PROBLEM 1.1.* Simplify each of the following expressions and write the answer in both rectangular form and polar form. The first one is done for you. Summarize your answers in a table, like the one shown below, but also describe your approach and **show your work** in each case.

	RECTANGULAR	POLAR
(a) $z = j(2 + 2j)$	$-2 + 2j$	$2\sqrt{2}e^{j0.75\pi}$
(b) $z = (1 + j)^3$	$-2 + 2j$	$2\sqrt{2}\, e^{(3\pi/4)j}$
(c) $z = e^{j12\pi}e^{j\pi/6} + e^{j15\pi}e^{-j\pi/6}$	$0 + j$	$e^{(\pi/2)j}$
(d) $z = \dfrac{3 - \sqrt{3}j}{3 + \sqrt{3}j}$	$\frac{1}{2} - \frac{\sqrt{3}}{2}j$	$e^{(-\pi/3)j}$
(e) $z = (1 + j)^4 - (1 - j)^4$	0	0
(f) $z = j\cos(\pi/3) - \sin(\pi/3)$	$-\frac{\sqrt{3}}{2} + \frac{1}{2}j$	$e^{(5\pi/6)j}$

$\frac{5\pi}{6}$

$$(1+j)^3 \Rightarrow \left(\sqrt{2}\, e^{\pi/4\, j}\right)^3 \Rightarrow 2\sqrt{2}\; e^{3\pi/4\, j}$$

$\tan^{-1}(1) = \pi/4 \qquad x = 2\sqrt{2}\cos\left(\frac{3}{4}\pi\right) + 2\sqrt{2}\sin\left(\frac{3\pi}{4}\right)$

$$e^{12\pi j}e^{\frac{\pi}{6}j} + e^{15\pi j}e^{-\pi/6\, j} \Rightarrow e^{j\left(\frac{73\pi}{6}\right)} + e^{j\left(\frac{89}{6}\pi\right)}$$

$$= \cos\left(\frac{73\pi}{6}\right) + j\sin\left(\frac{73\pi}{6}\right) + \cos\left(\frac{89\pi}{6}\right) + j\sin\left(\frac{89}{6}\pi\right)$$

$$\frac{\sqrt{3}}{2} + j\frac{1}{2} + -\frac{\sqrt{3}}{2} + \frac{1}{2}j \Rightarrow 0 + j = e^{\frac{1}{2}\pi}$$

$\sqrt{9+3}$

$\operatorname{atan}\left(\frac{\sqrt{3}}{3}\right)$

$$z = \frac{3-\sqrt{3}j}{3+\sqrt{3}j} \Rightarrow \frac{\cancel{\sqrt{12}}\, e^{-\frac{\pi}{6}j}}{\cancel{\sqrt{12}}\, e^{\pi/6\, j}} \Rightarrow e^{\left(-\frac{\pi}{6} - \frac{\pi}{6}\right)j} \Rightarrow e^{-\frac{\pi}{3}j} \Rightarrow \cos\left(-\frac{\pi}{3}\right) + j\sin\left(-\frac{\pi}{3}\right)$$

$$\frac{1}{2} - \frac{\sqrt{3}}{2}j$$

$$z = (1+j)^4 - (1-j)^4$$

$$\left(\sqrt{2}e^{\pi/4}\right)^4 - \left(\sqrt{2}\, e^{-\pi/4 j}\right)^4$$

$$4\, e^{\pi j} - 4e^{-\pi j}$$

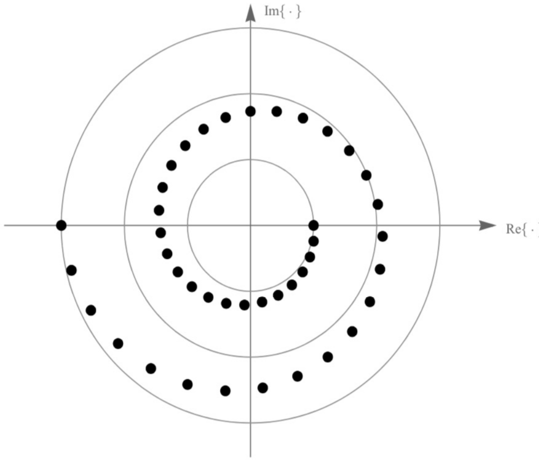
$$4\cos(\pi) + 4j\sin(\pi) - 4\cos(-\pi) - 4j\sin(-\pi)$$

$$-4 + 0 - (-4) - 0 \Rightarrow 0$$

$$j\cos(\pi/3) - \sin(\pi/3)$$

$$\frac{1}{2}j - \frac{\sqrt{3}}{2}$$

PROBLEM 1.2.* The figure below shows the locations in the complex plane of an unspecified complex number z when raised to the powers of $k \in \{0, 1, 2, 3, \ldots 40\}$. In other words, it show the locations in the complex plane of z^0, z^1, z^2, z^3, z^4, z^5, ... and z^{40}:



Find z. Specify the anwer in polar form, $z = re^{j\varphi}$, with $r > 0$ and $\varphi \in (-\pi, \pi]$ to ensure that the answer is unique.

More details regarding the figure:

- The axes are not labeled.
- The large gray circles are centered at the origin and are concentric with radii r_1, $2r_1$, and $3r_1$, for some unspecified r_1.
- The innermost point intersects both the real axis and the inner concentric circle, while the outermost point intersects both the real axis and the outer circle.

$$z^0 = r_1 \Rightarrow (r e^{j0\pi})^0 \Rightarrow r^0 e^0 \Rightarrow r^0 = 1 = r_1$$

$$z^{40} = -3r_1 \Rightarrow (re^{j\theta})^{40} \Rightarrow r^{40} e^{j40\theta} = -3r_1$$

$$r^{40} = 3$$

$$e^{j40\theta} = -1$$

$$\cos(40\theta) + j\sin(40\theta) = -1$$

$$40\theta = 3\pi$$

$$\boxed{\theta = \frac{3\pi}{40}}$$

$$\sqrt[40]{3} = \frac{1255}{1221}$$

$$\boxed{z = \frac{1255}{1221} e^{j\left(\frac{3\pi}{40}\right)}}$$

PROBLEM 1.3.* The equation $|z - 6 - 8j| = 10$ specifies a shape in the complex plane.

$z = 6 + 8j \Rightarrow 10e^{.9273j}$

(a) Describe the shape, in words.

(b) Draw a picture of the shape in the complex plane. Carefully label the scale of both axes.

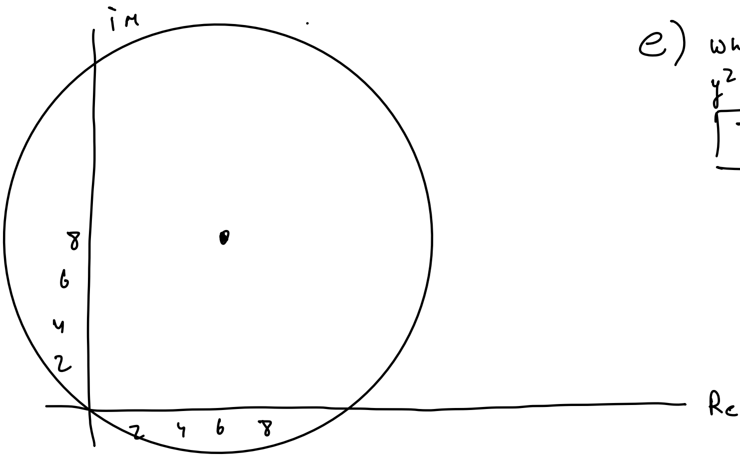
(c) Of all of the values of z that satisfy this equation, which has the largest real part?

(d) Specify all of the real values of z that satisfy this equation, if any.

(e) Specify all of the imaginary values of z that satisfy this equation, if any.

a) This is a circle centered @ $10e^{.9273j}$ w/ radius 10

b)



e) when $x = 0$

$y^2 - 16y = 0$

$\boxed{Im\{z\} = 0 + 16}$

c) $z = Re^{.9273j}$

$Re\{z\} = r\cos(.9273j)$

This is the center

largest $Re\{z\}$ will be center + radius

$\Rightarrow$ z w/ largest Real is

$\boxed{z = 10e^{.9273j} + 10}$

D) $z = x + yi \Rightarrow |x + yi - 6 - 8j| = 10 \Rightarrow |(x-6) + (y-8)j| = 10$

$\Rightarrow \left(\sqrt{(x-6)^2 + (y-8)^2}\right)^2 = (10) \Rightarrow (x-6)^2 + (y-8)^2 = 10^2$

all Real of z is when $y = 0$

$\Rightarrow x^2 - 12x + 36 + 54 \Rightarrow x^2 - 12x + 0 \Rightarrow \boxed{Re\{z\} = 0 + 12}$

PROBLEM 1.4.* Consider the complex number $z = e^{j0.4\pi}$.

(a) Sketch the locations of z, z^2, z^3, z^4, and z^5 in the complex plane. Include the unit circle in the sketch for reference.

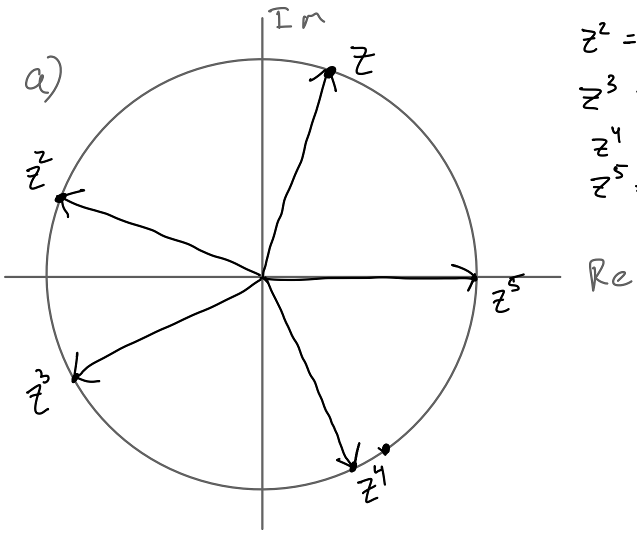
(b) Specify the three smallest positive integers n for which $z^n = 1$.

(c) Specify the three smallest positive integers N for which $\sum_{k=0}^{N-1} z^k = 1$.

(d) Specify the three smallest positive integers m for which $\sum_{k=0}^{4} z^{mk} = 0$.

(e) Specify the three smallest positive integers m for which $\sum_{k=0}^{4} z^{mk} \neq 0$.

a)



$z^2 = e^{j(.8)\pi}$
$z^3 = e^{j(1.2)\pi}$
$z^4 = e^{j(1.6)\pi}$
$z^5 = e^{j(2\pi)}$

$z^{0,2,4,6,8}$
$z^6 = e^{j(2.4\pi)}$
$z^7 = e^{j(2.8\pi)}$
$z^8 = e^{j(3.2\pi)}$
$z^9 = e^{j(3.6\pi)}$

0 4 8 12 16

$z^{0,3,6,9,12}$
z

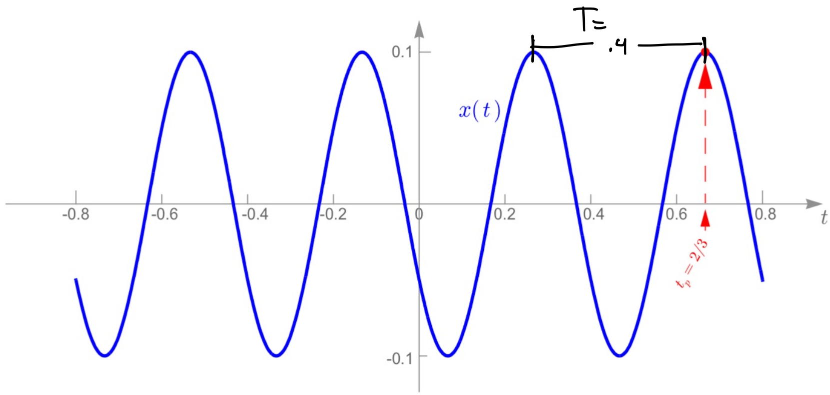
b) $n \in \{5, 10, 15\}$

c) $N \in \{1, 6, 11\}$

D) $m \in \{1, 2, 3\}$

E) $m \in \{5, 10, 15\}$

PROBLEM 1.5.* The waveform shown below can be represented by $x(t) = \mathrm{Re}\{Xe^{j\omega_0 t}\}$:



As indicated in the figure, it achieves a peak at time $t_p = 2/3$.

(a) Find $\omega_0 \geq 0$.

(b) Find the complex phasor X, expressed in polar form.

(c) The waveform can also be written in *standard form* as $x(t) = A\cos(2\pi f_0 t + \varphi)$. Find the amplitude A, the Hertzian frequency f_0 (in Hz), and the phase φ. (*Standard form* means that $A \geq 0$, $f_0 \geq 0$, and $-\pi < \varphi \leq \pi$, which makes the answers unique.)

(d) There are infinitely many values for t_0 for which the given signal can be written as:

$$x(t) = A\cos(2\pi f_0(t - t_0)).$$

Of these, specify the three that are *smallest*, when constrained to be positive ($t_0 > 0$).

---

a) $T = \frac{2\pi}{\omega} \Rightarrow \omega_0 = \frac{2\pi}{T} \Rightarrow \frac{2\pi}{.4} = \boxed{5\pi \text{ rad/sec}}$

$X = Ae^{j(-2\pi f t_0)}$

B) $\omega_0 = 5\pi$ rad/sec $\quad x(t) = Xe^{j5\pi t}$ $\quad \boxed{X = .1e^{\frac{10\pi}{3}j}}$ where ϕ is Phase of $x(t)$

c) $A = 0.1$ $\quad \phi = -2\pi f_0 t_p \Rightarrow -2\pi(2.5)\left(\frac{2}{3}\right) = -\frac{10\pi}{3}$ rad

$f = \frac{\omega}{2\pi} = \frac{5\pi}{2\pi} = 2.5$ Hz

D) $t_0 = \frac{2}{3} - \frac{2}{5} \Rightarrow \frac{10}{15} - \frac{6}{15} = \boxed{\frac{4}{15}, \frac{10}{15}, \frac{16}{15}}$

: :

$$A\cos(\omega(t-t_m)) = A\cos(\omega t + \phi)$$

$$\Rightarrow -\omega t_m = \phi \Rightarrow \phi = -\left(\tfrac{1}{2}\pi\right)(.5)$$

measured Peak of MatlaB Plot

$$\Rightarrow -\tfrac{1}{2}\pi\left(\tfrac{1}{2}\right)$$

$$\Rightarrow \boxed{-\frac{\pi}{4} = \phi}$$

RUDRA Goel 1/13/2024

leoG ardUR

```
%% Your code here
ccsum2 = zeros(1,500);
tt2 = dt*[1:1:500]; % generate the timestamps
for kx = 1:length(XX)
    Ak = abs(XX(kx));
    phik = angle(XX(kx));
    ccsum2 = Ak*cos(2*pi*freq*tt2 + phik);
```

See Following Pages

$t_{p_1} = .3459$ $\qquad t_{p_2} - t_{p_1}$ = Pitch Period = .0074s

$t_{p_2} = .3385$ $\qquad \Delta t_{p21} = .0074$

$(\Delta t_{p21})^{-1} = 135.14$ Hz

See following Pages

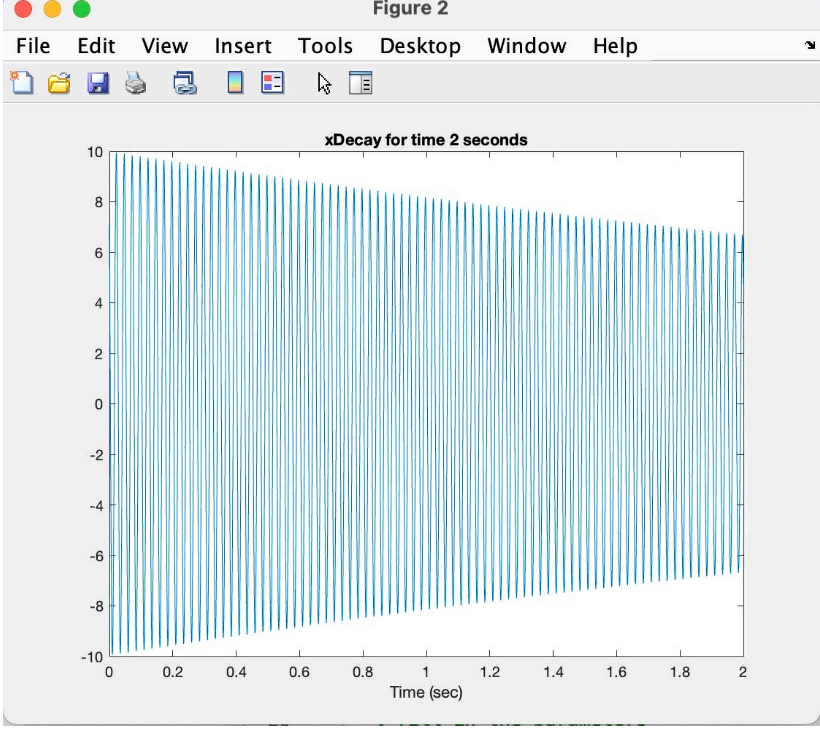


Figure 2
File Edit View Insert Tools Desktop Window Help
xDecay for time 2 seconds
10
8
6
4
2
0
-2
-4
-6
-8
-10
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2
Time (sec)

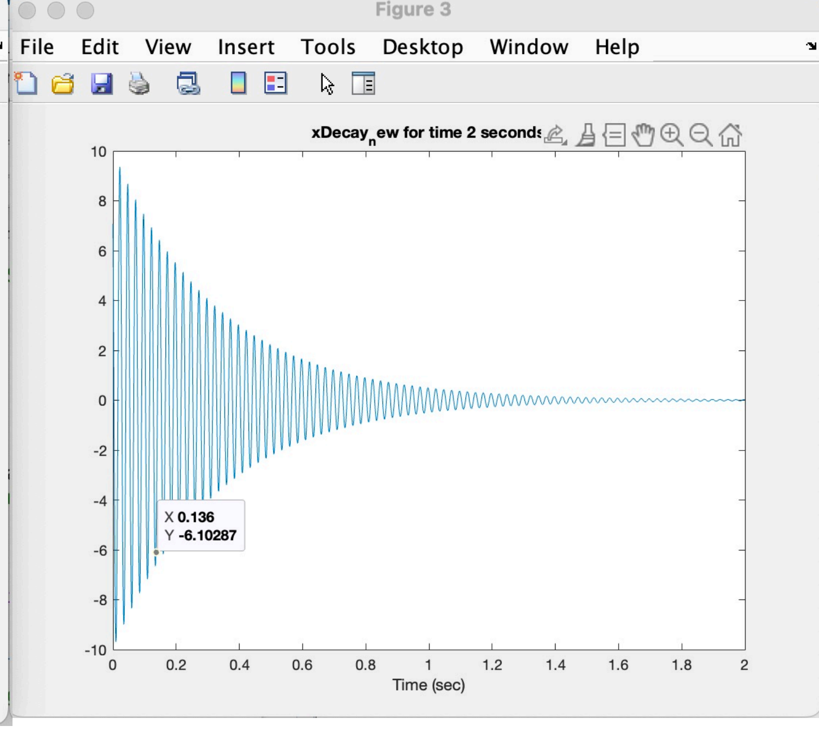


Figure 3
File Edit View Insert Tools Desktop Window Help
xDecay_new for time 2 seconds
10
8
6
4
2
0
-2
-4
-6
-8
-10
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2
Time (sec)
X 0.136
Y -6.10287

P 3.3

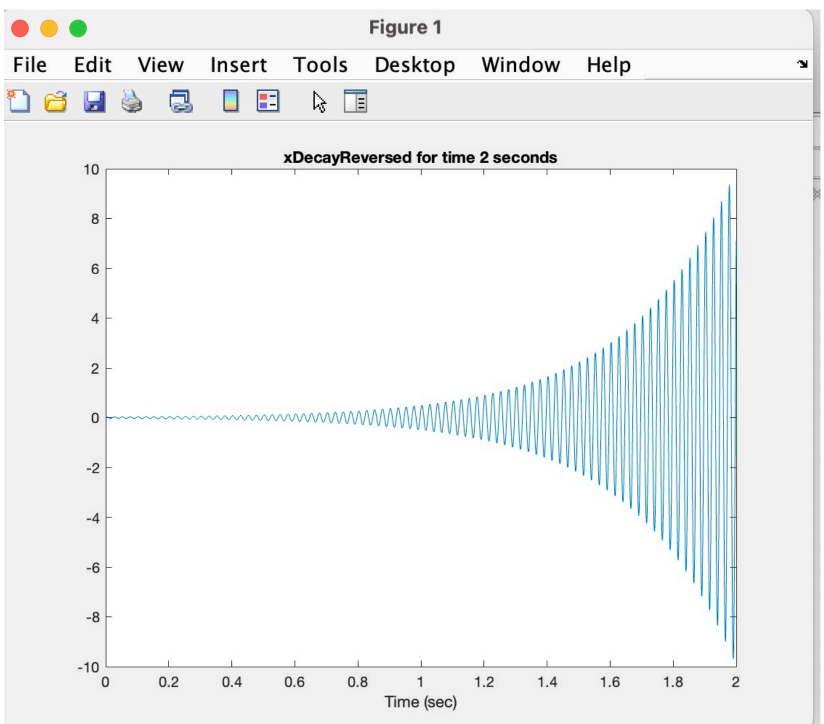


Figure 1
File Edit View Insert Tools Desktop Window Help
xDecayReversed for time 2 seconds
10
8
6
4
2
0
-2
-4
-6
-8
-10
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2
Time (sec)

P 3.5.1

PROBLEM 2.1.* Let $x(t)$ be the following sinsoid plus a constant:

$$2\pi f_0 (t - t_0) = 4\pi t - .4\pi$$

$$x(t) = 10 + 9\cos(4\pi t - 0.4\pi).$$

$$2\pi f_0 t - 2\pi f_0 t_0 = 4\pi t - .4\pi$$

$f_0 = 2 \quad T = \frac{1}{2} = .5 \quad t_0 = \frac{1}{10}$

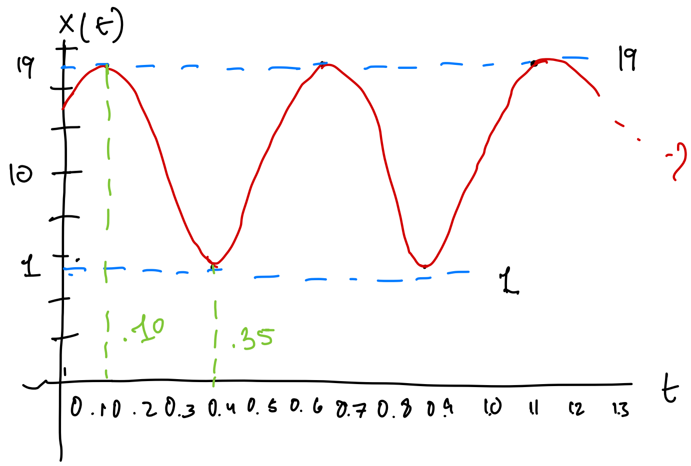
(a) Sketch $x(t)$ versus time t for time in the range $0 < t < 2$, *carefully* labeling important values for the time axis (like the times of the peaks) and important values for the vertical axis (like the value at the peaks and at the valleys).

(b) What is the time of the peak that happens closest to time zero?

(c) Evaluate the following integral:

$$\int_0^7 x(t)dt.$$

a)



b) $\boxed{t = .1 \text{ seconds}}$

c) $\int_0^7 x(t)\,dt \Rightarrow \int_0^7 10 + 9\cos(4\pi t - .4\pi)\,dt \Rightarrow 10\int_0^7 dt + 9\int_0^7 \cos(4\pi t - .4\pi)\,dt$

$70 + \frac{9}{4\pi}\int_0^7 \cos(U)\,dU \qquad U = 4\pi t - .4\pi \qquad dU = 4\pi \qquad \Rightarrow 70 + \frac{9}{4\pi}\left[\sin(4\pi t - .4\pi)\right]_0^7$

$70 + \frac{9}{4\pi}(1. \ldots E{-15}) = \boxed{70}$

PROBLEM 2.2.* Find a pair of sinusoids $s_1(t)$ and $s_2(t)$ that satisfy both of the following for all t:

let $S_1 = r_1 e^{j\phi_1}$

$S_2 = r_2 e^{j\theta_2}$

$$\sqrt{3}\,s_1(t) + s_2(t) = \cos(2048\pi t),$$

$$\sqrt{3}\,s_1(t) - s_2(t) = \sin(2048\pi t).$$

$\frac{e^{j\theta_1} + e^{-j\theta_1}}{2} = \cos(\theta_1)$

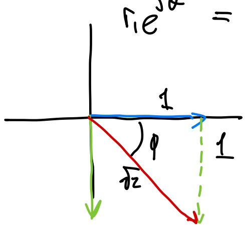
$\frac{e^{j\theta_1} - e^{-j\theta_1}}{2j} = \sin(\theta_1)$

Write your answers in standard form.

$2\sqrt{3}\,S_1(t) = \cos(2048\pi t) + \cancel{\sin(2048\pi t)} + \cos(2048\pi t - \pi/2)$

$r_1 e^{j\theta} = 1e^{j(0)} + 1e^{-j(\pi/2)}$

$\theta = -\pi/4$



$2\sqrt{3}\,r_1 = \sqrt{2}$

$r_1 = \frac{\sqrt{2}}{2\sqrt{3}} \Rightarrow \frac{\sqrt{6}}{6}$

$S_1(t) = \frac{\sqrt{6}}{6} e^{j(-\pi/4)}$

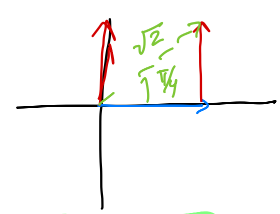
$2S_2(t) = \cos(2048\pi t) - \sin(2048\pi t)$

$e^{0j} - e^{-\pi/2 j}$

$2S_2(t) = e^{0j} + e^{-3\pi/2 j} \quad \theta = \pi/4$

$2r_1 = \sqrt{2} \Rightarrow R_1 = \sqrt{2}/2$

$S_2(t) = \frac{\sqrt{2}}{2} e^{j(\pi/4)}$



$S_1(t) = \frac{\sqrt{6}}{6}\cos(2048\pi t - \frac{\pi}{4})$

$S_2(t) = \frac{\sqrt{2}}{2}\cos(2048\pi t + \pi/4)$

Made with Goodnotes

PROBLEM 2.3.* Suppose that the following equation is true for all t:

$$\sin(2026\pi t + 2026.5\pi) + \cos(2\pi f_0 t - \tfrac{\pi}{6}) + \cos(2\pi f_0(t - t_0)) = A\cos(2\pi f_0 t).$$

(a) Find $f_0 > 0$.

(b) There are multiple possibilities for the value of t_0; find the *smallest* $t_0 > 0$.

(c) Find $A > 0$.

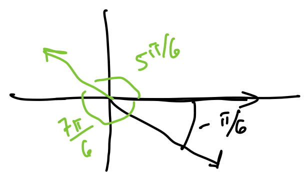
$2026\pi - 2026\pi = \phi = 0$

a) $\cos(2026\pi t + \cancel{2026.5\pi - \frac{\pi}{2}}) + \cos(2\pi f_0 t - \pi/6) + \cos(2\pi f_0(t - t_0)) = A\cos(2\pi f_0 t)$

$f_0 = 1013\ Hz$ $\quad e^{j0} + e^{j(-\pi/6)} + e^{-2026\pi t_0} = Ae^{j(0)}$ $\quad 2026\pi t - 2026\pi t_0$ $\quad \phi = 0$

$A e^{j(0)}$

$A + 0j$

b) 

$-2026\pi t_0 = \frac{5\pi}{6}$ X

$-2026\pi t_0 = \frac{-7\pi}{6} \Rightarrow \boxed{t_0 = \frac{7}{12156}}$

c) $\underbrace{1}_{e^{0j}} + \underbrace{\cancel{\frac{\sqrt{3}}{2}} - \cancel{\frac{1}{2}j}}_{e^{-\pi/6 j}} + \underbrace{-\cancel{\frac{\sqrt{3}}{2}} + \cancel{\frac{1}{2}j}}_{e^{-7\pi/6 j}} = A \Rightarrow \boxed{A = 1}$

PROBLEM 2.4.* Define $x(t)$ as the following sum of sinusoids:

$\pi t - \pi t_0$ $\quad \pi t - \pi t_0 - .1\pi$ $\quad \phi$ $\quad \Rightarrow \phi = -\pi(t_0 + .1)$

$$x(t) = \cos(\pi t) + \cos(\pi t - 0.1\pi) + \cos(\pi t - 0.2\pi) + \cos(\pi t - 0.3\pi) + \cos(\pi t - 0.4\pi).$$

(a) Find the smallest positive value for the delay $t_0 \geq 0$ so that the delayed signal can be written as:

$$x(t - t_0) = A\sin(\pi t)$$

$X(t) = 1 + e^{-.1\pi} + e^{-.2\pi} + e^{-.3\pi} + e^{-.4\pi}$

$\Rightarrow Ae^{-\pi/2} = A\cos(\pi t - \pi/2$

for some unspecified positive constant A.

(b) Find the positive constant $A > 0$ in part (a).

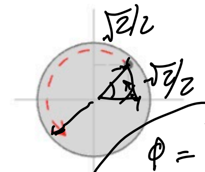
a) $Ae^{-\pi/2 j} = e^{-j\pi t_0} + e^{-j\pi(t_0 + .1)} + e^{-j\pi(t_0 + .2)} + e^{-j\pi(t_0 + .3)} + e^{j\pi(t_0 + .4)}$

$-\frac{5}{2} - \cancel{\frac{1}{2}} = -t_0 - t_0 - .1 - t_0 - .2 - t_0 - .3 - t_0 - .4$

$\Rightarrow -5t_0 - 1 = \frac{-5}{2} \Rightarrow -5t_0 = -\frac{3}{2} \Rightarrow t_0 = 3/10$

b) $4.5201 = A$

PROBLEM 2.5.* The magnetic disk in a hard drive spins in a counterclockwise direction at a rate of 7200 rpm (rotations per minute). The disk has a defect that appears as a large black dot that is a distance 4 (in unspecified units) from the center of the disk. Let $x(t)$ and $y(t)$ denote the horizontal and vertical coordinates, respectively, of the black dot at time t.



$\phi = \frac{\pi}{4}$

$$\frac{7200 \text{ rot}}{1 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}}$$

$$\frac{7200}{60} = \frac{\text{rot}}{\text{sec}} = \text{Hz}$$

$$120 \text{ Hz} = f_0$$

The figure shows a snapshot taken at time 0, at which time the horizontal and vertical coordinates of the black dot are identical, as illustrated in the figure, so that $x(0) = y(0)$.

(a) Write the sum $s(t) = x(t) + y(t)$ as a function of t as a single sinusoid, in standard form.

$e^{j(0)}$ $\quad e^{j(\pi/2)}$ $\quad a^2+b^2 = 4$ $\quad \cos(240\pi t + \frac{\pi}{2})$

(b) Find the smallest positive t_0 for which the following equation is true for all time t.

$$x(t) + y(t - t_0) = 0.$$

$\rightarrow \cos(240\pi t - 240\pi t_0 + \frac{\pi}{2})$

$\sin(240\pi t - 240\pi t_0 + \pi/4)$

(c) Find the real positive constants β and A and f_0 for which for which the following equation is true for all t:

$$x(t) + \beta y(t) = A\cos(2\pi f_0 t - \frac{\pi}{12}).$$

$\frac{\sqrt{2}}{2} A = 4$

$A = \frac{8}{\sqrt{2}}$

$= 4\sqrt{2}$

$x\sqrt{2}$ $\quad x$ $\quad x$

$x\sqrt{2} = 4$

$\Rightarrow x = 4/\sqrt{2}$

x

$\frac{4\sqrt{2}}{2} = \boxed{2\sqrt{2}}$

$4 \cdot 2 = 8$

a) $S(t) = x(t) + y(t)$

$$x(t) = A\cos(240\pi t + \pi/4)$$

$$y(t) = B\sin(240\pi t + \pi/4)$$

$$\sqrt{\frac{A^2}{2} + \frac{B^2}{2}} = 4 \Rightarrow \frac{A^2}{2} + \frac{B^2}{2} = 16$$

$x(0) = y(0)$

$\frac{A\cancel{\sqrt{2}}}{\cancel{2}} = \frac{B\cancel{\sqrt{2}}}{\cancel{2}} = A = B$

$A^2 + B^2 = 32$

$2A^2 = 32$

$A^2 = 16 \Rightarrow A = 4$

$x(t) = 4\cos(240\pi t + \pi/4) \rightarrow 4e^{j\pi/4}$

$y(t) = 4\sin(240\pi t + \pi/4) \rightarrow 4\cos(240\pi t + \pi/4 - \pi/2) \rightarrow 4e^{-j\pi/4}$

$S(t) = 4e^{j(\pi/4)} + 4e^{-j(\pi/4)} \Rightarrow 4(e^{j\pi/4} + e^{-j\pi/4})$

$= 4(2\cos(240\pi t + \pi/4)) \Rightarrow 8\cos(240t + \pi/4)$

b) $S(t) = 4\cos(240\pi t + \pi/4) + 4\cos(240\pi t - 240\pi t_0 - \pi/4)$

$\cancel{240\pi t} + \frac{\pi}{4} - \pi = \cancel{240\pi t} - 240\pi t_0 - \pi/4$

$\frac{\pi}{2} - \pi = -240\pi t_0$

$-\cancel{\pi}/2 = -240\cancel{\pi} t_0$

$\Rightarrow \boxed{\frac{1}{480} = t_0}$

c) $4\cos(240\pi t + \pi/4) + \beta 4\cos(240\pi t - \pi/4) = A\cos(240\pi t - \pi/12)$ $\quad f_0 = 120$

$4e^{j\pi/4} + \beta 4 e^{j(-\pi/4)} = Ae^{j(-\pi/12)}$

$4\cos(\pi/4) + j4\sin(\pi/4) + \beta 4\cos(-\pi/4) + j\beta 4\sin(-\pi/4) = A\cos(-\pi/12) + Aj\sin(-\pi/12)$

$2\sqrt{2} + j2\sqrt{2} = -B2\sqrt{2} = jB2\sqrt{2} + A\cos(-\pi/12) + Aj\sin(-\pi/12)$

$A\cos(-\pi/12) - B2\sqrt{2} = 2\sqrt{2}$

$Aj\sin(-\pi/12) + Bj2\sqrt{2} = j2\sqrt{2}$

$\boxed{A = 8 \quad B = 1.7321}$

Made with Goodnotes

PROBLEM 3.1.* Consider the signal shown below. It achieves a peak value of 17 at time –0.5. Sketch the (two-sided) *spectrum* for this signal, carefully labeling the frequency (in Hz) and complex coefficient for each line.

$x(t) = A\cos(2\pi f_0(t - t_D)) + C$

$A = 3.5$

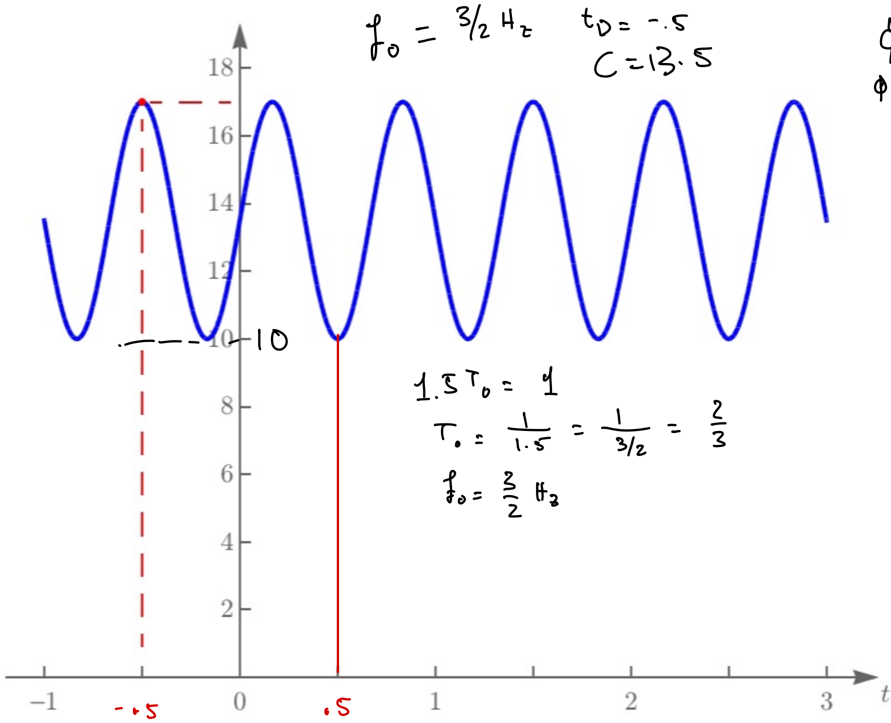
$f_0 = 3/2$ Hz $\quad t_D = -.5$

$C = 13.5$

$\phi = -2\pi f_0 t_D$

$\phi = 3\pi(\frac{1}{2}) = \frac{3}{2}\pi$ rad $= \frac{3}{2}\pi - 2\pi \Rightarrow -\frac{\pi}{2}$

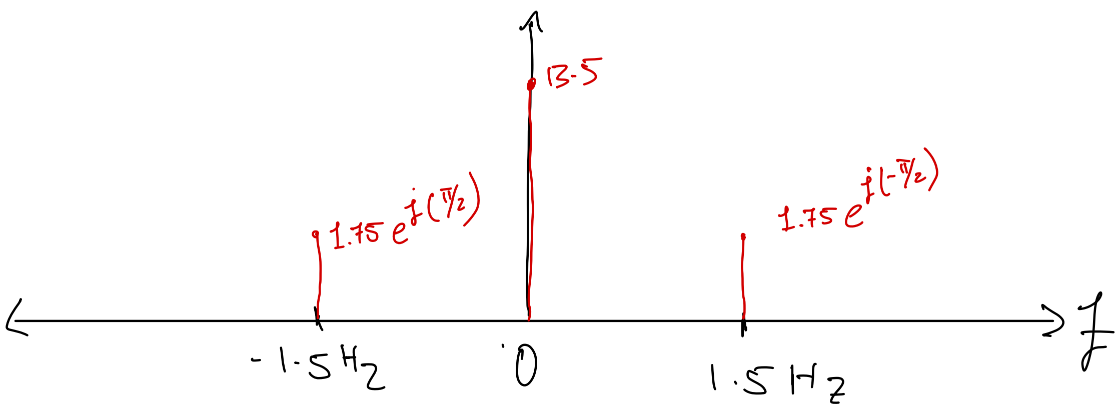
$X = 3.5\, e^{j(-\pi/2)}$



$1.5\, T_0 = 1$

$T_0 = \frac{1}{1.5} = \frac{1}{3/2} = \frac{2}{3}$

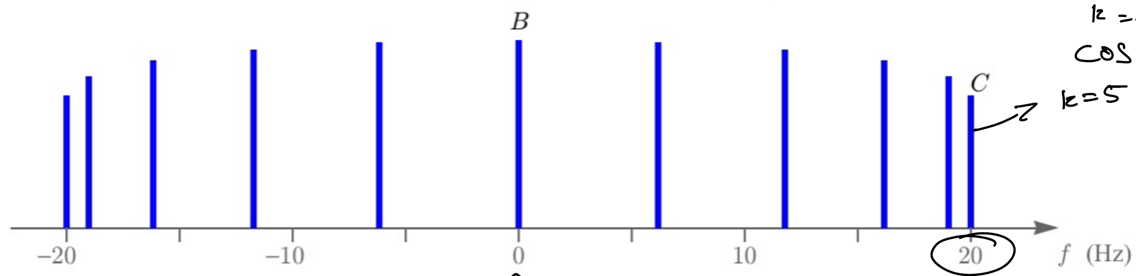
$f_0 = \frac{3}{2}$ Hz



$e^{j\omega t}$ $k=5$ $e^{j2\pi(20\cdot\sin(\frac{\pi}{2}))t}$ $\Rightarrow e^{j(2\pi)(20)t}$

PROBLEM 3.2.* Suppose the (unlabeled) spectrum for $x(t) = \sum_{k=-5}^{5} \cos(0.05\pi k)e^{jA\pi\sin(0.1\pi k)t}$ is as shown below, where A is real and positive but otherwise unspecified:

$k=0$



$k=5 \Rightarrow \cos(.05\pi(5))$

$k=5 \Rightarrow \cos(\pi/4)$

$\sqrt{2}/2$

$1e^{jA\pi(0)t} \Rightarrow 1$

(a) Find A.

