MONASH University

Rob J Hyndman

Forecasting:

Principles and Practice

2. The forecaster's toolbox

OTexts.com/fpp/2/

Outline

1 **Time series graphics**

2 Seasonal or cyclic?

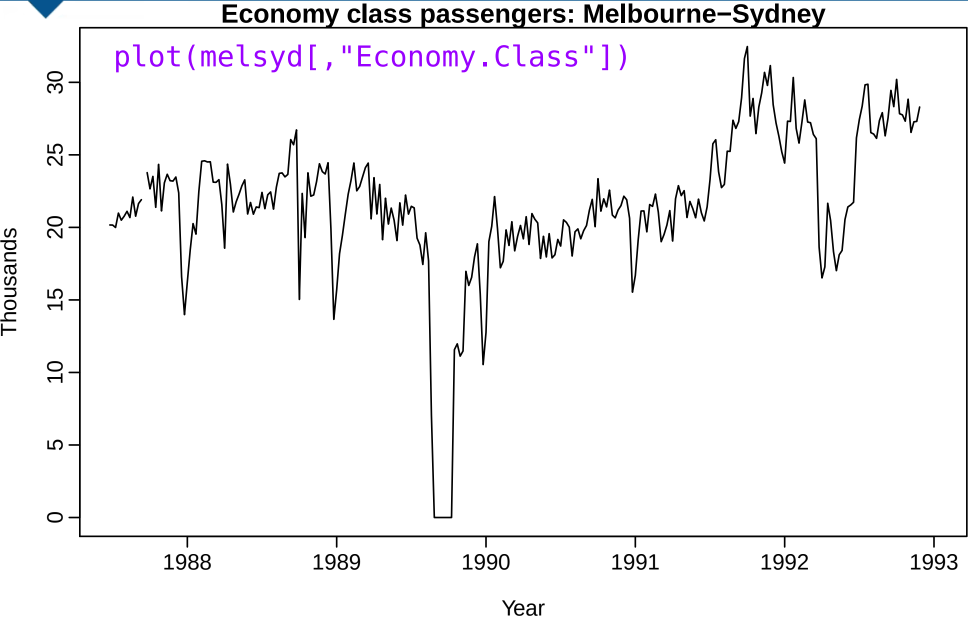
3 Autocorrelation

4 Forecast residuals

5 White noise

6 Evaluating forecast accuracy

Time series graphics



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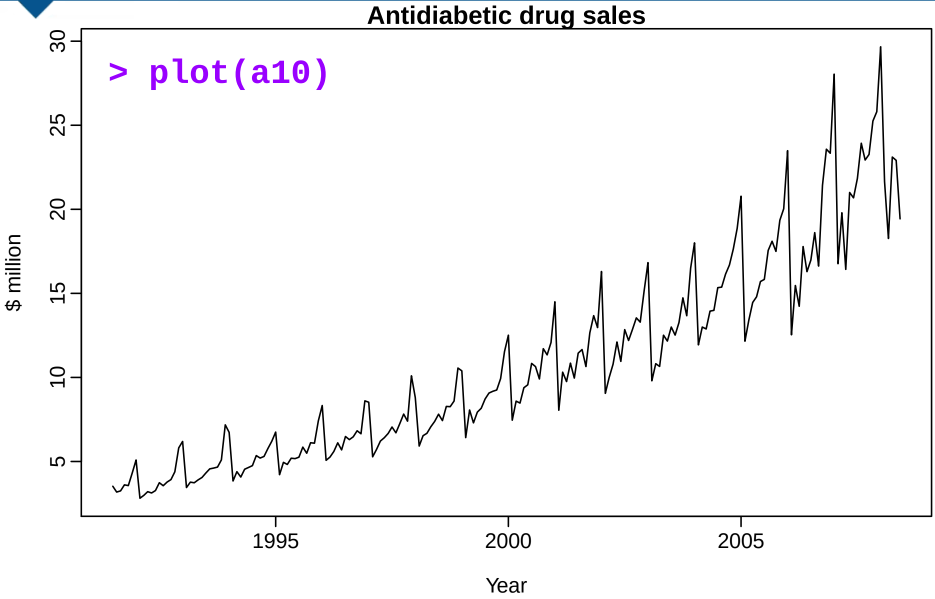

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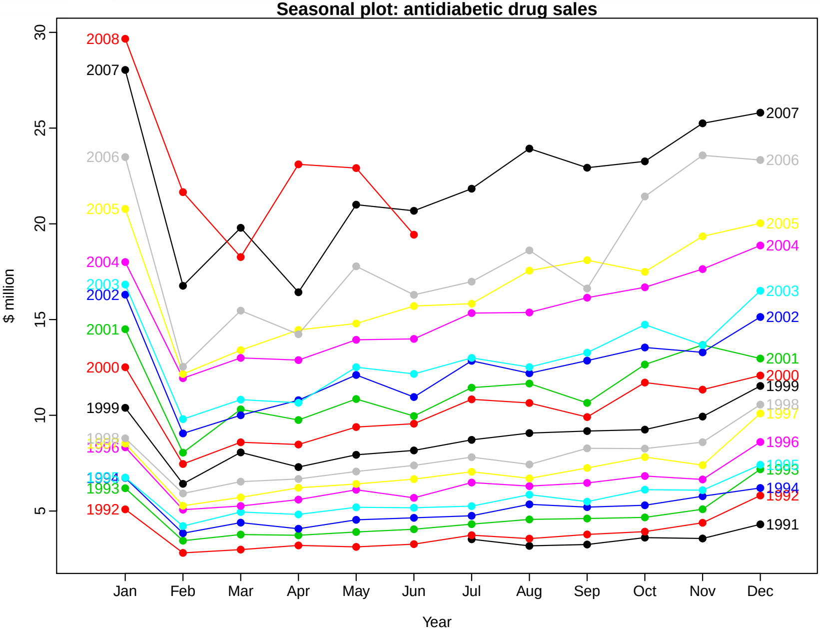

Seasonal plots

- Data plotted against the individual “seasons” in which the data were observed. (In this case a “season” is a month.)
- Something like a time plot except that the data from each season are overlapped.
- Enables the underlying seasonal pattern to be seen more clearly, and also allows any substantial departures from the seasonal pattern to be easily identified.
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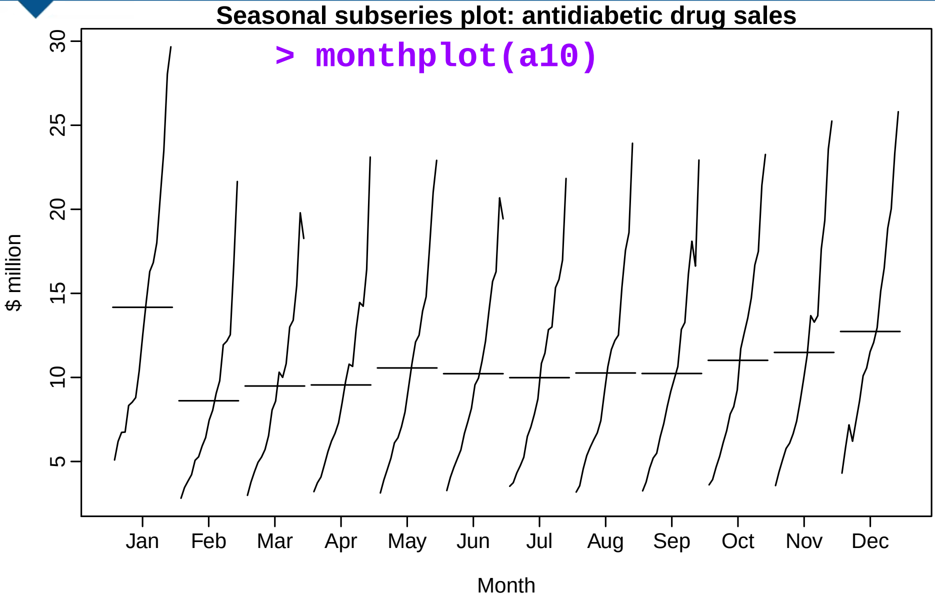

Seasonal subseries plots

- Data for each season collected together in time plot as separate time series.
- Enables the underlying seasonal pattern to be seen clearly, and changes in seasonality over time to be visualized.
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Quarterly Australian Beer Production

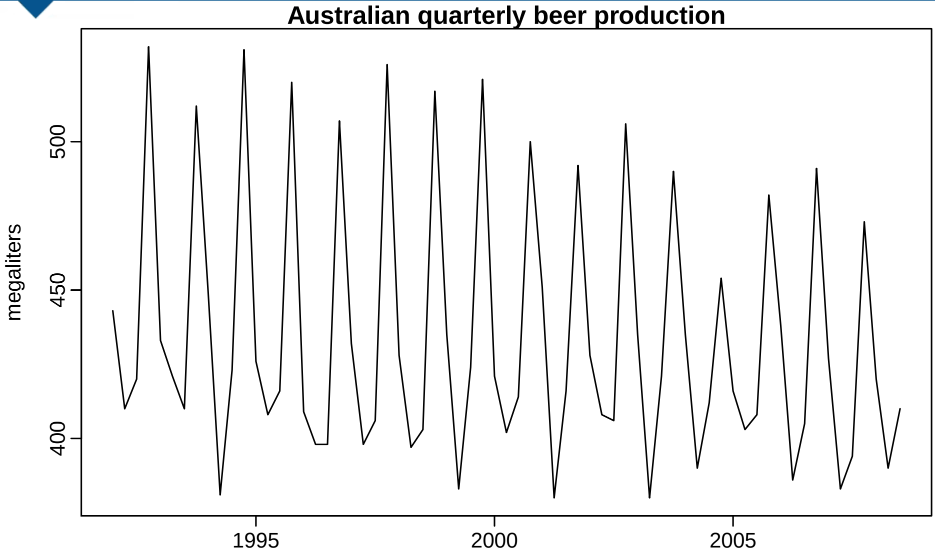
```
beer <- window(ausbeer,start=1992)

plot(beer)

seasonplot(beer,year.labels=TRUE)

monthplot(beer)
```

