
11.2.5 Using the Law of Total Probability with Recursion

A very useful technique in the analysis of Markov chains is using law of total probability. In fact, we have already used this when finding n -step transition probabilities. In this section, we will use this technique to find **absorption probabilities**, **mean hitting times**, and **mean return times**. We will introduce this technique by looking at an example. We will then provide the general formulas. You should try to understand the main idea here. This way, you do not need to memorize any formulas. Let's consider the Markov chain shown in Figure 11.12.

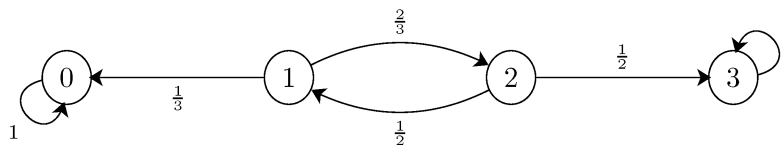


Figure 11.12 - A state transition diagram.

The state transition matrix of this Markov chain is given by the following matrix.

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & \frac{2}{3} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Before going any further, let's identify the classes in this Markov chain.

Example 11.9

For the Markov chain given in Figure 11.12, answer the following questions: How many classes are there? For each class, mention if it is recurrent or transient.

Solution

There are three classes: Class 1 consists of one state, state 0, which is a recurrent state. Class two consists of two states, states 1 and 2, both of which are transient. Finally, class three consists of one state, state 3, which is a recurrent state.