MONASH University

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Functional time series

with applications in demography

5. Forecasting functional time series via PLS

Outline

Functional PLS

- PCA components are designed to explain historical variation. They do not necessarily provide the best predictors.
- Partial least squares extracts uncorrelated latent components cores by maximizing the covariance between predictors and response.
- Response is $s_t(x)$ and predictor is $s_{t-1}(x)$.
- We want to predict $s_t(x)$ using
- This is a functional ARH(1).
- How to choose $b(x,u)$?

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Functional weighted PLS

Outer relationship

$$\hat{\boldsymbol{f}}^*(x) = \sum_{k=1}^{\infty} \beta_k \psi_k(x), \quad \text{and} \quad \hat{\boldsymbol{g}}^*(x) = \sum_{k=1}^{\infty} \beta_k \phi_k(x)$$

- $\boldsymbol{W} = \text{diagonal}(w_1, \dots, w_T), \quad w_t = \kappa(1-\kappa)^{T-t}$
- $\boldsymbol{f}^*(x) = \boldsymbol{W}[s_1^*(x), \dots, s_{T-1}^*(x)]'$ and $\boldsymbol{g}^*(x) = \boldsymbol{W}[s_2^*(x), \dots, s_T^*(x)]'$ are weighted decentralized functional predictors and responses
- β_k denotes common latent component scores
- $\psi_k(x)$ and $\phi_k(x)$ are latent components of predictors and responses respectively

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$$\beta_k = \int \boldsymbol{f}^*(x) w_k(x) dx = \int \boldsymbol{g}^*(x) m_k(x)\, dx,$$

Compute β_k iteratively:

1. Let $\boldsymbol{f}_0^*(x) = \boldsymbol{f}^*(x)$ and $\boldsymbol{g}_0^*(x) = \boldsymbol{g}^*(x)$
2. Obtain $w_k(x)$ iteratively, starting with $w_k^{(0)}(x) = 1$:
   $w_k^{(i)}(x) = \iint w_k^{(i-1)}(v)[\hat{\boldsymbol{f}}_{k-1}^*(v)]' \hat{\boldsymbol{g}}_{k-1}^*(u)[\hat{\boldsymbol{g}}_{k-1}^*(u)]' \hat{\boldsymbol{f}}_{k-1}^*(x)\, dv\, du$
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Computationally equivalent approach

- Discretize $s_t^*(x)$ on a dense grid of q equally spaced points.
- Denote discretized $s_t^*(x)$ as $T \times q$ matrix $\boldsymbol{G}^*$ and let $\boldsymbol{G} = \boldsymbol{W}\boldsymbol{G}^*$.
- Define $\boldsymbol{G}_{k-1}$ and $\boldsymbol{F}_{k-1}$ analogously
- $w_1(x) =$ largest eigenvector of $\boldsymbol{G}'\boldsymbol{F}$
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Functional autoregression coefficient:

$$b(x,u) = \sum_{k=1}^{\infty} w_k(u)\phi_k(x)$$

By orthogonality of β_k:

$$\phi_k(x) = (\beta_k'\beta_k)^{-1}\beta_k'\hat{\boldsymbol{g}}^*(x).$$

Therefore

$$\hat{b}(x,u) = \sum_{k=1}^{K} w_k(u)(\beta_k'\beta_k)^{-1}\beta_k'\hat{\boldsymbol{g}}^*(x),$$

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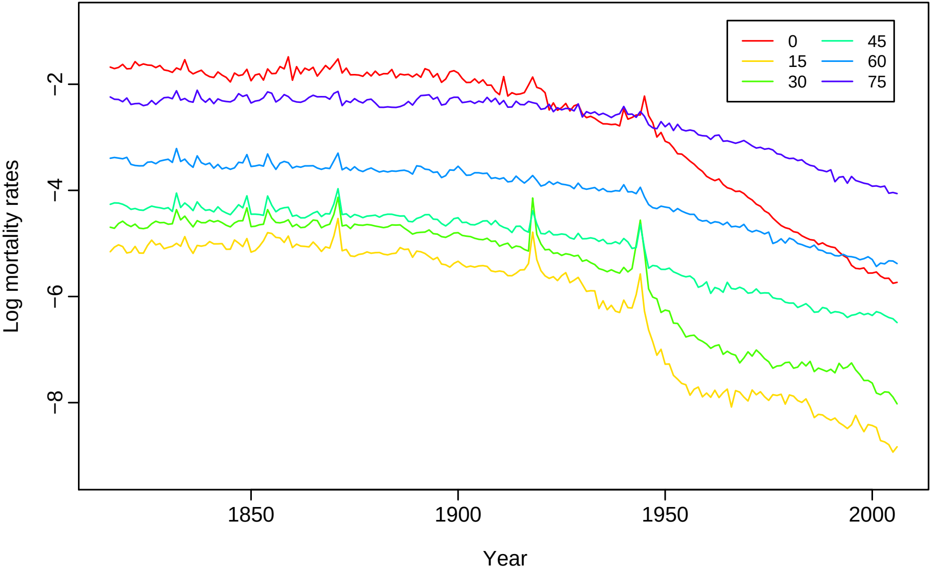
Forecasted curves:

$$\hat{f}_{t+1|t}(x) = \hat{\mu}(x) + \int [\hat{f}_t(u) - \hat{\mu}(u)]\, \hat{b}(x,u)\, du.$$

For $\hat{f}_{t+h|t}(x)$ where $h > 1$, apply iteratively.

