$$E_p = \frac{T_1}{p \cdot T_p}$$

$$E_p = 1 \Rightarrow T_1 = p \cdot T_p$$

$$\Rightarrow T_p = \frac{T_1}{p}$$

En clase vimos:

$$E_{filas} = E_{columnas} = O\left(\frac{1}{\sqrt{p}}\right)$$

Por demostrar

$$E_{bloques} = O\left(\frac{1}{\log_2(p)}\right)$$

Particionar matriz A

en r filas

y c columnas

$p = r \cdot c$ (ejemplo $r=2$ $c=3$)

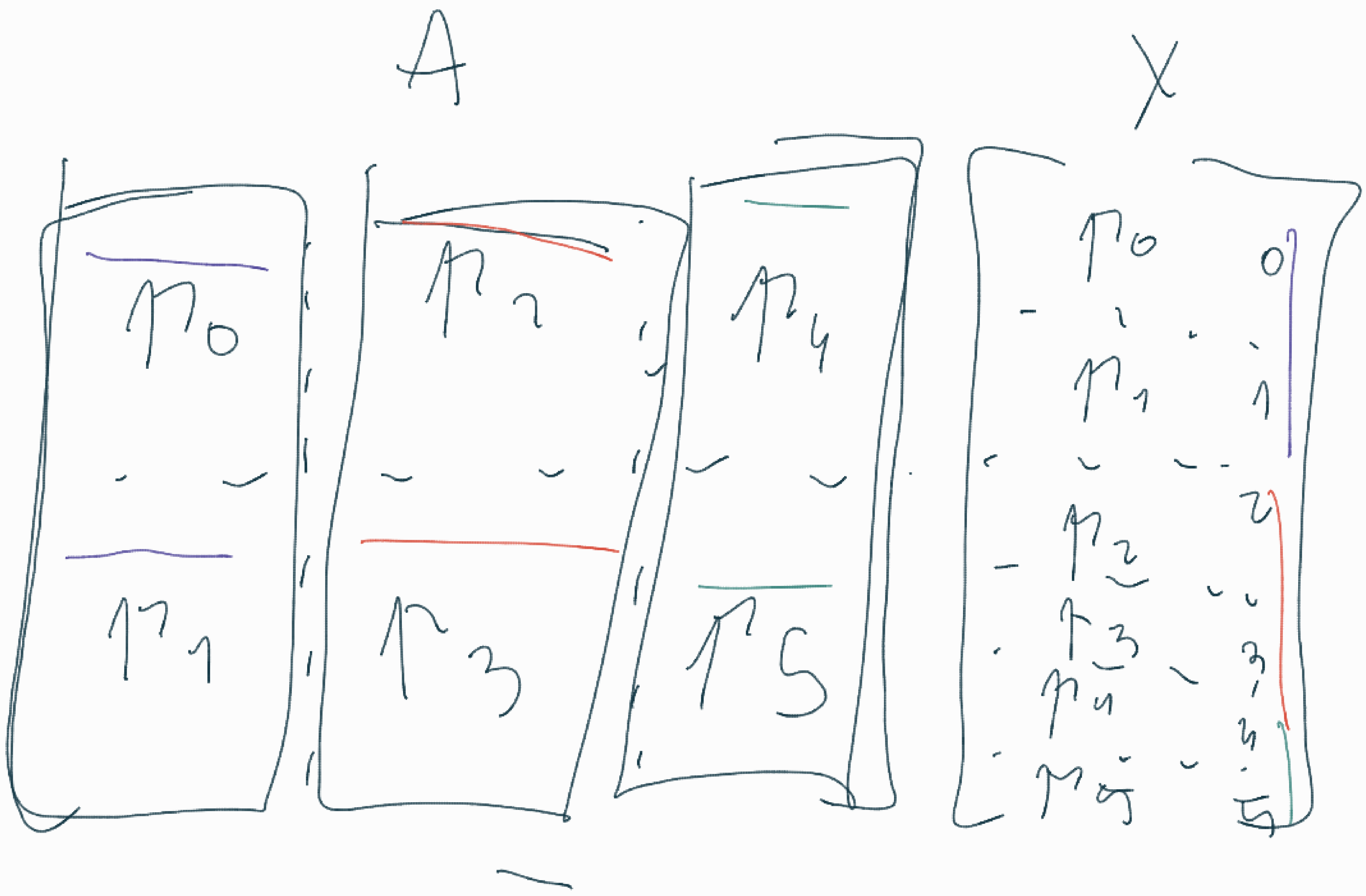
$$\overset{A}{\left[\begin{array}{c:c:c} p_0 & p_2 & p_4 \\ \hdashline p_1 & p_3 & p_4 \end{array}\right]} \overset{x}{\left[\begin{array}{c} p_0 \\ \hdashline p_1 \\ \hdashline p_2 \\ p_3 \\ p_4 \\ p_5 \end{array}\right]} = \overset{y}{\left[\begin{array}{c} \text{Veremos} \\ \text{después} \end{array}\right]}$$

