Some Important Distributions

Discrete Distributions

$X \sim Bernoulli(p)$

PMF:

$$P_X(k) = \begin{cases} p & \text{for } k = 1 \\ 1-p & \text{for } k = 0 \end{cases}$$

CDF:

$$F_X(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1-p & \text{for } 0 \leq x < 1 \\ 1 & \text{for} 1 \leq x \end{cases}$$

Moment Generating Function (MGF):

$$M_X(s) = 1 - p + pe^s$$

Characteristic Function:

$$\phi_X(\omega) = 1 - p + pe^{i\omega}$$

Expected Value:

$$EX = p$$

Variance:

$$\operatorname{Var}(X) = p(1-p)$$

$X \sim Binomial(n,p)$

PMF:

$$P_X(k) = \binom{n}{k} p^k (1-p)^{n-k} \quad \text{for } k = 0, 1, 2, \cdots, n$$

Moment Generating Function (MGF):

$$M_X(s) = (1 - p + pe^s)^n$$

Characteristic Function:

$$\phi_X(\omega) = (1 - p + pe^{i\omega})^n$$

Expected Value:

$$EX = np$$

Variance:

$$\operatorname{Var}(X) = np(1-p)$$

MATLAB:

R = binornd(n,p)

$X \sim Geometric(p)$

PMF:

$$P_X(k) = p(1-p)^{k-1} \quad \text{for } k = 1, 2, 3, \dots$$

CDF:

$$F_X(x) = 1 - (1-p)^{\lfloor x \rfloor} \quad \text{for } x \geq 0$$

Moment Generating Function (MGF):

$$M_X(s) = \frac{pe^s}{1-(1-p)e^s} \quad \text{for } s < -\ln(1-p)$$

Characteristic Function:

$$\phi_X(\omega) = \frac{pe^{i\omega}}{1-(1-p)e^{i\omega}}$$

Expected Value:

$$EX = \frac{1}{p}$$

Variance:

$$\operatorname{Var}(X) = \frac{1-p}{p^2}$$

MATLAB:

R = geornd(p)+1

$X \sim Pascal(m,p)$ (Negative Binomial)

PMF:

$$P_X(k) = \binom{k-1}{m-1} p^m (1-p)^{k-m} \quad \text{for } k = m, m+1, m+2, m+3, \ldots$$

Moment Generating Function (MGF):

$$M_X(s) = \left(\frac{pe^s}{1-(1-p)e^s} \right)^m \quad \text{for} \quad s < -\log(1-p)$$

Characteristic Function:

$$\phi_X(\omega) = \left(\frac{pe^{i\omega}}{1-(1-p)e^{i\omega}} \right)^m$$

Expected Value:

$$EX = \frac{m}{p}$$

Variance:

$$\operatorname{Var}(X) = \frac{m(1-p)}{p^2}$$

MATLAB:

R = nbinrnd(m,p)+1

$X \sim Hypergeometric(b, r, k)$

PMF:

$$P_X(x) = \frac{\binom{b}{x}\binom{r}{k-x}}{\binom{b+r}{k}} \quad \text{for } x = \max(0, k-r), \max(0, k-r)+1, \ldots, \min(k, b)$$

Expected Value:

$$EX = \frac{kb}{b+r}$$

Variance:

$$\mathrm{Var}(X) = \frac{kbr}{(b+r)^2}\frac{b+r-k}{b+r-1}$$

MATLAB:

R = hygernd($b+r$,b,k)