(b) Find B.

(c) find C.

A) $A = 40$

C) $C = \frac{\sqrt{2}}{2}e^{j(0)}$

$\Rightarrow \frac{\sqrt{2}}{2}$

B) $B = 1$

$\omega = A\pi\sin(.1\pi k)$

$\omega = 2\pi(\frac{A}{2}\sin(.1\pi k))$

$f_0 = \frac{A}{2}\sin(.1\pi k)$

$20 = \frac{A}{2}\sin(\overset{.5\pi}{\cancel{.1\pi k}}) \rightarrow 1 \Rightarrow A = 40$

as $k\uparrow$, $\sin(.1\pi k)\uparrow$

PROBLEM 3.3.* For each signal below, sketch its spectrum.
Carefully label the frequency (in Hz) and complex coefficient for each line.

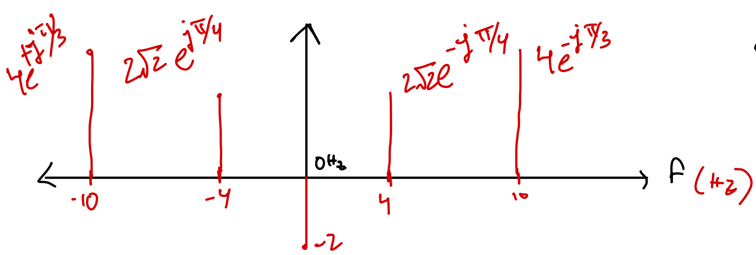
$4\cos(2\pi(4)t - \frac{\pi}{2})$ $4\cos(2\pi(4)t)$ $8\cos(2\pi(10)t - \pi/3) \Rightarrow 8e^{-\pi/3 j}$

(a) $x(t) = -2 + 4\sin(8\pi t) + 4\cos(8\pi t) + 8\cos(20\pi t - \frac{\pi}{3})$.

(b) $x(t) = \sum_{k=0}^{4} 6\cos(10\pi t + k0.5\pi)$.

(c) $x(t) = \sum_{k=0}^{4} 6\cos(10k\pi t + 0.5\pi)$.

a)



$4e^{j(-\pi/2)} + 4$ $x(t) = -2 + 4\sqrt{2}e^{-j(\pi/4)}e^{-j2\pi(4)t} + 8e^{-\pi/3 j}e^{j2\pi(10)t}$

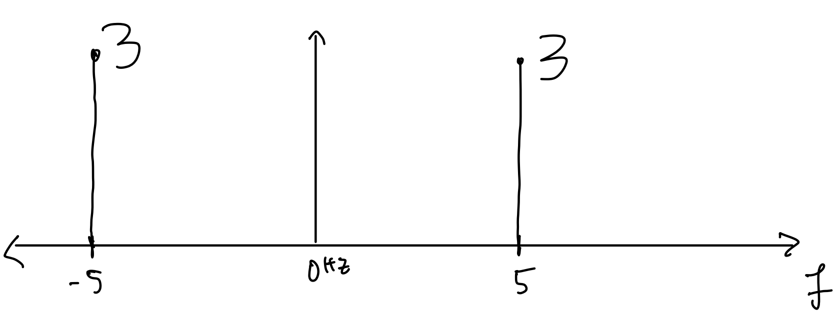
$4\cos(-\frac{\pi}{2}) + 4j\sin(-\frac{\pi}{2}) + 4$

$\cancel{4(0)} \rightarrow 0 + 4(j)(-1) + 4$

$4 - 4j \Rightarrow \phi = -\pi/4$

$A = \sqrt{32} \Rightarrow 4\sqrt{2}$

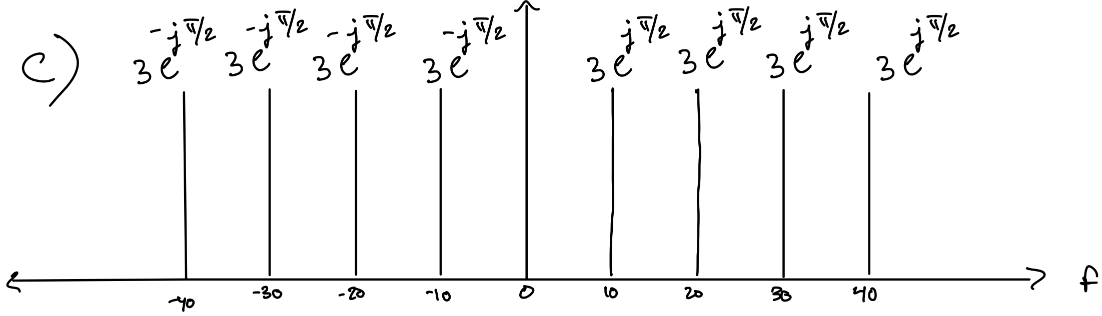
b)



$\sum_{k=0}^{4} 6e^{jk\pi/2} \Rightarrow [1 + \cancel{e^{j\frac{\pi}{2}}} + \cancel{e^{j\pi}} + \cancel{e^{j3\pi/2}} + \cancel{e^{j2\pi}}]6$

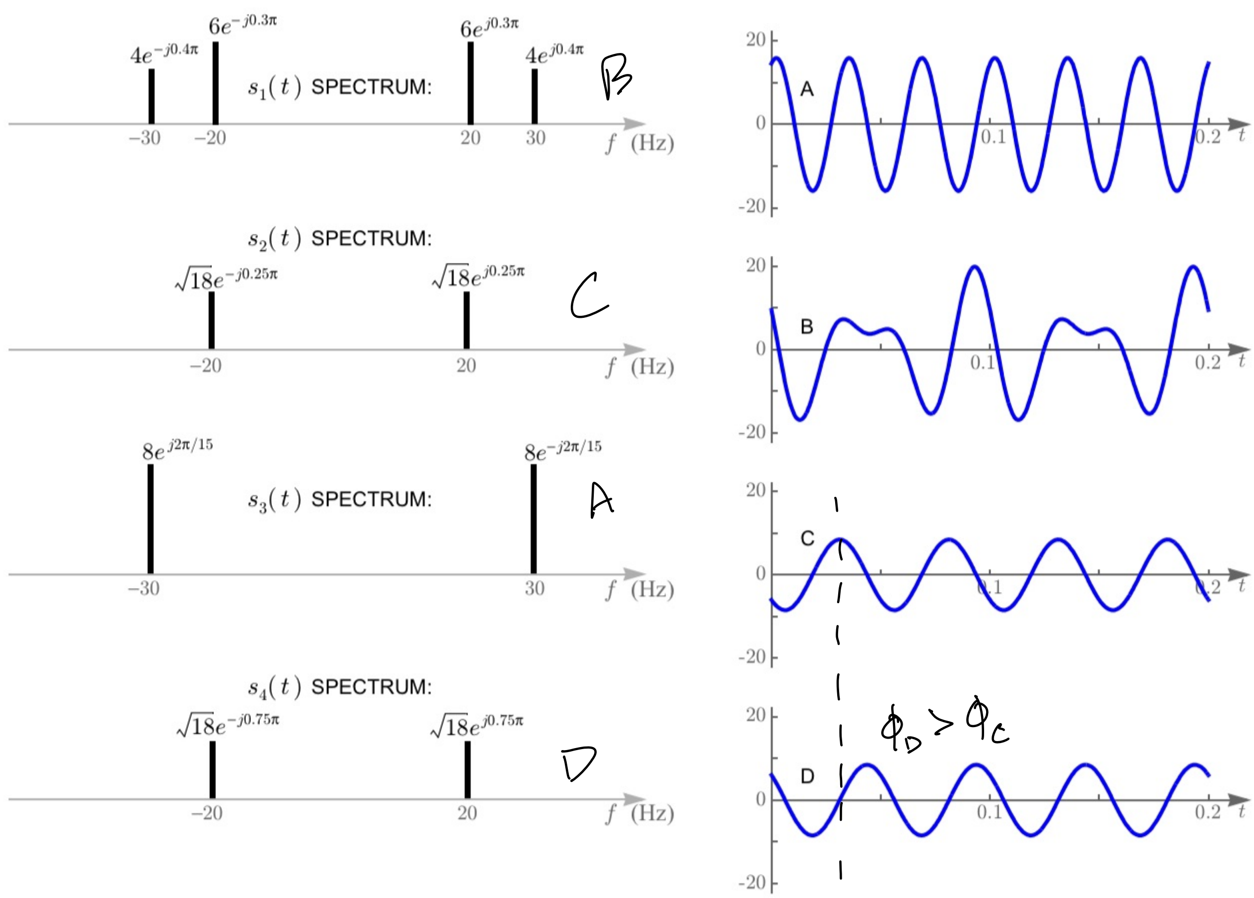
$\Rightarrow [6] \Rightarrow x(t) = 6e^{j(2\pi)(5)t}$

c)



$$x(t) = 0 + 6e^{j\frac{\pi}{2}}\left[e^{j2\pi(10)t} + e^{j2\pi(20)t} + e^{j2\pi(30)t} + e^{j2\pi(40)t}\right]$$

PROBLEM 3.4.* Shown on the left are the spectra for four different signals $s_1(t)$, $s_2(t)$, $s_3(t)$, and $s_4(t)$:



(a) Shown on the right are the corresponding time-domain plots, labeled A through D, but in a scrambled order. Match each spectrum to its corresponding plot. *Justify each answer.* In addition, summarize your answers in a table like this, by writing letters from A through D into the second column:

$s_1(t)$	B	→ only non simple sinusoid
$s_2(t)$	C	→ highest f
$s_3(t)$	A	→ $\phi_D > \phi_C$
$s_4(t)$	D	→

(b) Write an equation for $s_2(t)$ in standard form, $s_2(t) = A_2\cos(2\pi f_2 t + \varphi_2)$, with $A_2 > 0$, $f_2 > 0$, and $\varphi_2 \in (-\pi, \pi]$. $S_2(t) = 2\sqrt{18}\cos(2\pi(20)t + \pi/4)$

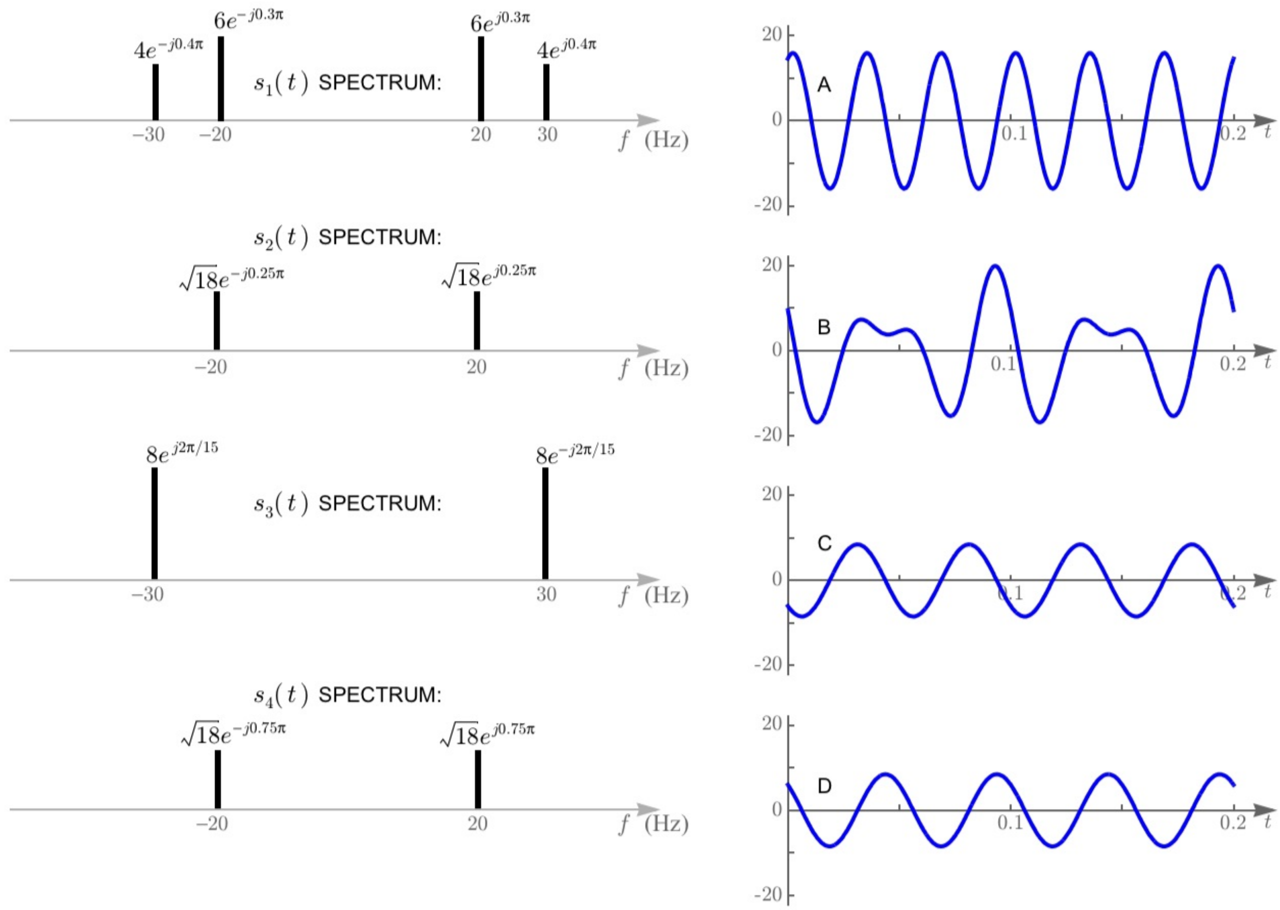
(c) Find the smallest positive delay $t_0 > 0$ for which adding a *delayed* version of the first sinusoid to the sum of all of the others yields a *single* sinusoid, namely: →

$$s_1(t - t_0) + s_2(t) + s_3(t) + s_4(t) = A\cos(2\pi f_0 t + \varphi),$$

for some unspecified constants A, f_0, and φ.

(d) Find the corresponding $A > 0$, $f_0 > 0$, and $\varphi \in (-\pi, \pi]$ in part (c).

PROBLEM 3.4.* Shown on the left are the spectra for four different signals $s_1(t)$, $s_2(t)$, $s_3(t)$, and $s_4(t)$:



c) Cancel Out 20 Hz Signal

$$12 e^{j(.3\pi - 2\pi(20)(t_0))} + \cancel{8e^{j(.4\pi - 2\pi(30)(t_0))}} + 2\sqrt{18}\, e^{-j\pi/4} + \cancel{16 e^{-j2\pi/15}} + 2\sqrt{18}\, e^{j3\pi/4}$$

$$12 e^{j40\pi t}\left(e^{.3\pi} e^{-40\pi t_0}\right) + 2\sqrt{18}\, e^{j40\pi t}\left(e^{.25\pi j} + e^{.75\pi j}\right)$$

$$\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}j + -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}j$$

$$12 e^{j40\pi t} e^{(.3\pi - 40\pi t_0)j} + 12 e^{j40\pi t} e^{\pi/2\, j} \Rightarrow \pi \text{ offset Phasor}$$

$\sqrt{2}\, j \Rightarrow \theta = \pi/2$

$\sqrt{2}\, e^{\pi/2\, j}$

$$.3\pi - 40\pi t_0 = -\frac{\pi}{2}$$

$$-40 t_0 = -.8$$

$$\boxed{t_0 = .02}$$

D) $$8e^{j(.4\pi - 60\pi(.02))} + 16 e^{-j2\pi/15}$$

$A = 16.1240$

$f_0 = 30$

$\phi = .4274$ rad $\approx \frac{362\pi}{2661}$

PROBLEM 3.5.* Define a pair of sinusoids by:

$$s_3(t) = A_3\cos(2\pi f_3 t + \varphi_3),$$
$$s_4(t) = A_4\cos(2\pi f_4 t + \varphi_4),$$

where $A_3 = 3$, $A_4 = 4$, $f_3 = 3$ Hz, $f_4 = 4$ Hz, $\varphi_3 = 0.3\pi$, $\varphi_4 = 0.4\pi$.

(a) Sketch the spectrum for the sum $x(t) = s_3(t) + s_4(t)$ of the two sinusoids.

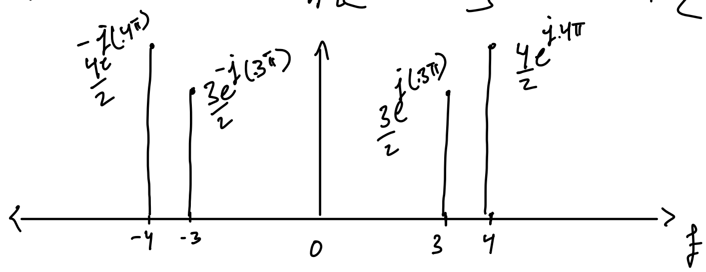
(b) Sketch the spectrum for the product $y(t) = s_3(t)s_4(t)$ of the two sinusoids.

(c) Write the product $y(t) = s_3(t)s_4(t)$ from part (b) as the *sum* of sinusoids, in the form:

$$y(t) = A_1\cos(2\pi f_1 t + \varphi_1) + A_2\cos(2\pi f_2 t + \varphi_2).$$

To make the answers unique, assume $f_1 < f_2$, and assume that the parameters are in standard form ($A_i > 0$, $f_i > 0$, and $\varphi_i \in (-\pi, \pi]$).

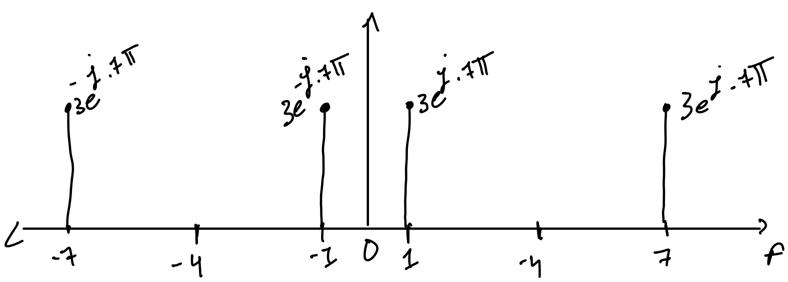
a) $x(t) = 3e^{j(.3\pi)}\,\mathbb{R}e\{e^{j(2\pi)(3)t}\} + 4e^{j(.4\pi)}\,\mathbb{R}e\{e^{j(2\pi)(4)t}\}$



b) $y(t) = S_3(t)\cdot S_4(t) \Rightarrow \overset{3\text{Hz}}{AM(t)} \cdot \cos(2\pi \overset{4\text{Hz}}{f_c} t + \phi_c)$

$f_c = 4$ Hz

$f_\Delta = 3$ Hz $\Rightarrow$ $f_1 = 1$ Hz, $f_2 = 7$ Hz



c) $x(t) = 3e^{j.7\pi}\left(e^{j2\pi(3)t} + e^{-j2\pi(3t)}\right)\left(e^{j(2\pi)(4)t} + e^{-j(2\pi)(4)t}\right)$

$3e^{j(\phi_c+\phi_\Delta)}\left(e^{j2\pi(f_\Delta)t} + e^{-j(2\pi)(f_\Delta)t}\right)\left(e^{j(2\pi)f_c t} + e^{-j2\pi f_c t}\right)$

$3e^{j(\phi_c+\phi_\Delta)}\left(e^{j2\pi(f_c+f_\Delta)t} + e^{j2\pi(f_c-f_\Delta)t} + \cancel{e^{j2\pi(f_c+f_\Delta)t}}\, e^{-j2\pi(f_c-f_\Delta)t} + e^{-j(f_c+f_\Delta)t}\right)$

$\underbrace{3e^{j2\pi(f_c+f_\Delta)t + (\phi_c+\phi_\Delta)} + 3e^{-j2\pi(f_c+f_\Delta)t + (\phi_c+\phi_\Delta)}} + 3e^{j2\pi(f_c-f_\Delta)t + (\phi_c+\phi_\Delta)} + 3e^{-j2\pi(f_c-f_\Delta)t + (\phi_c+\phi_\Delta)}$

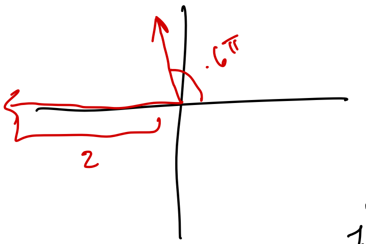
$6\cos(2\pi\underbrace{(f_c+f_\Delta)}_{f_2}t + \underbrace{\phi_c+\phi_\Delta}_{\phi_2}) + 6\cos(2\pi\underbrace{(f_c-f_\Delta)}_{f_1}t + \underbrace{\phi_c+\phi_\Delta}_{\phi_1})$

$x(t) = 6\cos(2\pi(7)t + .7\pi) + 6\cos(2\pi(1)t + .7\pi)$

~~π~~ ~~1.1π~~ ~~1.2π~~ ~~1.3π~~ ~~1.4π~~ ~~1.5π~~ ~~1.6π~~ ~~1.7π~~ ~~1.8π~~ 1.9π

1

1.9π



$$e^{j(.1+.5)\pi} \Rightarrow e^{j(.6\pi)} \; -2 \Rightarrow 2e^{j(-\pi)}$$

$$e^{j\left(\frac{\pi 6}{10}\right)} + 2e^{j(-\pi)} = \frac{1}{x-jy}$$

$$\cos(.6\pi) + j\sin(.6\pi) - 2$$

$$\cancel{(A+Bj)^{-1} = \frac{1}{x-jy} = x-jy} = \frac{1}{A+Bj}$$

k	f_0	ϕ
1	0	$\pi/2$
2	98	$2\pi/3$
3	140	π
4	168	$4\pi/3$

PROBLEM 4.1.* Let $x(t) = 8\sum_{k=1}^{4}\cos\left(2\pi(7k^3 - 70k^2 + 259k - 196)t + k\pi/3\right)$.

(a) Sketch the two-sided spectrum for $x(t)$,
carefully labeling the frequency (in Hz) and complex coefficient (in polar form) of each line.

(b) Find the smallest positive real T for which the following equation is true for all time t:

$$x(t) = x(t + T).$$

(c) The signal $x(t)$ is periodic. Find its fundamental period T_0.

(d) The signal $x(t)$ is periodic. Find its fundamental frequency f_0, in Hz.

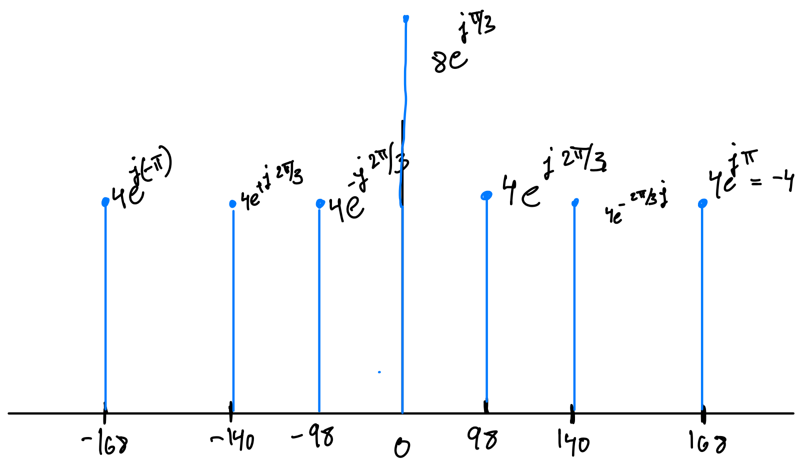
(e) Which harmonics are present in $x(t)$?
(The k-th harmonic is *present* when its spectrum has a line at kf_0).

(f) The signal can be written as the Fourier series $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi k f_0 t}$.

Specify all of the nonzero FS coefficients $\{a_k\}$ in a table in the shown format, with the first column specifying the integer k, and the second column specifying the corresponding FS coefficient a_k. Only nonzero coefficients should appear in the table.

k	a_k
⋮	⋮

a)



b) $2\pi(7k^3 - 70k^2 + 259k - 196)T = 2\pi \quad \forall k \in \{1, 2, 3, 4\}$

$\Rightarrow T(7k^3 - 70k^2 + 259k - 196) = 1$

$k=1$: $0T = 0$

$k=2$: $98T = \alpha$ s.t. these vars are integers

$k=3$: $140T = \beta$

$k=4$: $168T = \lambda$ $\Rightarrow f_0 = \frac{1}{T} = GCD(98, 140, 168)$

$\Rightarrow \boxed{T = 1/14\,s}$

c) $GCD(98, 140, 168) \Rightarrow 14 = f_0$

$T_0 = 1/f_0 \Rightarrow \boxed{1/14\,s}$

D) $f_0 = 1/T_0 = \boxed{14\,Hz}$

E) The 0^{th}, 7^{th}, 10^{th}, and 12^{th} Harmonics and -7^{th}, -10^{th}, -12^{th} harmonics are present

f)

k	a_k
-12	$4e^{j(-\pi)}$
-10	$4e^{j(2\pi/3)}$
-7	$4e^{j(-2\pi/3)}$
0	$8e^{j(\pi/3)}$
7	$4e^{j(2\pi/3)}$
10	$4e^{j(-2\pi/3)}$
12	$4e^{j(\pi)}$

PROBLEM 4.2.* Consider the signal $s(t) = \sqrt{\pi e} + \sqrt{\pi}\cos(2.5\sqrt{\pi^3 e}\,t) + \sqrt{e^3}\cos(3\sqrt{\pi^3 e}\,t)$.

(a) Is it periodic? (YES or NO.)

(b) If NO, explain why not.
If YES, specify its fundamental frequency f_0 (in Hz).

$\frac{3\sqrt{\pi^3 e}}{2\pi} = f$

$\frac{2.5\sqrt{\pi^3 e}}{2\pi}$ is f

a) yes, it is periodic

B) The fundamental frequency f_0 of this signal

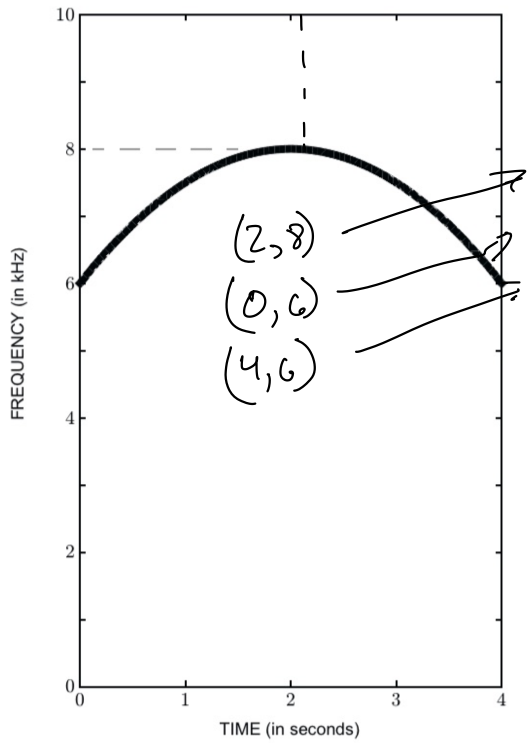
will Be the $GCD\left(\frac{2.5\sqrt{\pi^3 e},\ 3\sqrt{\pi^3 e}}{2\pi}\right)$

$\Rightarrow \quad \Rightarrow \quad \frac{.5\sqrt{\pi^3 e}}{2\pi} \Rightarrow \boxed{\frac{\sqrt{\pi^3 e}}{4\pi}\ Hz}$

PROBLEM 4.3.* Find positive numerical values for the constants A, B, and C so that the spectrogram of

$$x(t) = \frac{AB}{C}\sin\left(A(t-2) + Bt^2 - Ct^3 - A^2BC + 0.3\pi\right)$$

for time in the range $0 < t < 4$ looks like this:



$$f_i(t) = \frac{\partial}{\partial t}\varphi(t)\frac{1}{2\pi}$$

$$= (A + 2Bt - 3Ct^2)\frac{1}{2\pi}$$

$$f_i(0) = 6 = \frac{A}{2\pi} + 0 + 0$$

$$\Rightarrow A = 12\,000\,\pi$$

$$f_i(2) = 8000 = (12\,000\,\pi + 4B - 12C)\frac{1}{2\pi}$$

$$16\,000\pi - 12\,000\pi = 4B - 12C$$

$$4\,000\pi = 4B - 12C \to 4000\pi - 4000\pi$$

$$-4000\pi / -12$$

$$f_i(4) = (6000)\cancel{2\pi} = \cancel{12000\pi} + 8B - 48C$$

$$0 = 8B - 48C$$

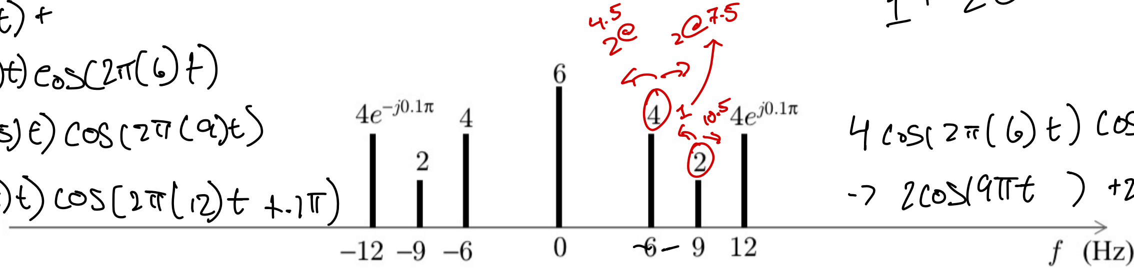
$$B = 2000\,\pi$$

$$C = \frac{1000\pi}{3}$$

PROBLEM 4.4.* Shown below is the spectrum of a signal $s(t)$:

$1 + 2e^{j(.1\pi)}$

$6\cos(2\pi(1.5)t) +$
$8\cos(2\pi(1.5)t)\cos(2\pi(6)t)$
$4\cos(2\pi(1.5)t)\cos(2\pi(9)t)$
$8\cos(2\pi(1.5)t)\cos(2\pi(12)t + .1\pi)$



$4\cos(2\pi(6)t)\cos(2\pi(1.5)t)$
$\to 2\cos(9\pi t) + 2\cos(15\pi t)$

$s(t) = 6 + 8\cos(2\pi(6)t) + 4\cos(2\pi(9)t) + 8\cos(2\pi(12)t + .1\pi)$

(a) Is it periodic? If NO, explain why not. If YES, specify its fundamental frequency f_0 (in Hz).

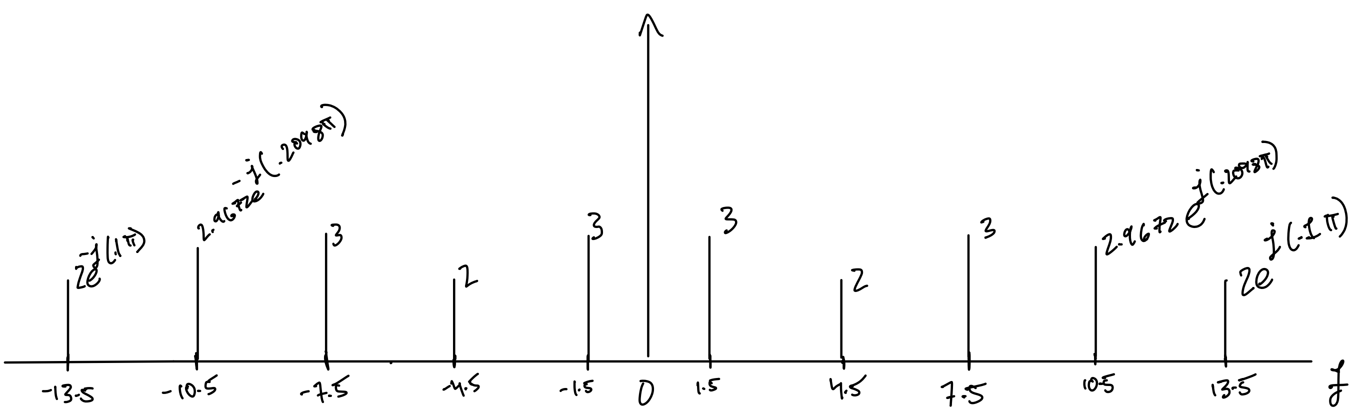
(b) Sketch the spectrum for $x(t) = s(t)\cos(3\pi t)$, carefully labeling the frequency and coefficient of each line.

(c) Sketch the spectrum for $y(t) = s(t)\cos(60\pi(t - 0.01))$, carefully labeling the frequency and coefficient of each line.

(d) Does there exist a frequency f_1 for which $s(t)\cos(2\pi f_1 t)$ is *not* periodic? If YES, specify such an f_1 (in Hz). If NO, explain.

a) yes, all frequencies present are integer multiples of a fundamental frequency $f_0 = 3\,Hz$

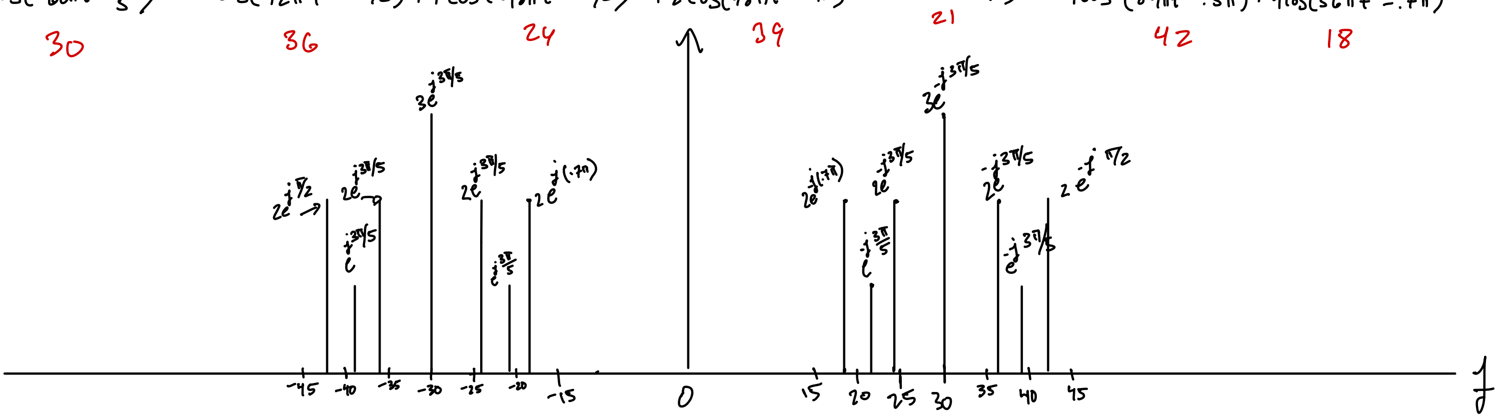
b) $x(t) = \cos(3\pi t)\,s(t) \Rightarrow \cos(2\pi(1.5)t)\,s(t)$



c) $x(t) = \cos(2\pi(30)(t - .01))\,s(t) \Rightarrow \cos(60\pi t - 3\pi/5)\,s(t)$ $f \in \{30, 36, 24, 39, 42, 84, 36\}$

$6\cos(60\pi t - \frac{3\pi}{5}) + 4\cos(72\pi t - 3\pi/5) + 4\cos(48\pi t - 3\pi/5) + 2\cos(78\pi t - 3\pi/5) + 2\cos(42\pi t - 3\pi/5) + 4\cos(84\pi t - .5\pi) + 4\cos(36\pi t - .7\pi)$

30 36 24 39 21 42 18

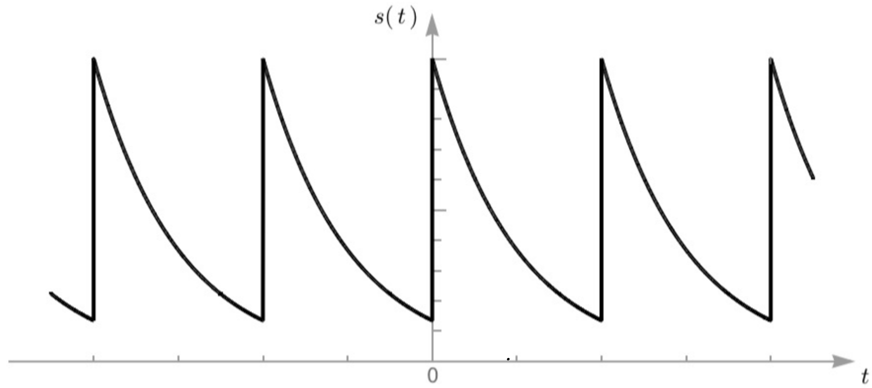


D) for the signal $x(t) = \cos(2\pi f_1 t) S(t)$ to break its periodicity, the resulting signal $x(t)$, when decomposed as a sum of sinusoids, will not have a fundamental frequency. in other words, the GCD of all frequencies in $x(t)$ must not exist. Therefore, for the frequencies in $S(t)$ ($f_s \in \{0, 6, 9, 12\}$) the expression $(f_s - f_1)$ or $(f_s + f_1)^*$ must result in an irrational number. * By identity $\cos\beta \cos\alpha = (\cos(\beta-\alpha) + \cos(\beta+\alpha))\frac{1}{2}$

Since the frequencies in $S(t)$ are rational, having an α that is irrational would make the decomposed frequencies some rational # $\pm$ an irrational. This would result in an irrational # $\Rightarrow$ There does exist a f_1 to make $S(t)\cos(2\pi f_1 t)$ non periodic.

PROBLEM 4.5.* Consider the periodic signal $s(t)=\sum_{k=-\infty}^{\infty}\frac{e^{jk\pi t}}{1+jk\pi}$ shown below (neither the time-axis scale nor the vertical scale is specified):

$1+jk\pi \Rightarrow \sqrt{1+k^2\pi^2}\, e^{j\tan^{-1}(k\pi)}$



(a) Find the fundamental period T_0 (in seconds) and the fundamental frequency f_0 (in Hz).

