1.2.4 Functions

We often need the concept of functions in probability. A function f is a rule that takes an input from a specific set, called the **domain**, and produces an output from another set, called **co-domain**. Thus, a function *maps* elements from the domain set to elements in the co-domain with the property that each input is mapped to exactly one output. For a function f, if x is an element in the domain, then the function value (the output of the function) is shown by $f(x)$. If A is the domain and B is the co-domain for the function f, we use the following notation:

$$f : A \rightarrow B.$$

Example 1.6

- Consider the function $f : \mathbb{R} \rightarrow \mathbb{R}$, defined as $f(x) = x^2$. This function takes any real number x and outputs x^2. For example, $f(2) = 4$.
- Consider the function $g : \{H, T\} \rightarrow \{0, 1\}$, defined as $g(H) = 0$ and $g(T) = 1$. This function can only take two possible inputs H or T, where H is mapped to 0 and T is mapped to 1.

The output of a function $f : A \rightarrow B$ always belongs to the co-domain B. However, not all values in the co-domain are always covered by the function. In the above example, $f : \mathbb{R} \rightarrow \mathbb{R}$, the function value is always a positive number $f(x) = x^2 \geq 0$. We define the **range** of a function as the set containing all the possible values of $f(x)$. Thus, the range of a function is always a subset of its co-domain. For the above function $f(x) = x^2$, the range of f is given by

$$\text{Range}(f) = \mathbb{R}^+ = \{x \in \mathbb{R} | x \geq 0\}.$$

Figure 1.14 pictorially shows a function, its domain, co-domain, and range. The figure shows that an element x in the domain is mapped to $f(x)$ in the range.

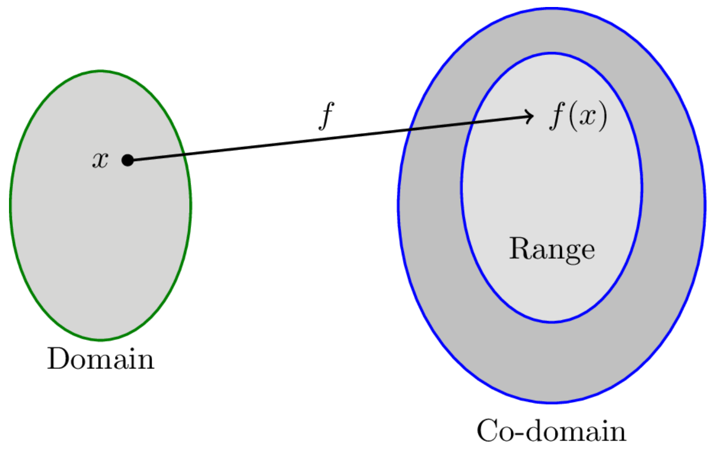

Fig.1.14 - Function $f : A \to B$, the range is always a subset of the co-domain.
