

## Solutions to Quant Sectional Test # 8

Sol 1  $x=3$  and  $y=2$  eliminates option A ;  $x=5$  and  $y=3$  eliminates B ;  $x=17$  and  $y=3$  eliminates option D ;  $x=43$  and  $y=23$  eliminates option E. Hence option (c) is correct.

Alternative sol : Suppose  $x - y = 13$ . Now, let  $x$  and  $y$  be distinct prime numbers, both greater than 2. Then both  $x$  and  $y$  are odd numbers since the only even prime is 2. Hence,  $x = 2k + 1$ , and  $y = 2h + 1$ , for some positive integers  $k$  and  $h$ . And  $x - y = (2k + 1) - (2h + 1) = 2k - 2h = 2(k - h)$ . Hence,  $x - y$  is even. This contradicts the assumption that  $x - y = 13$ , an odd number. Hence,  $x$  and  $y$  cannot both be greater than 2. Next, suppose  $y = 2$ , then  $x - y = 13$  becomes  $x - 2 = 13$ . Solving yields  $x = 15$ . But 15 is not prime. Hence, there does not exist prime numbers  $x$  and  $y$  such that  $x - y = 13$ .

Sol2. Since the sum of negative numbers is negative,  $x + y$  is negative. Since the quotient of an even number of negative numbers is positive,  $y/x$  is positive. Hence, Column B is larger than Column A. Hence option (b) is correct.

Sol3. Suppose there were 8 people on the bus choice (c). Then after the first stop, there would be 4 people left on the bus. After the second stop, there would be 2 people left on the bus. After the third stop, there would be only one person left on the bus. Hence, on the third stop the next to last person would have exited the bus. The answer is (c).

Sol4. The length of  $x$  cannot be determined because there is no indication of the overlapping length of the rectangle to the left of  $x$ . If you can't determine  $x$ , you can't determine  $x + y$ . Hence option (d) is correct.

Sol5 As weight of 12 Adults = 20 children or 3 Adults = 5 children or 9 Adults = 15 children. Thus 3 Adults can board the elevator. Hence option (a) is correct.

Sol6. The range for excellent condition (white part of the bar) for Bob Feller's card extends from \$30 to \$100. Hence option (d) is correct.

Sol7. In September, a Mantle card realized 120% of its start-of-season price. The price range at the start of season for a near-mint Mantle card was \$400 to \$600. Therefore, increasing its range by 20% would indicate a new range of \$480 to \$720. Hence option (d) is correct.

Sol8. Roy Campanella had a greater range for near-mint condition (\$50 to \$100, or a range of \$50) compared to the range for excellent condition (\$20 to \$50, or \$30). Campanella is the only card whose near-mint-condition range exceeded its excellent-condition range. Hence option (e) is correct.

Sol9. An excellent-condition Willie Mays card had a top price of \$300 at the start of the season and realized 120% of its price at the end of the season. So the highest price for such a card was \$360. A near-mint-condition Hank Aaron card started the season with a top price of \$350 and reached 105% of its price in October, or \$367.50. So the difference in the highest price of these cards was \$7.50. Hence option (a) is correct.

Sol10. Now  $(x+1)(x+C) = x^2 + (1+C)x + C$ , on comparing with the given equation, we get  $C$  as 5, thus  $K$  is 6. Hence option (c) is correct.

Sol11. If  $AC = CE$  and  $AC \perp FE$ , then  $EACE$  and  $ECEF$  are right angles. If  $BD = CF$  and  $AE \perp BD$ , then all angles formed at points  $G$  and  $H$  are right angles. Because  $H$  is the midpoint of  $BD$ ,  $BH = HD$ , and therefore,  $CH$  is not only a median perpendicular to its opposite side, but also an angle bisector. Angles  $FCE$  and  $FCA$  are therefore each  $45^\circ$ . Similarly, angles  $CAE$  and  $EFC$  are also  $45^\circ$  each. So, there are three equal  $45^\circ - 45^\circ - 90^\circ$  triangles: triangles  $FGE$ ,  $CGE$ , and  $AGC$ . Flipping the shaded quadrangle  $ABHG$  into its equal space  $GHDE$  shows that two of the equal triangles are shaded, and one is not. Therefore, the ratio of shaded area to un-shaded area is 2 to 1.