Outline

French female mortality rates



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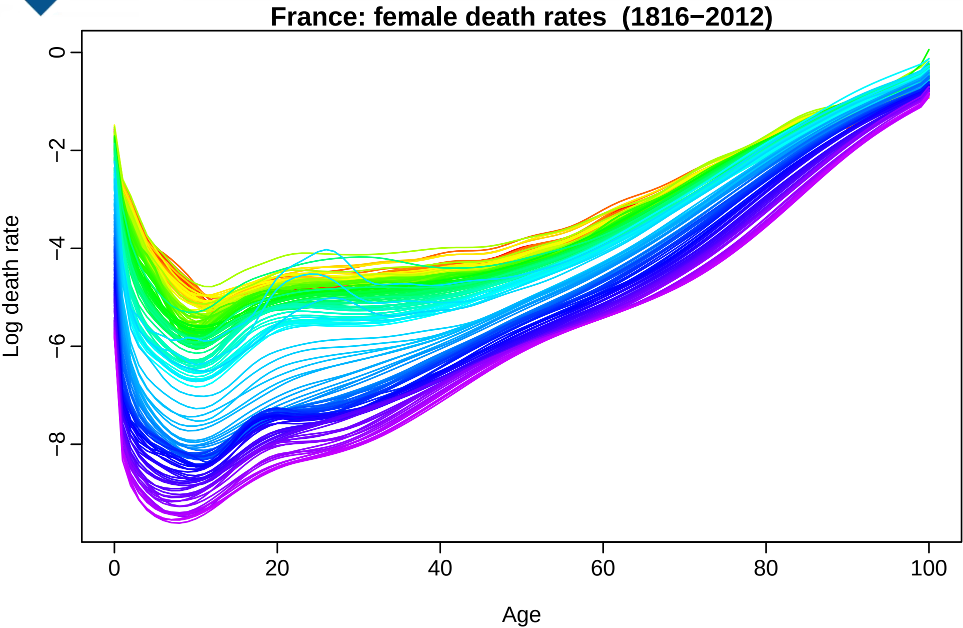
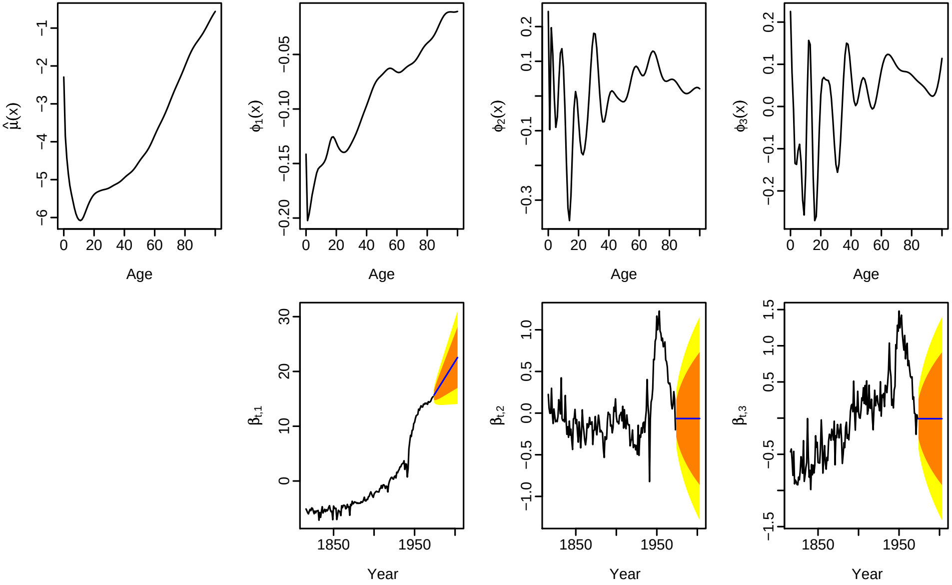


French mortality rate models



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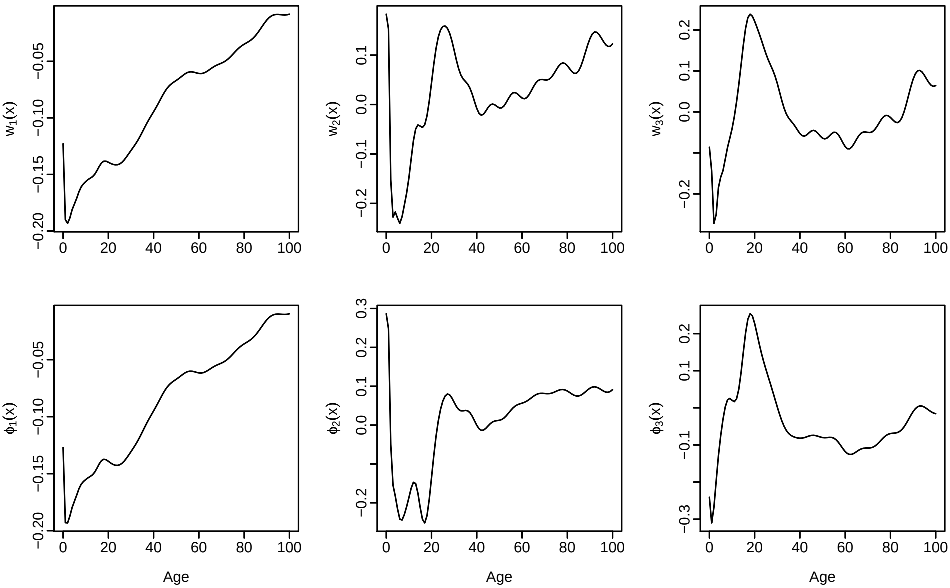

