## 11.1.2 Basic Concepts of the Poisson Process

The Poisson process is one of the most widely-used counting processes. It is usually used in scenarios where we are counting the occurrences of certain events that appear to happen at a certain rate, but completely at random (without a certain structure). For example, suppose that from historical data, we know that earthquakes occur in a certain area with a rate of $2$ per month. Other than this information, the timings of earthquakes seem to be completely random. Thus, we conclude that the Poisson process might be a good model for earthquakes. In practice, the Poisson process or its extensions have been used to model [24]

– the number of car accidents at a site or in an area;
– the location of users in a wireless network;
– the requests for individual documents on a web server;
– the outbreak of wars;
– photons landing on a photodiode.

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**Poisson random variable:** Here, we briefly review some properties of the Poisson random variable that we have discussed in the previous chapters. Remember that a discrete random variable $X$ is said to be a *Poisson* random variable with parameter $\mu$, shown as $X \sim Poisson(\mu)$, if its range is $R_X = \{0, 1, 2, 3, \dots\}$, and its PMF is given by

$$P_X(k) = \begin{cases} \frac{e^{-\mu}\mu^k}{k!} & \text{for } k \in R_X \\ 0 & \text{otherwise} \end{cases}$$

Here are some useful facts that we have seen before:

1. If $X \sim Poisson(\mu)$, then $EX = \mu$, and $\mathrm{Var}(X) = \mu$.
2. If $X_i \sim Poisson(\mu_i)$, for $i = 1, 2, \cdots, n$, and the $X_i$'s are independent, then

$$X_1 + X_2 + \cdots + X_n \sim Poisson(\mu_1 + \mu_2 + \cdots + \mu_n).$$

3. The Poisson distribution can be viewed as the limit of binomial distribution.