Designing active acoustic metamaterials

Georg Fischer[1], Albert Zhykhar[1], Jan Küller[1], Agostino Troll[2]

[1] *Fraunhofer Institute for Digital Media Technology IDMT, 98693 Ilmenau, Germany, Email: georg.fischer@idmt.fraunhofer.de*

[2] *Fraunhofer Institute for Building Physics IBP, 70569 Stuttgart, Germany, Email: agostino.troll@ibp.fraunhofer.de*

Introduction

In recent years metamaterials gained a lot of interest from the research community in acoustics. One potential benefit of acoustic metamaterials is that they allow to design compact absorber systems which are efficient at low frequencies. For this case resonators can be used as building blocks of the metamaterial. From that follows, that the effective frequency bandwidth of metamaterials is narrow by nature. To design metamaterials that are also effective for broadband or time variant scenarios, incorporating active components is a promising way[1].

To develop an active metamaterial it is advantageous to combine methods from different disciplines such as acoustics and electronics. In this article we will propose one way to systematically model active acoustic metamaterials making use of the transfer-matrix-method (TMM). We focus ourselves on metamaterials made of periodically arranged locally resonant structures referred to as unit cells that are applied to one-dimensional waveguides. Here, the active components are realized as electrodynamic loudspeakers. The designs are simulated with the proposed methods and evaluated by a finite element simulation.

Metamaterials

It is well known that the ability to have bandgaps, and hence to exhibit sound absorption for particular frequency bands, in multilayer materials can be attributed either to Bragg scattering or to local resonances of individual layers. Here we will focus on the mechanism that is based on the locally resonances feature, whereas the Bragg effect could also be modelled with the proposed methods.

Acoustical properties of one-dimensional periodically structured acoustical materials as well as of unit cells constituting them can effectively be described with the TMM approach. In this method an acoustic wave propagation through a medium is fully determined by a transfer matrix $\mathbf{T}$. Considering fluid-like material confined in ducts of finite cross-sectional area the transfer matrix relates the pressure p and volume velocity v at the inlet and at the outlet of a duct as

$$\begin{bmatrix} p \\ v \end{bmatrix}_{in} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} p \\ v \end{bmatrix}_{out}. \tag{1}$$

For a simple case of a duct of cross section S and of the length l the transfer matrix $\mathbf{T}_{\text{Duct}}$ is given by

$$\mathbf{T}_{\text{Duct}} = \begin{bmatrix} \cos(kl) & jZ_0 \sin(kl) \\ \frac{j}{Z_0}\sin(kl) & \cos(kl) \end{bmatrix}. \tag{2}$$

Here $Z_0 = \rho c/S$ is the normalized characteristic acoustic impedance, with ρ being the density and c the sound speed of the material, and $k = \omega/c$ is the wavenumber at the angular frequency $\omega = 2\pi f$. In the case of unit cells of the length l, they can be modelled as an equivalent one-dimensional fluid-like material with complex and frequency-dependent effective parameters using transfer matrix $\mathbf{T}_{\text{UC}}$ having the same structure as the matrix 2. Other advantages of the TMM approach are that it allows to combine metamaterial structures with common acoustic elements like ducts of different geometries or include other building blocks like conventional absorbers to model and evaluate more real-life one-dimensional acoustic systems. Another important advantage is the ability to incorporate complex acoustic elements whose transfer matrix can be computed separately by a FEM software like COMSOL.

The analysis of the metamaterial as an infinite acoustic periodic medium arranged from unit cells provides us with the dispersion relation [2, 3]

$$|\mathbf{T}_{\text{UC}}(\omega) - e^{jkl}\mathbf{I}| = 0. \tag{3}$$

The dispersion relation is completely defined by the unit cell transfer matrix $\mathbf{T}_{\text{UC}}(\omega)$ and describes propagation regimes under ideal (infinite and periodic) conditions.

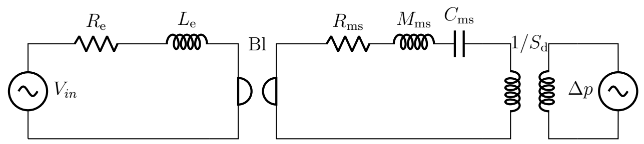

Figure 1: Electrical circuit representing the electrical (left) and mechanical (middle) properties of an electrodynamic loudspeaker. R_e and L_e are representing the electrical parameters, R_{ms}, L_{ms} and C_{ms} are parameters that describe the membrane dynamics. The conversion from the electrical to the mechanical domain is done by applying the force factor Bl. The conversion from the mechanical to the acoustical domain is done by the membrane area S_d. An acoustic pressure results in a force of $F = S_d \Delta p$ on the membrane, where $\Delta p = p^+ - p^-$ is the difference of pressure in front (p^+) and pressure behind (p^-) the membrane.

