

Minimum Wages and Firms' Labor Market Power*

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Abstract

This paper examines the sectoral and aggregate implications of the minimum wage where firms compete for workers in an oligopsonistic setting. Using administrative matched employer-employee data, I first study Uruguay's experience with the minimum wage, which increased its statutory value by 86% in real terms in 2005, raising the share of binding workers from 3% to 15% of the total workforce. At the firm level, I show that directly affected firms experienced employment declines, although these were significantly attenuated for firms with initially higher market shares. To explain these responses as well as evaluate the welfare effects of the minimum wage, I then calibrate a model with heterogeneous firms, where firms engage in oligopsonistic competition featuring strategic interactions in local labor markets. The calibrated model reveals substantial pre-policy labor market distortions: an aggregate wage markdown of 67% and a corresponding 3% reduction in total factor productivity. I find moreover that once the minimum wage is implemented, it increases the aggregate wage markdown while worsening aggregate TFP, though the magnitudes are small. Overall, labor market power generates welfare losses of 6.6%, driven mainly by wider markdowns and, to a lesser extent, by misallocation.

Key words: Minimum wages, labor reallocation, efficiency.

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1 Introduction

Minimum wage policies represent one of the most prevalent forms of labor market regulation globally. The International Labor Organization reports that 90% of countries maintain minimum wage laws, directly affecting 19% of wage earners worldwide (ILO, 2020). Despite this widespread adoption, economists continue to debate the policy's actual impact on employment outcomes (Card and Krueger, 2000; Neumark and Wascher, 2000; Dube *et al.*, 2010; Jha *et al.*, 2024). Traditional competitive economic theory predicts that binding minimum wages should reduce employment levels, yet decades of empirical research have consistently found minimal employment effects, with employment elasticities approaching zero in many studies (Dube and Zipperer, 2024).

One leading explanation for these limited employment effects is that labor markets operate under monopsonistic or oligopsonistic competition, where firms with market power suppress both wages (markdowns) and employment below competitive levels.¹ Under oligopsonistic competition with heterogeneous firm productivities, markdowns vary across firms because each faces a different elasticity of labor supply. Specifically, firms with higher wage-bill shares encounter lower labor supply elasticities, granting them greater wage-setting power. When productivity differs across firms, a binding minimum wage may alleviate market power distortions for some firms while creating employment rationing among others. Consequently, the joint distribution of markdowns and productivities across firms is needed to understand the aggregate effects of minimum wage policies.

The goal of this paper is to study the effects of an actual episode of a minimum wage increase from the lens of a framework with oligopsonistic competition with heterogeneous firms. I first use matched employer-employee data from Uruguay covering 1997–2009—a period when the country reintroduced its national minimum wage in 2005 and raised it by 144 percent in real terms (86 percent in the first year)—to estimate the policy's effect on firm-level employment. My estimation results are consistent with an oligopsonistic labor market in which larger firms face lower labor supply elasticities. Then, I develop a model of oligopsonistic competition following Berger *et al.* (2025), and I calibrate it using Uruguayan microdata. My calibration strategy adapts to the fact that many of the Uruguayan local labor markets have a small number of firms. The estimated elasticities of labor supply across firms and between sectors imply that firms possess significant wage-setting power, leading to non-negligible wage markdowns. According to the calibrated

¹In a monopsony—a situation with only one employer—it has been well established since Robinson (1969) that a minimum wage can theoretically achieve first-best outcomes by offsetting welfare losses from labor market power.

model, prior to the minimum wage policy, firms exhibited an average wage markdown of 67 percent, which depressed aggregate productivity by 3 percent and welfare by 6.6 percent, relative to perfect competition. Following the implementation of the minimum wage, the aggregate markdowns and inefficiency rose slightly. This is because despite the large increase in the minimum wage it was binding on only the low-wage firms. My work shows that under imperfect competition, wage and employment responses to minimum wage changes need not move in opposite directions. Yet, despite the rich heterogeneity in firm-level adjustments, the aggregate impact remains modest because the firms directly affected by the policy account for a relatively small fraction of total productivity.

Using matched employer-employee data, I study how Uruguayan firms responded to the reintroduction of the minimum wage. I assess the effects of the change in minimum wage policy on employment by using a difference-in-differences specification. I find that firms that were initially paying below the new statutory minimum wage reduced employment by about 16% and increased average wages by 150% within four years of the policy. However, this adjustment was not uniform. Firms initially paying below the minimum wage reduced employment by roughly 17%, but those also directly affected and with higher wage-bill shares in their local labor markets experienced a 7% decline in employment levels. Furthermore, local labor markets in Uruguay are highly concentrated, with relatively few employers accounting for most employment. Taken together, main findings on heterogeneous responses to minimum wages and concentrated labor markets are consistent with oligopsonistic competition.