Time series graphics



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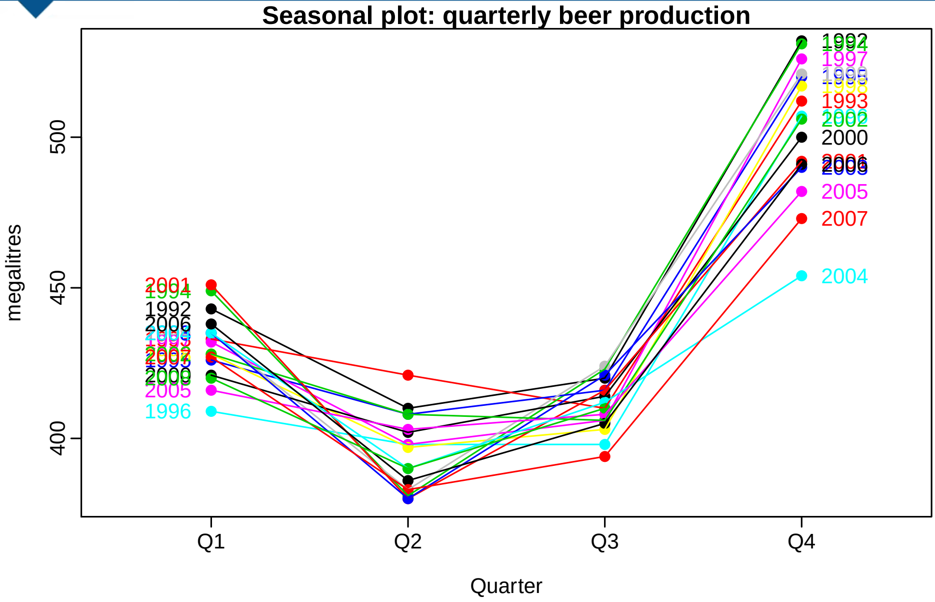

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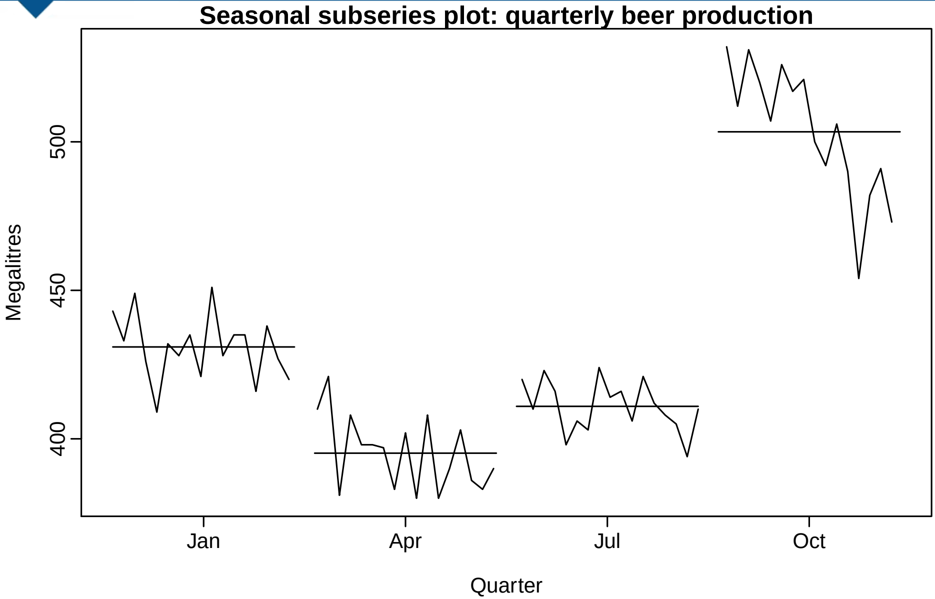


Time series graphics

- **Time plots**

  R command: `plot` or `plot.ts`
- **Seasonal plots**

  R command: `seasonplot`
- **Seasonal subseries plots**

  R command: `monthplot`
- **Lag plots**

  R command: `lag.plot`
- **ACF plots**

  R command: `Acf`

Outline

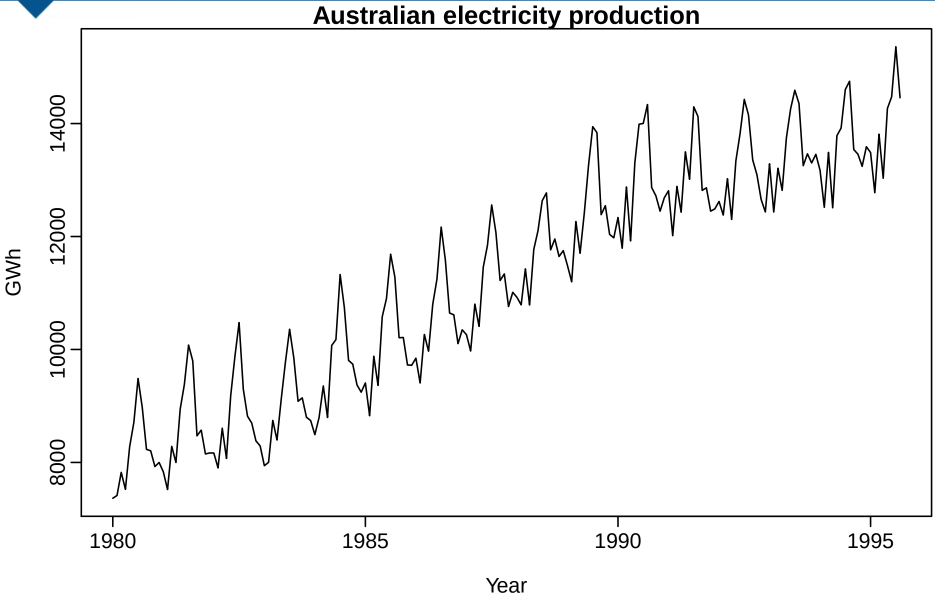
Time series patterns

Trend pattern exists when there is a long-term increase or decrease in the data.

Seasonal pattern exists when a series is influenced by seasonal factors (e.g., the quarter of the year, the month, or day of the week).

Cyclic pattern exists when data exhibit rises and falls that are *not of fixed period* (duration usually of at least 2 years).

Time series patterns



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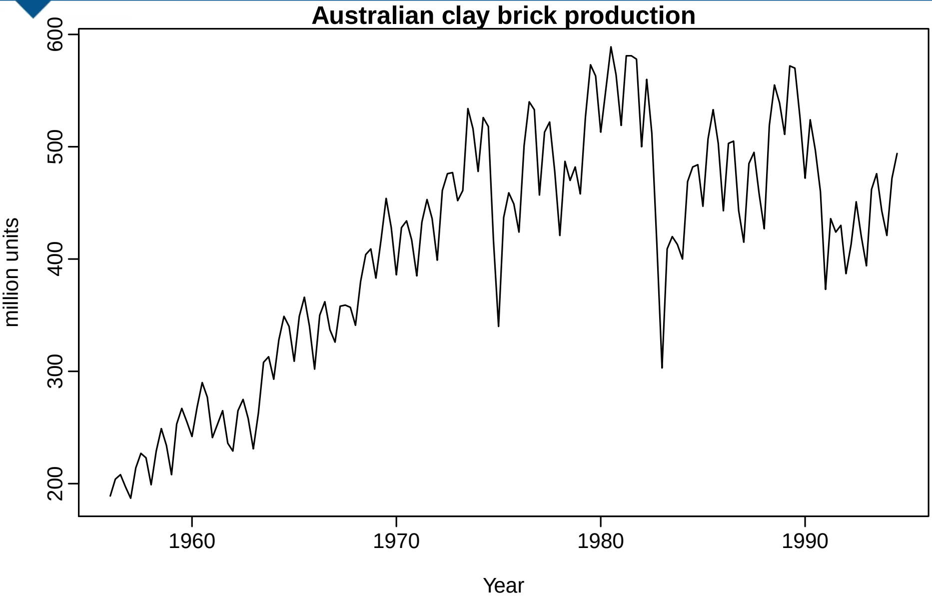

