
8.2.4 Asymptotic Properties of MLEs

We end this section by mentioning that MLEs have some nice asymptotic properties. By asymptotic properties we mean properties that are true when the sample size becomes large. Here, we state these properties without proofs.

Asymptotic Properties of MLEs

Let $X_1, X_2, X_3, \dots, X_n$ be a random sample from a distribution with a parameter θ . Let $\hat{\Theta}_{ML}$ denote the maximum likelihood estimator (MLE) of θ . Then, under some mild regularity conditions,

1. $\hat{\Theta}_{ML}$ is asymptotically consistent, i.e.,

$$\lim_{n \rightarrow \infty} P(|\hat{\Theta}_{ML} - \theta| > \epsilon) = 0.$$

- Item $\hat{\Theta}_{ML}$ is asymptotically unbiased, i.e.,

$$\lim_{n \rightarrow \infty} E[\hat{\Theta}_{ML}] = \theta.$$

2. As n becomes large, $\hat{\Theta}_{ML}$ is approximately a normal random variable. More precisely, the random variable

$$\frac{\hat{\Theta}_{ML} - \theta}{\sqrt{\text{Var}(\hat{\Theta}_{ML})}}$$

converges in distribution to $N(0, 1)$.
