

10.1 A) 4 Pt DFT coefficients of  $[z \ 1 \ 0 \ 1]$

$$X[n] = 1 + \cos\left(\frac{\pi n}{2}\right) \quad X(e^{j\omega}) = 2 + e^{-j\omega} + e^{-j3\omega}$$

$$X[0] = X(e^{j(0)}) = z + 1 + 1 = 4$$

$$\boxed{[4 \ z \ 0 \ 2]} = X[k]$$

$$X[1] = X(e^{j(\pi/2)}) = z + -j + j = 2$$

$$X[2] = X(e^{j(\pi)}) = z - 1 - 1 = 0$$

$$X[3] = X(e^{j(3\pi/2)}) = z + j + -j = 2$$

10.1 B) 8 Pt DFT for  $x[n] = [4 \ 4 \ 4 \ 4 \ 4 \ 4 \ 4 \ 4]$

$$x[n] = 4 \quad \forall n \geq 0 \quad X(e^{j\omega}) = \sum_{n=0}^{N-1} 4e^{-jn\omega}$$

$$X[0] = 32$$

$$\frac{2\pi}{8} \quad \frac{4\pi}{8} \quad \frac{6\pi}{8}$$

$$X[1] = 4 + 4e^{j2\pi/8} + 4e^{j4\pi/8} + 4e^{j6\pi/8} + 4e^{j8\pi/8} + 4e^{j10\pi/8} + 4e^{j12\pi/8} + 4e^{j14\pi/8}$$

$$\cancel{4} + \cancel{4[\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}j]} + \cancel{4j} + \cancel{4(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}j)} + \cancel{-4} + \cancel{4[-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}j]} + \cancel{-4j} + \cancel{4[\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}j]}$$

$$X[2] = 4 \sum_{n=0}^7 e^{j2n(2\pi)/8} \Rightarrow z = re^{j\theta} \Rightarrow \theta = \frac{(2)(2\pi)}{8} = 4 \sum_{n=0}^{N-1} z^n = 0$$

Roots of  $z^n = 1 \Rightarrow n=k \cdot 2\pi j$   $\Rightarrow X[k] = \begin{cases} 32 & k=0 \\ 0 & k \neq 0 \end{cases}$

10.1 C)  $X[n] = 8f[n] = X(e^{j\omega}) = 8$

$$\boxed{X[k] = 8 \quad \forall k}$$

10.1 D)  $X[n] = [3 \ 1 \ 3 \ 1 \ \dots]$

$$X[n] = 2 + \cos(\pi n)$$

$$X(e^{j\omega}) = 2f[n] + f[n-\pi] + f[n+\pi]$$

$$X[0] = 2$$

$$X[1] = 0 \quad \omega = 2\pi/8$$

$$X[2] = 0 \quad 4\pi/8$$

$$X[3] = 0 \quad 6\pi/8$$

$$\boxed{X[4] = 1 \quad 8\pi/8}$$

$$\boxed{X[k] = 8 \begin{cases} 2 & k=0 \\ 1 & k=4 \\ 0 & otherwise \end{cases}}$$

$$[0.1e) X[k] = 4 \text{ for } \Rightarrow X(e^{j\omega}) = 4$$

$$X[n] = 4 \delta[n] \Rightarrow X[n] = \begin{cases} 4 & n=0 \\ 0 & n \leq 7 \end{cases}$$

$$[0.1f) X[k] = [8, 0, 0, 0, 0, 0, 0, 0]$$

$$X(e^{j\omega}) = 8 \delta[\omega] \Rightarrow \text{DC offset}$$

$$X[n] = [1 1 1 1 1 1 1 1]$$

$$[0.1g) X[k] = 9 \quad 5j \quad -1 \quad -5j$$

$$X[0] = \frac{1}{4} (9 + 5j - 1 - 5j)$$

$$X[1] = \frac{1}{4} (9 + 5j e^{j2\pi(1)(1)/4} - e^{j(2\pi)(2)(1)/4} - 5j e^{j2\pi(3)(1)/4})$$

$$\frac{9-5}{9-5} + 1 - 5 = 0$$

$$X[2] = Y_4 (9 + 5j e^{j4\pi/4} - e^{j4\pi(2)/4} - 5j e^{j4\pi(3)/4})$$

$$9 + -5j - 1 + 5j = 2$$

$$X[3] = Y_4 (9 + 5j e^{j6\pi/4} - e^{j6\pi(2)/4} - 5j e^{j6\pi(3)/4})$$

$$Y_4 (9 + 5j(-j)) + 1 - 5j(j) =$$

$$X[n] = 2 \quad 0 \quad 2 \quad 5$$

$$[0.1h) X[k] = [0 \quad 0 \quad 8 \quad 0 \quad 0 \quad 0 \quad 8 \quad 0]$$

$$@ \omega = 4\pi/8 \quad X(e^{j\omega}) = 8$$

$$@ \omega = 12\pi/8 \quad X(e^{j\omega}) = 8 \quad 3\pi/2$$

$$X(e^{j\omega}) = 8 \delta[\omega - \pi/2] + 8 \delta[\omega + \pi/2]$$

$$X[n] = [8 \cos(\pi/2n) + 8 \cos(-\pi/2n)] \forall n$$

$$X[n] = [2 \quad 0 \quad -2 \quad 0 \quad 2 \quad 0 \quad -2 \quad 0]$$

$$10.2 A+B \times [k] = 30 \quad 6+jB \quad 0 \quad 0 \quad 0 \quad 12 e^{j\phi}$$

