11.1.3 Merging and Splitting Poisson Processes

Merging Independent Poisson Processes:

Let $N_1(t)$ and $N_2(t)$ be two independent Poisson processes with rates λ_1 and λ_2 respectively. Let us define $N(t) = N_1(t) + N_2(t)$. That is, the random process $N(t)$ is obtained by combining the arrivals in $N_1(t)$ and $N_2(t)$ (Figure 11.5). We claim that $N(t)$ is a Poisson process with rate $\lambda = \lambda_1 + \lambda_2$. To see this, first note that

$$\begin{aligned} N(0) &= N_1(0) + N_2(0) \\ &= 0 + 0 = 0. \end{aligned}$$

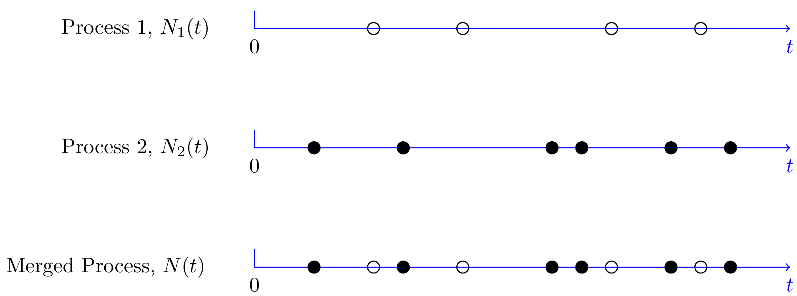

Figure 11.5 - Merging two Poisson processes $N_1(t)$ and $N_2(t)$.

Next, since $N_1(t)$ and $N_2(t)$ are independent and both have independent increments, we conclude that $N(t)$ also has independent increments. Finally, consider an interval of length τ, i.e, $I = (t, t + \tau]$. Then the numbers of arrivals in I associated with $N_1(t)$ and $N_2(t)$ are $Poisson(\lambda_1 \tau)$ and $Poisson(\lambda_2 \tau)$ and they are independent. Therefore, the number of arrivals in I associated with $N(t)$ is $Poisson\big((\lambda_1 + \lambda_2)\tau\big)$ (sum of two independent Poisson random variables).