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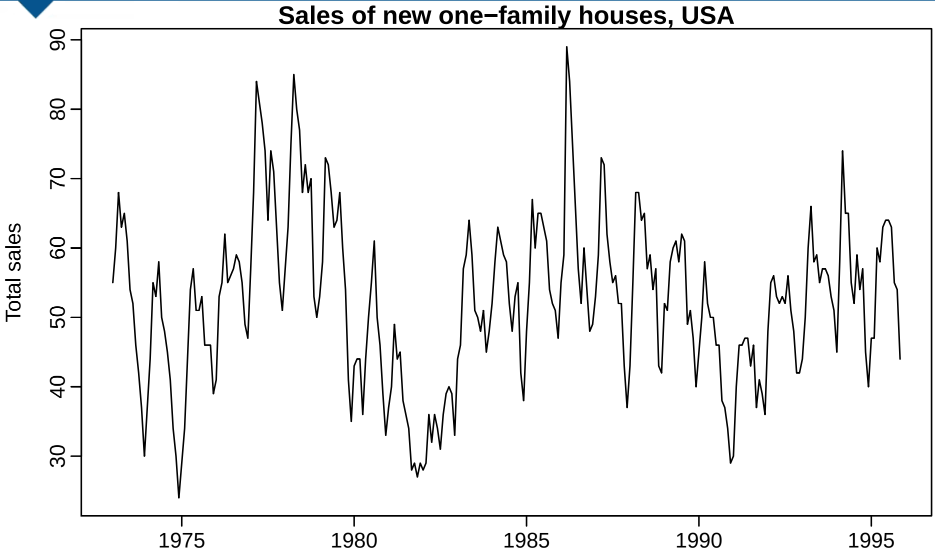

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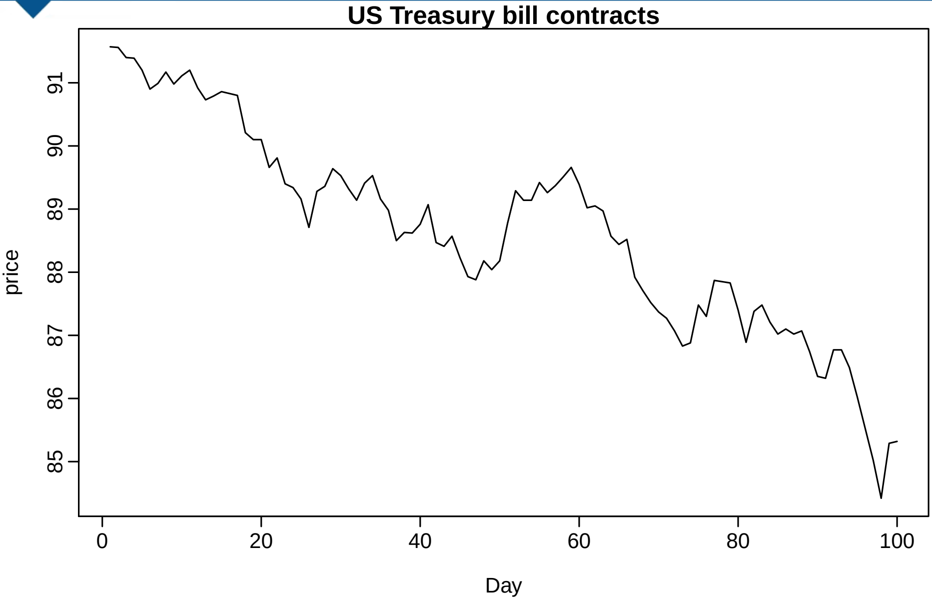

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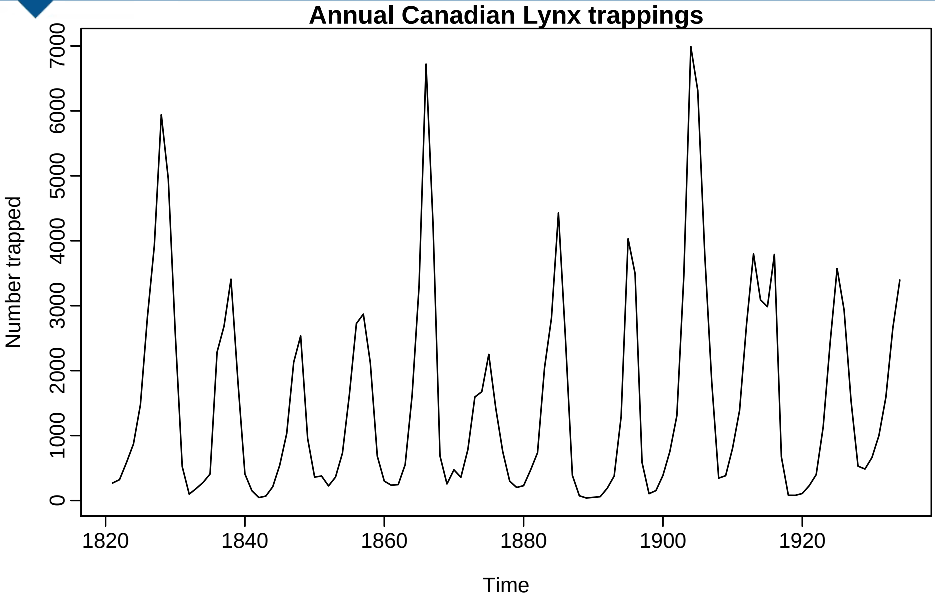

Seasonal or cyclic?

Differences between seasonal and cyclic patterns:

- seasonal pattern constant length; cyclic pattern variable length
- average length of cycle longer than length of seasonal pattern
- magnitude of cycle more variable than magnitude of seasonal pattern

The timing of peaks and troughs is predictable with seasonal data, but unpredictable in the long term with cyclic data.

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Autocorrelation

Covariance and **correlation**: measure extent of **linear relationship** between two variables (y and X).

Autocovariance and **autocorrelation**: measure linear relationship between **lagged values** of a time series y.

We measure the relationship between:
y_t and y_{t-1}
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etc.

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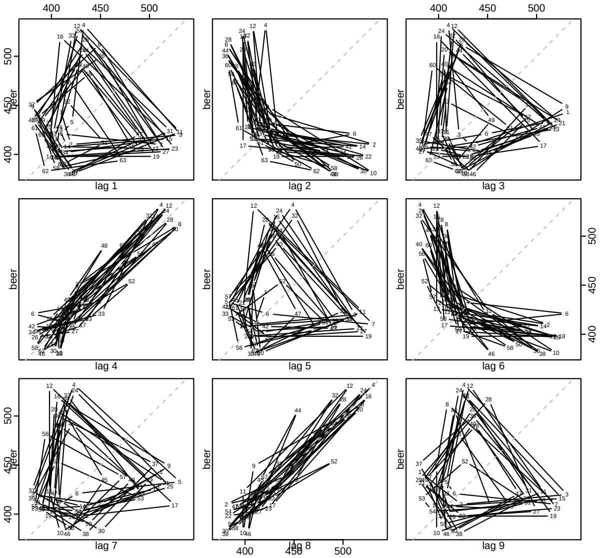
Example: Beer production

```
> lag.plot(beer,lags=9)
```



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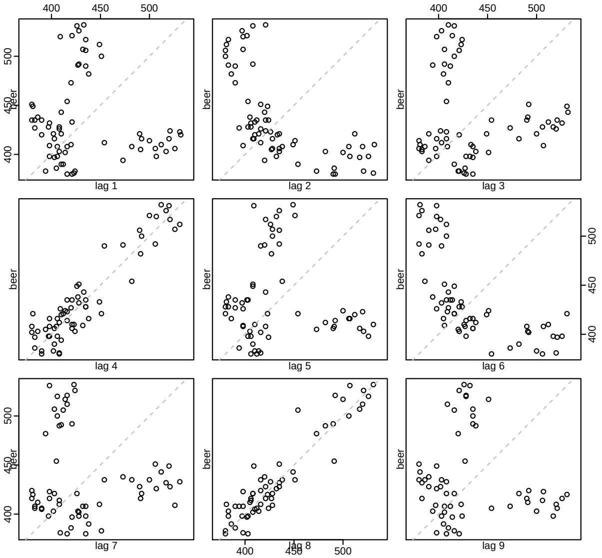
```
> lag.plot(beer,lags=9,do.lines=FALSE)
```



Lagged scatterplots

- Each graph shows y_t plotted against y_{t-k} for different values of k.
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Autocorrelation

We denote the sample autocovariance at lag k by c_k and the sample autocorrelation at lag k by r_k. Then define

$$c_k = \frac{1}{T}\sum_{t=k+1}^{T}(y_t - \bar{y})(y_{t-k} - \bar{y})$$

and $$r_k = c_k/c_0$$

- r_1 indicates how successive values of y relate to each other
- r_2 indicates how y values two periods apart relate to each other
- r_k is *almost* the same as the sample correlation between y_t and y_{t-k}.

