8.4.3 Hypothesis Testing for the Mean

Here, we would like to discuss some common hypothesis testing problems. We assume that we have a random sample X_1,X_2,...,X_n from a distribution and our goal is to make inference about the mean of the distribution μ. We consider three hypothesis testing problems. The first one is a test to decide between the following hypotheses:

H_0: $\mu = \mu_0$,

H_1: $\mu \neq \mu_0$.

In this case, the null hypothesis is a simple hypothesis and the alternative hypothesis is a *two-sided* hypothesis (i.e., it includes both $\mu < \mu_0$ and $\mu > \mu_0$). We call this hypothesis test a *two-sided* test. The second and the third cases are *one-sided* tests. More specifically, the second case is

H_0: $\mu \leq \mu_0$,

H_1: $\mu > \mu_0$.

Here, both H_0 and H_1 are one-sided, so we call this test a *one-sided* test. The third case is very similar to the second case. More specifically, the third scenario is

H_0: $\mu \geq \mu_0$,

H_1: $\mu < \mu_0$.

In all of the three cases, we use the sample mean

$$\overline{X} = \frac{X_1 + X_2 + \ldots + X_n}{n}$$

to define our statistic. In particular, if we know the variance of the X_i's, $\mathrm{Var}(X_i) = \sigma^2$, then we define our test statistic as the normalized sample mean (assuming H_0):

$$W(X_1, X_2, \cdots, X_n) = \frac{\overline{X} - \mu_0}{\sigma / \sqrt{n}}.$$