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A Tutorial on Preview Control Systems

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Abstract: This paper gives a tutorial on preview control problems. The concept of preview control is first explained by using some practical examples. We then formulate and solve the H^∞ preview control problem based on the state augmentation technique. The application of the H^∞ preview control to the servomechanism design is also discussed. Some numerical examples show the effectiveness of the preview control.

Keywords: preview control, finite future information, H^∞ preview control, servomechanism design

1. Introduction

In many control systems, it is required that the outputs should track the reference signals in the presence of exogenous disturbances. If the future information of the reference signals or the disturbances is available, then we can greatly improve the performance of transient responses. This kind of control problems, for which future information on the reference signals and/or disturbances is utilized, is called a *preview control problem* [1]. Since the original work by Tomizuka [1] was published, a number of research works on the preview control problem have been reported from various viewpoints[1)−22)]. In particular, the H^∞ preview control[13)−19)] and the robust preview control[20)−22)] have been extensively studied over recent years.

The purpose of this paper is to introduce the notion of the preview control, and demonstrate the effectiveness of H^∞ preview control. The organization of this paper is as follows. Section 2 gives some examples for motivation of preview control problems. In Section 3, we will formulate and solve the H^∞ preview control problem. We also discuss the application of the H^∞ preview control to the servomechanism design, and give simulation results in Section 4. In Section 5, we will give concluding remarks.

2. Motivating Examples

To understand the concept and the motivation of preview control, we consider the following two examples in which the finite future information of reference signal and disturbance are utilized for control.

Example 1:
We consider the task of driving a car along a winding road. It is impossible for the driver to know complete information on the status of the road from the starting point to the destination. If the driver looks at a short range just before his/her eyes, then he/she can only steer the handle late after he/she recognizes a curve. This will make the car deviate from the lane, and will lead to a car crash. In order to drive safely, the driver usually tries to look ahead as far as possible, and steer the handle properly based on the obtained future information on the lane. This means that the driver is unconsciously doing *the control with previewed reference signal*, since the lane can be treated as a reference signal or a reference trajectory which should be tracked by the car. This observation will be helpful for the design of an automatic driving system.

Example 2:
As an industrial example, we consider a rolling stand of a tandem cold mill in steel making works[18)]. The purpose in control of the rolling stand is to maintain the accurate thickness of the exit strip and to prevent the strip from looping or being torn down. There exist several disturbances in the single rolling stand, and the entry thickness variation is a typical disturbance. However, the entry thickness variation is previewable, because the thickness of the incoming strip is normally measured by using an X-ray gauge meter before the strip comes into the rolling stand. Hence, we can improve the control performance (disturbance attenuation) by making use of the previewed information. This is *the control with previewed disturbance*.

Besides the above examples, the preview control scheme has been successfully applied to a variety of practical engineering problems such as motor control[11)], robotics[12)], power plant[3)], process control[5, 6, 7)], e.t.c.

3. H^∞ Preview Control Problem

3.1 Problem formulation

We consider the preview control for a general discrete-time system described by

$$x(t+1) = Ax(t) + Bu(t) + Ed(t) \tag{1a}$$
$$z(t) = Cx(t) + Du(t) \tag{1b}$$

where $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$ are the state vector and the control input, respectively. The signal $z(t) \in \mathbb{R}^p$ denotes the controlled output or the tracking error. Moreover, $d(t) \in \mathbb{R}^l$ denotes the exogenous signal which can be considered as the reference signal or the disturbance.

We make the following assumptions for this system.

(A1) (A, B) is stabilizable.

(A2) $\begin{bmatrix} A - e^{j\theta}I & B \\ C & D \end{bmatrix}$ has full column rank for any $\theta \in [0, 2\pi)$.

(A3) The values of $d(t)$, $d(t+1)$, $\cdots$, $d(t+h)$ are available for control, where h is a nonnegative constant that is called *a preview length*.