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Autocorrelation

Results for first 9 lags for beer data:

r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	r_9
-0.126	-0.650	-0.094	0.863	-0.099	-0.642	-0.098	0.834	-0.116

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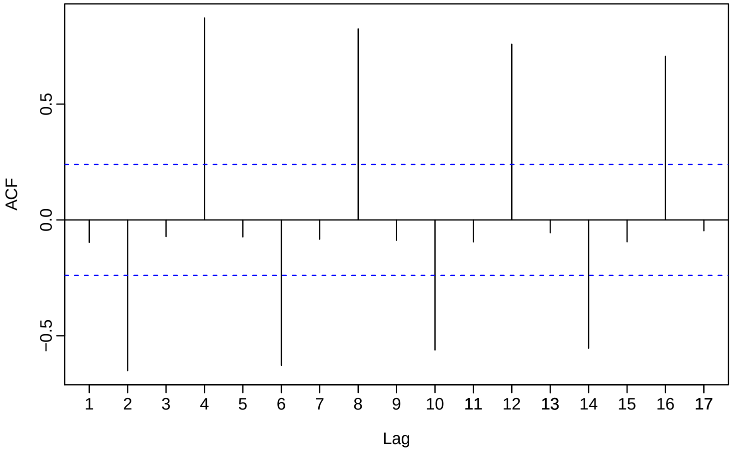

Autocorrelation

- r_4 higher than for the other lags. This is due to **the seasonal pattern in the data:** the peaks tend to be **4 quarters** apart and the troughs tend to be **2 quarters** apart.
- r_2 is more negative than for the other lags because troughs tend to be 2 quarters behind peaks.
- Together, the autocorrelations at lags 1, 2, . . . , make up the *autocorrelation* or ACF.
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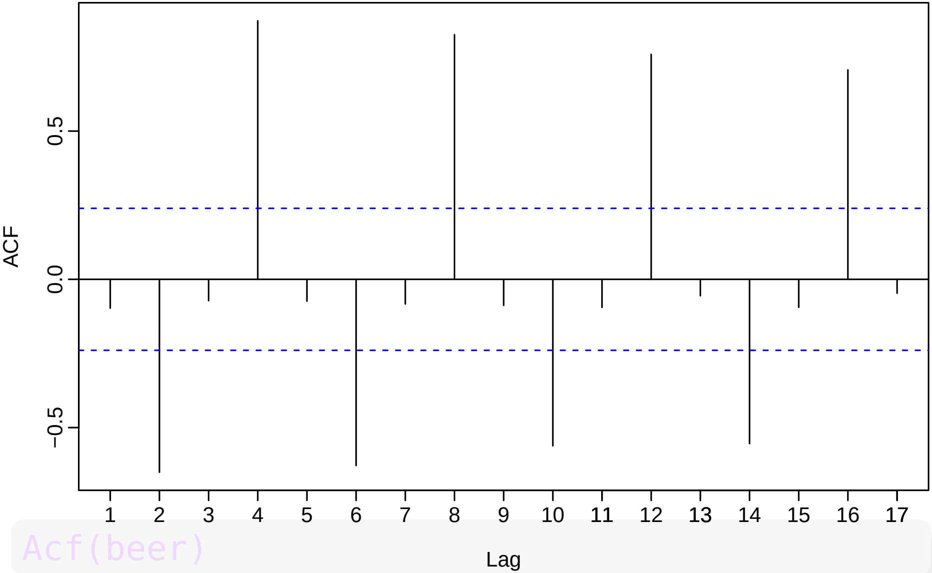
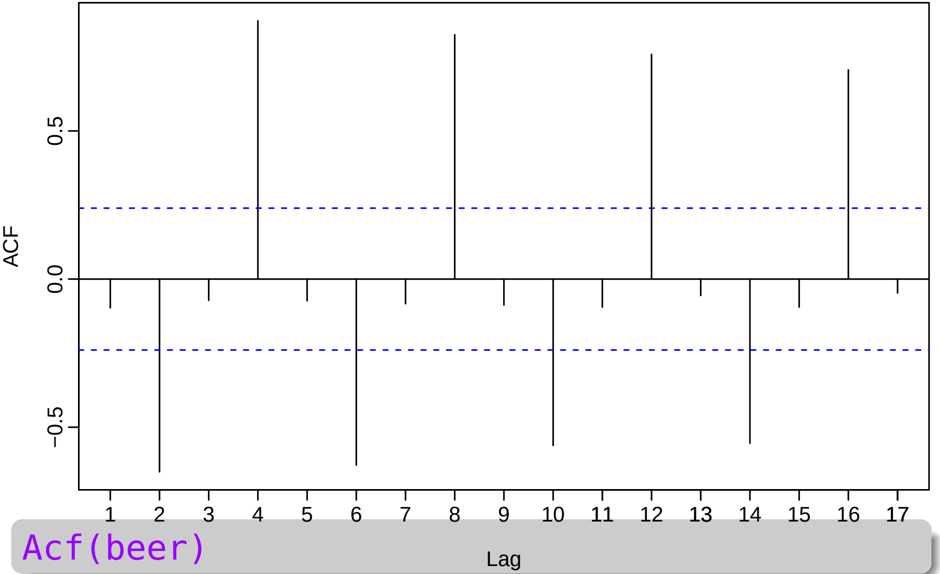



```
Acf(beer)
```

ACF

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Recognizing seasonality in a time series

If there is seasonality, the ACF at the seasonal lag (e.g., 12 for monthly data) will be **large and positive**.

- For seasonal monthly data, a large ACF value will be seen at lag 12 and possibly also at lags 24, 36, . . .
- For seasonal quarterly data, a large ACF value will be seen at lag 4 and possibly also at lags 8, 12, . . .

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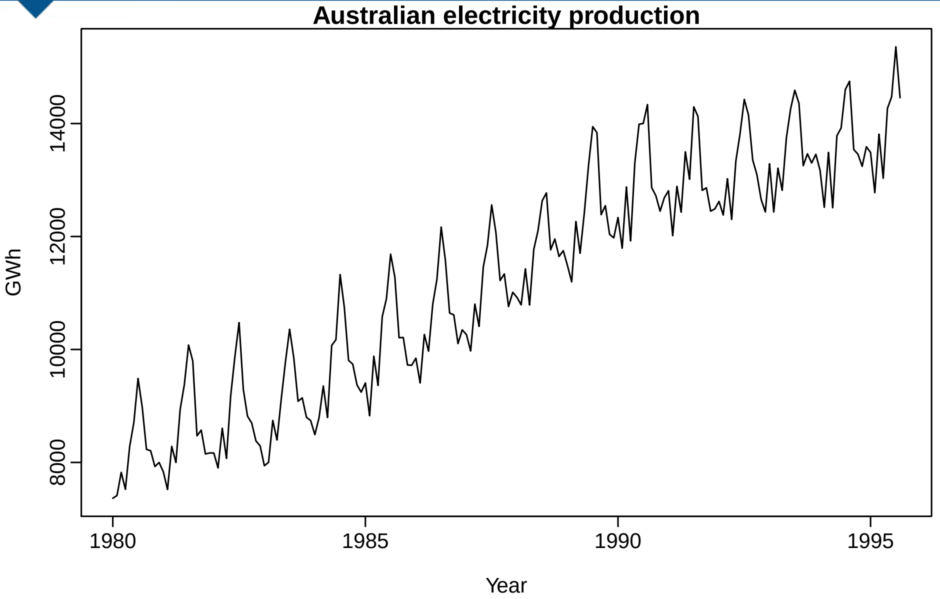

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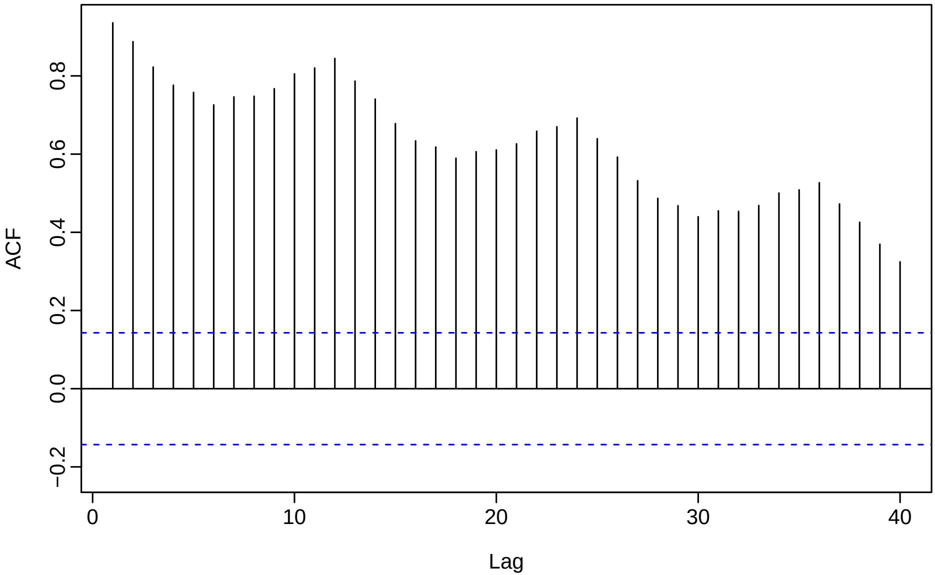

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Time plot shows clear trend and seasonality.

The same features are reflected in the ACF.

- The slowly decaying ACF indicates trend.
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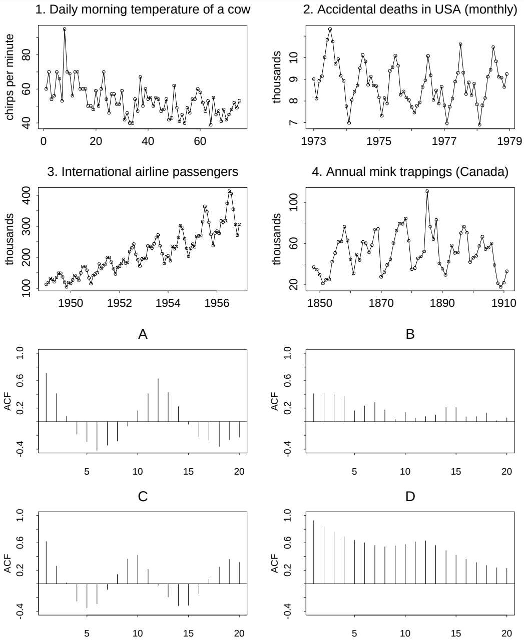
Which is which?



Outline

Forecasting residuals

Residuals in forecasting: difference between observed value and its forecast based on all previous observations: $e_t = y_t - \hat{y}_{t|t-1}$.

Assumptions

1. $\{e_t\}$ uncorrelated. If they aren't, then information left in residuals that should be used in computing forecasts.
2. $\{e_t\}$ have mean zero. If they don't, then forecasts are biased.

