10.1.3 Multiple Random Processes

We often need to study more than one random process. For example, when investing in the stock market you consider several different stocks and you are interested in how they are related. In particular, you might be interested in finding out whether two stocks are positively or negatively correlated. A useful idea in these situations is to look at **cross-correlation** and **cross-covariance** functions.

For two random processes $\big\{X(t), t \in J\big\}$ and $\big\{Y(t), t \in J\big\}$:

– the **cross-correlation** function $R_{XY}(t_1, t_2)$, is defined by

$$R_{XY}(t_1, t_2) = E[X(t_1)Y(t_2)], \quad \text{for } t_1, t_2 \in J;$$

– the **cross-covariance** function $C_{XY}(t_1, t_2)$, is defined by

$$\begin{aligned} C_{XY}(t_1, t_2) &= \text{Cov}\big(X(t_1), Y(t_2)\big) \\ &= R_{XY}(t_1, t_2) - \mu_X(t_1)\mu_Y(t_2), \quad \text{for } t_1, t_2 \in J. \end{aligned}$$

To get an idea about these concepts suppose that $X(t)$ is the price of oil (per gallon) and $Y(t)$ is the price of gasoline (per gallon) at time t. Since gasoline is produced from oil, as oil prices increase, the gasoline prices tend to increase, too. Thus, we conclude that $X(t)$ and $Y(t)$ should be positively correlated (at least for the same t, i.e., $C_{XY}(t, t) > 0$).

Example 10.6

Let A, B, and C be independent normal $N(1, 1)$ random variables. Let $\big\{X(t), t \in [0, \infty)\big\}$ be defined as

$$X(t) = A + Bt, \quad \text{for all } t \in [0, \infty).$$

Also, let $\big\{Y(t), t \in [0, \infty)\big\}$ be defined as

$$Y(t) = A + Ct, \quad \text{for all } t \in [0, \infty).$$