Impedance control

Acoustic impedance control has a long history, see e.g. [4, 5, 6]. Often the goal is to achieve high absorption at low frequencies, while maintaining a small form factor of the absorber, e.g. for room mode damping. Acoustic impedance control can be seen as way to control the ratio of acoustic velocity and sound pressure at the location of the sensor-actuator configuration.

Here the sensor is a microphone that feeds back the sound pressure signal, through a control filter to an electrodynamic loudspeaker, that is used as the (volume) veloc-

ity source. Opposed to feedforward active noise control schemes, the error and primary microphone are the same and the actuator controls the local sound field at one point in front of the actuator. The control filter is designed based on the actuator dynamics, that can be modeled by a lumped element model (LEM) (s. figure 1). When staying in the low frequency regime the LEM of the electrodynamic loudspeaker is also a good approximation to describe the influence of the actuator on the acoustic domain.
One promising signal processing strategy that fits well to a design approach was introduced in [7]. Here the actuator model is a current drive for the loudspeaker. With this the electrical part of the LEM can be neglected and the model of the electrodynamic loudspeaker reduces to the set of mechanical parameters and the force factor. Furthermore, by using a closed enclosure, the velocity of the loudspeaker membrane together with the compliance of the closed enclosure can be used to model the acoustic pressure behind the membrane p^{-}. This reduces the problem to only sensing the pressure in front of the loudspeaker. With these simplifications the control gain, to achieve a desired impedance of Z_d is

$$G(\omega) = \frac{S_d}{Bl}\left(1 - \frac{Z_{ls}(\omega)}{Z_d(\omega)}\right). \quad (4)$$

Furthermore the controller can be implemented in the time domain. By calculating the analytic signal with the Hilbert transform, the middle frequency can be shifted to the zeroth frequency. Then, a lowpass filter can be used to specify the bandwidth of the impedance controller. With this approach the shape, the middle frequency as well as the absolute value of the impedance of the active unit cell can be designed. Please, have a look at [7] for further details.

Simulation Example

By using the knowledge mentioned above the procedure to design an active acoustic metamaterial can now be described. For the example considered here the goal is to achieve a high transmission loss in a duct system at a certain design frequency. All methods were implemented as a MATLAB/Simulink toolbox. A special care was taken to design a flexible hierarchy of objects where a duct system could be modeled by an assembly of blocks representing single acoustic elements connected together. This simplifies modelling of compound acoustic systems and allows easiy realization of all the advantages of TMM in the software.
The simulation procedure can be split into three steps. The first step is to simulate a single active unit cell. In the second step the behaviour of the infinite array can be analysed based on the corresponding dispersion relation. The purpose here is to evaluate if a stopband to be produced with the current design. In the third step a finite array of a fixed number of active unit cells is simulated to predict the achievable transmission loss.

Step one: Single active unit cell
For the example we use the presented control strategy from section two . Our goal is to achieve a high transmission loss at 200 Hz, with a small bandwidth of 2 Hz.

Table 1: A summary of parameters used in the simulations.

Parameter	value	description
simulation parameters:		
f_s	8000 Hz	sampling frequency
T	20 °C	temperature of fluid
fluid	air	fluid type
loudspeaker parameters:		
R_{e}	4.69 Ω	electrical voice coil resistance
L_{e}	0.01 mH	frequency independent part of voice coil inductance
Bl	3.49 N/A	force factor of the el. dyn. loudspeaker
R_{ms}	1.68 kg/s	mechanical resistance of driver losses
M_{ms}	1.10 g	mechanical mass of driver diaphragm
C_{ms}	0.57 mm/N	mechanical compliance of driver suspension
S_{d}	7.07 cm^2	area of the diaphragm
controller parameters:		
f_{c}	200 Hz	midfrequency of the control filter
BW	2 Hz	bandwidth of the control filter
geometrical parameters:		
d_{duct}	10 cm	diameter of the duct
l_1	25 cm	distance between sound source an first unit cell
l_2	25 cm	distance between last unit cell and duct boundary
l	5 cm	distance between unit cells

The bandwidth is controlled with an infinite impulse response lowpass of order 4. The parameters of the simulation are summarized in table 1. In order to analyze the behaviour of the actuator we implement the dynamic equations from the LEM (see Figure 1) together with the signal processing steps in the software Simulink. As input signal we choose a sinus sweep with amplitude one, serving as an incident sound pressure of 94 $\mathrm{dB}_{\mathrm{SPL}}$. After simulating the response of the loudspeaker the membrane velocity and the pressure can be extracted into Matlab. Note that here no nonlinearities are incorporated in the loudspeaker model. Now, the resulting acoustic impedance can be calculated as $Z_{ls}(\omega) = \frac{P(\omega)}{S_d V(\omega))}$. Here, $P(\omega)$ and $V(\omega)$ are corresponding to the acoustic pressure and the membrane velocity after transformation into the frequency domain. Note, that in the remainder of the paper the frequency dependence is omitted.

