MONASH University

Rob J Hyndman

Forecasting:

Principles and Practice

7. Non-seasonal ARIMA models

OTexts.com/fpp/8/

Outline

Autoregressive models

Autoregressive (AR) models:

$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + e_t,$$

where e_t is white noise. This is a multiple regression with **lagged values** of y_t as predictors.

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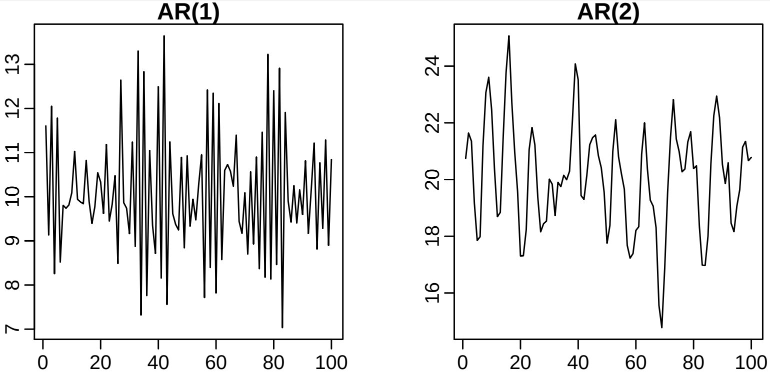

AR(1) model

$$y_t = 2 - 0.8y_{t-1} + e_t$$

$e_t \sim N(0, 1)$
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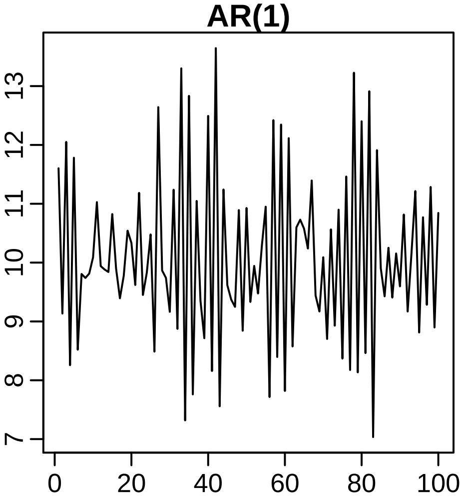

AR(1) model

$$y_t = c + \phi_1 y_{t-1} + e_t$$

- When $\phi_1 = 0$, y_t is **equivalent to WN**
- When $\phi_1 = 1$ and $c = 0$, y_t is **equivalent to a RW**
- When $\phi_1 = 1$ and $c \neq 0$, y_t is **equivalent to a RW with drift**
- When $\phi_1 < 0$, y_t tends to **oscillate between positive and negative values**.

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AR(2) model

$$y_t = 8 + 1.3y_{t-1} - 0.7y_{t-2} + e_t$$

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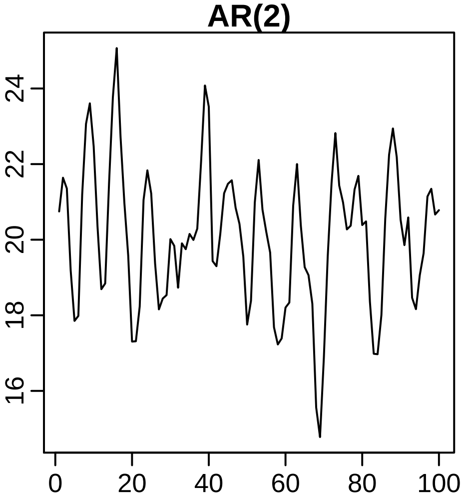

Stationarity conditions

We normally restrict autoregressive models to stationary data, and then some constraints on the values of the parameters are required.

General condition for stationarity

Complex roots of $1 - \phi_1 z - \phi_2 z^2 - \cdots - \phi_p z^p$ lie outside the unit circle on the complex plane.

- For $p = 1$: $-1 < \phi_1 < 1$.
- For $p = 2$:
  $-1 < \phi_2 < 1 \qquad \phi_2 + \phi_1 < 1 \qquad \phi_2 - \phi_1 < 1.$
- More complicated conditions hold for $p \geq 3$.
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Moving Average (MA) models

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$$y_t = c + e_t + \theta_1 e_{t-1} + \theta_2 e_{t-2} + \cdots + \theta_q e_{t-q},$$

where e_t is white noise. This is a multiple regression with **past *errors*** as predictors. *Don't confuse this with moving average smoothing!*

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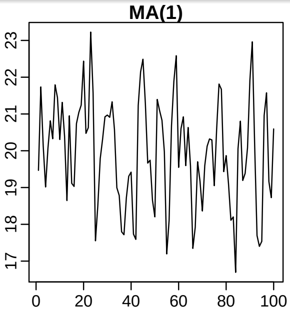

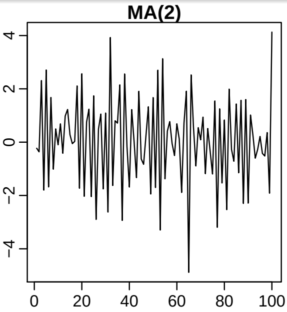

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$$y_t = 20 + e_t + 0.8e_{t-1}$$