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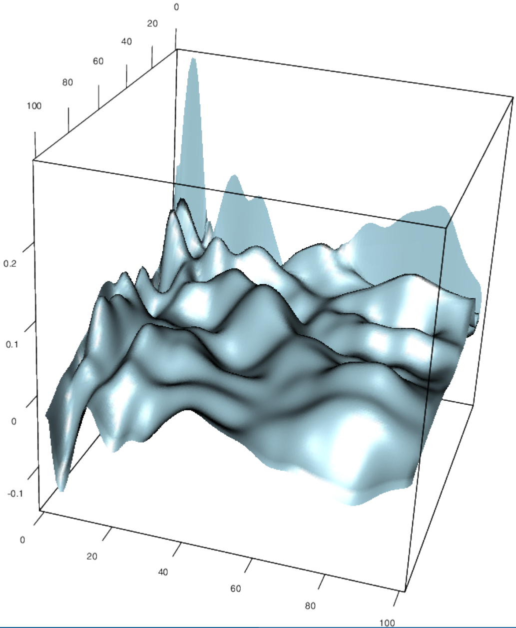
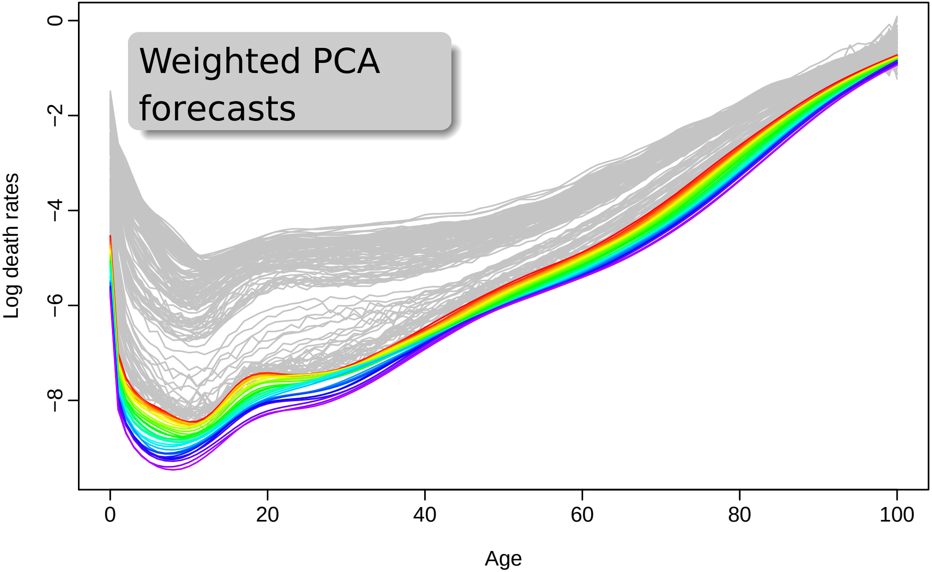


French mortality rate forecasts



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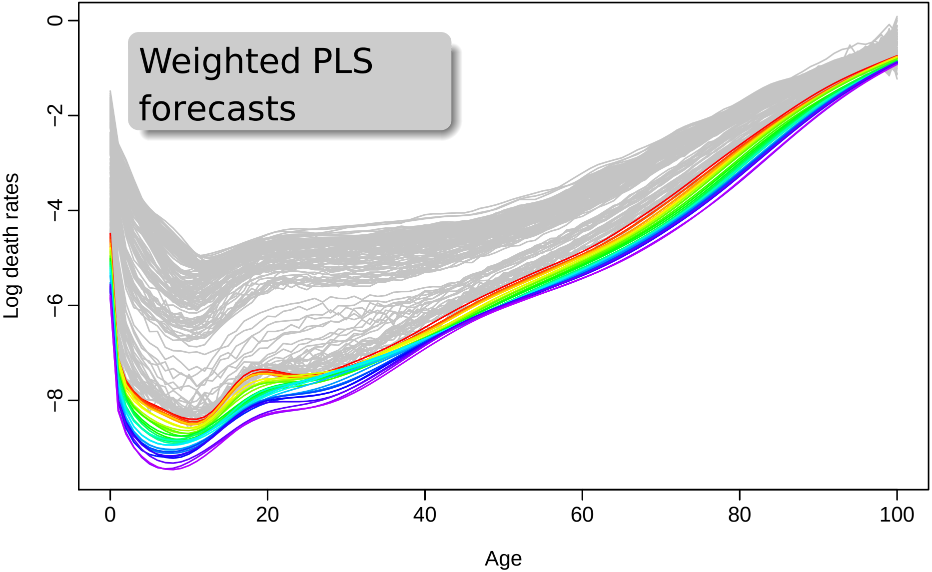