Useful properties (for prediction intervals)

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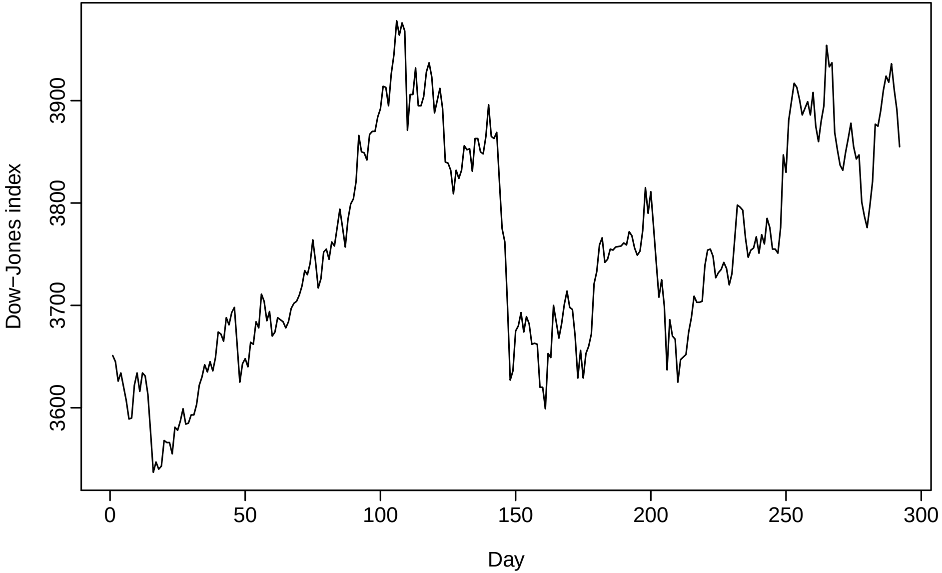

Forecasting Dow-Jones index

Naïve forecast:

$$\hat{y}_{t|t-1} = y_{t-1}$$

$$e_t = y_t - y_{t-1}$$

Note: e_t are one-step-forecast residuals

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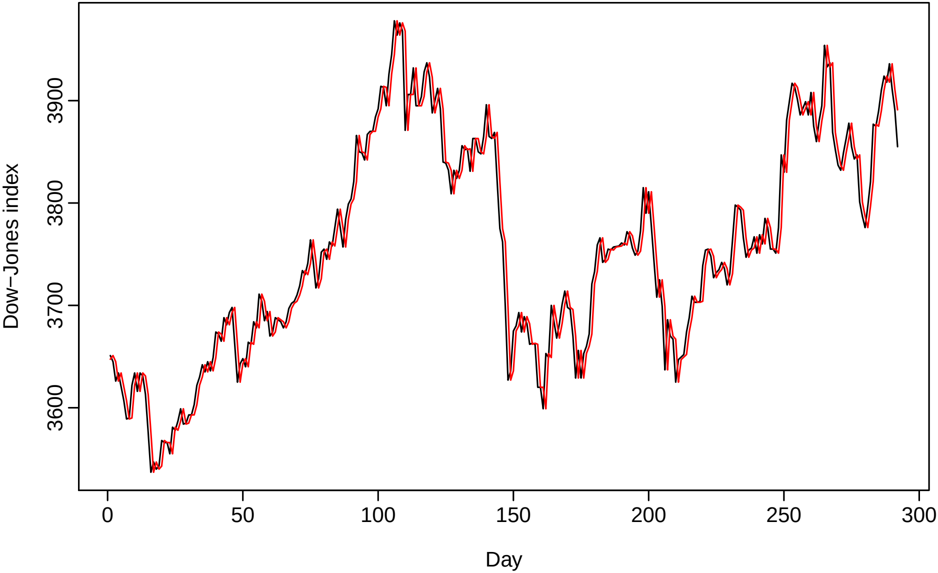

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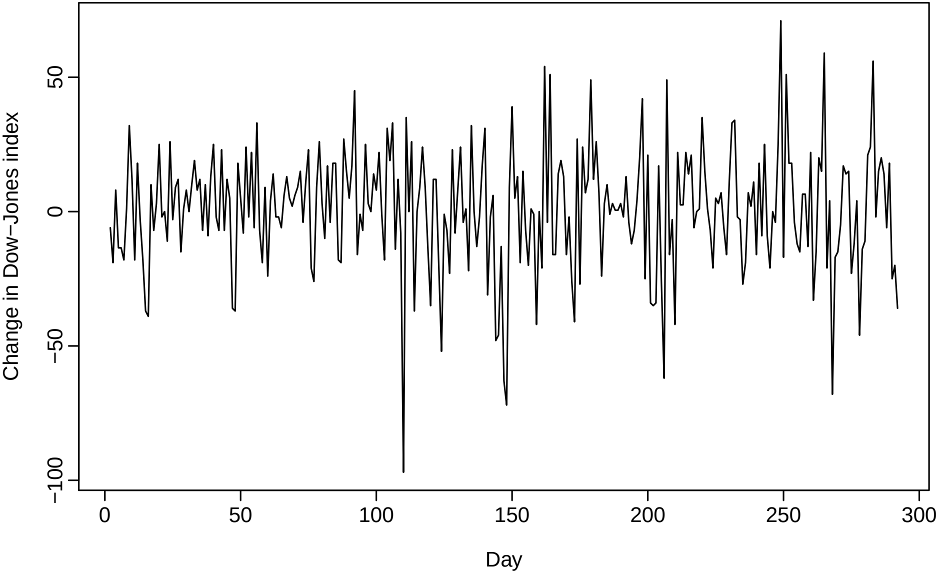

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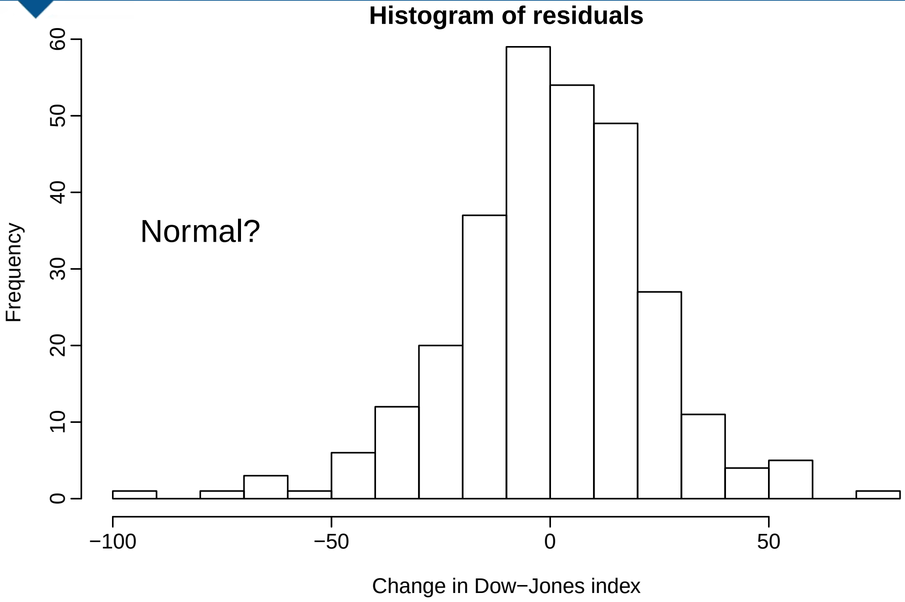

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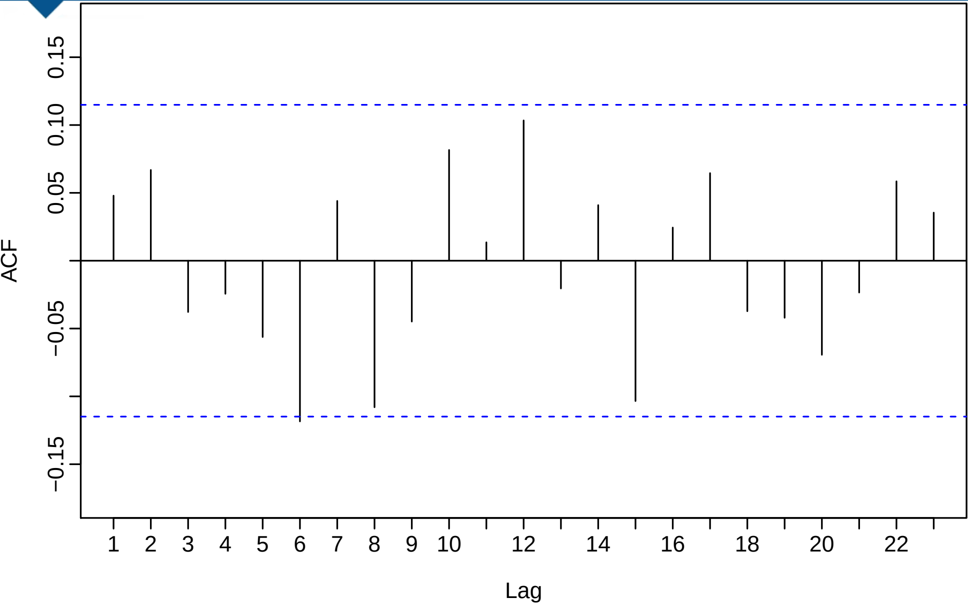


