Informed Search Algorithms
– 4.1 & 4.2 –

Outline

- ♢ Best-first search
- ♢ A* search
- ♢ Heuristics

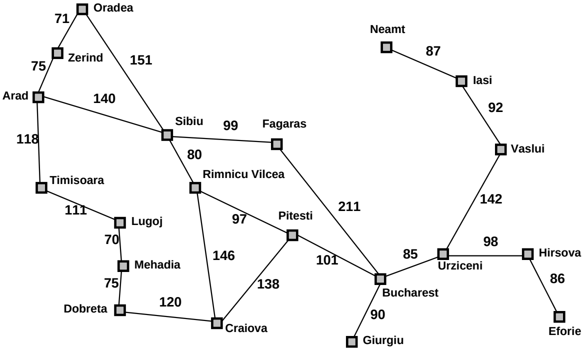
Review: Tree search

function TREE-SEARCH(*problem*, *fringe*) **returns** a solution, or failure
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[*problem*]), *fringe*)
    loop do
        if *fringe* is empty **then return** failure
        node ← REMOVE-FRONT(*fringe*)
        if GOAL-TEST[*problem*] applied to STATE(*node*) succeeds **return** *node*
        fringe ← INSERTALL(EXPAND(*node*, *problem*), *fringe*)

Best-first search

- Idea: use an evaluation function for each node to estimate of "desirability"
  $\Rightarrow$ Expand most desirable unexpanded node
- Implementation: fringe is a queue sorted in decreasing order of desirability
- Special cases:
  - greedy search
  - A^* search

Romania with step costs in km



Straight−line distance
to Bucharest

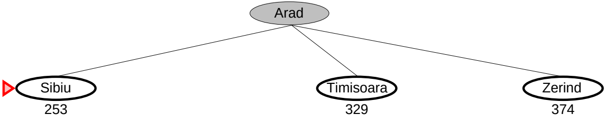
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Greedy search

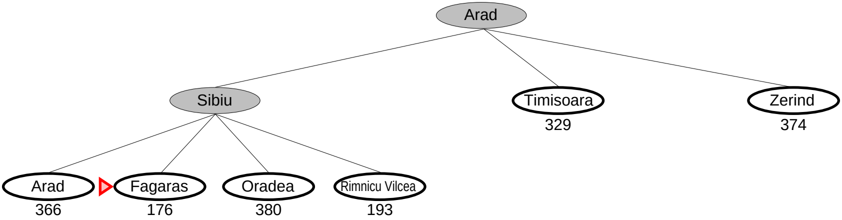
- Evaluation function $h(n)$ (**h**euristic) $=$ estimate of cost from n to the closest goal
  E.g., $h_{\mathrm{SLD}}(n)$ $=$ straight-line distance from n to Bucharest
- Greedy search expands the node that **appears** to be closest to goal

Greedy search example

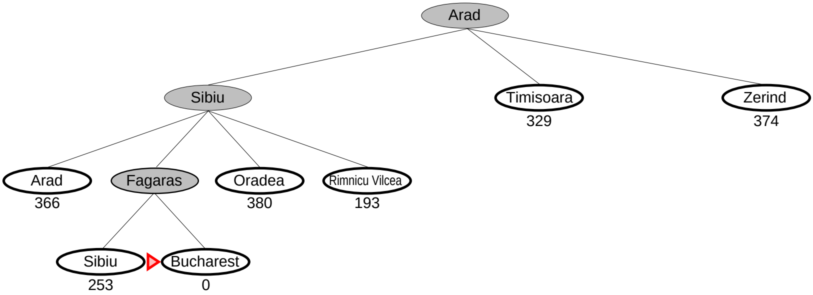
Greedy search example



Greedy search example



Greedy search example



Properties of greedy search

Complete??

Properties of greedy search

Complete?? No–can get stuck in loops, e.g., with Oradea as goal,

Iasi $\rightarrow$ Neamt $\rightarrow$ Iasi $\rightarrow$ Neamt $\rightarrow$

Complete in finite space with repeated-state checking

Time??

Properties of greedy search

Complete?? No–can get stuck in loops, e.g.,

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Complete in finite space with repeated-state checking
Time?? $O(b^m)$, but a good heuristic can give dramatic improvement

Space??

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Space?? $O(b^m)$—keeps all nodes in memory

Optimal??

