8.3.0 Interval Estimation (Confidence Intervals)

Let X_1, X_2, X_3, $\ldots$, X_n be a random sample from a distribution with a parameter θ that is to be estimated. Suppose that we have observed $X_1 = x_1$, $X_2 = x_2$, $\cdots$, $X_n = x_n$. So far, we have discussed point estimation for θ. The point estimate $\hat{\theta}$ alone does not give much information about θ. In particular, without additional information, we do not know how close $\hat{\theta}$ is to the real θ. Here, we will introduce the concept of **interval estimation**. In this approach, instead of giving just one value $\hat{\theta}$ as the estimate for θ, we will produce an interval that is likely to include the true value of θ. Thus, instead of saying

$$\hat{\theta} = 34.25,$$

we might report the interval

$$[\hat{\theta}_l, \hat{\theta}_h] = [30.69, 37.81],$$

which we hope includes the real value of θ. That is, we produce two estimates for θ, a *high estimate* $\hat{\theta}_h$ and a low estimate $\hat{\theta}_l$. In interval estimation, there are two important concepts. One is the **length** of the reported interval, $\hat{\theta}_h - \hat{\theta}_l$. The length of the interval shows the precision with which we can estimate θ. The smaller the interval, the higher the precision with which we can estimate θ. The second important factor is the **confidence level** that shows how confident we are about the interval. The confidence level is the probability that the interval that we construct includes the real value of θ. Therefore, high confidence levels are desirable. We will discuss these concepts in this section.