ECE 3030: Physical Foundations of Computer Engineering
Spring 2024
Homework 6—Total points 100
Due on Thursday 4/10/2024 at 11.59am. In case of a late submission, you will be penalized by 50 points for each day after the submission deadline has passed. You will receive no score if you submit after the solution has been posted.

Q1 In class, we derived the following equations (square law model with correction for subthreshold current) relating the drain current I_D, the drain voltage V_D and the gate voltage V_G in a long-channel MOSFET.

$$\frac{I_D}{W} = \begin{cases} I_{sub-V_t} e^{\frac{q(V_G - V_t)}{mkT}} (1 - e^{\frac{-qV_D}{kT}}); \text{ when } V_G < V_t \text{ (sub-threshold)} \\ I_{sub-V_t} (1 - e^{\frac{-qV_D}{kT}}) + \mu C_{ox} \frac{1}{L}((V_G - V_t)V_D - \frac{1}{2}V_D^2); \text{ when } V_G - V_t > V_D \text{ (linear)} \\ I_{sub-V_t} (1 - e^{\frac{-qV_D}{kT}}) + \mu C_{ox} \frac{1}{2L}(V_G - V_t)^2; \text{ when } V_G - V_t < V_D \text{ (saturation)} \end{cases}$$

We have also defined the on-current I_{ON} and the off-current I_{OFF} as follows.

$$\begin{aligned} I_{ON} &= I_D(V_G = V_D = V_{DD}) \\ I_{OFF} &= I_D(V_G = 0, V_D = V_{DD}) \end{aligned}$$

Here, V_{DD} is the power supply voltage, and $qV_{DD} >> kT$.

Q1.1 Based on these relations, find expressions for I_{ON} and I_{OFF} in terms of V_{DD}, V_t, I_{sub-V_t}, μ, C_{ox}, W and L. Note that when $V_G = V_D = V_{DD}$, the MOSFET operates in the saturation region. [5 pts]

Q1.2 Based on the derived expressions, find how I_{ON} and I_{OFF} would change if V_t is increased. [5 pts]

Solution to Q1:

Q2

Q2.1 : $I_{OFF} = I_D(V_G=0, V_D=V_{DD}) = W I_{sub-V_t}\, e^{-\frac{qV_t}{mKT}} \left(1 - e^{-\frac{qV_{DD}}{KT}}\right)$
2.2

$qV_{DD} >> KT \quad e^{-\frac{qV_{DD}}{KT}} \cong 0$

$$\underline{I_{OFF} = W I_{sub-V_t}\, e^{-\frac{qV_t}{mKT}}}$$

$\underline{V_t \uparrow \quad I_{OFF} \downarrow}$

$$I_{ON} = I_D(V_G = V_D = V_{DD}) = W I_{sub-V_t}\left(1 - e^{-\frac{qV_{DD}}{KT}}\right) + \mu C_{ox}\frac{W}{2L}(V_{DD}-V_t)^2$$

[when $V_G = V_D = V_{DD}$ MOSFET in saturation]

$$\underline{I_{ON} = W I_{sub-V_t} + \mu C_{ox}\frac{W}{2L}(V_{DD}-V_t)^2}$$ [$\because qV_{DD} >> KT$]

$\underline{V_t \uparrow \quad (V_{DD}-V_t)\downarrow \quad I_{ON}\downarrow}$

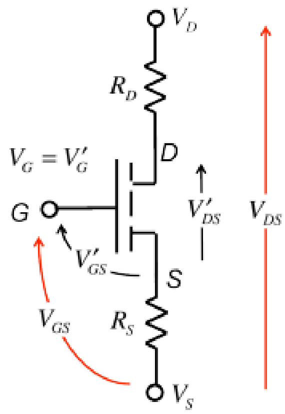
Q2 Consider an n-type MOSFET with N_A=7×10^{18} m^3. The gate length of the MOSFET L=2 μm, width W=12 μm and the oxide thickness t_{ox}= 8 nm. Take N_C=N_V=10^{25} m^{-3}, E_G=1.12 eV, n_i=1.5×10^{16} m^{-3}, kT=0.026 eV, vacuum permittivity $\varepsilon_\circ$=8.854×10^{-12} F/m, dielectric constant of oxide ε_{ox}=4, dielectric constant of silicon ε_{Si}=12, electron mobility μ_n=230×10^{-4} m^2/Vs, hole mobility μ_p=83 ×10^{-4} m^2/Vs.