The purpose of the preview control is to design a controller in the form of

$$u(t) = K_x x(t) + \sum_{i=0}^{h} K_{di}\, d(t+i) \tag{2}$$

so that the following quadratic performance index is made satisfactorily small even in the presence of the exogenous input d.

$$J = \|z\|_2^2 = \sum_{t=0}^{\infty} \|z(t)\|^2 \tag{3}$$

Note that the first and second terms on the right-hand side of (2) represent the state feedback and preview compensations, respectively.

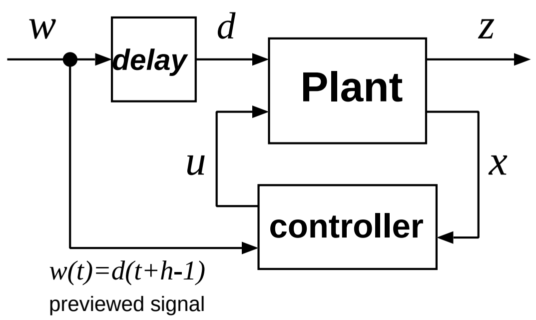

Figure 1: Preview control problem

3.2 Solution via state augmentation

We will solve the preview control problem based on the state augmentation technique.

We first defining

$$w(t) := d(t+h-1).$$

Then, the system of (1) is described as a system with time-delay in the exogenous signal, namely

$$\begin{aligned} x(t+1) &= Ax(t) + Bu(t) + Ew(t-h-1), && (4a)\\ z(t) &= Cx(t) + Du(t). && (4b) \end{aligned}$$

Then, the block diagram of the preview control problem is depicted in Figure 1.

Let $x_d(t)$ be the vector which represents the previewed information that is available for control, namely

$$x_d(t) = \begin{bmatrix} d(t) \\ d(t+1) \\ \vdots \\ d(t+h) \end{bmatrix} = \begin{bmatrix} w(t-h-1) \\ w(t-h) \\ \vdots \\ w(t-1) \end{bmatrix} \in \mathbb{R}^{l(h+1)}.$$

It is then easily seen that

$$x_d(t+1) = A_d x_d(t) + B_d w(t), \tag{5}$$

where

$$A_d = \begin{bmatrix} 0 & I & & \mathrm{O} \\ & 0 & \ddots & \\ & & \ddots & I \\ \mathrm{O} & & & 0 \end{bmatrix}, \; B_d = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ I \end{bmatrix}. \tag{6}$$

Now, define the augmented state vector

$$\xi(k) = [x^T(t) \;\; x_d^T(t)]^T. \tag{7}$$

Putting (1) and (5) together, we obtain

$$\begin{aligned} \xi(t+1) &= F\xi(t) + Gu(t) + Lw(t), && (8a)\\ z(t) &= H\xi(t) + Du(t), && (8b) \end{aligned}$$

where

$$F = \left[\begin{array}{c|cc} A & E & 0 \\ \hline 0 & \multicolumn{2}{c}{A_d} \end{array}\right], \; G = \left[\begin{array}{c} B \\ \hline 0 \end{array}\right], \; L = \left[\begin{array}{c} 0 \\ \hline B_d \end{array}\right], \tag{9a}$$

$$H = \left[\begin{array}{c|cc} C & 0 & 0 \end{array}\right]. \tag{9b}$$

It is clearly seen that the preview controller (2) is a state feedback law for the augmented system (8).

Notice that the modes corresponding to x_d is stable and uncontrollable from u. Hence, it is easy to verify that, under the assumption (A1), (F, G) is stabilizable independently of the preview length h. In other words, the stability of the closed-loop system does not depend on the preview action, because it is essentially a feedforward control.

Also, the matrix $\begin{bmatrix} F - e^{j\theta}I & G \\ H & D \end{bmatrix}$ has full column rank for every $\theta \in \mathbb{R}$ under the assumption (A2).

Since we can utilize only h step future values of $d(t)$, it is reasonable to regard $w(t) = d(t+h+1)$ as the disturbance to the system. Then, the H^∞ performance criterion

$$\sup_{w \in \ell^2} \frac{\|z\|_2}{\|w\|_2} < \gamma \tag{10}$$

naturally arises from this observation, where $\gamma > 0$ is a prescribed performance level. That is, the control $u(t)$ act on the system so that $\|z\|_2$, or J, is minimized, while $w(t)$ tries to maximize it.

