Assignment 5

AI1110: Probability and Random Variables

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PROBABILITY, RANDOM VARIABLES, AND STOCHASTIC PROCESSES
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Question

Example 6-50

Suppose that the random variables x and y are $N(0, 0, \sigma_1^2, \sigma_2^2, r)$. As we know,

$$E\{x^2\} = \sigma_1^2 \tag{1}$$
$$E\{x^4\} = 3\sigma_1^4 \tag{2}$$

Furthermore, $f(y|x)$ is a normal density with mean $\frac{r\sigma_2 x}{\sigma_1}$ and variance $\sigma_2\sqrt{1-r^2}$. Hence,
Using (6-244), we shall show that

$$E\{xy\} = r\sigma_1\sigma_2 \tag{3}$$
$$E\{x^2y^2\} = E\{x^2\}E\{y^2\} + 2E^2\{xy\} \tag{4}$$

Solution

Proof.

$$E\{xy\} = E\{xE\{y|x\}\} \tag{5}$$

$$= E\{r\sigma_2 \frac{x^2}{\sigma_1}\} \tag{6}$$

$$= r\sigma_2 \frac{\sigma_1^2}{\sigma_1} \tag{7}$$

Solution(contd.)

Now, we know that,

$$E\{x^2y^2\} = E\{x^2 E\{y^2|x\}\} \tag{8}$$
$$= E\{x^2[r^2\sigma_2^2\frac{x^2}{\sigma_1^2} + \sigma_2^2(1-r^2)]\} \tag{9}$$
$$= 3\sigma_1^4 r^2\frac{\sigma_2^2}{\sigma_1^2} + \sigma_1^2\sigma_2^2(1-r^2) \tag{10}$$
$$= \sigma_1^2\sigma_2^2 + 2r^2\sigma_1^2\sigma_2^2 \tag{11}$$

and the proof is complete.