1.2.5 Solved Problems: Review of Set Theory

Problem 1

Let A, B, C be three sets as shown in the following Venn diagram. For each of the following sets, draw a Venn diagram and shade the area representing the given set.

a. $A \cup B \cup C$
b. $A \cap B \cap C$
c. $A \cup (B \cap C)$
d. $A - (B \cap C)$
e. $A \cup (B \cap C)^c$

Solution

Figure 1.15 shows Venn diagrams for these sets.

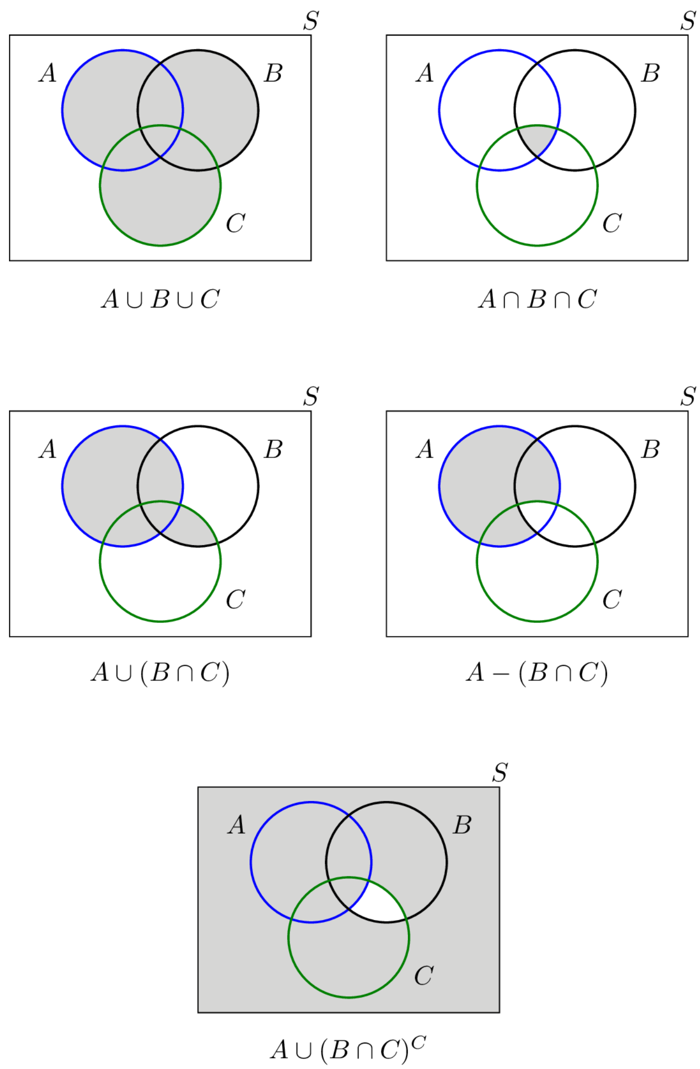

Fig.1.15 - Venn diagrams for different sets.

Problem 2

Using Venn diagrams, verify the following identities.

a. $A = (A \cap B) \cup (A - B)$

b. If A and B are finite sets, we have

$$|A \cup B| = |A| + |B| - |A \cap B| \tag{1.2}$$

Solution

Figure 1.16 pictorially verifies the given identities. Note that in the second identity, we show the number of elements in each set by the corresponding shaded area.

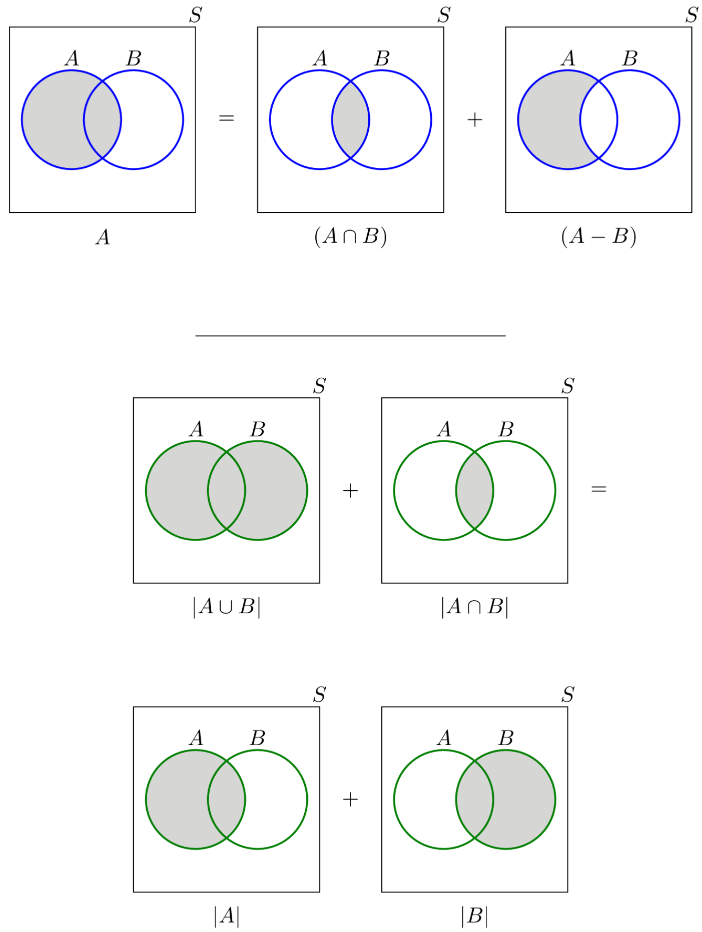

Fig.1.16 - Venn diagrams for some identities.

Problem 3

Let $S = \{1, 2, 3\}$. Write all the possible partitions of S.

Solution

Remember that a partition of S is a collection of nonempty sets that are disjoint and their union is S. There are 5 possible partitions for $S = \{1, 2, 3\}$:

1. $\{1\}, \{2\}, \{3\}$;

2. $\{1,2\},\{3\}$;
3. $\{1,3\},\{2\}$;
4. $\{2,3\},\{1\}$;
5. $\{1,2,3\}$.

Problem 4

Determine whether each of the following sets is countable or uncountable.

a. $A = \{x \in \mathbb{Q} | -100 \leq x \leq 100\}$
b. $B = \{(x,y) | x \in \mathbb{N}, y \in \mathbb{Z}\}$
c. $C = (0, 0.1]$
d. $D = \{\frac{1}{n} | n \in \mathbb{N}\}$

Solution

a. $A = \{x \in \mathbb{Q} | -100 \leq x \leq 100\}$ is **countable** since it is a subset of a countable set, $A \subset \mathbb{Q}$.
b. $B = \{(x,y) | x \in \mathbb{N}, y \in \mathbb{Z}\}$ is **countable** because it is the Cartesian product of two countable sets, i.e., $B = \mathbb{N} \times \mathbb{Z}$.
c. $C = (0, .1]$ is **uncountable** since it is an interval of the form $(a,b]$, where $a < b$.
d. $D = \{\frac{1}{n} | n \in \mathbb{N}\}$ is **countable** since it is in one-to-one correspondence with the set of natural numbers. In particular, you can list all the elements in the set D, $D = \{1, \frac{1}{2}, \frac{1}{3}, \cdots\}$.

Problem 5

Find the range of the function $f : \mathbb{R} \to \mathbb{R}$ defined as $f(x) = \sin(x)$.

Solution

For any real value x, $-1 \leq \sin(x) \leq 1$. Also, all values in $[-1,1]$ are covered by $\sin(x)$. Thus, Range$(f) = [-1,1]$.
