10.2.3 Power in a Frequency Band

Here, we would like show that if you integrate $S_X(f)$ over a frequency range, you will obtain the expected power in $X(t)$ in that frequency range. Let's first define what we mean by the expected power "in a frequency range." Consider a WSS random process $X(t)$ that goes through an LTI system with the following transfer function (Figure 10.7):

$$H(f) = \begin{cases} 1 & f_1 < |f| < f_2 \\ \\ 0 & \text{otherwise} \end{cases}$$

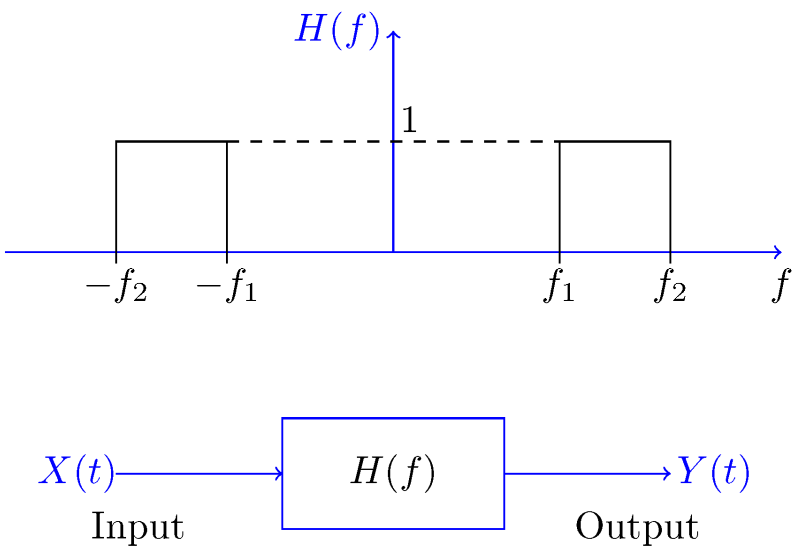

Figure 10.7 - A bandpass filter.

This is in fact a bandpass filter. This filter eliminates every frequency outside of the frequency band $f_1 < |f| < f_2$. Thus, the resulting random process $Y(t)$ is a filtered version of $X(t)$ in which frequency components in the frequency band $f_1 < |f| < f_2$ are preserved. The expected power in $Y(t)$ is said to be the expected power in $X(t)$ in the frequency range $f_1 < |f| < f_2$.

Now, let's find the expected power in $Y(t)$. We have

$$S_Y(f) = S_X(f)|H(f)|^2 = \begin{cases} S_X(f) & f_1 < |f| < f_2 \\ \\ 0 & \text{otherwise} \end{cases}$$

Thus, the power in $Y(t)$ is