Math-1564 K *Midterm 1*

Name (Print). ______________________________ Student-ID (Print):______________________________

Remarks:

- Show all your work and justify your answers where required to get full credit.
- This quiz is closed-book, closed-notes and no calculators are allowed.
- Simplify your answers unless told otherwise.

1. For each of following statement, indicate if it is true or false by filling the appropriate circles (no checkmarks).
   (a) ○ **T** ○ **F** $\varnothing \subseteq \{\varnothing\}$.
   (b) ○ **T** ○ **F** $\{\varnothing\} \in \{\varnothing\}$.
   (c) ○ **T** ○ **F** $(A \cup B) \cap C = A \cup (B \cap C)$.
   (d) ○ **T** ○ **F** Any 3×2 matrix A with two pivotal positions has a non-trivial solution to system $A\boldsymbol{x} = \mathbf{0}$.
   (e) ○ **T** ○ **F** If a linear system has more equations than unknowns then the system cannot have a unique solution.
   (f) ○ **T** ○ **F** A subset of a linearly dependet set of vectors is linearly dependent.
   (g) ○ **T** ○ **F** If matrices A, B are row equivalent then systems $(A|b), (B|b)$ have the same number of solutions.

2. Define the following terms mathematically. You need to be precise and succinct.
   (a) Function f from domain $\mathcal{D}$ to codomain $\mathcal{C}$.
   (b) Equivalence class $[a]$ given equivalence relation $\sim$ on set S.
   (c) Linear independence.

   Please review all definitions we have covered so far.

3. Is it possible that vectors $\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3$ are linearly dependent, but the vectors $\boldsymbol{w}_1, \boldsymbol{w}_2, \boldsymbol{w}_3$ are linearly independent? Here $\boldsymbol{w}_1 = \boldsymbol{v}_1 - \boldsymbol{v}_2, \boldsymbol{w}_2 = \boldsymbol{v}_2 + \boldsymbol{v}_3,$ and $\boldsymbol{w}_3 = \boldsymbol{v}_3 + \boldsymbol{v}_1$. Justify your answer.

4. Define relation $\sim$ on set $S := \mathbb{R}^2 \setminus \{\mathbf{0}\}$ such that for $x, y \in S, x \sim y$ if $x \in span(y)$. Show that $\sim$ is an equivalence relation. Interpret how $\mathbb{R}^2/\sim$ forms a partition of S.

5. Given matrix $M \in M_{m\times n}(\mathbb{R})$, $rank(M)$ is the number of pivots of $ref(M)$. Show that $rank(A|b) = rank(A)$ if and only if system $(A|b)$ is consistent.

6. Consider subspace U in vector space V. Let 0_U be the additive identity of U and 0_V the additive identity of V. Show that $0_U = 0_V$. Carefully justify your steps.

7. Let U, W be two subspaces of vector space V. x, y are two elements in V. Show that if $x + U = y + W$ then $U = W$. Here $x + U = \{x + u | u \in U\}$ and $y + W = \{y + w | w \in W\}$