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Complete in finite space with repeated-state checking

Time?? $O(b^m)$, but a good heuristic can give dramatic improvement

Space?? $O(b^m)$—keeps all nodes in memory

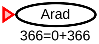
Optimal?? No

A* search

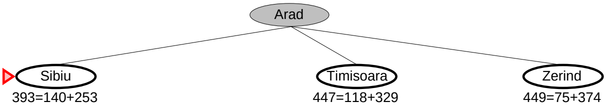
Idea: avoid expanding paths that are already expensive

- Evaluation function $f(n) = g(n) + h(n)$
- $g(n)$ = cost so far to reach n
- $h(n)$ = estimated cost to goal from n
- $f(n)$ = estimated total cost of path through n to goal
- A* search uses an admissible heuristic
  - $h(n) \leq h^*(n)$ where $h^*(n)$ is the **true** cost from n.
    (Also require $h(n) \geq 0$, so $h(G) = 0$ for any goal G.)
  - $h_{\mathrm{SLD}}(n)$ never overestimates the actual road distance
- Theorem: A* search is optimal

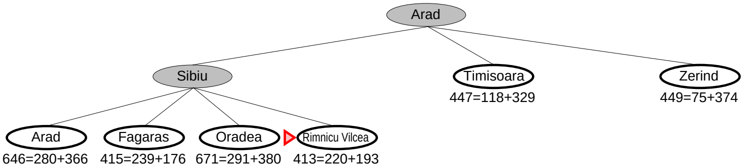
A* search example



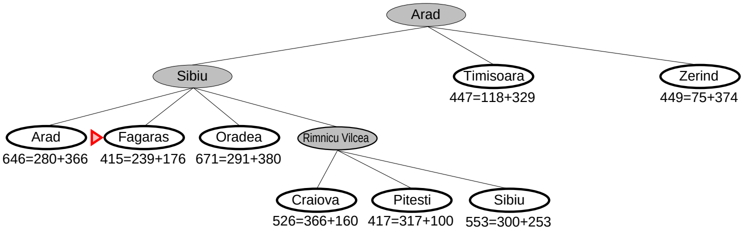
A* search example



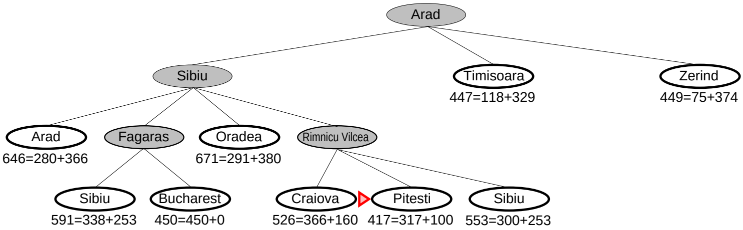
A* search example



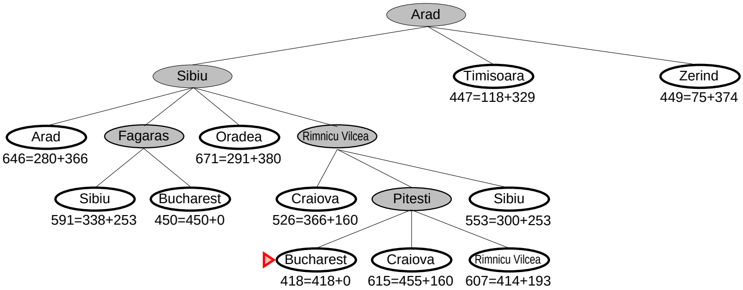
A* search example



A* search example

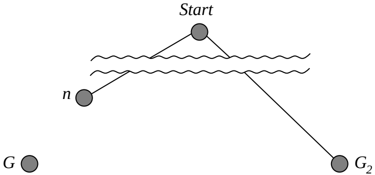


A* search example



Optimality of A* (standard proof)

Suppose some suboptimal goal G_2 has been generated and is in the queue. Let n be an unexpanded node on a shortest path to an optimal goal G.



$$
\begin{array}{rcll}
f(G_2) & = & g(G_2) & \text{since } h(G_2) = 0 \\
 & > & g(G_1) & \text{since } G_2 \text{ is suboptimal} \\
 & \geq & f(n) & \text{since } h \text{ is admissible}
\end{array}
$$

Since $f(G_2) > f(n)$, A* will never select G_2 for expansion

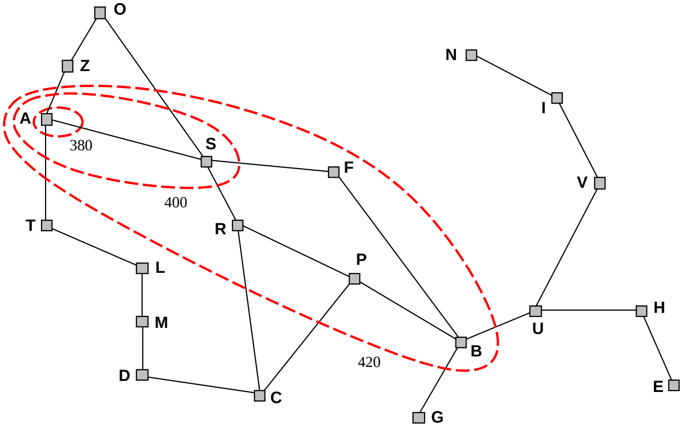
Optimality of A* (more useful)

Lemma: A* expands nodes in order of increasing f value*

Indeed, it gradually adds "f-contours" of nodes (cf. breadth-first adds layers)

Contour i has all nodes with $f = f_i$, where $f_i < f_{i+1}$

Optimality of A* (more useful)



Properties of A*

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Complete?? Yes, unless there are infinitely many nodes with $f \leq f(G)$
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Time?? Exponential in [relative error in h $\times$ length of soln.]

