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# No cure no pay contracts with limited liability

Rud Faden

*University of Copenhagen*

[rud.faden@econ.ku.dk](mailto:rud.faden@econ.ku.dk)

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## Abstract

This paper examines a principal agent model in which a risk neutral physician makes an ex-ante effort choice while receiving payment from a risk neutral principal. It is assumed that the physician is subject to limited liability, such that he cannot be punished for bad outcome, but only rewarded for good outcomes.

## 1 MODEL

There is a risk neutral physician that cures a continuum of patients with a value to the principal of  $y \in [0, \infty)$  and receives payment  $R(y)$ . The physician cures patient with effort  $e$ . I assume that the probability of curing a patient is monotonically increasing in effort (monotone likelihood ratio property), such that

$$\frac{\partial}{\partial y} \left( \frac{g_e(y | e)}{g(y | e)} \right) > 0 \quad (1)$$

for all  $e > 0$  and  $y \geq 0$ . In addition I assume that  $E[y | e = 0] = 0$ . Further I also assume that the distribution functions satisfies convexity of the distribution function (CDFC).<sup>1</sup>

The principal observes the physicians value of cured patients ex-ante. Therefore the hospital specifies a contract that pays the physician  $R(y)$ .

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<sup>1</sup>The CDFC assumption implies that  $\frac{\partial^2 G(y|e)}{\partial e^2} \geq 0$ . See Rogerson (1985, p. 1362)

Further it is assumed that the payoff is bounded by limited liability. This implies that (i) the principal cannot be required to pay the physician more than the value of the production, and (ii) the physician cannot be required to make payments to the hospital. Therefore the payment function is bounded by  $0 \leq R(y) \leq y$ .

Let  $U(y, e)$  denote the physicians continuous and twice differential utility function given by

$$U(R, e) = \int_0^\infty R(y)g(y|e) dy - C(e) \quad (2)$$

and the principals payoff is given by

$$V(y, R(y)) = \int_0^\infty (y - R(y))g(y|e) dy \quad (3)$$

## 2 OPTIMAL PAYMENT WITH A RISK NEUTRAL AGENT

Unfortunately neither existence or uniqueness of a solution to eq. (2) can be guaranteed. To reduce the ambiguity I follow Innes (1990) and introduce the first assumption.

**Assumption 2.1.** [Innes (1990)] *There exists a finite  $e_{\max}$  such that*

$$\lim_{e \rightarrow e_{\max}} E[U(y, e) | e] < E[U(y, 0) | 0]$$

Assumption 2.1 indicates that there is some disutility to effort and that this effort is increases as  $e \rightarrow e_{\max}$ . Therefore the physicians effort choice can be limited to  $[0, e_{\max}]$

**Lemma 2.2.** [Innes (1990)] *For all  $R(y)$  functions satisfying limited liability  $0 \leq R(y) \leq y$ , (i)  $E[U(y, e) | e]$  is bounded from above and below on the domain  $e \in [0, e_{\max}]$ , and therefore there exists at least one solution to eq. (2)*

*Proof.* For all  $R(y)$  satisfying the limited liability constraint  $E[U(y, e) | e] \geq E[U(R(y), e) | e]$ , and for  $e \in [0, e_{\max}]$ ,  $E[U(R(y), e) | e] \geq E[U(0, e) | e] \geq E[U(R(y) - k^*, e) | e]$ ,  $k^* = E[R(y) | e_{\max}]$ . Since  $E[U(R(y), e) | e]$  and thereby also  $E[U(R(y) - k^*, e) | e]$  is continuous on the compact set  $e \in [0, e_{\max}]$ , implying result (i). Result (ii) then follows from the extreme value theorem.  $\square$

Lemma 2.2 implies that there exists an optimal level of effort given a eligible payoff function.

