Game playing

Outline

- Games
- Perfect play
  - minimax decisions
  - α–β pruning
- Resource limits and approximate evaluation
- Games of chance
- Games of imperfect information

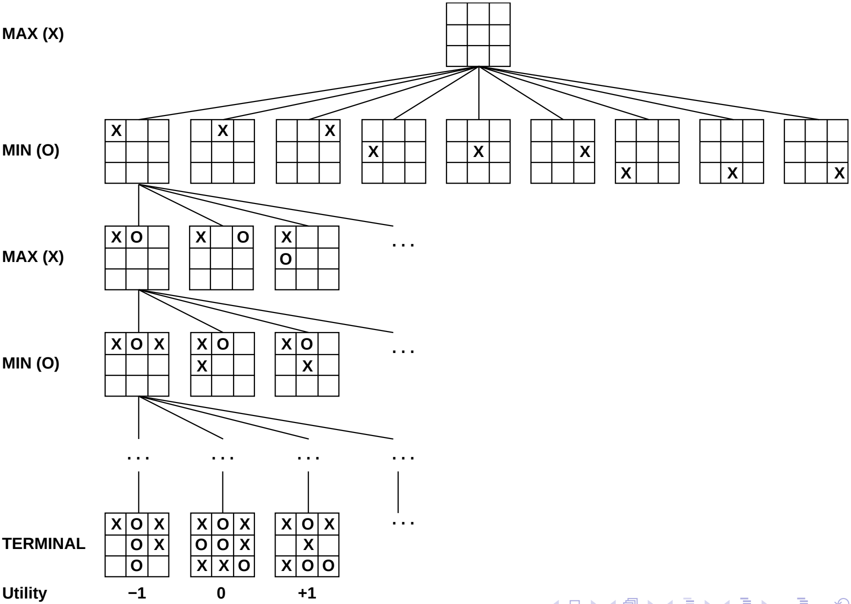
Games vs. search problems

- Unpredictable opponent $\Rightarrow$ solution is a strategy specifying a move for every possible opponent reply
- Time limits $\Rightarrow$ unlikely to find goal, must approximate
- Plan of attack:
  - Computer considers possible lines of play (Babbage, 1846)
  - Algorithm for perfect play (Zermelo, 1912; Von Neumann, 1944)
  - Finite horizon, approximate evaluation (Zuse, 1945; Wiener, 1948; Shannon, 1950)
  - First chess program (Turing, 1951)
  - Machine learning to improve evaluation accuracy (Samuel, 1952–57)
  - Pruning to allow deeper search (McCarthy, 1956)

Types of games

	deterministic	chance
perfect information	chess, checkers, go, othello	backgammon monopoly
imperfect information	battleships, blind tictactoe	bridge, poker, scrabble nuclear war

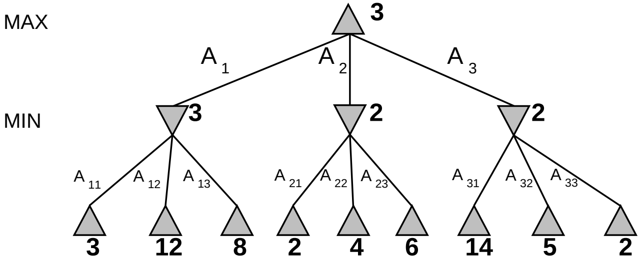
Game tree (2-player, deterministic, turns)



Minimax: Perfect play for deterministic, perfect-information games

Idea: choose move to position with highest minimax value = best achievable payoff against best play

Example 2-ply game:



Minimax algorithm

```
function MINIMAX-DECISION(state) returns an action
    inputs: state, current state in game

    return the a in ACTIONS(state) maximizing MIN-VALUE(RESULT(a, state))
function MAX-VALUE(state) returns a utility value
    if TERMINAL-TEST(state) then return UTILITY(state)
    v ← −∞
    for a, s in SUCCESSORS(state) do v ← MAX(v, MIN-VALUE(s))
    return v
function MIN-VALUE(state) returns a utility value
    if TERMINAL-TEST(state) then return UTILITY(state)
    v ← ∞
    for a, s in SUCCESSORS(state) do v ← MIN(v, MAX-VALUE(s))
    return v
```

Properties of minimax

Complete?? Yes, if tree is finite (chess has specific rules for this)

Optimal?? Yes, against an optimal opponent. Otherwise??

