Absorption Probabilities

Consider a finite Markov chain $\{X_n, n=0,1,2,\cdots\}$ with state space $S=\{0,1,2,\cdots,r\}$. Suppose that all states are either absorbing or transient. Let $l \in S$ be an absorbing state. Define

$$a_i = P(\text{absorption in } l | X_0 = i), \quad \text{for all } i \in S.$$

By the above definition, we have $a_l = 1$, and $a_j = 0$ if j is any other absorbing state. To find the unknown values of a_i's, we can use the following equations

$$a_i = \sum_k a_k p_{ik}, \quad \text{for } i \in S.$$

In general, a finite Markov chain might have several transient as well as several recurrent classes. As n increases, the chain will get absorbed in one of the recurrent classes and it will stay there forever. We can use the above procedure to find the probability that the chain will get absorbed in each of the recurrent classes. In particular, we can replace each recurrent class with one absorbing state. Then, the resulting chain consists of only transient and absorbing states. We can then follow the above procedure to find absorption probabilities. An example of this procedure is provided in the Solved Problems Section (See Problem 2 in Section 11.2.7).

Mean Hitting Times:

We now would like to study the expected time until the process hits a certain set of states for the first time. Again, consider the Markov chain in Figure 11.12. Let's define t_i as the number of steps needed until the chain hits state 0 or state 3, given that $X_0 = i$. In other words, t_i is the expected time (number of steps) until the chain is absorbed in 0 or 3, given that $X_0 = i$. By this definition, we have $t_0 = t_3 = 0$.

To find t_1 and t_2, we use the law of total probability with recursion as before. For example, if $X_0 = 1$, then after one step, we have $X_1 = 0$ or $X_1 = 2$. Thus, we can write

$$\begin{aligned} t_1 &= 1 + \frac{1}{3}t_0 + \frac{2}{3}t_2 \\ &= 1 + \frac{2}{3}t_2. \end{aligned}$$