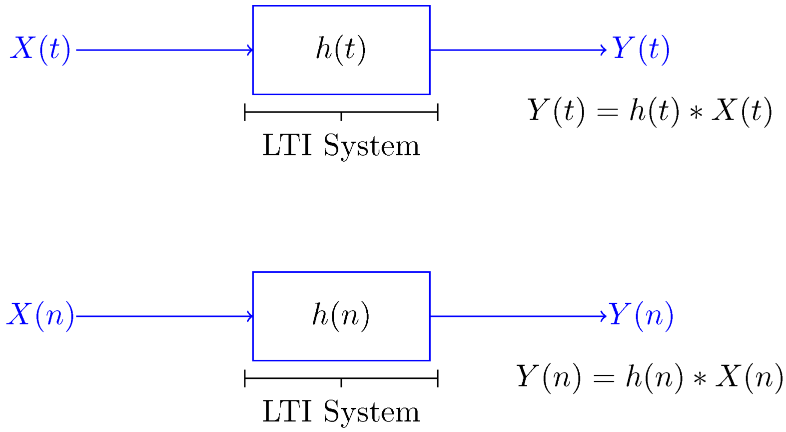

Figure 10.6 - LTI systems.

LTI Systems with Random Inputs:

Consider an LTI system with impulse response $h(t)$. Let $X(t)$ be a WSS random process. If $X(t)$ is the input of the system, then the output, $Y(t)$, is also a random process. More specifically, we can write

$$\begin{aligned} Y(t) &= h(t) * X(t) \\ &= \int_{-\infty}^{\infty} h(\alpha) X(t-\alpha)\, d\alpha. \end{aligned}$$

Here, our goal is to show that $X(t)$ and $Y(t)$ are jointly WSS processes. Let's first start by calculating the mean function of $Y(t)$, $\mu_Y(t)$. We have

$$\begin{aligned} \mu_Y(t) = E[Y(t)] &= E\left[\int_{-\infty}^{\infty} h(\alpha) X(t-\alpha)\, d\alpha\right] \\ &= \int_{-\infty}^{\infty} h(\alpha) E[X(t-\alpha)]\, d\alpha \\ &= \int_{-\infty}^{\infty} h(\alpha) \mu_X\, d\alpha \\ &= \mu_X \int_{-\infty}^{\infty} h(\alpha)\, d\alpha. \end{aligned}$$

We note that $\mu_Y(t)$ is not a function of t, so we can write

$$\mu_Y(t) = \mu_Y = \mu_X \int_{-\infty}^{\infty} h(\alpha)\, d\alpha.$$

Let's next find the cross-correlation function, $R_{XY}(t_1, t_2)$. We have