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DAA Assignment

Q.1 using master theorem

Ans. (A) $T(n) = 9T(n/3) + n$

$\log_3 9 = 2$, $K = 1$, $P = 0$

$\log_b a > K$

$\therefore$ complexity of function is $O(n^2)$

(B) $T(n) = 9T\left(\frac{2n}{3}\right) + 1$

$\log_{3/2} 9 = 5.9675$, $K = 0$, $P = 0$

$\log_b a > K$

$\therefore O(n^{5.9675})$

$\approx O(n^6)$

$\therefore$ complexity of function is : $O(n^6)$

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→ using iterative method :

$$
\begin{aligned}
(A)\ T(n) &= 9T(n/3) + n \\
&= 9[9T(n/3)(3) + n/3] + n \\
&= 81T(n/9) + 3n + n \\
&= 3^4 T(n/3^2) + 3n + n \\
&= 3^4[9T((n/3^2)/3) + n/3^2] + 3n + n \\
&= 3^4[3^2 T(n/3^3) + n/3^2] + 3n + n \\
&= 3^6 T(n/3^3) + 3^2 n + 3n + n \\
&= (3^2)^2 T(n/3^3) + 3^2 n + 3^1 n + n \\
&= (3^k)^2 T(n/3^k) + (3^{k-1} n + 3^{k-2} n + \dots) \\
&= (3^k)^2 T(n/3^k) + (3^{k-1} n + 3^{k-2} n + \dots + 3^1 n)
\end{aligned}
$$

Assume that $\frac{n}{3^k} = 1$, (or $3^k = n$)

$\therefore 3^k = n$

$\Rightarrow k \log 3 = \log n$

or $k = \log_3^n$

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$\therefore T(n) = (3^k)^2 T(n/3^k) + n[3^{k-1} + \ldots + 3^1]$

$= n^2 T(1) + n[3^{\log n - 1} + 3^{\log n - 2} + \ldots + 3]$

$= n^2 T(1) + n\, O(3^{\log n - 1})$ $[\because T(1) = 1]$

$= n^2 + n\,O(n)$

$= O(n^2)$

Q.2. $T(n) = T(n-1) + T(n-2) \rightarrow (1)$

$T(n-1) = T(n-2) + T(n-3)$

$T(n-2) = T(n-3) + T(n-4)$

$\therefore T(n) = T(n-2) + T(n-3) + T(n-3) + T(n-4)$

$= T(n-2) + 2T(n-3) + T(n-4)$

$= T(n-3) + T(n-4) + 2T(n-3) + T(n-4)$

$\therefore T(n) = 3T(n-3) + 2T(n-4) \rightarrow (2)$

$T(n-3) = T(n-4) + T(n-5)$

$T(n-4) = T(n-5) + T(n-6)$

$\therefore T(n-3) = 2T(n-5) + T(n-6)$

$\therefore T(n) = 8T(n-5) + 5T(n-6) \rightarrow (3)$

$\therefore T(n-5) = T(n-6) + T(n-7)$

$T(n-6) = T(n-7) + T(n-8)$

$T(n-5) = 2T(n-7) + T(n-8)$

$\therefore T(n) = 21T(n-7) + 13T(n-8) \rightarrow (4)$

$\rightarrow \therefore T(n) = 1T(n-1) + 1T(n-2) +$
$+ 2T(n-4) + 3T(n-3) + 5T(n-6)$
$+ 8T(n-5) + \ldots$

The above function is of fibonaci series

$\therefore$ complexity of function is $O(2^n)$

Q.3. $T(n) = T(n/3) + T(2n/3) + n$

n — n time

$n/3 \quad 2n/3$ → n time

$n/3^2 \quad 2n/3^2 \quad 2n/3^2 \quad 2^2 n/3^2$ → n times

$n/3^3 \quad 2n/3^3 \quad \frac{2n}{3^3} \quad \frac{2^2 n}{3^3}$

⋮

$\frac{n}{3^k}, \frac{2^k n}{3^k} = \frac{n}{(3/2)^k}$ → n time

upto k steps

i.e. total kn.

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$$n \rightarrow \frac{2n}{3^2} \rightarrow \frac{2^2 n}{3^2} \rightarrow \frac{2^3 n}{3^3} \cdots \rightarrow \frac{2^k n}{3^k}$$

$$= \frac{n}{(3/2)^k}$$

$$\therefore \frac{n}{(3/2)^k} = 1$$

$$\Rightarrow n = (3/2)^k$$

$$\therefore k = \log_{\frac{3}{2}} n$$

$\therefore$ complexity of function $= O(n \log n)$.

Q.4. $T(n) = T(n-1) + n$ using Substitution and iteration methods.