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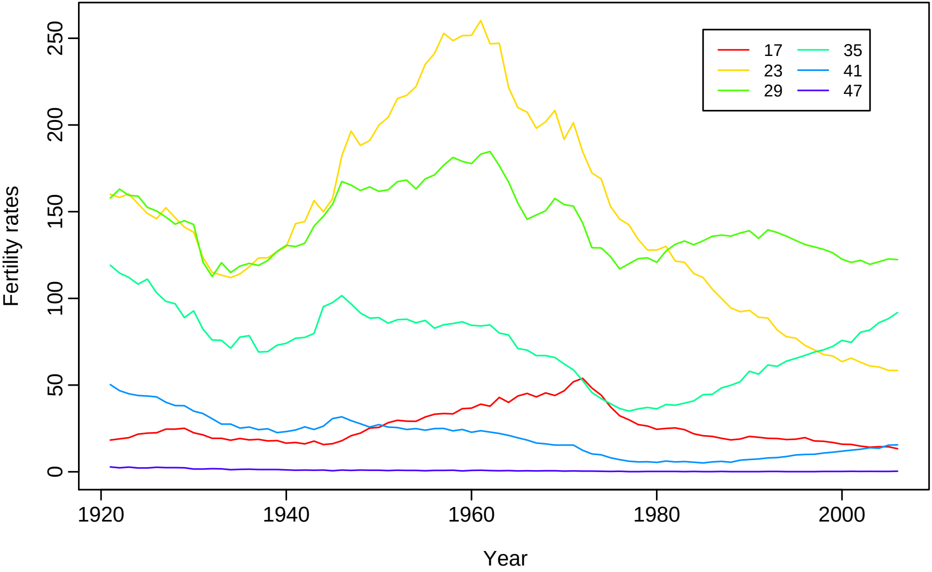
Australian fertility rates



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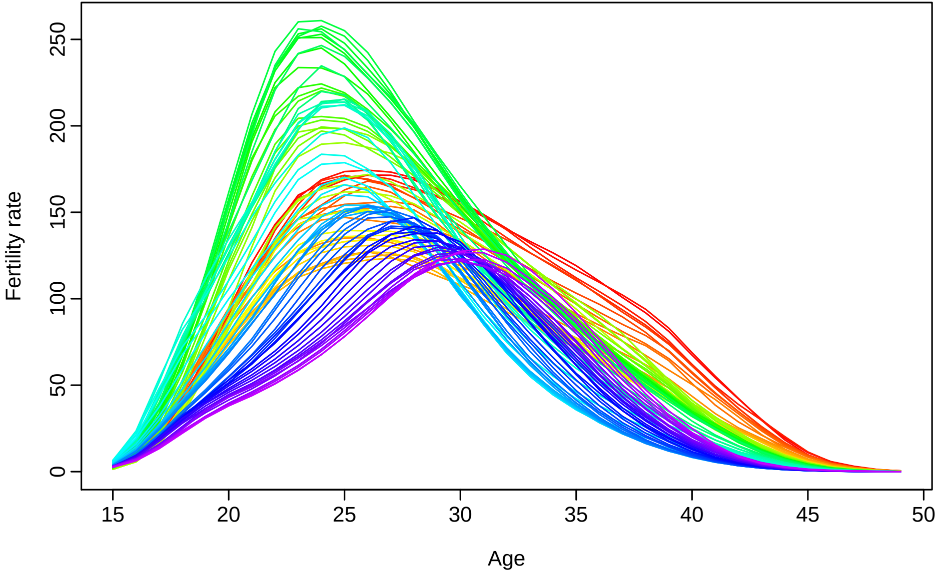

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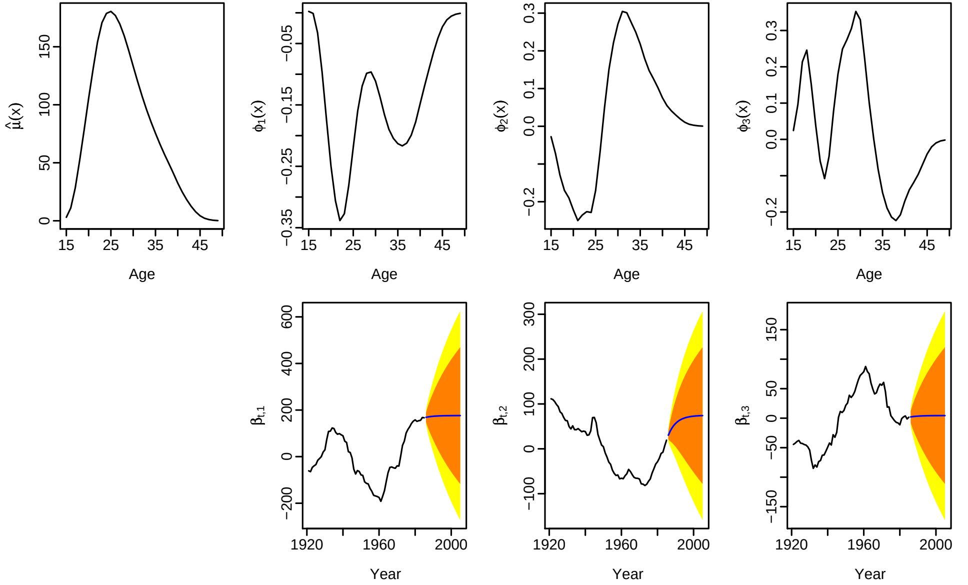

