Superradiance via synchronization

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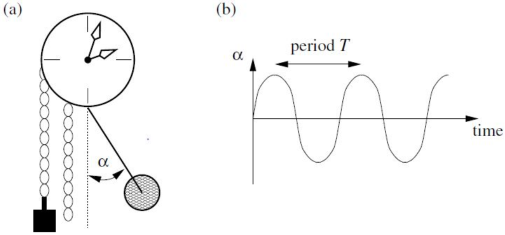
Part I

1. Synchronization
2. Quantum synchronization of a driven oscillator
3. Qubit + driven oscillator

Self Sustained Oscillator

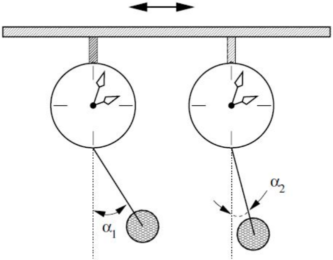
- Characterized by frequency and time-period
- Self sustained oscillators
  - Have an internal source of energy
  - Continue to oscillate with their natural frequency in the absence of any external disturbance
  - eg:- pendulum clock

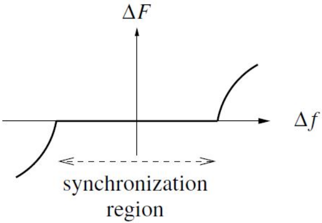


A. Pikovsky, Synchronization: A universal concept in nonlinear sciences

What is synchronization?

- Adjustment of an oscillator's rythm in response to an external signal/perturbation





Signs of synchronization

- Frequency entrainment, $f_1 \to f_2$
- Phase locking, $\Delta\phi \to$ *constant*

Synchronization of coupled pendulum clocks

Synchronization scenarios

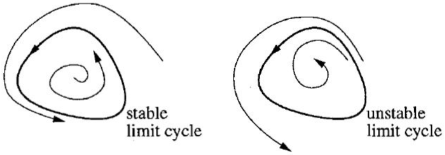
- Essential - low detuning and weak coupling
- Cases of synchronization
  - Two self sustained oscillators with slightly different frequencies coupled to each other
  - An oscillator driven by an external harmonic drive with a slightly detuned frequency
  - Several oscillators coupled to each other

van der Pol oscillator

Consider the van-der Pol oscillator described by

$$\ddot{x} + \mu(x^2 - 1)\dot{x} + x = 0$$

- Represents a non-linear oscillator
- Characterised by a stable limit cycle in phase space
- In fact, non-linearity is essential for stable limit cycle
- Good candidates for studying synchronization



Strogatz, Nonlinear dynamics and chaos

Mathematical formulation (open quantum systems)

- Damping implies non-unitary time evolution
- Such systems are open quantum systems
- Described by density matrix, $\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$
- Evolution of density matrices is governed by quantum master equations

Lindblad equation

$$\dot{\rho} = -i[H, \rho] + \sum_{i=1}^{N^2-1} \gamma_i (A_i \rho A_i{}^\dagger - \frac{1}{2}\{A_i^\dagger A_i, \rho\})$$

Quantum synchronization of driven vdP oscillator

Classical case: $\ddot{x} + (-\gamma_1 + \gamma_2 x^2)\dot{x} + \omega_0^2 x = \Omega \cos(\omega_d t)$

Quantum case: Evolution equation in the rotating frame given by

$$\frac{d\rho}{dt} = -i[-\Delta \hat{b}^\dagger \hat{b} + i\Omega(\hat{b} - \hat{b}^\dagger), \rho] + \gamma_1 D[\hat{b}^\dagger]\rho + \gamma_2 D[\hat{b}^2]\rho$$

where $\hbar = 1$. D are the Lindblad dissipators given by

$$D[\hat{b}^\dagger]\rho = \hat{b}^\dagger \rho \hat{b} - \frac{1}{2}\{\hat{b}\hat{b}^\dagger, \rho\}$$

negative damping adds a photon at rate $\sim \gamma_1$

$$D[\hat{b}^2]\rho = \hat{b}^2 \rho \hat{b}^{\dagger 2} - \frac{1}{2}\{\hat{b}^{\dagger 2}\hat{b}^2, \rho\}$$

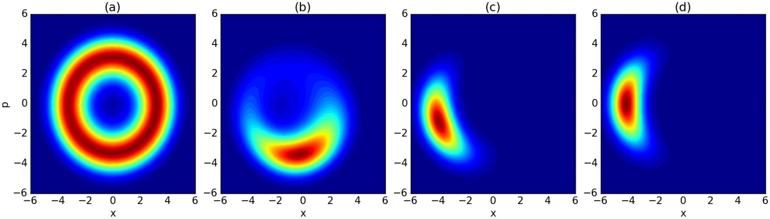
non-linear damping removes two photons at rate $\sim \gamma_2$