Forecasting Dow-Jones index

```
fc <- rwf(dj)

res <- residuals(fc)

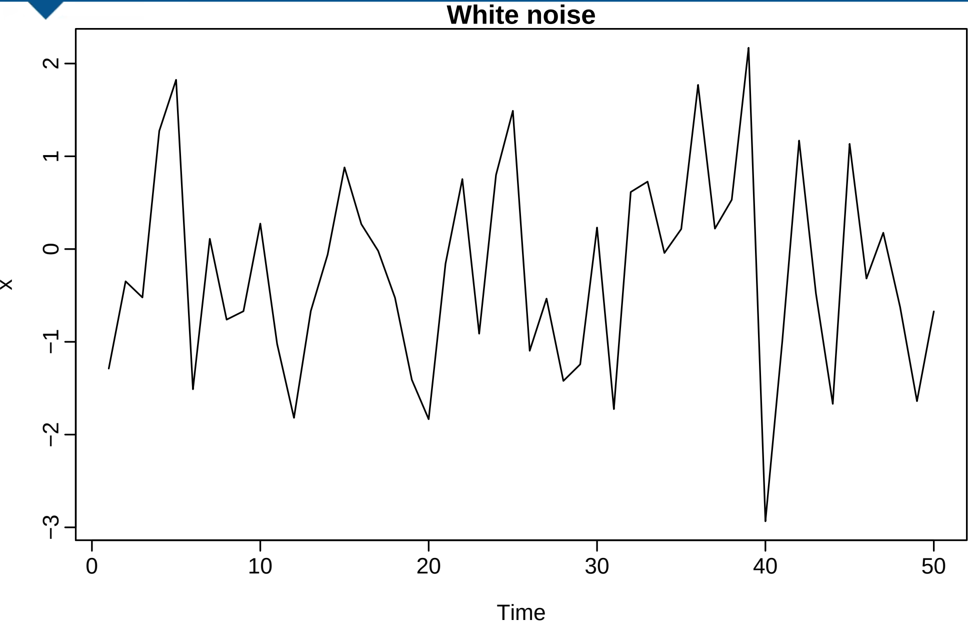
plot(res)

hist(res,breaks="FD")

Acf(res,main="")
```

Outline

Example: White noise



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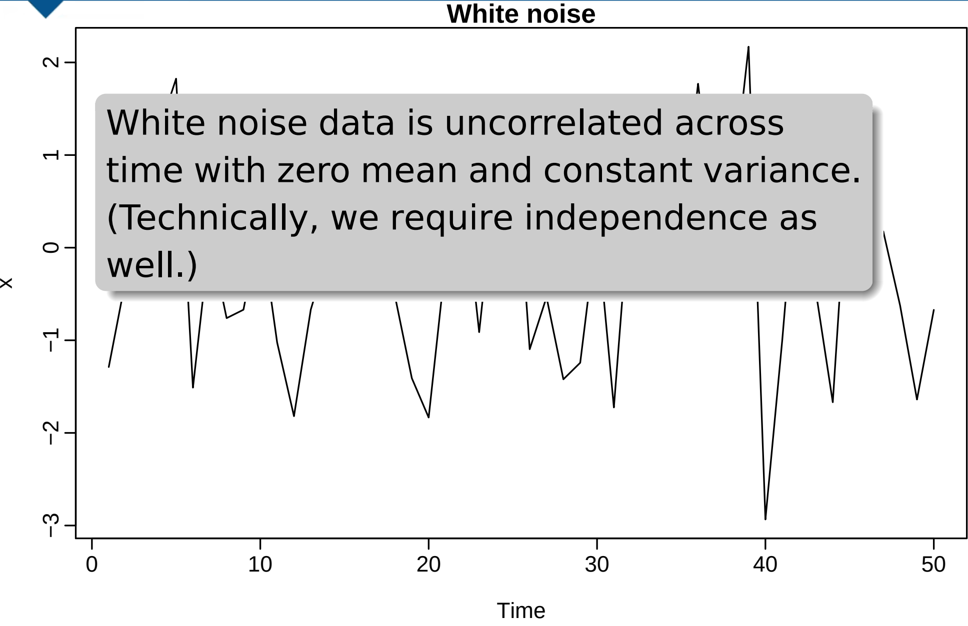

White noise data is uncorrelated across time with zero mean and constant variance. (Technically, we require independence as well.)

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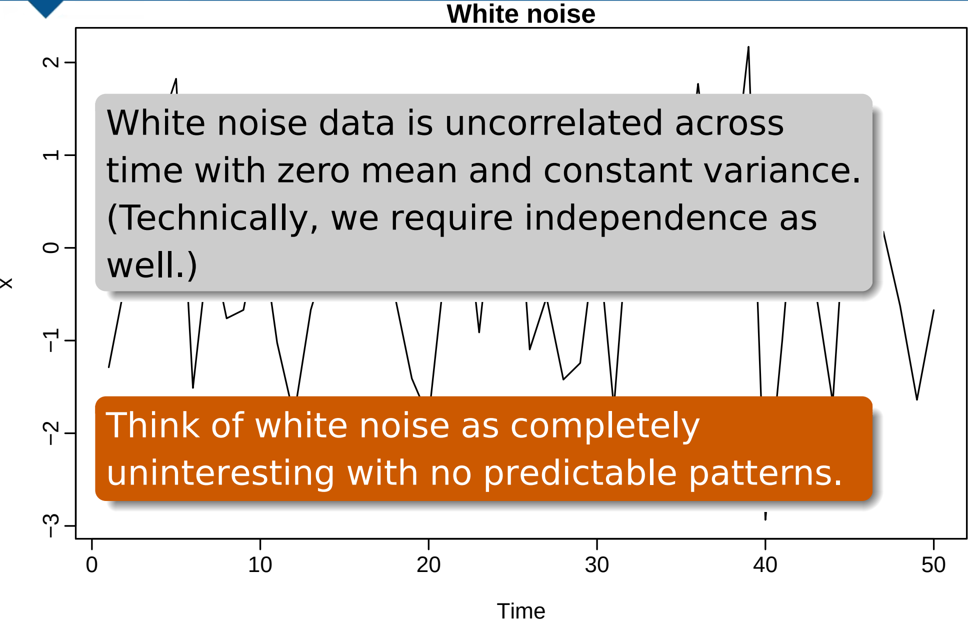

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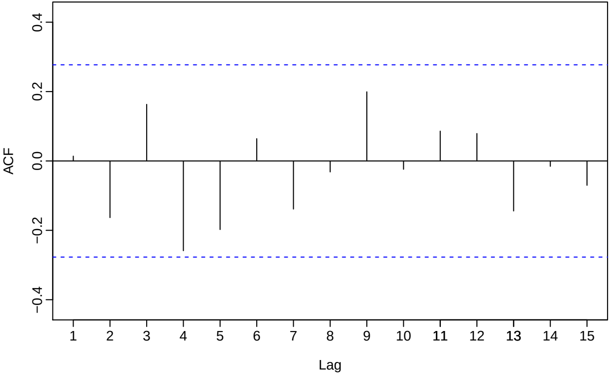
Think of white noise as completely uninteresting with no predictable patterns.

Example: White noise

$$
\begin{aligned}
r_1 &= 0.013 \\
r_2 &= -0.163 \\
r_3 &= 0.163 \\
r_4 &= -0.259 \\
r_5 &= -0.198 \\
r_6 &= 0.064 \\
r_7 &= -0.139 \\
r_8 &= -0.032 \\
r_9 &= 0.199 \\
r_{10} &= -0.240
\end{aligned}
$$



Sample autocorrelations for white noise series.
For uncorrelated data, we would expect each autocorrelation to be close to zero.

Sampling distribution of autocorrelations

Sampling distribution of r_k for white noise data is asymptotically $\mathrm{N}(0,1/T)$.

- 95% of all r_k for white noise must lie within $\pm 1.96/\sqrt{T}$.
- If this is not the case, the series is probably not WN.
- Common to plot lines at $\pm 1.96/\sqrt{T}$ when plotting ACF. These are the *critical values*.

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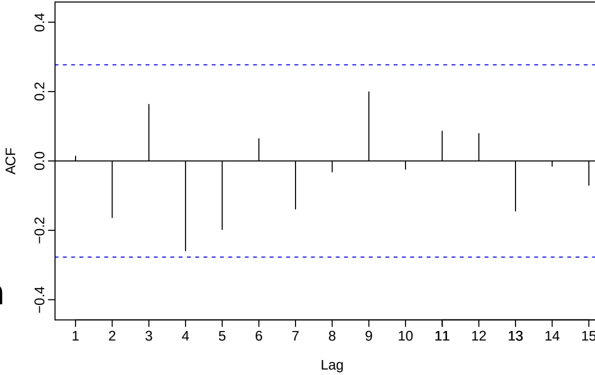
Autocorrelation

Example:

$T = 50$ and so critical values at $\pm 1.96/\sqrt{50} = \pm 0.28$.

All autocorrelation coefficients lie within these limits, confirming that the data are white noise.



(More precisely, the data cannot be distinguished from white noise.)

