11/08

Tuesday, November 7, 2023 16:52

1. We have established that if A is not invertible then det(A) = 0. Can you show the converse? In the future, you can add this result to the ever-growing list of invertibility theorem.

2. Find the determinant of the following matrices with the proposed methods.
   a. $\begin{pmatrix} 1 & 9 & 8 & 7 \\ 0 & 2 & 9 & 6 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & 4 \end{pmatrix}$. Use Gauss eliminiation and cofactor expansion. Generalize the result.
   b. $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 3 & 3 \\ 2 & 2 & 5 \end{pmatrix}$. Use Gauss eliminiation, Sarrus's Rule and cofactor expansion.
   c. Discuss the computational cost of finding the determinant with Laplace/Cofactor expansion vs row reduction. If you are asked to make a program to find determinant, which algorithm would you use? What if the matrix is large?

3. Let $A \in M\_n(\mathbb{R})$. What is det(-A)?

4. If Q is an orthogonal matrix, what can you say about det(Q)?

5. Let $T \in \mathscr{L}(V)$ where V is finite-dimensional. We define the determinant of T to be equal to det$[T]_{\mathscr{B}}$ for some basis $\mathscr{B}$.
   a. Note that in this definition we call it the determinant instead of a determinant. This suggests that det of T is the same regardless what $\mathscr{B}$ one chooses. Show this fact.
   b. Determine the determinant of T where $T: \mathbb{R}_2[x] \to \mathbb{R}_2[x]$ given by $f \mapsto 2f + 3f'$

6. Determinant as an expansion factor.
   a. Consider the parallelogram, call it R, determined by vector $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$, $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$. Under the map $T_A: \mathbb{R}^2 \to \mathbb{R}^2$ defined by $x \mapsto Ax$, where $A = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$. What is the area of the image of R under T_A?
   b. Given that linear map $Tx = Ax$ from $\mathbb{R}^n \to \mathbb{R}^n$, where $A \in M_n(\mathbb{R})$. Show that det(A) gives the expansion factor of T for n-paralellepiped determined by $v_1, \ldots, v_n \in \mathbb{R}^n$, i.e.
   $Vol(Av_1, \ldots, Av_n) = |det(A)|\, Vol(v_1, \ldots, v_n)$.
   Where Vol(…) gives the "volume" of n-paralellepipids.

This can be shown in multiple ways. I suggest row reduction
class.

Please stress the general rule for triangular(upper/lower) an

I believe it is n! vs n^3 . Please tell them just because we hav
necessarily the best when matrix size is big. Row reduction is

Please stress polarity of n is very important.

Please also relate 4 with 6, since Q does not change area but m

We have covered det(A^-1) = 1/det(A). The result follows from
$[T]_{\mathscr{B}} = [id]_{C \to B}[T]_{\mathscr{C}}[id]_{\mathscr{B} \to C}$

This is useful for their multi-variable calc.
Please also draw a unit square and show them how the expans

Please also bring their attention to |detA| since it can be a neg
consideration.