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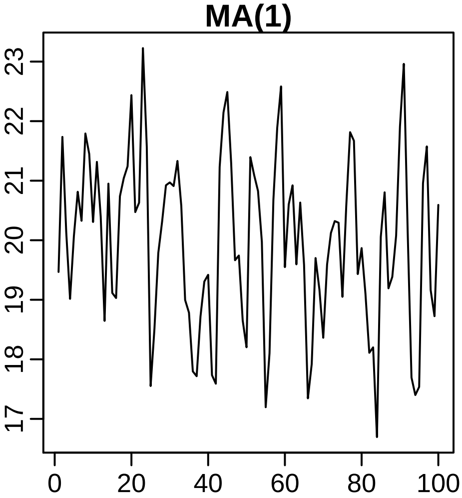

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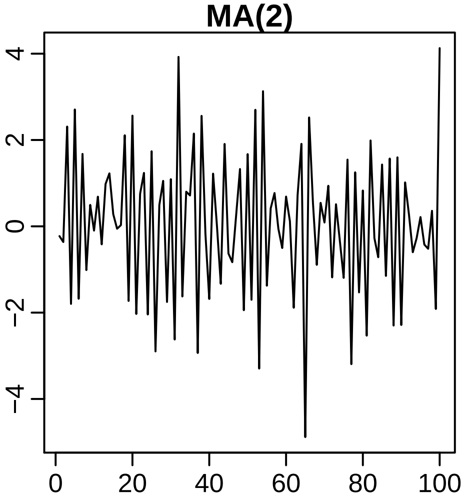

Invertibility

- Any MA(q) process can be written as an AR(∞) process if we impose some constraints on the MA parameters.
- Then the MA model is called “invertible”.
- Invertible models have some mathematical properties that make them easier to use in practice.
- Invertibility of an ARIMA model is equivalent to forecastability of an ETS model.

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General condition for invertibility

Complex roots of $1 + \theta_1 z + \theta_2 z^2 + \cdots + \theta_q z^q$ lie outside the unit circle on the complex plane.

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Autoregressive Moving Average models:

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- Predictors include both **lagged values of y_t and lagged errors.**
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Autoregressive Integrated Moving Average models

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- $(1 - B)^d y_t$ follows an ARMA model.

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ARIMA(p, d, q) model

AR: $p =$ order of the autoregressive part
I: $d =$ degree of first differencing involved
MA: $q =$ order of the moving average part.

- White noise model: ARIMA(0,0,0)
- Random walk: ARIMA(0,1,0) with no constant
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- AR(p): ARIMA(p,0,0)
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Backshift notation for ARIMA

- ARMA model:

$$y_t = c + \phi_1 B y_t + \cdots + \phi_p B^p y_t + e_t + \theta_1 B e_t + \cdots + \theta_q B^q e_t$$

or $$(1 - \phi_1 B - \cdots - \phi_p B^p) y_t = c + (1 + \theta_1 B + \cdots + \theta_q B^q) e_t$$

- ARIMA(1,1,1) model:

$$(1 - \phi_1 B) \; (1 - B) y_t \; = \; c + (1 + \theta_1 B) e_t$$

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$$\uparrow$$

First
difference

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$$\underset{\substack{\uparrow \\ \text{AR(1)}}}{(1 - \phi_1 B)} \quad (1 - B) y_t \quad = \quad c + (1 + \theta_1 B) e_t$$

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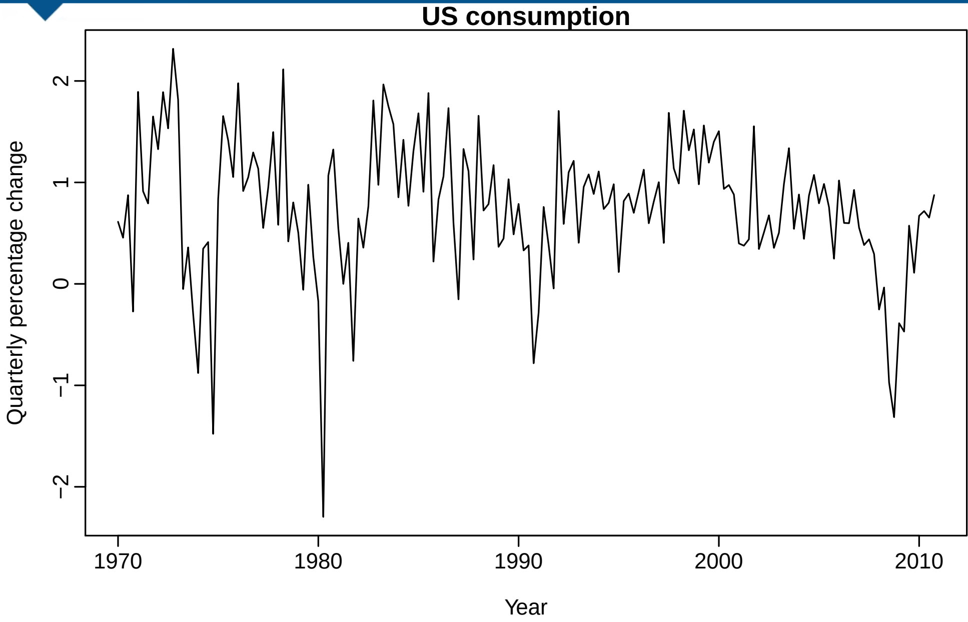
Written out:

