

Translate in propositional logic the following:

Logic is **either** too boring **or** too difficult. For **either** it is part of mathematics **or** it is part of philosophy. And **unless** it is NOT part of mathematics it is too difficult. **Only if** it is boring will it be part of philosophy.

First, let us assign propositional letters:

p: logic is too boring

q: logic is too difficult

r: logic is part of mathematics

s: logic is part of philosophy

Logic is **either** too boring **or** too difficult:

For **either** it is part of mathematics **or** it is part of philosophy:

... **unless** it is NOT part of mathematics it is too difficult:

**Only if** it is boring will it be part of philosophy:

**either** p **or** q

$r \rightarrow p, s \rightarrow q$

**unless**  $\sim r, q$

**Only if** p, s

Now we need to consider the translations of 'either-or', 'unless', and 'only if'

1. either-or is usually translated by an XOR. Thus  

$$\text{either } p \text{ or } q = (p \vee q) \wedge \sim(p \wedge q) = (p \vee q) \wedge (\sim p \vee \sim q) =$$

$$(p \vee q) \wedge \sim p \vee (p \vee q) \wedge \sim q = p \wedge \sim p \vee q \wedge \sim p \vee p \wedge \sim q \vee q \wedge \sim q = F \vee q \wedge \sim p \vee p \wedge \sim q \vee F =$$

$$p \wedge \sim q \vee q \wedge \sim p$$
2. Next, 'Only if p, s'. Recall 's if and only if p'. Our sentence is then 's only if p', which is the 2<sup>nd</sup> half of what you have often seen in expressions of the 's if and only if p', which is translated as  $(p \rightarrow s) \wedge (s \rightarrow p)$ . Thus, 's only if p' is translated by  $s \rightarrow p$ . Equivalently we can translate as  $\sim p \rightarrow \sim s$ , but it is better to avoid the proliferation of negations.
3. Next, 'Unless p, q'. First, write this as '**q unless p**'. We can interpret this as ' $\sim p \rightarrow q$ '. To see this, we decide first that the best translation is an implication. Also, we realize that in this implication there should not be more than one negation. So, we have the following four possibilities (and their truth tables):

		q unless p			
		(1)	(2)	(3)	(4)
p	q	$p \rightarrow \sim q$	$q \rightarrow \sim p$	$\sim p \rightarrow q$	$\sim q \rightarrow p$
T	T	F	F	T	T
T	F	T	T	T	T
F	T	T	T	T	T
F	F	T	T	F	F

The usual interpretation for 'q unless p' is  $\sim p \rightarrow q$ . Maybe an easier way to see this is to translate "I eat an apple unless I eat an orange". Natural restating of this is "If I do not eat an orange then I eat an apple"

Thus, letting  $q$  = 'I eat an apple' and  $p$  = 'I eat an orange', the sentence becomes  $\sim p \rightarrow q$ .

With the above, we can now continue the translation of our Natural Language sentence:

Logic is <b>either</b> too boring <b>or</b> too difficult:	<b>either</b> $p$ <b>or</b> $q$	$p \vee q$
... <b>either</b> it is part of mathematics <b>or</b> it is part of philosophy:	<b>either</b> $r$ <b>or</b> $s$	$r \vee s$
... <b>unless</b> it is NOT part of mathematics it is too difficult:	<b>unless</b> $\sim r, q$ Denote $\sim r$ by $t$ to obtain Unless $t, q$	$\sim q \rightarrow t$ Or restoring $\sim r$ $\sim q \rightarrow \sim r$
<b>Only if</b> it is boring will it be part of philosophy:	<b>Only if</b> $p, s$	$s \rightarrow p$

From this point on one should be able to translate the argument:

$p \vee q, \sim p, r \rightarrow p, s \rightarrow q, \sim q \rightarrow \sim r, \vdash s \rightarrow p$

To continue, one may like to use:

Use:  $a \wedge b \vee c = (a \vee c) \wedge (b \vee c)$

$a$	$b$	$c$	$a \wedge b$	$a \wedge b \vee c$	$a \vee c$	$b \vee c$	$(a \vee c) \wedge (b \vee c)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	F	T	F	F
F	T	T	F	T	T	T	T
F	T	F	F	T	T	T	T
F	F	T	F	T	T	T	T
F	F	F	F	F	F	F	F

However, since there are 4 propositions, a semantic proof of this argument would need a truth table with 16 rows!