(Q3.1) Calculate $\phi_B = |E_F - E_i|$, the oxide capacitance C_{ox}, the maximum depletion width W_{max} and the threshold voltage V_t. [5 pts]

(Q3.2) Calculate the drain current I_D for the following six cases. [30 pts]

1. V_G=3 V, V_D=2 V.
2. V_G=0.2 V, V_D=1 V.
3. V_G=2.2 V, V_D=2 V.
4. V_G=2 V, V_D=1 V.
5. V_G=0.5 V, V_D=0.5 V.
6. V_G=1.5 V, V_D=2 V.

Q3 Real transistors have parasitic series resistances at the source and drain. As shown in the figure below, the result is that the voltages applied to the terminals of the device are not the voltages on the terminals of the intrinsic device. Modify the square law MOSFET equations to include the effects of source and drain series resistances.



Prob. 3 Figure.

Solution to Q3:

The voltages in the square law expressions are the intrinsic voltages – not the voltages applied to the terminals. Using prime to denote the intrinsic voltages:

$$I_D = \frac{W\bar{\mu}_n C_{ox}}{L}\left[\left(V_{GS}' - V_T\right)V_{DS}' - \frac{V_{DS}'^2}{2}\right] \qquad 0 \le V_{DS}' < V_{Dsat} \quad V_{GS}' \ge V_T$$

$$I_D = \frac{W\bar{\mu}_n C_{ox}}{2L}\left(V_{GS}' - V_T\right)^2 \qquad V_{DS}' \ge V_{Dsat} \qquad V_{GS}' \ge V_T$$

Note that we use V_{GS}' instead of V_G' and V_{DS}' instead of V_{GS}' because with a series resistance, the intrinsic source is not at ground potential.

Straightforward circuit analysis gives the intrinsic voltages as

$$V_{GS}' = V_G - I_D R_S$$
$$V_{DS}' = V_{DS} - I_D\left(R_S + R_D\right)$$

Inserting these voltages in the square law theory gives the answer. Note that I_D ends up on both sides of the equations, and it is not completely trivial to plot the IV with series resistance.

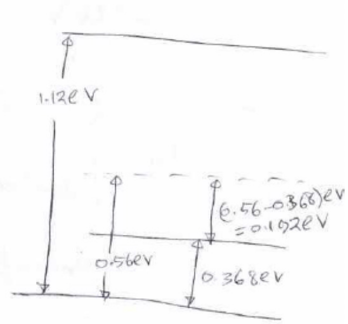
Solution to Q2:

$$p = N_A = N_V e^{\frac{E_V - E_F}{k_B T}}$$

$$\Rightarrow E_V - E_F = k_B T \ln \frac{N_A}{N_V}$$

$$= 0.026 \ln \frac{7\times10^{18}}{10^{25}}$$

$$= -0.368\,eV$$



ⓐ $q\psi_B = |E_i - E_F| = (0.55-0.368)\,eV = 0.192\,eV$

ⓑ $$C_{ox} = \frac{\varepsilon_0 \varepsilon_{ox}}{t_{ox}} = \frac{8.854\times10^{-12}\,F/m \times 4}{8\times10^{-9}\,m}$$

$$= 4.427\times10^{-3}\,F/m^2$$

ⓒ $$W_{max} = \sqrt{\frac{2\varepsilon_0\varepsilon_{si}(2\psi_B)}{qN_A}} = \sqrt{\frac{2\times12\times8.854\times10^{-12}\times2\times0.192}{1.6\times10^{-19}\times7\times10^{18}}}$$

$$= 8.536\,\mu m$$

(1) $V_G = 3V$, $V_D = 2V$, $(V_G - V_T) = 2.614V > V_D \rightsquigarrow$ cond^n (ii)

