$X \sim Poisson(\lambda)$

PMF:

$$P_X(k) = \frac{e^{-\lambda}\lambda^k}{k!} \quad \text{for } k = 0, 1, 2, \cdots$$

Moment Generating Function (MGF):

$$M_X(s) = e^{\lambda(e^s - 1)}$$

Characteristic Function:

$$\phi_X(\omega) = e^{\lambda\left(e^{i\omega} - 1\right)}$$

Expected Value:

$$EX = \lambda$$

Variance:

$$\mathrm{Var}(X) = \lambda$$

MATLAB:

R = poissrnd($\lambda$)

## Continuous Distributions

$X \sim Exponential(\lambda)$

PDF:

$$f_X(x) = \lambda e^{-\lambda x}, \quad x > 0$$

CDF:

$$F_X(x) = 1 - e^{-\lambda x}, \quad x > 0$$

Moment Generating Function (MGF):

$$M_X(s) = \left(1 - \frac{s}{\lambda}\right)^{-1} \quad \text{for} \quad s < \lambda$$

Characteristic Function:

$$\phi_X(\omega) = \left(1 - \frac{i\omega}{\lambda}\right)^{-1}$$

Expected Value:

$$EX = \frac{1}{\lambda}$$

Variance:

$$\mathrm{Var}(X) = \frac{1}{\lambda^2}$$

MATLAB:

R = exprnd($\mu$), where $\mu = \frac{1}{\lambda}$.

$X \sim Laplace(\mu, b)$

PDF:

$$f_X(x) = \frac{1}{2b}\exp\left(-\frac{|x-\mu|}{b}\right) = \begin{cases} \frac{1}{2b}\exp\left(\frac{x-\mu}{b}\right) & \text{if } x < \mu \\ \frac{1}{2b}\exp\left(-\frac{x-\mu}{b}\right) & \text{if } x \geq \mu \end{cases}$$

CDF:

$$F_X(x) = \begin{cases} \frac{1}{2}\exp\left(\frac{x-\mu}{b}\right) & \text{if } x < \mu \\ 1 - \frac{1}{2}\exp\left(-\frac{x-\mu}{b}\right) & \text{if } x \geq \mu \end{cases}$$

Moment Generating Function (MGF):

$$M_X(s) = \frac{e^{\mu s}}{1 - b^2 s^2} \quad \text{for} \quad |s| < \frac{1}{b}$$

Characteristic Function:

$$\phi_X(\omega) = \frac{e^{\mu i \omega}}{1 + b^2 \omega^2}$$

Expected Value:

$$EX = \mu$$

Variance:

$$\mathrm{Var}(X) = 2b^2$$

## $X \sim N(\mu, \sigma^2)$ (Gaussian Distribution)

PDF:

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

CDF:

$$F_X(x) = \Phi\left(\frac{x-\mu}{\sigma}\right)$$

Moment Generating Function (MGF):

$$M_X(s) = e^{\mu s + \frac{1}{2}\sigma^2 s^2}$$

Characteristic Function:

$$\phi_X(\omega) = e^{i\mu\omega - \frac{1}{2}\sigma^2\omega^2}$$

Expected Value:

$$EX = \mu$$

Variance:

$$\mathrm{Var}(X) = \sigma^2$$

MATLAB:

Z = randn, R = normrnd($\mu$,$\sigma$)

$X \sim Beta(a, b)$

PDF:

$$f_X(x) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{(a-1)}(1-x)^{(b-1)}, \quad \text{for } 0 \leq x \leq 1$$

Moment Generating Function (MGF):

$$M_X(s) = 1 + \sum_{k=1}^{\infty} \left( \prod_{r=0}^{k-1} \frac{a+r}{a+b+r} \right) \frac{s^k}{k!}$$

Expected Value:

$$EX = \frac{a}{a+b}$$

Variance:

$$\mathrm{Var}(X) = \frac{ab}{(a+b)^2(a+b+1)}$$

MATLAB:

R = betarnd($a$,$b$)

$X \sim \chi^2(n)$ (Chi-squared)

Note:

$$\chi^2(n) = Gamma\left(\frac{n}{2}, \frac{1}{2}\right)$$

PDF:

$$f_X(x) = \frac{1}{2^{\frac{n}{2}}\Gamma\left(\frac{n}{2}\right)} x^{\frac{n}{2}-1} e^{-\frac{x}{2}}, \quad \text{for } x > 0.$$

Moment Generating Function (MGF):

$$M_X(s) = (1-2s)^{-\frac{n}{2}} \quad \text{for} \quad s < \frac{1}{2}$$

Characteristic Function:

$$\phi_X(\omega) = (1-2i\omega)^{-\frac{n}{2}}$$

Expected Value:

$$EX = n$$

Variance:

$$\text{Var}(X) = 2n$$

MATLAB:

R = chi2rnd($n$)

## $X \sim T(n)$ (The $t$-Distribution)

PDF:

$$f_X(x) = \frac{\Gamma(\frac{n+1}{2})}{\sqrt{n\pi}\Gamma\left(\frac{n}{2}\right)}\left(1 + \frac{x^2}{n}\right)^{-\frac{n+1}{2}}$$

Moment Generating Function (MGF):

$$\text{undefined}$$

Expected Value:

$$EX = 0$$

Variance:

$$\mathrm{Var}(X) = \frac{n}{n-2} \quad \text{for} \quad n > 2, \quad \infty \quad \text{for } 1 < n \leq 2, \quad \text{undefined} \quad \text{otherwise}$$

MATLAB:

R = trnd($n$)

$X \sim Gamma(\alpha, \lambda)$

PDF:

$$f_X(x) = \frac{\lambda^\alpha x^{\alpha-1} e^{-\lambda x}}{\Gamma(\alpha)}, \quad x > 0$$

Moment Generating Function (MGF):

$$M_X(s) = \left(1 - \frac{s}{\lambda}\right)^{-\alpha} \quad \text{for} \quad s < \lambda$$

Expected Value:

$$EX = \frac{\alpha}{\lambda}$$

Variance:

$$\mathrm{Var}(X) = \frac{\alpha}{\lambda^2}$$

MATLAB:

R = gamrnd($\alpha$,$\lambda$)

$X \sim Erlang(k, \lambda)$ $[= Gamma(k, \lambda)]$, $k > 0$ is an integer

PDF:

$$f_X(x) = \frac{\lambda^k x^{k-1} e^{-\lambda x}}{(k-1)!}, \quad x > 0$$

Moment Generating Function (MGF):

$$M_X(s) = \left(1 - \frac{s}{\lambda}\right)^{-k} \quad \text{for} \quad s < \lambda$$

Expected Value:

$$EX = \frac{k}{\lambda}$$

Variance:

$$\mathrm{Var}(X) = \frac{k}{\lambda^2}$$

$X \sim Uniform(a, b)$

PDF:

$$f_X(x) = \frac{1}{b-a}, \quad x \in [a, b]$$

CDF:

$$F_X(x) = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & x \in [a, b) \\ 1 & \text{for } x \geq b \end{cases}$$

Moment Generating Function (MGF):

$$M_X(s) = \begin{cases} \frac{e^{sb} - e^{sa}}{s(b-a)} & s \neq 0 \\ 1 & s = 0 \end{cases}$$

Characteristic Function:

$$\phi_X(\omega) = \frac{e^{i\omega b} - e^{i\omega a}}{i\omega(b-a)}$$

Expected Value:

$$EX = \frac{1}{2}(a+b)$$

Variance:

$$\mathrm{Var}(X) = \frac{1}{12}(b-a)^2$$

MATLAB:

U = rand or R = unifrnd($a$,$b$)