Step two: Dispersion relation
After extracting the acoustic impedance of the active controlled unit cell the acoustic model with periodic boundary conditions can be analysed. For this we have to plug in the acoustic impedance into the transfer matrix, that represents the unit cell. For our case of setting up a metamaterial out of resonators sitting at the side of a duct the transfer matrix $\mathbf{T}_{\mathrm{UC}}$ for a unit cell is set up as

$$\mathbf{T}_{\mathrm{UC}} = \mathbf{T}_{\mathrm{Duct}} \begin{bmatrix} 1 & 0 \\ \frac{1}{Z_d} & 1 \end{bmatrix} \mathbf{T}_{\mathrm{Duct}} \quad (5)$$

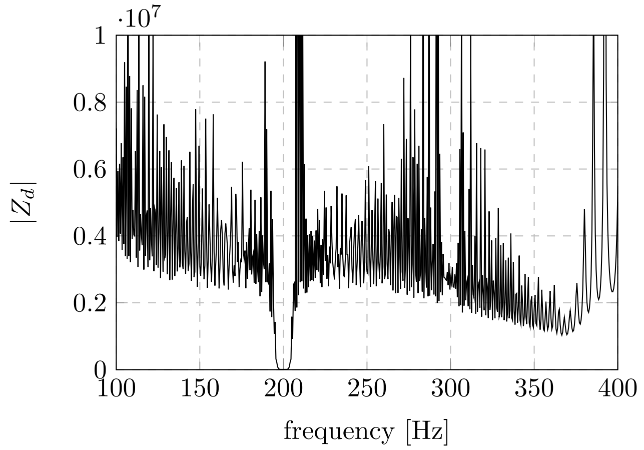

Figure 2: The acoustic impedance of the active controlled loudspeaker, where the impedance is controlled in a small region at a midfrequency of 200 Hz.

Here the length of the duct segments l corresponds to the periodic distance of the unit cells. In our simulations the surrounding fluid is air at 20 °C. That results in the specific acoustic impedance $Z_0 = 413,45\,\frac{\mathrm{sPa}}{\mathrm{m}}$. Using equation 3 the so called dispersion relation can be obtained. It is visualized in figure 3 Here a bandgap is developing at 200 Hz. One has to admit, that for a real bandgap, i.e. that of an ideal resonator, the value for the normalised wave number should reach one. Nevertheless, the procedure results in a stopband behaviour, which can be seen by plotting the transmission loss (s. figure 5).

Step three: Finite array of unit cells
With the obtained unit cell transfer matrix $\mathbf{T}_{\mathrm{UC}}$, the more complex and realistic model can be assembled. The transmission matrix for the resulting finite array can be expressed as

$$\mathbf{T}_{\mathrm{total}} = \mathbf{T}_{\mathrm{Duct}}\mathbf{T}_{\mathrm{UC}}\mathbf{T}_{\mathrm{UC}}\mathbf{T}_{\mathrm{UC}}\mathbf{T}_{\mathrm{UC}}\mathbf{T}_{\mathrm{Duct}}. \tag{6}$$

For active muffler systems it can be benificial to implement a one port source and a termination in the simulation. With this it is possible to examine different states of the active components, by defining different sound pressures in the simulation. The one port source is defined by a source pressure strength and a source impedance. The termination can be defined solely by the complex reflection coefficient r – in our case, we chose an anechoic termination and set $r = 0$.
The resulting pressure difference along the duct system can than be obtained by extracting the pressure and velocity at the different one and two port matrices.