$$y_t = c + y_{t-1} + \phi_1 y_{t-1} - \phi_1 y_{t-2} + \theta_1 e_{t-1} + e_t$$

US personal consumption



US personal consumption

```
> fit <- auto.arima(usconsumption[,1],
    max.P=0,max.Q=0,D=0)

ARIMA(0,0,3) with non-zero mean
Coefficients:
         ma1     ma2     ma3  intercept
      0.2542  0.2260  0.2695     0.7562
s.e.  0.0767  0.0779  0.0692     0.0844

sigma^2 estimated as 0.3856:  log likelihood=-154.73
AIC=319.46   AICc=319.84   BIC=334.96
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AIC=319.46   AICc=319.84   BIC=334.96
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ARIMA(0,0,3) or MA(3) model:

$$y_t = 0.756 + e_t + 0.254e_{t-1} + 0.226e_{t-2} + 0.269e_{t-3},$$

where e_t is white noise with standard deviation $0.62 = \sqrt{0.3856}$.

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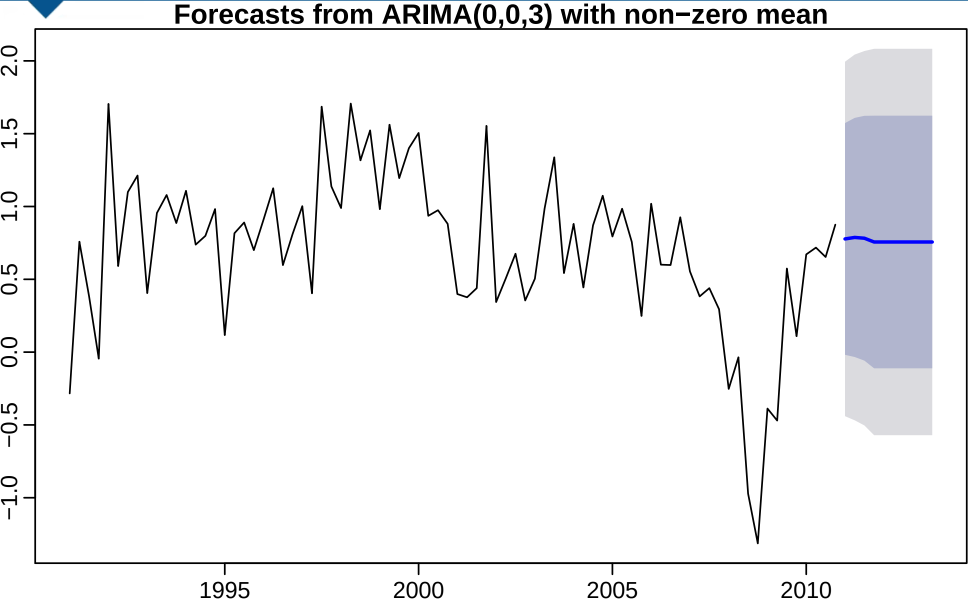

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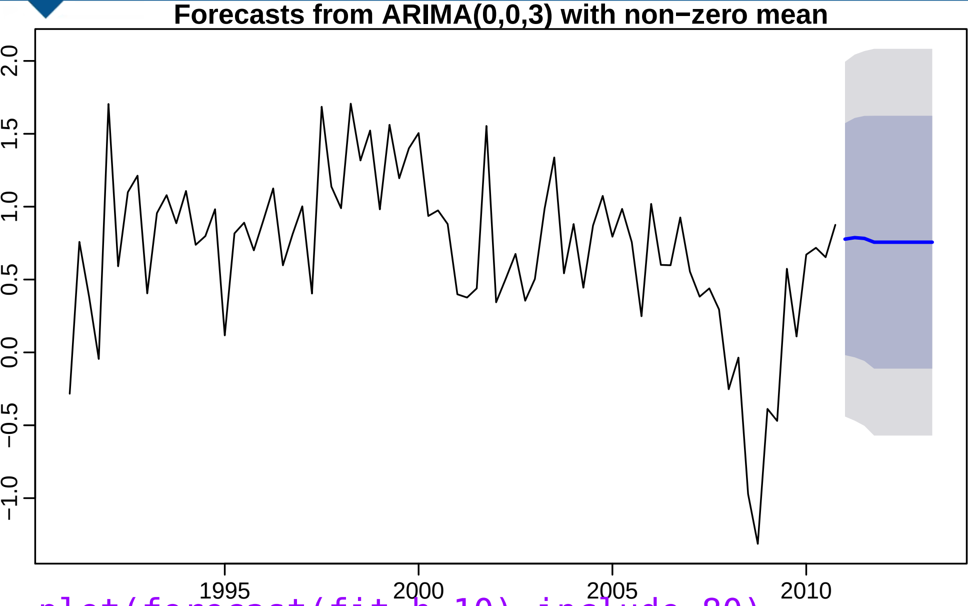


