8.2.0 Point Estimation

Here, we assume that θ is an unknown parameter to be estimated. For example, θ might be the expected value of a random variable, $\theta = EX$. The important assumption here is that θ is a fixed (non-random) quantity. To estimate θ, we need to collect some data. Specifically, we get a random sample X_1, X_2, X_3, ..., X_n such that X_i's have the same distribution as X. To estimate θ, we define a point estimator $\hat{\Theta}$ that is a function of the random sample, i.e.,

$$\hat{\Theta} = h(X_1, X_2, \cdots, X_n).$$

For example, if $\theta = EX$, we may choose $\hat{\Theta}$ to be the sample mean

$$\hat{\Theta} = \overline{X} = \frac{X_1 + X_2 + \ldots + X_n}{n}.$$

There are infinitely many possible estimators for θ, so how can we make sure that we have chosen a good estimator? How do we compare different possible estimators? To do this, we provide a list of some desirable properties that we would like our estimators to have. Intuitively, we know that a good estimator should be able to give us values that are "close" to the real value of θ. To make this notion more precise we provide some definitions.