Motivated by the empirical facts, and to study the efficiency and employment effects of Uruguay's minimum wage, I then develop a model of heterogeneous firms with strategic interactions. The model draws from [Berger et al. \(2025\)](#) and operates through both direct wage effects and indirect spillover mechanisms that propagate through strategic firm interactions. At low levels, the minimum wage compresses markdowns among directly affected firms and generates positive spillovers to others, as higher wages and employment at these constrained firms increase competitive pressure in local labor markets. As the policy tightens further, however, it begins to impose rationing constraints on more firms, reversing the initial wage gains and leading to employment losses concentrated among low-productivity producers. This nonlinear adjustment captures both the efficiency gains and the distortions that emerge as the minimum wage rises.

I then calibrate the model to the Uruguayan economy before the implementation of the minimum wage reform. I target three key moments: the share of workers earning the minimum wage (15%), the average firm size (21.2 employees), and the labor share (62%) to inform the productivity shifter, the labor disutility shifter, and the decreasing return

to scale parameter, respectively. The model closely matches the targeted moments.

The model broadly captures the distribution of employment across firm sizes, although it overpredicts the employment share of medium-size firms and slightly underpredicts that of large firms. Regarding wages, it tends to overstate the upward slope of the wage distribution across employment deciles, implying average log wages approximately 26% higher than those observed in the data.

To validate the model, I compare the standard deviation of log productivity with empirical estimates. The model generates a standard deviation equal to 1.09, which falls within the empirically observed range of 1.0–1.2 for Uruguayan firms, and reproduces key features of the interquartile and decile-based dispersion of productivity before the minimum wage policy

Prior to the implementation of the minimum wage policy, the calibrated model displays heterogeneity in wage markdowns across firms. More productive firms tend to exert greater oligopsony power, thus facing lower labor supply elasticities, and therefore setting lower wages relative to marginal products. For instance, workers in large employers take home 44% of their marginal revenue product of labor, while workers in small firms take home 88%. Taking together, the aggregate wage markdown in the Uruguayan economy implies that a representative household takes home 67% of their marginal revenue product of labor, depressing aggregate productivity by 3%.

After the introduction of the minimum wage, the model shows that binding wage floors reduce the degree of wage markdowns, particularly among low-wage firms. This compression slightly improves allocative efficiency for a small subset of firms that were previously paying below their marginal products. However, the policy primarily binds among low-wage, small firms, restricting their ability to adjust employment and production. As the minimum wage rises beyond the productivity of some of these firms, rationing constraints emerge, forcing employment reductions and generating new distortions captured by shadow markdowns. In contrast, large employers—who account for most employment, market power, and efficiency losses—are largely unaffected, as they already paid wages above the new floor. Quantitatively, the positive spillover effects on higher-wage firms are not large enough to offset the direct negative effects on constrained low-wage firms. Consequently, aggregate productivity and average markdowns remain roughly constant in the model. The model also reproduces the empirical finding that directly affected firms reduced employment by about 21% four years after the policy was implemented.

Related literature. This paper contributes to several strands of literature. First, it

advances our understanding of macroeconomic and development issues in settings with oligopsonistic competition. Second, it adds to the growing literature examining firm-level responses to minimum wage policies where firms exert market power. Finally, it contributes to the study of labor market policies in Uruguay, providing the first macroeconomic general equilibrium analysis of the country's minimum wage reform.

Oligopsonistic competition has emerged as an important framework for studying macroeconomic and development issues. Berger *et al.* (2022) develop a general equilibrium model estimating that labor market power reduced U.S. welfare by 7.6%, while Ar-mangué-Jubert *et al.* (2025) find that suppressing market power in less developed economies would increase output per capita by 44%. The framework has been used to explain issues ranging from wage dispersion and minimum wage effects (Bhaskar *et al.*, 2002), trade-related welfare losses (Gutiérrez, 2022), labor market concentration in trade liberalization (Felix, 2021), firm-level wage bargaining and its potential to mitigate distortions (Azkarate-Ascasua and Zerecero, 2024), and gender wage gaps (Sharma, 2023). My contribution is to calibrate and measure the level of market power and misallocation *before* the implementation of a binding minimum wage. This is important, because in the presence of price controls, labor demand may not be optimal and observed market share are not informative of the level of labor market power in binding firms.

