6.1.3 Moment Generating Functions

Here, we will introduce and discuss **moment generating functions (MGFs)**. Moment generating functions are useful for several reasons, one of which is their application to analysis of sums of random variables. Before discussing MGFs, let's define moments.

Definition 6.2. The ***n*th moment** of a random variable X is defined to be $E[X^n]$. The ***n*th central moment** of X is defined to be $E[(X - EX)^n]$.

For example, the first moment is the expected value $E[X]$. The second central moment is the variance of X. Similar to mean and variance, other moments give useful information about random variables.

The moment generating function (MGF) of a random variable X is a function $M_X(s)$ defined as

$$M_X(s) = E\left[e^{sX}\right].$$

We say that MGF of X exists, if there exists a positive constant a such that $M_X(s)$ is finite for all $s \in [-a, a]$.

Before going any further, let's look at an example.

Example 6.3

For each of the following random variables, find the MGF.

a. X is a discrete random variable, with PMF

$$P_X(k) = \begin{cases} \frac{1}{3} & k = 1 \\ \\ \frac{2}{3} & k = 2 \end{cases}$$