have derived for the LINEAR REGION, we can also find the current in the SATURATION REGION.

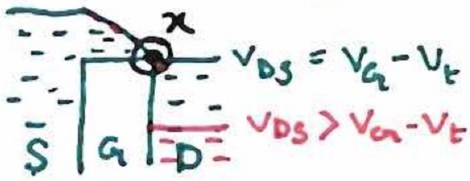
SATURATION REGION CURRENT

Let us first define :-

V_{DSat} ⟶ which is the Drain Saturation Voltage

∴ V_{DSat} is the V_{DS} value where the channel is no longer inverted in the vicinity of the Drain.

Let us go back to our WATER MODEL and see what happens in the Saturation Region :-



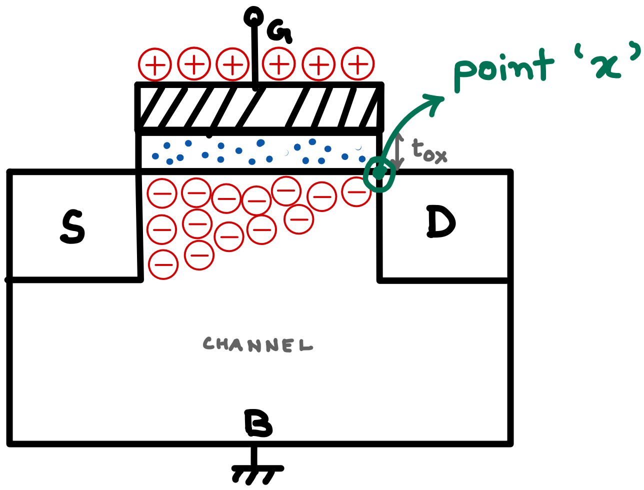
Here at point x, in the Saturation region there is almost no WATER. But water still flows to the Drain.

This happens because of the potential head.

We know ⟶ in our analogy
WATER is equivalent to charge
+
FLOW OF WATER is equivalent to current.

NOW, let's consider,
potential head to be equivalent to Electric Field

Drawing the MOSFET, we have,



Here, similar to the WATER MODEL, at point 'x', in the SATURATION REGION, there are no electrons (e^-s) i.e. NO INVERSION

But, we will have current (as we had flow of water) because of the electric field, and the electrons will be swept to the Drain.

When V_{DS} keeps increasing and reaches :-

$$V_{DS} \geq V_{GS} - V_{th}$$

i.e. $V_D - V_S \geq V_G - V_S - V_{th}$

or, $-V_D \leq -V_G + V_{th}$

or, $V_G - V_D \leq V_{th}$

i.e $V_{GD} \leq V_{th}$

∴ When V_{GD} is less than the threshold voltage V_t / V_{th}, then inversion will no longer occur in the drain side (cut-off) and we will not have mobile charges here anymore.

But all the e^-s that started in the source, will get swept away to the drain with the pull of the electric field (similar to the potential head)

Here, $V_{Dsat} = V_G - V_{th}$

or, $\boxed{V_{Dsat} = V_{Gt}}$

where, $V_{Gt} = V_G - V_t$

∴ Here, the saturation current I_{Dsat} is given by :-

$$I_{Dsat} = \beta\left(V_{Gt} - \frac{V_{Dsat}}{2}\right)V_{Dsat}$$

∵ $V_{Dsat} = V_{Gt}$, we have

$$\boxed{I_{Dsat} = \frac{\beta V_{Gt}^2}{2}}$$

Let us now write down the equations for the current in all the regions.

$\boxed{\text{REGION (I)}}$ → $I_{DS} \simeq 0$ → when $V_{GS} < V_{th}$

→ CUT-OFF REGION

In more advanced classes, you will see that the current in the CUT-OFF REGION is NOT EXACTLY '0', but very, very small. This is called the LEAKAGE CURRENT or Sub-threshold current.

Hence, we will say that, I_{DS} is almost or approximately equal to '0' in the CUT-OFF REGION.