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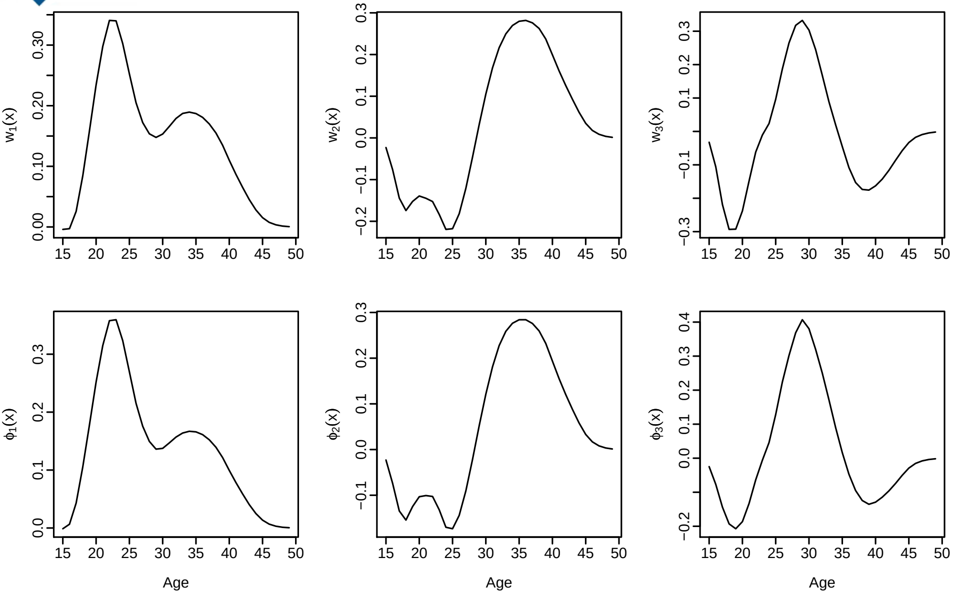

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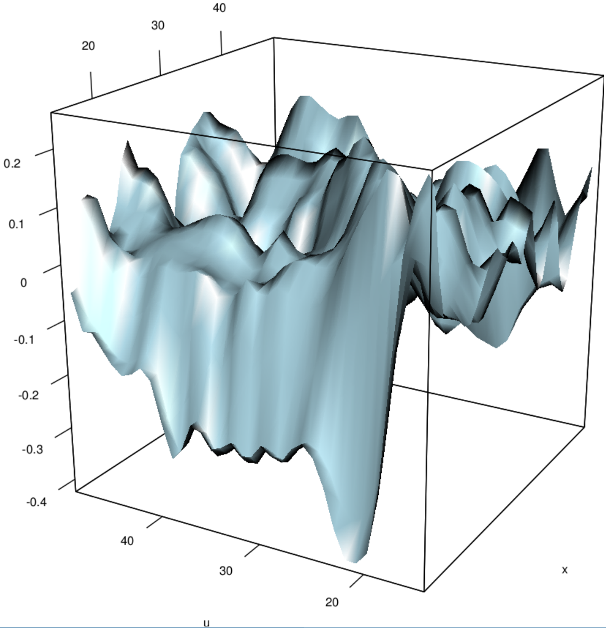

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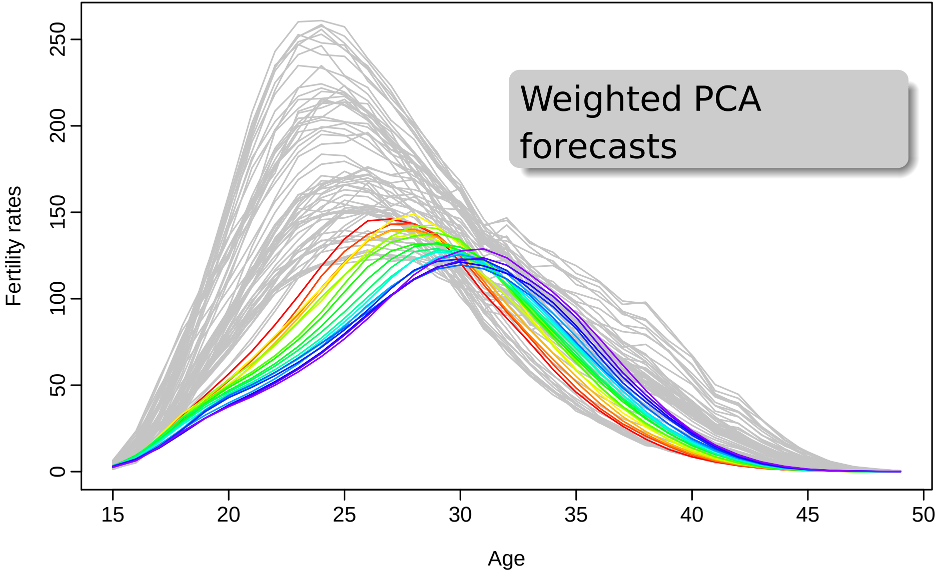

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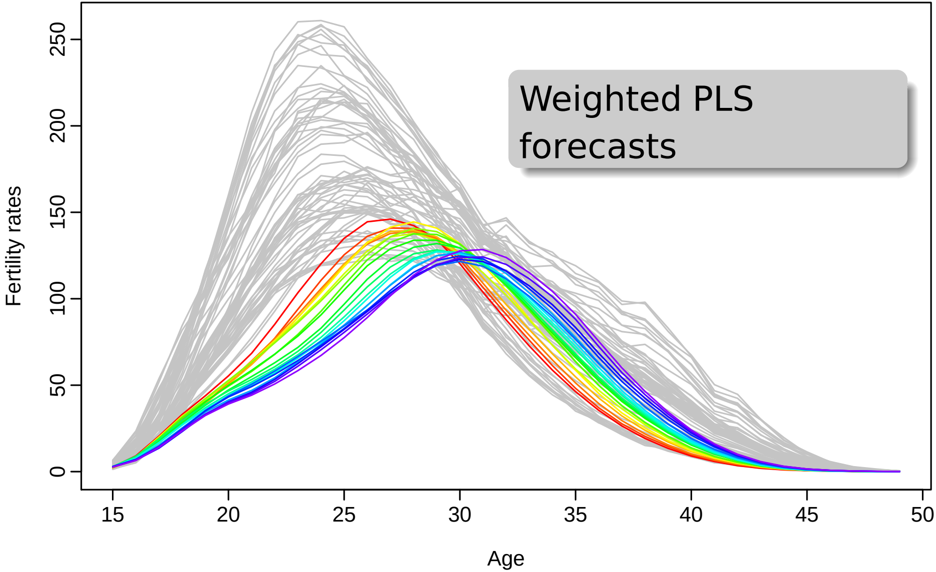