Consequently, by applying the standard H^∞ control theory (Theorem A.1 in Appendix) to the system (8), we obtain the following result.

Theorem 1 *Let a positive constant γ be given. There exists a stabilizing state feedback control with preview action satisfying the H^∞ performance level (10) if and only if there exists a positive semi-definite stabilizing solution P to the following algebraic Riccati equation such that $W := \gamma^2 I - L^T PL > 0$.*

$$P = F^T PF - \begin{bmatrix} G^T PF + D^T H \\ L^T PF \end{bmatrix}^T V^{-1} \begin{bmatrix} G^T PF + D^T H \\ L^T PF \end{bmatrix} + H^T H \tag{11a}$$

$$V = \begin{bmatrix} D^T D & 0 \\ 0 & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} G^T \\ L^T \end{bmatrix} P \begin{bmatrix} G & L \end{bmatrix} \tag{11b}$$

In this case, one of the desired H^∞ preview controllers is given by (2) with

$$\begin{bmatrix} K_x & K_{d0} & \cdots & K_{dh} \end{bmatrix} = -(D^T D + G^T \hat{P} G)^{-1}(G^T \hat{P} F + D^T H),$$
$$\hat{P} = P + PLW^{-1}L^T P.$$

Next, we consider the achievable performance of the H^∞ preview control obtained by Theorem 1. In the following, we assume that $R := D^T D > 0$ for simplicity of notations. As shown in Theorems A.1 and A.2 in Appendix, the stabilizing solution to the ARE (11) can be obtained by computing the generalized eigenvectors for the Hamiltonian pencil $\lambda \mathsf{L} - \mathsf{H}$, where

$$\mathsf{L} = \begin{bmatrix} I & GR^{-1}G^T - \gamma^{-2}LL^T \\ 0 & (F - GR^{-1}D^T H)^T \end{bmatrix}, \tag{12a}$$

$$\mathsf{H} = \begin{bmatrix} F - GR^{-1}D^T H & 0 \\ -H^T(I - DR^{-1}D^T)H & I \end{bmatrix}. \tag{12b}$$

Theorem 2[19] *Suppose that D has full column rank. Assume that there exists a positive semi-definite stabilizing solution to the ARE (11) satisfying $W > 0$ for a given γ. Then, we have*

$$\gamma > \gamma_{low}, \tag{13}$$

where γ_{low} is the infimal value of γ for which the Hamiltonian pencil

$$\lambda \begin{bmatrix} I & BR^{-1}B^T - \gamma^{-2}EE^T \\ 0 & \hat{A}^T \end{bmatrix} - \begin{bmatrix} \hat{A} & 0 \\ -\hat{Q} & I \end{bmatrix}, \tag{14}$$
$$\hat{A} = A - BR^{-1}D^T C, \;\; \hat{Q} = C^T(I - DR^{-1}D^T)C$$

has no finite eigenvalue on the unit circle.

Proof: It follows from (9) and (12) that

$$\lambda \mathsf{L} - \mathsf{H} =$$
$$\left[\begin{array}{ccccc|ccccc}
\lambda I - \hat{A} & -E & 0 & \cdots & 0 & \lambda BR^{-1}B^T & & & & 0 \\
 & \lambda I & -I & \ddots & \vdots & & 0 & & & \\
 & & \lambda I & \ddots & 0 & & & \ddots & & \\
 & & & \ddots & -I & & & & 0 & \\
0 & & & & \lambda I & 0 & & & & -\lambda\gamma^{-2}I \\
\hline
-\hat{Q} & & & & 0 & \lambda \hat{A}^T - I & & & & 0 \\
 & 0 & & & & \lambda E^T & -I & & & \\
 & & 0 & & & & \lambda I & -I & & \\
 & & & \ddots & & & & \ddots & \ddots & \\
0 & & & & 0 & 0 & & & \lambda I & -I
\end{array}\right].$$

