Assignment 6

AI1110: Probability and Random Variables

Indian Institute of Technology Hyderabad

Rudransh Mishra
AI21BTECH11025

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PROBABILITY, RANDOM VARIABLES, AND STOCHASTIC PROCESSES
Athanasios Papoulis

Example 9-38 Show that if R(r) is the inverse Fourier transform of a function $S(\omega)$ and $S(\omega) \geq 0$, then, for any a$_i$,

$$\sum_{i,k} a_i a_k^* R(\tau_i - \tau_k) \geq 0 \tag{1}$$

Proof. We know that,

$$\int_{-\infty}^{\infty} S(\omega)|\sum_{i} a_i e^{j\omega\tau_i}|d(\omega) \geq 0 \tag{2}$$

And,

$$\int_{-\infty}^{\infty} S(\omega)|\sum_{i} a_i e^{j\omega\tau_i}|d(\omega) \tag{3}$$

$$= \int_{-\infty}^{\infty} S(\omega)|\sum_{i,k} a_i a_k^* e^{j\omega(\tau_i-\tau_k)}|d(\omega) \tag{4}$$

Now, we know that,

$$\int_{-\infty}^{\infty} S(\omega)|\sum_{i,k} a_i a_k^* e^{j\omega(\tau_i-\tau_k)}|d(\omega) \tag{5}$$

$$= \sum_{i,k} a_i a_k^* R(\tau_i - \tau_k) \tag{6}$$

From (2) and (6),

$$\sum_{i,k} a_i a_k^* R(\tau_i - \tau_k) \geq 0 \tag{7}$$

and the proof is complete.

—x-x-x-x-x-x—