10.2.2 Linear Time-Invariant (LTI) Systems with Random Inputs

Linear Time-Invariant (LTI) Systems:

A **linear time-invariant (LTI)** system can be represented by its **impulse response** (Figure 10.6). More specifically, if $X(t)$ is the input signal to the system, the output, $Y(t)$, can be written as

$$Y(t) = \int_{-\infty}^{\infty} h(\alpha)X(t-\alpha)\ d\alpha = \int_{-\infty}^{\infty} X(\alpha)h(t-\alpha)\ d\alpha.$$

The above integral is called the *convolution* of h and X, and we write

$$Y(t) = h(t) * X(t) = X(t) * h(t).$$

Note that as the name suggests, the impulse response can be obtained if the input to the system is chosen to be the unit impulse function (delta function) $x(t) = \delta(t)$. For discrete-time systems, the output can be written as (Figure 10.6)

$$\begin{aligned} Y(n) &= h(n) * X(n) = X(n) * h(n) \\ &= \sum_{k=-\infty}^{\infty} h(k)X(n-k) = \sum_{k=-\infty}^{\infty} X(k)h(n-k). \end{aligned}$$

The discrete-time unit impulse function is defined as

$$\delta(n) = \begin{cases} 1 & n = 0 \\ & \\ 0 & \text{otherwise} \end{cases}$$

For the rest of this chapter, we mainly focus on continuous-time signals.