Assignment 8

AI1110: Probability and Random Variables

Indian Institute of Technology Hyderabad

Rudransh Mishra
AI21BTECH11025

19 June 2022

PROBABILITY, RANDOM VARIABLES, AND STOCHASTIC PROCESSES
Athanasios Papoulis

Example 15.2

Consider a population that is able to produce new offspring of like kind. For each member let $p_k, k = 0, 1, 2, \ldots$ represent the probability of creating k new members. The direct descendents of the nth generation form the (n + 1)st generation. The members of each generation are independent of each other. Suppose X_n represents the size of the nth generation. It is clear that X_n depends only on $X_n - 1$ since $X_n = \sum_{i=1}^{x_n - 1} Y_i$, where Y_i represents the number of offspring of the i th member of the (n - 1) generation, and the manner in which the value of $X_n - 1$ was reached is of no consequence. Thus X_n represents a Markov chain.

Example(cont.)

Nuclear chain reactions, survival of family surnames, gene mutations, and waiting lines in a queueing system are all examples of branching processes. In a nuclear chain reaction, a particle such as a neutron scores a hit with probability p, creating m new particles, and $q = 1 - p$ represents the probability that it remains inactive with no descendants. In that case, the only possible number of descendants is zero and m with probabilities q and p. If P is close to one, the number of particles is likely to increase indefinitely, leading to an explosion, whereas if P is close to zero the process may never start.