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Saturday, November 18, 2023 18:56

1. For a matrix A that is diagonalizable, we can write $A = PDP^{-1}$. Is this decomposition unique?

2. Consider 6×6 matrix A with eigenvalues 0, 1,2. It is known that $E_{\{\lambda=1\}}$ is of dimension 3 and $E_{\{\lambda=0\}}$ is of dimension 2. Determine the characteristic polynomial of A. Is A diagonalizable?

3. Let $\lambda_1, \ldots, \lambda_n$ be complex eigenvalues of matrix A, counting multiplicities. Show that
   a. $detA = \prod_{i=i}^{n} \lambda_i$
   b. $trA = \sum_{i=1}^{n} \lambda_i$

4. Square matrix A is said to be nilpotent if there exits $k \in \mathbb{N}$ such that $A^k = (0)$.
   a. Show that if A is nilpotent, then its only eigenvalue is 0.
   b. Give an example of a non-zero 3×3 matrix A such that $A^2 = (0)$.

5. Let A be a symmetric real valued matrix. Show that
   a. All of A's eigenvalues are real.
   b. Eigenspaces of A are orthogonal to each other.

6. Diagonalize the following matrices if possible
   a. $\begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$
   b. $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

We have covered that matrix A is diagonaliable iff
1. geomMulti of each eigenvalue sums to n
2. geomMulti = algeMult for each eigenvalue
3. There exists an eigenbasis for V

Please cover this since I will need this for spectral theorer

Please remind them that b is symmetric and its eigenspace orthogonal to each other as shown in 5b