After appropriate permutations of columns and rows, application of the identity $\det \begin{bmatrix} M_1 & M_2 \\ M_3 & M_4 \end{bmatrix} = \det M_4 \det(M_1 - M_2 M_4^{-1} M_3)$ to $\lambda \mathsf{L} - \mathsf{H}$ yields

$$\det(\lambda \mathsf{L} - \mathsf{H}) =$$
$$(-\lambda)^{(h+1)l} \det \left[\begin{array}{c|c} \lambda I - \hat{A} & \lambda(BR^{-1}B^T - \gamma^{-2}EE^T) \\ \hline -\hat{Q} & \lambda \hat{A}^T - I \end{array}\right] \tag{15}$$

It is clear from Theorem A.2 that $\lambda \mathsf{L} - \mathsf{H}$ must not have any eigenvalue on the unit circle if the H^∞ preview control problem is solvable. Hence, from (15), $\gamma > \gamma_{low}$ must be satisfied for the existence of an preview H^∞ controller. ■

Remark 1: In this section, a solution to the H^∞ preview control problem is derived in terms of the Riccati equation because the Riccati-based approach is useful for characterizing the lower bound of the performance of the H^∞ preview controller as shown in Theorem 2. The approach based on linear matrix inequalities (LMIs) is also applicable to the preview control problem.

Remark 2: We can solve the H^∞ preview control problem for a continuous-time system based on the same idea as the discrete-time case, though the theory in the continuous-time case involves an infinite dimensional augmented system and operator Riccati equations. Interested readers should refer to Kojima and Ishijima[15)−17)] for the details.

3.3 Numerical example

Example 3:

We consider the system described by

$$x(t+1) = \begin{bmatrix} 1.1052 & 0 \\ 0.1223 & 1.3499 \end{bmatrix} x(t) + \begin{bmatrix} 0.1052 \\ 0.0057 \end{bmatrix} u(t)$$
$$+ \begin{bmatrix} 0 \\ 0.1166 \end{bmatrix} w(t),$$
$$z(t) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t).$$

By the bisection method, we obtain $\gamma_{low} = 0.3162$ for this system. It should also be noted that the state-feedback H^∞ control problem without preview if and only if $\gamma > \gamma_{\mathrm{noprev}} := 5.4755$. Figure 2 shows the achievable performance of the preview H^∞ controller for various preview length. Clearly, the preview controller gives much better performance than the non-preview controller. Note that, in the case of $h = 0$, since the current value of $w(t)$ is used for control, the preview controller is better than the non-preview controller. We also see from the figure that, the longer the preview length h is, the better the performance is improved. Moreover, the lower bound γ_{low} is attained for the finite preview length $h = 11$. A similar result for the continuous-time case was reported by Kojima and Ishijima[15)].

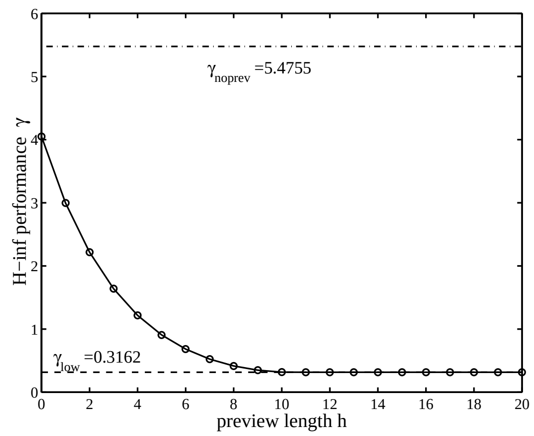

Figure 2: Achievable H^∞ performance of preview control

4. Application to Servomechanism Design

In this section, we will consider the application of the H^∞ preview control technique in the previous section to *the servomechanism design with previewed reference signal*.

Suppose that the plant is given by

$$x(t+1) = A_p x(t) + B_p u(t), \quad (16a)$$
$$y(t) = C_p x(t) + D_p u(t) \quad (16b)$$

under the following assumptions.

(B1) (A_p, B_p) is stabilizable.

(B2) (C_p, A_p) has no unobservable mode on the unit circle.

(B3) $\begin{bmatrix} A_p - I & B_p \\ C_p & D_p \end{bmatrix}$ has full row rank.

