Math-1564 K *Midterm 2*

Name (Print). ______________________________ Student-ID (Print):______________________________

Remarks:

- Show all your work and justify your answers where required to get full credit.
- This quiz is closed-book, closed-notes and no calculators are allowed.
- Simplify your answers unless told otherwise.

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1. For each of following statement, indicate if it is true or false by filling the appropriate circles (no checkmarks).
   (a) Any two planes that pass through the origin in $\mathbb{R}^3$ are isomorphic.
   (b) Linear map $\frac{d^2}{dx^2} : \mathbb{R}_3[x] \to \mathbb{R}_1[x]$ defined as $p(x) \mapsto p'(x)$ is surjective.
   (c) $\mathcal{L}(\mathbb{R}^3, \mathbb{R}^5) \cong \mathbb{R}^8$.
   (d) If $A, B \in M_n(\mathbb{R})$ are both invertible then $A + B$ is also invertible.
   (e) Let $U = \{A \in M_3(\mathbb{R}) | A = A^T\}$. Then dim$U = 6$.
   (f) All 2×4 matrices have non-trivial null space.
   (g) Let $A \in M_n(\mathbb{R})$. If $A^2 = 0$, the zero matrix then $(I - A)^{-1} = I + A$.
   (h) If v is in subspace U, then $Proj_U(v) = v$.
   (i) $Ax = b$ has no solution iff $b \notin Col(A)$.
   (j) If $M^T M = I_2$ then M is an orthogonal matrix.

2. Please review defintions we have covered in class so far.

3. Let U be a subspace with ordered basis $\mathcal{B} =< e^x, e^{-2x}, xe^x >$ in vector space of continuous functions on $\mathbb{R}$. Consider map

$$\begin{aligned} T : U &\to U \\ f &\mapsto f'' + f' - 2f \end{aligned}$$

   (a) Determine $[T]_{\mathcal{B}\to\mathcal{B}}$.
   (b) Determine Ker$T$.
   (c) Use the above information to solve differential equation $f''(x) + f'(x) - 2f(x) = -6e^x$

4. Let $U = span\{u, v\}$ where $u = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}, v = \begin{pmatrix} 3 \\ 0 \\ 8 \end{pmatrix}$.
   (a) Determine an orthonormal basis for U with Gram-Schmidt process.
   (b) Determine $Proj_U(w)$, where $w = \begin{pmatrix} 5 \\ 6 \\ 5 \end{pmatrix}$

5. Let $T \in \mathcal{L}(V)$ and consider $T^2 \in \mathcal{L}(V)$. Show that
   (a) $KerT \subseteq KerT^2$
   (b) $ImT^2 \subseteq ImT$
   (c) $KerT = KerT^2$ iff $ImT^2 = ImT$

6. Consider $A \in M_n(\mathbb{R})$ that is invertible.
   (a) Show A^T is invertible.
   (b) Show $(A^T)^{-1} = (A^{-1})^T$.