

Question 4

Sunday, November 5, 2023

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4. Four data points in \mathbb{R}^3 with coordinates are given as follows.

$$(-1, 2, 9), (0, 1, 1), (2, 0, 0), (1, 2, -1)$$

Determine coefficients c_1, c_2 such that the plane $z = c_1x + c_2y$ best fits the data.

$$\begin{bmatrix} c_1 & c_2 \\ -1 & 2 \\ 0 & 1 \\ 2 & 0 \\ 1 & 2 \end{bmatrix} \begin{matrix} z \\ 9 \\ 1 \\ 0 \\ -1 \end{matrix} \text{ has no solutions.}$$

We must take x^* , least square solution

$Ax = z$ is inconsistent

We must find $\text{proj}_{\text{col}(A)} \vec{z}$

$$\vec{z} = \begin{bmatrix} 9 \\ 1 \\ 0 \\ -1 \end{bmatrix}$$

$$\text{where } \text{col}(A) = \text{Span} \left\{ \begin{bmatrix} -1 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \\ 2 \end{bmatrix} \right\}$$

$$= \sum_{i=1}^2 \frac{\langle \vec{z}, \vec{q}_i \rangle}{\langle \vec{q}_i, \vec{q}_i \rangle} \vec{q}_i \Rightarrow$$

$$\text{proj}_{\vec{q}_1} \vec{z} = \frac{-9 + 0 + 0 + -1}{1 + 0 + 4 + 1} = \frac{-10}{6} = -\frac{5}{3} \begin{bmatrix} -1 \\ 0 \\ 2 \\ 1 \end{bmatrix}$$

$$\text{proj}_{\vec{q}_2} \vec{z} = \frac{18 + 1 + 0 + -2}{4 + 1 + 0 + 4} = \frac{17}{9} = \frac{17}{9} \begin{bmatrix} 2 \\ 1 \\ 0 \\ 2 \end{bmatrix}$$

$$x^* = \begin{bmatrix} -5/3 \\ 17/9 \end{bmatrix}$$

$$c_1 = -5/3 \quad c_2 = 17/9$$