(b) Evaluate the integral:

$$\frac{1}{T_0}\int_0^{T_0} s(t)\,dt.$$

a) $e^{jk\pi t} \Rightarrow e^{j2\pi k\left(\frac{1}{2}\right)t} \Rightarrow f_0 = \frac{1}{2}$

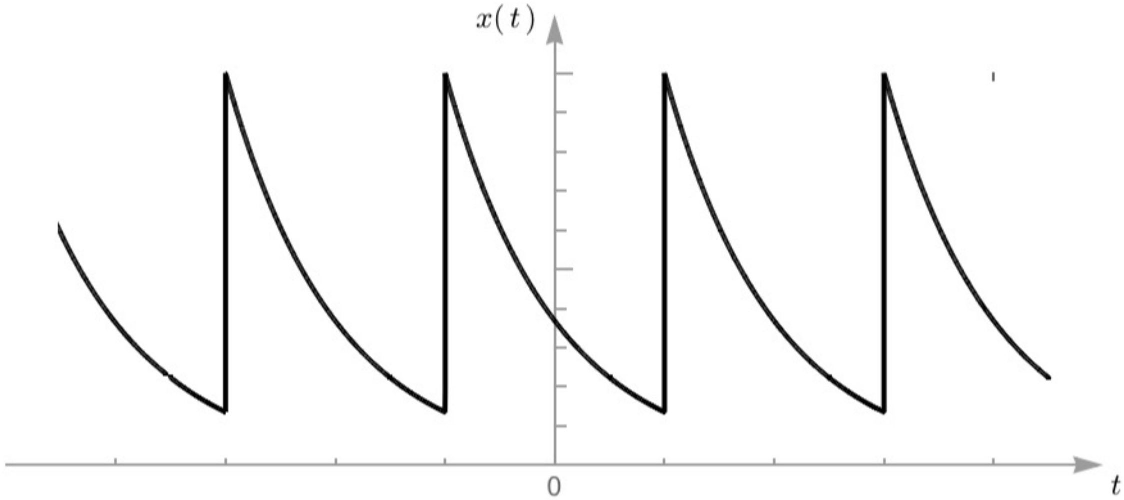
$\frac{1}{1+jk\pi} = a_k \Rightarrow s(t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi k\left(\frac{1}{2}\right)t}$

$T_0 = 2$ seconds , $f_0 = \frac{1}{2}$ Hz

B) $a_k = \frac{1}{T_0}\int_0^{T_0} s(t)\, e^{-j2\pi f_0 k t}\,dt \Rightarrow$ when $k=0 \Rightarrow \frac{1}{T_0}\int_0^{T_0} s(t)\, \cancel{e^{-j(2\pi/T_0)0t}}^{=1}\, dt \Rightarrow \underbrace{\frac{1}{T_0}\int_0^{T_0} s(t)\,dt}_{=a_0}$

$k=0,\ s(t) = \frac{e^{j\pi(0)t}}{1+j(0)\pi} \Rightarrow 1 \Rightarrow \frac{1}{T_0}\int_0^{T_0} 1\,dt \Rightarrow \frac{1}{T_0}\left[T_0 - 0\right] \Rightarrow \frac{2}{2} = \boxed{1}$

(c) Shown below is a shifted version $x(t) = s(t - \frac{T_0}{2})$:



Find an expression for the k-th coefficient a_k in its Fourier series $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi k f_0 t}$. Simplify as much as possible.

$$a_k = \frac{1}{T_0}\int_0^{T_0} x(t) e^{-j2\pi f_0 t k} dt \qquad x(t) = \sum_{k=-\infty}^{\infty} \frac{e^{j\pi k(t-1)}}{1 + kj\pi} \Rightarrow \sum_{-\infty}^{\infty} \underbrace{\frac{e^{-jk\pi}}{1+kj\pi}}_{a_k} \underbrace{e^{jk\pi t}}_{\downarrow} = x(t)$$

$$e^{jk\pi(2)(\frac{1}{2})t}$$

$\uparrow$ f_0

$$\boxed{a_k \Rightarrow \left(\frac{1}{1+kj\pi}\right)\left(e^{-jk\pi\left(\frac{T_0}{2}\right)}\right)}$$

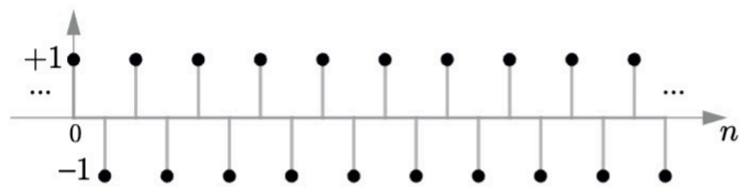
PROBLEM 5.1.* Download the WAV file `mystery.wav` from the canvas homework page.
Import it into MATLAB and listen to it using the `audioread` and `soundsc` commands.
Hidden in this audio file is a message. What is the message?
Specify the message as well as the MATLAB code you used to decode the message.

```
[xx, fs] = audioread("mystery.wav");

spectrogram(xx, 256, [], [], fs, 'yaxis');
```

PROBLEM 5.2.* Write each of the following as a discrete-time sinusoid in standard form $A\cos(\hat{\omega}n + \varphi)$, with $A \geq 0$, $0 \leq \hat{\omega} \leq \pi$, and $-\pi < \varphi \leq \pi$:

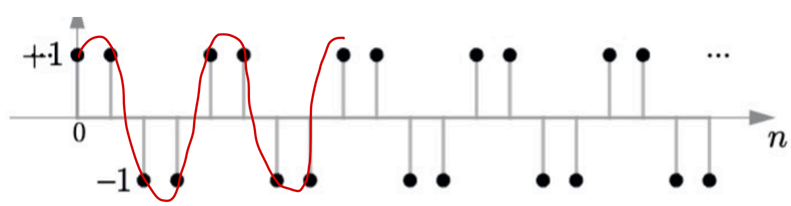
(a) $x_a[n] = \ldots 1, -1, 1, -1, \ldots$



if $x_a[n] = A\cos(\hat{\omega}n + \phi)$ $A = 1$, $\hat{\omega} = \pi$, $\phi = 0$

$x_a[n] = \cos(\pi n)$

(b) $x_b[n] = \ldots 1, 1, -1, -1, 1, 1, -1, -1, \ldots$



$x_b[n] = A\cos(\hat{\omega}n + \phi)$ period is 4 samples $= f_0 = 1/4$

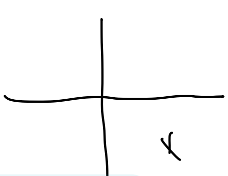
$\hat{\omega} = \frac{2\pi}{4} = \pi/2$

$n=0 \Rightarrow A\cos(\phi) = 1 \Rightarrow \cos(\phi) = 1/A$

$n=1 \Rightarrow A\cos(\pi/2 + \phi) = 1 \Rightarrow \cancel{\cos(\pi/2)\cos(\phi)} - \sin(\pi/2)\sin(\phi) = 1/A \Rightarrow \sin(\phi) = -1/A$

$n=2 \Rightarrow A\cos(\pi + \phi) = -1 \Rightarrow \cos(\pi)\cos(\phi) - \cancel{\sin(\pi)\sin(\phi)} = -1/A \Rightarrow \cos(\phi) = +1/A$

$n=3 \Rightarrow A\cos(3\pi/2 + \phi) = -1 \Rightarrow \cancel{\cos(3\pi/2)\cos(\phi)} - \sin(3\pi/2)\sin(\phi) = -1/A \Rightarrow \sin(\phi) = -1/A$



$\cos(-\pi/4) = \sqrt{2}/2$

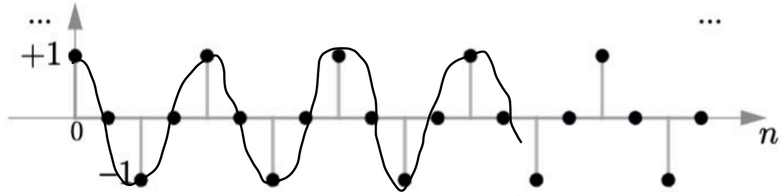
$\sin(-\pi/4) = -\sqrt{2}/2$ $\Rightarrow \boxed{\phi = -\pi/4}$ $\boxed{A = 2/\sqrt{2}}$

$\frac{\sqrt{2}}{2} = \frac{1}{A}$

$x[n] = \frac{2}{\sqrt{2}}\cos\left(\frac{\pi}{2}n - \pi/4\right)$

Made with Goodnotes

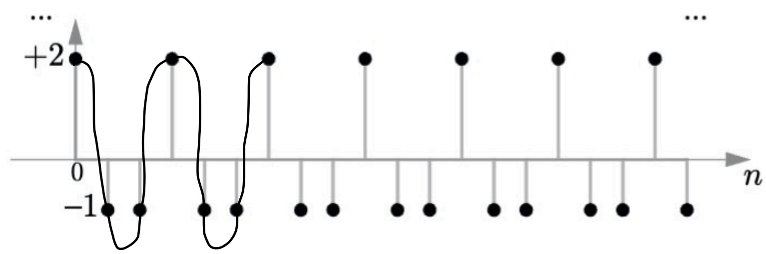
(c) $x_c[n] = \ldots 1, 0, -1, 0, 1, 0, -1, 0, \ldots$



$\hat{\omega} = 2\pi\left(\frac{1}{4}\right)$ $\cos\left(\frac{\pi}{2}n + \phi\right)$ $A = 1$

$x_c[n] = \cos\left(\frac{\pi}{2} n\right)$

(d) $x_d[n] = \ldots 2, -1, -1, 2, -1, -1, \ldots$



$\hat{\omega} = 2\pi\left(\frac{1}{3}\right) \Rightarrow x[n] = 2\cos\left(2\pi\left(\frac{1}{3}\right)n\right.$

$$\left.\begin{array}{lll} n=0 & \Rightarrow & 2 \cdot \cos(0 + \phi) = 2 \\ n=1 & \Rightarrow & 2 \cdot \cos\left(\frac{2\pi}{3} + \phi\right) = -1 \\ n=2 & \Rightarrow & 2 \cdot \cos\left(\frac{4\pi}{3} + \phi\right) = -1 \end{array}\right\} \phi = 0$$

$x[n] = 2 \cdot \cos\left(\frac{2\pi}{3} n\right)$

PROBLEM 5.3.* Write each of the following as a discrete-time sinusoid in standard form $A\cos(\hat{\omega}n + \varphi)$, with $A \geq 0$, $0 \leq \hat{\omega} \leq \pi$, and $-\pi < \varphi \leq \pi$.

$\hat{\omega} = \frac{M}{N_0}(2\pi)$

(a) $x_a[n] = \text{Re}\{\sqrt{2}\,e^{j(0.2\pi n + \pi/3)}\}$.

(b) $x_b[n] = -\sin(9.3\pi(n-4) + 0.3\pi)$.

(c) $x_c[n] = \sqrt{2} + \cos(2026\pi n) + \sqrt{2}\sin(2026(2\pi)n)$.

(d) $x_d[n] = \cos(7n) - \sin(7n) + \sqrt{2}\cos(7n + 0.25\pi)$

a) $x[n] = \sqrt{2}\cos(.2\pi n + \frac{\pi}{3})$

b) $x[n] = -\sin(9.3\pi n - 4(9.3\pi) + .3\pi)$

$= \sin(-9.3\pi n + 36.9\pi) \Rightarrow \cos(-9.3\pi + 36.4\pi)$

$\hat{\omega} = -9.3\pi + 2\pi l \; (l = 5)$

$= .7\pi$

$\phi = 36.4\pi - 2\pi l \; (l = 18)$

$\phi = .4\pi$

$x[n] = \cos(.7\pi n + .4\pi)$

c) $x[n] = \sqrt{2} + \underbrace{\cos(2026\pi n)}_{\forall n \in \mathbb{N},\ \text{is } 1} + \sqrt{2}\sin(2026(2\pi)n)$

$\Rightarrow (1 + \sqrt{2}) + \sqrt{2}\cos(2026(2\pi)n - \pi/2)$

$\sqrt{2}(\cancel{\cos(2026(2\pi)n)\cos(-\pi/2)} + \cancel{\sin(2026(2\pi)n)\sin(\pi/2)})$

$0 \qquad 0$

$\boxed{x[n] \Rightarrow 1 + \sqrt{2}}$

D) $x[n] = \cos(7n) - \sin(7n) + \sqrt{2}\cos(7n + .25\pi)$

$\hat{\omega} = 7 \qquad -\cos(7n - \pi/2)$

$e^{j(0)} + (-1)e^{j(-\pi/2)} + \sqrt{2}e^{j(.25\pi)} \Rightarrow 2.8284\cos(7n + \pi/4)$

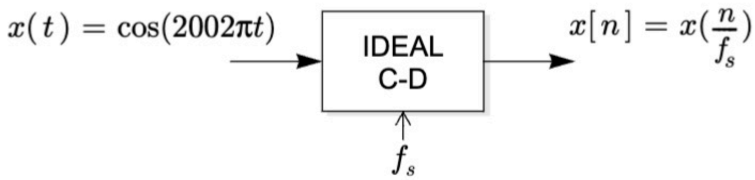
$\hookrightarrow e^{j(\pi/2)}$

$7 - 2\pi l\,(l = 1) = .7168 = \hat{\omega}$

$x[n] \approx \underbrace{2.8284}_{(2\sqrt{2})}\cos(.7168n + \pi/4)$

Made with Goodnotes

PROBLEM 5.4.* A continuous-time sinusoid $x(t) = \cos(2002\pi t)$ is sampled with sampling rate f_s, resulting in a discrete-time sequence:



(a) Find the fundamental period N_0 for $x[n]$ when the sampling rate is $f_s = 3003$ Hz.

(b) Find the fundamental period N_0 for $x[n]$ when the sampling rate is $f_s = 1092$ Hz.

(c) Find the fundamental period N_0 for $x[n]$ when the sampling rate is $f_s = 507006.5$ Hz.

(d) Answer *TRUE* or *FALSE*, and explain your reasoning:
In order for $x[n]$ to be periodic in this example, the sampling rate must be *rational*.

(e) Find the smallest integer-valued sampling rate f_s for which $x[n]$ is periodic with fundamental period $N_0 = 8$.

(f) Specify three distinct integer-valued sampling rates $f_s > 0$ for which $x[n]$ is periodic with fundamental period $N_0 = 60$. (Hint: There are eight.)

a) $\omega = 2\pi \frac{f_0}{f_s} = \frac{1001}{3003}(2\pi) = \frac{1}{3}(2\pi) \Rightarrow N_0 = 3$ Samples

b) $2\pi\left(\frac{1001}{1092}\right) \Rightarrow N_0 = 12$ Samples

$\frac{11}{12}$

c) $\hat{\omega} = 2\pi\left(\frac{1001}{507006.5}\right) \Rightarrow$

$\frac{2002}{1014013} \Rightarrow \frac{2}{1013} \Rightarrow$ $\boxed{N_0 = 1013 \text{ Samples}}$

D) if f_s were not to be Rational (irrational) Then the term $\hat{\omega} = 2\pi \frac{f_0}{f_s}$ would result in M/N Being irrational. Specifically, N_0 would Not Be Rational $\Rightarrow$ Signal is Non periodic $\Rightarrow$ f_s must Be Rational (True)

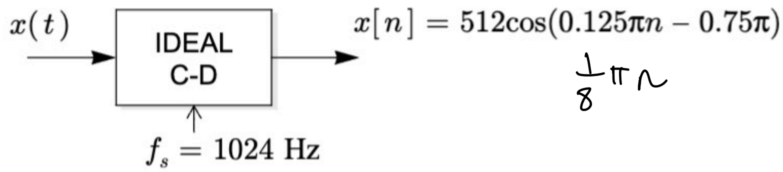
E) $\hat{\omega} = 2\pi\left(\frac{M}{N_0}\right) \Rightarrow \cancel{2\pi}\left(\frac{M}{8}\right) = \cancel{2\pi}\left(\frac{1001}{f_s}\right) \Rightarrow f_s = \frac{8008}{M} \Rightarrow$ $\frac{1001}{8008}$ can be $\frac{1}{8}$

$\Rightarrow \boxed{f_s = 8 \text{ Hz}}$

PROBLEM 5.5.* In Prob. 5.4 the C-D input is specified, and you are asked about the output. Here we consider the reverse: the *output* is specified, and you are asked about the input.

Suppose that a continuous-time signal $x(t)$ is sampled at a sampling rate of 1024 samples/sec, resulting in the discrete-time sinusoidal signal $x[n]$ indicated below:

$f_s = 1024$



$\frac{1}{8}\pi n$

Knowing the C-D output $x[n]$ does not uniquely determine the C-D input $x(t)$; there are many continuous-time signals $x(t)$ that when sampled would produce this $x[n]$.
If we add a constraint that the input $x(t)$ is a sinusoid whose frequency is less than 2 kHz, then there are only four possible inputs. Name all four. In other words, specify four different continuous-time sinsusoidal signals (all in standard form)

$$x_1(t) = A_1\cos(2\pi f_1 t + \varphi_1),$$
$$x_2(t) = A_2\cos(2\pi f_2 t + \varphi_2),$$
$$x_3(t) = A_3\cos(2\pi f_3 t + \varphi_3),$$
$$x_4(t) = A_4\cos(2\pi f_4 t + \varphi_4)$$

that could have produced this particular $x[n]$, subject to the constraint that $0 < f_i < 2$ kHz in all four cases.

1. $\frac{\cancel{2\pi} f_1}{f_s} = 2\pi \frac{M}{N_0} = \hat{\omega} \Rightarrow \cancel{2\pi}\left(\frac{1}{16}\right)$ $\quad x(t) = 512\cos(2\pi(64)t - .75\pi)$

   $\frac{f_1}{1024} = \frac{1}{16} \Rightarrow f_1 = 64\ Hz$

2. $\cancel{2\pi}\frac{f_1}{1024} = \hat{\omega} + 2\pi \Rightarrow 2.125\pi \Rightarrow \cancel{2\pi}\left(\frac{17}{16}\right) \Rightarrow f_2 = 1088\ Hz$

   $x(t) = 512\cos(2\pi(1088)t - .75\pi)$

3. $\cancel{2\pi}\frac{f_1}{1024} = \hat{\omega} + 4\pi \Rightarrow 4.125\pi = \cancel{2\pi}\left(\frac{33}{16}\right)$ ~~X~~ 2

   $\Rightarrow \hat{\omega} - 2\pi = -1.875\pi \Rightarrow \cancel{2\pi}\left(-\frac{15}{16}\right) \Rightarrow -960$ $\quad x(t) = 512\cos(2\pi(960)t + .75\pi)$

4. $2\pi\frac{f_1}{1024} = \hat{\omega} - 4\pi = -3.875\pi \Rightarrow 2\pi\left(-\frac{31}{16}\right) : f_1 = -1984$ $\quad x(t) = 512\cos(2\pi(1984)t + .75\pi)$

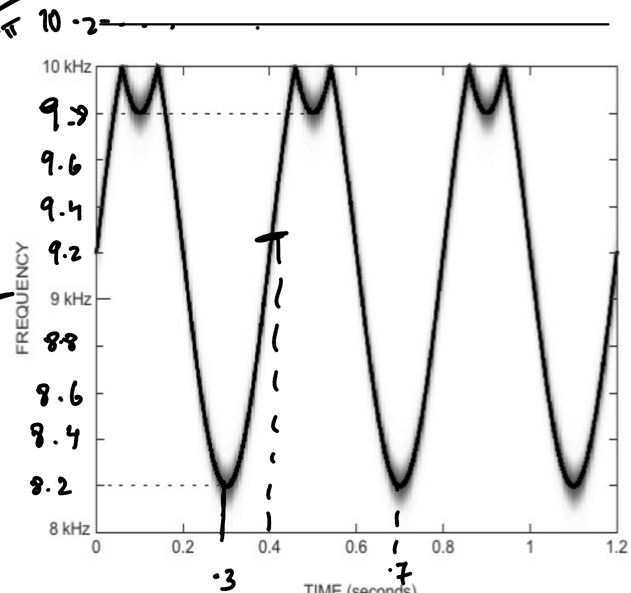
PROBLEM 6.1.* Suppose that running the following MATLAB code:

```
A =
B =
C =
fs =
dur =
t = 0:1/fs:dur;
x = cos(2*pi*B*t + A*cos(C*t));
spectrogram(x, hanning(300), [], 2^12, fs, 'yaxis');
```

leads to the following plot:

Pea @ 11.8 => $A = \frac{1000}{5\pi}$

$f_i(t) = \frac{\partial \phi(t)}{\partial t} \Big/ 2\pi$



$\Delta t = T = .4$ sec

(a) Specify numerical values for the unspecified variables fs and dur.

(b) Specify a numerical value for C, with the constraint that $0 < C < 6\pi$.

(c) Specify one possible set of numerical values for the unspecified variables A and B, with the constraint that $0 < B < 10000$.

(d) Specify a second set of numerical values for the unspecified variables A and B, this time with the constraint that $10000 < B < 20000$.

a)

$f_s = 20$ kHz

since the plot (spectrogram) cuts off @ 10k Hz

dur = 1.2 seconds from Graph

b)

$x = \cos(\psi(t))$

$\psi(t) = 2\pi B t + A\cos(Ct)$

$\frac{\partial \psi}{\partial t} = \frac{2\pi B - AC\sin(Ct)}{2\pi}$

$\frac{A(5\pi)}{2\pi}\sin(5\pi t)$

$T = .4$

$f = \frac{1}{.4} = 2.5$ Hz

$\boxed{C = 5\pi}$

c) midline is 9200

=> $\boxed{B = 9200}$

Swing from 8.2 to 10.2 => 2000/2 is Amplitude

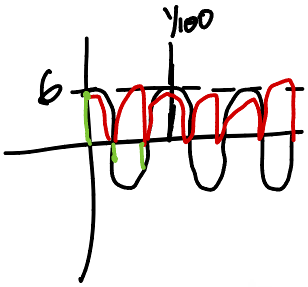
Amplitude is $\frac{AC}{2\pi} = A(2.5) \Rightarrow$ | $A = -400$

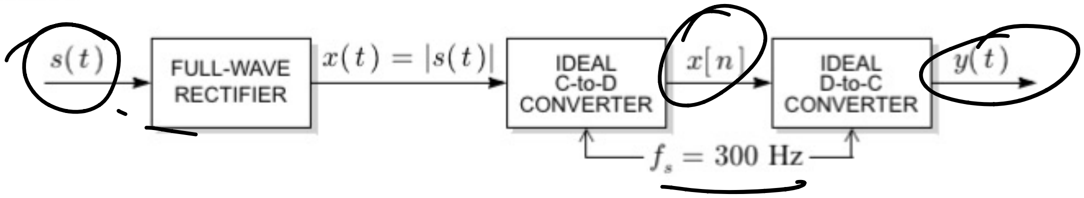
D) $\boxed{400 = A}$

$\boxed{B = 10.8 \text{ kHz}}$

h

PROBLEM 6.2.* A full-wave rectifier is a device whose output is the absolute value of its input. Suppose that a rectified version of $s(t) = 6\cos(200\pi t)$ is input to the cascade of an ideal C-D converter and an ideal D-C converter, both with sampling rate $f_s = 300$ Hz, as shown below:





(a) Find an expression for the output $y(t)$, simplified as much as possible.

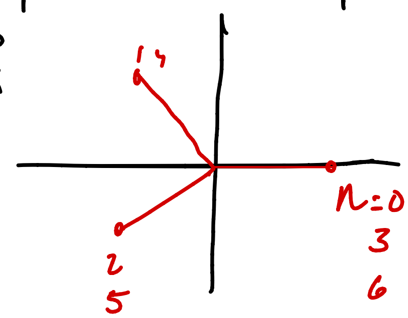
(*Hint 1*: This problem is easiest to solve in the time domain: first determine the values $x[0]$, $x[1]$, $x[2]$, etc. before considering what $y(t)$ might look like. *Hint 2*: The solution to HW5 Prob. 5.2(d) may be useful.)

(b) Sketch $x(t) = |s(t)|$ and $y(t)$ on the same scale for time in the range $0 < t < 0.03$. Confirm that the two curves intersect at the sampling times. Label the intersection of the two curves at time 0 by $x[0]$, the intersection at time $1/f_s$ by $x[1]$, the intersection at time $2/f_s$ by $x[2]$, and similarly label the intersection of the two curves at time n/f_s by $x[n]$ for all $n \in \{0, 1, \ldots 9\}$.

a) $|6\cos(2\pi(100)t)| \rightarrow x[n] = \left|6\cos\left(2\pi\left(\frac{100}{300}\right)n\right)\right| \Rightarrow \left|6\cos\left(\frac{2\pi}{3}n\right)\right|$

$x[n] = \{ |6|, |-3|, |-3|, |6|, |-3|, \ldots \}$

$x[n] = \{ 6, 3, 3, 6, 3, \ldots \}$ $\omega = 3$



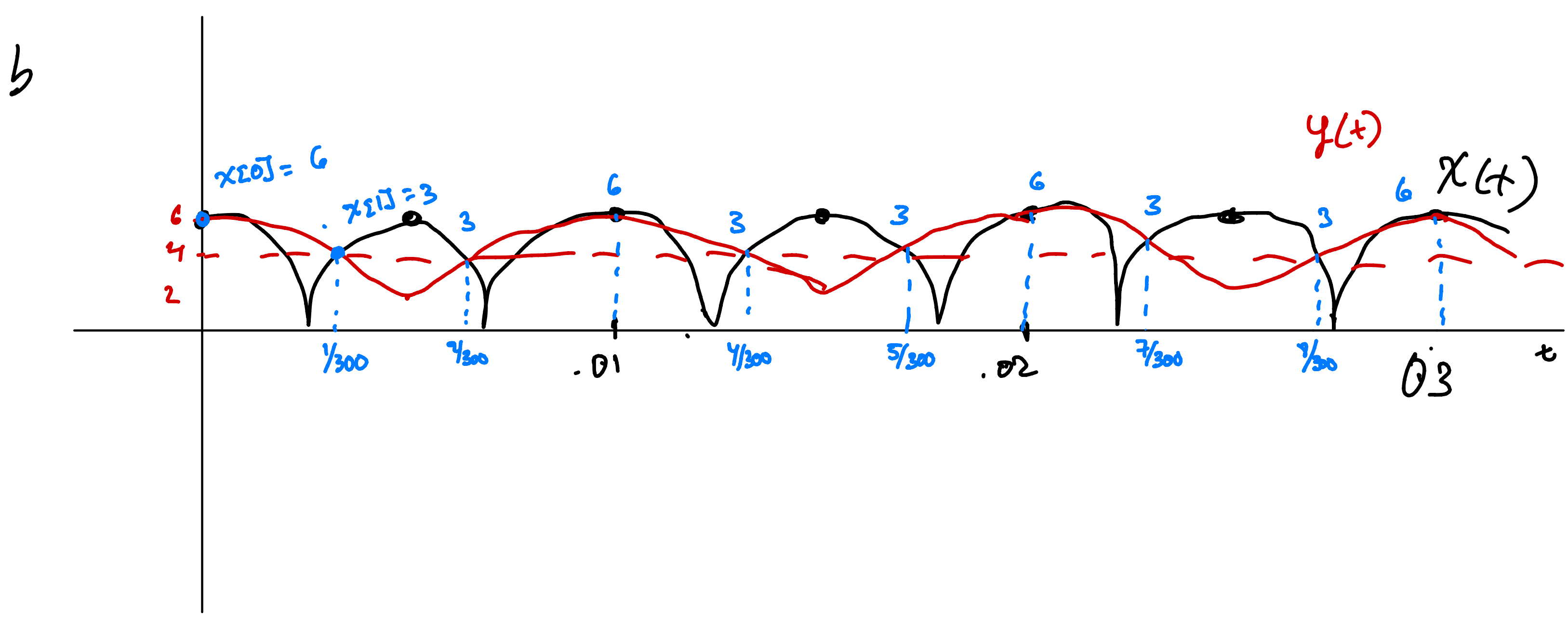
$x[n] = \left|6\cos\left(\frac{2\pi}{3}n\right)\right|$ $n = t f_s = 300t$

$\boxed{y(t) = 4 + 2\cos(200\pi t)}$

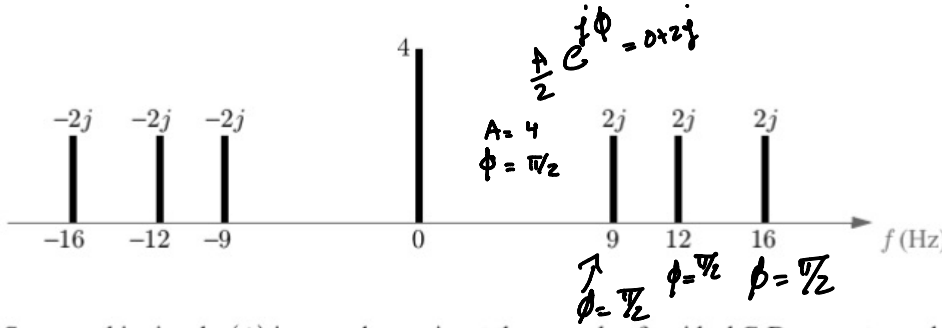
$\cos\left(\frac{2\pi}{3}(300)t\right)$

$\rightarrow \cos(200\pi t)$

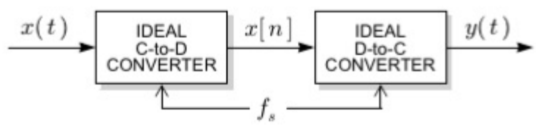
b



PROBLEM 6.3.* Consider a signal $x(t)$ whose spectrum is shown below:



Suppose this signal $x(t)$ is passed as an input the cascade of an ideal C-D converter and an ideal D-C converter, both with the same unspecified sampling rate f_s, as shown below:



(a) If $f_s = 40$ Hz, is the output $y(t)$ periodic?
If *YES*, specify its fundamental frequency f_0. If *NO*, explain why not.

(b) If $f_s = 1.1$ Hz, is the output $y(t)$ periodic?
If *YES*, specify its fundamental frequency f_0. If *NO*, explain why not.

(c) Find the largest sampling rate f_s for which the output $y(t)$ is periodic with fundamental frequency $f_0 = 3$ Hz.

(d) Find the largest sampling rate f_s for which the output $y(t)$ is periodic with fundamental frequency $f_0 = 9$ Hz.

(e) Find the largest sampling rate f_s for which the output $y(t)$ is periodic with fundamental frequency $f_0 = 12$ Hz.

a) Periodicity is Guaranteed if $\hat{\omega}_0 = 2\pi\left(\frac{f_0}{f_s}\right)$ can be reduced and is Rational, and the Signal $x(t)$ is itself Periodic. $x(t)$ has fundamental frequency of 1 Hz $\Rightarrow$ $y(t)$ is also Periodic since the Sampling Rate will result in Perfect Reconstruction of $x(t)$.

b) $f_{0_1} = 9 \Rightarrow f_{1y} = \min_l |9 - 1.1l|$ $l = 8 \Rightarrow f_{1y} = .2 \Rightarrow y(t) = \cos(2\pi(.2)t + \phi_1)$

$= 12 \Rightarrow f_{1y} = \min_l |12 - 1.1l|$ $l = 11 \Rightarrow f_{1y} = .1 \Rightarrow y(t) = \cos(2\pi(.1)t + \phi_2)$

$16 \Rightarrow f_{1y} = \min_l |16 - 1.1l|$ $l = 15 = f_{1y} = .5 \Rightarrow y(t) = \cos(2\pi(.5)t + \phi_3)$

Yes with $\boxed{f_0 = .1\text{ Hz}}$

c)

Since 9 and 12 Hz are already fundamental to 3 — we need 16 Hz to fold to a multiple of 3 Hz or 0. if we Sample @ 32 Hz, Then with the 90° deg Phase Shift, the Signal will be sampled every time $\cos(2\pi(16)t + \pi/2)$ crosses the x axis Resulting in samples of only zero.
$\Rightarrow$ $y(t)$ only preserves 9 Hz and 12 Hz, Both multiples of 3.

D) Periodic for $f_0 = 9$ Hz

$\Rightarrow$ want 12 Hz signal to cancel 16 Hz aliased signal.

$24 < f_s < 32$ $\Rightarrow$ $\min_l |16 - f_s l| = 12$

$|16 - f_s| = 12 \Rightarrow f_s = \cancel{4}$ or 28

if $f_s = 28$ Hz $\Rightarrow$ for $f_1 = 16$ Hz, the output is

$4\cos(2\pi(16-28)t + \pi/2) \Rightarrow 4\cos(2\pi(-12)t + \pi/2) \Rightarrow$

$4\cos(2\pi(12) - \pi/2)$ which is 180° out of Phase with original $4\cos(2\pi(12) + \pi/2)$ signal $\Rightarrow$ only 9 Hz Signal Remains

$\Rightarrow$ fundamental frequency of $y(t)$ is 9 Hz.

E) Same as 6.2D but Get 16 to fold to 9 Hz

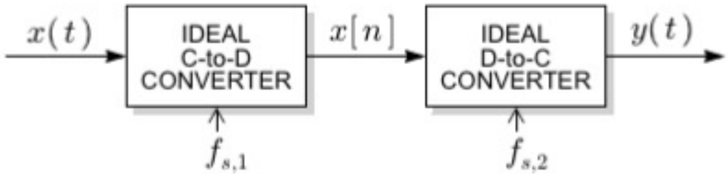
$\min_l |16 - f_s l| = 9$

$|16 - f_s| = 9 \Rightarrow f_s = 25$ or ~~7 Hz~~

$4\cos(2\pi(16-25)t + \pi/2)$
$= 4\cos(-2\pi(9) + \pi/2)$
$= 4\cos(2\pi(9) - \pi/2)$

only $4\cos(2\pi(12) + \pi/2) = y(t)$ Remains $\Rightarrow$ $f_s = 25$

PROBLEM 6.4.* Consider the following cascade of an ideal C-D converter with sampling rate $f_{s,1}$ and an ideal D-C converter with sampling-rate parameter $f_{s,2}$:



While it is common for the two parameters to be identical ($f_{s,2} = f_{s,1}$), here we consider the case where they are different.

(a) Suppose that $x(t)$ is the recording of a pop song that starts at time 0 and ends at time 240, a total of 4 minutes. A radio broadcaster wants to use the above-pictured system to shorten the song from 4 minutes to 3 minutes (180 seconds) to leave more time for commercials. If $f_{s,1} = 44.1$ kHz, find $f_{s,2}$ so that $y(t)$ starts at time 0 and ends at time 180.

(b) Using $f_{s,2} \neq f_{s,1}$ will not only change the duration of the song, it will also change the pitch. Suppose a segment of the song consists of a single tone at $f_0 = 800$ Hz, namely $x(t) = \cos(1600\pi t)$ for $t \in (1, 1.1)$; the corresponding tone for $y(t)$ will be at a different frequency $f_0' \neq 800$ Hz.

Find f_0' when $f_{s,1} = 44.1$ kHz, and when $f_{s,2}$ is chosen according to part (a).

a) $x[n] = A\cos\left(2\pi\left(\frac{f_0}{f_s}\right)n + \phi\right)$ $\quad 0 < t < 240$

$n \in \{0 : 1/44.1\text{kHz} : 240\}$

$t = n f_s$ $\quad$ n has $(44.1E3)(240)$ samples

$\boxed{f_{s2} = 58.8\text{kHz}}$ $\quad$ $240(44.1e3) = 180 \cdot f_s$

b) $x(t) = \cos(2\pi(800)t)$ $\quad f_0 = 800$ $\quad \hat{\omega}_0 = \frac{8\cancel{00}}{441\cancel{00}}$

$x[n] = \cos\left(2\pi\left(\frac{8}{441}\right)n\right)$

$y(t) = x[n = t f_s] = \cos\left(2\pi\left(\frac{8}{441}\right)(t)(58.8e3)\right)$

$y(t) = \cos(2\pi(1066.7)t) \Rightarrow \boxed{f_1 = 1067 \text{ Hz}}$

PROBLEM 6.5.* The following difference equation defines an LTI discrete-time system:

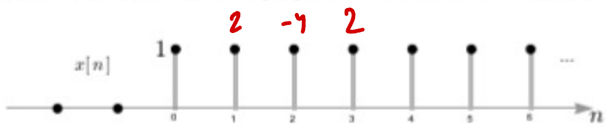
$$y[n] = 2x[n] - 4x[n-1] + 2x[n-2].$$

(a) Find the filter coefficients $\{b_k\}$ in the FIR representation $y[n] = \sum_{k=0}^{M} b_k x[n-k]$.

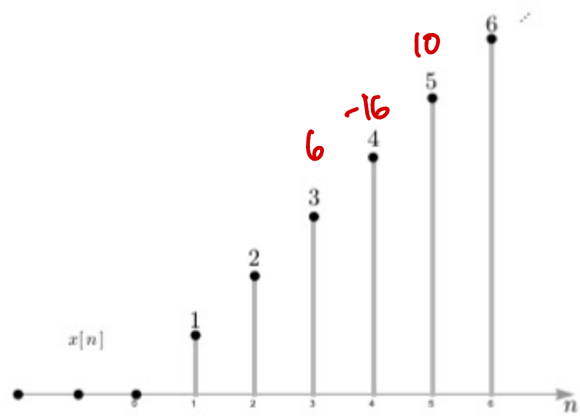
(b) Sketch a *stem plot* of the impulse response $h[n]$. Label each nonzero value.

