Transit Network Timetabling Problem: A Case Study of the Transport Operator in Cinu (Colombia)

Andrés Rueda López[1,], Juan Sebastian Agudelo Gomez[b]

[a] Universidad de Los Andes, , Bogotá, , Colombia
[b] Universidad de Los Andes, , Bogotá, , Colombia

Abstract

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1. Introduction

Public transport systems (PTS) play an important role in the development of urban populations and have a significant influence on the quality of life for residents. In recent years, the need to develop more efficient PTS has grown, as it has become evident that the economic and environmental performance of cities can be improved by efficiently connecting resources and mass population mobility (7). One of the main concerns in developing PTS and making accurate operational decisions is measuring the temporal and spatial coverage of the systems for the population. While spatial coverage or accessibility refers to ensuring that a large number of people have access to a transit station within a short distance, temporal coverage means that PTS are available when customers need the service (11).

The increasing demand for PTS, driven by the rapid growth of urban populations, implies that PTS planning becomes exponentially more complex. If this systmes are unable to adapt rappidly and in an efficient way to those changes, it can result in an increase of waiting times or PTS congestions (13). For this reason, it is essential that Public Transport Companies (PTC) allocate some of their resources to develop smart planning strategies that can quickly and accurately adapt to the needs of users.

A robust plan is the base for a PTC to satisfy the demand, give users the best possible experience, and guarantee a fair working policy for their drivers. A common transit operations planning process consists of four stages: first, designing the route network system; second, developing a timetable for each route; third, scheduling vehicles to match the timetable trips; and finally, crew scheduling (4).

This paper focuses on the second stage of the robust planning process by designing a bus timetable for one route of the *Transport Operator of Cinu (Colombia)*, dealing with what is known as a *Transit Network Timetabling Problem* (TNTP). This involves scheduling a timetable to operate over pre-planned routes, where all vehicles depart from and arrive at a single depot. This well-know problem can be typically be solved in two possible ways, periodic timetabling represented as $trips/hour$ and non-periodic timetabling where each trip has a fixed departure time (12).

The proposed solution should benefit the company by either reducing operational costs or increasing revenue through an efficient and replicable implementation across different routes. However, because the company provides a public service, it is equally important that the resulting solution guarantees temporal accessibility for users. Since users always have a preferred departure time, aligning the service with this preference in demand behavior will increase user satisfaction and should also be considered in the problem solution (5). The common strategies to achieve a solution for this kind of problems involve exact and/or approximate methods.

Exact methods often formulate timetabling as an optimization problem. For complex combinatorial problems, classic optimization techniques such as column generation are commonly used. These methods enable the creation of variables without the need to explore the entire solution space directly, facilitating the generation of efficient schedules or shifts (6). Additionally, given the mixed-integer nature of these problems, computational acceleration techniques such as branch-and-cut can be applied, utilizing both branching and cutting to enhance solution efficiency (16). It has also been shown that these techniques are beneficial when applied to scheduling problems formulated in networks with resource constraints (14).

Approximation methods based on heuristics offer an alternative for reducing the complexity of solving transit network timetable problems (TNTP). Heuristics often frame TNTP as a graph problem, where nodes represent feasible departure times and arcs indicate possible sequences of these times, transforming the solution into a shortest path problem. In

Email addresses: a.ruedal@uniandes.edu.co (Andrés Rueda López), js.agudelo10@uniandes.edu.co (Juan Sebastian Agudelo Gomez)

[1]

(9), a multi-objective label-correcting algorithm is proposed to minimize both user waiting time (service metric) and the number of empty seats (operations metric) by comparing near-optimal solutions using a Pareto strategy.

In recent years, *Reinforcement Learning* (RL) has emerged as a method to solve timetabling problems in real-time. RL treats timetabling as a Markov Decision Process (MDP), where future decisions depend only on the current state (3). RL agents interact with simulated environments, receiving rewards for actions like departure timing. In (2) is proposed a reward function that aims to optimize bus utilization while minimizing waiting times, using a Deep Q-Network (DQN). However, RL agents may select infeasible actions, requiring a masking step to ensure that only valid actions are considered (10).

When dealing with large-scale scenarios, formulating and solving optimization problems can become challenging. In these cases, it is common to adopt a standard formulation. Most bus scheduling formulations aim to minimize user waiting times while adhering to resource constraints (8). One formulation that follows this for a a single-direction bus line is presented in (15) as follows:

$$(Q) \quad \min \sum_{p \in \mathcal{P}} w_p \tag{1}$$

$$x_1 = D_f \tag{2}$$

$$x_k = D_l \tag{3}$$

$$x_k \leq x_{k+1} \tag{4}$$

$$V_{1k} = x_k \tag{5}$$

$$V_{mk} = V_{m-1,k} + \lambda_{m-1,j} \tag{6}$$

$$\tau_p = \min_k (V_{Bpk} > A_p) \tag{7}$$

$$w_p = V_{B_p} \tau_p - A_p \tag{8}$$

Equation (1) minimizes passenger waiting times throughout the day. Equations (2) and (3) ensure the first and last trips are dispatched according to the schedule. Equation (4) maintains the order of trips. Equation (5) defines the dispatch time of the first trip. Equation (6) guarantees accurate arrival times at each stop. Equation (7) assigns passengers to buses, and Equation (8) calculates the waiting time for each passenger.

There are several extensions to the standard formulation discussed earlier, such as the one presented by (8), where the author introduces additional constraints, including vehicle availability for successive trips, vehicle capacities, and permitted dispatching headway variations.

This paper explores the application of the standard formulation proposed by (15) to optimize one bus line of the Transport Operator of Cinu (Colombia), using real-world data provided by the company. Beyond adapting the model to this specific case constrains, the paper introduces a novel reformulation, replacing the MIP approach from (15) with a network-based model. This reformulation leverages the computational efficiency of network optimization, particularly for integer problems, aiming to deliver scalable solutions that can be easily extended to larger transport networks, thus avoiding computational limitations (1).

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