9.1.7 Estimation for Random Vectors

The examples that we have seen so far involved only two random variables X and Y. In practice, we often need to estimate several random variables and we might observe several random variables. In other words, we might want to estimate the value of an unobserved random vector $\mathbf{X}$:

$$\mathbf{X} = \begin{bmatrix} X_1 \\ X_2 \\ . \\ . \\ . \\ X_m \end{bmatrix},$$

given that we have observed the random vector $\mathbf{Y}$,

$$\mathbf{Y} = \begin{bmatrix} Y_1 \\ Y_2 \\ . \\ . \\ . \\ Y_n \end{bmatrix}.$$

Almost everything that we have discussed can be extended to the case of random vectors. For example, to find the MMSE estimate of $\mathbf{X}$ given $\mathbf{Y} = \mathbf{y}$, we can write

$$\hat{\mathbf{X}}_M = E[\mathbf{X}|\mathbf{Y}] = \begin{bmatrix} E[X_1|Y_1, Y_2, \cdots, Y_n] \\ E[X_2|Y_1, Y_2, \cdots, Y_n] \\ . \\ . \\ . \\ E[X_m|Y_1, Y_2, \cdots, Y_n] \end{bmatrix}.$$

However, the above conditional expectations might be too complicated computationally. Therefore, for random vectors, it is very common to consider simpler estimators such as the linear MMSE. Let's now discuss linear MMSE for random vectors.

Linear MMSE for Random Vectors: