Shaped RF Pulses: Part 2

Calculating RF Power

Recap

For introduction to shaped RF pulses, please visit:

https://github.com/rudysynex/NMR-CONCEPTS/blob/main/pdf_slides/Shaped%20RF%20Tutorial.pdf

The amplitude of a hard pulse

- The **flip-angle** of a hard pulse of duration **τ** seconds and amplitude $\boldsymbol{B_1}$ (**T**, Tesla), for a nucleus with gyromagnetic ratio $\boldsymbol{\gamma}$ (radians.T^{-1}.sec^{-1}) is given by:

$$\boldsymbol{\theta} \text{ (radians)} = \boldsymbol{\gamma . B_1 . \tau} \qquad \text{... Eq. 1}$$

- Rearranging the above, we get:

$$\boxed{\boldsymbol{\gamma . B_1}} = \boldsymbol{\theta / \tau} \text{ (rad.sec}^{-1}\text{)} \qquad \text{... Eq. 2}$$

This is the rotating frame precession frequency $\boldsymbol{\omega_1}$ in rad.sec^{-1}

- Dividing $\boldsymbol{\omega_1}$ by $2\boldsymbol{\pi}$ converts it to $\boldsymbol{v_1}$ Hz, giving:

$$\boldsymbol{v_1 = \gamma . B_1} / 2\boldsymbol{\pi} = \boldsymbol{\theta} / 2\boldsymbol{\pi . \tau} \text{ (Hz)} \qquad \text{... Eq. 3}$$

- **Pulse amplitude or power** is often expressed in units of **kHz**, as it's independent of the instrument.

Example: Amplitude of a 90° pulse

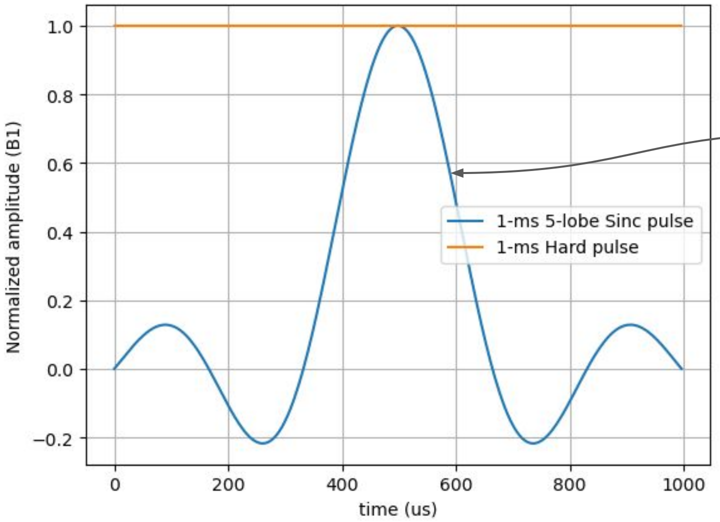
What is the amplitude required for a **10-μs** hard/square RF pulse to achieve a **flip-angle of 90°**?

$\boldsymbol{\theta} = 90°$, $\boldsymbol{\tau} = 10 \times 10^{-3}$ ms

$\therefore$ Amplitude $= \boldsymbol{\theta} / 2\boldsymbol{\pi}.\boldsymbol{\tau} = 90°/(360° \times 10 \times 10^{-3}$ ms$) =$ **25 kHz**

But how does this apply to a shaped pulse?

The ‘Pulse Integral’



The $\boldsymbol{B_1}$ field of a shaped pulse **varies with time**, unlike a hard pulse. Thus, for **Eq. 1** to hold, the $\boldsymbol{B_1}$ field needs to be integrated over the pulse duration $\boldsymbol{\tau}$, giving a generalized form of Eq. 1:

$$\boldsymbol{\theta} = \boldsymbol{\gamma}\,{}_0\!\int^{\tau} B_1(t)\,\mathrm{d}t$$

$B_1(t)$ can be rewritten as $B_{1max}.f(t)$ where $f(t)$ is a time-varying factor:

$$\boldsymbol{\theta} = \boldsymbol{\gamma} B_{1\,max}.\,{}_0\!\int^{\tau} f(t)\,\mathrm{d}t = \boldsymbol{\gamma} B_{1\,max}\boldsymbol{F\tau}$$