$$I_D(V_G = 3V, V_D = 2V) = \mu_n C_{ox} \frac{W}{L}\left[(V_G - V_t)V_D - \frac{1}{2}V_D^2\right]$$

$$= 6.10926 \times 10^{-4}\left[2.614 \times 2 - \frac{1}{2}\cdot 2^2\right]$$

$$= 1.972\,mA$$

(2) $V_G = 0.2V$, $V_D = 1V$, $V_G < V_T \rightsquigarrow$ cond^n (i)

$$I_D(V_G = 0.2V, V_D = 1V) = \sim 0 \quad \text{(cut off)}$$

(3) $V_G = 2.2V$, $V_D = 2V$, $(V_G - V_t) = 1.814V < V_D \rightsquigarrow$ cond^n (iii)

$$I_D(V_G = 2.2V, V_D = 2V) = \mu_n C_{ox} \frac{W}{2L}(V_G - V_t)^2$$

$$= \frac{1}{2} \times 6.10926 \times 10^{-4}(1.814)^2$$

$$= 1.025\,mA$$

(4) $V_G = 2V$, $V_D = 1V$, $V_G - V_t = 1.614V > V_D \rightsquigarrow$ cond^n (ii)

$$I_D(V_G = 2V, V_D = 1V) = \mu_n C_{ox} \frac{W}{L}\left[(V_G - V_t)V_D - \frac{1}{2}V_D^2\right]$$

$$= 6.10926 \times 10^{-4}\left[1.614 \times 1 - \frac{1}{2}\cdot 1^2\right]$$

$$= 0.7502\,mA$$

$$V_t = \frac{\sqrt{4 \varepsilon_0 \varepsilon_{si} q N_A \varphi_B}}{C_{ox}} + 2\varphi_B$$

$$= \frac{\sqrt{4 \times 8.854 \times 10^{-12} \times 12 \times 1.6 \times 10^{-19} \times 7 \times 10^{18} \times 0.192}}{4.427 \times 10^{-3}}$$

$$+ 2 \times 0.193$$

$$= 0.386\ V$$

Q1-2.

$$I_D = \begin{cases} 0, & V_G < V_t \quad (\mathrm{i}) \\ \mu_n C_{ox} \frac{W}{L}\left[(V_G - V_t)V_D - \frac{1}{2}V_D^2\right], & (V_G - V_t) > V_D \quad (\mathrm{ii}) \\ \mu_n C_{ox} \frac{W}{2L}(V_G - V_t)^2, & (V_G - V_t) < V_D \quad (\mathrm{iii}) \end{cases}$$

Here, $\mu_n = 230 \times 10^{-4}\ m^2/Vs$, $\mu_p = 83 \times 10^{-4}\ m^2/Vs$

$W = 12\ \mu m$, $L = 2\ \mu m$, $V_t = 0.386\ V$,

$C_{ox} = 4.427 \times 10^{-3}\ F/m^2$

$$\therefore \mu_n C_{ox} \frac{W}{L} = 230 \times 10^{-4} \times 4.427 \times 10^{-3} \times \frac{13 \times 10^{-6}}{2 \times 10^{-6}}$$

$$= 6.10926 \times 10^{-4}\ F/Vs$$

(5) $V_G = 0.5V,\ V_D = 0.5V,\ (V_G - V_t) = 0.114V < V_D \rightsquigarrow$ cond^n (iii)

$$I_D(V_G = 0.5V, V_D = 0.5V) = \mu_n C_{ox} \frac{W}{2L}(V_G - V_t)^2$$

$$= \frac{1}{2} \times 6.10926 \times 10^{-4}\ (0.114)^2$$

$$= 3.97\ \mu A$$

(6) $V_G = 1.5V,\ V_D = 2V,\ (V_G - V_t) = 1.114V \ngtr < V_D \rightsquigarrow$ cond^n (iii)

$$I_D(V_G = 1.5V, V_D = \cancel{0.5}\ 2V) = \mu_n C_{ox} \frac{W}{2L}(V_G - V_t)^2$$

$$= \frac{1}{2} \times 6.10926 \times 10^{-4} (1.114)^2$$

$$= 0.379\ mA$$