Time complexity?? $O(b^m)$

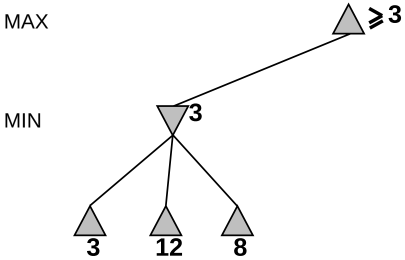
Space complexity?? $O(bm)$ (depth-first exploration)

For chess, $b \approx 35$, $m \approx 100$ for "reasonable" games

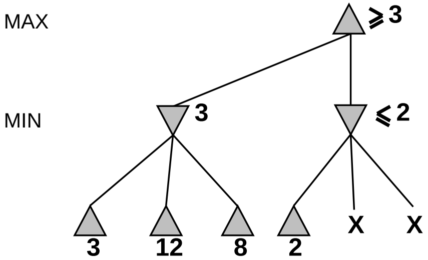
$\Rightarrow$ exact solution completely infeasible.

But do we need to explore every path?

α–β pruning example



α–β pruning example



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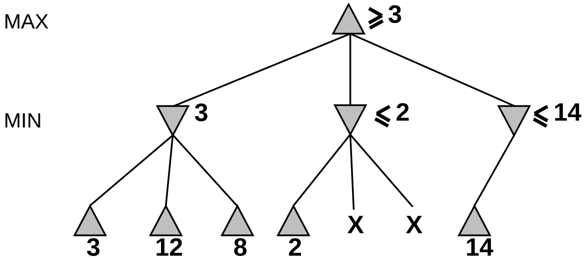

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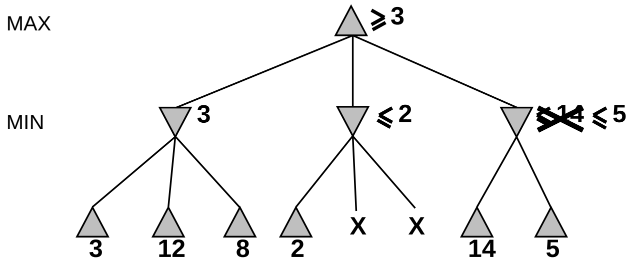
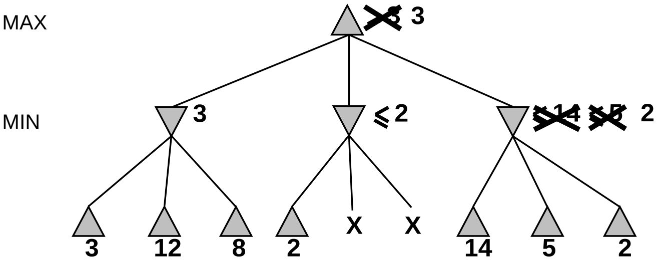
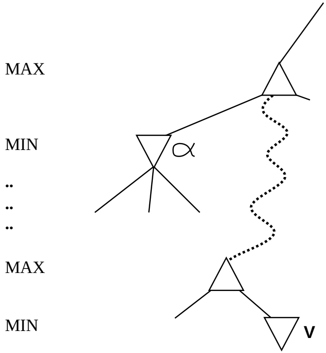


α–β pruning example



Why is it called α–β?



- α is the best value (to MAX) found so far off the current path
- If V is worse than α, MAX will avoid it $\Rightarrow$ prune that branch
- Define β similarly for MIN

The α–β algorithm

function ALPHA-BETA-DECISION(*state*) **returns** an action
  return the a in ACTIONS(*state*) maximizing MIN-VALUE(RESULT(a, *state*))
function MAX-VALUE(*state*, α, β) **returns** *a utility value*
  inputs: *state*, current state in game
    α, the value of the best alternative for MAX along the path to *state*
    β, the value of the best alternative for MIN along the path to *state*

  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow -\infty$
  for a, s in SUCCESSORS(*state*) **do**
    $v \leftarrow$ MAX(v, MIN-VALUE(s, α, β))
    if $v \geq \beta$ **then return** v
    $\alpha \leftarrow$ MAX(α, v)
  return v

Properties of α–β

- Pruning **does not** affect final result
- Good move ordering improves effectiveness of pruning
- With "perfect ordering," time complexity $= O(b^{m/2})$
  $\Rightarrow$ **doubles** solvable depth
- A simple example of the value of reasoning about which computations are relevant (a form of metareasoning)
- Unfortunately, 35^{50} is still impossible!

Resource limits

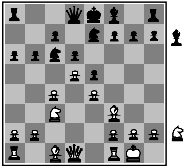
- Standard approach:
  - Use CUTOFF-TEST instead of TERMINAL-TEST: e.g., depth limit (perhaps add quiescence search)
  - Use EVAL instead of UTILITY: i.e., evaluation function that estimates desirability of position

Suppose we have 100 seconds, explore 10^4 nodes/second

$\Rightarrow 10^6$ nodes per move $\approx 35^{8/2}$

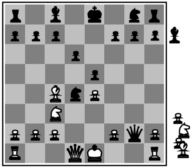
$\Rightarrow \alpha\text{–}\beta$ reaches depth 8 $\Rightarrow$ pretty good chess program