$k$	$\omega$	$X[k]$
0	0	30
1	$2\pi/6$	$6+jB$
2	$4\pi/6$	0
3	$6\pi/6$	0
4	$8\pi/6$	0
5	$10\pi/6$	$12e^{j\phi}$

conjugates

$$G - jB = 12 e^{j\phi}$$

$$12(\cos(\phi) + j\sin(\phi))$$

$$12 \cos \phi = 6$$

$$\boxed{\phi = -\pi/3}$$

$$-jB = j\sin(\pi/3)$$

$$\boxed{B = \frac{\sqrt{3}}{2}}$$

$$10.2 C) X[k] = 30 \quad 12e^{j\pi/3} \quad 0 \quad 0 \quad 0 \quad 12e^{j(-\pi/3)}$$

$$X[0] = \frac{1}{6}(30 + 12e^{j\pi/3} + 0 + 0 + 0 + 12e^{j(-\pi/3)})$$

$$\frac{1}{6}(42) = 7$$

$$X[1] = \frac{1}{6}(30 + 12e^{j\pi/3} e^{j2\pi/6} + 12e^{j(-\pi/3)} e^{j2\pi(5)/6} \\ 12e^{j4\pi/3})$$

$$= 3$$

$$X[2] = \frac{1}{6}(30 + 12e^{j\pi/3} e^{j2\pi(2)/6} + 12e^{j(-\pi/3)} e^{j(2\pi)(5)(2)/6}) \\ \frac{1}{6}(30 + 12e^{j\pi} + 12e^{j\pi/3})$$

$$X[2] = 1$$

$$X[3] = \frac{1}{6}(30 + 12e^{j\pi/3} e^{j2\pi(3)/6} + 12e^{j(-\pi/3)} e^{j2\pi(5)(3)/6}) \\ \frac{1}{6}(30 + 12e^{j5\pi/3} + 12e^{j11\pi/3})$$

$$= 3$$

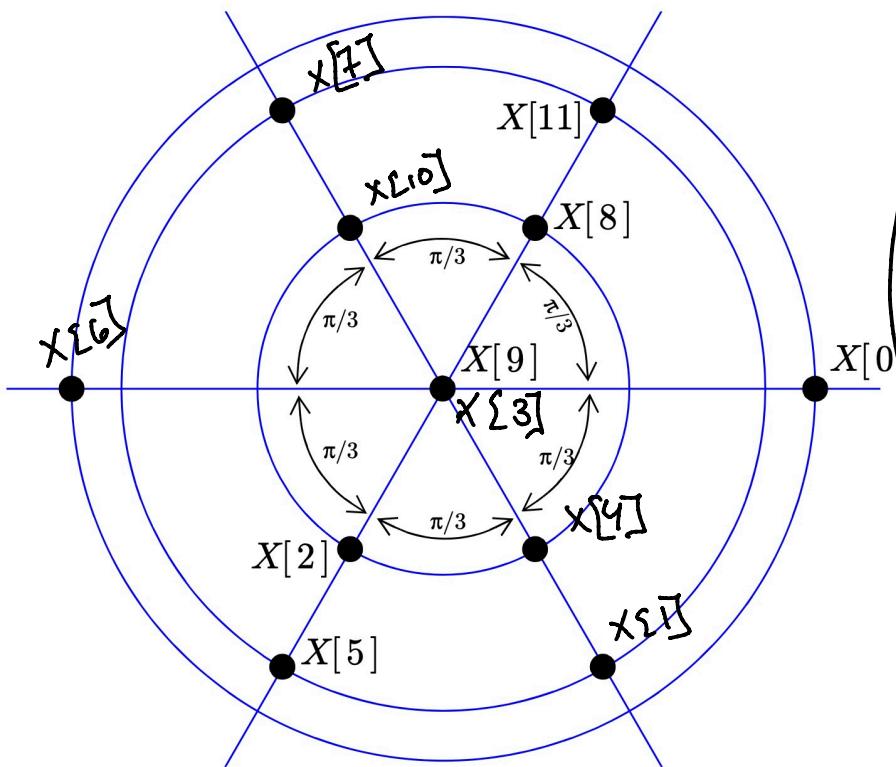
$$X[4] = \frac{1}{6}(30 + 12e^{j\pi/3} e^{j2\pi(4)/6} + 12e^{j(-\pi/3)} e^{j2\pi(5)(4)/6}) \\ \frac{1}{6}(30 + 12e^{j7\pi/3} + 12e^{j13\pi/3})$$

