Lecture 13.2

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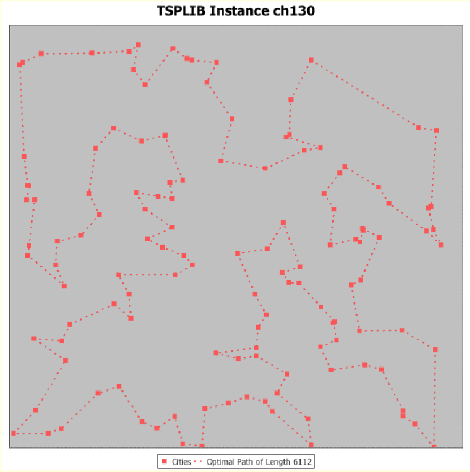
Branch & Bound Algorithms: TSP

Travelling Salespersons Problem (TSP)

- Recall: The goal in the TSP is to find the shortest path in a graph that starts at a given vertex, visits each vertex exactly once, and ends up at the starting vertex. Such a path is called optimal tour.
- Previously we have seen a dynamic programming algorithm to solve TSP; now we will see how the branch-and-bound technique can be used to solve TSP

GPS?

TSP



Cont.

- The state space tree for TSP can be as follows:
  - Start with a starting vertex on level 0 of the tree
  - Level 1 can have child nodes for each remaining vertex other than the starting vertex, level 2 can have child nodes for remaining vertices than do not fall on each path up to level 1 and so on
- To use best-first search, determine a bound for each node
- The bound in TSP can be a lower bound on the length of any tour that can be obtained by expanding beyond a given node - we can mark a node promising only if its bound is less than current minimum tour length

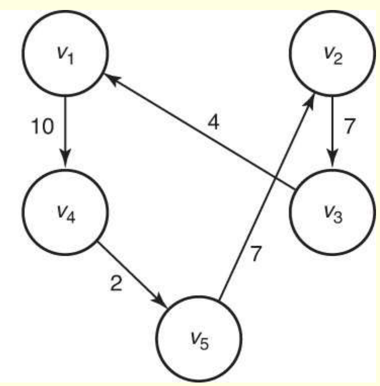
Cont.

- The method used in TSP to calculate the bound is to add up the minimum edges to every remaining vertex
- Calculate the bounds for the various children of the root, so you know which node to expand further i.e. the node with the minimum value needs to be expanded

TSP State Space Tree Example

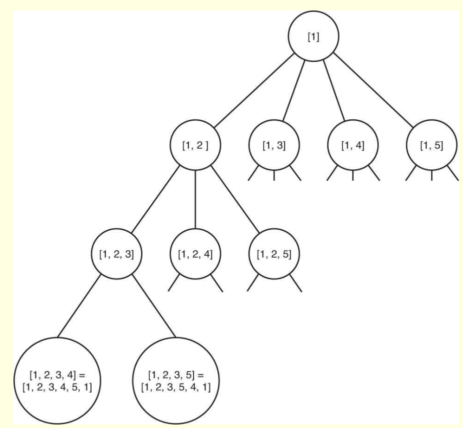
- Adjacency matrix for a graph with 5 vertices:

$$\begin{bmatrix} 0 & 14 & 4 & 10 & 20 \\ 14 & 0 & 7 & 8 & 7 \\ 4 & 5 & 0 & 7 & 16 \\ 11 & 7 & 9 & 0 & 2 \\ 18 & 7 & 17 & 4 & 0 \end{bmatrix}$$



If the starting vertex is 1

- Goal is to find the shortest path in a directed graph that starts at a given vertex, visits each vertex in the graph exactly once, and ends up back at the starting vertex. Such a path is called an optimal tour.



If starting from v1

- Lower bounds:

$$
\begin{array}{lll}
v_1 & minimum\,(14,4,10,20) & = 4 \\
v_2 & minimum\,(14,7,8,7) & = 7 \\
v_3 & minimum\,(4,5,7,16) & = 4 \\
v_4 & minimum\,(11,7,9,2) & = 2 \\
v_5 & minimum\,(18,7,17,4) & = 4
\end{array}
$$

Because a tour must leave every vertex exactly once, a lower bound on the length of a tour is the sum of these minimums. Therefore, a lower bound on the length of a tour is

$$4+7+4+2+4=21.$$

Suppose v1 → v2 is committed

$$
\begin{array}{lll}
v_1 & & 14 \\
v_2 & minimum\,(7,8,7) & = 7 \\
v_3 & minimum\,(4,7,16) & = 4 \\
v_4 & minimum\,(11,9,2) & = 2 \\
v_5 & minimum\,(18,17,4) & = 4
\end{array}
$$

To obtain the minimum for v_2 we do not include the edge to v_1, because v_2 cannot return to v_1. To obtain the minimums for the other vertices we do not include the edge to v_2, because we have already been at v_2. A lower bound on the length of any tour, obtained by expanding beyond the node containing [1, 2], is the sum of these minimums, which is

$$14+7+4+2+4=31.$$

Deterministic solution?