These are the standard assumptions of the servomechanism design problem. The purpose of control in this section is to design a type-1 servomechanism controller such that the output tracks the reference signal r with a good transient response in the sense that the quadratic performance index

$$J := \sum_{t=0}^{\infty} \left\{ e^T(t)Qe(t) + \tilde{u}^T(t)R\tilde{u}(t) \right\}, \quad Q, R \geq 0 \quad (17)$$
$$e(t) := r(t) - y(t), \quad \tilde{u}(t) := u(t+1) - u(t)$$

should be minimized for the reference signal $r(t)$. The physical interpretation of J is to achieve the fast asymptotic regulation of the tracking error $e(t)$ without excessive rate of change in the control input.

We assume that the current and h future values of the reference signal, i.e. $r(t)$,$r(t+1)$,$\cdots$, $r(t+h)$, are available for control $u(t)$. Note that, in the traditional servomechanism design, only the current reference signal $r(t)$, or the current tracking error $e(t)$, is utilized by the controller on the contrary to the preview control setting.

We define

$$\tilde{x}(t) = x(t+1) - x(t), \quad d(t) = r(t+1) - r(t).$$

It follows from (16) that

$$\tilde{x}(t+1) = A_p\tilde{x}(t) + B_p\tilde{u}(t), \quad (18)$$
$$e(t+1) = e(t) + d(t) - C_p\tilde{x}(t) - D_p\tilde{u}(t). \quad (19)$$

Combining these two equations, we get the augmented plant

$$\overline{x}(t+1) = A\overline{x}(t) + B\overline{u}(t) + Ed(t), \quad (20a)$$
$$z(t) = C\overline{x}(t) + D\overline{u}(t), \quad (20b)$$

where

$$\overline{x}(t) = \begin{bmatrix} \tilde{x}(t) \\ e(t) \end{bmatrix}, \quad z(t) = \begin{bmatrix} Q^{1/2}e(t) \\ R^{1/2}u(t) \end{bmatrix}$$

and

$$A = \begin{bmatrix} A_p & 0 \\ -C_p & I \end{bmatrix}, \quad B = \begin{bmatrix} B_p \\ -D_p \end{bmatrix}, \quad E = \begin{bmatrix} 0 \\ I \end{bmatrix},$$
$$C = \begin{bmatrix} 0 & Q^{1/2} \\ 0 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0 \\ R^{1/2} \end{bmatrix}.$$

In this augmented system, $\tilde{u}(t)$ and $z(t)$ are regarded as the new control input and controlled output, respectively.

The performance index J is also rewritten as

$$J = \sum_{t=0}^{\infty} \|z(t)\|^2 = \|z\|_2^2. \quad (21)$$

It is straightforward to show that, under the assumptions of (B1)–(B3), the augmented plant (20) satisfies the conditions (A1),(A2) in the previous section. Moreover, the h step future value of $d(t)$ is available for the control $\tilde{u}(t)$ since $r(0), \cdots, r(t+h)$ is previewable. This implies that (A3) is also fulfilled. Hence, we can obtain a servomechanism controller with preview action by applying Theorem 1 to the system (20) and the performance index (21). The resulting H^∞ preview controller for the augmented system has the form of

$$\tilde{u}(t) = K_x\tilde{x}(t) + K_e e(t) + \sum_{i=0}^{h} K_{ri}\, d(t),$$

where the feedback gain matrices are obtained by solving the ARE (11). Consequently, summing up the above equation yields

$$u(t) = \sum_{k=-1}^{t-1} \tilde{u}(k) = K_x x(t) + K_e \sum_{k=0}^{t-1} e(k) + \sum_{i=0}^{h} K_{ri}\, r(t). \quad (22)$$

In this equation, we have assumed that all signals are equal to zero for $t < 0$. The third term on the most right-hand side represents the preview action, while the first and second terms are the state feedback control and the integral action, respectively. The block diagram of the preview servomechanism is shown in Figure 3, in which σ denotes the time shift operator $\sigma x(t) = x(t+1)$.