Sol12.  $xy = \text{constant}$ . If  $x$  is increased by 50% i.e.  $3/2x$  then  $y$  should be  $2/3y$  to keep the product constant, a decrease of  $1/3y$  or 33.33%. Hence option (c) is correct.

Sol13. Circumference =  $2\pi \times 3.14 \times 5 = 31.4$  units. Side  $PQ = \sqrt{25 + 25} = 5\sqrt{2}$ . Thus perimeter of the square =  $4 \times 5 \times 1.4 = 28(\text{approx}) < 31.4$ . Hence option (a) is correct.

Sol14. I — Plugging in values for  $z$ : If  $z$  is 1,  $y$  would be  $2/3$  — not an integer and therefore not possible. If  $z$  is 2,  $y$  would be  $4/3$  — again, not possible.  $z$  cannot be 3, because 3 is divisible by 3. If  $z$  is 4,  $y$  would be  $8/3$  — not possible. You notice that because  $z$  cannot be a multiple of 3, all the values of  $y$  obtained by plugging in integers other than multiples of 3 are non-integers. So I,  $(2z)/3$ , cannot be a value of  $y$ . I is true.

II — If  $z$  is 1,  $y$  is  $2z + 1$ , or 3. If  $x + y = z$ , then  $x + 3 = 1$ , so  $x = -2$ , which is an even integer. This option,  $2z + 1$ , is a possible value of  $y$ . II is false.

III — If  $z$  is 4, then  $y$  is  $z/2$ , or 2. Because  $x + y = z$ ,  $x$  equals 2, which is an even integer. So  $z/2$  is a possible value of  $y$ . III is false. Hence option (a) is correct.

Sol15. Sum of first ' $n$ ' odd numbers is  $n^2$ , thus 625 is the required answer, hence option (a) is correct.

Sol16. In Column A, the  $x$  jars have  $15x$  marbles, and  $3x$  jars have  $20 \cdot 3x = 60x$  marbles. Hence, Column A has a total of  $15x + 60x = 75x$  marbles. Now, in Column B, the  $x$  jars have  $25x$  marbles, and  $2x$  jars have  $35 \cdot 2x = 70x$  marbles. Hence, Column B has a total of  $25x + 70x = 95x$  marbles. Thus, Column B is larger, hence option (b) is correct.

Sol17. Statement I could be true because  $-|0| = -(+0) = -(0) = 0$ . Statement II could be true because the right side of the equation is always negative [ $-|x| = -(a \text{ positive number}) = a \text{ negative number}$ ]. Now, if one side of an equation is always negative, then the other side must always be negative, otherwise the opposite sides of the equation would not be equal. Since Statement III is the opposite of Statement II, it must be false. But let's show this explicitly: Suppose  $x$  were positive. Then  $|x| = x$ , and the equation  $x = -x$  becomes  $x = -|x|$ . Dividing both sides of this equation by  $x$  yields  $1 = -1$ . This is contradiction. Hence,  $x$  cannot be positive. Hence option (d) is correct.

Sol18. Let 'abcd' be the number of the car, therefore telephone number will be 'dcba'.  $abcd + dcba = 16456$ . These are made up of only odd numbers. As  $d + a$  ends in 6, only two pairs (1,5) and (7,9) are possible. If we take (1,5) as the pair, then in the sum  $1bc5 + 5cb1 = 16456$ , we can not end up with 16 ( $1 + 5 + \text{carryover from } b+c$  can be max 1), therefore not possible. Thus  $7bc9 + 9cb7 = 16456$ . While performing addition, we see that  $1+b+c$  should be 5 or  $b + c = 4$ . Thus only of (1,3) is possible. Also if  $b=3$  then  $c=1$  or if  $b=1$ , then  $c=3$ . Hence option (d) is correct.

Sol19. If we see carefully, as all boxes are equi-spaced, we can easily apply pythagoras to find the distances,  $AB = \sqrt{(1+1)} = \sqrt{2}$ .  $BC = \sqrt{(9+16)} = 5$ ,  $CA = \sqrt{(25+4)} = \sqrt{29}$ . Hence option (a) is correct.

Sol20. The average speed at which car X traveled is  $\text{Total Distance} / 30$  and the average speed at which car Y traveled is  $\text{Total Distance} / 20$ . The two fractions have the same numerators, and the denominator for car Y is smaller. Hence, the average miles per hour at which car Y traveled is greater than the average miles per hour at which car X traveled. Hence option (b) is correct.

