

Figure 4.13 • A greedy solution and an optimal solution to the 0-1 Knapsack problem.

Suppose we have the items in Figure 4.13 and W = 30. First we determine which entries are needed in each row.

Determine entries needed in row 3: We need

$$P\left[3\right]\left[W\right]=P\left[3\right]\left[30\right].$$

Determine entries needed in row 2: To compute P [3] [30], we need

$$P\left[3-1\right]\left[30\right] = P\left[2\right]\left[30\right] \qquad \text{and} \qquad P\left[3-1\right]\left[30-w_3\right] = P\left[2\right]\left[10\right].$$

Determine entries needed in row 1:

To compute P[2][30], we need

$$P\left[2-1\right]\left[30\right] = P\left[1\right]\left[30\right] \qquad \text{and} \qquad P\left[2-1\right]\left[30-w_2\right] = P\left[1\right]\left[20\right].$$

To compute P[2][10], we need

$$P[2-1][10] = P[1][10]$$
 and $P[2-1][10-w_2] = P[1][0]$.

Next we do the computations.

Compute row 1:

$$\begin{split} P\left[1\right]\left[w\right] &= \begin{cases} \left. maximum\left(P\left[0\right]\left[w\right],\$50 + P\left[0\right]\left[w - 5\right]\right) \text{ if } w_1 = 5 \leq w \\ P\left[0\right]\left[w\right] & \text{if } w_1 = 5 > w, \end{cases} \\ &= \begin{cases} \$50 \text{ if } w_1 = 5 \leq w \\ \$50 \text{ if } w_1 = 5 > w. \end{cases} \end{split}$$

Therefore,

$$\begin{split} P\,[1]\,[0] &= \$0 \\ P\,[1]\,[10] &= \$50 \\ P\,[1]\,[20] &= \$50 \\ P\,[1]\,[30] &= \$50. \end{split}$$

Compute row 2:

$$P[2][10] = \begin{cases} maximum(P[1][10], \$60 + P[1][0]) & \text{if } w_2 = 10 \le 10 \\ P[1][10] & \text{if } w_2 = 10 > 10 \end{cases}$$

$$= \$60.$$

$$\begin{split} P\left[2\right]\left[30\right] &= \begin{cases} maximum(P\left[1\right]\left[30\right],\$60 + P\left[1\right]\left[20\right]) \text{ if } w_2 = 10 \leq 30 \\ P\left[1\right]\left[30\right] & \text{if } w_2 = 10 > 30 \\ &= \$60 + \$50 = \$110. \end{cases} \end{split}$$

Compute row 3:

$$P[3][30] = \begin{cases} maximum(P[2][30], \$140 + P[2][10]) & \text{if } w_3 = 20 \le 30 \\ P[2][30] & \text{if } w_3 = 20 > 30 \\ = \$140 + \$60 = \$200. \end{cases}$$