```
plot(forecast(fit,h=10),include=80)
```

Understanding ARIMA models

- If $c = 0$ and $d = 0$, the long-term forecasts will go to zero.
- If $c = 0$ and $d = 1$, the long-term forecasts will go to a non-zero constant.
- If $c = 0$ and $d = 2$, the long-term forecasts will follow a straight line.
- If $c \neq 0$ and $d = 0$, the long-term forecasts will go to the mean of the data.
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Forecast variance and d

- The higher the value of d, the more rapidly the prediction intervals increase in size.
- For $d = 0$, the long-term forecast standard deviation will go to the standard deviation of the historical data.

Cyclic behaviour

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- For cyclic forecasts, $p \geq 2$ and some restrictions on coefficients are required.
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Partial autocorrelations

Partial autocorrelations measure relationship between y_t and y_{t-k}, when the effects of other time lags — $1, 2, 3, \dots, k-1$ — are removed.

$$\alpha_k = k\text{th partial autocorrelation coefficient}$$
$$= \text{equal to the estimate of } b_k \text{ in regression:}$$
$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_k y_{t-k}.$$

- Varying number of terms on RHS gives α_k for different values of k.
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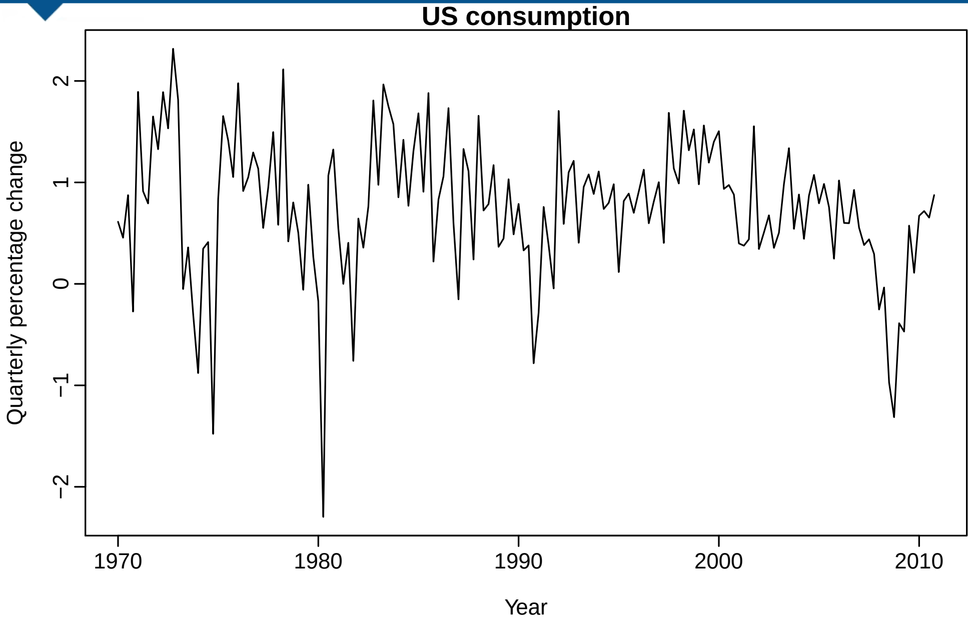
Example: US consumption



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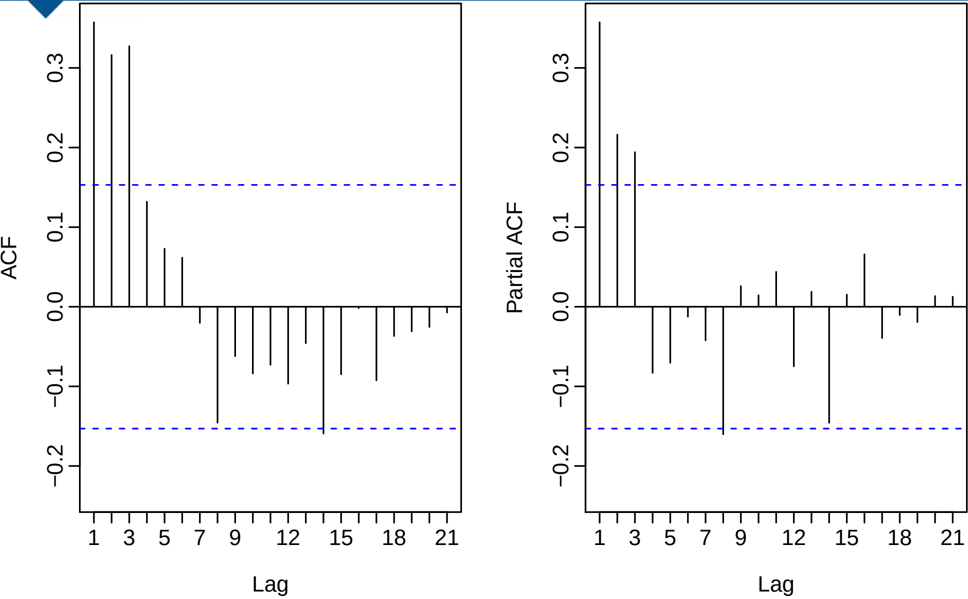

ACF and PACF interpretation

ARIMA(p,d,0) model if ACF and PACF plots of differenced data show:

- the ACF is exponentially decaying or sinusoidal;
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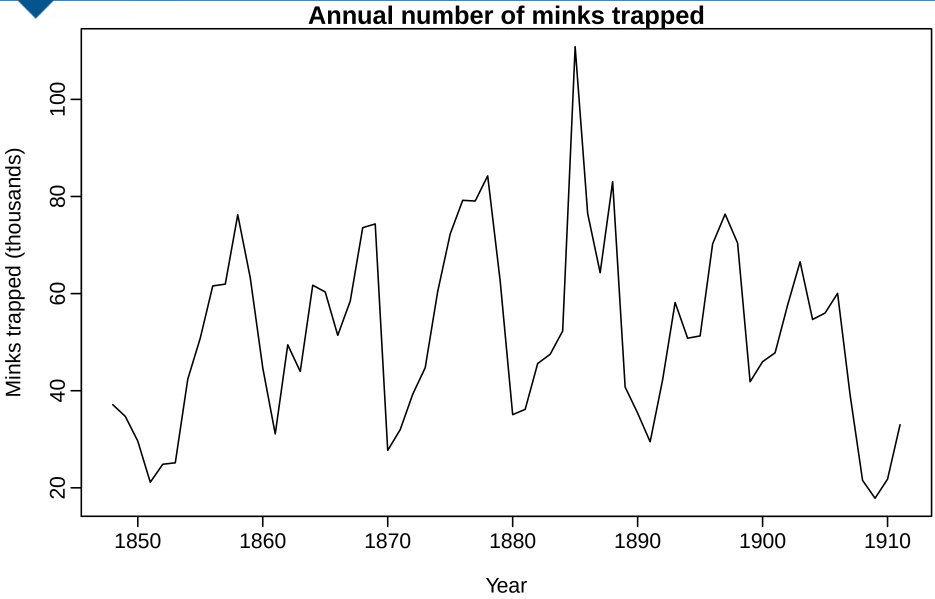

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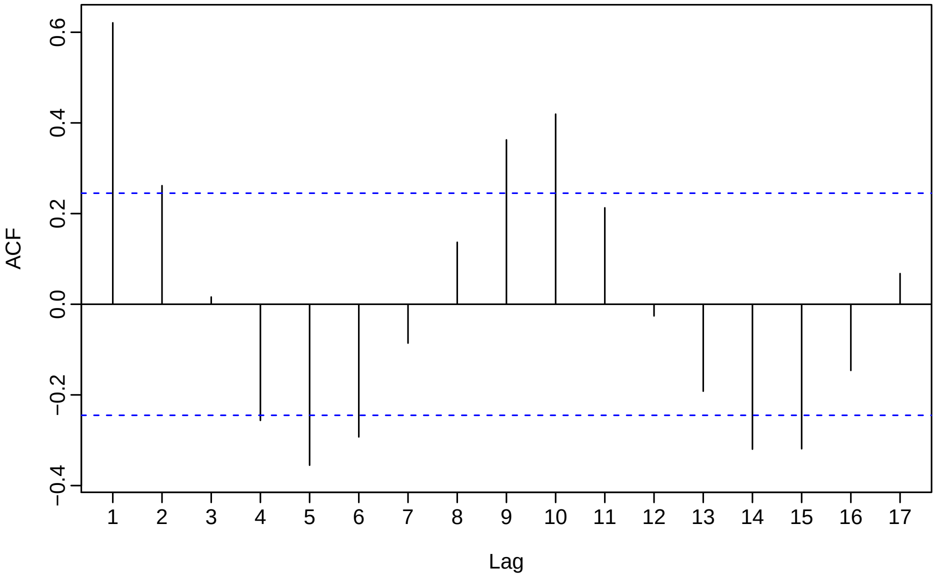

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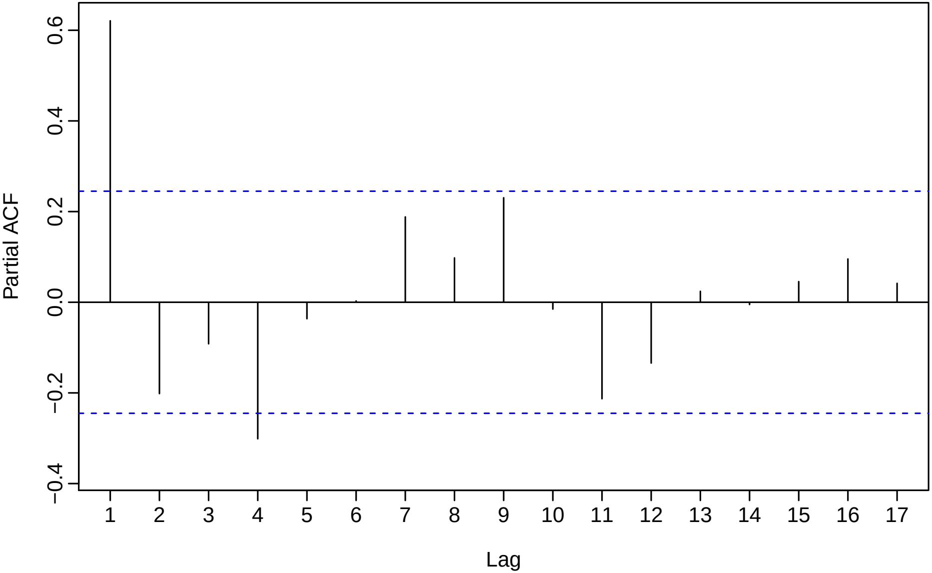

Outline

Maximum likelihood estimation

Having identified the model order, we need to estimate the parameters c, $\phi_1, \dots, \phi_p$, $\theta_1, \dots, \theta_q$.

- MLE is very similar to least squares estimation obtained by minimizing

$$\sum_{t=1}^{T} e_t^2.$$

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Information criteria

Akaike's Information Criterion (AIC):

$$\text{AIC} = -2\log(L) + 2(p+q+k+1),$$

where L is the likelihood of the data,

$k = 1$ if $c \neq 0$ and $k = 0$ if $c = 0$.

Corrected AIC:

$$\text{AIC}_\text{c} = \text{AIC} + \frac{2(p+q+k+1)(p+q+k+2)}{T-p-q-k-2}.$$

Bayesian Information Criterion:

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A non-seasonal ARIMA process

$$\phi(B)(1-B)^d y_t = c + \theta(B)\varepsilon_t$$

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Hyndman and Khandakar (JSS, 2008) algorithm:

- Select no. differences d and D via unit root tests.
- Select p, q by minimising AICc.
- Use stepwise search to traverse model space.

How does auto.arima() work?

$$\text{AIC} = -2\log(L) + 2(p + q + k + 1)$$

where L is the maximised likelihood fitted to the *differenced* data, $k = 1$ if $c \neq 0$ and $k = 0$ otherwise.

Step 1: Select current model (with smallest AIC) from:
ARIMA$(2, d, 2)$
ARIMA$(0, d, 0)$
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Step 2: Consider variations of current model:
- vary one of p, q, from current model by ± 1
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Model with lowest AIC becomes current model.

Repeat Step 2 until no lower AIC can be found.

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Choosing your own model