Ans Substitution method:

$$T(n) = T(n-1) + n \rightarrow (1)$$
$$T(n-1) = T(n-2) + n - 1.$$

$$\therefore T(n) = T(n-2) + n - 1 + n$$
$$= T(n-2) + 2n - 1$$
$$= T(n-3) + 3n - 2$$

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$\therefore T(n) = T(n-k) + k_n - (k-1)$
↳ const

Let $n - k = 1$
$\Rightarrow k = n-1$

$\therefore T(n-(n-1)) + n(n-1) \Rightarrow$
$T(1) = 1$

$= T(1) + n^2 - 1$
$= 1 + n^2 - 1$
$= n^2$

$\therefore$ complexity is $O(n^2)$

→ Iteration method:

$T(n) = T(n-1) + n$

Let,

$T(1) = 1$
$T(2) = T(2-1) + 2$
$= T(1) + 2$
$= 1 + 2$

$T(3) = T(2) + 3$
$= 1 + 2 + 3$

$T(4) = T(3) + 4$
$= 1 + 2 + 3 + 4$

$T(n) = 1+2+3+4+\dots(n-1)+n$

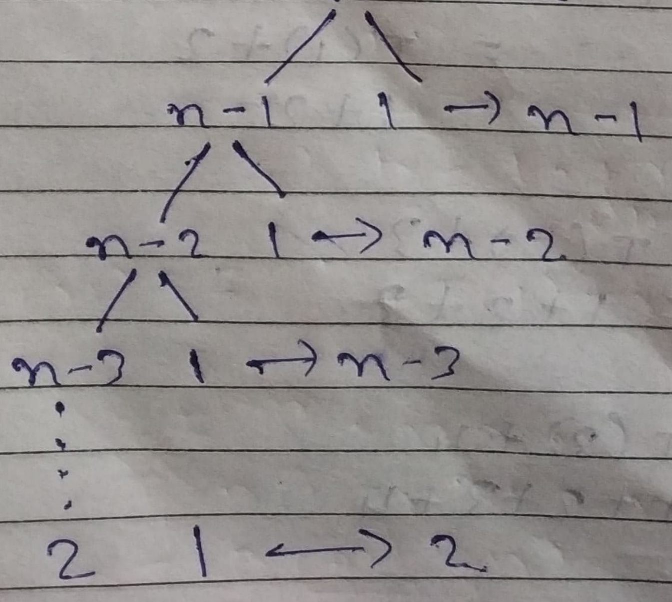
$\Rightarrow T(n) = \dfrac{n(n+1)}{2} = \dfrac{n^2}{2} + \dfrac{n}{2}$

$\therefore$ complexity is $O(n^2)$

Q.5

Ans

```
T(n) <- fun factorial (n)
        {
            if (n==1)
    1 <-    return 1;
            else
T(n-1) <-  return n*factorial
                     (n-1)
        }
```



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$\therefore T(n) = T(n-1) + 1$

$T(n) = (n + (n-1) + (n-2) + \dots + 2) + 1$

$= 1 + 2 + \dots + (n-1) n$

$= n \frac{(n+1)}{2}$

$= \frac{n^2}{2} + \frac{n}{2}$

$= O(n^2)$

Q.6. $T(n) = 5T(n/2) + n$

Reversion tree

$n \rightarrow n$ times

$5n/4 \quad n \rightarrow n$ times

$5n/4^2 \quad n/4 \rightarrow n$ times

$5n/4^3 \quad n/4^2 \rightarrow n$ times

$\frac{n}{4^k}$ (1) upto k
that is nk time

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$\therefore \frac{n}{4^k} = 1$

$\therefore n = 4^k$

$\Rightarrow k = \log_4 n \quad \therefore O(n^k)$

$\therefore$ complexity is $O(n\log n)$

→ Iteration method:

$$T(n) = 5T(n/4) + n$$
$$= 5\left[5T\left(\frac{n}{4^2}\right) + n\right] + n$$
$$= 5^2 T(n/4^2) + 5n + 5$$
$$= 5^3 T(n/4^3) + 5^2 n + 5n + 5$$
$$= 5^k T(n/4^k) + n[5^{k-1} + 5^{k-2} + \dots + 5]$$

Assume, $\frac{n}{4^k} = 1$

$\Rightarrow k = \log_4 n$

$$T(n) = 5^{\log n} + T(1) + n[5^{\log n - 1} + 5^{\log n - 2} + \dots + 5]$$
$$= 5^{\log n} + n\,O(n) \quad (T(1) = 1)$$
$$= O(\log n) + O(n)$$
$$= O(n\log n)$$

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Q.7 $T(n) = 49T(n/25) + n^{3/2}\log n$

Ans. using master theorem:

$\log_{25} 49 = 1.209$, $k = 3/2$, $P = 1$

$\therefore \log_b a < k$; $P > 0$

$\therefore O(n^k \log n^P)$

$\approx O(n^{3/2} \log n^1)$

$\approx O(n^2 \log n)$.

Q.8 work case analysis of quick sort recurrance relation.

Ans. $T(n) = T(n-1) + n$ [$\because T(0) = T(1) = 0$].

$\therefore T(n) = T(n-1) + n$

$T(n-1) = T(n-2) + n-1$

$T(n-2) = T(n-3) + n-2$

$\therefore T(n) = n + (n-1) + (n-2) + \ldots + 3 + 2$

$\approx \frac{n^2}{2}$

$\therefore$ complexity is $O(n^2)$.

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