$\boxed{\text{REGION (II)}}$ → $I_{DS} = \beta\left(V_{Gt} - \frac{V_{DS}}{2}\right)V_{DS}$

where, $\beta = \frac{\epsilon_{ox}\mu}{t_{ox}} \cdot \frac{W}{L}$

and $V_{Gt} = V_G - V_{th}$

→ when $V_{GS} > V_{th}$ + $V_{DS} < V_{DSat}$

→ LINEAR REGION

$\boxed{\text{REGION III}} \longrightarrow I_{DS} = \frac{\beta V_{Gt}^2}{2} \rightarrow$ when $V_{GS} > V_{th}$ AND, $V_{DS} \geq V_{Dsat}$

$\rightarrow$ SATURATION REGION

Let us now draw the current-voltage characteristics based on these equations (when $V_{GS} > V_{th}$) i.e when the MOSFET is ON

- In the LINEAR REGION,

$$I_{DS} = \beta\left(V_{Gt} - \frac{V_{DS}}{2}\right)V_{DS}$$

Here, $V_{DS} < V_{Dsat}$ or $V_{DS} < V_{Gt}$

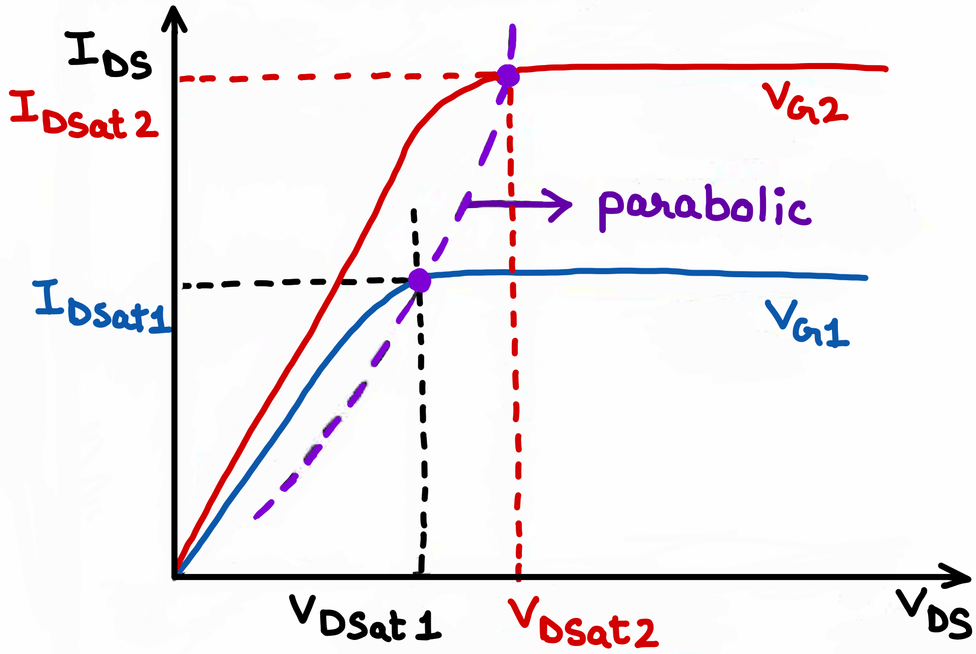
$\therefore \frac{V_{DS}}{2} \ll V_{Gt}$. Hence we can ignore the 2nd term.

$$\therefore I_{DS} \simeq \beta V_{Gt} V_{DS}$$

and hence I_{DS} varies LINEARLY with V_{DS}.

So if we draw the I-V characteristics of the MOSFET, I_{DS} vs V_{DS}

will be a LINEAR PLOT in this REGION :-



This plot will be linear for a particular $V_{Gt_1} = V_{G_1} - V_{th}$ until we reach $V_{DSat\,1}$.

Here, $V_{DSat_1} = V_{G_1} - V_{th}$

From here the current will saturate as we have seen (in the SATURATION REGION) i.e. the current I_{DS} will not change as

V_{DS} increase but will remain a constant value

$$I_{Dsat_1} = \frac{\beta V_{Gt_1}^2}{2}$$

Now, if we consider another gate voltage V_{G2} where $V_{G2} > V_{G1}$

Then, $V_{Gt_2} > V_{Gt_1}$

and ∴ $V_{Dsat2} > V_{Dsat1}$

Here, the saturation current I_{Dsat2} will also be greater than I_{Dsat_1}

Now, if we look at the different (V_{Dsat}, I_{Dsat}) points for the different V_G's, and join these points with a curve, we will see that this curve is PARABOLIC ($y = x^2$)

$$I_{Dsat} = \beta \frac{V_{Gt}^2}{2}$$

$$V_{Dsat} = V_{Gt}$$

$$\therefore \; I_{Dsat} = \beta \frac{V_{Dsat}^2}{2}$$

$$(y \propto x^2)$$

(see figure)

This I - V characteristics is known as the :-

$$\boxed{\text{OUTPUT CHARACTERISTICS}}$$

From the output characteristics in the SATURATION REGION, we have,

$$I_{DS} \propto V_{Gt}^2$$

i.e. $I_{DS} \propto (V_G - V_t)^2$

This is therefore called the SQUARE-LAW MODEL of the MOSFET.

Although this is a LONG-CHANNEL approximation, it works pretty well and even in modern times, this approximation holds.