Space??

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Complete?? Yes, unless there are infinitely many nodes with $f \leq f(G)$
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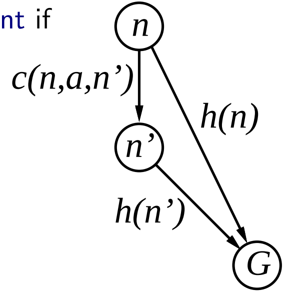
Space?? Keeps all nodes in memory
Optimal??

Properties of A*

Complete?? Yes, unless there are infinitely many nodes with $f \leq f(G)$
Time?? Exponential in [relative error in h $\times$ length of soln.]
Space?? Keeps all nodes in memory
Optimal?? Yes—cannot expand f_{i+1} until f_i is finished
A* expands all nodes with $f(n) < C^*$
A* expands some nodes with $f(n) = C^*$
A* expands no nodes with $f(n) > C^*$

Proof of lemma: Consistency

A heuristic is consistent if



$$h(n) \leq c(n, a, n') + h(n')$$

If h is consistent, we have

$$\begin{aligned} f(n') &= g(n') + h(n') \\ &= g(n) + c(n, a, n') + h(n') \\ &\geq g(n) + h(n) \\ &= f(n) \end{aligned}$$

i.e., $f(n)$ is nondecreasing along any path.

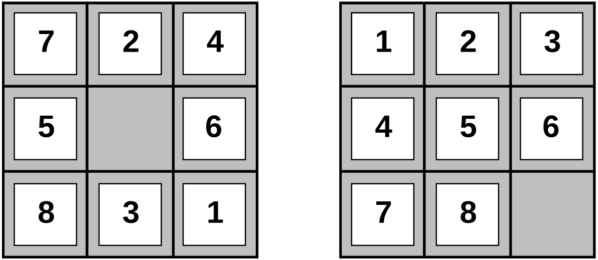
Admissible heuristics

E.g., for the 8-puzzle:

$h_1(n)$ = number of misplaced tiles

$h_2(n)$ = total Manhattan distance

(i.e., no. of squares from desired location of each tile)



Start State **Goal State**

$h_1(S) = ??$

$h_2(S) = ??$

Admissible heuristics

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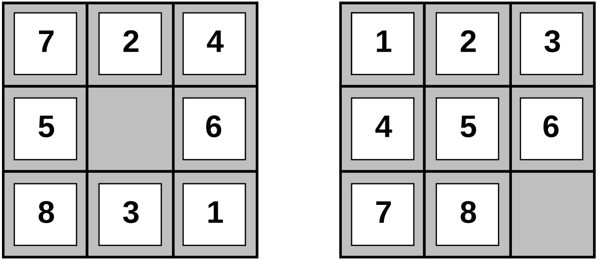


Start State **Goal State**

$h_1(S) = ??$ 6

$h_2(S) = ??$ $4+0+3+3+1+0+2+1 = 14$

Dominance

If $h_2(n) \geq h_1(n)$ for all n (both admissible) **then** h_2 dominates h_1 and is better for search

Typical search costs:

$d = 14$	IDS $=$ 3,473,941 nodes
	$A^*(h_1) = 539$ nodes
	$A^*(h_2) = 113$ nodes
$d = 24$	IDS $\approx$ 54,000,000,000 nodes
	$A^*(h_1) = 39{,}135$ nodes
	$A^*(h_2) = 1{,}641$ nodes

Given any admissible heuristics h_a, h_b,

$$h(n) = \max(h_a(n), h_b(n))$$

is also admissible and dominates h_a, h_b

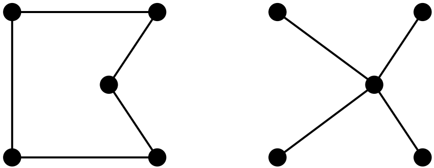
Relaxed problems – 1

- Admissible heuristics can be derived from the **exact** solution cost of a **relaxed** version of the problem
- If the rules of the 8-puzzle are relaxed so that a tile can move **anywhere**, then $h_1(n)$ gives the shortest solution
- If the rules are relaxed so that a tile can move to **any adjacent square**, then $h_2(n)$ gives the shortest solution

Key point: the optimal solution cost of a relaxed problem is no greater than the optimal solution cost of the real problem

Relaxed problems – 2

Well-known example: traveling salesperson problem (TSP)
Find the shortest tour visiting all cities exactly once



Minimum spanning tree can be computed in $O(n^2)$ and is a lower bound on the shortest (open) tour.

Summary

- Heuristic functions estimate costs of shortest paths
- Good heuristics can dramatically reduce search cost
- Greedy best-first search expands lowest h
  - incomplete and not always optimal
- A* search expands lowest $g + h$
  - complete and optimal
  - also optimally efficient (up to tie-breaks, for forward search)
- Admissible heuristics can be derived from exact solution of relaxed problems