$$= 7$$

$$X[5] = \frac{1}{6}(30 + 12e^{j2\pi} + 12e^{j8\pi}) = 9$$

$$\boxed{X[0] = [7 \quad 3 \quad 1 \quad 3 \quad 7 \quad 9]}$$

10.3 A



$$\begin{aligned}
 X[0] &= 2 \\
 X[1] &= -\gamma_2 - \delta_3 \\
 X[2] &= \sqrt{3}(-\gamma_2 - \delta_3/2)j \\
 X[3] &= \gamma_2 + \delta_3/2j \\
 X[4] &= 0 \\
 X[5] &= \sqrt{3}(\frac{1}{2} + \frac{\sqrt{3}}{2}j) \\
 X[6] &= \sqrt{3}(\frac{1}{2} - \frac{\sqrt{3}}{2}j) \\
 X[7] &= -\gamma_2 + \delta_3/2j \\
 X[8] &= \sqrt{3}(\gamma_2 + \delta_3/2)j \\
 X[9] &= \gamma_2 + \delta_3
 \end{aligned}$$

10.3 B)  $X[9] = X[3]$

10.3 C)  $X[n] = [0 \ 1 \ 0 \ | \ 0 \ 0 \ 0]$

10.4 A) Consider Symm. of DFTs state  $X[k] = X^*[N-k]$

$$X[33] = X[1024 - 33] \Rightarrow X[33] = X[993]$$

10.4 B)  $\begin{bmatrix} 1 & b & 1 \end{bmatrix}$  is a Nullity filled + can be thought of as nullity 1 frequency

for a 1024 pt DFT,  $X[33]$  is  $X(e^{j\omega})$

$$\text{Sampled at } \omega = \left(\frac{2\pi}{1024}\right)(33) = \frac{662}{1024}\pi \approx \frac{331}{512}\pi$$

$$b = \cos(\omega) = \boxed{-2 \cos\left(\frac{331}{512}\pi\right) \approx -0.8882}$$

$$10.5 A) a_0 = \frac{1}{c} (2 + 0 + 2 + 6 + 2 + 0) = \boxed{12}$$

$$a_1 = \frac{1}{6} (2 + 2 e^{-j(1)2\pi(2)/6} + 6 e^{-j(1)2\pi(2)/6} + 2 e^{-j2\pi(1)(4)/6})$$

$$2 + 2e^{-j2\pi/3} + 6e^{-j\pi} + 2e^{-j4\pi/3}$$

$$2 - 1 - 6 - 1 = -\frac{6}{6} = \boxed{-1}$$

$$a_2 = 1 \quad a_3 = 0 \quad a_4 = 1 \quad a_5 = -1$$

10.5 b)

$$X[n] = a_{-5} e^{-j2\pi(-5)n/6} + a_5 e^{-j2\pi(5)n/6} \rightarrow 2a_5 \cos\left(\frac{2\pi(5)n}{6}\right) \rightarrow 2a_5 \cos\left(\frac{5\pi}{3}n\right) + 2a_5 \cos\left(+\frac{\pi}{3}n\right)$$

$$a_{-4} e^{-j2\pi(-4)n/6} + a_4 e^{-j2\pi(4)n/6} \rightarrow 2a_4 \cos\left(\frac{2\pi(4)n}{6}\right) \rightarrow 2a_4 \cos\left(\frac{4\pi}{3}n\right) + 2a_4 \cos\left(+\frac{2\pi}{3}n\right)$$

$$a_{-3} e^{-j2\pi(-3)n/6} + a_3 e^{-j2\pi(3)n/6} \rightarrow \cancel{2a_3 \cos\left(\frac{2\pi(3)n}{6}\right)} \rightarrow 0$$

$$a_{-2} e^{-j2\pi(-2)n/6} + a_2 e^{-j2\pi(2)n/6} \rightarrow 2a_2 \cos\left(\frac{2\pi(2)n}{6}\right) \rightarrow 2a_2 \cos\left(\frac{2\pi}{3}n\right)$$

$$a_{-1} e^{-j2\pi(-1)n/6} + a_1 e^{-j2\pi(1)n/6} \rightarrow 2a_1 \cos\left(\frac{2\pi(1)n}{6}\right) \rightarrow 2a_1 \cos\left(\frac{\pi}{3}n\right)$$

$$a_0 e^{-j2\pi(0)n/6} \Rightarrow a_0 = B$$

$$(2a_1 + 2a_5) \cos\left(\frac{\pi}{3}n\right) + (2a_2 + 2a_4) \cos\left(\frac{2\pi}{3}n\right) - 2 + 2 \quad 4 \cos\left(\frac{2\pi}{3}n\right) - 4 \cos\left(\frac{\pi}{3}n\right) +$$

$$\boxed{B=2}$$

$$A_1=4 \quad \omega_1 = 2\pi/3$$

$$A_2=4 \quad \omega_2 = \pi/3$$

$$10.5 c) X[k] = [12 \quad -6 \quad 6 \quad 0 \quad 6 \quad -6]$$

$$10.5 d) a_{12} = \frac{1}{N} X[k]$$

10.5 e) verified