Bruder et al. Phys. Rev. Lett., Mar 2014

Quantum synchronization of driven vdP oscillator

- Steadystate is calculated numerically
- Wigner function in phase space

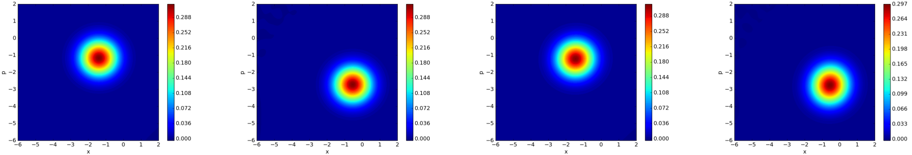
$$W_{ss}(x,p) = \frac{1}{\pi}\int_{-\infty}^{\infty} dy e^{-2ipy} \langle x+y|\hat{\rho}_{ss}|x-y\rangle$$



$$\frac{\Delta}{\gamma_1} = (a)\ 16 \quad (b)\ 0.6 \quad (c)\ 0.1 \quad (d)\ 0$$

Adding a qubit to driven oscillator

- Can coupling with a qubit introduce essential non-linearity in the system for synchronization (Fazio et al. 2015)
- Replaced nonlinear term with a qubit interaction term
- $H = \Delta\left(b^\dagger b + \frac{\sigma_z}{2}\right) + \Omega(b^\dagger + b) + g(b^\dagger + b)\sigma_x$



$\frac{g}{\gamma_1} = 0.1,\ \frac{\Delta}{\gamma_1} = 0.6$ $\frac{g}{\gamma_1} = 0.01,\ \frac{\Delta}{\gamma_1} = 0.1$ $\frac{g}{\gamma_1} = 1,\ \frac{\Delta}{\gamma_1} = 0.6$ $\frac{g}{\gamma_1} = 0.1,\ \frac{\Delta}{\gamma_1} = 0.1$

Part II

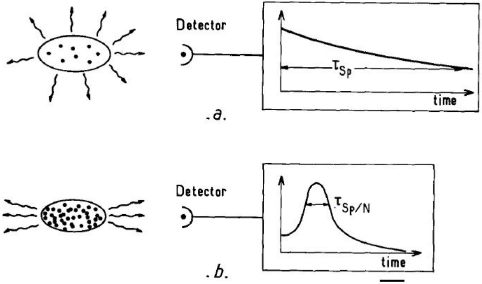
1. Introduction to superradiance
2. Superradiance in qubits coupled to a synchronized oscillator

Motivation

- Use synchronization for quantum technology
- Robust mechanism for controlling oscillator
- Produce collective interactions of atoms using synchronization

Superradiance

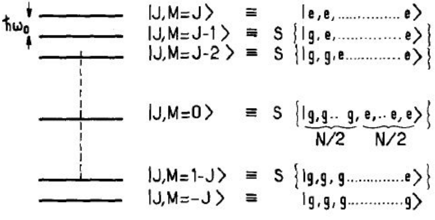
- Ordinary fluorescence: atoms de-excite independently
- Collective emissions: superradiance
- Atomic dipoles get phase locked
- Radiation emitted in a short outburst



Gross and Haroche, Superradiance

Dicke Superradiance

- Dimensions of the cavity should be less than the wavelength of radiation, $d < \lambda$
- State should be symmetrical
- Collective state can be represented as $|JM\rangle$, $J = \sum_i j_i$
- N qubits in a cavity (spin 1/2 particles), $J_{max} = N/2$

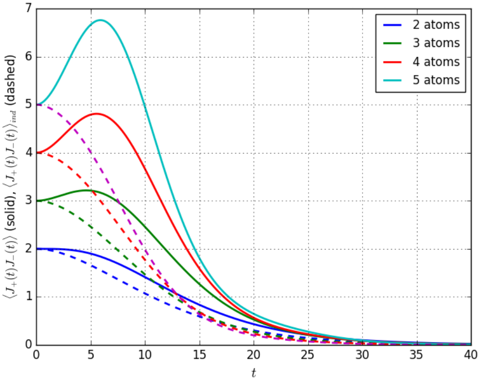


Dicke Superradiance

- Rate of independent emissions, $W_I = \Gamma \sum_i \langle D_i^+ D_i^- \rangle$
- Rate of collective emissions, $W_N = \Gamma \langle D^+ D^- \rangle$
- $\langle D^+ D^- \rangle = \langle JM | D^+ D^- | JM \rangle = (J+M)(J-M+1)$
- $\langle (\sum_i D_i^+)(\sum_j D_j^-) \rangle = N(N-1)\langle D_i^+ D_j^- \rangle + \sum_i \langle D_i^+ D_i^- \rangle$
- $\langle JM | D_i^+ D_j^- | JM \rangle = \frac{J^2 - M^2}{N(N-1)}$