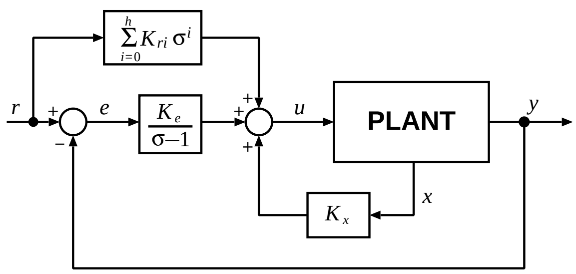

Figure 3: Servomechanism with preview action

Example 4:

We consider the plant (16) with

$$A_p = \begin{bmatrix} 0.9752 & 0.0248 & 0.1983 & 0.0017 \\ 0.0248 & 0.9752 & 0.0017 & 0.1983 \\ -0.2459 & 0.2459 & 0.9752 & 0.0248 \\ 0.2459 & -0.2459 & 0.0248 & 0.9752 \end{bmatrix},$$

$$B_p = \begin{bmatrix} -0.0199 \\ -0.0001 \\ -0.1983 \\ -0.0017 \end{bmatrix}, \quad C_p = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}, \quad D_p = 0.$$

We chose the weighting matrices Q and R by trial and errors as $Q = 20$ and $R = 1$. By the bisection method, we obtained the optimal value of H^∞ *non*-preview control as $\gamma_{\text{noprev}} = 27.3601$.

We carried out the simulation for the reference signal

$$r(t) = \begin{cases} 1, & (t \geq 20) \\ 0, & (t < 20) \end{cases}$$

The simulation results for the LQI optimal servo controller and the H^∞ controller without preview ($\gamma = 30$) are depicted in Figure 4. Figure 5 illustrates the simulation results of the H^∞ preview servomechanism system with $\gamma = 22 < \gamma_{\text{noprev}}$ for $h = 1, 5$, and 10. We can see from the figures that, in comparison with the traditional LQI optimal design, the transient response of the preview control system is significantly improved as the preview length h increases.

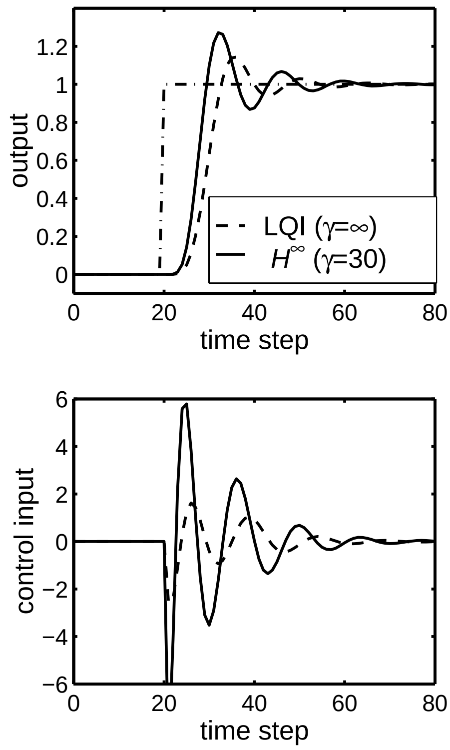

Figure 4: Simulation of servomechanisms without preview

5. Concluding Remarks

In this paper, we have given an introduction to preview control problems. In particular, we formulated the H^∞ preview control problem, and solved the problem based on the state augmentation technique: we first construct an augmented system whose state vector contains the previewed information, and then apply the standard H^∞ control theory to the augmented system. It was also shown by the numerical examples that the preview control scheme has the advantage of improving the control performance by utilizing the finite future information on the exogenous signals (reference signal, disturbance).