$$E_p = \frac{T_1}{p \cdot T_p}$$

$$T_1 = 0\alpha + 0\beta$$
$$n(n + n \cdot 1)\gamma$$

$$T_1 = n(2n\cancel{m})\gamma$$

$$\boxed{T_1 = 2n^2\gamma} \leftarrow$$



1) Allgather sobre los X_i con $i \in$ columna

$$\sim \log_2(\text{cantidad procesos})\,\alpha + (\text{cant procesos} - 1) \times \text{cant datos}\ \beta$$

$$\sim \log_2(r)\,\alpha + (r-1)\frac{n}{p}\,\beta$$

Minimum Spanning tree

	p0	p1	p2	p3
t0	a			
t1	a	a		
t2	a	a	a	a
t3				

$\log(p)$ pasos en que todos tengan los datos.

$\Rightarrow \log(p) \cdot \alpha$ Tiempo

2) Local Mat. Vec

$$
\begin{array}{ccc}
A & x & y \\
\left[\begin{array}{c|c|c} r_0 & r_1 & r_4 \\ \hline r_1 & r_3 & r_4 \end{array}\right]
&
\left[\begin{array}{c} r_0 \\ r_1 \\ \hline r_2 \\ r_3 \\ r_4 \\ r_5 \end{array}\right]
= &
\left[\begin{array}{c} S_0 + S_1 + S_4 \\ \hline S_1 + S_3 + S_4 \end{array}\right]
\end{array}
$$

T

$$\frac{n}{r}\left(\frac{n}{c} + \frac{n}{c}\cancel{m}\right)\gamma$$

$$2\frac{n^2}{rc}\gamma = \boxed{2\frac{n^2}{p}\gamma}$$

3) Reduce - Scatter

$$\sim \log_2(\text{cant proces})\alpha + \frac{(\text{cant proces} - 1)}{\text{cant procesadores}} \cdot \text{cant datos}$$

$$\times (\beta + \gamma)$$

$$\sim \log_2(c)\alpha + \frac{c-1}{c} \cdot \frac{n}{r}(\beta + \gamma)$$

$$\sim \underline{\log_2}(c)\alpha + n\frac{c-1}{r}(\beta + \gamma)$$

$$T_p = \alpha\left(\log(c) + \log(r)\right)$$

$$+ \beta\left(n\frac{c-1}{r} + (r-1)\frac{n}{r}\right)$$

$$+ \gamma 2\frac{n^2}{r^2} + \gamma \cdot n\frac{c-1}{r}$$

$$T_p = \alpha \log(\cancel{c \cdot r}\, p)$$
$$+ \beta \frac{n}{p} (c + r)$$
$$+ \gamma\, 2 \frac{n^2}{p} + \gamma \frac{nc}{p}$$

$$T_p = \alpha \log(p)$$
$$+ \beta \left(\frac{n}{r} + \frac{n}{c} \right)$$
$$+ \underline{\gamma \frac{2n^2}{p}} + \gamma \frac{n}{r}$$

Para $r = c = \sqrt{p}$

$$T_p = \alpha \log(p) + \beta \frac{2n}{\sqrt{p}} + \gamma \frac{2n^2}{p} + \gamma \frac{n}{\sqrt{p}}$$

$$T_p = \alpha \log(p) + \frac{n}{\sqrt{p}} (2\beta + \gamma) + \gamma \frac{2n^2}{p}$$

$$E_p = \frac{T_1}{p \cdot T_p} = \frac{2n^2 \gamma}{\alpha p \log(p) + \sqrt{p}\, n (2\beta + \gamma) + 2\gamma n^2}$$

$$\lim_{n \to \infty} E_p = 1, \quad \lim_{p \to \infty} E_p = 0$$

Supongamos $\underbrace{p \sim n^2}$

$\Rightarrow \exists\, a\ \mathbb{R}_+ \quad a > 0$

tq $\quad ap = n^2$

$\Rightarrow n = \sqrt{ap}$

$$E_p = \frac{2n^2\gamma}{\alpha p \log(n) + \sqrt{p}\, n\,(2\beta+\gamma) + 2\gamma n^2}$$

$$E_p = \frac{2\,\cancel{a}\,\cancel{p}\,\gamma}{\frac{\cancel{a}\,\alpha\,\cancel{p}\log(p)}{a} + \cancel{p}\,\frac{\cancel{a}\sqrt{a}}{a}(2\beta+\gamma) + 2\gamma\,\cancel{a}\,\cancel{p}}$$

$$E_n = \frac{2\gamma}{\frac{\alpha \log(n)}{n} + \frac{1}{\sqrt{n}}(2\beta + \gamma) + 2\gamma}$$

$$E_n \in O\left(\frac{1}{\log(n)}\right)$$