Firm-level responses to minimum wages have been studied in Lamadon *et al.* (2022), Hurst *et al.* (2022), Harasztsosi and Lindner (2019), and Berger *et al.* (2025). Lamadon *et al.* (2022) provide comprehensive evidence on imperfect competition in the U.S. labor market using matched employer-employee data, documenting substantial variation in firm wage premia and rent-sharing practices that complement the monopsony framework for understanding wage determination in concentrated labor markets. Hurst *et al.* (2022) examine the distributional consequences of minimum wage policies using a heterogeneous agent model, finding that while minimum wages reduce inequality in the short run, their long-term effects depend critically on firm adjustment through capital investment and technology adoption. Harasztsosi and Lindner (2019) use administrative data from Hungary to examine firm-level responses to a large minimum wage increase, finding that the policy led to employment reductions primarily among small firms while larger firms adjusted through price increases and productivity improvements, providing a methodological foundation for firm-level analysis.

Closely related to my paper, Berger *et al.* (2025) constructs an oligopsonistic competition model with labor rationing constraints for the U.S. labor market, finding that minimum wage efficiency gains remain modest because small firms face the greatest constraints while larger competitors retain substantial market power. My model draws from

Berger *et al.* (2025), but I calibrate it to an actual change in minimum wages, and, in addition, I study empirically the effects of minimum wage on employment comparing reduced-form estimates with model-based implied responses. Delgado-Prieto (2024) further extend the oligopsonistic competition framework to incorporate firm organizational structures, showing how internal firm characteristics can moderate the employment effects of minimum wage policies. While these studies focus primarily on welfare decompositions and theoretical mechanisms, my analysis emphasizes the empirical validation of oligopsonistic models against granular administrative data, revealing that the model accurately match employment responses of firms directly affected by the policy.

Finally, this paper relates to a growing literature on wage-setting institutions in Uruguay. Casacuberta and Gandelman (2023) use production function estimation to recover firm-level markups and markdowns, showing that collective bargaining and minimum wage policies reduce markdowns but partially offset these gains through higher markups. Blanchard *et al.* (2023) document that Uruguay's wage policies compressed income inequality without significant disemployment effects. To the best of my knowledge, my paper is the first to analyze these policies from a macroeconomic perspective, by embedding Uruguay's minimum wage reform in a general equilibrium model of oligopsonistic competition and empirically validating its predictions using administrative data.

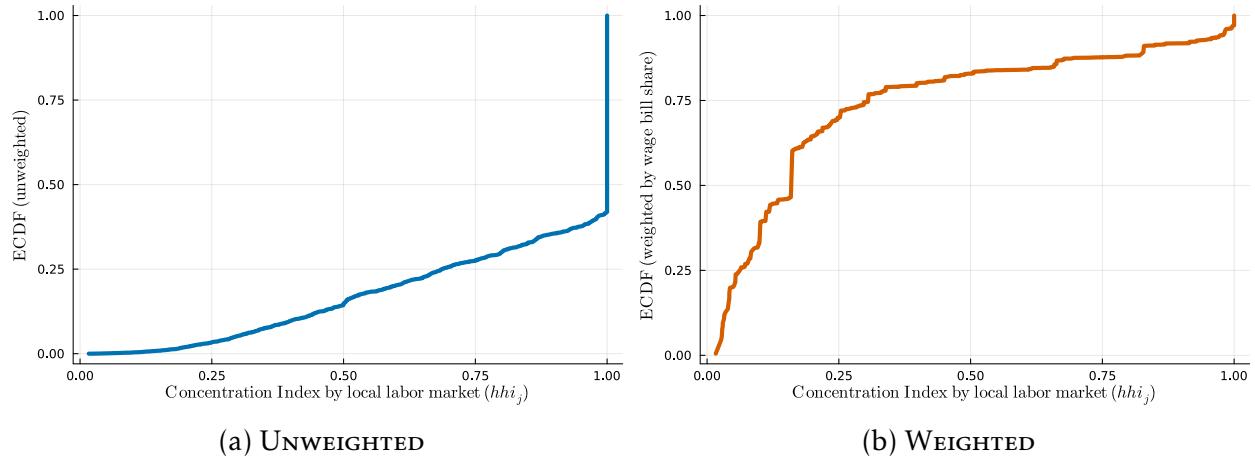
2 Data

The main data source for this paper is an administrative matched employer-employee dataset covering the universe of formal workers in Uruguay from 1997 to 2013, provided by the Uruguayan Social Security Administration (BPS, for its acronym in Spanish). The dataset includes detailed monthly information on employment, hours worked, wages, and industry for all private and public workers. I apply several restrictions to focus the analysis: public workers are excluded because the paper studies minimum wages under profit-maximizing firms, and attention is limited to the non-agricultural sector due to its relatively high level of informality. The data are then collapsed at the *firmid* level of the worker's payroll. In addition, I incorporate a secondary dataset containing information on firm locations; however, the location is not observed for all firms individually. From an initial sample of 14,880 firms, location information can be merged for 11,371 firms.