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Forecast accuracy comparisons

$$\text{MSE}_t = \frac{1}{p}\sum_{i=1}^{p}\left[y_t(x_i) - \hat{y}_{t|t-1}(x_i)\right]^2.$$

- Averaged over last m years of observed data.
- For French female mortality, $m = 30$
- For Australian fertility, $m = 20$
- For comparison, compare random walk: $y_{t+1|t}(x) = y_t(x)$.

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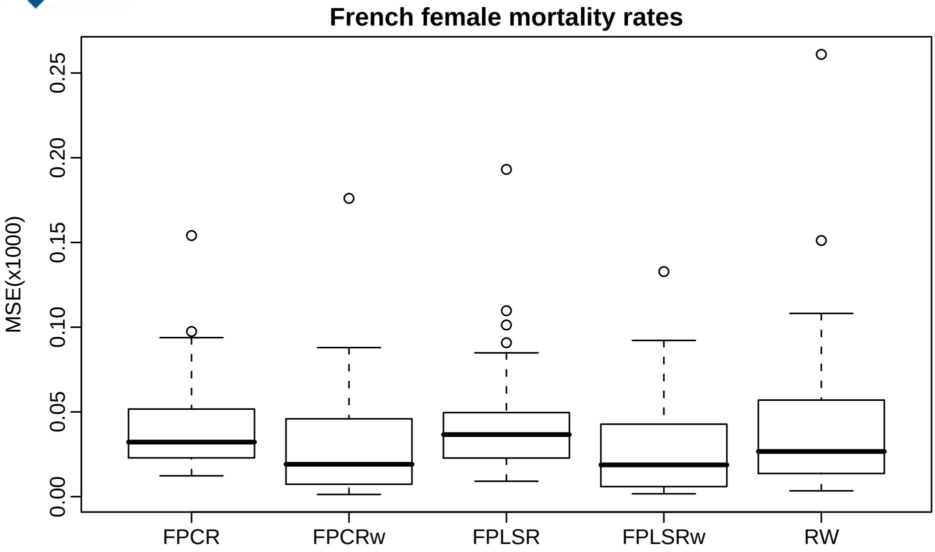

Forecast accuracy comparisons

MSE: French female mortality rates ($x1000$)

K	FPC	FPC$_w$	FPLSR	FPLSR$_w$	RW
1	0.5956	0.0293	0.5994	0.0607	
2	0.0537	0.0310	0.0738	0.0288	
3	0.0316	0.0310	0.0445	0.0288	
4	0.0296	0.0311	0.0428	0.0288	
5	0.0287	0.0311	0.0472	0.0297	
6	0.0425	0.0311	0.0474	0.0291	0.0437

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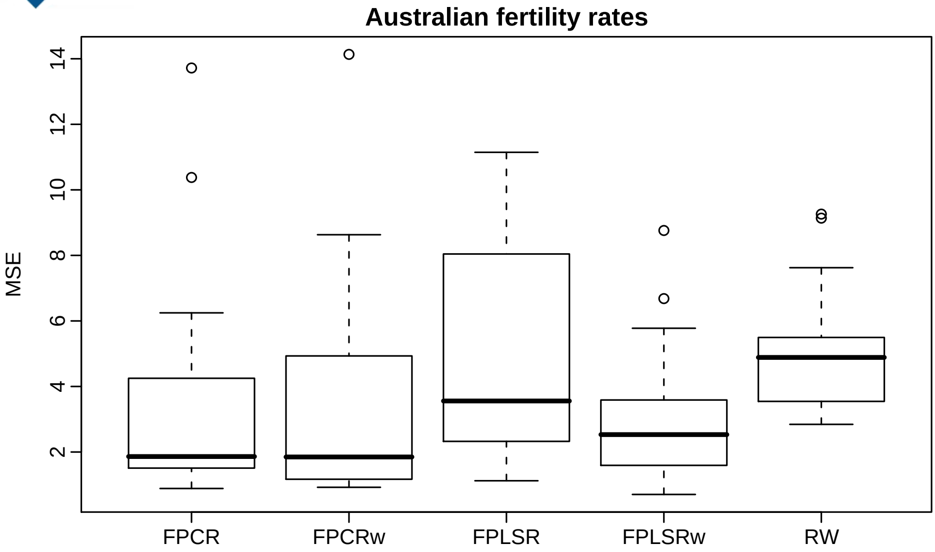

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MSE: Australian fertility rates

K	FPC	FPC$_w$	FPLSR	FPLSR$_w$	RW
1	99.0611	16.7304	94.0311	53.8186	
2	56.3095	3.3019	54.3410	17.5883	
3	24.9330	3.2580	26.0428	10.2599	
4	15.6845	3.1995	19.7227	4.4818	
5	4.4495	3.2132	5.9299	4.0573	
6	3.4310	3.2123	4.9205	2.9046	4.9800

Computation time

- Weighted FPLSR more efficient than weighted FPC as FPC requires many univariate time series models.
- **Time to fit 100 replications:**