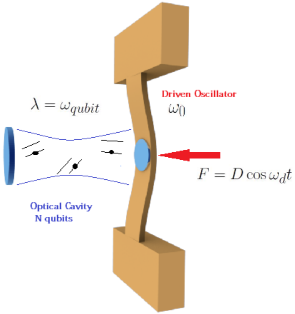
Dicke Superradiance

- Numerical illustration of superradiance
- Take $|\psi(0)\rangle = |ee...e\rangle$, i.e. $|JJ\rangle$ state



Superradiance in a cavity of qubits

- Qubits coupled to a driven vdP oscillator
- $d < \lambda$
- $\omega_{qubit} \approx \omega_d$



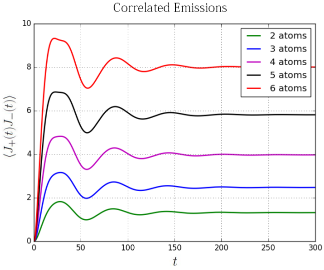
modified from Aspelmeyer et al. (Dec 2014)

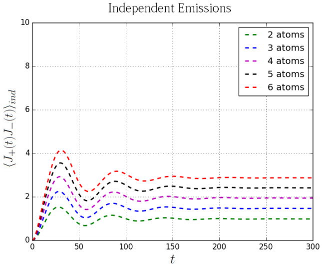
In the rotating frame,

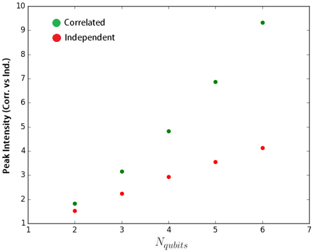
$$\hat{H} = \Delta \hat{r}^\dagger \hat{r} + D(\hat{r}^\dagger + \hat{r}) + ig_r(\hat{r}\hat{J}_+ - \hat{r}^\dagger \hat{J}_-) + ig_c(\hat{c}\hat{J}_+ - \hat{c}^\dagger \hat{J}_-)$$

$$\frac{d\rho}{dt} = -i[H,\rho] + \gamma_1 D[\hat{r}^\dagger]\rho + \gamma_2 D[\hat{r}^2]\rho + \kappa D[\hat{c}]\rho$$

Measurements of $\langle J_+ J_- \rangle$





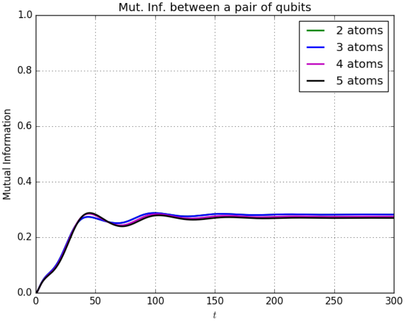


Mutual Information

Mutual Information

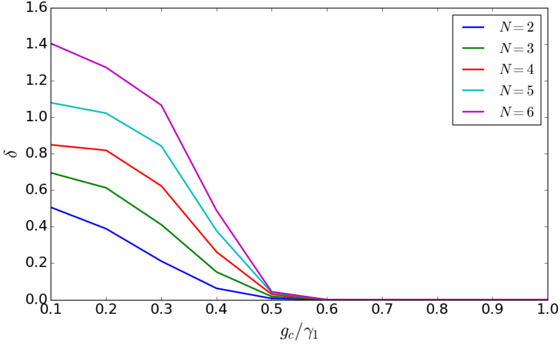
$$I = S(\rho_A) + S(\rho_B) - S(\rho)$$

where, $S(\rho) = -Tr(\rho \log \rho)$ is the von-Neumann entropy



Quantum phase transition

- The Dicke model shows a phase transition into the superradiant regime beyond a critical value of the atom-field coupling constant, g_c (Sayak et al. May 2015)
- Signatures of phase transition in our model



$$\delta \equiv \langle J_+ J_- \rangle_{max} - \langle J_+ J_- \rangle_{steadystate}$$

Conclusions

- Frequency entrainment of vdP leads to efficient pumping of qubits
- Different systems of qubits can be controlled using the same resonator
- Tunability in frequency
- Experimental verification may lead to technological implications

Thank You!