Market concentration. The Uruguayan labor market exhibits a relatively high degree of employment concentration. I define a local labor market as the intersection of ISCO rev-3 occupations and municipalities (departamentos). A typical local labor market has

an average of 9.8 competitors and a median of 2 employers, with a wage-bill-weighted Herfindahl-Hirschman Index (HHI) of 0.26 and an arithmetic mean of 0.66.² Concentration varies geographically: by 2004, Montevideo, the capital, accounts for 75% of all formal workers and exhibits substantially lower levels of concentration (wage-bill-weighted HHI of 0.19) compared to other locations, where the HHI reaches 0.9. High concentration may generate strategic interactions among firms, such that a minimum wage policy affects not only the firm's marginal cost directly but also indirectly via changes in competitors' market shares under Cournot competition. Figure 1 presents the empirical cumulative distribution of HHI across local labor markets, both unweighted and weighted by each market's share of the total wage bill. Most labor markets are highly concentrated, with over 50% effectively behaving as full monopsonies; however, the wage-bill share of these extremely concentrated markets is limited. In contrast, labor markets with HHI below 0.25, though less numerous, account for roughly three-quarters of total wages paid.

Figure 1: EMPIRICAL DISTRIBUTION OF CONCENTRATION



Notes: These figures show the empirical cumulative distribution of concentration indexes across local labor markets. The panel a) displays the unweighted distribution while panel b) displays weighted by wage bill share. Concentration at each local labor market is measured using the herfindahl-hirschman index (hhii). The concentration index is computed as $hhii_j = \sum_{i=1}^{M_j} s_{ij}^2$ and wage-bill share of local labor market is $s_j = (w_j n_j) (\int_0^1 w_j n_j dj)^{-1}$.

Labor market regulations. Established by law in 1969, the National Minimum Wage (NMW) was designed to serve as a wage floor for all workers, regardless of sector³. However, be-

²For comparison, Berger *et al.* (2025) reports that a typical U.S. local labor market has an average of 113 firms and an HHI of 0.11.

³The second pillar of wage regulation in Uruguay consists of Collective Bargaining Agreements (CBAs), a system of sector-level wage negotiations introduced in 1948. While CBAs were reinstated in July 2005 following the election of Uruguay's first left-wing government, they were not fully implemented across all sectors until 2009 (Bergolo *et al.*, 2025).

fore 2005, the NMW also functioned as a unit of account for calculating taxes and social transfers, which limited its effectiveness as a labor market instrument. Since any increase in the NMW entailed higher public expenditures, its role as a wage floor was constrained. To address this, in January 2005 the government introduced a new accounting unit (the BPC, for its acronym in Spanish) for taxes and subsidies. This reform allowed for a significant nominal increase in the NMW—58% that year. Additionally, the newly elected government raised the NMW again by 22% in July 2005, resulting in an 86% real increase in its statutory value between July 2004 and July 2005. As a consequence, the share of binding workers rose from just 3% of the workforce in 2004 to 15% in 2005.

Heterogeneous employment response to the minimum wage. When firms are directly exposed to the minimum wage, they tend to reduce employment. However, for firms with significant labor market power, this effect may be mitigated or even reversed. To test this hypothesis, I estimate regression (1), which includes an interaction between treatment status and a dummy for “large” firms, defined as those with a local market share above 15%, and controls for local labor market times year fixed effects. The coefficient of interest, ξ_t , captures whether larger treated firms respond differently in terms of employment. The regression is then:

$$\frac{y_{it} - y_{i2004}}{y_{i2004}} = \alpha_{jt} + \beta_t T_i + \xi_t (T_i \times big_i) + \gamma_t X_{it} + \varepsilon_{it} \quad (1)$$

Table (1) presents results from the pooled specification. The coefficient on $After \times Treat$ indicates that treated firms reduced employment by about 16% following the reform. However, the triple interaction term $After \times Treat \times s_{ij} > 0.15$ is positive (0.107) and statistically significant at the 10% level, suggesting that among treated firms, those with larger initial market shares experienced a substantially smaller contraction in employment. In fact, their decline is roughly offset, implying that while small firms shrank, larger treated firms either maintained or modestly expanded employment.

