3.1.1 Random Variables

In general, to analyze random experiments, we usually focus on some numerical aspects of the experiment. For example, in a soccer game we may be interested in the number of goals, shots, shots on goal, corners kicks, fouls, etc. If we consider an entire soccer match as a random experiment, then each of these numerical results gives some information about the outcome of the random experiment. These are examples of *random variables*. In a nutshell, a random variable is a real-valued variable whose value is determined by an underlying random experiment.

Let's look at an example.

Example 3.1
I toss a coin five times. This is a random experiment and the sample space can be written as

$$S = \{TTTTT, TTTTH, \ldots, HHHHH\}.$$

Note that here the sample space S has $2^5 = 32$ elements. Suppose that in this experiment, we are interested in the number of heads. We can define a random variable X whose value is the number of observed heads. The value of X will be one of $0, 1, 2, 3, 4$ or 5 depending on the outcome of the random experiment.

In essence, a random variable is a real-valued function that assigns a numerical value to each possible outcome of the random experiment. For example, the random variable X defined above assigns the value 0 to the outcome $TTTTT$, the value 2 to the outcome $THTHT$, and so on. Hence, the random variable X is a function from the sample space S=$\{TTTTT$,$TTTTH$, $\cdots$,$HHHHH\}$ to the real numbers (for this particular random variable, the values are always integers between 0 and 5).

Random Variables:
A random variable X is a function from the sample space to the real numbers.

$$X : S \to \mathbb{R}$$

We usually show random variables by capital letters such as X, Y, and Z. Since a random variable is a function, we can talk about its range. The range of a random variable X, shown by Range(X) or R_X, is the set of possible values for X. In the above example, Range$(X) = R_X = \{0, 1, 2, 3, 4, 5\}$.

The range of a random variable X, shown by Range(X) or R_X, is the set of possible values of X.

Example 3.2
Find the range for each of the following random variables.

1. I toss a coin 100 times. Let X be the number of heads I observe.
2. I toss a coin until the first heads appears. Let Y be the total number of coin tosses.
3. The random variable T is defined as the time (in hours) from now until the next earthquake occurs in a certain city.

Solution

1. The random variable X can take any integer from 0 to 100, so $R_X = \{0, 1, 2, \dots, 100\}$.
2. The random variable Y can take any positive integer, so $R_Y = \{1, 2, 3, \dots\} = \mathbb{N}$.
3. The random variable T can in theory get any positive real number, so $R_T = [0, \infty)$.
