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# Physician incentive contracting with limited liability

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## Abstract

This paper examines a principal-agent model in which a risk neutral physician makes an ex-ante effort choice. The physician receives payment, only if he cures the patients (e.g. the DRG-value). In the production, the physician employs costly hospital. In his contract with the hospital, the physician is subject to statutory limited liability. Given these liability limits, the question is then which contract the hospital optimally offers the physician.

## 1 MODEL

There is a risk neutral physician that cures a continuum of patients with a monetary value of  $y \in [0, \infty)$ . The physician cures patient with effort  $e$ . I assume that the probability of curing a patient is monotonically increasing in effort (monotone likelihood ratio property), such that

$$\frac{\partial}{\partial y} \left( \frac{g_e(y | e)}{g(y | e)} \right) > 0 \quad (1)$$

for all  $e > 0$  and  $y \geq 0$ . In addition I assume that  $E[y | e = 0] = 0$

The principal observes the physicians value of cured patients ex-ante. Therefore the hospital specifies a contract that requires the physician to repay the hospital as a function of production  $R(y)$ .

Further it is assumed that the physicians payoff is bounded by limited liability. This implies that (i) the physician cannot be required to pay the hospital more than

the value of the production, and (ii) the hospital cannot be required to make payments to the physician. Therefore the payment function is bounded by  $0 \leq R(y) \leq y$ .

Let  $U(y, e)$  denote the physicians continuous and twice differential utility function given by

$$U(e) = y - R(y) - C(e) \quad (2)$$

Given eq. (2) and a fixed payment function  $R(y)$ , the physician will choose effort to solve the following problem

$$\max_y \int_0^\infty (y - R(y))g(y|e) dy - C(e) \quad (3a)$$

$$\text{s.t. } \int_0^\infty R(y)g(y|e) dy \geq R_L^0 \quad (3b)$$

$$E[V(R, e) | e] \leq E[V(R, e^*) | e] \quad (3c)$$

$$0 \leq R(y) \leq y \quad (3d)$$

**Proposition 1.1.** *If  $R$  solves the maximization problem in eq. (3a), then there is some threshold value  $y^*$ , such that*

$$R^*(y) = \begin{cases} y & \text{for } y < y^* \\ 0 & \text{for } y \geq y^* \end{cases}$$

*Proof.* Replacing eq. (3c) with it's first order condition, and writing down the Lagrangian I get that

$$\mathcal{L} = (y - R(y)) (g(y|e) + \mu g_e(y|e)) - \mu C_e(e) + \lambda R(y)g(y|e) - \lambda C(e) \quad (4)$$

which can also be written as

$$\begin{aligned} & (\lambda - 1)R(y)g(y|e) - \mu R(y)g_e(y|e) + \\ & y(g(y|e) + \mu g_e(y|e)) - \mu C_e(e) - \lambda C(e) \end{aligned} \quad (5)$$

Only the first to terms of eq. (5) depends on the size of  $R(y)$  and thereby determines the maximum. If the first term is larger than the second term

$$(\lambda - 1)g(y|e) \geq \mu f_e(y|e) \quad (6)$$

the whole expression is increasing in  $R(y)$ , therefore  $R(y)$  should be set to it's maximum  $R(y) = y$ . If the opposite is true

$$(\lambda - 1)g(y|e) < \mu f_e(y|e) \quad (7)$$

the expression is decreasing in  $R$ , and  $R(y)$  should be set to it's minimum  $R(y) = 0$ . Note that eq. (6) can be written as

$$\frac{g_e(y|e)}{g(y|e)} \leq \frac{\lambda - 1}{\mu}, \quad \lambda \neq 1, \mu \neq 0 \quad (8)$$

Given eq. (1), the left hand side of eq. (8) is increasing in  $y$  this inequality is satisfied for some  $y$  below a threshold level  $y^*$ . Above  $y^*$  eq. (6) holds. Thereby the payment function is 0 for some  $y < y^*$  and  $y$  for some level above  $y \geq y^*$  as stated in proposition 1.1

□

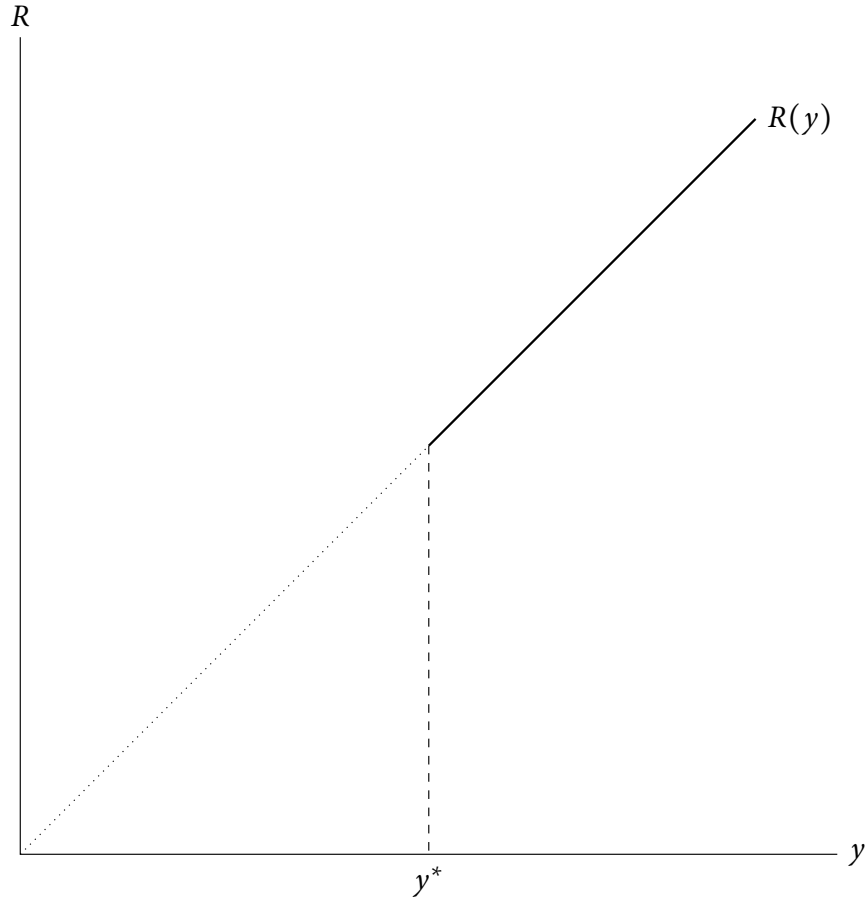


Figure 1: The payment function  $R(y)$ . Before  $y^*$  the physician is payed nothing and after  $y^*$  the physician is the payed maximum of  $R(y)$