7.2.1 Convergence of a Sequence of Numbers

Before discussing convergence for a sequence of random variables, let us remember what convergence means for a sequence of real numbers. If we have a sequence of real numbers $a_1, a_2, a_3, \cdots$, we can ask whether the sequence converges. For example, the sequence

$$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \cdots, \frac{n}{n+1}, \cdots$$

is defined as

$$a_n = \frac{n}{n+1}, \qquad \text{for } n = 1, 2, 3, \cdots$$

This sequence converges to 1. We say that a sequence a_1, a_2, a_3, $\cdots$ converges to a limit L if a_n approaches L as n goes to infinity.

Definition 7.2.
A sequence a_1, a_2, a_3, $\cdots$ converges to a limit L if

$$\lim_{n \to \infty} a_n = L.$$

That is, for any $\epsilon > 0$, there exists an $N \in \mathbb{N}$ such that

$$|a_n - L| < \epsilon, \qquad \text{for all } n > N.$$