Assignment 6

AI1110: Probability and Random Variables

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15 June 2022

PROBABILITY, RANDOM VARIABLES, AND STOCHASTIC PROCESSES
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Question

Example 9-38

Show that if R(r) is the inverse Fourier transform of a function $S(\omega)$ and $S(\omega) \geq 0$, then, for any a_i,

$$\sum_{i,k} a_i a_k^* R(\tau_i - \tau_k) \geq 0 \tag{1}$$

Proof

Proof. We know that,

$$\int_{-\infty}^{\infty} S(\omega) | \sum_i a_i e^{j\omega\tau_i} |, d(\omega) \geq 0 \tag{2}$$

And,

$$\int_{-\infty}^{\infty} S(\omega) | \sum_i a_i e^{j\omega\tau_i} |, d(\omega) = \int_{-\infty}^{\infty} S(\omega) | \sum_{i,k} a_i a_k^* e^{j\omega(\tau_i - \tau_k)} |, d(\omega) \tag{3}$$

Proof(contd.)

Now, we know that,

$$\int_{-\infty}^{\infty} S(\omega)|\sum_{i,k} a_i a_k^* e^{j\omega(\tau_i - \tau_k)}|, d(\omega) = \sum_{i,k} a_i a_k^* R(\tau_i - \tau_k) \tag{4}$$

From (2) and (4),

$$\sum_{i,k} a_i a_k^* R(\tau_i - \tau_k) \geq 0 \tag{5}$$

and the proof is complete.