Solution

We need to find the Fourier transform of $R_X(\tau)$. We can do this by looking at a Fourier transform table or by finding the Fourier transform directly as follows.

$$\begin{aligned} S_X(f) &= \mathcal{F}\{R_X(\tau)\} \\ &= \int_{-\infty}^{\infty} e^{-a|\tau|} e^{-2j\pi f\tau}\, d\tau \\ &= \int_{-\infty}^{0} e^{a\tau} e^{-2j\pi f\tau}\, d\tau + \int_{0}^{\infty} e^{-a\tau} e^{-2j\pi f\tau}\, d\tau \\ &= \frac{1}{a - j2\pi f} + \frac{1}{a + j2\pi f} \\ &= \frac{2a}{a^2 + 4\pi^2 f^2}. \end{aligned}$$

Cross Spectral Density:

For two jointly WSS random processes $X(t)$ and $Y(t)$, we define the *cross spectral density* $S_{XY}(f)$ as the Fourier transform of the cross-correlation function $R_{XY}(\tau)$,

$$S_{XY}(f) = \mathcal{F}\{R_{XY}(\tau)\} = \int_{-\infty}^{\infty} R_{XY}(\tau) e^{-2j\pi f\tau}\, d\tau.$$