Evaluation

In order to evaluate the simulation approach, a finite element (FE) model in COMSOL was built. Here the unit cells are modeled as lumped element speakers that are coupled to the FE acoustic module. As feedback signal the pressure is measured at a node near the speakers and weighted by the control filter, that was precalculated in Simulink. The noise source is modeled as a loudspeaker. The termination of the duct is modeled as a perfectly matched layer which corresponds to the anechoic ending

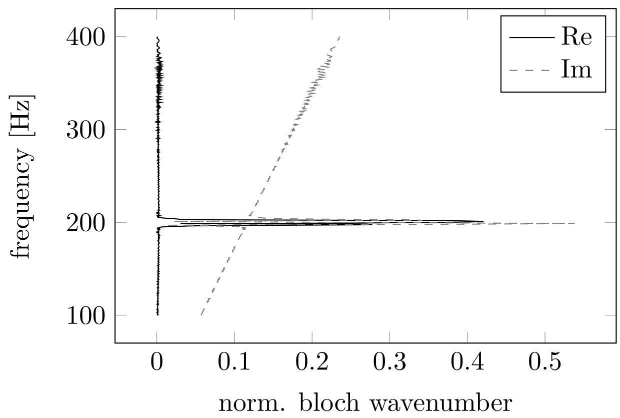

Figure 3: The dispersion relation of the active unit cell. The normalized Bloch wavenumber is $\frac{kl}{\pi}$

of the TMM model. The acoustics are modeled with all thermo-viscous boundaries and losses applied.
By comparing the pressure of incident P_{in} and outgoing P_{out} wave, the resulting transmission loss is calculated by

$$\mathrm{TL} = 20\log\left(\left|\frac{P_{in}}{P_{out}}\right|\right). \tag{7}$$

The peak of the transmission loss is in good agreement between the FE model and the TMM model. While the transmission loss for the TMM model is flat the curve in the FE model is quite chaotic. The reason for that might be that additional losses are present in the FE model, that are not implemented by the simple TMM approach. Furthermore in the TMM the microphone positions don't play a role since only the resulting acoustic impedance is used in the acoustical domain. In the FE model the feedback signal incorporates the full spatial sound field that is present at the measurement nodes.

Discussion

We presented a way to design and evaluate active acoustic metamaterials. While the approach delivers a fairly good agreement with a FE model and it should give the engineer a toolset to plan the application of an active metamaterial there are some restrictions and downsides of the overall procedure.

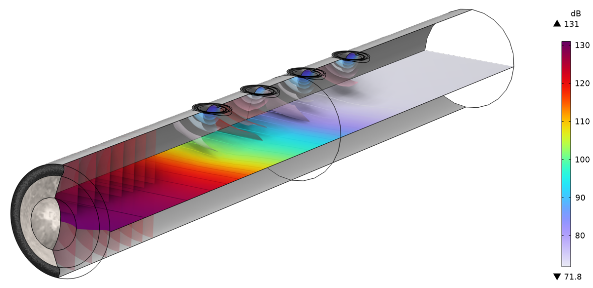

Figure 4: The simulation model in COMSOL. Here the acoustic pressure along the duct is visualized. The sound source is a loudspeaker with the same diameter as the duct.

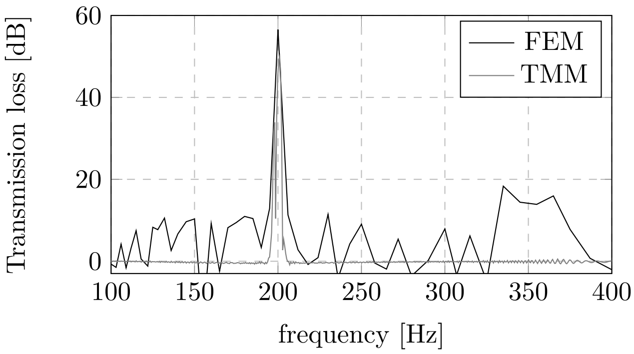

Figure 5: Simulation results of the transmission loss for four active unit cells. The black curve is the result of the FEM simulation, gray the result of the TMM simulation.

The first restriction is that the procedure does not take crosstalk between the unit cells into account. When keeping the acoustic impedance of the unit cells positive, the cross talk should be negligible. This corresponds to a fully decentralized control scheme whereas a central controller may yield better performance [8] - possibly at higher costs. Furthermore the presented methods only hold for long wavelengths, i.e. plane waves and one dimensional geometries – with this the results for grazing incident sound and high-order modes in the acoustical systems will be inaccurate.

Future work

With the presented simulation strategy different controllers and actuators can be further investigated. Especially the effect of nonlinear behaviour of the actuators should be considered. Furthermore the simulations need to be evaluated against measurements.
The simulation strategy can also be extended to cover two- or three dimensional geometries.
Also the relation between actuator dynamics and control effort to obtain a band gap can be further analysed. This might result in methods to optimize the actuator design for active unit cells.

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