10/4

Tuesday, October 03, 2023 4:53 PM

We have covered rank-nullity theorem and basic examples of matrix multiplication, row/col rank of a matrix and concluded that they are the same.

1. $A = \begin{pmatrix} 1 & 1 & 2 & 4 \\ 1 & -3 & 3 & 1 \\ 1 & 9 & 0 & 10 \end{pmatrix}$. Find its Col(A), Row(A), Nul(A) and their dimensions. What is rank(A)?
2. Let $A \in M_n(\mathbb{R})$ such that $A_{i,j} = 0$ if i+j is odd and 1 if i+j is even . $n \geq 2$. What is rankA? Give a basis for Col(A).
3. If matrix $AB = (0)$, where (0) denotes the zero matrix. Show that Col(B) is a contained inside of Nul(A).
4. Let $v = \begin{pmatrix} 1 \\ 0 \\ 5 \\ 9 \end{pmatrix}$, $w = \begin{pmatrix} 0 \\ 1 \\ 9 \\ 2 \end{pmatrix}$.
    a. Determine $v^T w$. This is the alternative definition of dot product.
    b. Determine vv^T. Determine its rank.
    c. Show that in general, rank(vv^T) = 1, if $v \in \mathbb{R}^n$ and is non-zero.
5. Let A, B, C be matrices of appropriate sizes. Show that $A(B + C) = AB + AC$.
6. Let $T \in \mathscr{L}(V, W)$. Show that
    a. If T is surjective dimV $\geq$ dimW
    b. If T is injective then dimV $\leq$ dimW
    c. If T is an isomorphism, dimV = dimW
    d. If T is surjective and dimV=dimW, then T is also injective.
7. True or False
    a. For any $m \times n$ matrices, rank(A+B) = rank(A) + rank(B)
    b. For any $m \times n$ matrices, rank(A+B) $\leq$ rank(A) + rank(B)
    c. For any $m \times n$ matrices, rank(AB) $=$ rank(A) rank(B)

Aran, when I did row space in class, I kept the rows in their c
$A \in M_{m \times n}(\mathbb{R})$, Row(A) is inside $M_{1 \times n}(\mathbb{R})$

These are essentially corollaries from rank-nullity.