Question 7

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7. Let $A \in M_n(\mathbb{R})$ that is invertible with eigen-pair (λ, v).

(a) Is v an eigenvector of A^5? What's its corresponding eigenvalue? Generalize.

$A\vec{v} = \lambda\vec{v}$

$A^2v \Rightarrow \quad AAv = A\lambda v = \lambda Av = \lambda(\lambda v) = \lambda^2\vec{v}$

Do this iteratively on A^n for $n=5$

The corresponding eigenvalue for A^5 is λ^5

Generally $A^n\vec{v} = \lambda^n\vec{v}$

(b) Is v an eigenvector of A^{-1}? What's its corresponding eigenvalue?

$A^{-1}A\vec{v} = A^{-1}\lambda\vec{v}$

$I_n\vec{v} = \lambda A^{-1}v$

$A^{-1}v = \frac{1}{\lambda}\vec{v}I_n \Rightarrow A^{-1}\vec{v} = \frac{1}{\lambda}\vec{v}$ ∴ yes

$\frac{1}{\lambda}$ is eigenvalue

(c) Is v an eigenvector of $A^2 + 3A + 6I_n$? What's its corresponding eigenvalue?

$(A^2 + 3A + 6I_n)\vec{v}$

$A^2\vec{v} + 3A\vec{v} + 6I_nv$

$I_nv = \lambda v$
$\lambda = 1$

$\lambda^2 V + 3\lambda V + 6$ eigenvalue is $(\lambda^2 + 3\lambda + 6)$