```
tsdisplay(internet)
adf.test(internet)
kpss.test(internet)
kpss.test(diff(internet))
tsdisplay(diff(internet))
fit <- Arima(internet,order=c(3,1,0))
fit2 <- auto.arima(internet)
Acf(residuals(fit))
Box.test(residuals(fit), fitdf=3, lag=10,
  type="Ljung")
tsdiag(fit)
forecast(fit)
plot(forecast(fit))
```

Modelling procedure

1. Plot the data. Identify any unusual observations.
2. If necessary, transform the data (using a Box-Cox transformation) to stabilize the variance.
3. If the data are non-stationary: take first differences of the data until the data are stationary.
4. Examine the ACF/PACF: Is an AR(p) or MA(q) model appropriate?
5. Try your chosen model(s), and use the AIC$_c$ to search for a better model.
6. Check the residuals from your chosen model by plotting the ACF of the residuals, and doing a portmanteau test of the residuals. If they do not look like white noise, try a modified model.
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The automated algorithm only takes care of steps 3–5. So even if you use it, you will still need to take care of the other steps yourself.

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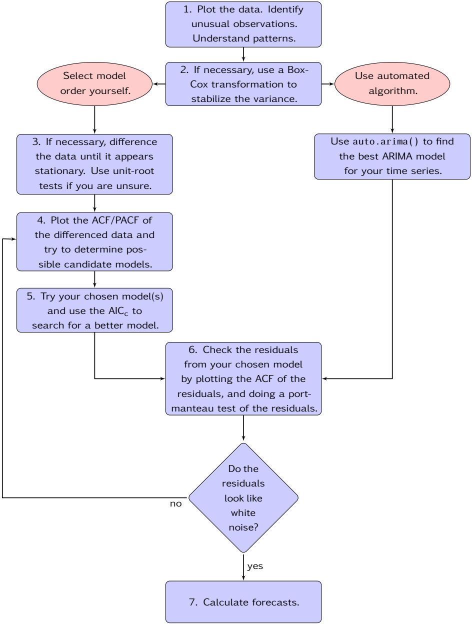
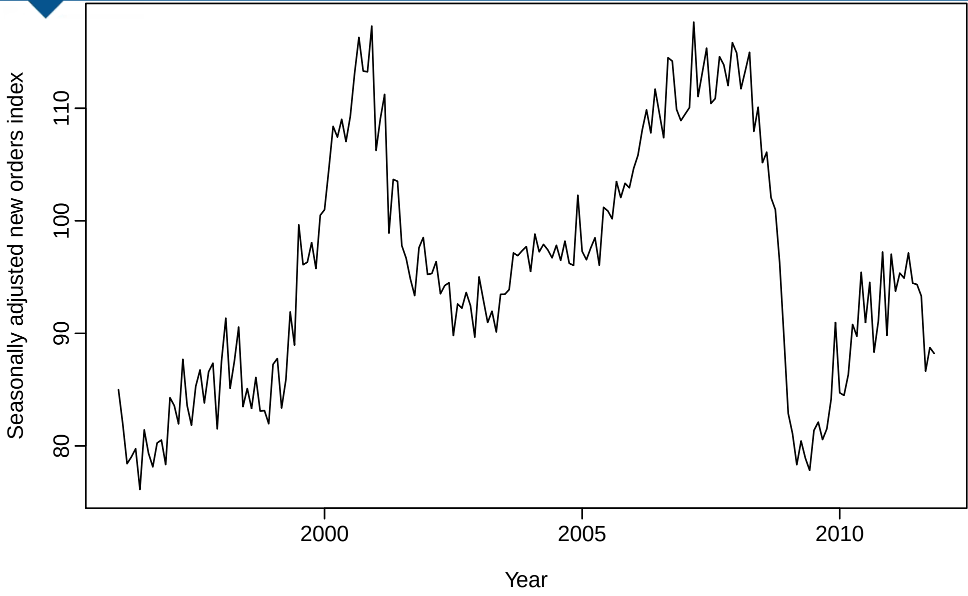


Seasonally adjusted electrical equipment



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1. Time plot shows sudden changes, particularly big drop in 2008/2009 due to global economic environment. Otherwise nothing unusual and no need for data adjustments.
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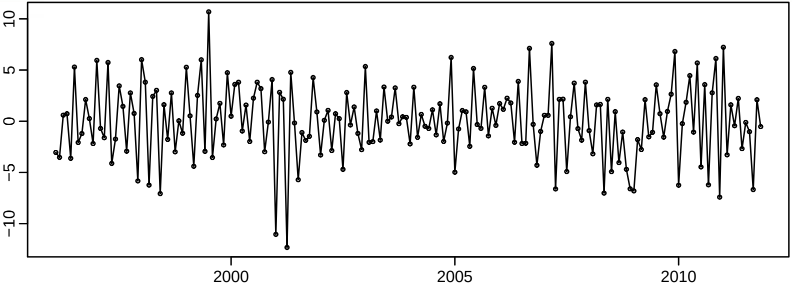

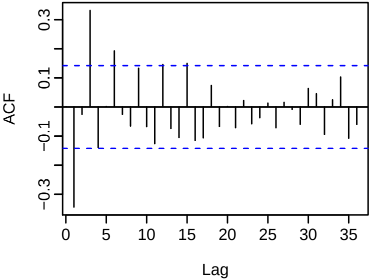

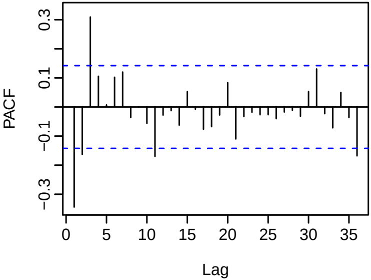

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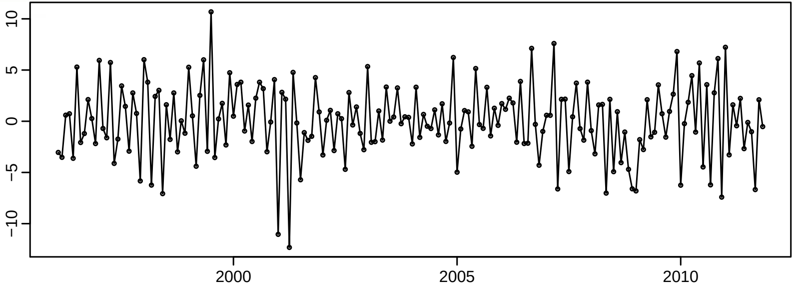

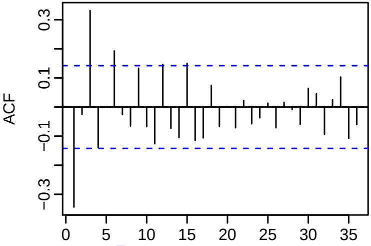

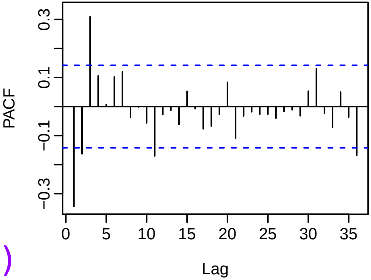