(c) Sketch a stem plot of the output $y[n]$ when the input is $x[n] = \delta[n-1] + \delta[n-2] + \delta[n-3]$, as sketched below:

(d) Sketch a stem plot of the output $y[n]$ when the input is the unit step, $x[n] = u[n]$:

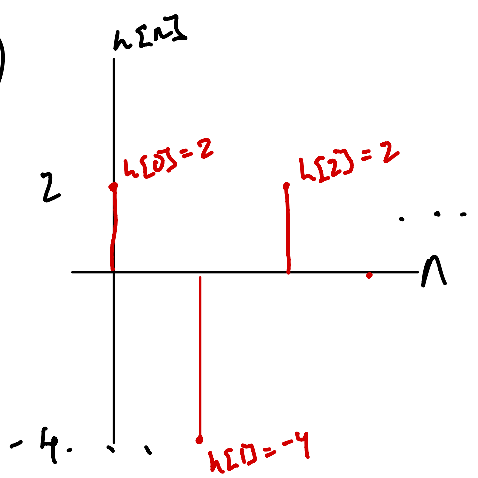


(e) Sketch a stem plot of the output $y[n]$ when the input is the unit ramp, $x[n] = nu[n]$, as sketched below:



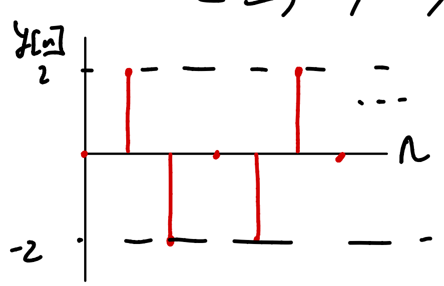
a) $b_k \in \{2, -4, 2\}$

b)

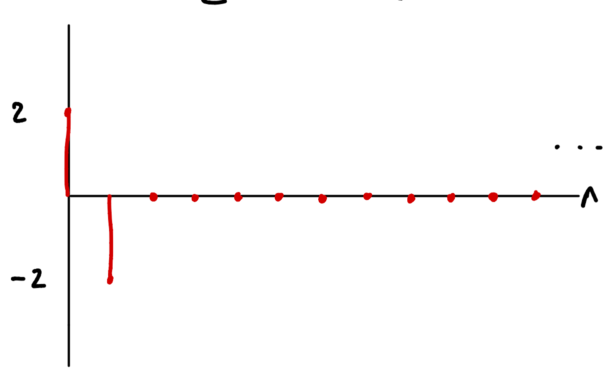


c) $x[n] = \{0, 1, 1, 1\}$

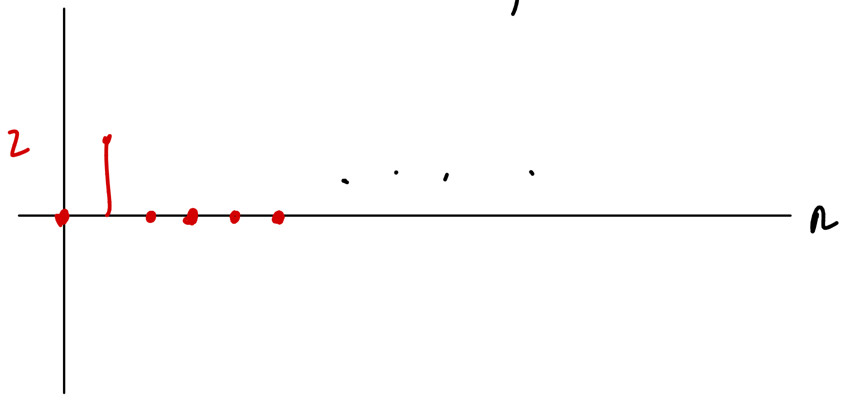
$y[n] = \{0, 2, -2, 0, -2, 2\}$



D) $x = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$



e) $x = \begin{cases} n, & n \geq 0 \\ 0, & n < 0 \end{cases}$



$y[n] = 2x[n] - 4x[n-1] + 2x[n-2]$

$x[n-1] = x[n-2] + 1$

$x[n] = x[n-2] + 2$

$\Rightarrow$

$\cancel{2x[n-2]} \cancel{+4} - \cancel{4x[n-2]} \cancel{-4} + \cancel{2x[n-2]}$

$= 0$ for $n \geq 2$

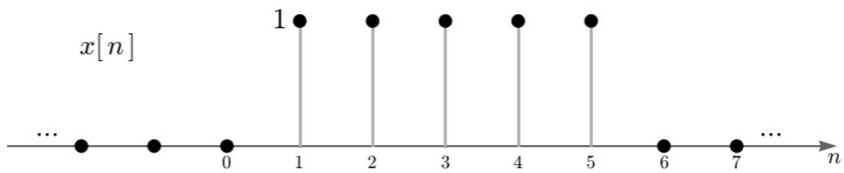
PROBLEM 7.1.* The following difference equation defines an LTI discrete-time system:

$$y[n] = x[n] + 2x[n-1] + x[n-2].$$

(a) Sketch a stem plot of the impulse response $h[n]$. Label each nonzero value.

(b) Find the filter coefficients $\{b_k\}$ in the FIR representation $y[n] = \sum_{k=0}^{M} b_k x[n-k]$.

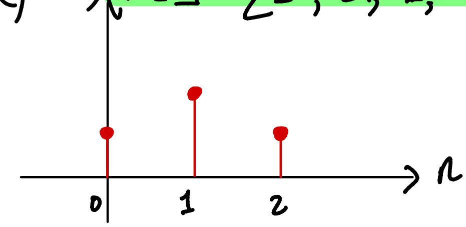
(c) Sketch a stem plot of the output $y[n]$ when the input is $x[n] = u[n-1] - u[n-6]$, the rectangular sequence whose stem plot is shown below. Use convolution.



(d) Sketch a stem plot of the output $y[n]$ when the input is the plus-minus sequence (a maximum-frequency sinusoid) $x[n] = (-1)^n = \cos(\pi n)$.

(You don't need Chap. 6 to solve this problem, you can stick to the time domain and use the tools of Chap. 5.)

a) $h[n] = \{1, 2, 1, 0, 0, \dots\}$



b) $b_k \in \{1, 2, 1\}$

c) $x[n] = \{\underline{0}, 1, 1, 1, 1, 1, 0, 0, \dots\}$

$y[n] = h[n] * x[n]$

=> 0 1 1 1 1 1 0 0

1 2 1 → 0

1 2 1 → 1

1 2 1 → 3

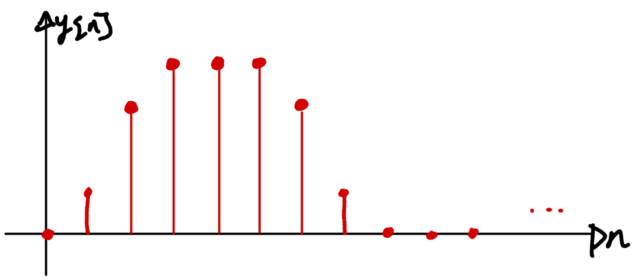
1 2 1 → 4

1 2 1 → 4

1 2 1 → 4

1 2 1 → 3

1 2 1 → 1



D) $x[n] = \cos(\pi n) = \{1, -1, 1, -1, 1, -1, \dots\}$

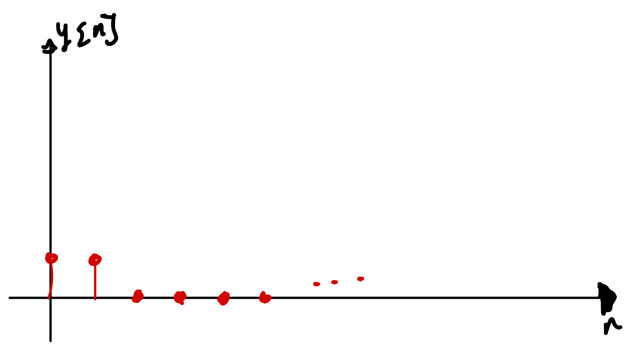
1 2 1 → 1 $n=0$

1 2 1 → 1 $n=1$

1 2 1 → 0 $n=2$

1 2 1 → 0 $n=3$

⋮



PROBLEM 7.2.* Suppose we want to convolve the following two signals:

$x[n] = \{2, 1, 3, -1, -2, 1, 2, -2, 1\}$

$$x[n] = 2\delta[n] + \delta[n-1] + 3\delta[n-2] - \delta[n-3] - 2\delta[n-4]$$
$$+ \delta[n-5] + 2\delta[n-6] - 2\delta[n-7] + \delta[n-8],$$

$$h[n] = 2\delta[n] + \delta[n-1]. \quad h[n] = \{2, 1\}$$

There are many ways to implement convolution, but for this problem you must implement it using a *convolution table*. Even then you have an extra degree of freedom that follows from the fact that convolution is commutative: You can view either of the above two signals as the filter input, and the other as the filter impulse response, it doesn't matter which is which, the filter output will be the same. The point of this problem is to implement it both ways, to confirm that they yield the same answer, and to observe that one way is easier than the other.

(a) Implement the convolution using a "tall, narrow" table that has lots of rows: the zero-th row represents $x[0]h[n]$, the next row represents $x[1]h[n-1]$, the next $x[2]h[n-2]$, and in general the k-th row (for $k \in \{0, 1, 2, \ldots\}$) represents $x[k]h[n-k]$.
The convolution is found by adding all of the rows.

(b) Implement the convolution the other way, using a "short, fat" table with only two main rows: the zero-th row represents $h[0]x[n]$, the next row represents $h[1]x[n-1]$. The convolution is found by adding these two rows.

(c) Verify that the answers in (a) and (b) are identical.

(d) In your opinion, which was easier to implement, (a) or (b)?

a)

x[n] \ h[n]	2	1
2	4	2
1	2	1
3	6	3
−1	−2	−1
−2	−4	−2
1	2	1
2	4	2
−2	−4	−2
1	2	1

$y[n]: \{4, 4, 7, 1, -5, 0, 5, -2, 0, 1\}$

c) Signals match ✓

D) B was easier to implement

⇒ column vector signal is 2nd in conv eqn

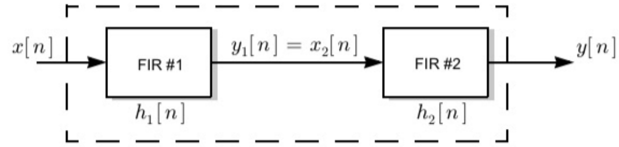
$$y[n] = \sum_k x[k]\, h[n-k]$$

b)

h[n] \ x[n]	2	1	3	−1	−2	1	2	−2	1
2	4	2	6	−2	−4	2	4	−4	2
1	2	1	3	−1	−2	1	2	−2	1

$h[n] = \{4, 4, 7, 1, -5, 0, 5, -2, 0, 1\}$

PROBLEM 7.3.* The figure below depicts a *cascade* connection of two FIR filters, in which the output $y_1[n]$ of the first filter is the input $x_2[n]$ to the second filter, and the output of the second filter is the *overall* output $y[n]$:



The "overall" filter formed by this cascade has input $x[n]$ and output $y[n]$, as depicted by the dashed box.

(a) Find the overall difference equation (relating $y[n]$ to $x[n]$) when:

- the first filter is a two-point averager
- the second filter is a first difference filter

(b) Find the overall step response (i.e., find $y[n]$ when $x[n] = u[n]$) when:

- the first filter is a five-point averager
- the second filter is a first difference filter

(c) Find the second impulse response $h_2[n]$ when the difference equations defining the first filter and the overall filter are:

$$y_1[n] = 2x[n] + 2x[n-1] - x[n-2],$$
$$y[n] = 4x[n] + 10x[n-3] - x[n-4] - x[n-5].$$

Specify your answer in two forms: (i) as a stem plot of $h_2[n]$ vs n; and (ii) by listing numerical values for $h_2[0]$, $h_2[1]$, $h_2[2]$, $h_2[3]$, and $h_2[4]$.

a) $y_1[n] = \frac{1}{2}\left(x[n] + x[n-1]\right) \Rightarrow h_1[n] = \{\frac{1}{2}, \frac{1}{2}\}$

$y_2[n] = x[n] - x[n-1] \rightarrow h_2[n] = \{1, -1\})$

$h[n] = h_1[n] * h_2[n] = \{1/2, 0, -1/2\}$

$y[n] = \frac{1}{2}x[n] - 1/2\, x[n-2]$

b) $h_1[n] = \{1/5, 1/5, 1/5, 1/5, 1/5\}$ $h_2[n] = \{1, -1\}$

$h[n] = \{1/5, 0, 0, 0, 0, -1/5\}$

1/5
1/5
1/5
1/5
1/5
0

$S[n] = \{1/5, 1/5, 1/5, 1/5, 1/5, 0, 0, \ldots\}$

$S[n] = \begin{cases} .2 & \text{for } 0 \le n \le 4 \\ 0 & \text{for } n \ge 5 \end{cases}$

c) $h_1[n] = \{\underline{2}, 2, -1\}$ $\quad h[n] = \{\underline{4}, 0, 0, 10, -1, -1\}$

$h_2[n] = ?$

The convolved signal is dimension $N + M - 1$ where N is samples in $h_1[n]$ and M is samples in h_2

$\Rightarrow 6 = 3 + M - 1 \Rightarrow M = 4 \Rightarrow h_2$ has 4 elements

$h[n] = h_1 * h_2 \Rightarrow \sum_k h_1[k]\, h_2[n-k]$

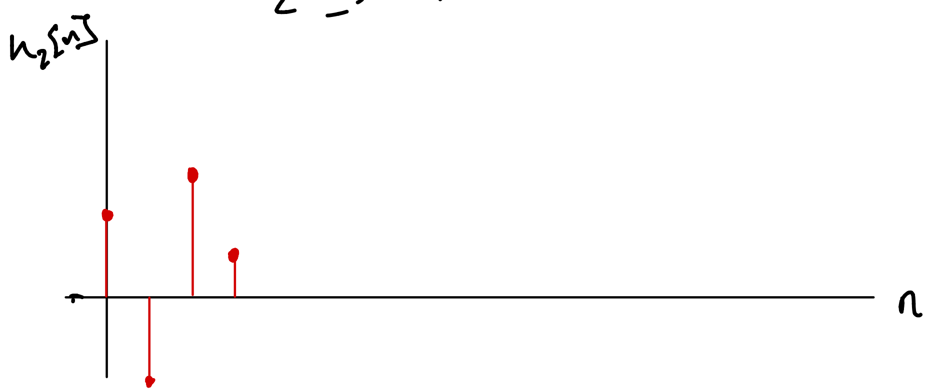
	a	b	c	d
2	2a	2b	2c	2d
2	2a	2b	2c	2d
-1	-a	-b	-c	-d

$2a = 4$

$2a + 2b = 0$
$-a + 2b + 2c = 0$
$-b + 2c + 2d = 10$
$-c + 2d = -1$
$-d = -1$

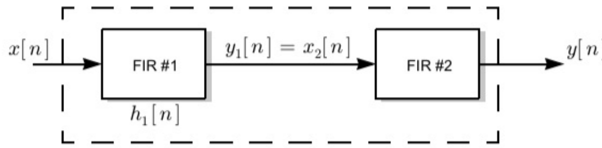
$$\begin{array}{cccc|c} a & b & c & d & \\ \hline 2 & 0 & 0 & 0 & 4 \\ 2 & 2 & 0 & 0 & 0 \\ -1 & 2 & 2 & 0 & 0 \\ 0 & -1 & 2 & 2 & 10 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & -1 \end{array}$$

$h_2[n] = \{\underline{2}, -2, 3, 1\}$



PROBLEM 7.4.* Consider the following serial cascade of two FIR filters, where

- the first filter has impulse response $h_1[n] = 6\delta[n] + 12\delta[n-1]$;
- the second filter is a six-point averager:



(a) Find numerical values for the following overall outputs:

$$y[0],\ y[1],\ \ldots\ y[8]$$

when the overall input signal is $x[n] = 2\delta[n] - \delta[n-1]$.

(b) Find numerical values for the following overall inputs:

$$x[0]$$
$$x[1]$$
$$x[2]$$

when the overall output signal is $y[n] = 2\delta[n] + 4\delta[n-1] - 2\delta[n-6] - 4\delta[n-7]$.

a) $h_1[n] = \{6, 12\}$ $h_2[n] = \{1/6, 1/6, 1/6, 1/6, 1/6, 1/6\}$

$h[n] = \{1, 3, 3, 3, 3, 3, 2\}$

$x[n] = \{2, -1\}$ $y[n] = x[n] * h[n]$

$y[n] = \{\underline{2}, 5, 3, 3, 3, 3, 1, -2, 0\}$

b) $y[n] = \{\underline{2}, 4, 0, 0, 0, 0, -2, -4\}$

$4 = 6 - 2$

$0 = 6 - 6$

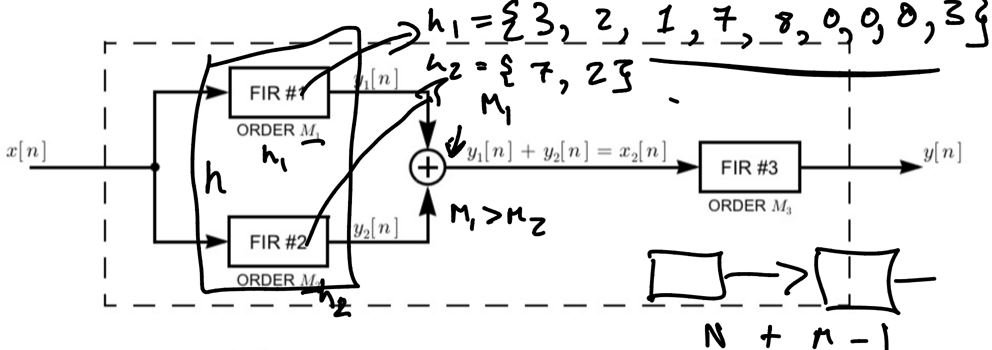
$x[0] = 2$

$8 = 7 + M - 1$

$x[1] = -2$

$x[2] = 0$

PROBLEM 7.5.* The figure below shows an overall filter constructed from three building-block filters using a combination of a parallel cascade and serial cascade:



The overall input $x[n]$ is fed as an input to both the first and second building-block filters, whose outputs are added before being fed to the last building block filter, yielding an overall output $y[n]$. Let M_i denote the order[1] of the i-th building-block filter, for $i \in \{1, 2, 3\}$, and let M denote the order of the *overall* filter (from $x[n]$ to $y[n]$), as indicated by the dashed box.

(a) Find M in the special case when all three building block filters are 8-point averagers.

(b) For the special case when $M_1 \geq M_2$ and $h_1[M_1] + h_2[M_1] \neq 0$:
Find an equation for the overall order M,
expressed as a function of the unspecified building-block orders $M_1 \geq M_2$, M_2, and M_3.

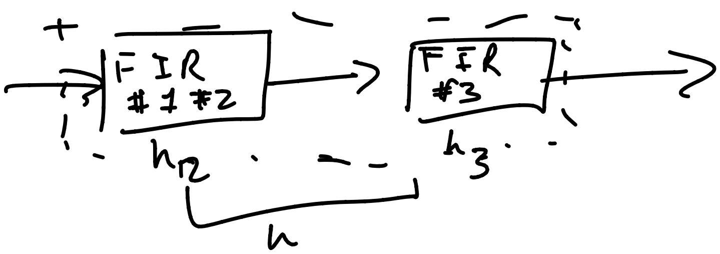
(c) How might the answer in part (b) change without the constraint $h_1[M_1] + h_2[M_1] \neq 0$?

(d) Find the overall impulse response $h[n]$ when:

- FIR#1 is a first difference filter
- FIR#2 is a two-point averaging filter
- FIR#3 is a ten-point averaging filter

a) When all Building Filters are 8-pt averagers

Since #1 and #2 are parallel summed =>



$h = h_{12} * h_3$

order of h is $N + M - 1$

$\Rightarrow 8 + 8 - 1 = \boxed{15}$

b) $M_1 + M_3 - 1$

c) without the constraint $h_1[M_1] + h_2[M_1] \neq 0$, meaning $M_1 = M_2$, Then The overall order is still $M_1 + M_3 - 1 = M$ Since $M_1 = M_2$ in This case.

D) $h_1[n] = x[n] - x[n-1] \Rightarrow \{1, -1\}$

$h_2[n] = \frac{1}{2} x[n] + \frac{1}{2} x[n-1] \Rightarrow \{\frac{1}{2}, \frac{1}{2}\}$

$h_3[n] = \{\frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}\}$

$h_{12}[n] = \{\frac{3}{2}, -\frac{1}{2}\} * h_3[n]$

$h[n] = \{\frac{3}{20}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, -\frac{1}{20}\}$

3.1 a) $f_s \geq 2 f_{o\,max} \Rightarrow$ if $f_{max} = 15$ Hz

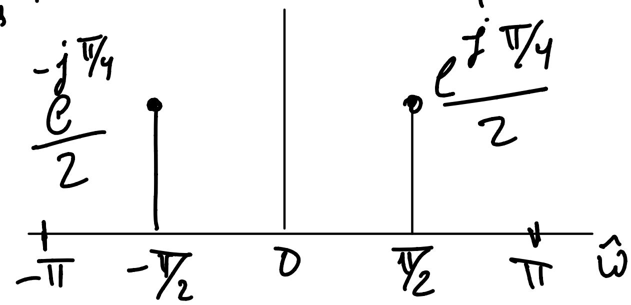
$\Rightarrow f_s \geq 30$ Hz

3.1 B) Signal $x(n) = \cos(2\pi(15)t + \phi)$ $\phi = \pi/4$

$t = \frac{n}{f_s}$ $\hat{\omega} = 2\pi\left(\frac{f_0}{f_s}\right) \Rightarrow 2\pi\left(\frac{15}{20}\right) = \frac{6\pi}{4}$

Reduce $\hat{\omega}$ s.t. $-\pi \leq \hat{\omega} \leq \pi \Rightarrow \hat{\omega} = \frac{\pi}{2}$

$x[n] = \cos\left(\frac{\pi}{2}n + \pi/4\right)$



3.1 c) $A = 1$

$\Rightarrow$ complex amplitude is $\frac{1}{2}e^{j\pi/4}$

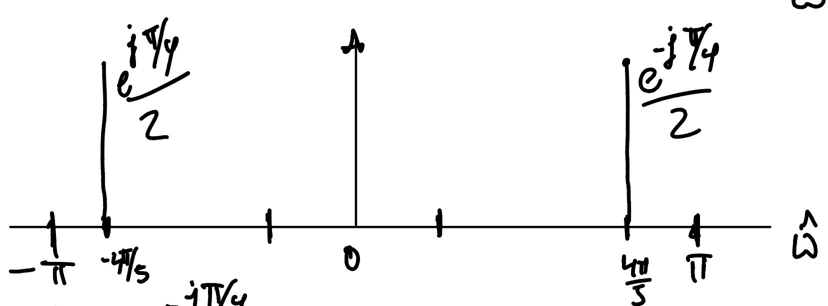
3.1 D) $f_0 = 12$ Hz $f_s = 20$ Hz $\phi = \pi/4$

$\cos(-\frac{4\pi}{5}n + \pi/4) = \cos(\frac{4\pi}{5} - \pi/4)$

$\hat{\omega} = \frac{12}{20}(2\pi) - 2\pi l$

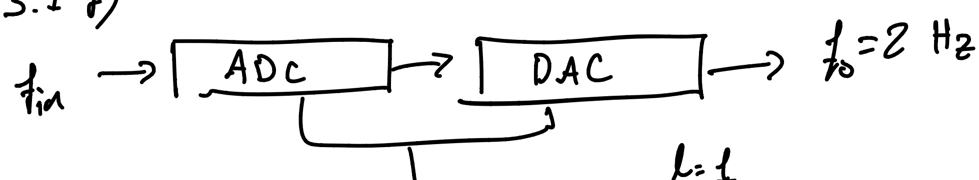
$= \frac{3}{5}(2\pi) - 2\pi l$

$= \frac{6\pi}{5} - 2\pi l \Rightarrow -\frac{4\pi}{5}$, $l = 1$



3.1 E) $\frac{e^{-j\pi/4}}{2}$

3.1 f)



$f_{in} = \begin{cases} f_0 + l f_s \\ -f_0 + l f_s \end{cases}$

$l = 1$

$f_{in} = 22$ Hz $\phi = -3\pi/4$

$f_{in} = 18$ Hz $\phi = 3\pi/4$

$l = 2$ $f_{in} = 42$ Hz $\phi = -3\pi/4$

Made with Goodnotes

3.1 f) $f_{in} = 15$ Hz $\phi = \pi/4$

$$-\pi < \hat{\omega} < 0 \Rightarrow \hat{\omega} = 2\pi\left(\frac{15}{f_s}\right) - 2\pi l$$

$$-\cancel{\pi} < \hat{\omega} = \cancel{2\pi}\left(\frac{15}{\frac{4}{f_s}} - l\right) < 0$$

$f_s = 19$ Hz

$$\frac{15}{20} = \frac{3}{4}(2\pi) = \frac{6}{4}\pi - 2\pi l = \frac{-2\pi}{4} = \frac{-\pi}{2} \Rightarrow$$

$$\frac{15}{18} \quad \frac{5}{6}(2\pi) \quad \frac{10}{6}\pi - 2\pi l \qquad -10\pi \quad 2\pi(-5)$$

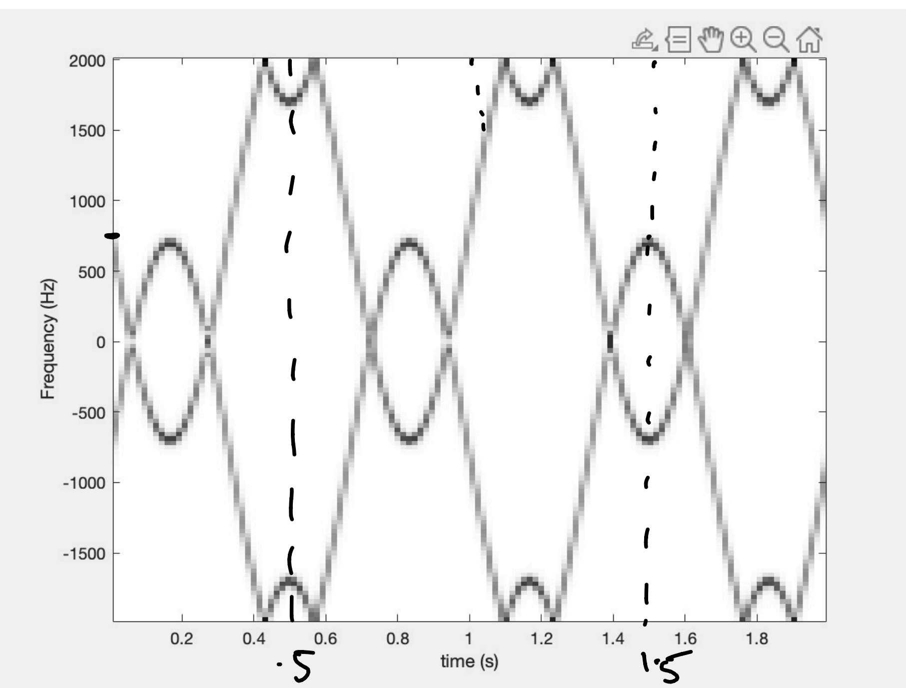
$$\frac{-2\pi}{6} = \frac{-\pi}{3}(18)t = -6\pi \quad 2\pi(-3)$$

$$\frac{15}{19}(2\pi) \quad \frac{30}{19}\pi - 2\pi l$$

$$-\frac{8\pi}{\cancel{19}}\cancel{(19)}$$

3.2 a) $x(t) = A\cos(2\pi f_c t + \alpha\cos(2\pi \beta t + \gamma))$

$A = 2$, $f_c = 800$ Hz, $\alpha = 1000$ Hz, $\beta = 1.5$, $\gamma = 0$, $f_s = 4000$

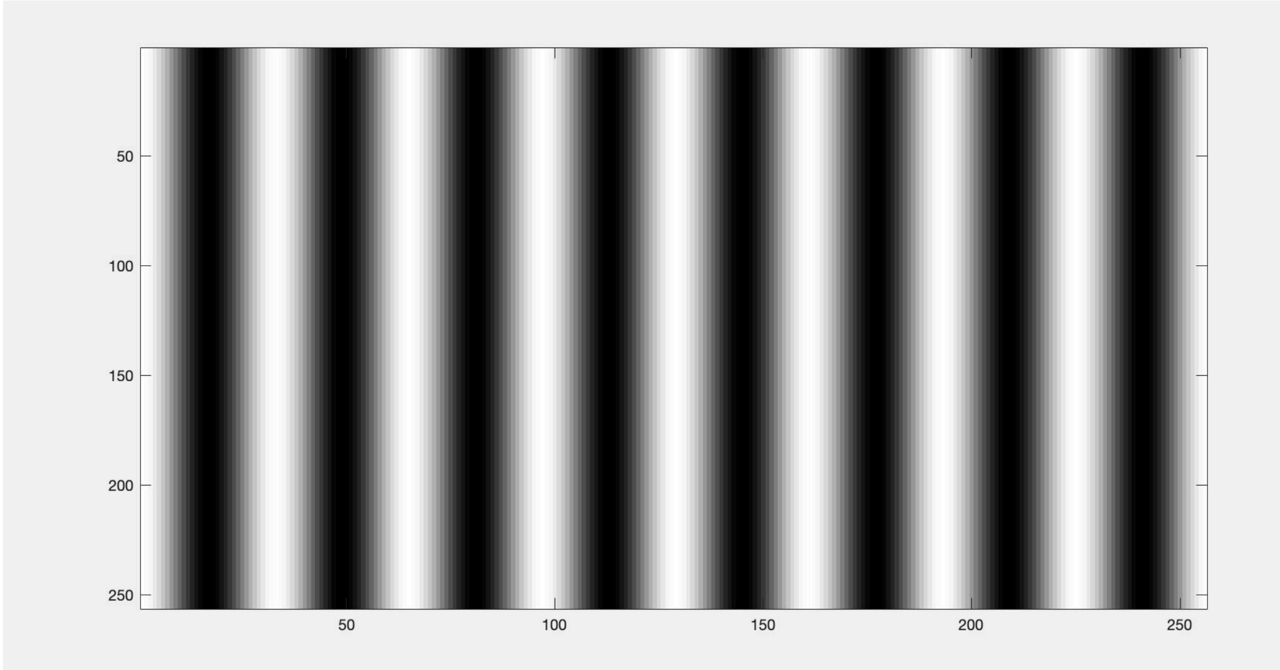


f_i is correct within the range $|0 \text{ \& } 2000 \text{ Hz}|$

Given that the f_i is $\frac{d\psi(t)}{dt}\frac{1}{2\pi} = f_c + -\alpha(\beta)\sin(2\pi\beta t)$

800 is lifted by max 1500 which would pass 2000 Hz and alias the signal as f_s is 4000.

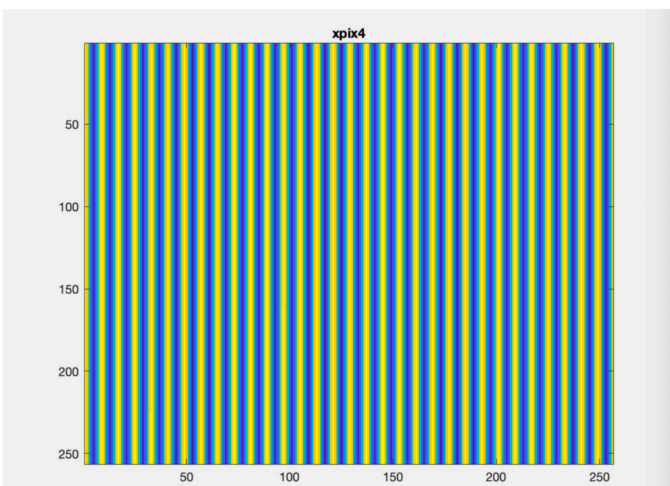
3.3 a)

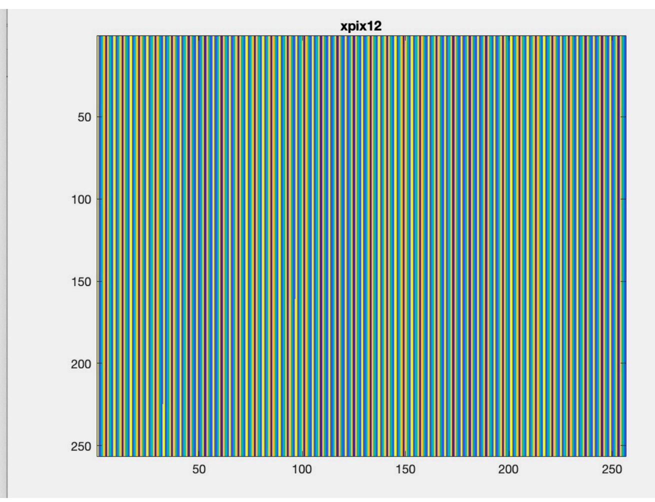


3.3 B)

Black is -1 while white is 1

3.4 a)

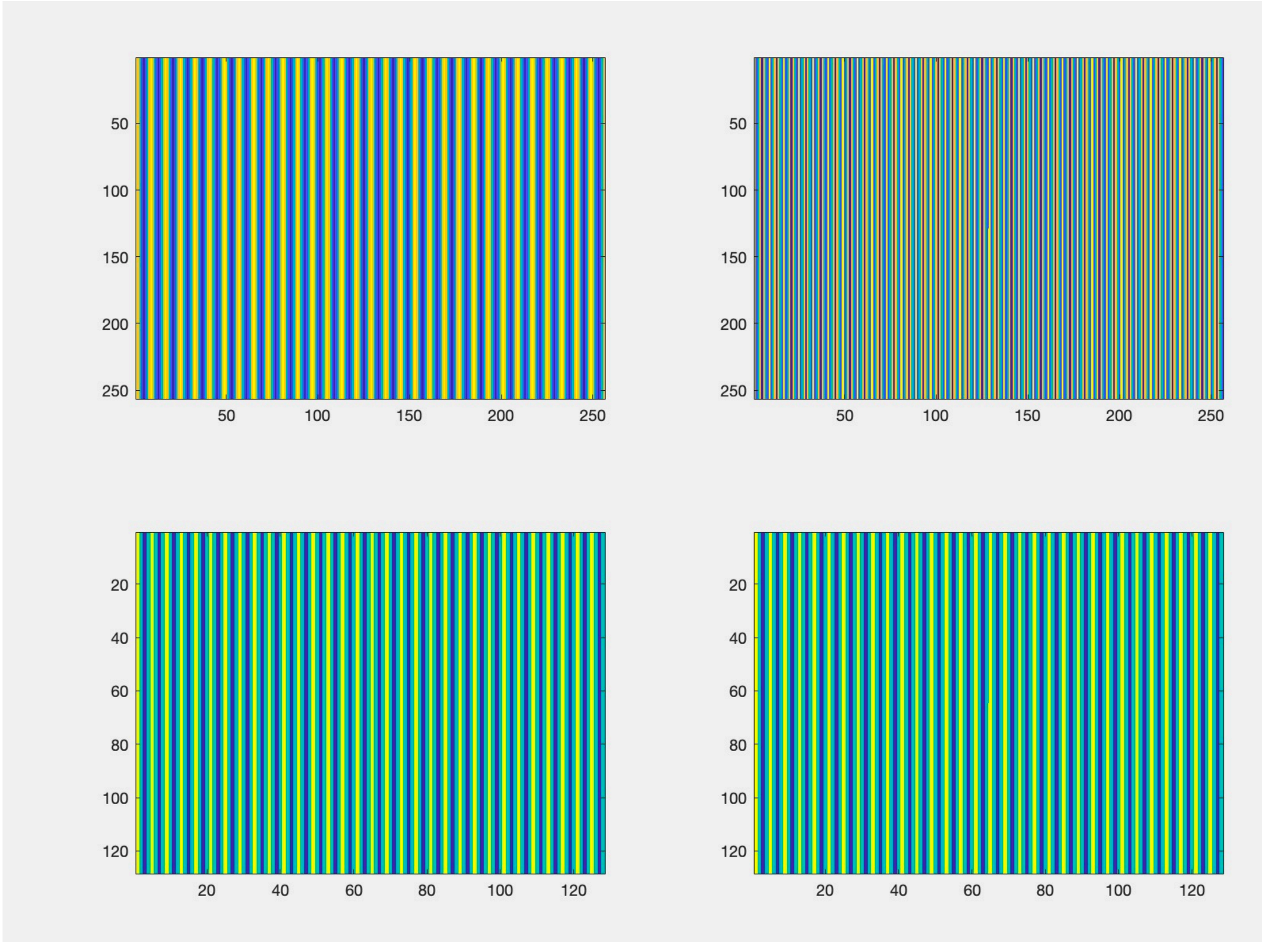




- xpix 12 is higher frequency Cause Because the cosine function oscillates between -1 and 1 3 times as fast as xpix 4
  ⇒ shorter horizontal period

3.4 b) Next page

3.4 B)

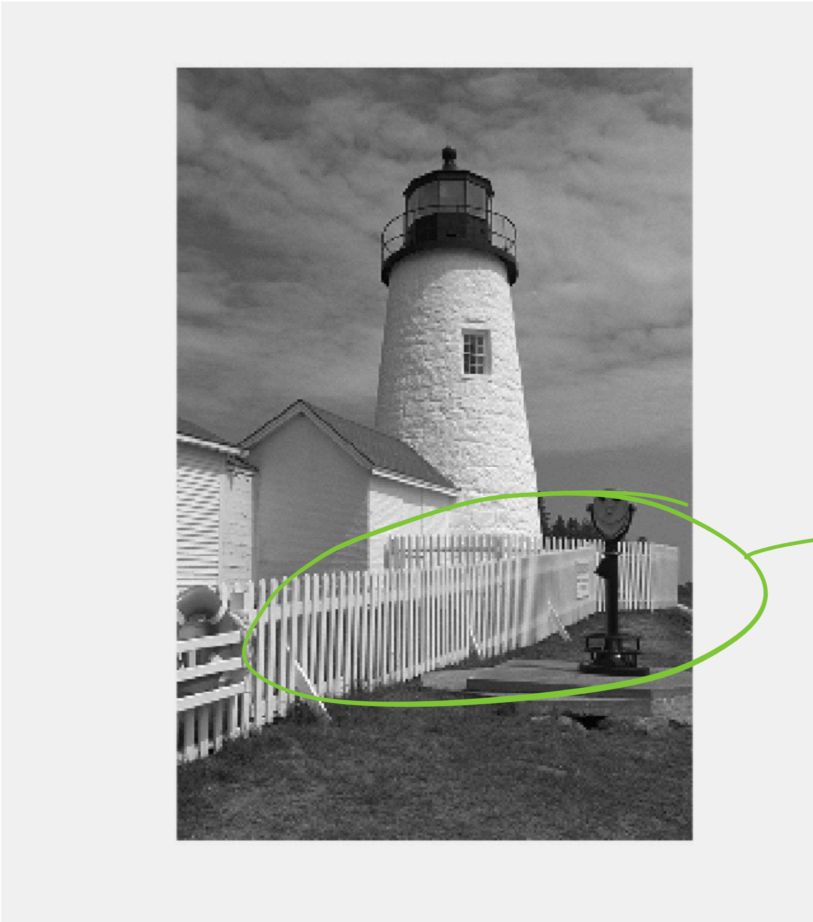


every other sample is removed and essentially the sampling rate is halved. considering how the first signal was $\hat{\omega} = 2\pi\left(\frac{4}{32}\right)$ and the second $\hat{\omega} = 2\pi\left(\frac{12}{32}\right)$

If we half the f_s to be $16 \Rightarrow \hat{\omega}_4 = 2\pi\left(\frac{4}{16}\right) = \frac{\pi}{2}$

and $\hat{\omega}_{12} = 2\pi\left(\frac{12}{16}\right) = 2\pi\left(\frac{3}{4}\right) = \frac{6\pi}{4} - 2\pi l = -\frac{2\pi}{4} = -\pi/2$ a signal of same aliased frequency.

3-5 A) Down Sampled size is 312 x 214



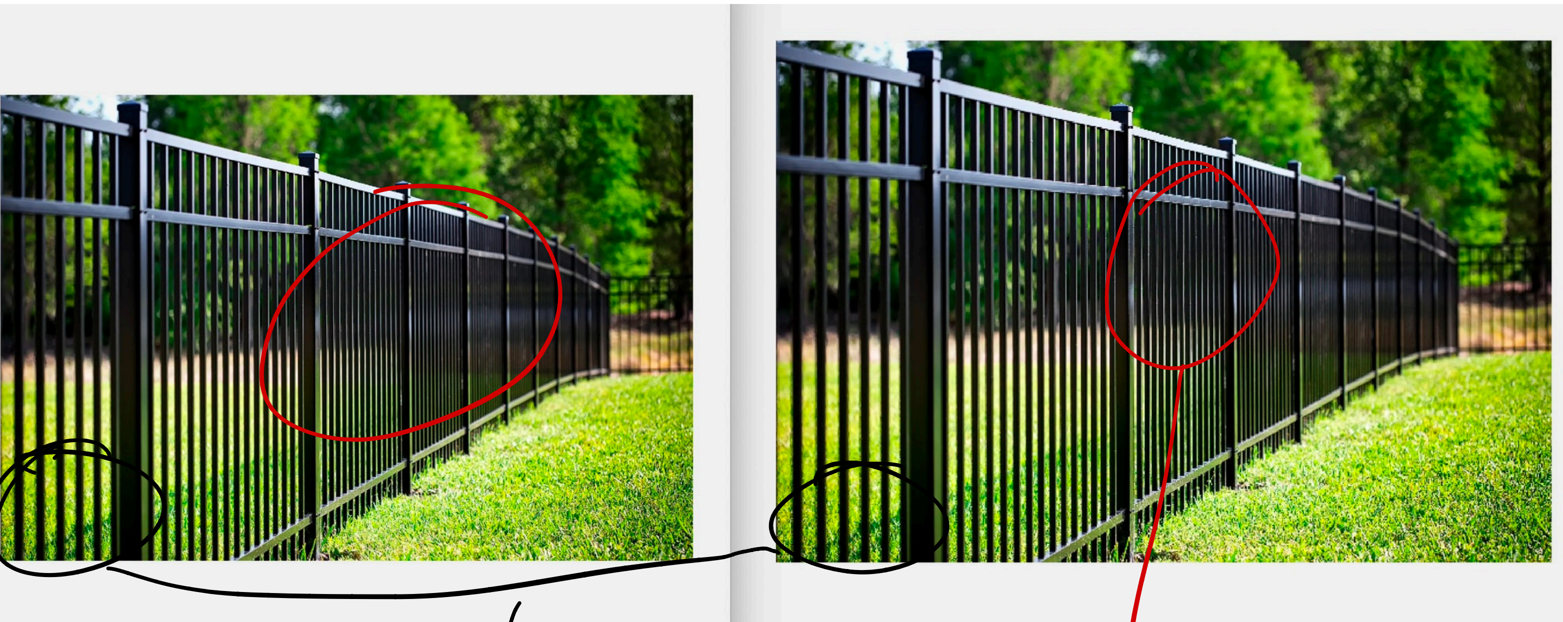
3-5 B)

most significant effects of aliasing

it appears blurry and almost skips details

The small fence holes/planks are increasing in frequency at that region and the down sampled image cannot reconstruct it well.