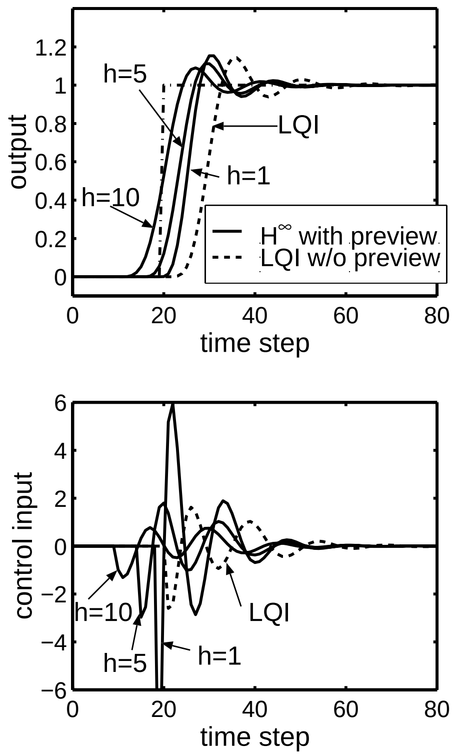

Figure 5: Simulation of H^∞ preview servomechanism

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Appendix: State Feedback H^∞ Control

We will briefly review the basic results on the standard state feedback H^∞ control problem [23].

We consider the standard state-feedback H^∞ control problem for the plant

$$x(t+1) = Ax(t) + Bu(t) + Ed(t), \qquad (23a)$$
$$z(t) = Cx(t) + Du(t), \qquad (23b)$$

under the following assumptions.

(A1) (A, B) is stabilizable.

(A2) $\begin{bmatrix} A - e^{j\theta} I & B \\ C & D \end{bmatrix}$ has full column rank for every $\theta \in [0, 2\pi)$.

Theorem A.1: *There exists a state-feedback controller achieving the H^∞ norm bound*

$$\sup_{d \in \ell^2} \frac{\|z\|_2}{\|d\|_2} < \gamma$$

for a prescribed performance level $\gamma > 0$ if and only if there exists a stabilizing solution $X \geq 0$ to the following ARE with $W := \gamma^2 I - EXE^T > 0$.

$$X = A^T XA - \begin{bmatrix} B^T XA + D^T C \\ E^T XA \end{bmatrix}^T V^{-1} \begin{bmatrix} B^T XF + D^T C \\ E^T XA \end{bmatrix} + C^T C, \qquad (24a)$$

$$V = \begin{bmatrix} D^T D & 0 \\ 0 & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} B^T \\ E^T \end{bmatrix} X \begin{bmatrix} B & E \end{bmatrix}. \qquad (24b)$$

Moreover, if such a solution exists, one of the feedback gain achieving the H^∞ performance is given by $u(t) = Kx(t)$ with

$$K = -(D^T D + B^T \hat{X} B)^{-1} (B^T \hat{X} A + D^T C),$$
$$\hat{X} = X + XEW^{-1} E^T X.$$

In addition to (A1) and (A2), we assume that $R := D^T D > 0$ holds for simplicity. We define

$$\mathsf{L} = \begin{bmatrix} I & BR^{-1}B^T - \gamma^{-2} EE^T \\ 0 & \hat{A}^T \end{bmatrix}, \quad \mathsf{H} = \begin{bmatrix} \hat{A} & 0 \\ -\hat{Q} & I \end{bmatrix},$$
$$\hat{A} = A - BR^{-1} D^T C, \quad \hat{Q} = C^T (I - DR^{-1} D^T) C.$$

We define Λ as the Jordan matrix associated with the stable finite eigenvalues of $\lambda \mathsf{L} - \mathsf{H}$. Let $V = \mathrm{col}(V_1, V_2)$ be a matrix formed by the generalized eigenvectors corresponding to the stable finite eigenvalues. Then, we have $\mathsf{L} V \Lambda = \mathsf{H} V$, or equivalently

$$\begin{bmatrix} I & BR^{-1}B^T - \gamma^{-2} EE^T \\ 0 & \hat{A}^T \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \Lambda = \begin{bmatrix} \hat{A} & 0 \\ -\hat{Q} & I \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}.$$

Theorem A.2: *There exists a stabilizing solution to the ARE (24) if and only if*

(i) $\lambda \mathsf{L} - \mathsf{H}$ has no eigenvalues on the unit circle, and

(ii) V_1 is invertible.

If these conditions are satisfied, the stabilizing solution of (24) is uniquely obtained by $X = V_2 V_1^{-1}$.