10.1.2 Mean and Correlation Functions

Since random processes are collections of random variables, you already possess the theoretical knowledge necessary to analyze random processes. From now on, we would like to discuss methods and tools that are useful in studying random processes. Remember that expectation and variance were among the important statistics that we considered for random variables. Here, we would like to extend those concepts to random processes.

Mean Function of a Random Process:

Mean Function of a Random Process

For a random process $\big\{X(t), t \in J\big\}$, the **mean function** $\mu_X(t) : J \rightarrow \mathbb{R}$, is defined as

$$\mu_X(t) = E[X(t)]$$

The above definition is valid for both continuous-time and discrete-time random processes. In particular, if $\big\{X_n, n \in J\big\}$ is a discrete-time random process, then

$$\mu_X(n) = E[X_n], \quad \text{for all } n \in J.$$

Some books show the mean function by $m_X(t)$ or $M_X(t)$. Here, we chose $\mu_X(t)$ to avoid confusion with moment generating functions. The mean function gives us an idea about how the random process behaves on average as time evolves. For example, if $X(t)$ is the temperature in a certain city, the mean function $\mu_X(t)$ might look like the function shown in Figure 10.3. As we see, the expected value of $X(t)$ is lowest in the winter and highest in summer.