11-01

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In class I defined that $T \in \mathscr{L}(\mathbb{R}^n)$ is an rthogonal transformatio

We defined an orthogonal matrix is a square matrix whose cols fo

1. Let $T \in \mathscr{L}(\mathbb{R}^n)$ is an orthogonal transformation.
    a. Show that T preserves orthogonality.
    b. Show further that $[T]_{\mathscr{E} \to \mathscr{E}}$ is an orthogonal matrix, i.e., its cols form an orthonormal set.
    c. Show that if $A \in M_n(\mathbb{R})$ is an orthgonal matrix, then the map
    $$T_A : \mathbb{R}^n \to \mathbb{R}^n$$
    $$x \mapsto Ax$$
    Is an orthogonal transformation.

2. Let $A, B \in M\_n(\mathbb{R})$ that are orthogonal
    a. Show AB is also orthogonal
    b. Does A^{-1} exist? Is it orhtogonal?

3.
    a. Let $A \in Mn(\mathbb{R})$. Show that A is orthogonal iff $A^T A = I_n$ iff $A^{-1} = A^T$.
    b. Show that the transformation in 1.c in this case also preserves dot product (angle-preserving)
4. Determine the forms of all orthogonal matrices in $M_2(\mathbb{R})$.

$$\begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix}, \begin{pmatrix} cos\theta & sin\theta \\ sin\theta & -cos\theta \end{pmatrix}$$

5. True or False
    a. Let $A \in M_{m\times n}(\mathbb{R})$. If $A^T A = I_m$, then $A^T A = I_n$.
    b. Let $A \in M_n(\mathbb{R})$. If $A^T A = I_n$, then $A^T A = I_n$.
    c. A reflection transformation is an orthogonal transformation.

No. Consider (e_1, e_2) in $\mathbb{R}^3$
Yes.
Yes.