This equation might look a little confusing at first, but it is just another way of writing the law of total expectation (Equation 5.4). To better understand it, let's solve Example 5.7 using this terminology. In that example, we want to find EX. We can write

$$\begin{aligned} E[X] &= E[E[X|N]] \\ &= E[Np] \qquad\qquad \text{(since } X|N \sim Binomial(N,p)) \\ &= pE[N] = p\lambda. \end{aligned}$$

Equation 5.7 is called the *law of iterated expectations*. Since it is basically the same as Equation 5.4, it is also called the law of total expectation [3].

$$\boxed{\text{Law of Iterated Expectations: } E[X] = E[E[X|Y]]}$$

Expectation for Independent Random Variables:

Note that if two random variables X and Y are independent, then the conditional PMF of X given Y will be the same as the marginal PMF of X, i.e., for any $x \in R_X$, we have

$$P_{X|Y}(x|y) = P_X(x).$$

Thus, for independent random variables, we have

$$\begin{aligned} E[X|Y=y] &= \sum_{x \in R_X} x P_{X|Y}(x|y) \\ &= \sum_{x \in R_X} x P_X(x) \\ &= E[X]. \end{aligned}$$

Again, thinking of this as a random variable depending on Y, we obtain

$$E[X|Y] = E[X], \text{ when } X \text{ and } Y \text{ are independent.}$$

More generally, if X and Y are independent then any function of X, say $g(X)$, and Y are independent, thus

$$E[g(X)|Y] = E[g(X)].$$

Remember that for independent random variables, $P_{XY}(x,y) = P_X(x)P_Y(y)$. From this, we can show that $E[XY] = EXEY$.