Evaluation functions



Black to move

White slightly better



White to move

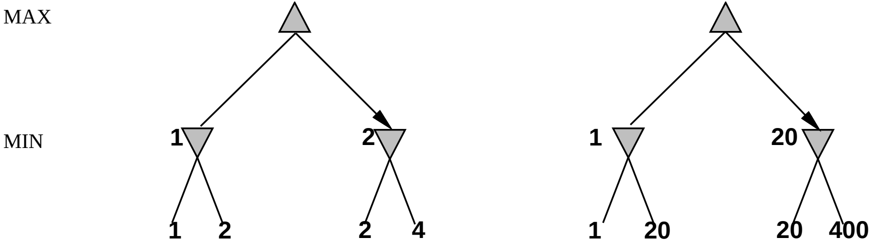
Black winning

For chess, typically linear weighted sum of features

$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

e.g., $w_1 = 9$ with
$f_1(s) =$ (number of white queens) – (number of black queens),
etc.

Digression: Exact values don't matter



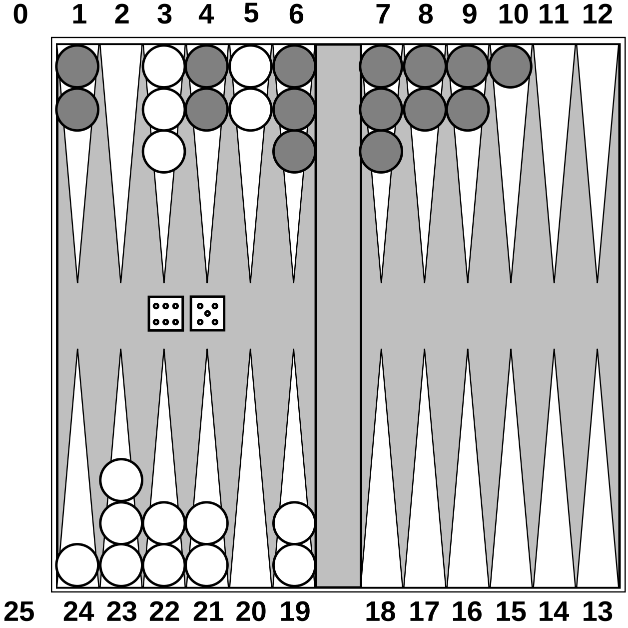
Behaviour is preserved under any **monotonic** transformation of EVAL

Only the order matters: payoff in deterministic games acts as an ordinal utility function

Deterministic games in practice

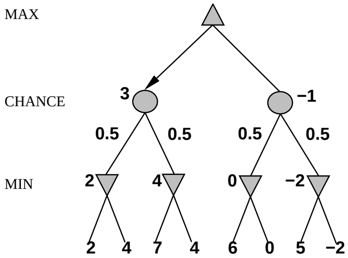
- Checkers: Chinook ended 40-year-reign of world champion Marion Tinsley in 1994.
  - Used endgame database of perfect play for all positions with ≤ 8 pieces on the board: 443,748,401,247 positions.
- Chess: Deep Blue defeated world champion Gary Kasparov in a six-game match in 1997.
  - It searches 200 million positions / sec + very sophisticated evaluation,+ undisclosed methods for extending some lines of search up to 40 ply.
- Othello: human champions refuse to compete against computers, who are too good.
- Go: human champions refused to compete against computers, who were too bad. In go, $b > 300$, so most programs use pattern knowledge bases to suggest plausible moves. Then in 2016 ....
  http://www.scientificamerican.com/article/how-the-computer-beat-the-go-master

Nondeterministic games: backgammon



Nondeterministic games in general

In nondeterministic games, chance is introduced by dice, card-shuffling
Simplified example with coin-flipping:



Algorithm for nondeterministic games

Expectminimax gives perfect play

Just like MINIMAX, except we must also handle chance nodes:

. . .

if state is a MAX node **then**

return highest ExpectMinimax-Value of Successors(state)

if state is a MIN node **then**

return the lowest ExpectMinimax-Value of Successors(state)

if state is a CHANCE node **then**

return average of ExpectMinimax-Value of Successors(state)

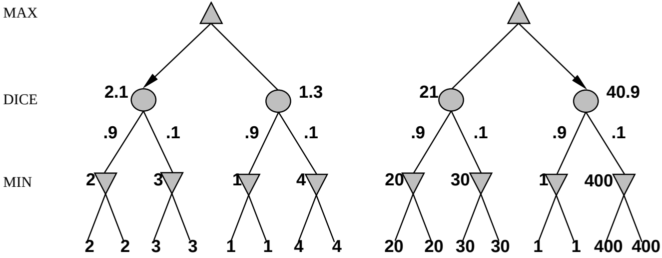
. . .