Example: Pigs slaughtered

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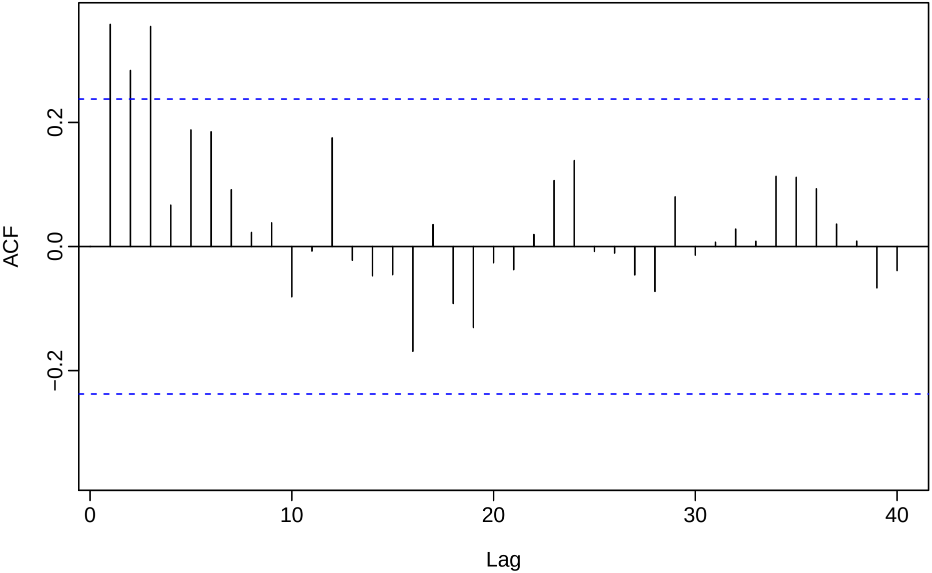

Example: Pigs slaughtered

Monthly total number of pigs slaughtered in the state of Victoria, Australia, from January 1990 through August 1995. (Source: Australian Bureau of Statistics.)

- Difficult to detect pattern in time plot.
- ACF shows some significant autocorrelation at lags 1, 2, and 3.
- r_{12} relatively large although not significant. This may indicate some slight seasonality.

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ACF of residuals

- We assume that the residuals are white noise (uncorrelated, mean zero, constant variance). If they aren't, then there is information left in the residuals that should be used in computing forecasts.
- So a standard residual diagnostic is to check the ACF of the residuals of a forecasting method.
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Dow-Jones naive forecasts revisited

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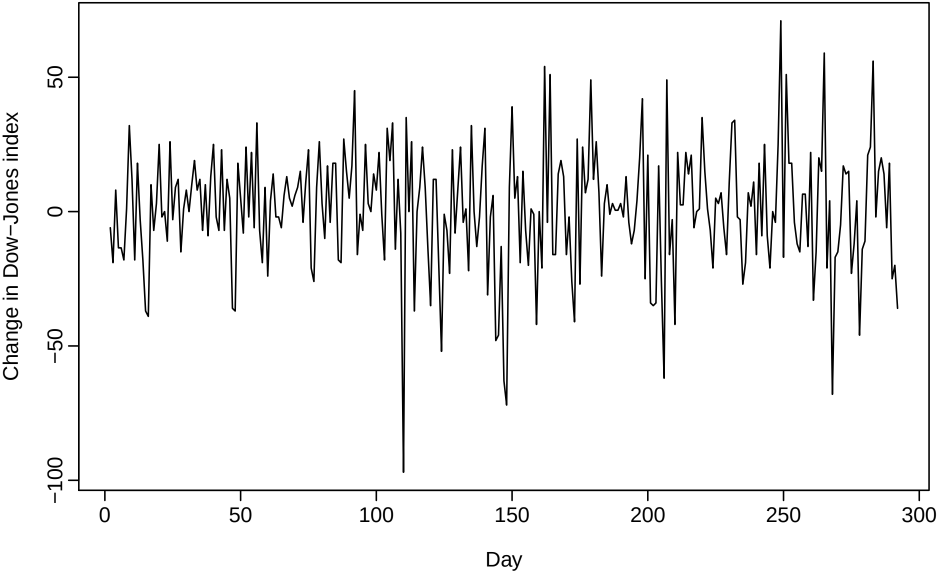
Forecasting Dow-Jones index



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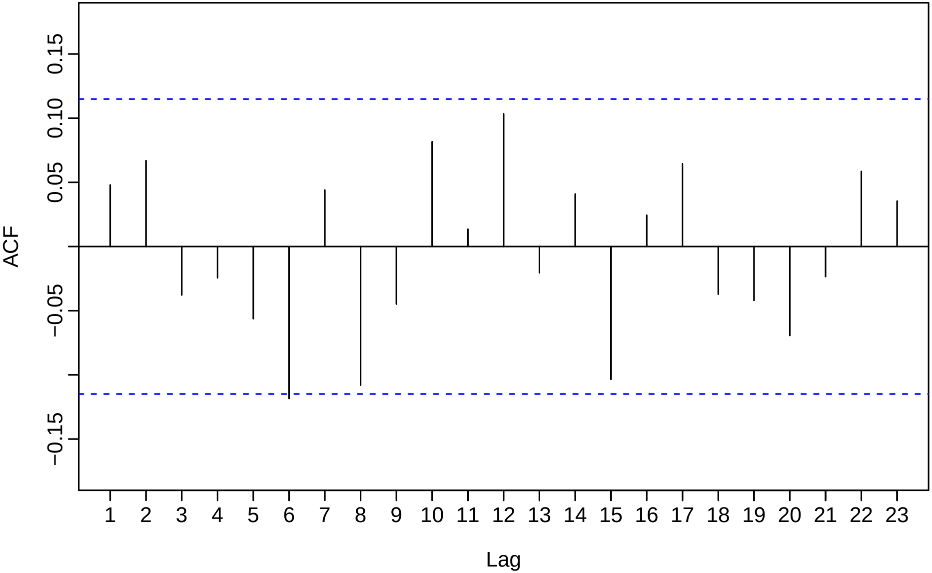

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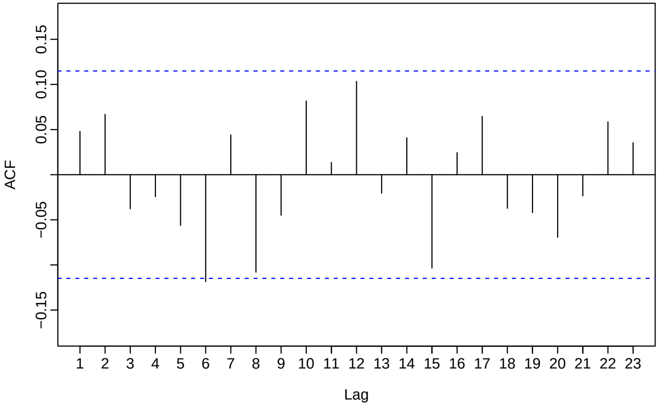

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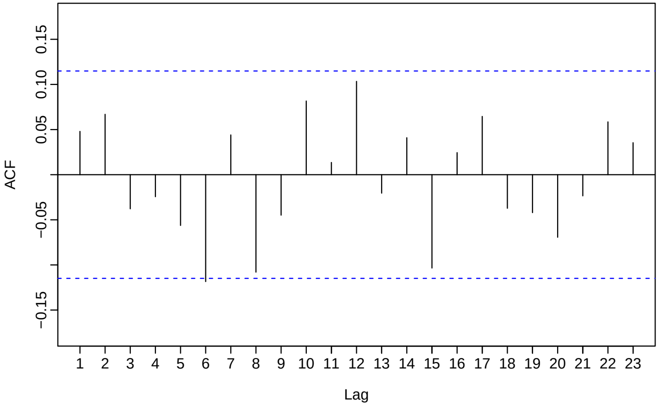

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$$Q = T \sum_{k=1}^{h} r_k^2$$

where h is max lag being considered and T is number of observations.

- My preferences: $h = 10$ for non-seasonal data, $h = 2m$ for seasonal data.
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- If some r_k values large (positive or negative), Q will be large.

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Ljung-Box test

$$Q^* = T(T+2)\sum_{k=1}^{h}(T-k)^{-1}r_k^2$$

where h is max lag being considered and T is number of observations.

- My preferences: $h = 10$ for non-seasonal data, $h = 2m$ for seasonal data.
- Better performance, especially in small samples.

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- If data are WN, Q^* has χ^2 distribution with $(h - K)$ degrees of freedom where $K =$ no. parameters in model.
- When applied to raw data, set $K = 0$.
- For the Dow-Jones example,

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res <- residuals(naive(dj))

# lag=h and fitdf=K
> Box.test(res, lag=10, fitdf=0)
  Box-Pierce test
X-squared = 14.0451, df = 10, p-value = 0.1709
> Box.test(res, lag=10, fitdf=0, type="Lj")
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Exercise

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2. Test if the residuals are white noise.