Own Image



Other regions of the fence (closer to us) is not aliased as the horizontal frequency of Green -> Black is very low so even w/ a factor of 3 down sampled => image is not aliased in that region.

in here you can see faint shapes ⋂ ⋂ in the rails of the fence => skips the non black regions when down sampling by a factor of 3.

CRIB Sheet 2026

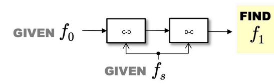
$$\cos\alpha\cos\beta = \frac{\cos(\alpha+\beta)+\cos(\alpha-\beta)}{2}$$

$$\sin\alpha = \cos(\alpha - \pi/2)$$

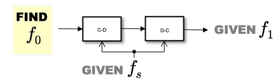
$$\cos(-\alpha) = \cos\alpha$$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j(2\pi)(k)f_0 t}$$ fourier coefficients

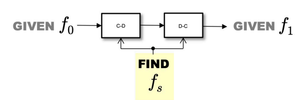
$$a_k = \frac{1}{T_0}\int_0^{T_0} x(t)\, e^{-j(2\pi)(k)f_0 t}\, dt$$ analysis fourier coefficients



$$f_1 = \min_\ell |f_0 - \ell f_s| = |f_0 - \ell^* f_s|$$



$$f_0 = \pm f_1 + \ell f_s$$



$$f_s = \frac{f_0 \pm f_1}{\ell} \geq 2f_1$$

$$f_0 = GCD(f_1, f_2, \ldots, f_n)$$

if $f_n\ \forall n \in \mathbb{R}$ is irrational ⇒ signal $x(t)$ is non periodic

if $x(t) = x(-t)$ (even) ⇒ $a_k \in \mathbb{R}\ \forall k$

if $x(-t) = -x(t)$ (odd) ⇒ $a_k \in \mathbb{C}\ \forall k$

$$f_i(t) = \frac{1}{2\pi}\frac{\partial \psi(t)}{\partial t}$$

$$\hat{\omega} = 2\pi(f/f_s)\ ; \quad \hat{\omega} \in (-\pi, \pi]$$

$$\hat{\omega} = \min(\hat{\omega} \pm 2\pi l)$$

$x[n] = \cos(\hat{\omega} n)$ periodic when

1. $x[0] = x[N]$
2. $\hat{\omega} N = M 2\pi$, $M \in \mathbb{Z}$
3. $\hat{\omega} = 2\pi \frac{M}{N}$, $\frac{M}{N}$ is Rational

$$x(t) \longrightarrow \boxed{ADC} \longrightarrow x[n]$$

$t = n/f_s$

$n = t f_s$

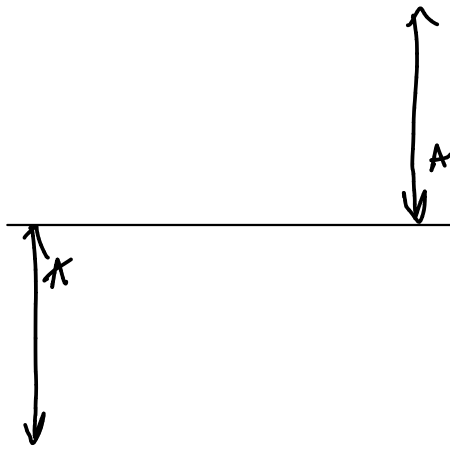
a system is linear when

$$\alpha x_1[n] + \beta x_2[n] = \alpha y_1[n] + \beta y_2[n]$$

a system is Time-invariant

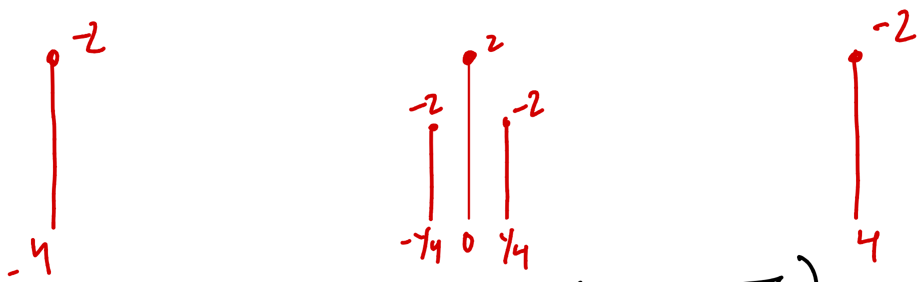
$$x[n-n_0] = y[n-n_0]$$

$T_0 = 4$



$1/4$

2



$$\frac{2}{.5} \Rightarrow 4 \text{ Hz}$$

$$x(t) = 2 + \cos(2\pi(4) - \pi/2) + -2\cos(2\pi(1/4))$$

$$y(t) = 2 + \cos\left(2\pi\left(\frac{4}{40}\right)n - \pi/2\right) - \cos\left(2\pi\left(1/160\right)\right)$$

$h[n] = \{0, 1\}$

$n = 1$ Delay filter

$y[n] = k\cos(\Omega n)$

$y[n] = x[n-1]$

$\cos(\pi(n-1))$

$\cos(\pi n - \pi)$

$e^{j(-\pi)} \cos(\pi n)$

1 −1 1 −1 . . .

0 0 ⟶

1 1 −1 1 −1

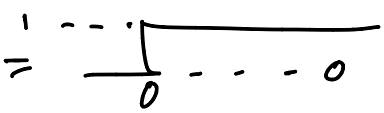
$\cos(\pi n)$

0 1 −1 1 −1 1 −1

$$7(x^2[n-1] - 2x[n-1] + 1) \Rightarrow 7x^2[n-1] - 14x[n-1] + 7$$

Non-LTI as if $x[n] = \{0\}$ Then $w[n] = 7 \neq 0 \Rightarrow$ Not linear or homogeneous.

No since if the input is delayed by α Then output should also only be delayed by α But is $(n-\alpha)w[n-\alpha]$

$x =$ 

$n = -3 \xrightarrow{w} 7$

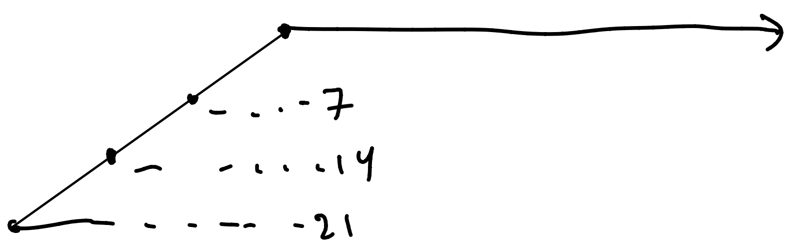
$= -2 \xrightarrow{w} 7$

$= -1 \xrightarrow{w} 7$

$= 0 \xrightarrow{w} 7$

$= 1 \rightarrow 0$

$$\left.\begin{array}{l} = 2 \\ = 3 \\ = 4 \end{array}\right\} \rightarrow 0$$



$$x = \cos\left(2\pi(7000)t + \pi/3\right)$$

$$\hat{\omega} = 2\pi\left(\frac{7000}{8000}\right)$$

$$\frac{2\pi \cdot 7}{8} = \frac{14}{8}\pi - 2\pi = \frac{14}{8}\pi - \frac{16}{8}\pi$$

$$-\frac{2}{8}\pi = -\frac{1}{4}\pi$$

$$\Rightarrow x[n] = \cos\left(2\pi\left(\tfrac{1}{8}\right)n - \pi/3\right)$$

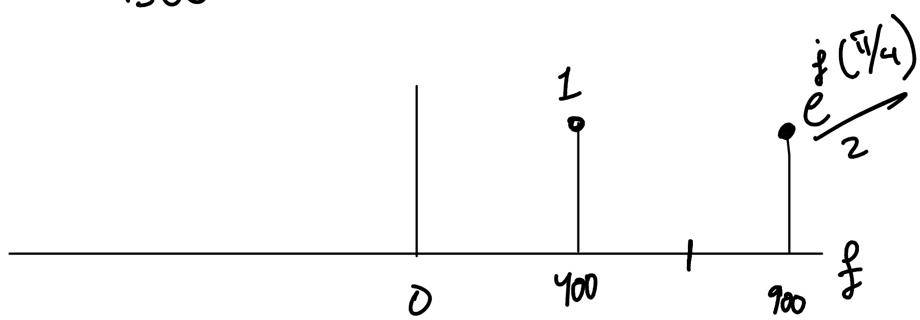
$$n = tf_s$$

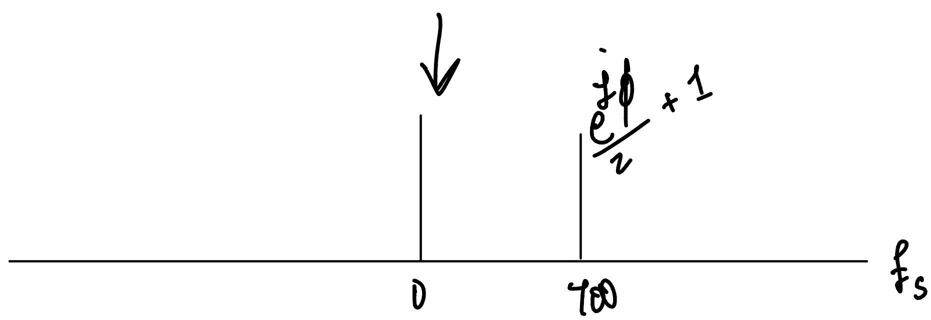
$$3000 = \left(\tfrac{1}{8}\right) f_s$$

$$\boxed{f_s = 24000}$$

$2\pi(400)t \quad 2\pi(900)t + \pi/4$

1300





$800 < f_s < 1800$ fold 900 into 400 Hz

$\cos(2\pi(\frac{900}{f_s}) n + \pi/4)$ want f_s, have f_o, f_i

$f_s = \frac{f_o \pm f_i}{l}$ $\quad \frac{900 \pm 400}{2l} < 1600 \quad 2\pi(\frac{900}{1300})$

$\frac{1300}{l}$

$\frac{18}{13}\pi - 2\pi \Rightarrow \frac{18}{13}\pi - \frac{26}{13}\pi$

$-\frac{8}{13}\pi \Rightarrow \cos(2\pi(\frac{7}{13})n - \pi/4)$

$e^{j\phi} + 1$

$e^{j(\pi/4)} + 1 = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} j + 1 \Rightarrow \frac{2+\sqrt{2}}{2} \cancel{*} \frac{\sqrt{2}}{2} j$

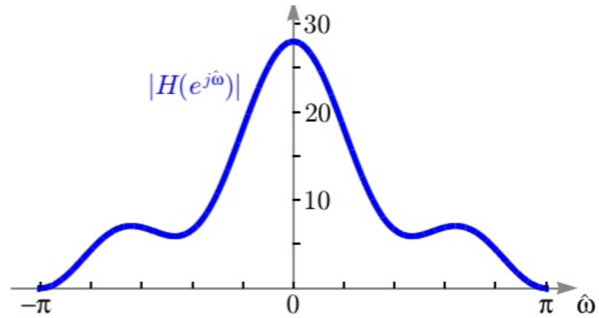
fs = 1300

$2\pi(800)t - \pi/4 - \phi$

PROBLEM 8.1.* Consider an LTI system whose impulse response is $h[n]$ and whose frequency response is:

$$H(e^{j\hat{\omega}}) = e^{-3j\hat{\omega}}\big(10 + 10\cos(\hat{\omega}) + 4\cos(2\hat{\omega}) + 4\cos(3\hat{\omega})\big),$$

with magnitude response $|H(e^{j\hat{\omega}})|$ shown here:



(a) Compare the sum of the impulse response coefficients $\sum_{k=-\infty}^{\infty} h[k]$ to the frequency response evaluated at zero frequency $H(e^{j0})$.

(b) Find the system output $y[n]$ when the system input is:

$$x[n] = 0.25 + 2\cos(\tfrac{\pi}{3}n) + 2\cos(\tfrac{\pi}{2}n) + 4\cos(\tfrac{2\pi}{3}n).$$

Find α and β so that the difference equation relating the input $x[n]$ to the output $y[n]$ can be written as:

$$y[n] = \alpha x[n] + \alpha x[n-1] + \beta x[n-2] + (\alpha\beta)x[n-3] + \beta x[n-4] + \alpha x[n-5] + \alpha x[n-6].$$

a) h[n] can be derived from expanding $H(e^{j\omega})$ and finding the coefficients of each exponential

$$e^{-3j\omega}(10 + 5e^{j\omega} + 5e^{-j\omega} + 2e^{j2\omega} + 2e^{-j2\omega} + 2e^{j3\omega} + 2e^{-j3\omega})$$

$$\cancel{10e^{-j3\omega}} + \cancel{5e^{-j2\omega}} + \cancel{5e^{-j4\omega}} + \cancel{2e^{-j\omega}} + \cancel{2e^{-j5\omega}} + \cancel{2e^{0}} + \cancel{2e^{-j6\omega}}$$

$h[n] = \{\underline{2}, 2, 5, 10, 5, 2, 2\} \Rightarrow \sum h[n] = 28$ Same

$H(e^{j\omega})\big|_{\hat{\omega}=0} = e^{3j0}(10 + 10\cos(0) + 4\cos(2(0)) + 4\cos(3(0))) = 28$

b) if $x[n]$ is of $A\cos(\hat{\omega}n + \phi) \Rightarrow$

$y[n] = |H(e^{j\omega})| \cdot A\cos(\hat{\omega}n + \phi + \angle H(e^{j\omega}))$

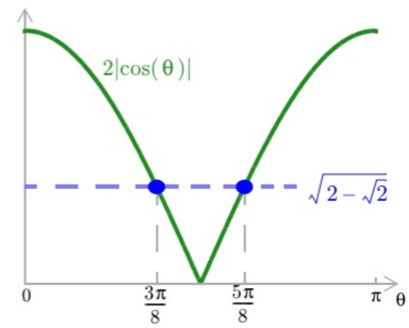
$\hat{\omega}$	$\lvert H(e^{j\omega})\rvert$	$\angle H(e^{j\omega})$
0	28	0
$\pi/3$	9	$-\pi$
$\pi/2$	6	$-3\pi/2$
$2\pi/3$	7	-2π

$y[n] = 7 - 18\cos(\pi/3\, n) + 12\cos(\tfrac{\pi}{2}n - 3\pi/2) + 28\cos(\tfrac{2\pi}{3}n)$

Solved in Part (a) $\boxed{\alpha = 2 \\ \beta = 5}$

PROBLEM 8.2.* Consider an FIR filter defined by the following difference equation:

$$y[n] = x[n] + \sqrt{2-\sqrt{2}}\,x[n-1] + x[n-2].$$



Hint: The accompanying plot of $2|\cos(\theta)|$ versus θ shows how the constant $\sqrt{2-\sqrt{2}} \approx 0.765$ is related to $3\pi/8$ and $5\pi/8$:

(a) Find the filter output $y[n]$ when the filter input is the sinusoid $x[n] = \dfrac{1}{\sqrt{2-\sqrt{2}}}\sin(\pi n/2)$.

$\cos(\frac{\pi}{2}n - \pi/2)$

(b) This FIR filter is called a "nulling" filter because it nulls a sinusoidal input for a particular sinusoid frequency. Which frequency does this filter null? In other words, for what positive value of $\hat{\omega}_0$ (in the range 0 to π) will an input of the form $x[n] = A\cos(\hat{\omega}_0 n + \varphi)$ result in the all-zero output, $y[n] = 0$ for all n?

1 - 2 - 1 filter $\Rightarrow$ $H(e^{j\omega}) = e^{-j\omega}(b_1 + 2\cos(\omega))$

a) Given $\hat{\omega}$ of $x[n]$ is $\pi/2$ $\Rightarrow$ $|H(e^{j\omega})| = \sqrt{2-\sqrt{2}} + 2\cancel{\cos(\pi/2)}^{0}$

$= .765$

$\angle H(e^{j\omega}) = -\pi/2$

$\Rightarrow y[n] = \dfrac{\sqrt{2-\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\cos(\frac{\pi}{2}n - \pi) \Rightarrow y[n] = \cos(\frac{\pi}{2}n - \pi)$

$\Rightarrow \boxed{-\cos(\frac{\pi}{2}n)}$

b) $b_1 + 2\cos(\omega) = 0$

$\Rightarrow \sqrt{2-\sqrt{2}} = -2\cos(\hat{\omega})$

$\omega = \cos^{-1}\left(\dfrac{-\sqrt{2-\sqrt{2}}}{2}\right) \Rightarrow \boxed{.625\pi}$

Made with Goodnotes

PROBLEM 8.3.* Consider an LTI system whose frequency response is $H(e^{j\hat{\omega}}) = e^{-2j\hat{\omega}}(2 - 2\cos(2\hat{\omega}))$.

$2 - 2\cos(\pi)$

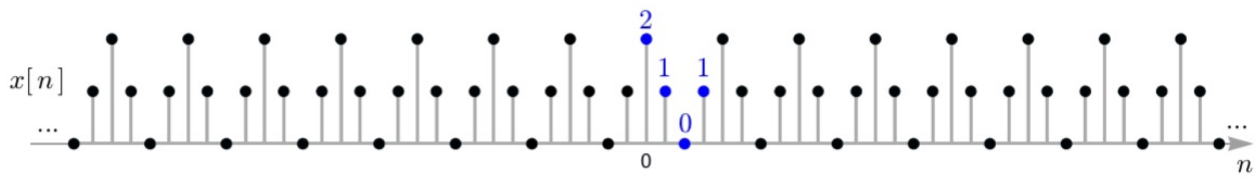
(a) Is this system an FIR filter?
If *YES*, find the difference equation that defines the FIR filter.
If *NO*, explain why not.

(b) Find the *dc gain* of the system.

(c) Find the output $y[n]$ in response to the constant input sequence $x[n] = 16$, for all n.

(d) Find the output in response to the "plus-minus" input sequence $x[n] = \cos(\pi n) = (-1)^n$.

(e) Find the output $y[n]$ in response to the "periodic extension" of [2 1 0 1],
i.e., an input $x[n]$ that is periodic with period 4 and satisfies $[x[0], \ldots x[3]] = [2, 1, 0, 1]$, as illustrated below:



(f) Specify a sinusoidal input $x[n]$ in *standard form*[1] that would result in the following output:

$$y[n] = 12\cos(\tfrac{\pi}{3}n).$$

(g) Specify numeric values for the constants a and b so that a system input of the form:

$$x[n] = a\delta[n] + b\delta[n-2] + a\delta[n-4] + \frac{a}{b} + a\cos(\pi n + b)$$

will result in the following system output:

$$y[n] = -\delta[n] + 2\delta[n-4] - \delta[n-9].$$

a) $H(e^{j\omega}) = e^{-j2\omega}(2 - e^{j2\omega} - e^{-j2\omega}) = 2e^{-j2\omega} - e^{0} - e^{-j4\omega}$

$\Rightarrow h[n] = \{\underline{-1}, 0, 2, 0, -1\}$

$\boxed{\text{Yes}}$ $\quad y[n] = -x[n] + 2x[n-2] - x[n-4]$

b) DC Gain is $\sum_{k}^{N} h[k] \Rightarrow 0$

c) $\forall n \in \mathbb{Z}$, $x[n] = 16 \Rightarrow y[n] = 0$ Since @ $\omega = 0$ for x, $f[n] = 0$

D) if $x[n] = (-1)^n$, $y[n] = 0$ Since for $n = 0, 2, 4$, $x[n]$ is even and 1 and thus suming over that interval results in $y[n] = 0$

E) $x[n] = 1 + \cos(\frac{\pi}{2}n)$ Given $\hat{\omega} = \pi/2 \Rightarrow |H(e^{j\omega})| = 4$ & $\angle H(e^{j\omega}) = -\pi$

$\Rightarrow x[n] = 4\cos(\frac{\pi}{2}n - \pi) = \boxed{-4\cos(\frac{\pi}{2}n)}$

f) $y = 12\cos(\frac{\pi}{3}n)$ $\qquad \hat{\omega} = \frac{\pi}{3}$

$$2 - 2\cos(2\omega) = \pm 12$$

$$2 - 2\cos(\tfrac{2\pi}{3}) = 2 - 2(-\tfrac{1}{2}) = 2 + 1 = 3 = |H(e^{j\omega})|$$

$$\angle H(e^{j\omega}) = -2\omega = -\frac{2\pi}{3}$$

$$\boxed{x[n] = 4\cos(\tfrac{\pi}{3}n + \tfrac{2\pi}{3})}$$

G) $x[n] = \alpha\,\delta[n] + \beta\,\delta[n-2] + \alpha\,\delta[n-4] + \frac{\alpha}{\beta} + \alpha\cos(\pi n + \beta)$

$x[n]$ has frequency $\omega = \pi$ and 0 (0 for a/b and π for $\cos(\dots)$ terms)

$$H(e^{j0}) = 0 \quad \& \quad H(e^{j\pi}) = 0$$

$\Rightarrow$ $x[n]$ is nulled @ $\omega = \pi$ and $0 \Rightarrow x[n] = a\,\delta[n] + B\,\delta[n-2]$ to $\delta[n-4]$

$y[0] = -1 = -a \Rightarrow \boxed{a = 1}$

$y[4] = 2 = -1 + 2b - 1$

$2 = -2 + 2B \Rightarrow \boxed{B = 2}$

x \ $h[n]$	-1	0	2	0	-1
a	-a	0	2a	0	-a
0	0	0	0	0	0
B	-B	0	2b	0	-B
0	0	0	0	0	0
a	-a	0	2a	0	-a

PROBLEM 8.4.* Consider the following FIR filter:

$$y[n] = x[n] + Ax[n-1] + Bx[n-2] + Ax[n-3] + x[n-4].$$

Find numeric values for the unspecified constants A and B so that the output in response to:

$$x[n] = 2.5\cos(0.25\pi n + 0.25\pi) + 5\cos(0.5\pi n + 0.5\pi)$$

is $y[n] = 0$, for all n. $\omega = \frac{\pi}{4}$ and $\frac{\pi}{2}$

$$h[n] = \{1, A, B, A, 1\}$$

$$H(e^{j\omega}) = 1 + Ae^{-j\omega} + Be^{-j2\omega} + Ae^{-j3\omega} + e^{-j4\omega}$$

$$e^{-j2\omega}\left(e^{j2\omega} + Ae^{j\omega} + B + Ae^{-j\omega} + e^{-j2\omega}\right) \Rightarrow e^{-j2\omega}\left(B + 2A\cos(\omega) + 2\cos(2\omega)\right)$$

$$|H(e^{j\omega})| = B + 2A\cos(\omega) + 2\cos(2\omega)$$

$$0 = B + 2A\cos(\tfrac{\pi}{4}) + 2\cancel{\cos(\tfrac{\pi}{2})}^{\to 0} \quad \text{for} \quad \hat{\omega} = \pi/4$$

$$0 = B + 2A\cancel{\cos(\tfrac{\pi}{2})}^{\to 0} + 2\cos(\pi) \quad \text{for} \quad \hat{\omega} = \pi/2$$

$$\boxed{B = 2} \qquad 2 + \cancel{2}A\left(\tfrac{\sqrt{2}}{\cancel{2}}\right) = 0 \Rightarrow A\sqrt{2} = -2 \Rightarrow \boxed{A = -\frac{2}{\sqrt{2}}}$$

PROBLEM 8.5.* Consider a generalization of the FIR nulling filter from Prob. 8.2, defined by the difference equation:

$$y[n] = x[n] + b_1 x[n-1] + x[n-2].$$

In Prob. 8.2 the coefficient b_1 was $\sqrt{2-\sqrt{2}}$, in this problem it could be anything.

(a) Write an equation for the filter's frequency response in the form:

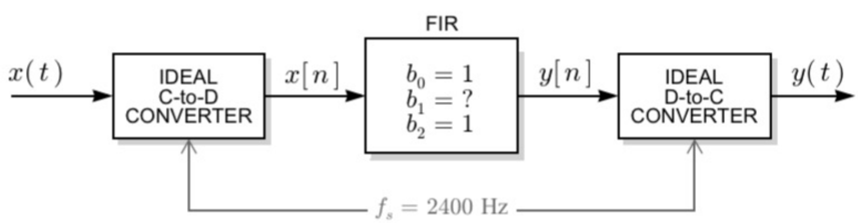
$$H(e^{j\hat{\omega}}) = e^{-Cj\hat{\omega}}\big(B + A\cos(\hat{\omega})\big)$$

for some constants A, B, and C that may depend on the unspecified coefficient b_1.
(*Hint:* factor out $e^{-j\hat{\omega}}$ and then apply Euler's relation to what's left.)

(b) In order for this filter to null a sinusoid with frequency $\hat{\omega}_0$, what must the coefficient b_1 be? Express b_1 as a function of the digital frequency $\hat{\omega}_0$ to be nulled.

When combined with ideal C-D and D-C converters, this filter can also be used to null a *continuous-time* sinusoidal input.

Consider the cascade below, where a continuous-time input is sampled at $f_s = 2400$ Hz, filtered in discrete time, and then converted back to continuous time (also with $f_s = 2400$ Hz):



(c) If $y(t) = 0$ for all t when $x(t) = 0.16\cos(1600\pi t + 0.16\pi)$, what is b_1?
(d) If $y(t) = 0$ for all t when $x(t) = 0.52\cos(5200\pi t + 0.52\pi)$, what is b_1?

a) $h[n] = \{1, b_1, 1\} \Rightarrow H(e^{j\omega}) = 1 + b_1 e^{-j\omega} + e^{-j2\omega}$

$H(e^{j\omega}) = e^{-j\omega}(e^{+j\omega} + b_1 + e^{-j\omega}) \Rightarrow \boxed{e^{-j\omega}(b_1 + 2\cos(\omega))}$

$\boxed{C = 1 \quad B = b_1 \quad A = 2}$

B) $\boxed{b_1 = -2\cos(\omega_0)}$

c) $x(t) = .16\cos(2\pi(800)t + .16\pi)$

$x[n] = .16\cos\left(2\pi\left(\frac{800}{2400}\right)n + .16\pi\right)$

$x[n] = .16\cos\left(\frac{2\pi}{3}n + .16\pi\right)$ $\quad \hat{\omega} = \frac{2\pi}{3} \Rightarrow$ null $\frac{2\pi}{3}$

$b_1 = -2\cos\left(\frac{2\pi}{3}\right) \Rightarrow -2\left(-\frac{1}{2}\right) = \boxed{1}$

D) $x(t) = .52\cos(2\pi(2600)t + .52\pi) \Rightarrow x[n] = .52\cos\left(2\pi\left(\frac{2600}{2400}\right)n + .52\pi\right)$

$.52\cos\left(2\pi\left(\frac{13}{12}\right)n + .52\pi\right) \Rightarrow \omega = \frac{26}{12}\pi - 2\pi \Rightarrow \frac{26}{12}\pi - \frac{24}{12}\pi = \frac{2}{12}\pi = \pi/6$

$.52\cos(\pi/6\, n + .52\pi) \Rightarrow \omega$ to null is $\pi/6$

$b_1 = -2\cos(\pi/6) = \frac{\sqrt{3}}{2}(-2) = \boxed{-\sqrt{3}}$

Lab 5 - FIR Filtering

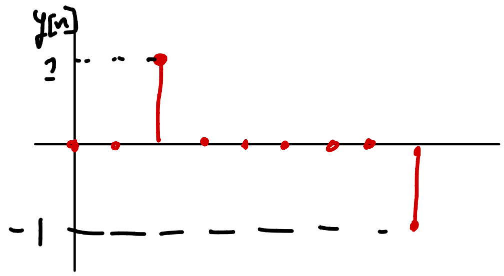
3.1 a) $\delta[n-3] * \delta[n-5] \Rightarrow \delta[n-8]$

$\delta[n-a] * \delta[n-b] \Rightarrow \delta[n-(a+b)]$

3.1 B) $x[n] = -3\{0, 0, 1, 1, 1, 1, 1, 1, 0, 0,\}$

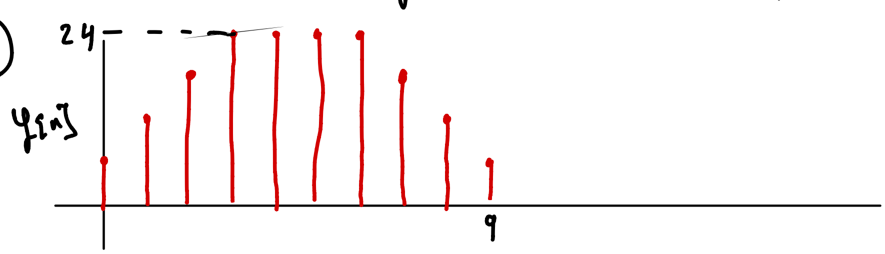
$h[n] = \{1, -1\}$

$$y[n] = \begin{cases} 1 & n = 2 \\ -1 & n = 8 \\ 0 & \text{otherwise} \end{cases} \quad \text{or} \quad y[n] = \delta[n-2] - \delta[n-8]$$



3.1 C) The filter computes the Difference Between Consecutive values ⇒ any consecutive (1, 1) Pair will convolve to zero except for the edges of the Signal).

3.1 D)



3.1 E) max amplitude : 24, length: $7+4-1 = 10$

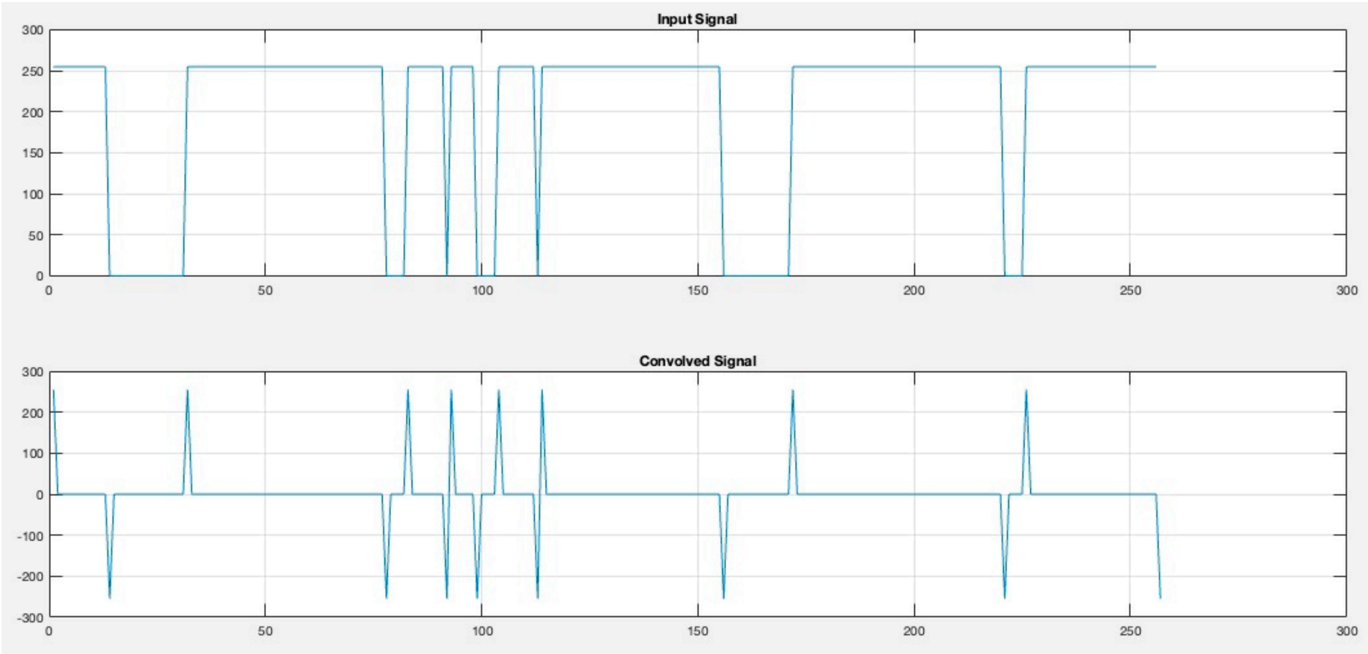
3.1 F) $y[n] = 1$ @ $n = 7, 20, 32, 45$

-1 @ $n = 13, 25, 38,$

@ $y[n] = 1$, the Signal $x[n]$ transitions From $0 \to 1$ (a POS $- 0 \Rightarrow +1$)

@ $y[n] = -1$, The Signal $x[n]$ transitions From $1 \to 0$ (a neg $- 0 \Rightarrow -1$)

3.2 c)



impulses Negative indicate a white -> Black transition + vice versa Positive => a Black -> white transition

indices = find(yyi) => indices(3) - indices(2)

3.4 b) @ $\omega = .2\pi$, $|H(e^{j\omega})| = .374$

$\angle H(e^{j\omega}) = -1.885$ or $-.6\pi$

3.4 c)

Time Delay of output signal $y[n]$

$(\phi_{y[n]} - 2\pi) = \phi_{x[n]} + \angle H(e^{j\omega})$

3.4 D) $y[n] = 1.8\,(|H(e^{j\omega})|)\cos(2\pi(.1)(n - \phi_x + \frac{\angle H(e^{j\omega})}{2\pi}))$

$= .673\cos(.2\pi(n-6)) \Rightarrow n_y = 6$

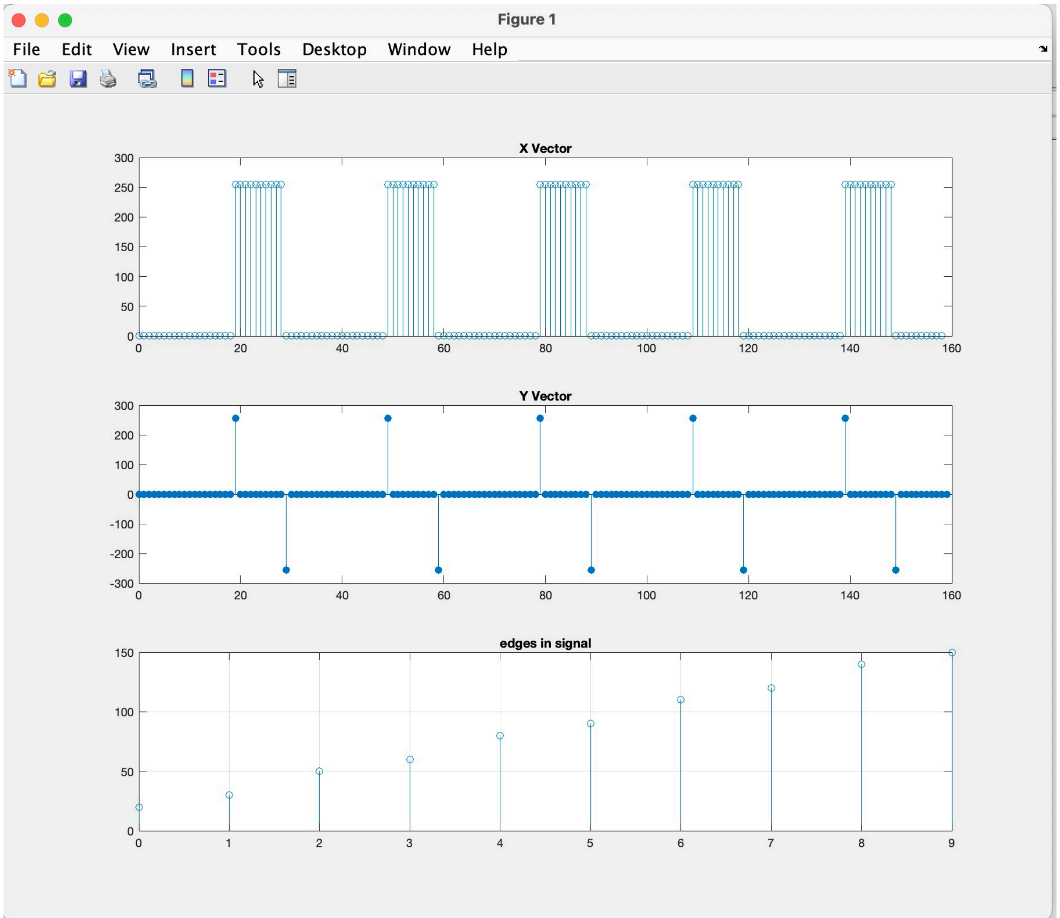


Figure 1
File Edit View Insert Tools Desktop Window Help
X Vector
Y Vector
edges in signal

