4.2.1 Uniform Distribution

We have already seen the uniform distribution. In particular, we have the following definition:

A continuous random variable X is said to have a *Uniform* distribution over the interval $[a, b]$, shown as $X \sim Uniform(a, b)$, if its PDF is given by

$$f_X(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & x < a \text{ or } x > b \end{cases}$$

We have already found the CDF and the expected value of the uniform distribution. In particular, we know that if $X \sim Uniform(a, b)$, then its CDF is given by equation 4.1 in example 4.1, and its mean is given by

$$EX = \frac{a+b}{2}$$

To find the variance, we can find EX^2 using LOTUS:

$$\begin{aligned} EX^2 &= \int_{-\infty}^{\infty} x^2 f_X(x) dx \\ &= \int_a^b x^2 \left(\frac{1}{b-a}\right) dx \\ &= \frac{a^2+ab+b^2}{3}. \end{aligned}$$

Therefore,

$$\begin{aligned} \text{Var}(X) &= EX^2 - (EX)^2 \\ &= \frac{(b-a)^2}{12}. \end{aligned}$$