5.1.1 Joint Probability Mass Function (PMF)

Remember that for a discrete random variable X, we define the PMF as $P_X(x) = P(X = x)$. Now, if we have two random variables X and Y, and we would like to study them jointly, we define the **joint probability mass function** as follows:

The **joint probability mass function** of two discrete random variables X and Y is defined as

$$P_{XY}(x, y) = P(X = x, Y = y).$$

Note that as usual, the comma means "and," so we can write

$$\begin{aligned} P_{XY}(x, y) &= P(X = x, Y = y) \\ &= P\big((X = x) \text{ and } (Y = y)\big). \end{aligned}$$

We can define the joint range for X and Y as

$$R_{XY} = \{(x, y) | P_{XY}(x, y) > 0\}.$$

In particular, if $R_X = \{x_1, x_2, \ldots\}$ and $R_Y = \{y_1, y_2, \ldots\}$, then we can always write

$$\begin{aligned} R_{XY} &\subset R_X \times R_Y \\ &= \{(x_i, y_j) | x_i \in R_X, y_j \in R_Y\}. \end{aligned}$$

In fact, sometimes we define $R_{XY} = R_X \times R_Y$ to simplify the analysis. In this case, for some pairs (x_i, y_j) in $R_X \times R_Y$, $P_{XY}(x_i, y_j)$ might be zero. For two discrete random variables X and Y, we have

$$\sum_{(x_i, y_j) \in R_{XY}} P_{XY}(x_i, y_j) = 1$$