```
> tsdisplay(diff(eeadj))
```

4. PACF is suggestive of AR(3). So initial candidate model is ARIMA(3,1,0). No other obvious candidates.
5. Fit ARIMA(3,1,0) model along with variations: ARIMA(4,1,0), ARIMA(2,1,0), ARIMA(3,1,1), etc. ARIMA(3,1,1) has smallest AIC_c value.

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Seasonally adjusted electrical equipment

```
> fit <- Arima(eeadj, order=c(3,1,1))
> summary(fit)
Series: eeadj
ARIMA(3,1,1)

Coefficients:
         ar1     ar2     ar3      ma1
      0.0519  0.1191  0.3730  -0.4542
s.e.  0.1840  0.0888  0.0679   0.1993

sigma^2 estimated as 9.532:  log likelihood=-484.08
AIC=978.17   AICc=978.49   BIC=994.4
```

Seasonally adjusted electrical equipment

6. ACF plot of residuals from ARIMA(3,1,1) model look like white noise.

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Acf(residuals(fit))
Box.test(residuals(fit), lag=24,
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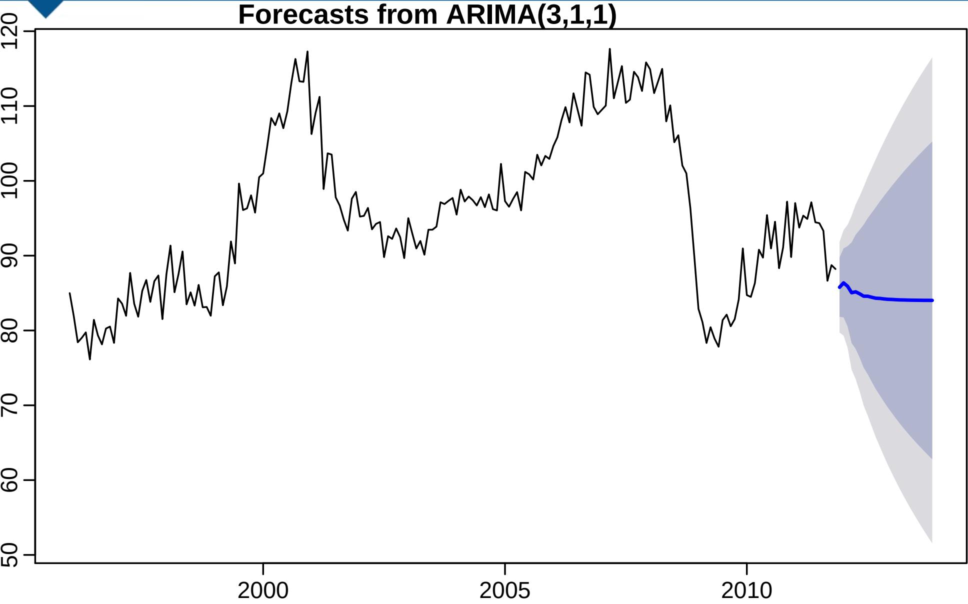

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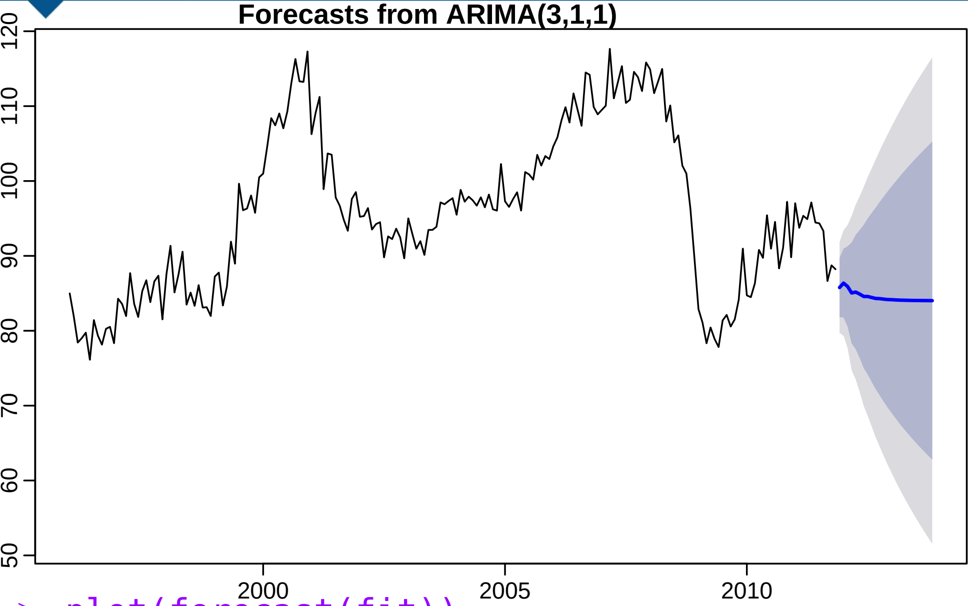


```
> plot(forecast(fit))
```

Outline

Point forecasts

1. Rearrange ARIMA equation so y_t is on LHS.
2. Rewrite equation by replacing t by $T + h$.
3. On RHS, replace future observations by their forecasts, future errors by zero, and past errors by corresponding residuals.

Start with $h = 1$. Repeat for $h = 2, 3, \dots$.

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Point forecasts

ARIMA(3,1,1) forecasts: Step 1

$$(1 - \phi_1 B - \phi_2 B^2 - \phi_3 B^3)(1 - B) y_t = (1 + \theta_1 B) e_t,$$

$$\left[1 - (1 + \phi_1)B + (\phi_1 - \phi_2)B^2 + (\phi_2 - \phi_3)B^3 + \phi_3 B^4\right] y_t = (1 + \theta_1 B) e_t,$$

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$$y_t = (1 + \phi_1) y_{t-1} - (\phi_1 - \phi_2) y_{t-2} - (\phi_2 - \phi_3) y_{t-3} - \phi_3 y_{t-4} + e_t + \theta_1 e_{t-1}.$$

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95% forecast interval

$$\hat{y}_{T+h|T} \pm 1.96\sqrt{v_{T+h|T}}$$

where $v_{T+h|T}$ is estimated forecast variance.

- $v_{T+1|T} = \hat{\sigma}^2$ for all ARIMA models regardless of parameters and orders.
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