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```
beer <- window(ausbeer,start=1992)
fc <- snaive(beer)
res <- residuals(fc)
Acf(res)
Box.test(res, lag=8, fitdf=0, type="Lj")
```

Outline

Measures of forecast accuracy

Let y_t denote the tth observation and $\hat{y}_{t|t-1}$ denote its forecast based on all previous data, where $t = 1, \dots, T$. Then the following measures are useful.

$$\text{MAE} = T^{-1} \sum_{t=1}^{T} |y_t - \hat{y}_{t|t-1}|$$

$$\text{MSE} = T^{-1} \sum_{t=1}^{T} (y_t - \hat{y}_{t|t-1})^2 \qquad \text{RMSE} = \sqrt{T^{-1} \sum_{t=1}^{T} (y_t - \hat{y}_{t|t-1})^2}$$

$$\text{MAPE} = 100T^{-1} \sum_{t=1}^{T} |y_t - \hat{y}_{t|t-1}|/|y_t|$$

- MAE, MSE, RMSE are all scale dependent.
- MAPE is scale independent but is only sensible if $y_t \gg 0$ for all t, and y has a natural zero.

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Proposed by Hyndman and Koehler (IJF, 2006)

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$$Q = (T-1)^{-1}\sum_{t=2}^{T}|y_t - y_{t-1}|$$

works well. Then MASE is equivalent to MAE relative to a naive method.

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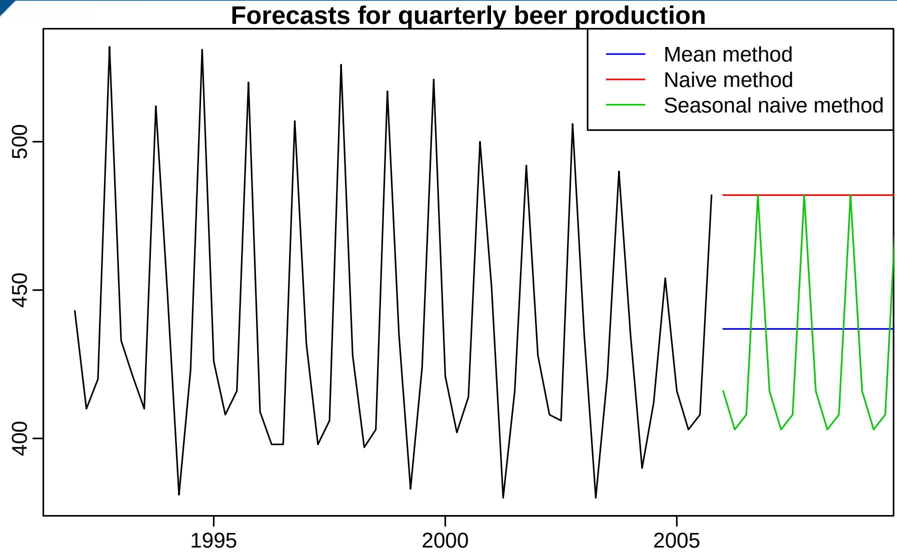
For seasonal time series,

$$Q = (T - m)^{-1} \sum_{t=m+1}^{T} \left| y_t - y_{t-m} \right|$$

works well. Then MASE is equivalent to MAE relative to a seasonal naive method.

Measures of forecast accuracy



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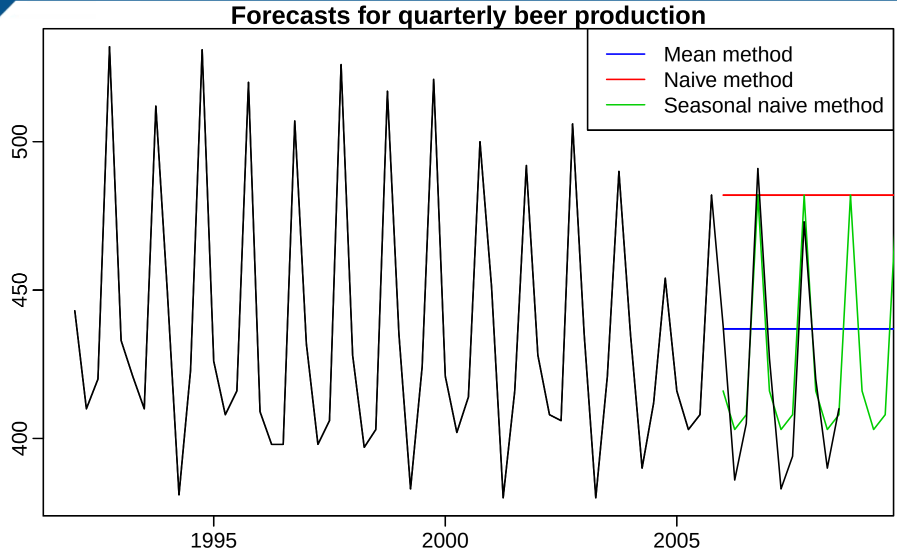


Measures of forecast accuracy

Mean method

RMSE	MAE	MAPE	MASE
38.0145	33.7776	8.1700	2.2990

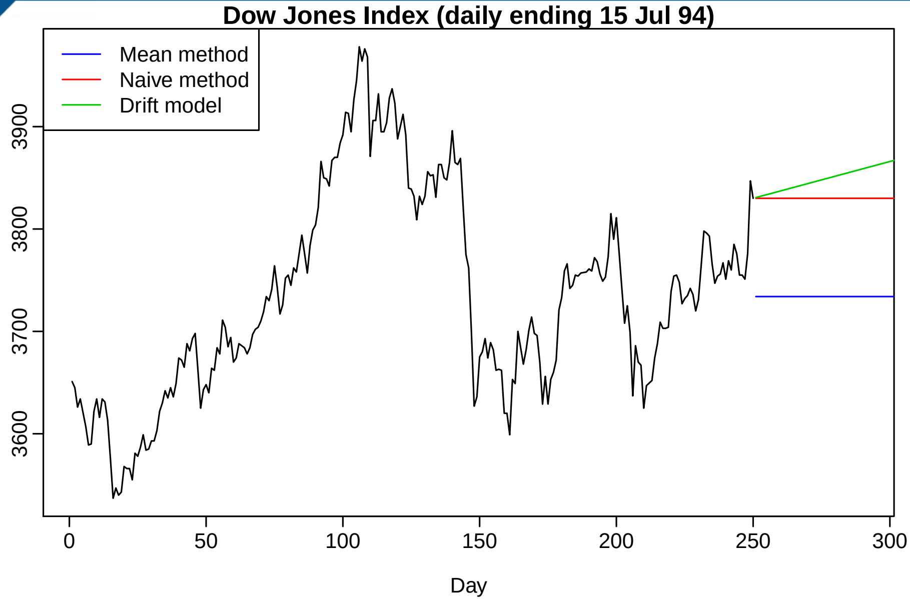
Naïve method

RMSE	MAE	MAPE	MASE
70.9065	63.9091	15.8765	4.3498

Seasonal naïve method

RMSE	MAE	MAPE	MASE
12.9685	11.2727	2.7298	0.7673

Measures of forecast accuracy



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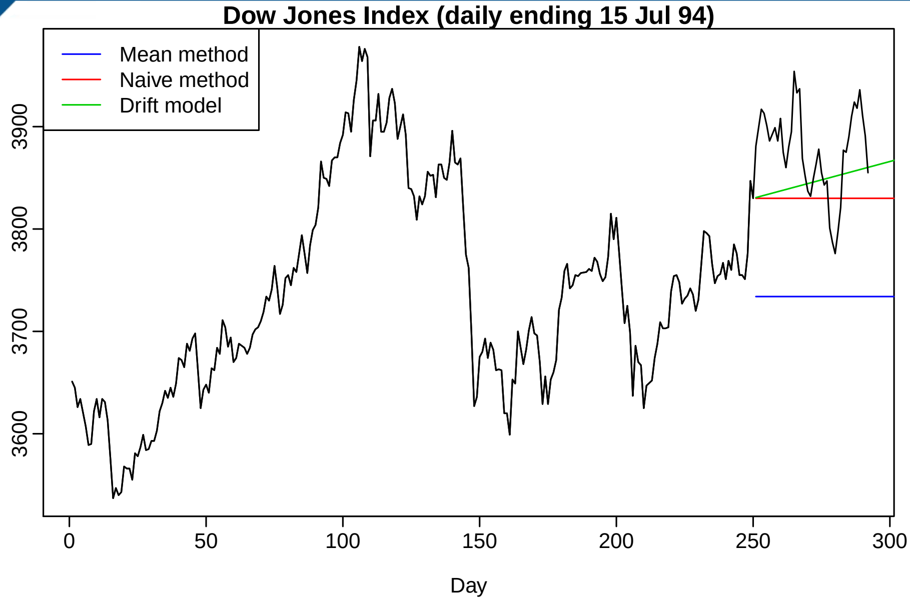

Measures of forecast accuracy

Mean method

```
    RMSE       MAE      MAPE     MASE
148.2357  142.4185   3.6630   8.6981
```

Naïve method

```
    RMSE       MAE      MAPE     MASE
 62.0285   54.4405   1.3979   3.3249
```

Drift model

```
    RMSE       MAE      MAPE     MASE
 53.6977   45.7274   1.1758   2.7928
```

Training and test sets

Available data

Training set (e.g., 80%)	Test set (e.g., 20%)

- The test set must not be used for *any* aspect of model development or calculation of forecasts.
- Forecast accuracy is based only on the test set.

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```
beer3 <- window(ausbeer,start=1992,end=2005.99)
beer4 <- window(ausbeer,start=2006)

fit1 <- meanf(beer3,h=20)
fit2 <- rwf(beer3,h=20)

accuracy(fit1,beer4)
accuracy(fit2,beer4)
```

In-sample accuracy (one-step forecasts)

```
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- A model which fits the data well does not necessarily forecast well.
- A perfect fit can always be obtained by using a model with enough parameters. (Compare R^2)
- Over-fitting a model to data is as bad as failing to identify the systematic pattern in the data.
- Problems can be overcome by measuring true *out-of-sample* forecast accuracy. That is, total data divided into "training" set and "test" set. Training set used to estimate parameters. Forecasts are made for test set.
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Poll: true or false?

1. Good forecast methods should have normally distributed residuals.
2. A model with small residuals will give good forecasts.
3. The best measure of forecast accuracy is MAPE.
4. If your model doesn't forecast well, you should make it more complicated.
5. Always choose the model with the best forecast accuracy as measured on the test set.