3.3.2

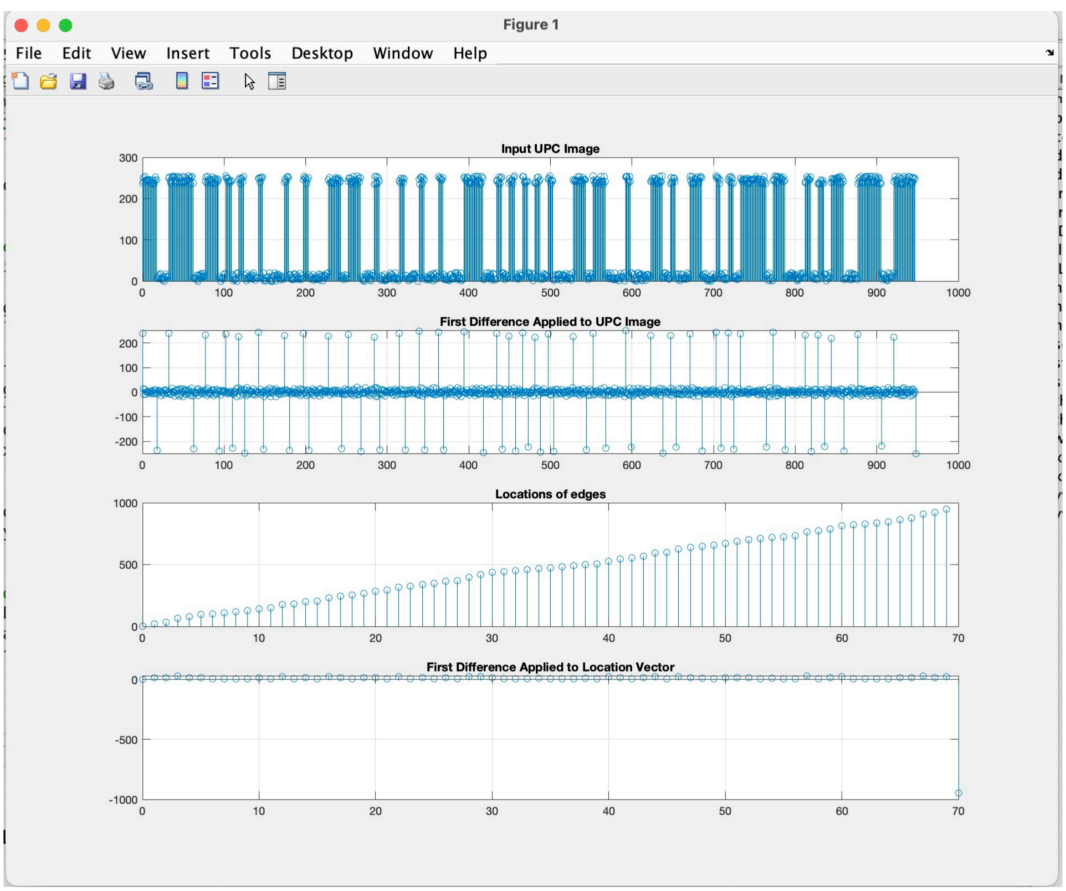
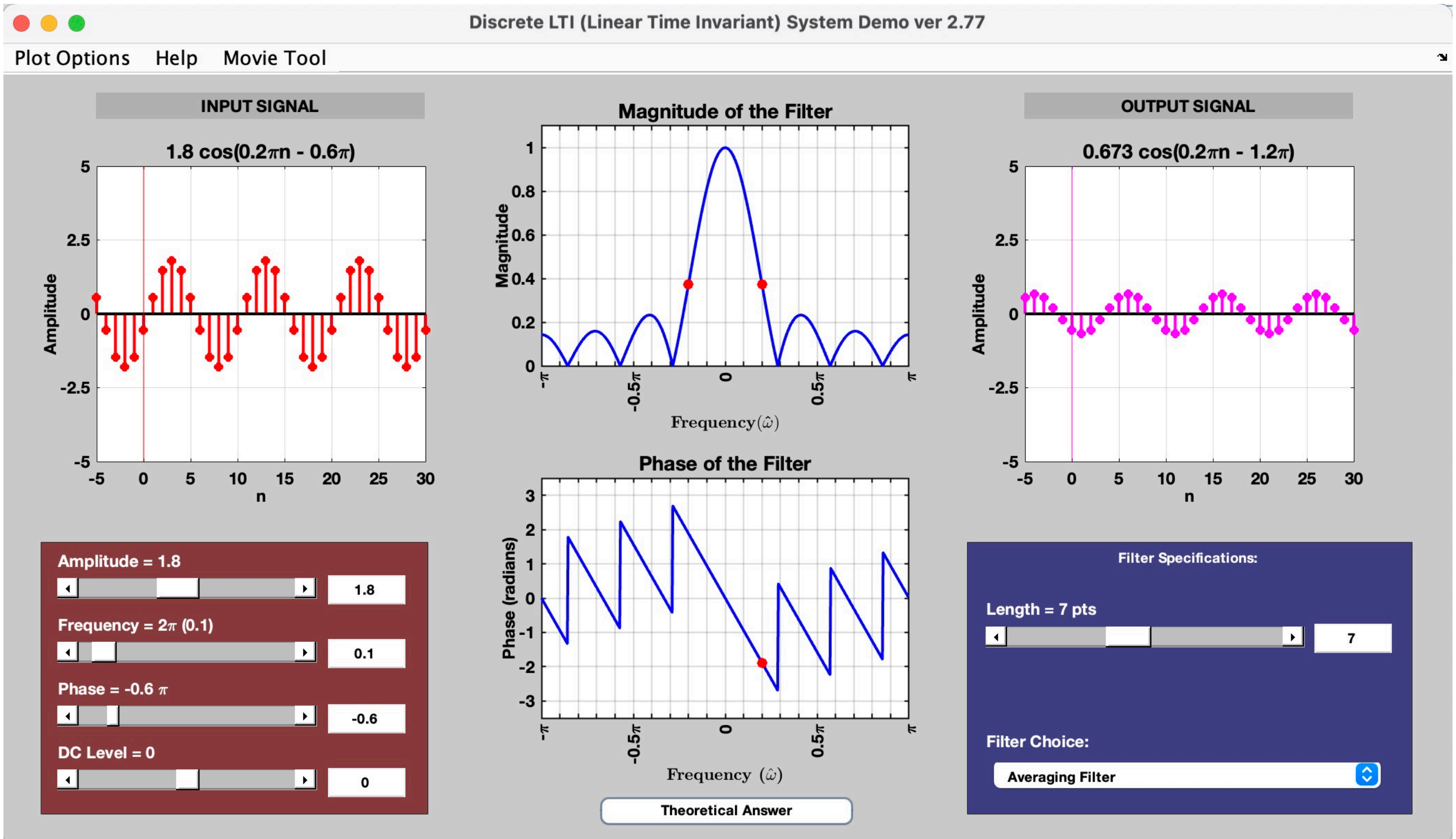


Figure 1
File Edit View Insert Tools Desktop Window Help
Input UPC Image
First Difference Applied to UPC Image
Locations of edges
First Difference Applied to Location Vector

3.3.4

3.4



Discrete LTI (Linear Time Invariant) System Demo ver 2.77
Plot Options
Help
Movie Tool
INPUT SIGNAL
1.8 cos(0.2πn - 0.6π)
Amplitude
n
Magnitude of the Filter
Magnitude
Frequency($\hat{\omega}$)
Phase of the Filter
Phase (radians)
Frequency ($\hat{\omega}$)
Theoretical Answer
OUTPUT SIGNAL
0.673 cos(0.2πn - 1.2π)
Amplitude
n
Amplitude = 1.8
1.8
Frequency = 2π (0.1)
0.1
Phase = -0.6 π
-0.6
DC Level = 0
0
Filter Specifications:
Length = 7 pts
7
Filter Choice:
Averaging Filter

PROBLEM 9.1.* Sketch the frequency response $H(e^{j\hat{\omega}})$ corresponding to each impulse response.

(Hint: all frequency responses are real-valued. Use Table 7.1 along with the *linearity* and *modulation* DTFT properties from Table 7-2).

Label all important heights and frequencies.

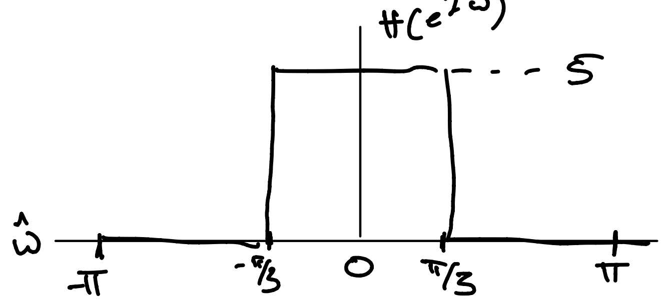
(a) $h[n] = 3\dfrac{\sin(\pi n/3)}{0.6\pi n}$.

(b) $h[n] = \delta[n] - \cos(\pi n)\dfrac{\sin(\pi n/3)}{\pi n}$.

(c) $h[n] = 4\delta[n] + \dfrac{\sin(0.2\pi n)}{0.25\pi n} - \dfrac{\sin(0.6\pi n)}{0.25\pi n} + 2\cos(0.6\pi n)\dfrac{\sin(0.2\pi n)}{0.25\pi n} + 2\cos(0.9\pi n)\dfrac{\sin(0.1\pi n)}{0.25\pi n}$.

a) $h[n] = 3\dfrac{\sin(\frac{\pi}{3}n)}{(.6)\pi n} = 5\cdot\dfrac{\sin(\pi/3\, n)}{\pi n}$ $\quad \hat{\omega}_b = \pi/3$

$H(e^{j\omega}) = 5\left[U(\omega+\pi/3) - U(\omega-\pi/3)\right]$

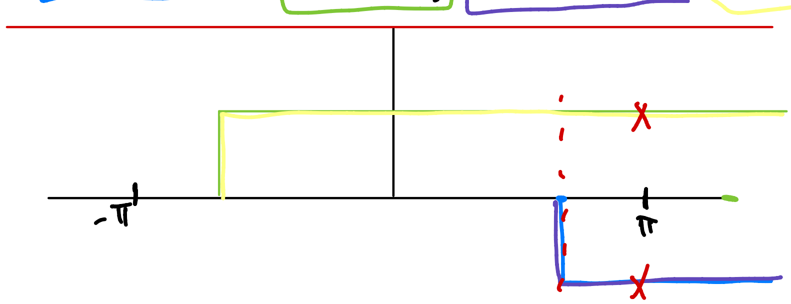


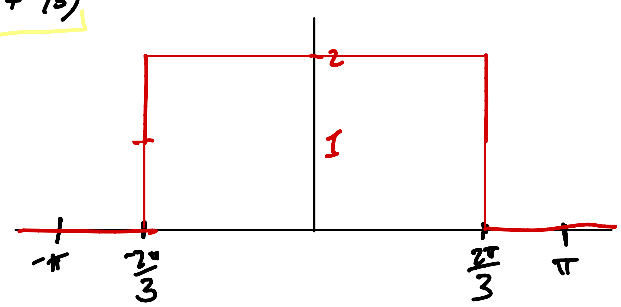
b) $h[n] = \delta[n] - \cos(\pi n)\dfrac{\sin(\frac{\pi}{3}n)}{\pi n}$ $\quad$ Let $x[n] = \dfrac{\sin(\frac{\pi}{3}n)}{\pi n} \overset{DTFT}{\Rightarrow} X(e^{j\omega}) = U(\omega+\frac{\pi}{3}) - U(\omega-\pi/3)$

$h[n] \Rightarrow \delta[n] - \cos(\pi n)\,x[n] \overset{DTFT}{\Rightarrow} H(e^{j\omega}) = 1 - \left(\frac{1}{2}X(e^{j(\omega-\pi)}) + \frac{1}{2}X(e^{j(\omega+\pi)})\right)$

$H(e^{j\omega}) = 1 - \left[\frac{1}{2}U(\overset{\omega-\pi}{\omega_1}+\pi/3) - \frac{1}{2}U(\omega_1-\pi/3) + \frac{1}{2}U(\overset{\omega+\pi}{\omega_2}+\pi/3) - \frac{1}{2}U(\omega_2-\pi/3)\right]$

$H(e^{j\omega}) = \underline{1} - \underbrace{\frac{1}{2}U(\omega-\frac{2\pi}{3})} + \underbrace{\frac{1}{2}(\omega - \overset{+2\pi/3}{\cancel{\frac{4\pi}{3}}})} \underbrace{- \frac{1}{2}U(\omega + \overset{-2\pi/3}{4\pi/3})} + \underbrace{\frac{1}{2}U(\omega+2\pi/3)}$





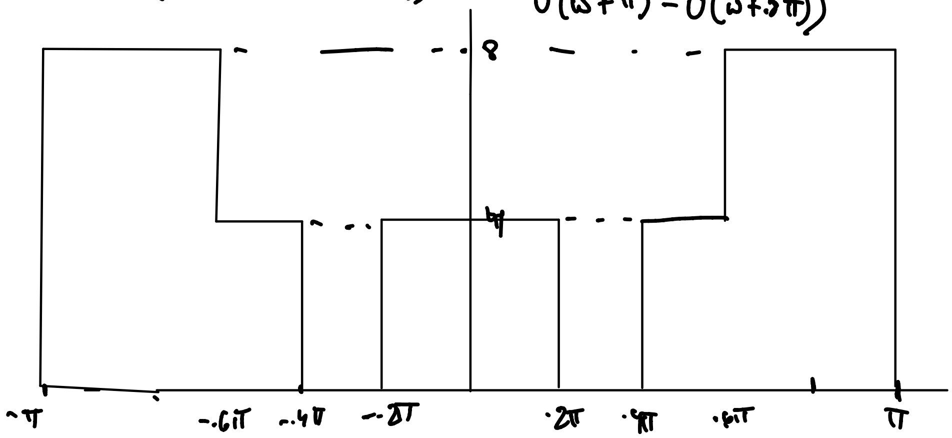
c) $h[n] = 4\delta[n] + \frac{1}{4}\frac{\sin(.2\pi n)}{\pi n} - \frac{1}{4}\frac{\sin(.6\pi n)}{\pi n} + 2\cos(.6\pi n)\left(\frac{1}{4}\right)\frac{\sin(.2\pi n)}{\pi n}$

$+ 2\cos(.9\pi n)\left(\frac{1}{4}\right)\frac{\sin(.1\pi n)}{\pi n}$

$U(\omega + .1\pi n) -$
$U(\omega - .1\pi n)$

$$H(e^{j\omega}) = 4 + \frac{1}{4}\Big(U(\omega+.2\pi) - U(\omega-.2\pi)\Big) - \frac{1}{4}\Big(U(\omega+.6\pi) - U(\omega-.6\pi)\Big)$$

$$+\frac{1}{4}\Big(U(\omega-.4\pi) - U(\omega-.8\pi) + U(\omega+.8\pi) - U(\omega+.4\pi)\Big) + -\frac{1}{4}\Big(U(\omega-.8\pi) - U(\omega-\pi) + U(\omega+\pi) - U(\omega+.8\pi)\Big)$$



PROBLEM 9.2.* Determine the discrete-time sequence $x[n]$ that corresponds to each of the following discrete-time Fourier transforms. Answer in the form of a carefully labeled stem plot. (*Hints:* Although you could use the inverse-DTFT integral, it is not necessary. Use the tables instead. Euler might be useful for part (b) and part (c).)

(a) $X(e^{j\hat{\omega}}) = 32 + 16e^{-2j\hat{\omega}}$.

(b) $X(e^{j\hat{\omega}}) = 4\cos^2(2\hat{\omega})$.

(c) $X(e^{j\hat{\omega}}) = 6je^{-6j\hat{\omega}}\sin(2\hat{\omega})$.

(d) $X(e^{j\hat{\omega}}) = \dfrac{\sin(3.5\hat{\omega})}{\sin(0.5\hat{\omega})}$. $\dfrac{\sin(7(.5\omega))}{\sin(.5\omega)}$

(e) $X(e^{j\hat{\omega}}) = \dfrac{1 - e^{-j5\hat{\omega}}}{1 - e^{-j\hat{\omega}}}$.

a) $x[n] = 32\delta[n] + 16\delta[n-2]$

b) $X(e^{j\omega}) = 4\cos^2(2\omega) \Rightarrow 4\left(\dfrac{e^{j2\omega} + e^{-j2\omega}}{2}\right)^2 \Rightarrow \dfrac{\cancel{4}(e^{j2\omega} + e^{-j2\omega})^2}{\cancel{4}}$

$e^{j2\omega}e^{j2\omega} + \cancel{e^{-j2\omega}e^{j2\omega}} + \cancel{e^{j2\omega}e^{-j(2\omega)}} + e^{-j2\omega}e^{-j2\omega} + 2$ $\quad e^0 \to 1 \quad e^0 \to 1$

$X(e^{j\omega}) = e^{j4\omega} + e^{-j4\omega} \xrightarrow{\text{IDTFT}} x[n] = \delta[n-4] + \delta[n+4] + 2\delta[n]$

c) $X(e^{j\omega}) = 6je^{-j6\omega}\sin(2\omega) \to 6\cancel{j}(e^{-j6\omega})\left(\dfrac{e^{j2\omega} + e^{-j2\omega}}{2\cancel{j}}\right) \to 3e^{-4\omega j} + 3e^{-8\omega j}$

$x[n] = 3\delta[n-4] + 3\delta[n-8]$

D) $X(e^{j\omega}) = \underbrace{\dfrac{\sin(7(\frac{1}{2}\omega))}{\sin(\frac{1}{2}\omega)} e^{-j\omega(7-1)/2}}$ $e^{j\omega(7-1)/2}$

let $Y(e^{j\omega})$ $e^{j\omega(7-1)/2}$

$e^{j3\omega}$

$y[n] = u[n] - u[n-7]$ $\Rightarrow n_d = -3$

$x[n] = y[n+3] = u[n+3] - u[n-4]$

E) $X(e^{j\omega}) = \dfrac{1 - e^{-j5\omega}}{1 - e^{-j\omega}} = \dfrac{1}{1 - e^{-j\omega}} - \dfrac{e^{-j5\omega}}{1 - e^{-j\omega}}$

$x[n] = u[n] - u[n-5]$

PROBLEM 9.3.* Evaluate the following convolutions.
Express your answer $y[n]$ as a function of n. Simplify as much as possible.

(a) $y_a[n] = \delta[n-2] * \delta[n-3]$.

(b) $y_b[n] = \cos(0.55\pi n) * \dfrac{\sin(0.4\pi n)}{\pi n}$.

(c) $y_c[n] = \left(\dfrac{\sin(0.7\pi n)}{\pi n} - \dfrac{\sin(0.4\pi n)}{\pi n}\right) * \left(\dfrac{\sin(0.3\pi n)}{\pi n} - \dfrac{\sin(0.2\pi n)}{\pi n}\right)$.

(d) $y_d[n] = \left(\dfrac{\sin(0.7\pi n)}{\pi n} - \dfrac{\sin(0.4\pi n)}{\pi n}\right) * \left(\dfrac{\sin(2\pi n/3)}{\pi n} - \dfrac{\sin(\pi n/3)}{\pi n}\right)$.

(e) $y_e[n] = \left(\delta[n] - \dfrac{\sin(0.4\pi n)}{\pi n}\right) * \dfrac{\sin(0.9\pi n)}{\pi n}$.

a) $y_a[n] \Rightarrow e^{-j\omega 2} e^{-j(3)\omega} \Rightarrow e^{-j5\omega} \longrightarrow \boxed{y_a[n] = \delta[n-5]}$

b) $\cos(.55 n\pi)$

$\dfrac{e^{j.55\pi n}}{2} + \dfrac{e^{-j.55\pi n}}{2}$

$\left[\tfrac{1}{2}\delta[\omega+.55\pi] + \tfrac{1}{2}\delta[\omega-.55\pi]\right](u[\omega+.4\pi) - u[\omega-.4\pi])$

$\underbrace{\tfrac{1}{2}\delta(\omega+.55\pi)u(\omega+.4\pi)}_{0} + \underbrace{\tfrac{1}{2}\delta(\omega-.55\pi)u(\omega+.4\pi)}_{0} - \underbrace{\tfrac{1}{2}u(\omega-.4\pi)\delta(\omega+.55\pi)}_{0} - \tfrac{1}{2}\delta[\omega-.55\pi)u(\omega-.4\pi)$

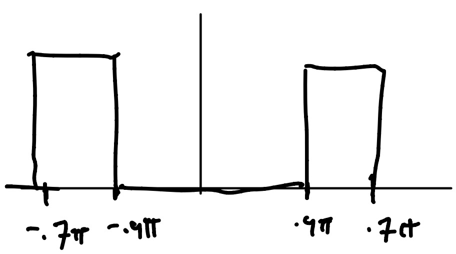
$-\tfrac{1}{2}\delta[\omega-.55\pi)$

$y_b[n] = \dfrac{1}{2\pi}\displaystyle\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}\,d\omega \Rightarrow 0 + 0 + 0 = \boxed{0}$

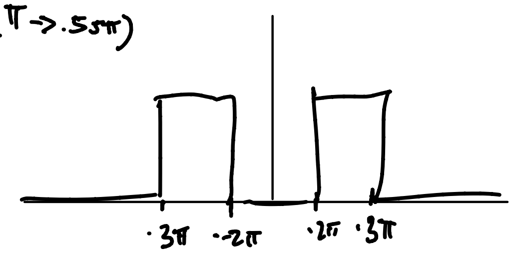
$[\pi \to .55\pi)$

No overlapping Regions

c)



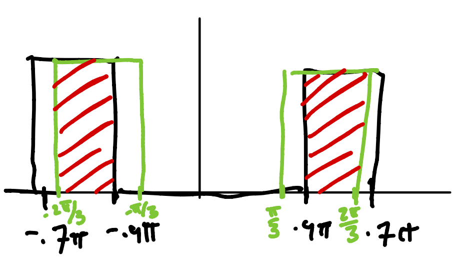
$\times$



$\Rightarrow$

$\boxed{y_c[n] = 0}$

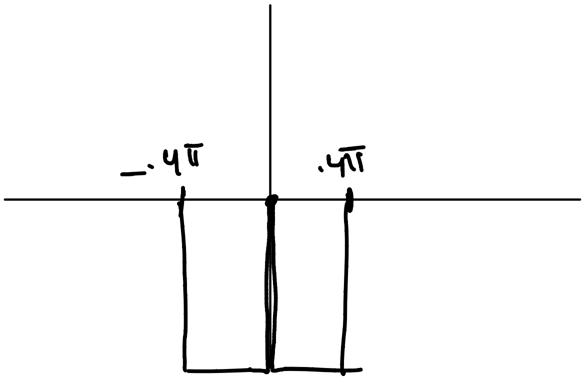
D)

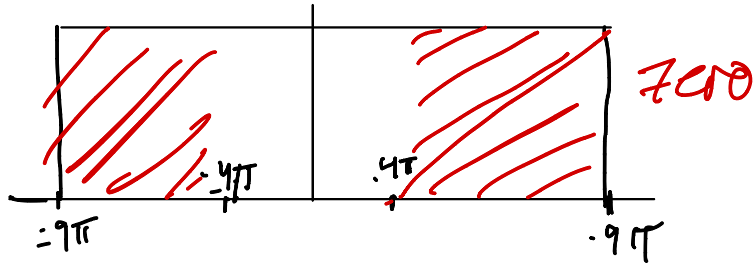


$\left[u[\omega+\tfrac{2\pi}{3}] - u[\omega-\tfrac{2\pi}{3}]\right] -$
$[u[\omega+.4\pi] - u[\omega-.4\pi]]$

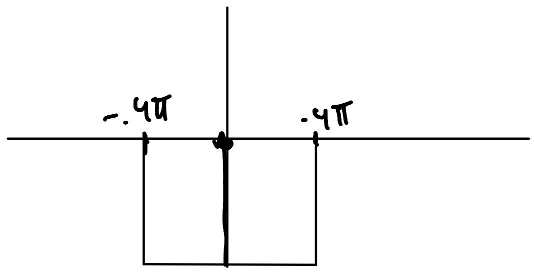
$\Rightarrow \boxed{y_D[n] = \dfrac{\sin(\frac{2\pi}{3}n)}{\pi n} - \dfrac{\sin(.4\pi n)}{\pi n}}$

e)





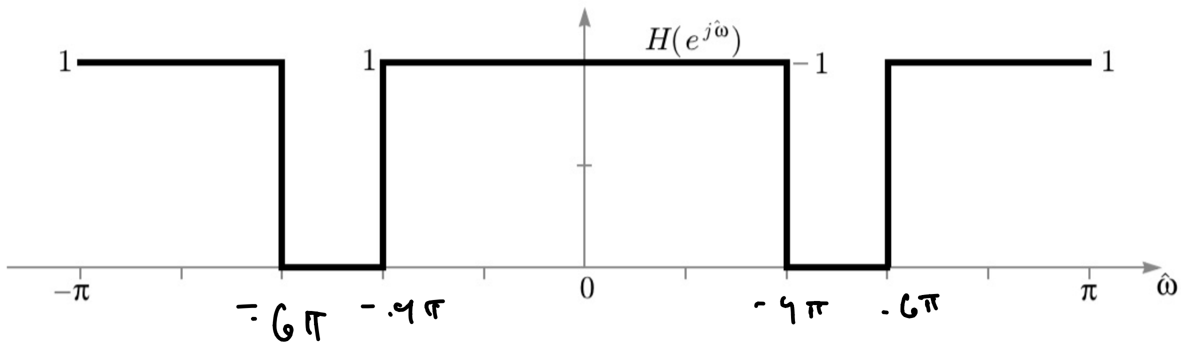
=>



Same thing

$$\boxed{y_e[n] = \frac{\sin(.9\pi n)}{\pi n} - \frac{\sin(.4\pi n)}{\pi n}}$$

PROBLEM 9.4.* Shown below is the frequency response $H(e^{j\hat{\omega}})$ of a *band-stop (or notch) filter* that rejects sinusoids whose frequencies are in the "band" between 0.4π and 0.6π, and passes everything else:



(a) If the filter input is the "...+ − + −..." sequence $x[n] = (-1)^n$, what is the output $y[n]$?

(b) If the filter input is the sinusoid $x[n] = \cos(0.5\pi n)$, what is the output $y[n]$?

In the remainder of the problem we explore four different ways to write the impulse response.

(c) The impulse response of this filter can be written as:

$$h[n] = A\frac{\sin(\hat{\omega}_1 n)}{\pi n} + B\cos(\pi n)\frac{\sin(\hat{\omega}_2 n)}{\pi n}.$$

Find numeric values for the constants A, B, $\hat{\omega}_1$ and $\hat{\omega}_2$.

a) $x[n] = (-1)^n \Rightarrow \cos(\pi n) = \omega = \pi \Rightarrow$ accepted

$\Rightarrow \boxed{y[n] = \cos(\pi n)}$

B) $y[n] = 0$

c) $h[n] = A\dfrac{\sin(\omega_1 n)}{\pi n} + \dfrac{B}{2}e^{j\pi n}\dfrac{\sin(\omega_2 n)}{\pi n} + \dfrac{B}{2}e^{-j\pi n}\dfrac{\sin(\omega_2 n)}{\pi n}$

$$\begin{cases} A & |\omega| < \omega_1 \\ 0 & \omega_1 < |\omega| < \pi \end{cases} + \begin{cases} B/2 & |\omega| < \omega_2 - \pi \\ 0 & \omega_2 - \pi < |\omega| < \pi \end{cases} + \begin{cases} B/2 & |\omega| < \omega_2 + \pi \\ 0 & \omega_2 + \pi < |\omega| < \pi \end{cases}$$

$\omega_1 = .4\pi$ $\quad$ $\omega_2 = .4\pi$

$A = 1$ $\quad$ $B = 2$

(d) The impulse response of this filter (also) can be written as:

$$h[n] = \delta[n] - A\cos(\hat{\omega}_1 n)\frac{\sin(\hat{\omega}_2 n)}{\pi n}.$$

Find numeric values for the constants A, $\hat{\omega}_1$, and $\hat{\omega}_2$.

(e) The impulse response of this filter (also) can be written as:

$$h[n] = \delta[n] - A\frac{\sin(\hat{\omega}_1 n)}{\pi n} + A\frac{\sin(\hat{\omega}_2 n)}{\pi n}.$$

Find numeric values for the constants A, $\hat{\omega}_1$, and $\hat{\omega}_2$.

(f) The impulse response of this filter can (also) be written as:

$$h[n] = A\cos(\hat{\omega}_1 n)\frac{\sin(0.2\pi n)}{\pi n} + B\cos(\hat{\omega}_2 n)\frac{\sin(0.2\pi n)}{\pi n}.$$

Find numeric values for the constants A, B, $\hat{\omega}_1$ and $\hat{\omega}_2$.

D) $1 - \begin{cases} \frac{1}{2}A & |\omega| < \omega_2 + \omega_1 \\ 0 & \omega_2+\omega_1 < |\omega| < \pi \end{cases} - \begin{cases} A/2 & |\omega| < \omega_2 - \omega_1 \\ 0 & \omega_2 - \omega_1 < |\omega| < \pi \end{cases}$

$A = 2$

$\omega_1 = .5\pi \quad \omega_2 = .1\pi$

E) $1 - \begin{cases} A & |\omega| < \omega_1 \\ 0 & \omega_1 < |\omega| < \pi \end{cases} + \begin{cases} A & |\omega| < \omega_2 \\ 0 & \omega_2 < |\omega| < \pi \end{cases}$

$A = 1 \quad \omega_1 = .6\pi$

$\omega_2 = .4\pi$

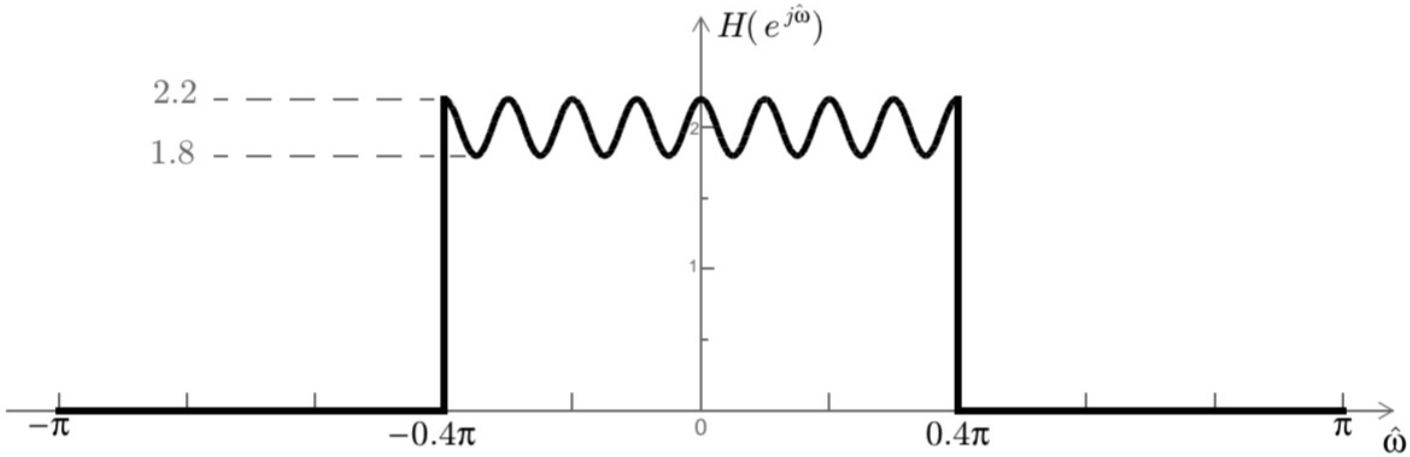
F) $\begin{cases} \frac{1}{2}A & |\omega| < .2\pi + \omega_1 \\ 0 & \omega_1 + .2\pi < |\omega| < \pi \end{cases} + \begin{cases} A/2 & |\omega| < .2\pi - \omega_1 \\ 0 & .2\pi - \omega_1 < |\omega| < \pi \end{cases} + \ldots \quad B$

$A = 2 \quad \omega_1 = .2\pi \quad \omega_2 = .8\pi$

$B = 2$

Made with Goodnotes

PROBLEM 9.5.* Consider the LTI filter defined by its frequency response $H(e^{j\hat{\omega}})$, which is the real-valued function of $\hat{\omega}$ shown below:



(This is just as it looks: $H(e^{j\hat{\omega}})$ oscillates sinusoidally between 1.8 and 2.2, undergoing precisely eight cycles as $\hat{\omega}$ ranges from -0.4π to 0.4π.)

(a) Find the filter output $y[n]$ in response to the filter input $x[n] = (\sqrt{2}\sin(0.25\pi n))^2$.

(b) Define $g[n]$ as the following sinc function:

$$g[n] = \frac{\sin(0.4\pi n)}{\pi n}.$$

The impulse response $h[n]$ of the above filter (i.e., the one defined by the above frequency-response plot) can be written in terms of this sinc function, according to:

$$h[n] = Ag[n] + Bg[n+D] + Cg[n-D].$$

Find numerical values for the unspecified constants A, B, C, and D.

a) $x[n] = 2\left(\frac{1}{2j}\left(e^{j\pi/4 n} - e^{-j\pi/4 n}\right)\right)^2$

$$-\frac{2}{4}\left(e^{j\pi/4 n}e^{j\pi/4 n} - \underbrace{\cancel{e^{-j\pi/4 n}e^{j\pi/4 n}}}_{-e^0} - \underbrace{\cancel{e^{j\pi/4 n}e^{-j\pi/4 n}}}_{-e^0} + e^{-j\pi/4 n}e^{-j\pi/4 n}\right)$$

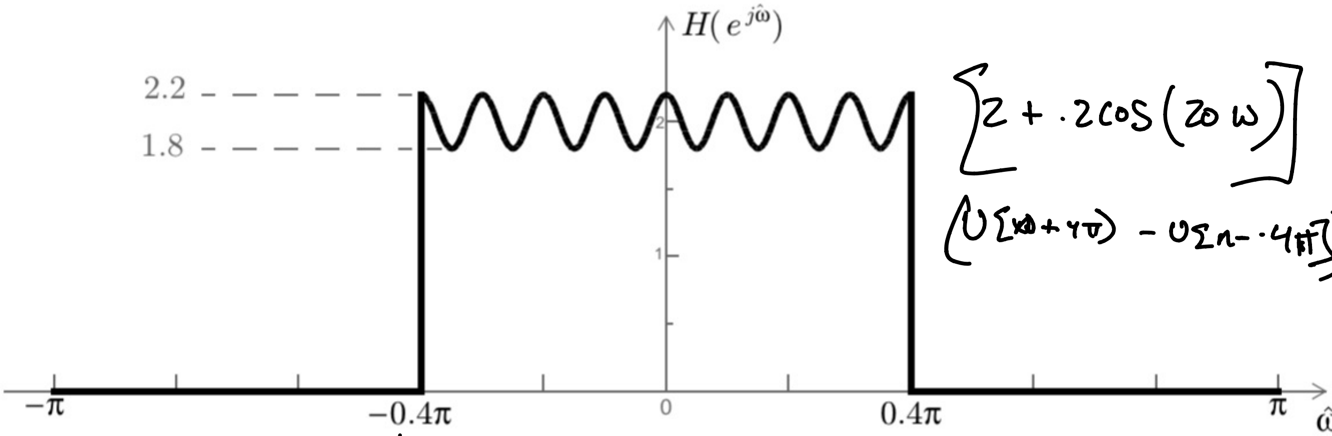
$$-\frac{1}{2}\left(e^{j.5\pi n} + e^{-j.5\pi n} - 2\right)$$

$1 - \cos(.5\pi n)$ $\quad \omega = \frac{1}{2}\pi \Rightarrow$ cos term attenuated

DC offset is 2.2 $\Rightarrow X(e^{j\omega}) \cdot H(e^{j\omega}) = Y(e^{j\omega})$

$$\boxed{y[n] = 2.2}$$

PROBLEM 9.5.* Consider the LTI filter defined by its frequency response $H(e^{j\hat{\omega}})$, which is the real-valued function of $\hat{\omega}$ shown below:



$\left[2 + .2\cos(20\omega)\right]$

$\left(U[\omega+.4\pi] - U[n-.4\pi]\right)$

$2\frac{\sin(.4\pi n)}{\pi n} + \frac{.2}{2}e^{j20\omega}U[\omega+.4\pi] - \frac{.2}{2}e^{j20\omega}U[n-.4\pi]$

(This is just as it looks: $H(e^{j\hat{\omega}})$ oscillates sinusoidally between 1.8 and 2.2, undergoing precisely eight cycles as $\hat{\omega}$ ranges from -0.4π to 0.4π.)

$+\frac{.2}{2}e^{-j20\omega}U[\omega+.4\pi] - \frac{.2}{2}e^{-j20\omega}U[\omega-.4\pi]$

(a) Find the filter output $y[n]$ in response to the filter input $x[n] = (\sqrt{2}\sin(0.25\pi n))^2$.

(b) Define $g[n]$ as the following sinc function:

$$g[n] = \frac{\sin(0.4\pi n)}{\pi n}.$$

$G(e^{j\hat{\omega}}) = U[\omega+.4\pi] - U[\omega-.4\pi]$

The impulse response $h[n]$ of the above filter (i.e., the one defined by the above frequency-response plot) can be written in terms of this sinc function, according to:

$$h[n] = Ag[n] + Bg[n+D] + Cg[n-D].$$

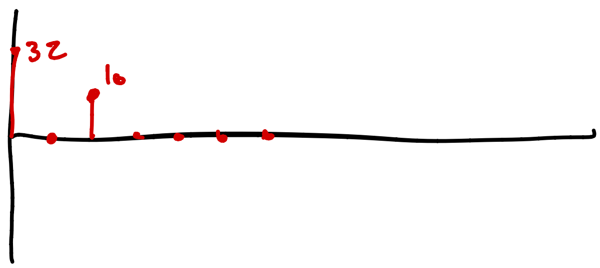
Find numerical values for the unspecified constants A, B, C, and D.

B) $A = 2$ $D = 20$

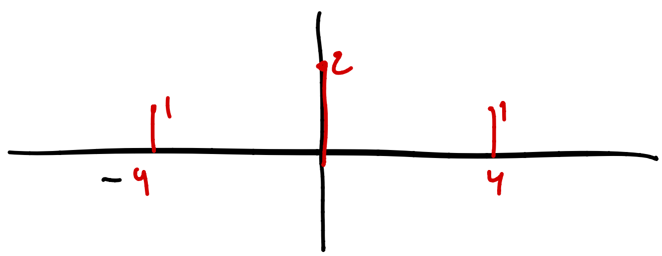
$B = .1$ $C = .1$

9.2 Stem Plots

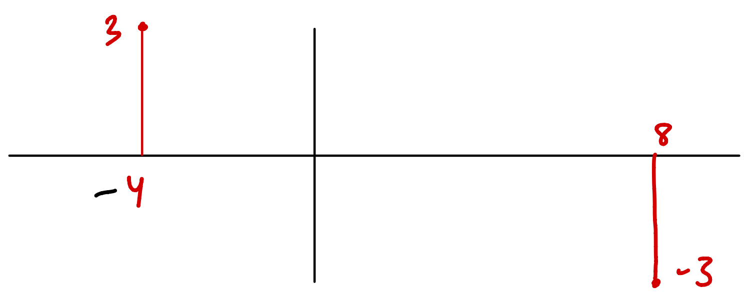
a).