Nondeterministic games in practice

- Dice rolls increase b: 21 possible rolls with 2 dice
- Backgammon $\approx$ 20 legal moves (can be 6,000 with 1-1 roll)

  $$\text{depth } 4 = 20 \times (21 \times 20)^3 \approx 1.2 \times 10^9$$
- As depth increases, probability of reaching a given node shrinks
  $\Rightarrow$ value of lookahead is diminished
  $\Rightarrow$ α–β pruning is much less effective
- TDGammon uses depth-2 search + very good Eval: $\approx$ world-champion level

Digression: Exact values DO matter



Behaviour is preserved only by positive linear transformation of EVAL

Hence EVAL should be proportional to the expected payoff

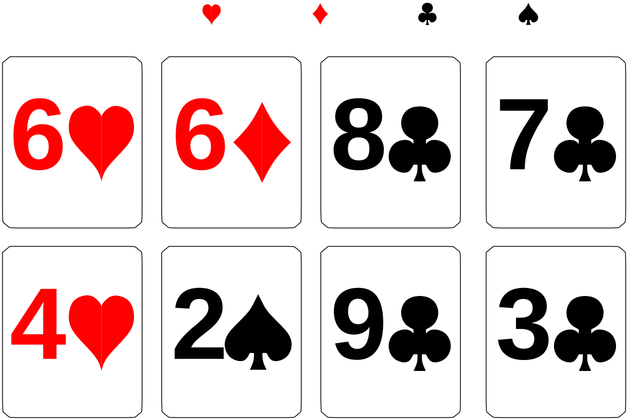
Games of imperfect information

E.g., card games, where opponent's initial cards are unknown

- Typically we can calculate a probability for each possible deal
- Seems just like having one big dice roll at the beginning of the game
- Idea: compute the minimax value of each action in each deal, then choose the action with highest expected value over all deals
- Special case: if an action is optimal for all deals, it's optimal.
- GIB, current best bridge program, approximates this idea by
    1. generating 100 deals consistent with bidding information
    2. picking the action that wins most tricks on average

Example

Four-card bridge/whist/hearts hand, MAX to play first



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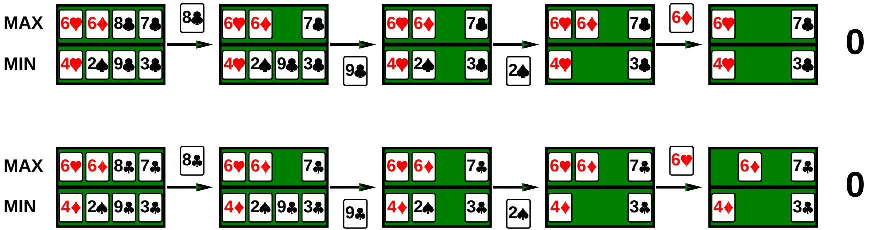

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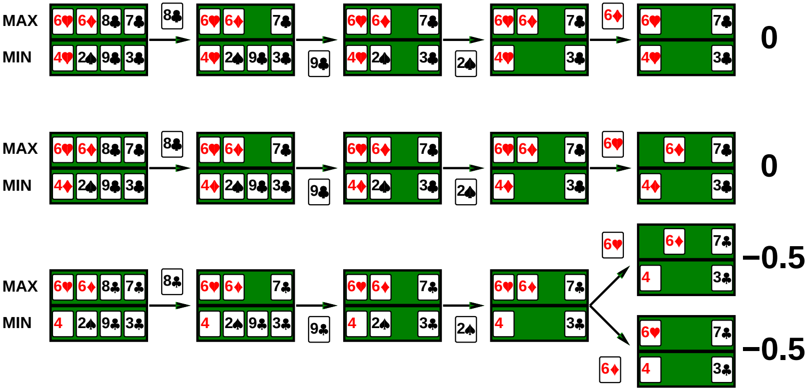

Commonsense example

1. Road A leads to a small heap of gold pieces
2. Road B leads to a fork:
   - take the left fork and you'll find a mound of jewels;
   - take the right fork and you'll be run over by a bus.

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1. Road A leads to a small heap of gold pieces
2. Road B leads to a fork:
   - guess correctly and you'll find a mound of jewels;
   - guess incorrectly and you'll be run over by a bus.

Proper analysis

- Intuition that the value of an action is the average of its values in all actual states is **WRONG**
- With partial observability, the value of an action depends on the information state or belief state the agent is in
- Can generate and search a tree of information states
- Leads to rational behaviors such as
  - Acting to obtain information
  - Signalling to one's partner
  - Acting randomly to minimize information disclosure

Summary

- Games are fun to work and illustrate several important points about AI
  - perfection is unattainable $\Rightarrow$ must approximate
  - good idea to think about what to think about
  - uncertainty constrains the assignment of values to states
  - optimal decisions depend on information state, not real state
- Games are to AI as grand prix racing is to automobile design