10.1.1 PDFs and CDFs

Consider the random process $\{X(t), t \in J\}$. For any $t_0 \in J$, $X(t_0)$ is a random variable, so we can write its CDF

$$F_{X(t_0)}(x) = P\big(X(t_0) \leq x\big).$$

If $t_1, t_2 \in J$, then we can find the joint CDF of $X(t_1)$ and $X(t_2)$ by

$$F_{X(t_1)X(t_2)}(x_1, x_2) = P\big(X(t_1) \leq x_1, X(t_2) \leq x_2\big).$$

More generally for $t_1, t_2, \cdots, t_n \in J$, we can write

$$F_{X(t_1)X(t_2)\cdots X(t_n)}(x_1, x_2, \cdots, x_n) = P\big(X(t_1) \leq x_1, X(t_2) \leq x_2, \cdots, X(t_n) \leq x_n\big).$$

Similarly, we can write joint PDFs or PMFs depending on whether $X(t)$ is continuous-valued (the $X(t_i)$'s are continuous random variables) or discrete-valued (the $X(t_i)$'s are discrete random variables).

Example 10.3

Consider the random process $\{X_n, n = 0, 1, 2, \cdots\}$, in which X_i's are i.i.d. standard normal random variables.

1. Write down $f_{X_n}(x)$ for $n = 0, 1, 2, \cdots$.
2. Write down $f_{X_m X_n}(x_1, x_2)$ for $m \neq n$.

Solution

1. Since $X_n \sim N(0, 1)$, we have

$$f_{X_n}(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}, \qquad \text{for all } x \in \mathbb{R}.$$

2. If $m \neq n$, then X_m and X_n are independent (because of the i.i.d. assumption), so

$$
\begin{aligned}
f_{X_m X_n}(x_1, x_2) &= f_{X_m}(x_1) f_{X_n}(x_2) \\
&= \frac{1}{\sqrt{2\pi}} e^{-\frac{x_1^2}{2}} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{x_2^2}{2}} \\
&= \frac{1}{2\pi} \exp\left\{ -\frac{x_1^2 + x_2^2}{2} \right\}, \qquad \text{for all } x_1, x_2 \in \mathbb{R}.
\end{aligned}
$$