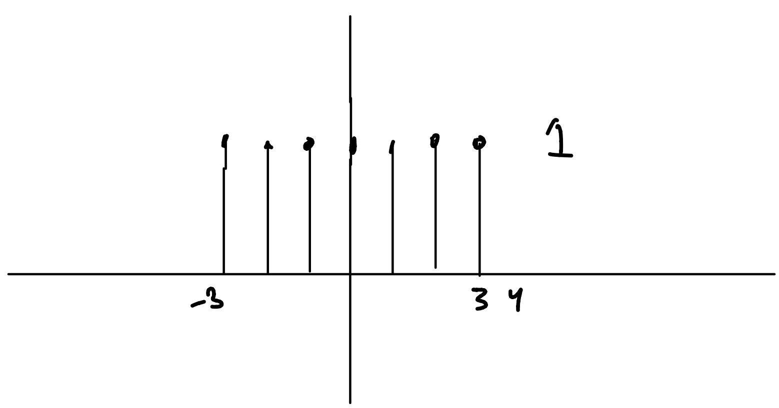
b)



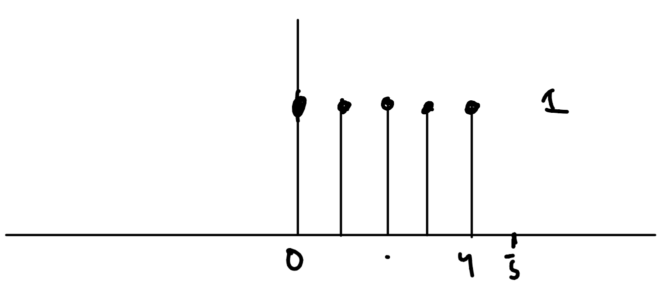
c)



D)



E)



10.1 A) $\underline{4\text{ Pt}}$ DFT coefficients of [2 1 0 1]

$$x[n] = 1 + \cos(\tfrac{\pi}{2}n) \qquad X(e^{j\omega}) = 2 + e^{-j\omega} + e^{-j3\omega}$$

$$\boxed{[4 \;\; 2 \;\; 0 \;\; 2] = X[k]}$$

$$X[0] = X(e^{j(0)}) = 2 + 1 + 1 = 4$$

$$X[1] = X(e^{j(\pi/2)}) = 2 + -j + j = 2$$

$$X[2] = X(e^{j(\pi)}) = 2 - 1 - 1 = 0$$

$$X[3] = X(e^{j(3\pi/2)}) = 2 + j + -j = 2$$

10.1 B) 8 Pt DFT for $x[n] = [4\ 4\ 4\ 4\ 4\ 4\ 4\ 4]$

$$x[n] = 4 \quad \forall\, n \geq 0 \qquad X(e^{j\omega}) = \sum_{n=0}^{N-1} 4e^{-j\omega n}$$

$$\frac{2\pi}{8} \quad \frac{4\pi}{8} \quad \frac{6\pi}{8}$$

$$X[0] = 32$$

$$X[1] = 4 + 4e^{j2\pi/8} + 4e^{j4\pi/8} + 4e^{j6\pi/8} + 4e^{j8\pi/8} + 4e^{j10\pi/8} + 4e^{j12\pi/8} + 4e^{j14\pi/8}$$

$$\cancel{4} + \cancel{4[\tfrac{\sqrt{2}}{2} + \tfrac{\sqrt{2}}{2}j]} + \cancel{4j} + \cancel{4(-\tfrac{\sqrt{2}}{2} + \tfrac{\sqrt{2}}{2}j)} + \cancel{-4} + \cancel{4[-\tfrac{\sqrt{2}}{2} + -\tfrac{\sqrt{2}}{2}j]} + \cancel{-4j} + \cancel{4[\tfrac{\sqrt{2}}{2} - \tfrac{\sqrt{2}}{2}j]}$$

$$0$$

$$X[2] = 4\sum_{n}^{7} e^{j2n(2\pi)/7} \Rightarrow z = re^{j\theta} \Rightarrow \theta = \frac{(2)(2\pi)}{N} = 4\underbrace{\sum_{0}^{N-1} z^n}_{\to 0} = 0$$

Roots of unity $\Rightarrow$ $\boxed{X[k] = \begin{cases} 32 & k=0 \\ 0 & k \neq 0 \end{cases}}$

10.1 C) $x[n] = 8\delta[n] = X(e^{j\omega}) = 8$

$$\boxed{X[k] = 8 \quad \forall k}$$

10.1 D) $x[n] = [3\ 1\ 3\ 1\ \ldots]$

$$x[n] = 2 + \cos(\pi n)$$

$$X(e^{j\omega}) = 2\delta[n] + \delta[n-\pi] + \delta[n+\pi]$$

$X[0] = 2$

$X[1] = 0 \qquad \omega = 2\pi/8$

$X[2] = 0 \qquad 4\pi/8$

$X[3] = 0 \qquad 6\pi/8$

$X[4] = 1 \qquad 8\pi/8$

$$\boxed{X[k] = 8\begin{cases} 2 & k=0 \\ 1 & k=4 \\ 0 & \text{otherwise} \end{cases}}$$

10.1 e) $X[k] = 4 \quad \forall k \Rightarrow X(e^{j\omega}) = 4$

$x[n] = 4\delta[n] \Rightarrow$ $\boxed{x[n] = \begin{cases} 4 & n=0 \\ 0 & 0<n \leq 7 \end{cases}}$

10.1 f) $X[k] = [8, 0, 0, 0, 0, 0, 0, 0]$

$X(e^{j\omega}) = 8\delta[\omega] \Rightarrow$ DC offset

$$\boxed{x[n] = [1\ 1\ 1\ 1\ 1\ 1\ 1\ 1]}$$

10.1 (G) $X[k] = 9 \quad 5j \quad -1 \quad -5j$

$x[0] = \frac{1}{4}(9 + 5j - 1 - 5j)$

$x[0] = 2$

$x[1] = \frac{1}{4}(9 + 5je^{j2\pi(1)(1)/4} - e^{j(2\pi)(2)(1)/4} - 5je^{j2\pi(3)(1)/4})$

$9 - 5 \quad +1 \quad -5 = 0$

$x[2] = \frac{1}{4}(9 + 5je^{j4\pi/4} - e^{j4\pi(2)/4} - 5je^{j4\pi(3)/4})$

$9 + -5j - 1 + 5j = 2$

$x[3] = \frac{1}{4}(9 + 5je^{j6\pi/4} - e^{j6\pi(2)/4} - 5je^{j6\pi(3)/4})$

$\frac{1}{4}(9 + 5j(-j) + 1 - 5j(j)) =$

$$\boxed{x[n] = 2 \quad 0 \quad 2 \quad 5}$$

10.1 h) $X[k] = [0 \quad 0 \quad 8 \quad 0 \quad 0 \quad 0 \quad 8 \quad 0]$

@ $\omega = 4\pi/8 \quad X(e^{j\omega}) = 8$

@ $\omega = 12\pi/8 \quad X(e^{j\omega}) = 8$ $\qquad 3\pi/2$

$X(e^{j\omega}) = 8\delta[\omega - \pi/2] + 8\delta[\omega + \pi/2]$

$x[n] = [8\cos(\pi/2\, n) + 8\cos(-\pi/2\, n)]\, 1/N$

$$\boxed{x[n] = [2 \quad 0 \quad -2 \quad 0 \quad 2 \quad 0 \quad -2 \quad 0]}$$

Made with Goodnotes

10.2 A+B) $X[k] = \quad 30 \quad 6+jB \quad 0 \quad 0 \quad 0 \quad 12e^{j\phi}$

k	ω	X[k]
0	0	30
1	2π/6	6 + jB
2	4π/6	0
3	6π/6	0
4	8π/6	0
5	10π/6	$12e^{j\phi}$

conjugates

$6 - jB = 12e^{j\phi}$

$12(\cos(\phi) + j\sin(\phi))$

$12\cos\phi = 6$

$\boxed{\phi = -\pi/3}$

$-jB = j\sin(\pi/3)$

$\boxed{B = \frac{\sqrt{3}}{2}}$

10.2 C) $X[k] = 30 \quad 12e^{j\pi/3} \quad 0 \quad 0 \quad 0 \quad 12e^{j(-\pi/3)}$

$x[0] = \frac{1}{6}(30 + 12e^{j\pi/3} + 0 + 0 + 0 + 12e^{j(-\pi/3)})$

$\frac{1}{6}(42) = 7$

$x[1] = 1/6\,(30 + 12e^{j\pi/3}e^{j2\pi/6} + 12e^{j(-\pi/3)}e^{j2\pi(5)/6}$

$1/6(30 + 12e^{j2\pi/3} + 12e^{j4\pi/3})$

$= 3$

$x[2] = 1/6(30 + 12e^{j\pi/3}e^{j2\pi(2)/6} + 12e^{j(-\pi/3)}e^{j(2\pi)(5)(2)/6})$

$1/6(30 + 12e^{j\pi} + 12e^{j\cancel{\pi/3}\,3\pi})$

$x[2] = 1$

$x[3] = 1/6(30 + 12e^{j(\pi/3)}e^{j2\pi(3)/6} + 12e^{j(-\pi/3)}e^{j2\pi(5)(3)/6})$

$1/6(30 + 12e^{j4\pi/3} + 12e^{j\cancel{\;}\,2\pi/3})$

$= 3$

$x[4] = 1/6(30 + 12e^{j\pi/3}e^{j2\pi(4)/6} + 12e^{j(-\pi/3)}e^{2\pi j(5)(4)/6})$

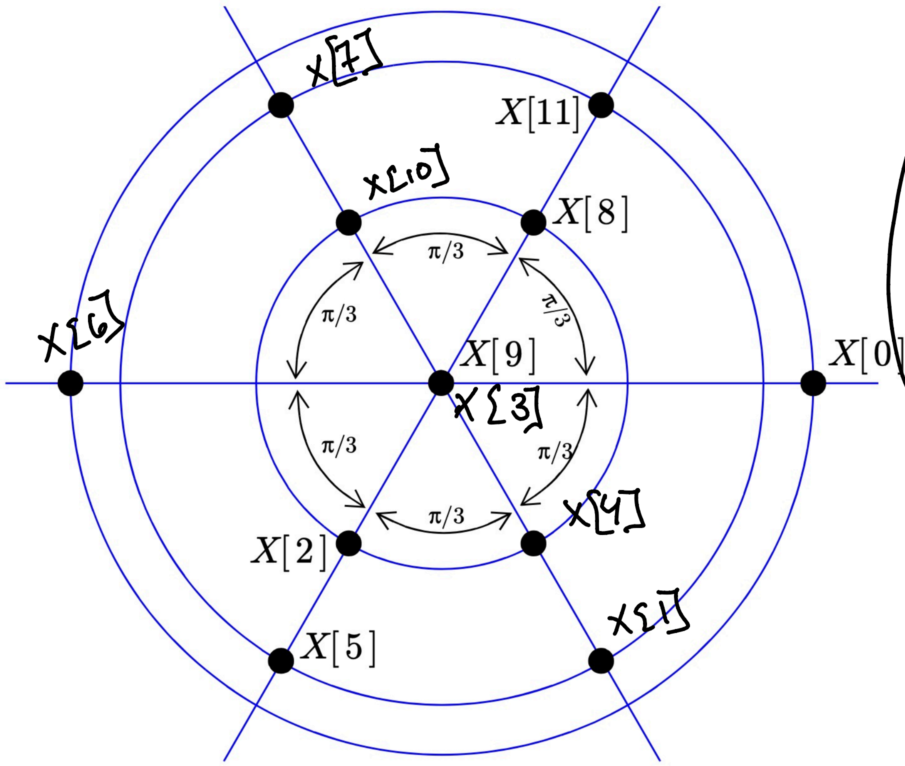
$1/6(30 + 12e^{j5\pi/3} + 12e^{j\cancel{\;}\,\pi/3}$

$= 7$

$x[5] = 1/6(30 + 12e^{j2\pi} + 12e^{j8\pi}) = 9$

$\boxed{x[n] = [7 \quad 3 \quad 1 \quad 3 \quad 7 \quad 9]}$

10.3 A



$X[0] = 2$
$X\ 1$
$X\ 2 = -\frac{1}{2} - \sqrt{3}$
$X\ 3$
$X\ 4$
$X\ 5 = \sqrt{3}\left(-\frac{1}{2} - \frac{\sqrt{3}}{2} j\right)$
$X\ 6$
$X\ 7$
$X\ 8 = \frac{1}{2} + \frac{\sqrt{3}}{2} j$
$X\ 9 = 0$
$X\ 10$
$X\ 11 = \sqrt{3}\left(\frac{1}{2} + \frac{\sqrt{3}}{2} j\right)$

10.3 B) $X[9] = X[3]$

10.3 C) $x[n] = [0\ 1\ 0\ 1\ 0\ 0\ 0]$

10.4 A) conjugate symmetry of DFTs state $X[k] = X^*[N-k]$

$X[331] = X[1024 - 331] \Rightarrow X[331] = X[693]$

10.4B) $[1\ b\ 1]$ is a nulling filter & can be thought of as nulling 1 frequency

for a 1024 pt DFT, $X[331]$ is $X(e^{j\omega})$

sampled at $\omega = \left(\frac{2\pi}{1024}\right)(331) = \frac{662}{1024}\pi \Rightarrow \frac{331}{512}\pi$

$b = -2\cos(\omega) = \boxed{-2\cos\left(\frac{331}{512}\pi\right) \approx .8882}$

10.5 A) $a_0 = \frac{1}{6}(2+0+2+6+2+0) = \boxed{2}$

$a_1 = \frac{1}{6}\left(2 + 2e^{-j(1)2\pi(2)/6} + 6e^{-j(1)2\pi(3)/6} + 2e^{-j2\pi(1)(4)/6}\right)$

$2 + 2e^{-j2\pi/3} + 6e^{-j\pi} + 2e^{-j4\pi/3}$

$2 \quad 2(-\tfrac{1}{2} + \cancel{\tfrac{\sqrt{3}}{2}j}) \quad -6 \quad + 2(-\tfrac{1}{2} - \cancel{\tfrac{\sqrt{3}}{2}j})$

$2 - 1 - 6 - 1 = -\frac{6}{6} = \boxed{-1}$

$a_2 = 1 \quad a_3 = 0 \quad a_4 = 1 \quad a_5 = -1$

10.5 B)

$x[n] = a_{-5}e^{-j2\pi(-5)n/6} + a_5 e^{-j2\pi(5)n/6} \rightarrow 2a_5 \cos\left(\frac{2\pi(5)n}{6}\right) \rightarrow 2a_5\cos\left(\frac{5\pi}{3}n\right)$ $+ 2a_5\cos\left(+\frac{\pi}{3}n\right)$

$a_{-4}e^{-j(2\pi)(-4)n/6} + a_4 e^{-j2\pi(4)n/6} \rightarrow 2a_4\cos\left(\frac{2\pi(4)n}{6}\right) \rightarrow 2a_4\cos\left(\frac{4\pi}{3}n\right)$ $+2a_4\cos\left(+\frac{2\pi}{3}n\right)$

$a_{-3}e^{-j(2\pi)(-3)n/6} + a_3 e^{-j(2\pi)(3)n/6} \rightarrow \cancel{2a_3\cos\left(\frac{2\pi(3)n}{6}\right)} \rightarrow 0$

$a_{-2}e^{-j(2\pi)(-2)n/6} + a_2 e^{-j(2\pi)(2)n/6} \rightarrow 2a_2\cos\left(\frac{2\pi(2)n}{6}\right) \rightarrow 2a_2\cos\left(\frac{2\pi}{3}n\right)$

$a_{-1}e^{-j2\pi(-1)n/6} + a_1 e^{-j(2\pi)(n)/6} \rightarrow 2a_1\cos\left(\frac{2\pi}{6}n\right) \rightarrow 2a_1\cos\left(\frac{\pi}{3}n\right)$

$a_0 \cancel{e^{-j(2\pi)(0)n/6}} \rightarrow 1 \quad \Rightarrow a_0 = B$

$(2a_1 + 2a_5)\cos\left(\frac{\pi}{3}n\right) + (2a_2 + 2a_4)\cos\left(\frac{2\pi}{3}n\right)$

$-2 + -2 \qquad 4\cos(2\pi/3\, n)$

$-4\cos\left(\frac{\pi}{3}n\right) +$

$B = 2$

$A_1 = 4 \qquad \omega_1 = 2\pi/3$

$A_2 = 4 \qquad \omega_2 = \pi/3$

10.5 C) $X[k] = [12 \quad -6 \quad 6 \quad 0 \quad 6 \quad -6]$

10.5 D) $a_k = \frac{1}{N}X[k]$

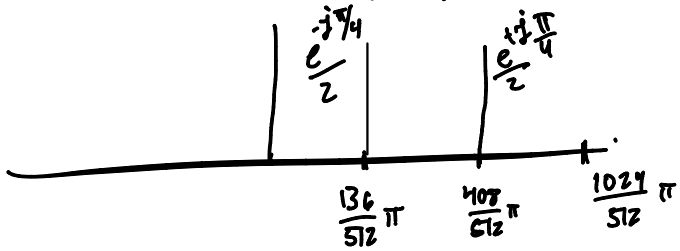
10.5 E) verified

10.6) $x[n] = \cos\left(\frac{17\pi}{64} n - \pi/4\right)$

$\omega = \frac{17}{64}\pi$ $\qquad e^{-j\pi/4}$

$$\Rightarrow X(e^{j\omega}) = \frac{1}{2}\delta\left[\omega + \frac{17}{64}\pi\right] + \frac{1}{2}\delta\left[\omega - \frac{17}{64}\pi\right]$$

DFT Samples $X(e^{j\omega})$ @ $\omega = \frac{2\pi}{512}k \quad k \in \{0, 1, \ldots, 511\}$



$X[k] = X\left(e^{j\frac{2\pi}{N}k}\right) \Rightarrow \frac{e^{j(-\pi/4)}}{2} = X\left(e^{j\frac{2\pi}{512}68}\right) = X[68]$

$\frac{e^{j(\pi/4)}}{2} = X\left(e^{j\frac{2\pi}{512}(204)}\right) = X[204]$

$X[k] = 0$ for all else k

PROB. 11.1.* Consider an LTI filter with impulse response $h[n] = \sqrt{2}\,\delta[n] + 0.5\delta[n-3]$.

(a) Find the system function $H(z)$ for this system.

(b) Sketch the pole-zero plot for $H(z)$.

(c) With the help of MATLAB, sketch its magnitude response $|H(e^{j\hat{\omega}})|$ versus $\hat{\omega}$. Make a connection between the locations of the zeros in the pole-zero plot and the frequencies for which $|H(e^{j\hat{\omega}})|$ is small.

a) $h[n] = [\sqrt{2}, 0, 0, 1/2]$

$$\sqrt{2} + \frac{1}{2}z^{-3} = H(z)$$

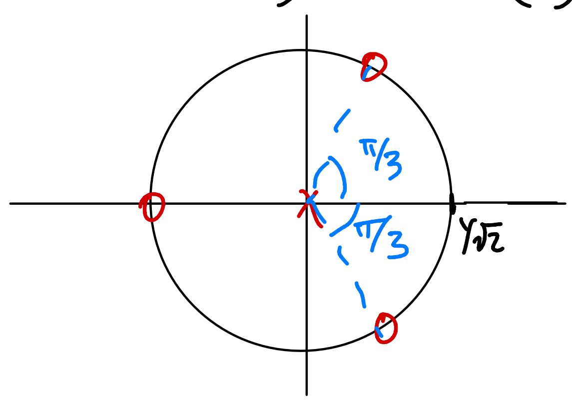
b) $$H(z)\left(\frac{z^3}{z^3}\right) = \frac{\sqrt{2}z^3 + \frac{1}{2}}{z^3} \Rightarrow \frac{\sqrt{2}\left(z^3 + \frac{1}{2\sqrt{2}}\right)}{z^3} \Rightarrow \frac{\sqrt{2}\left(z^3 + \frac{\sqrt{2}}{4}\right)}{z^3}$$

$$z = \left(re^{j\theta}\right)^3 = -\frac{\sqrt{2}}{4} \Rightarrow r^3 e^{j3\theta} = -\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) \Rightarrow re^{j\theta} = \left(\frac{-1}{2}\right)^{1/3}\left(\frac{\sqrt{2}}{2}\right)^{1/3}$$

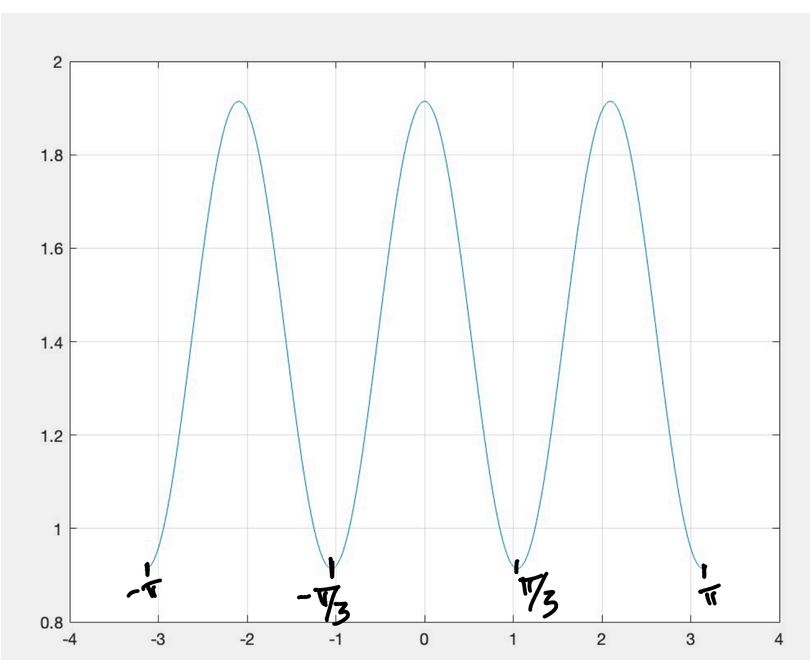
$$z = -\frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}(2)} \pm \frac{\sqrt{3}}{2\sqrt{2}}i$$

$$\frac{1}{\sqrt{2}}\left(\frac{1}{2} \pm \frac{\sqrt{3}}{2}i\right)$$

$\theta = \pi/3$



c)



Not zero @ locations but close since zeros occur @ $r = 1$ but $\frac{1}{\sqrt{2}} = r < 1 \Rightarrow$ close to zero

PROB. 11.2.* Consider an FIR system whose system function is $H(z) = 1 - z^{-2}$.

(a) Find the difference equation relating the filter output $y[n]$ to the input $x[n]$.

(b) Find the filter impulse response $h[n]$.

(c) Find the filter frequency response $H(e^{j\hat{\omega}})$.

(d) Find the dc gain of the filter.

(e) Sketch the magnitude response $|H(e^{j\hat{\omega}})|$. (*Hint:* It should look like a rectified sinusoid.)

(f) Sketch the pole-zero plot.

a) $H(z) = 1 - z^{-2} \Rightarrow h[n]: [1 \quad 0 \quad -1]$

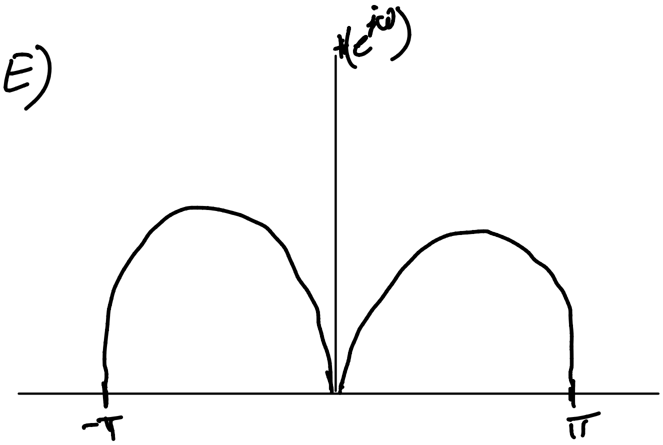
$y[n] = x[n] - x[n-2]$

b) $h[n] = \delta[n] - \delta[n-2]$

c) $H(e^{j\omega}) = 1 - e^{-2j\omega}$

D) $H(e^{j\omega})\Big|_{\omega=0} = 0$

E)



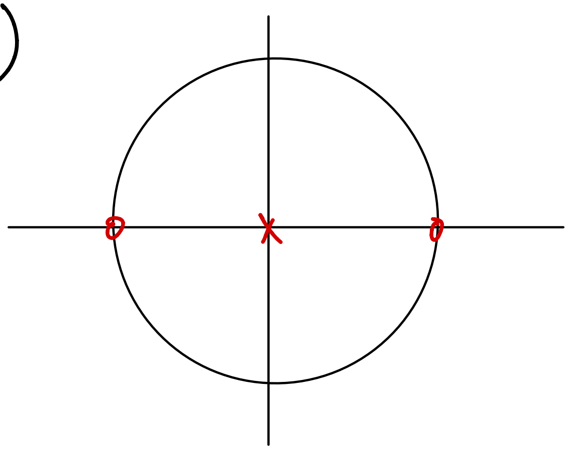
$H(e^{j\omega}) = e^{-j\omega}(e^{j\omega} - e^{-j\omega})$

$e^{-j\omega}(2i\sin(\omega))$

$e^{-j\omega}(2i\sin(\omega))$

$H(z) = \dfrac{z^2 - 1}{z^2}$

f)



PROB. 11.3.* A first-order LTI system whose system function is of the form $H(z) = 1 - e^{j\hat{\omega}_0}z^{-1}$ completely nulls out a complex exponential input signal of the form $e^{j\hat{\omega}_0 n}$.
The concept of *cascading* systems can be used to null out multiple complex exponential signals. For example, the system function $(1 - e^{j0.3\pi}z^{-1})(1 - e^{-j0.3\pi}z^{-1})$ nulls both $e^{j0.3\pi n}$ and $e^{-j0.3\pi n}$, and because of the Euler relation, it also nulls the real sinusoid of the form $A\cos(0.3\pi n + \varphi)$, regardless of the sinusoid amplitude or phase.

(a) Design the coefficients $\{b_k\}$ of an FIR filter of minimal order that nulls out the signal:

$$x_a[n] = \sqrt{2}\cos(0.2\pi n + 0.3\pi).$$

(b) Design the coefficients $\{b_k\}$ of an FIR filter of minimal order that nulls out the signal:

$$x_b[n] = \sqrt{2}\sum_{k=0}^{4}\cos(0.2\pi k)\cos(0.2\pi kn + 0.3\pi).$$

(c) Sketch the pole-zero plot of the system function for the filter of part (b).

a) to null out freq $\omega_0 = .2\pi n$

we design the system function as:

$$H(z) = (1 - e^{-j(.2\pi)}z^{-1})(1 - e^{-j(-.2\pi)}z^{-1})$$

$$1 - e^{-j.2\pi}z^{-1} - e^{+j.2\pi}z^{-1} + e^{j(.2\pi - .2\pi)}z^{-2}$$

$$1 - (e^{-j.2\pi} + e^{j.2\pi})z^{-1} + z^{-2}$$

$$1 - 2\cos(.2\pi)z^{-1} + z^{-2}$$

$$b_k = \{1, -2\cos(.2\pi), 1\}$$

b) $\omega = 0, .2\pi, .4\pi, .6\pi, .8\pi$ to Null

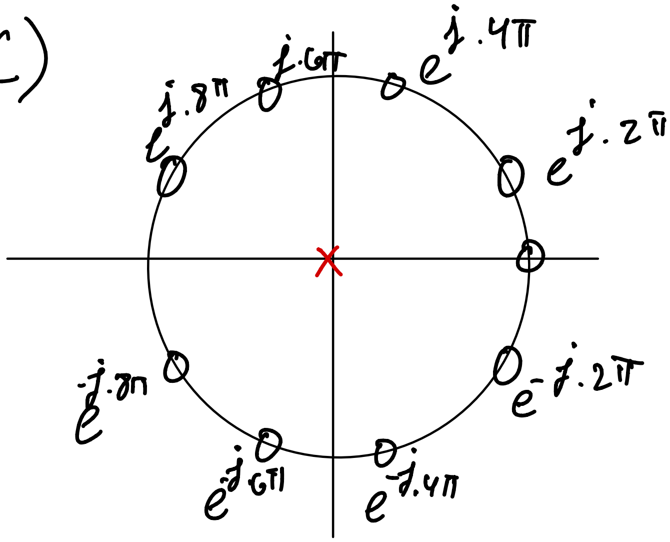
$$(1 - 2\cos(.\pi)z^{-1} + z^{-2})(1 - 2\cos(.2\pi)z^{-1} + z^{-2})(1 - z^{-1})$$

or convolve each individual filter

$$\Rightarrow b_k = \{\underset{b_0}{1}, \underset{b_1}{-1}, \underset{b_2}{1}, \underset{b_3}{-1}, \underset{b_4}{1}, \underset{b_5}{-1}, \underset{b_6}{1}, \underset{b_7}{-1}, \underset{b_8}{1}, \underset{b_9}{-1}\}$$

Made with Goodnotes

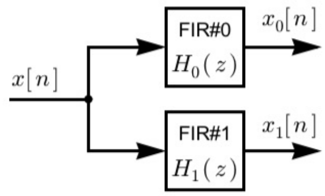
C)



PROB. 11.4.* Let $x[n]$ be an unspecified periodic sequence with fundamental period $N_0 = 3$. Applying the Euler relation to its DFS representation, such a signal can be written as a constant plus a sinusoid at the fundamental frequency $\frac{2\pi}{N_0} = \frac{2\pi}{3}$, according to:

$$x[n] = \underbrace{B}_{x_0[n]} + \underbrace{A\cos(2\pi n/3 + \varphi)}_{x_1[n]},$$

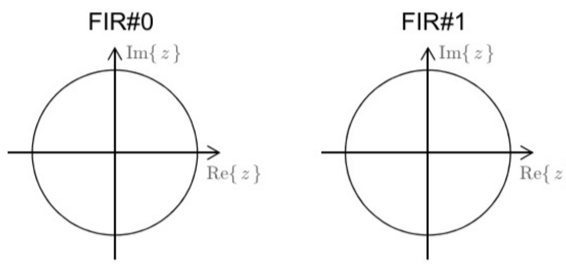
where $x_0[n]$ is the constant component, and $x_1[n]$ is the sinusoidal component. We can separate $x[n]$ into these components using a bank of FIR filters, as shown below:



(a) Design the coefficients $\{b_0, b_1, b_2\}$ of FIR#0 so that its output is $x_0[n]$.

(b) Design the coefficients $\{b_0, b_1, b_2\}$ of FIR#1 so that its output is $x_1[n]$.

(c) Let $H_i(z)$ be the system function of the i-th FIR filter. Sketch the pole-zero plot for both of the system functions $H_0(z)$ and $H_1(z)$. Align them side-by-side like this:



(d) Find a *simplified* expression for the sum of the component system functions:

$$H(z) = H_0(z) + H_1(z),$$

where $H_i(z)$ is the system function of the i-th FIR filter. (Simplify the answer as much as possible.)

a) must null frequencies $\omega = \frac{2\pi}{3}$ and DC gain must be 1

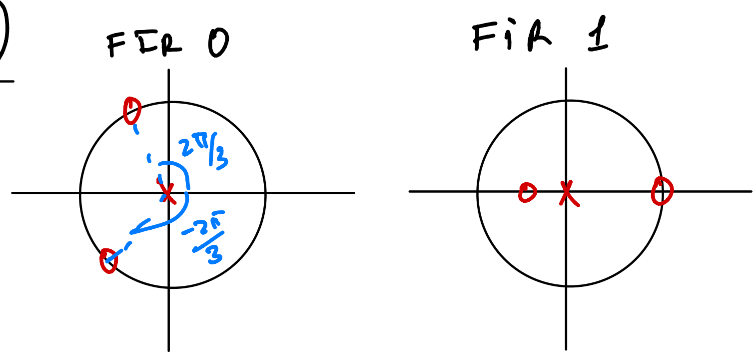
$$\Rightarrow \left(1 - z^{-1}(e^{j2\pi/3})\right)\left(1 - z^{-1}(e^{-j2\pi/3})\right)$$

$$\Rightarrow \{1 \quad -2\cos\left(\tfrac{2\pi}{3}\right) \quad 1\} \Rightarrow b_k = \{1/3 \quad 1/3 \quad 1/3\}$$

b) $x_1[n] = x[n] - x_0[n] \Rightarrow H(z) = 1 - \frac{1}{3}(1 + z^{-1} + z^{-2}) \Rightarrow \frac{2}{3} - \frac{z^{-1}}{3} - \frac{z^{-2}}{3} \Rightarrow b_k = \{2/3, -1/3, -1/3\}$

c) $H_0(z) = \left(\frac{1}{3}\right)\frac{(z - e^{j2\pi/3})(z - e^{-j2\pi/3})}{z^2}$

$H_1(z) = \frac{2z^2 + z + 1}{z^2} = \frac{(2z+1)(z-1)}{3z^2}$

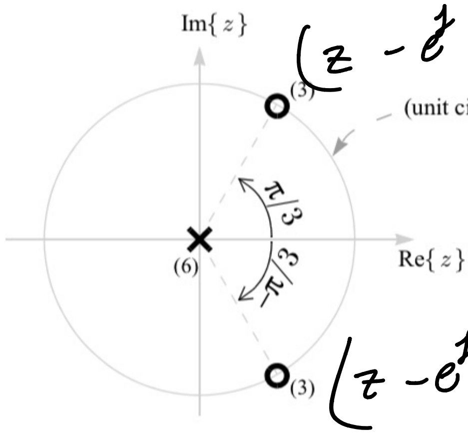


Made with Goodnotes

D) $\dfrac{z^2 + z + 1}{3z^2} + \dfrac{(z+1)(z-1)}{2z^2}$

$$\frac{z^2 \cancel{+z} \cancel{+1} + 2z^2 \cancel{-z} \cancel{-1}}{3z^2} \quad \frac{3z^2}{3z^2} = \boxed{1}$$

PROBLEM 11.5.* Consider an LTI filter whose system function $H(z)$ has the pole-zero plot shown below, with three zeros (multiplicity 3) on the unit circle with angle $\pi/3$, and similarly three zeros at the conjugate location, and six poles at the origin:



$(z - e^{j\pi/3})^3$

$(z - e^{j(-\pi/3)})^3$

Assume further that the evaluating $H(z)$ at $z = 1$ yields $H(1) = 1$.
(A pole-zero plot is invariant to an arbitrary scaling constant, some extra constraint like this is needed to uniquely define a filter.)

(a) Find the dc gain of the filter.

(b) Write the difference equation relating the filter output $y[n]$ to its input $x[n]$.

(c) Let $H_1(z) = 1 + b_1 z^{-1} + b_2 z^{-2}$ be the system function of an order-two FIR filter. Find real values for the coefficients b_1 and b_2 so that the filter with system function $H(z)$ — having the above pole-zero plot — can be constructed by cascading three copies of the $H_1(z)$ filter in series, as illustrated below:

$$x[n] \longrightarrow \boxed{H_1(z)} \longrightarrow \boxed{H_1(z)} \longrightarrow \boxed{H_1(z)} \longrightarrow y[n]$$

(d) Use MATLAB to generate a plot of the magnitude response $|H(e^{j\hat{\omega}})|$ versus $\hat{\omega}$.

(e) Looking at the plot from part (d), how are nulling frequencies of the frequency response (the values of $\hat{\omega}$ where the magnitude response is zero) related to the locations of the zeros in the given pole-zero plot?

a) Given that $H(z)\big|_{z=1} = 1$

we know $z = re^{j\omega} = 1 \Rightarrow \omega = 0$ meaning

DC gain is 1

b) $H(z) = \dfrac{(z - e^{j\pi/3})^3 (z - e^{-j\pi/3})^3}{z^6} \Rightarrow \left[\dfrac{(z - e^{j\pi/3})(z - e^{-j\pi/3})}{z^2}\right]^3$

$b_k = [1 \;\; -2\cos(\pi/3) \;\; 1] * [1 \;\; -2\cos(\pi/3) \;\; 1] \ldots$ ↑ 3 cascaded filters

$[1 \;\; -1 \;\; 1] * [1 \;\; -1 \;\; 1] * [1 \;\; -1 \;\; 1] = [1 \;\; -3 \;\; 6 \;\; -7 \;\; 6 \;\; -3 \;\; 1]$

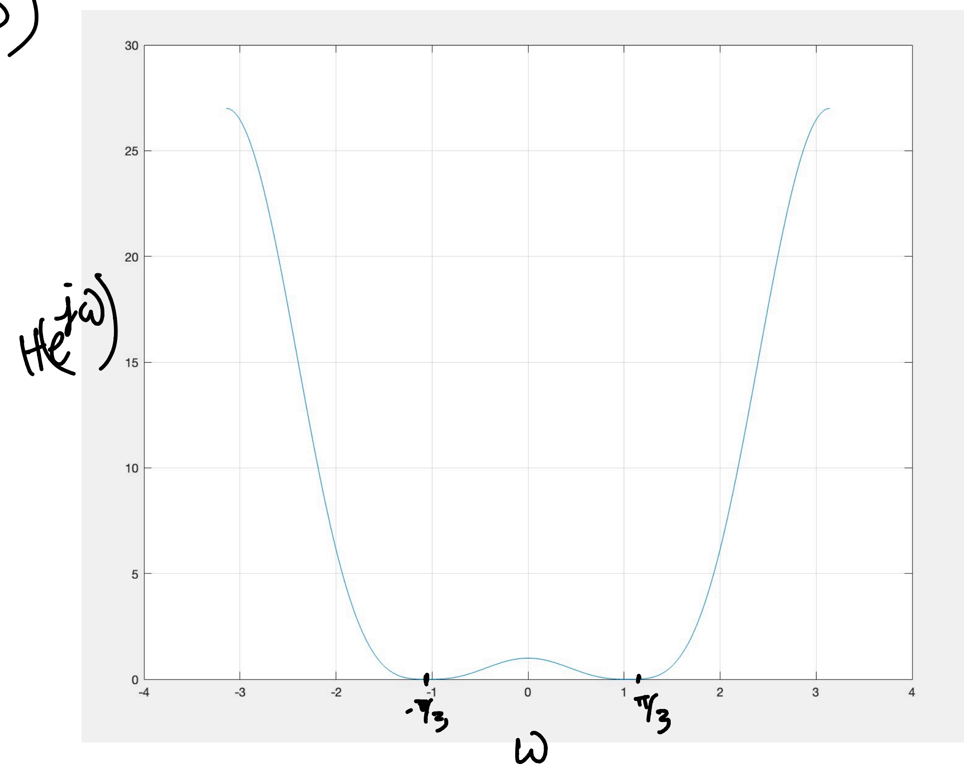
$y[n] = x[n] - 3x[n-1] + 6x[n-2] - 7x[n-3] + 6x[n-4] - 3x[n-5] + 1x[n-6]$

C) Solved in Part B

$b_1 = -1$

$b_2 = 1$

D)



E) The zeros of the pole-zero plot are @ $\pm\pi/3$ which is what $|H(e^{j\omega})|$ plot shows. Furthermore, the degree of each zero being 3 makes the frequencies in that region more heavily attenuated.

Made with Goodnotes

3.1.2) output signal for $w_c = .24\,(2\pi)$ is

A) $y[n] = 1.58\,(.46\pi n + .78\pi)$

This signal is attenuated by .1392 factor and is shifted around 1.5 ticks of the clock.

