1.3.1 Random Experiments

Before rolling a die you do not know the result. This is an example of a **random experiment**. In particular, a random experiment is a process by which we observe something uncertain. After the experiment, the result of the random experiment is known. An **outcome** is a result of a random experiment. The set of all possible outcomes is called the **sample space**. Thus in the context of a random experiment, the sample space is our *universal set*. Here are some examples of random experiments and their sample spaces:

- Random experiment: toss a coin; sample space: $S = \{heads, tails\}$ or as we usually write it, $\{H, T\}$.
- Random experiment: roll a die; sample space: $S = \{1, 2, 3, 4, 5, 6\}$.
- Random experiment: observe the number of iPhones sold by an Apple store in Boston in 2015; sample space: $S = \{0, 1, 2, 3, \cdots\}$.
- Random experiment: observe the number of goals in a soccer match; sample space: $S = \{0, 1, 2, 3, \cdots\}$.

When we repeat a random experiment several times, we call each one of them a **trial**. Thus, a trial is a particular performance of a random experiment. In the example of tossing a coin, each trial will result in either heads or tails. Note that the sample space is defined based on how you define your random experiment. For example,

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Example 1.7
We toss a coin three times and observe the sequence of heads/tails. The sample space here may be defined as

$$S = \{(H, H, H), (H, H, T), (H, T, H), (T, H, H), (H, T, T), (T, H, T), (T, T, H), (T, T, T)\}.$$

Our goal is to assign probability to certain **events**. For example, suppose that we would like to know the probability that the outcome of rolling a fair die is an even number. In this case, our event is the set $E = \{2, 4, 6\}$. If the result of our random experiment belongs to the set E, we say that the event E has occurred. Thus an event is a collection of possible outcomes. In other words, an event is a subset of the sample space to which we assign a probability. Although we have not yet discussed how to find the probability of an event, you might be able to guess that the probability of $\{2, 4, 6\}$ is 50 percent which is the same as $\frac{1}{2}$ in the probability theory convention.

Outcome: A result of a random experiment. Sample Space: The set of all possible outcomes. Event: A subset of the sample space.

Union and Intersection: If A and B are events, then $A \cup B$ and $A \cap B$ are also events. By remembering the definition of union and intersection, we observe that $A \cup B$ occurs if A or B occur. Similarly, $A \cap B$ occurs if both A and B occur. Similarly, if $A_1, A_2, \cdots, A_n$ are events, then the event $A_1 \cup A_2 \cup A_3 \cdots \cup A_n$ occurs if at least one of $A_1, A_2, \cdots, A_n$ occurs. The event $A_1 \cap A_2 \cap A_3 \cdots \cap A_n$ occurs if all of $A_1, A_2, \cdots, A_n$ occur. It can be helpful to remember that the key words "or" and "at least" correspond to unions and the key words "and" and "all of" correspond to intersections.