Given eq. (2) and a fixed payment function  $R(y)$ , the physician will choose effort to solve the following problem

$$\max_{R,e} \int_0^\infty R(y)g(y|e) dy - C(e) \quad (4a)$$

$$\text{s.t. } \int_0^\infty (y - R(y))g(y|e) dy \geq y_L^0 \quad (4b)$$

$$E[U(R, e) | e] \leq E[U(R, e^*) | e] \quad (4c)$$

$$0 \leq R(y) \leq y \quad (4d)$$

To find a solution to eq. (4) is difficult and few general results can be obtained. To make the problem more tractable the incentive comparability constraint in eq. (4c) is often replaced by the first order derivative.<sup>2</sup>

**Proposition 2.3.** *If  $R$  solves the maximization problem in eq. (4a), then there is some threshold value  $y^*$ , such that*

$$R^*(y) = \begin{cases} 0 & \text{for } y < y^* \\ y & \text{for } y \geq y^* \end{cases}$$

*Proof.* Replacing eq. (4c) with its (relaxed) first order condition, and writing down the Lagrangian I get that

$$\mathcal{L} = R(y)g(y|e) - C(e) + \mu[R(y)g_e(y|e) - C_e(e)] + \lambda[(y - R(y))g(y|e) - y_L^0] \quad (5)$$

which can also be written as

$$R(y) \left[ g(y|e) + \mu g_e(y|e) \right] + (y - R(y))\lambda g(y|e) + y g(y|e) - \mu C_e(e) - \lambda y_L^0 \quad (6)$$

Only the first two terms of eq. (6) depend on the size of  $R(y)$  and thereby determine the maximum. If the first term is larger than the second term

$$R(y) \left[ g(y|e) + \mu g_e(y|e) \right] > (y - R(y))\lambda g(y|e) \quad (7)$$

the whole expression is increasing in  $R(y)$ , therefore  $R(y)$  should be set to its maximum  $R(y) = y$ . If the opposite is true

$$R(y) \left[ g(y|e) + \mu g_e(y|e) \right] < (y - R(y))\lambda g(y|e) \quad (8)$$

the expression is decreasing in  $R$ , and  $R(y)$  should be set to its minimum  $R(y) = 0$ . Note that eq. (8) can be written as

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<sup>2</sup>This approach is in general only valid when the agent is risk neutral. In section 3 I will solve the problem when the agent is risk averse

$$1 + \mu \frac{g_e(y|e)}{g(y|e)} < \lambda \quad (9)$$

Given eq. (1), the left hand side of eq. (9) is increasing in  $y$  this inequality is satisfied for some  $y$  below a threshold level  $y^*$ . Above  $y^*$  eq. (7) holds. Thereby the payment function is 0 for some  $y < y^*$  and  $y$  for some level above  $y > y^*$  as stated in proposition 2.3. What remains is to show that the solution to eq. (4) is in fact an optimum. Following Rogerson (1985) it is sufficient to show that  $\mu > 0$ . If  $\mu \leq 0$ , payment is non-increasing in output. By assumption 2.1 the physician therefore chooses  $e = 0$ . However,  $e = 0$  is not feasible because of the assumption that  $E[y | e = 0] = 0$  and condition eq. (4b). Therefore for  $e > 0$ ,  $\mu$  must be positive.  $\square$