B) Because the phase slope is $\phi - 4\pi$

$\Rightarrow$ actual phase $\phi / .46\pi = 7 \Rightarrow \boxed{n_d = 7}$

c) Amplitude is .217

3.2 a) $S_p = \left| |H(e^{j\omega})| - 1 \right| \Rightarrow \omega_p = .2207\pi$

b) $\omega_s = .2793\pi$

c) $\omega_p = .14844\pi$

D) $\omega_s = .35156\pi$

e) for Rectangul filters yes

for Hamming filters yes

3.3) a) Rect $.0586\pi = \Delta\omega$

Hamming $.2027\pi = \Delta\omega$

B) This is true. Refer to Part A as the Rect. filter has more Ripples

c) $\omega_p = .44922\pi$ $\quad \Delta\omega = .0996\pi$

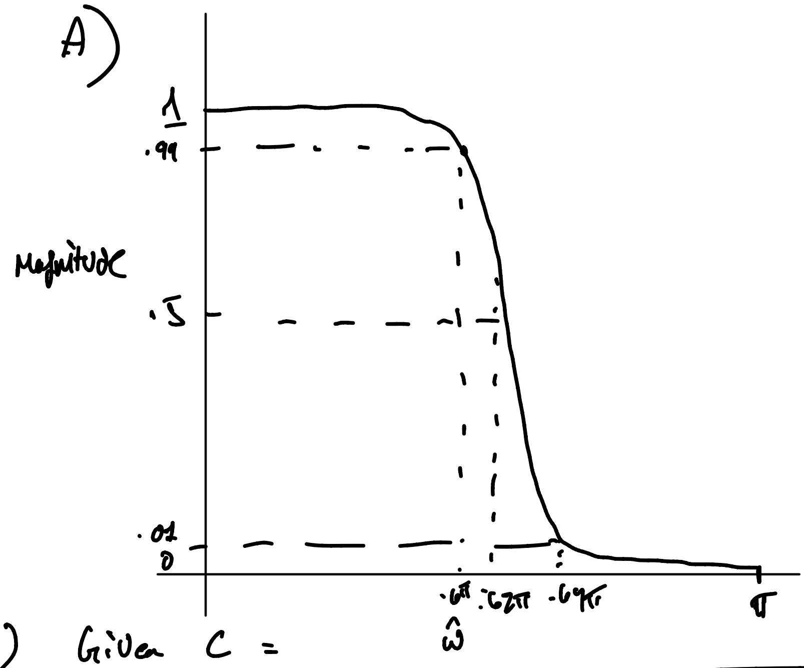
$\omega_s = .54883\pi$

D) When the order doubles $\Rightarrow \Delta\omega$ is $\sim 2x$

$\Delta\omega_1(30) \simeq \Delta\omega_2(60)$

$C = 5.9766$

3.4 A)



B) Given $C = 5.9766$

$\Delta w = .04 = \frac{C}{L}$

$\frac{5.9766}{.04} = \boxed{149.4150}$

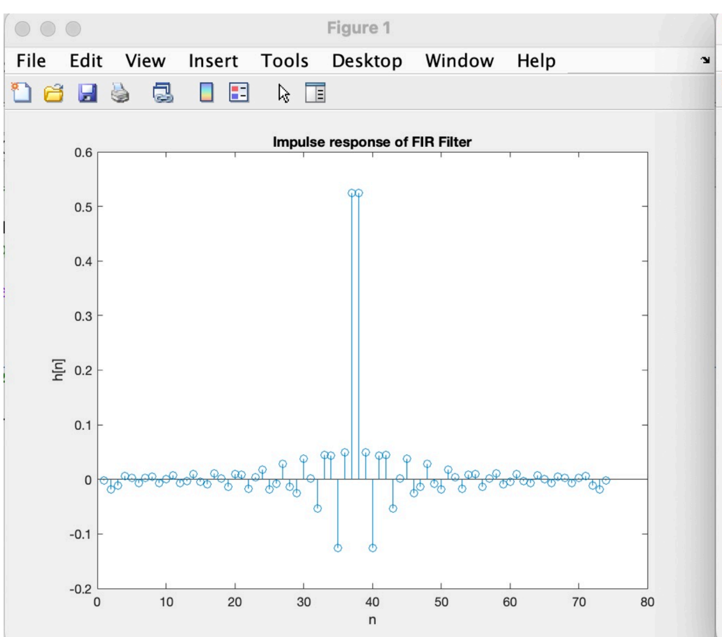
C) $w_{co} = .62\pi$

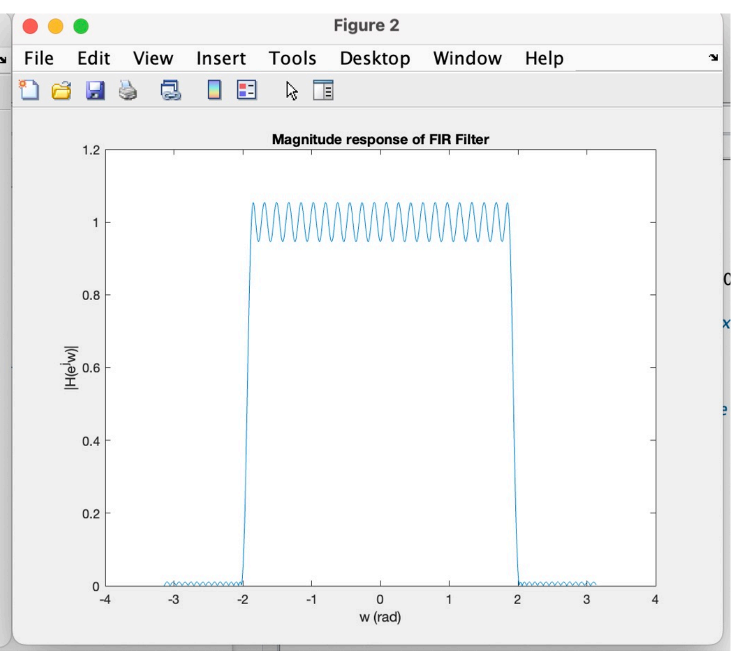
3.5) A) $2\pi \frac{f_c}{f_s}$

let $f_s = 10\,000$ Hz

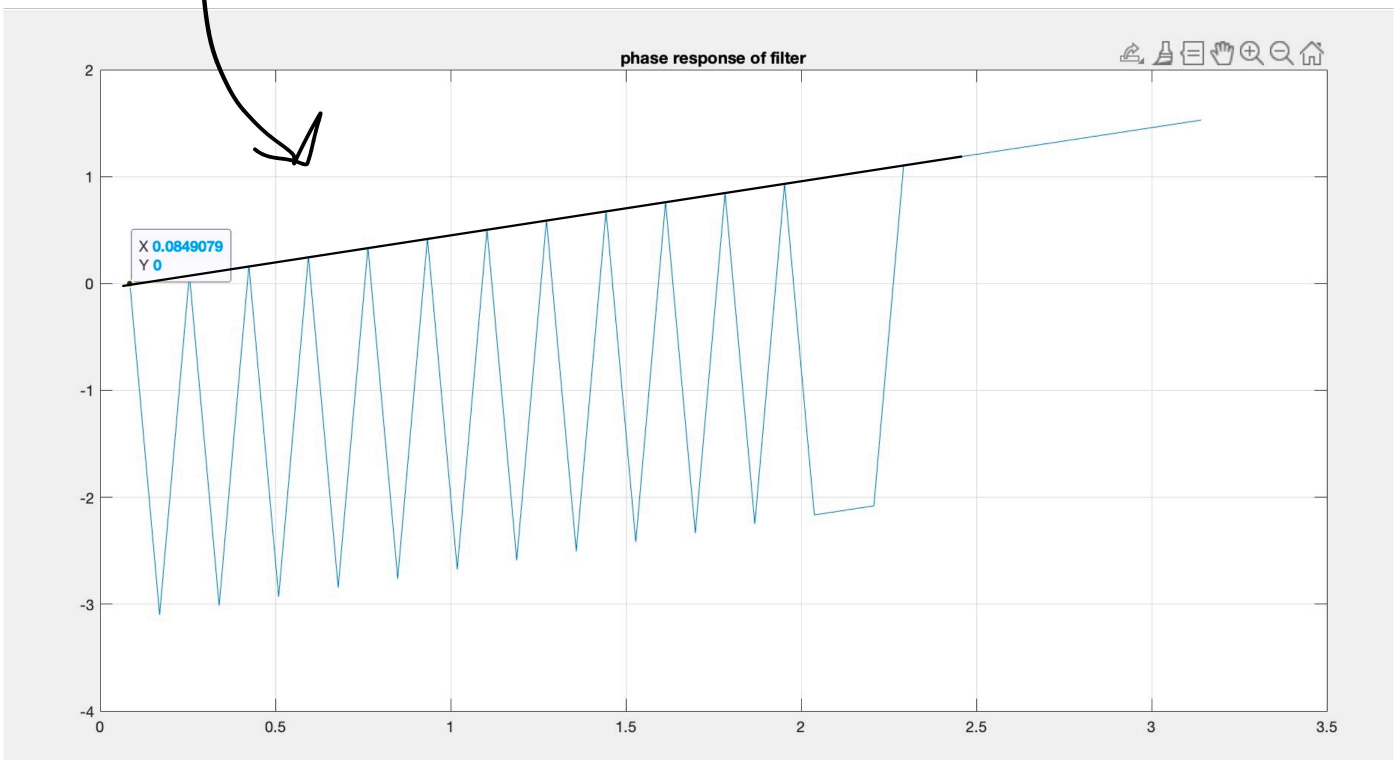
$f_p = 3000$ Hz

$f_s = 3200$ Hz

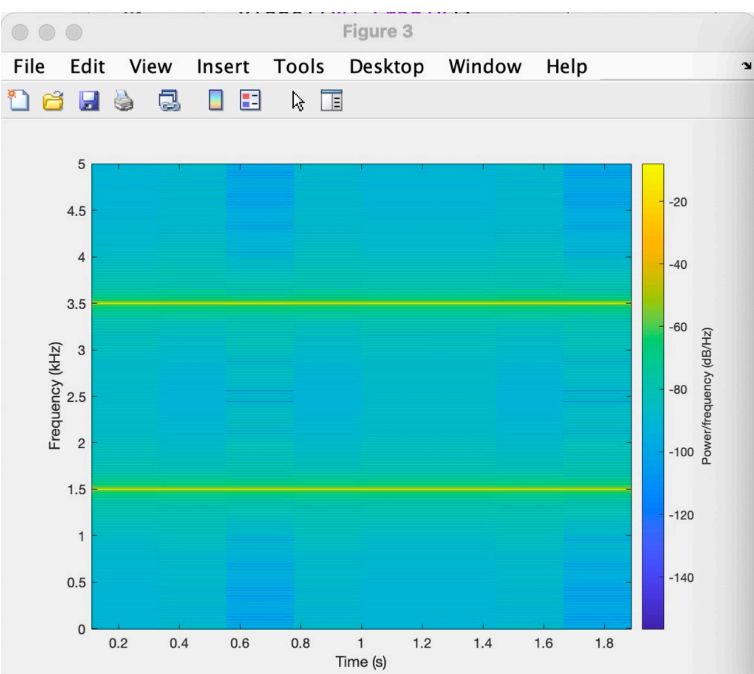


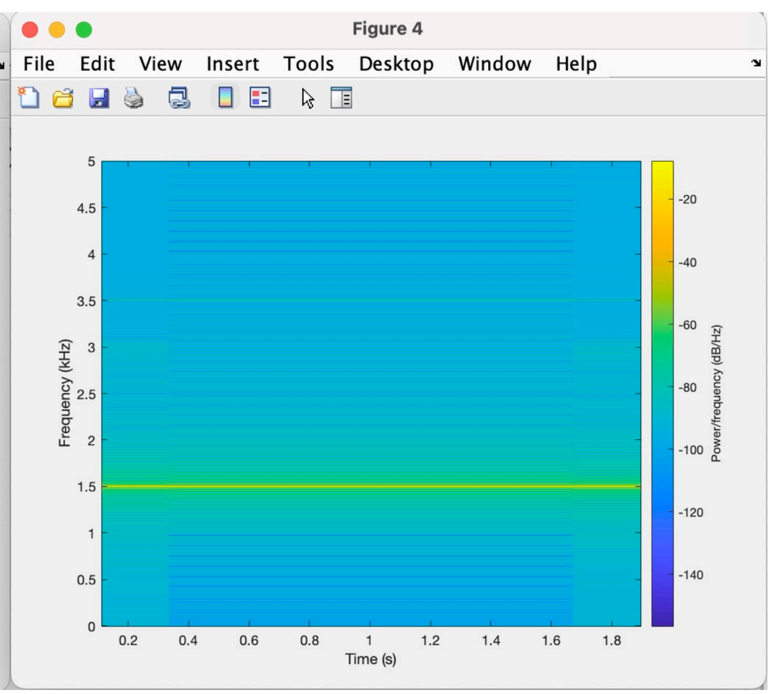


e) slope = .4865



f)





RUDRA GOEL

$A\cos(2\pi f t + \phi)$ $\qquad f_0 = 26\ \text{Hz} = GCD(\_\_\_)$

$x(t) = A\cos(2\pi(60)t - 2\pi(60)t_0) + \to Ae^{-j(2\pi 60 t_0)}$

$B\cos(2\pi t(F+43)) +$ $\qquad 13$

$B\cos(2\pi t(F-43))$ $\qquad 17$

60 Not factorable to 26 $\Rightarrow$

Need to cancel signal w/ $f = 60$ Hz $\qquad 1/120$

$A\cos(2\pi(60)t - 120\pi t_0) + B\cos(2\pi(60)t) + B\cos(2\pi(-20)t)$

$t_0 = \frac{1}{120}$

$B\sin(2\pi(\frac{1}{2})t) =$

$B\cos(2\pi(\frac{1}{2})t - \pi/2) = \sum_{k=0}^{M} \cos(2\pi(\frac{1}{2})t - \frac{k}{4}\pi)$ $\qquad 4$

$Be^{-j\pi/2} = \sum_{0=k}^{M} e^{-j\frac{k\pi}{4}}$ $\qquad 1+\sqrt{2}$

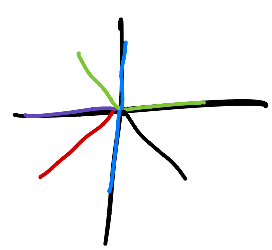
$B(0 + -j) = \cancel{(1+0)} +$

$(\cancel{\frac{\sqrt{2}}{2}} - \frac{\sqrt{2}}{2}j)$

$(0 - j)$

$(\cancel{-\frac{\sqrt{2}}{2}} - j\frac{\sqrt{2}}{2})$ $\qquad (-\sqrt{2} - 1)j$

$\cancel{(-1+0)}$



9000

6

6000π

−4000

1/2

$f_{max} = 4500$

$f_s = f_{max} \times 2$

$= 9000$

A = 2000

2000 = −2π F E

$\frac{-2000}{\pi} = E$

$\varphi = Dt + E\cos(2\pi f t) + T$

$f_i = \frac{1}{2\pi}\frac{d}{dt}\varphi(t) = D + -E(2\pi f)\sin(2\pi f t)$

$D - 2\pi f E \sin(2\pi(F)t)$

D=

5000

500 ↕

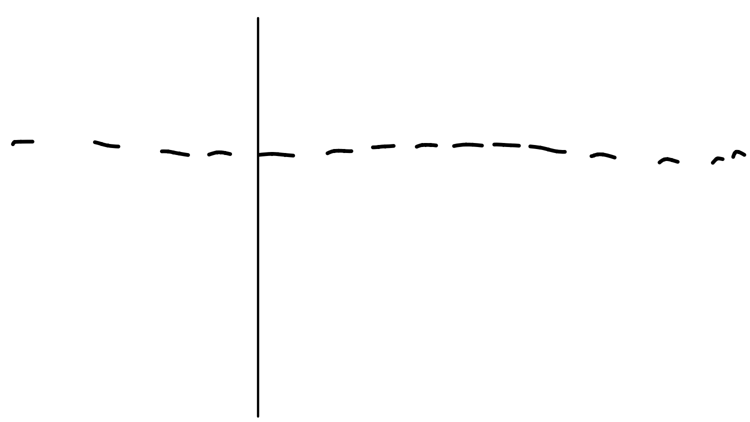
3 cycles in 6 seconds

1 cycle 2 second

⇒ $\frac{1}{2}$ Hz

5000 − 1000

$\frac{4000}{2}$



$\frac{1}{2}e^{j0}$

34

$12 - l5 \quad l=2 \Rightarrow f_1 = 2$ 2

$15 - l5 \quad l=3 \Rightarrow f_1 = 0$ 4

$17 - l5 \quad l=3 \Rightarrow f_1 = 2$ 2

$2\cos(2\pi(2)t) \rightarrow 2e^{j(0)} = 2$ 0

$2\cos(2\pi(2)t) \quad 2e^{j(0)} = 2$

1 −1   1 −1   1 −1   1 −1   1 −1   1 −1

0

0

1

−6

15

−20

15

−6

1

28

$y[n] = x(n) - 6x(n-1) + 15x(n-2) - 20x(n-3) + 15x(n-4)$
$- 6x(n-5) + x(n-6)$

$$\overset{1}{y[2]} = x[2] - \overset{-6(7)}{6x[1]} + \overset{15\ 1}{15x[0]} + 0 \ldots$$

$y[1] = x[1] - 6x[0] + 0 \ldots$ $1 = x[1] - 6$
$x[1] = 7$

$y[0] = x[0] + 0 \ldots$

$x[0] = 1$

$1 = x[2] - 42 + 15$

$h[n] = 1 \quad 1 \quad \beta \quad 1 \quad 1 \qquad\qquad 2$

$$1 + e^{-j\omega} + \beta e^{-j2\omega} + e^{-j3\omega} + e^{-j4\omega}$$

$$e^{-j2\omega}\left(e^{+j2\omega} + e^{j\omega} + \beta + e^{-j\omega} + e^{-j2\omega}\right)$$

$$\left(\beta + 2\cos(\omega) + 2\cos(2\omega)\right) e^{-j2\omega}$$

$$\angle H(e^{j\omega}) = -2\omega$$

$$\beta + \overset{0}{\cancel{2\cos(\tfrac{\pi}{2})}} + \overset{-2}{\cancel{2\cos(\pi)}} = 0$$

$$\beta + 2\cos\left(\tfrac{2\pi}{3}\right) + 2\cos\left(4\pi/3\right) = 0$$

$$\beta + 2\left(-\tfrac{1}{2} + \cancel{\tfrac{\sqrt{3}}{2} j}\right) + 2\left(-\tfrac{1}{2} - \cancel{\tfrac{\sqrt{3}}{2} j}\right) = 0$$

$$\beta - 1 - 1 = 0$$

X

$\cos(.4\pi n) + \cos(0) \Rightarrow$

$\cos(.2\pi n)\cos(.2\pi n) \Rightarrow \cos(.2\pi n + .2\pi n) + \cos(.2\pi n - .2\pi n)$

0

$e^{jC\pi}$ $e^{jD\pi}$

1

$\cancel{.2}$ .1

.2

.6

2

$[\quad]$

$\Sigma \frac{\sin(F\pi n)}{\pi n}\left[\frac{1}{2}\cos(.6\pi n) + \frac{1}{2}\cos(.2\pi n)\right]$

.2

.7

.3

.5

.1

1

.6

.2

$2\frac{\sin(P\pi n)}{\pi n}e^{j(.4\pi)} - 2\frac{\sin(\pi P n)}{\pi n}e^{j(.4\pi)}$

$$X[k] = \sum_{0}^{N-1} x[n]\, e^{-j 2\pi k n/N}$$

$2 = x_0 + x_1 + x_2 + x_3$

Since

$$X[k] = 2e^{(-j\, 2\pi\, k/16)3}$$

Where $2\delta[n-3] = x[n]$

0

0

0

2

$$H_1(z) = \frac{1}{1-.2z^{-1}}$$

$$\frac{\left(z+\frac{2}{3}\right)}{z^2}$$

$$\left(\frac{z^{-1}}{1-.2z^{-1}}\right)\left(15+Az^{-1}+Bz^{-2}\right)$$

$$z^{-1}+\frac{2}{3}z^{-2}$$

$$\Rightarrow h[n] = \left[0 \quad 1 \quad \frac{2}{3}\right]$$

$$= \frac{z+2/3}{z^2}$$

$$h_2[n] = [15 \quad A \quad B]$$

$$\left(15z^{-1}+Az^{-2}+Bz^{-3}\right)z^2 = (z+2/3)(1-.2z^{-1})$$

$$\frac{2}{3}-\frac{1}{5}$$

$$\frac{10}{15}-\frac{3}{15}$$

$$15z+A+Bz^{-1} = z+2/3-.2-\frac{2}{15}z^{-1}$$

$$14z+(B+2/15)z^{-1}+A-7/15=0$$

$$14z^2+(A-7/15)z+B+2/15=0$$

$$\left(z+\frac{2}{3}\right)(z+\alpha) \qquad \frac{2}{3}+\alpha = \frac{A-7/15}{14}$$

$$z^2+\left(\frac{2}{3}+\alpha\right)z+\frac{2}{3}\alpha=0 \qquad \frac{2}{3}\alpha = A$$

L

N

M

J

I

P

B

K

F

G

E

D

A

C

H

$\xi z \quad z^{X} d$

$f_s = 2000$ Hz

$f_1 =$

$\hat{\omega} = \frac{\pi}{2} = 2\pi\left(\frac{1}{4}\right) = 2\pi\left(\frac{f_{in}}{f_s}\right)$

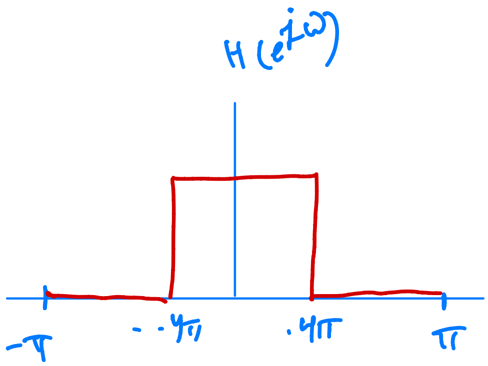
$\frac{1}{4} = \frac{x}{2000}$ $f_{in} = 500$ Hz

500 HZ

$$.4\pi = 2\pi\left(\frac{250}{f_s}\right)$$

$$.2 = \frac{250}{f_s}$$

1250 Hz



$2\pi\left(\frac{200}{300}\right)$ $\quad$ $2\pi\left(\frac{100}{300}\right)$

$2\pi\left(\frac{2}{3}\right)$ $\frac{4\pi}{3} - \frac{6\pi}{3}$ $\quad$ $\frac{2\pi}{3}$

$\frac{-2\pi}{3}$ $\quad$ $\frac{2\pi}{3}$

$\downarrow$

$\cos\left(\frac{2\pi}{3} - \pi/2\right)$

300 Hz

$$2\pi\left(\frac{200}{400}\right) \Rightarrow 2\pi\left(\frac{1}{2}\right) = \pi$$

$$2\pi\left(\frac{100}{400}\right) \Rightarrow 2\pi\left(\frac{1}{4}\right) = \frac{\pi}{2}$$

$\frac{v}{f_s} = 100$

$2\pi(100)$

350 Hz

$2\pi\left(\frac{250}{f_s}\right)$

$\frac{10}{7}\pi - \frac{14}{7}\pi = \left(\frac{-4}{7}\pi\right)$ 350 $\quad 200\pi$

$x[n] = \frac{4}{7}\pi$

$y[n] = \alpha y[n-1] + x[n] + .25y[n-2]$

$H(z) = \frac{Y(z)}{X(z)} \Rightarrow \alpha z^{-1} H(z) + 1 + .25z^{-2}H(z)$

$Y(z) = \alpha z^{-1} Y(z) + X(z) + .25z^{-2}Y(z)$

$H(z)(1 - \alpha z^{-1} - .25z^{-2}) = 1 \Rightarrow H(z) = \frac{1}{1 - \alpha z^{-1} - .25z^{-2}} = \frac{z^2}{z^2 - \alpha z - .25}$

$H(e^{j\omega}) = \frac{e^{j2\omega}}{e^{j2\omega} - \alpha e^{j\omega} - .25}$

$e^{j\omega}(e^{j\omega} - \alpha - .25e^{-j\omega})$

$$H(z) = \frac{1 + b_1 z^{-1} + z^{-2}}{1 - a_1 z^{-1}}$$

$$\omega = 2\pi\left(\frac{60}{8000}\right)$$

$$-1.99777 \qquad \left.\frac{1 + b_1 z^{-1} + z^{-2}}{1 + .3 z^{-1}}\right|_{z = e^{j}} = 0$$

$$\frac{1 + b_1 e^{-j(.015\pi)} + e^{-j(.03\pi)}}{\cancel{1 + .3 e^{-j(.015\pi)}}} = 0$$

$$b_1 = -2\cos\left(2\pi\left(\frac{60}{8000}\right)\right)$$

$$e^{-j\omega}\left(b_1 + 2\cos(\omega)\right)$$

3

1 1 1 = h[n]

1 1 1 1

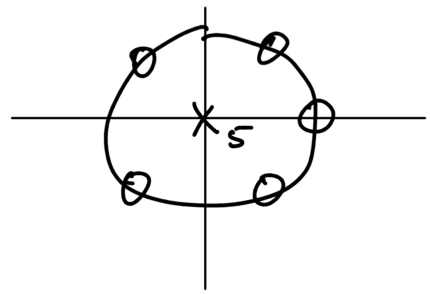
1 1 1 1 1

1 1 1 1 1

1 1 1 1 1

$2 - 2z^{-5}$ $\qquad$ $3 + 3z^{-5}$

$$H(z) = 2 - 2z^{-5} = \frac{2z^{5} - 2}{z^{5}}$$



$$2\,\frac{z^{5} - 1}{z^{5}}$$

$$z^{5} = 1 \Rightarrow z = \sqrt[5]{1}$$

$$6\delta[n] - 6\delta[n-10]$$

$H_1(z) = 2 - 2z^{-5} \rightarrow 2 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; -5$

$H_2(z) = 3 + 3z^{-5} \rightarrow 3 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 3$

$$H_{12}(z) = 6 - 6z^{-10}$$

0

2 $\quad Y(z) = 6z^{-2} - 6z^{-12}$

$$X(z) = z^{-B}$$

$$H(z) = 6 - 6z^{-10}$$

$$Y(z) = H(z)\,X(z) = z^{-B}6 - 6z^{-10-B}$$

0 $\quad$ 1 $\quad$ 1/6 $\quad$ 0

$$\frac{b_0 + b_{10}z^{-10}}{1 - a_1 z^{-1} - a_{10} z^{-10}}$$

$$\frac{1/6}{1 - z^{-10}}$$

$$H_3(z) \cdot H_{12}(z) = 1$$

$$H_3(z) = \frac{1}{6 - 6z^{-10}} \left(\frac{1/6}{1/6}\right)$$

$1 \quad -1.5 \quad .9$

$1 - 1.5z^{-1} + .9z^{-2}$

$1 + 2.7z^{-1} + .9z^{-2}$

F

E

$$\frac{z^2}{(z+.1+.1j)(z+.1-.1j)}$$

B

A

D

C

~~H~~ F ~~F~~ E

B A

D C

2

$e^{-j\omega}$ $e^{-j2\omega}$ $e^{-j3\omega}$ $e^{-4j\omega}$

3

5

$1 + z^{-1} + z^{-2} + z^{-3}$

$\delta[n] + \delta[n-1]$

$\delta[n-2]$

$\delta[n-3]$

6

0 1 0 2 0 1

7 $e^{-j\omega} + 2e^{-j3\omega} + e^{-j5\omega}$

$e^{-3\omega}$

1 $H(z) = \dfrac{1}{1-\frac{1}{2}z^{-1}}$

$\omega = \hat{\omega}_1 \qquad x_1 = \cos(\frac{\pi}{3} n)$

$x_2 = 2\cos(\frac{\pi}{2} n - \frac{\pi}{2})$

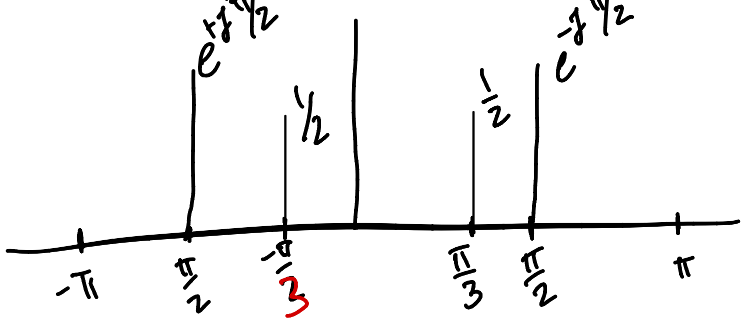
$-$

$0 \quad 0 \quad 1/2 \quad e^{-j\pi/2} \quad 0 \quad 0$

$0 \quad 0 \quad 1/2 \quad e^{j\pi/2} \quad 0 \quad 0$

$x = x_1 + x_2 = \cos(\frac{\pi}{3} n) + 2\cos(\frac{\pi}{2} n - \frac{\pi}{2})$

$2e^{-j\frac{\pi}{2}}$



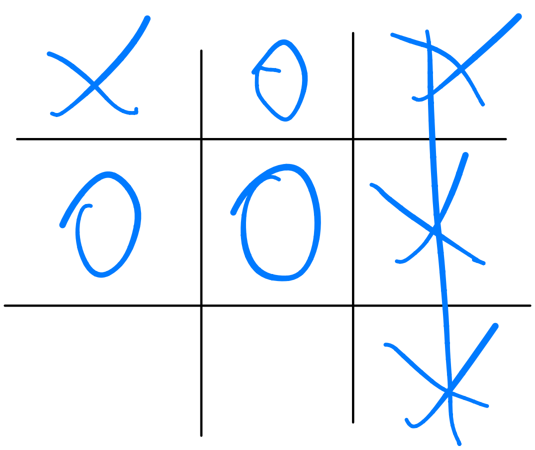
12 pt DFT is DTFT sampled @ $\frac{\pi}{12} n$

$\frac{2\pi}{12} n = \frac{\pi}{3} \Rightarrow n = 2$

$\frac{2\pi}{12} n = \frac{\pi}{2} \Rightarrow n = 3$

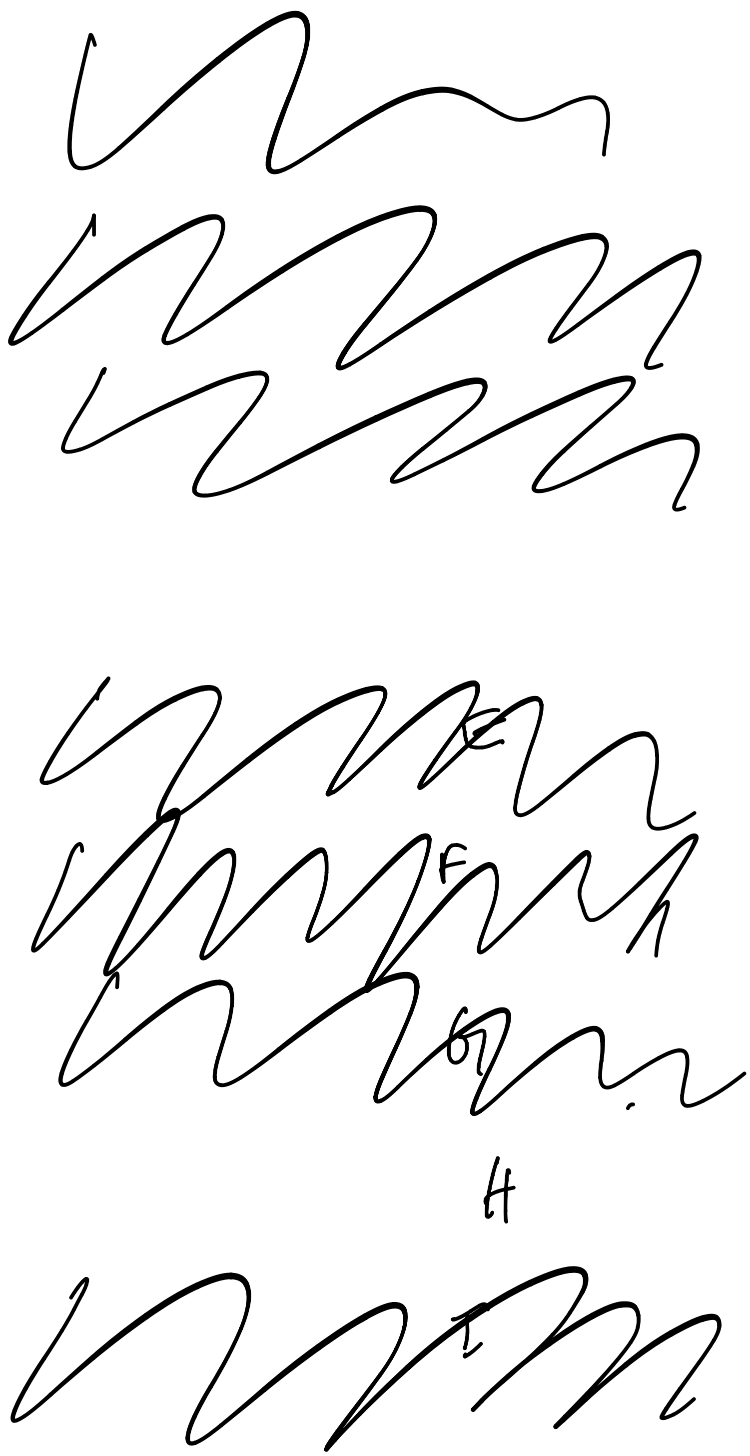
$\frac{2\pi}{12} n = \frac{4\pi}{3} \Rightarrow n = 8$

$\frac{2\pi}{12} n = \frac{3\pi}{2} \Rightarrow n = 9$



$x^*[k] = x[L-k]$

$0 \quad 6 \quad + \quad 5$



I

F

C

E

B X H

G

A X B

D

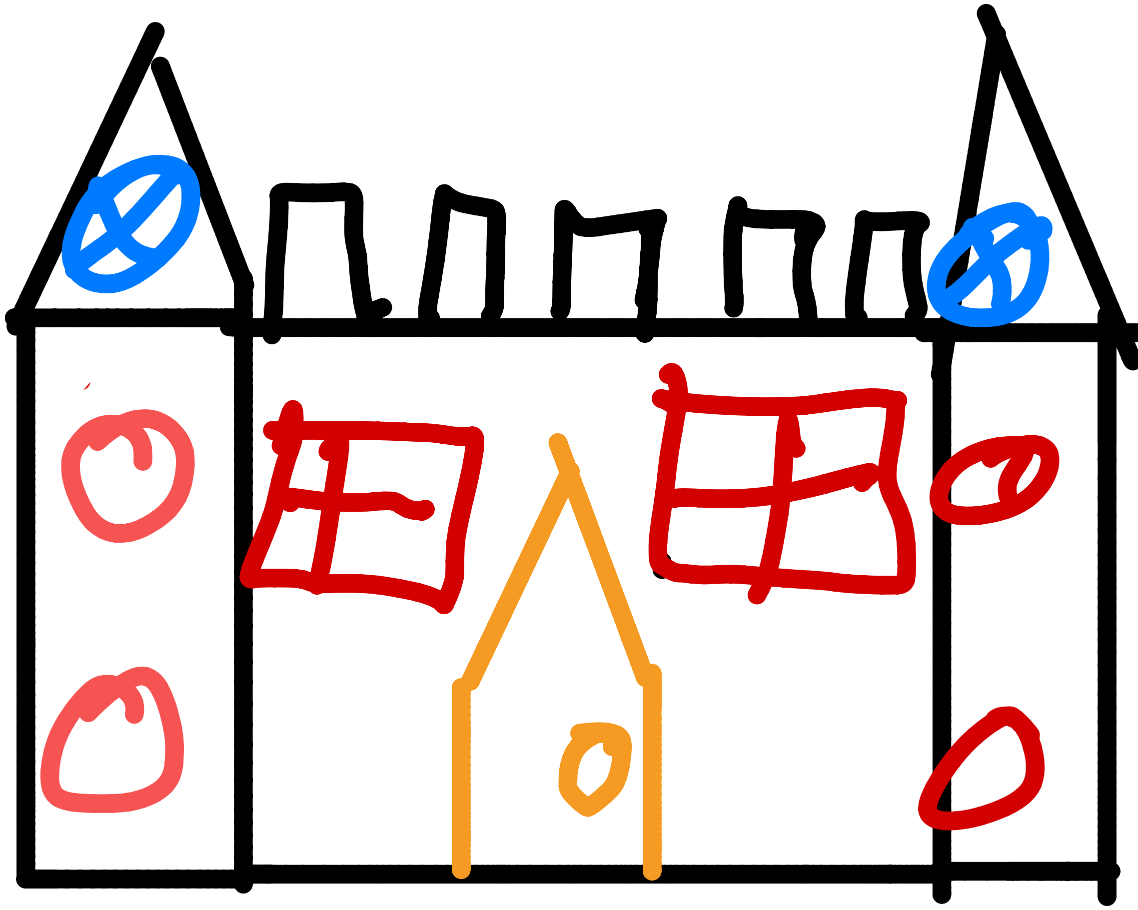
A

_ _ _ _ A

_ A _

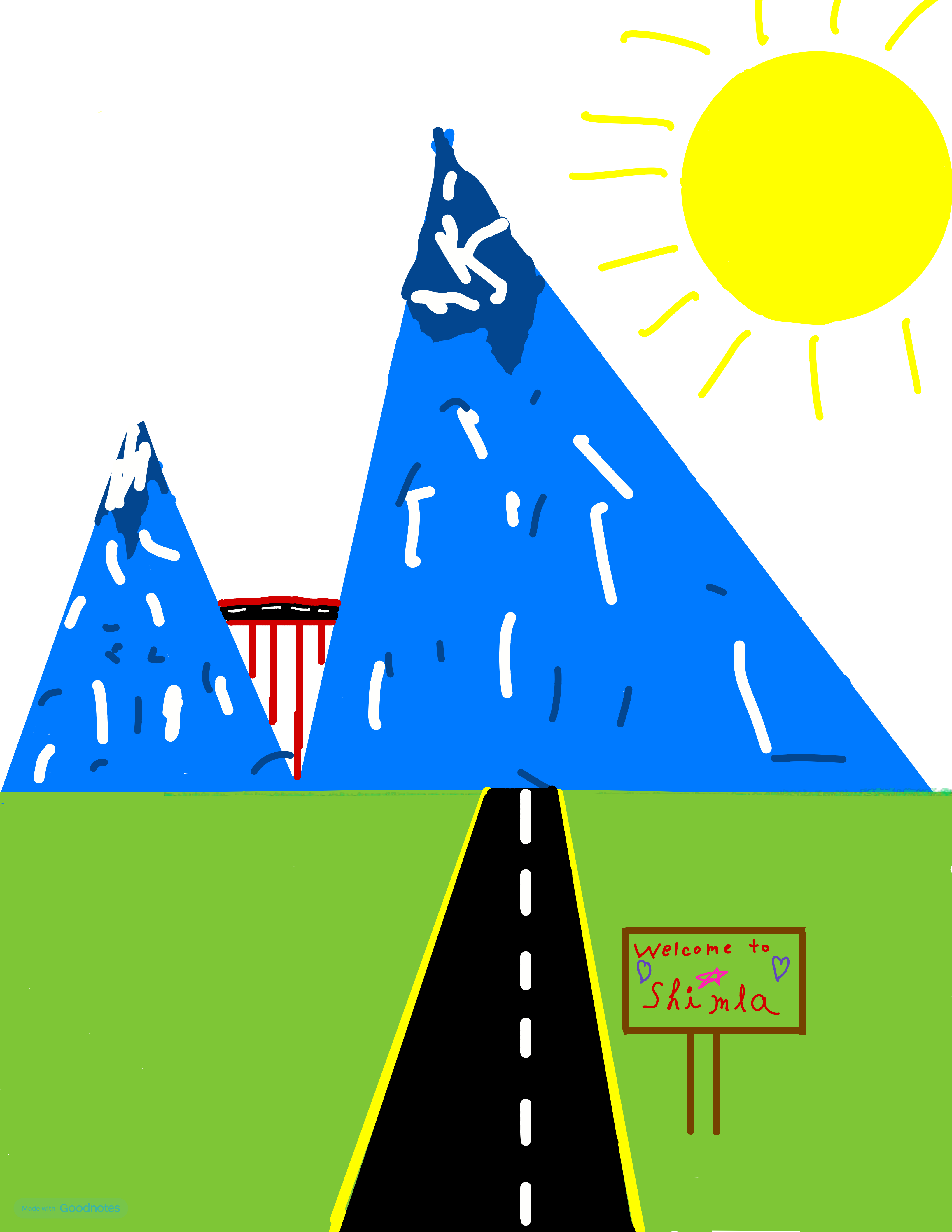
_ _ _ _ A _ _ E

A _ _ E



Pocchi's Art

Castle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