Method	Fertility data	Mortality data
FPC	34.1072	62.2797
FPC_w	33.1424	60.8426
FPLSR	0.4287	2.9184
$FPLSR_w$	0.4537	3.1602
RW	0.0000	0.0002

(Intel Xeon 2.33GHz processor)

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Outline

Sources of uncertainty

Functional PCA

1. smoothing error in estimating $s_t(x)$
2. error in estimating $\mu(x)$
3. error in forecasting the scores $\beta_{t,k}$
4. error in the model residuals $e_t(x)$
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Sources of uncertainty

Functional PLS

1. smoothing error in estimating $s_t(x)$
2. error in estimating $\mu(x)$
3. error in estimating $b(x, u)$
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Bootstrap curves for FPC

- Let $\hat{\xi}_{t,k} = \hat{\beta}_{t,k} - \hat{\beta}_{t|t-1,k}$ be 1-step errors of PC scores.
- $\{\xi_k^{(\ell)}\}$ sampled with replacement from $\{\hat{\xi}_{t,k}\}$.
- Simulate future sample paths of $\beta_{T+h|T,k}$ using these bootstrapped residuals: $\{\beta_{T+h|T,k}^{(\ell)}\}$.
- $\{e^{(\ell)}(x)\}$ sampled with replacement from residual functions $\{\hat{e}_1(x), \dots, \hat{e}_T(x)\}$.
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- Prediction intervals are produced from the bootstrap variants using percentiles.

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Bootstrap curves for FPLS

- Resample residuals and construct bootstrap samples:

$$\boldsymbol{f}^{(\ell)}(x) = \hat{\mu}(x) + \hat{\boldsymbol{f}}^{*}(x) + \boldsymbol{e}^{(\ell)}(x),$$
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- Construct weighted FPLSR model for each bootstrap sample.
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Coverage probability

Empirical coverage probability of 95% intervals

$$\frac{1}{mph}\sum_{t=T-m+1}^{T}\sum_{j=1}^{h}\sum_{i=1}^{p}1\left(\hat{y}_{t+j|t}^{(0.025)}(x_i)<y_{t+j}(x_i)<\hat{y}_{t+j|t}^{(0.975)}(x_i)\right)$$

- $\hat{y}_{t+j|t}^{(\alpha)}(x_i)=\alpha$-quantile from bootstrap samples
- $m=$ smallest number of observations used to fit a model

Method	Fertility data	Mortality data
FPC	98.00%	97.19%
FPLS	96.86%	97.23%

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Adjusted prediction intervals

- Compute $d(x) =$ difference between $(1-\alpha/2)$ and $(\alpha/2)$ quantiles of $\{\hat{f}_{t+1}(x) - \hat{f}_{t+1|t}(x);\ t = m, \dots, T-1\}$
- Let $[\ell_h(x), u_h(x)]$ be $100(1-\alpha)\%$ h-step-ahead prediction intervals obtained from bootstrap methods.
- Ideally, $u_1(x) - \ell_1(x) = d(x)$.

Adjusted prediction interval

$$\Big[0.5\{\ell_h(x)+u_h(x)\} - \{u_h(x)-\ell_h(x)\}p(x),\quad 0.5\{\ell_h(x)+u_h(x)\} + \{u_h(x)-\ell_h(x)\}p(x)\Big]$$

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Adjusted prediction intervals

	Fertility data		Mortality data	
	95%	adj 95%	95%	adj 95%
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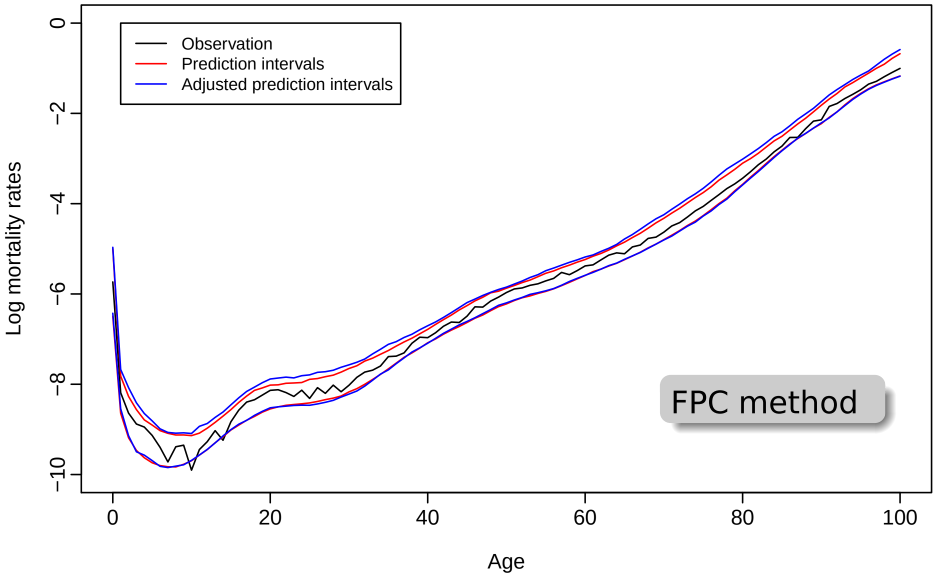