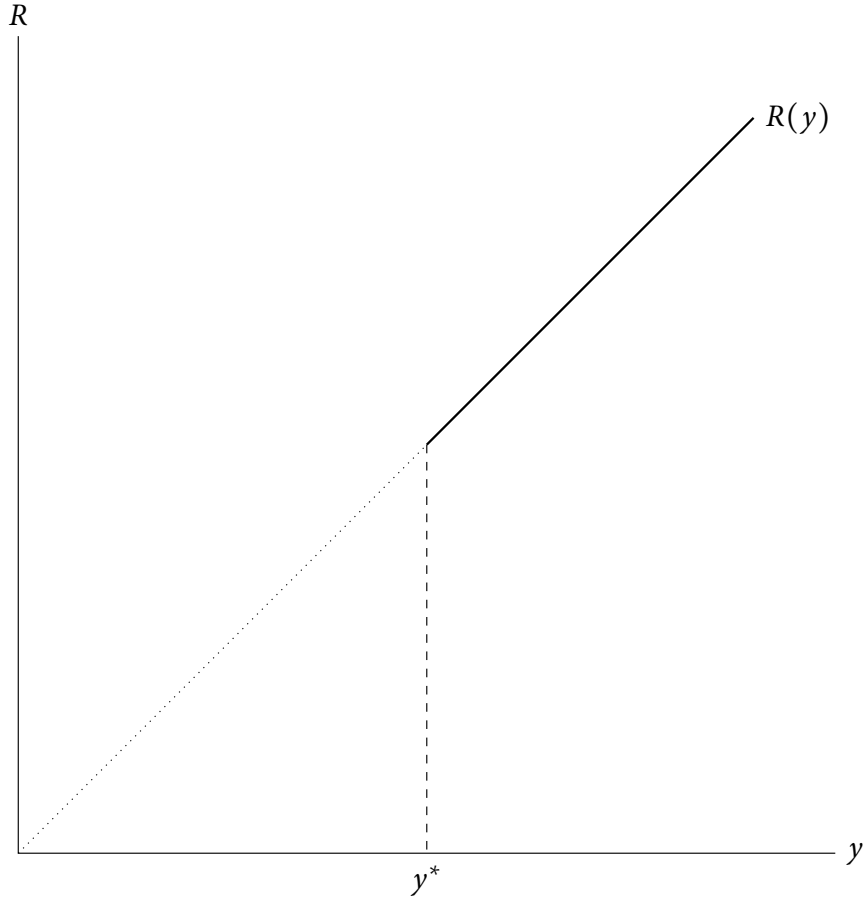


Figure 1: The payment function  $R(y)$ . Before  $y^*$  the physician is paid nothing and after  $y^*$  the physician is the paid maximum of  $R(y)$

## 2.1 An alternative proof

Give eq. (4) Lagrangian is

$$\begin{aligned} \mathcal{L} = & R(y)g(y|e) - C(e) + \mu[R(y)g_e(y|e) - C_e(e)] + \\ & \lambda[(y - R(y))g(y|e) - y_L^0] + \theta(y)R(y) + \eta(y)(y - R(y)) \end{aligned} \quad (10)$$

The first order conditions are

$$g(y|e) \left[ 1 - \lambda + \mu \frac{g_e(y|e)}{g(y|e)} \right] + \theta(y) - \eta(y) \quad (11a)$$

$$R(y) \left[ (1 - \lambda)g_e(y|e) + \mu g_{ee}(y|e) \right] - C_{ee}(e) \quad (11b)$$

Because of nonnegativity and complementary slackness conditions for  $\theta(y)$  and  $\eta(y)$ , eq. (11a) yields

$$\phi(y, e) = g(y|e) \left[ 1 - \lambda + \mu \frac{g_e(y|e)}{g(y|e)} \right] > 0 \Rightarrow R(y) = y \quad (12a)$$

$$\phi(y, e) = 0 \Rightarrow R(y) \in [0, y] \quad (12b)$$

$$\phi(y, e) < 0 \Rightarrow R(y) = 0 \quad (12c)$$

It is clear from eqs. (1) and (11a) that for  $y$  large enough eq. (12a) is always feasible. However, feasibility of eqs. (12a) and (12b) is not guaranteed, even for  $y = 0$

## 3 OPTIMAL PAYMENT WITH A RISK AVERSE AGENT

### REFERENCES

- Innes, Robert D. 1990. "Limited liability and incentive contracting with ex-ante action choices". *Journal of Economic Theory* 52 (1): 45–67. (Cit. on p. 2).
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