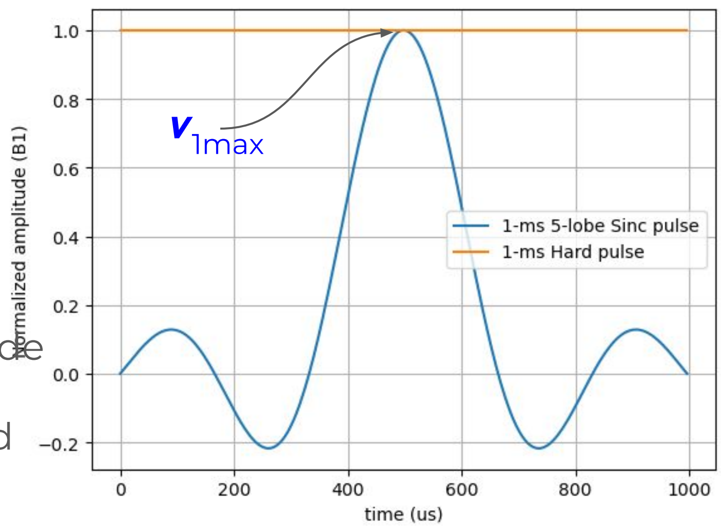
- **‘F’** is the **pulse integral** and it is the integral over the phase-sensitive RF pulse shape, i.e. area under the pulse, normalized to the range [0, 1], such that the **pulse integral of a hard RF pulse = 1** (*which makes sense as the B_1 is time invariant for a hard pulse*).

Example: Amplitude of a Sinc 90

- The pulse integral of a normalized 5-lobe Sinc pulse is **17.77%** of a Square function (hard pulse).
- The amplitude (in kHz) of a 1-ms, 90° pulse of this shape can be calculated as:

$\boldsymbol{v}_{1max}$(sinc) = 90°/(360° × 0.1777 × 1-ms) = **1.4 kHz**

- A hard 90° pulse of the same duration has an amplitude of **0.25 kHz** (*try the math yourself!*)
- Thus, to achieve same flip-angle, the **Sinc pulse** would require 1.4/0.25 = **5.6 times higher amplitude**.
- The higher amplitude can be achieved by proportionally increasing the RF current $\boldsymbol{I}$, but power (in Watts) scales as $\boldsymbol{I^2R}$ (R=resistance in Ω). So the **power level** needed from the RF amplifier would be $\mathbf{(5.6)^2}$ **= 31.36 times higher** than a square pulse of the same duration!



kHz → dB

- On NMR instruments power levels are often stated in **dB (decibel)**.
- If the power level (in dB) of a Hard/Square 90° pulse of a given duration is known, the required power (in dB) for a shaped pulse can be calculated using the following formula:

$$\mathbf{dB_{shaped}} = \mathrm{dB_{Hard}} + 20.\log_{10}(\boldsymbol{\nu}_{1\mathrm{shaped}}/\boldsymbol{\nu}_{1\mathrm{Hard}})$$

Known from calibration

- Using the same formula, you can also calculate the required amplitude for a **hard pulse of different duration** than the calibrated pulse. In fact, in this case the $\boldsymbol{\nu}_1$ ratio can be simplified to the ratio of pulse durations: $\boldsymbol{\tau}_{\mathbf{unknown}}/\boldsymbol{\tau}_{\mathbf{known}}$
- These methods are used to automatically calculate the required power for different shaped pulses on instruments, but there are limitations. It also assumes highly linearized RF amplifiers.

Limitations

- There are cases where the pulse-integral based method cannot be used reliably:
  - Highly inhomogeneous RF coils,
  - Adiabatic RF pulses (topic for another day)
- In these cases, manual calibration needs to be carried out by sweeping the RF pulse duration or amplitude.
- For shaped pulses, it is preferable to keep the duration constant and sweep the amplitude to maintain the same pulse bandwidth.
- Adiabatic RF pulses have a parameter called the ‘Q’ factor which determines their B_{1max}, but we will leave it for another tutorial.

Practice exercises

- Jupyter notebooks containing Python scripts to generate certain shaped pulses (sinc, gaussian, hermite etc.) and calculate their pulse integral + amplitude can be found at:

  https://github.com/rudysynex/NMR-CONCEPTS/tree/main/exercise_notebooks

- More notebooks will be added occasionally for other types of pulses.

Recommended resources to learn more about shaped pulses

- Robin de Graaf, *In vivo NMR Spectroscopy: Principles and Techniques*, 3rd Edition
- Timothy Claridge, *High-Resolution NMR Techniques in Organic Chemistry*, 3rd Edition