Question 1

Tuesday, November 14, 2023 9:55 PM

1. Show that similarity defines an equivalence relation on $M_n(\mathbb{R})$.

Reflexive: $\forall A \in M_n(\mathbb{R})$

if $A = SAS^{-1}$, $S = I_n$, $\Rightarrow \exists I_n$ s.t

$A = I_n A I_n^{-1} \Rightarrow I_n A = A$ $A \sim A$

Symmetric: $\forall A, B \in M_n(\mathbb{R})$

if $A = SBS^{-1}$, $S \in GL(\mathbb{R})$

$S^{-1}A = S^{-1}SBS^{-1} \Rightarrow S^{-1}A = BS^{-1}$

$\Rightarrow S^{-1}AS = BS^{-1} = B = S^{-1}AS$

$A \sim B$

Transitive: $\forall A, B, C \in M_n(\mathbb{R})$

if $A = SBS^{-1}$ and $B = PCP^{-1}$

$\Rightarrow A = S(PCP^{-1})S^{-1} \Rightarrow A = SPCP^{-1}S^{-1}$

let $D = SP$ $D^{-1} = P^{-1}S^{-1} \Rightarrow A = DCD^{-1}$

$n \times n$ $n \times n$

$\underline{A \sim C}$