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Tuesday, October 17, 2023 16:35

1. Let S be all vectors in $\mathbb{R}^3$ that are othorgonal to $\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$. Is S a subspace? If so, find a basis for S.

2. Consider two vectors in an inner-product space.
   a. Show that $< u+v, u-v > \; = \; ||u||^2 \; - \; ||v||^2$
   b. Show that the diagonals of a rhombus are orthogonal to each other.

3. Show that $< A, B > \; = tr\left(A^T B\right)$ defines an inner product on space $M_{m\times n}\left(\mathbb{R}\right)$. Try two small matrices and see that this inner product really computes.

4. Show that $< p\left(x\right),\; q\left(x\right) > \; = \int_{-1}^{1} p\left(x\right) q\left(x\right) dx$ defines an inner product on space of all real-valued continuous functions defined on $[-1, 1]$. Can you come up with two non-zero polynomials that are orthogonal with repect to this inner product?
5.
6. Consider the norm induced by an inner product, that is,
   $||x|| = \; < x, x >^{1/2}$
   a. Show that $||\lambda x|| = \; |\lambda| ||x||$ for any scalar $\lambda \in \mathbb{R}$.
   b. Show that $||x|| = 0$ if and only if $x = \; \vec{0}$
   c. Comment on the matrix norm induced by the inner product given by Q2. What does that norm look like?

7. Show that two unit vectors u, v are equal if $< u, v > \; = 1$.

Inner product over R definition I gave in class is
1. <x,y>=<y,x>
2. <x,ky>=k<x,y>
3. <x,y+z>=<x,y>+<x,z>
4. <x,x>= 0 and =0 iff x=0

I didn't give them the definition of norm in class, but this is essentially verifying it is a norm sans the triangular inequality condition, which I will cover Thursday. Please make sure you cover this one in studio.