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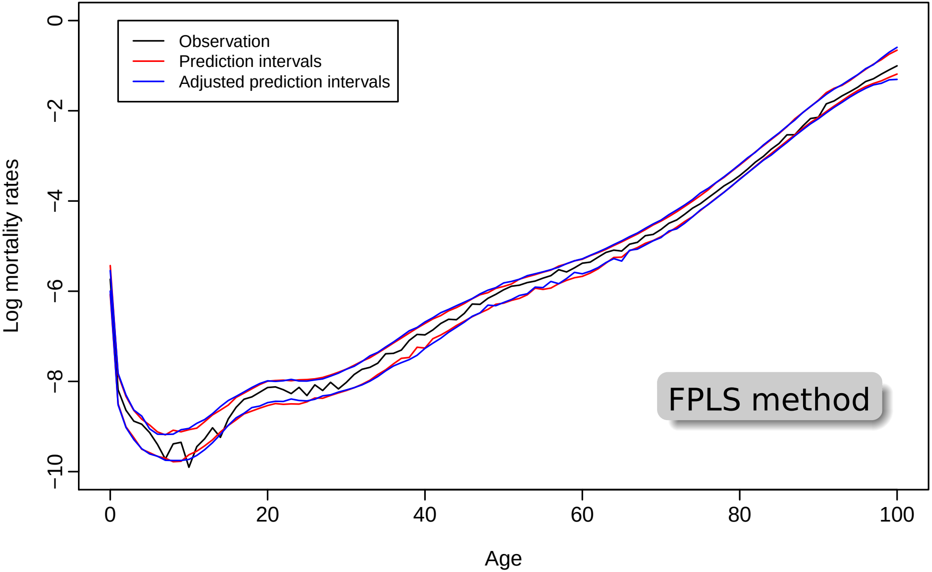

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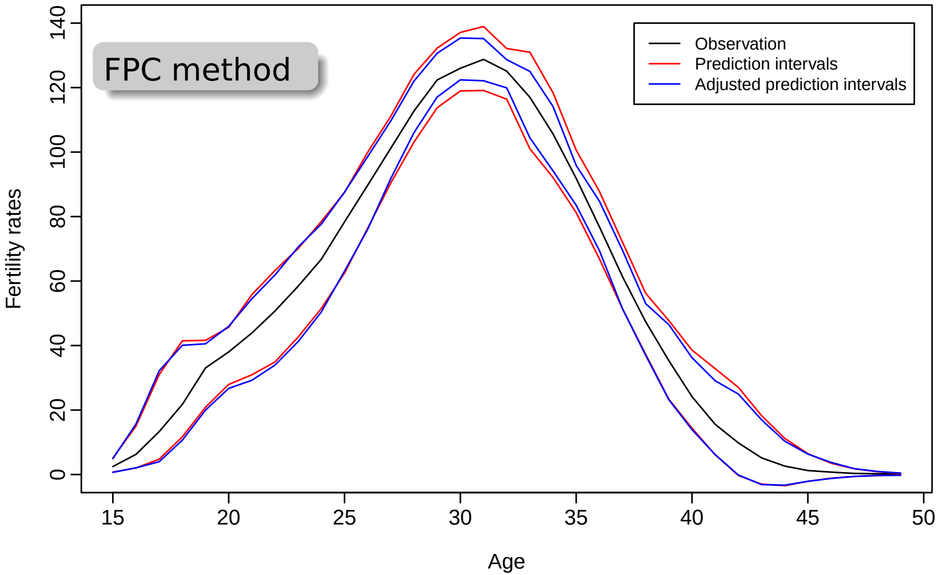

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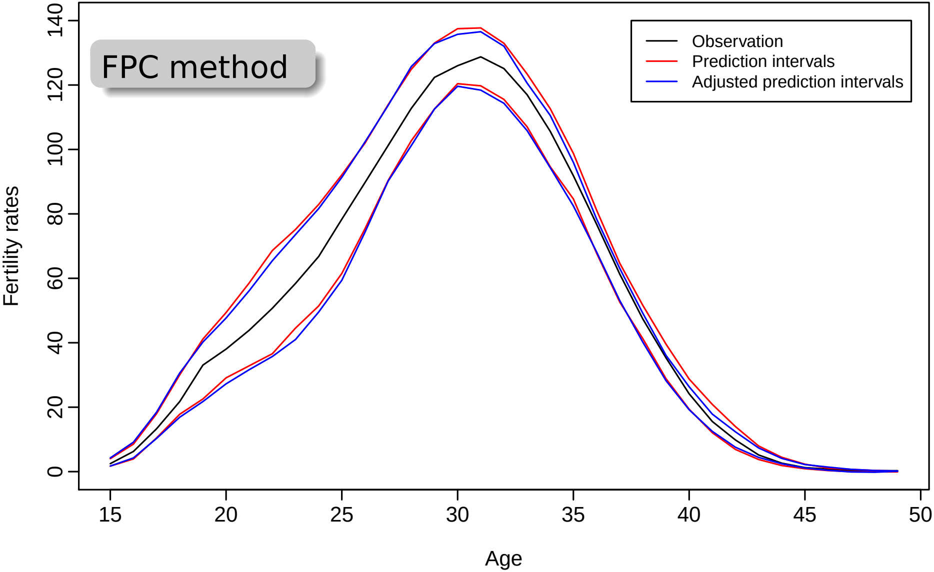

Outline

Comparisons

FPC advantages

1. allows more complex dynamics and higher order components;
2. eases interpretability of dynamic changes by separating out effects of a few orthogonal components;

FPLS advantages

1. latent components more suitable for prediction rather than variance decomposition.

Both methods implemented in `ftsa` package for R.

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Selected references

Hyndman, Shang (2009). “Forecasting functional time series (with discussion)”. *Journal of the Korean Statistical Society* **38**(3), 199–221.

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