Question 6

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6. Let $f(x)$ be a polynomial with real coefficients. Show that if $z \in \mathbb{C}$ is a root of $f(x)$ then so is its complex conjugate $\bar{z}$.

lemma: $\overline{z^n} = (\bar{z})^n$

Pf| if $z = re^{i\theta}$, complex #

Then $z^n = r^n e^{i\theta n}$

$\overline{z^n} = r^n e^{-i\theta n}$

But $(\bar{z})^n = (re^{-i\theta})^n \Rightarrow r^n e^{-i\theta n}$

$f(x) = a_0 + a_1 x + a_2 x^2 + \dots + a_k x^k$

if z is a Solution Then $f(z) = a_0 + a_1 z + \dots + a_k z^k = 0$

consider the complex conjugate of Both sides

$\bar{0} = \overline{a_0 + a_1 z + \dots + a_k z^k} \Rightarrow 0 = a_0 + a_1 \bar{z} + \dots + a_k \overline{z^k}$

But By lemma #1, $\overline{z^k} = \bar{z}^k$

$\Rightarrow 0 = a_0 + a_1 \bar{z} + a_2 \bar{z}^2 + \dots + a_k \bar{z}^k$ ∴ Thus $\bar{z}$ is also a complex conjugate.