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Sunday, November 12, 2023 11:45

1. For each of the following matrices, find the characteristic polynomial, eigenvalues and the corresponding eigenspace (find a spanning set of the eigenspace). Also determine the algebraic/geometric multiplicities of each eigenvalue.
   a. $\begin{pmatrix} 10 & -9 \\ 4 & -2 \end{pmatrix}$
   b. $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$
2. Show that matrix A is invertible iff 0 is not an eigenvalue of A. This should also be added to the invertibility theorem.
3. Let V be vector space of smooth infinitely differentiable functions. Consider differentiating map $T : V \to V$ defined as $p\left(x\right) \mapsto p'(x)$. Determine an eigenvector of this map.
4. Let $A, B \in M_n\left(\mathbb{R}\right)$ that are similar. You can use these results in the future.
   a. Show that they have the same determinant.
   b. Show that they have the same characteristic polynomial.

We have not gone over diagonalization theorem. They will have to l
Use the determinant might make it a little simpler.

For trace I think you can use cofactor expansion for this and carefull
contribute to the n-1 term's coefficient. Det is given by setting \lam

4. Show that $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ is not similar to any diagonal matrix.
5. Consider the characteristic polynomial $p_A\left(\lambda\right)$ of $n \times n$ matrix A.
   Show that $p_A(\lambda)$ is of the form
   $$(-1)^n\lambda^n + (-1)^{n-1}tr\left(A\right)\ \lambda^{n-1} +\ , \ldots ,\ + de\,t(A).$$
   In other words, its constant term is always det(A), and its λ^{n-1} has coefficient $\pm tr\left(A\right)$
   Write down the formula when A is a 2×2 matrix. You can use this result in the future.
6.
   a. If two matrices have the same det, trace and rank, are they necessarily similar?
   b. If two matrices have the same char poly, are they necessarily similar?

No. $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ $and\ I_2$