Table 1: EMPLOYMENT AND THE MINIMUM WAGE

	Change in employment	Change in employment
After	0.127*** (0.008)	0.111*** (0.009)
Treat	0.011 (0.008)	0.008 (0.009)
After x Treat	-0.162*** (0.022)	-0.172*** (0.024)
$s_{ij} > 0.15$		-0.027*** (0.009)
After x $s_{ij} > 0.15$		0.035** (0.016)
Treat x $s_{ij} > 0.15$		-0.019 (0.019)
After x Treat x $s_{ij} > 0.15$		0.107* (0.061)
Constant	-0.049*** (0.004)	-0.037*** (0.005)
R^2	0.03	0.04
Observations	70,533	70,533

Notes: This table reports the estimated coefficients from equation (1), where the dependent variable is the change in employment relative to its 2004 level. $Treat_i$ denotes treated firm, and big_i identifies firms with an initial wage-bill share larger than 0.15. Results are pooled across years. Robust standard errors clustered at the local labor market level are reported in parentheses.

3 Model

The model follows Berger *et al.* (2025) and considers an economy populated by a representative household that allocates workers across firms. On the production side, all firms produce a homogeneous good. The economy consists of a unit measure of local labor markets indexed by j , where each local labor market contains a fixed number of firms M_j that choose their labor demand. The finite number of firms generates strategic interactions, as each firm takes into account its effect on competitors' decisions. Specifically, I assume that firms within a sector engage in Cournot competition for labor.

3.1 Household

The household derives utility from the consumption of a final good (numeraire) and leisure. Her utility reads:

$$\mathcal{U}(C, N) = C - \frac{1}{\bar{\varphi}^{1/\varphi}} \frac{N^{1+\frac{1}{\varphi}}}{1 + \frac{1}{\varphi}} \quad (2)$$

where φ denotes the Frisch elasticity of labor supply. The labor disutility index N is defined as a nested CES aggregator. In the upper tier, local labor markets employment indexes n_j are aggregated with an elasticity of substitution θ , while within each local labor market, employment level of individual firms n_{ij} are aggregated with an elasticity of substitution η . I assume that household perceive jobs within a local labor market to be closer substitutes than across local labor markets, that is, $\eta > \theta$. Finally, $\bar{\varphi}$ is a labor disutility shifter calibrated to match the average firm size.

$$N = \left(\int_0^1 n_j^{\frac{\theta+1}{\theta}} dj \right)^{\frac{\theta}{\theta+1}}, n_j = \left(\sum_{i=1}^{M_j} n_{ij}^{\frac{\eta+1}{\eta}} \right)^{\frac{\eta}{\eta+1}}, C = \int_0^1 \sum_{i=1}^{M_j} c_{ij} dj \quad (3)$$

Because there is no capital, labor income and total profits Π is used for consumption. The budget constraint of the household can be written as:

$$C = \int_0^1 \sum_{i=1}^{M_j} w_{ij} n_{ij} + \Pi \quad (4)$$

Also, the household takes into account that a higher minimum wage shrink employment. This is the rationing constraint:

$$n_{ij} - \bar{n}_{ij} \leq 0 \quad (5)$$

where \bar{n}_{ij} will be determined as the intersection of the marginal revenue product of labor and the minimum wage \underline{w} .

Household labor supply. The Lagrangian of the household's utility maximization problem can be written as:

$$\mathcal{L} = \mathcal{U}(C, N) = C - \frac{1}{\bar{\varphi}^{1/\varphi}} \frac{N^{1+\frac{1}{\varphi}}}{1 + \frac{1}{\varphi}} + \lambda_{ij} \left(\int_0^1 \sum_{i=1}^{M_j} w_{ij} n_{ij} + \Pi - C \right) + \zeta_{ij} (\bar{n}_{ij} - n_{ij}) \quad (6)$$

Solving for the FOC's with respect to $\{C, n_{ij}\}$ delivers the usual labor-consumption trade-off with a complementary slackness condition:

$$w_{ij}\chi_{ij} = \left(\frac{n_{ij}}{n_j}\right)^{\frac{1}{\eta}} \left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} \left(\bar{\phi}N\right)^{\frac{1}{\varphi}}, \quad \zeta_{ij}(\bar{n}_{ij} - n_{ij}) = 0 \quad (7)$$

where $\zeta_{ij} = \lambda w_{ij}(1 - \chi_{ij})$ is the Lagrange multiplier of the rationing constraint $n_{ij} \leq \bar{n}_{ij}$

Therefore, the inverse labor supply schedule can be written as:

$$w(n_{ij}, \bar{n}_{ij}, \mathbf{S}) = \begin{cases} \left(\frac{n_{ij}}{n_j}\right)^{\frac{1}{\eta}} \left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} \left(\frac{N}{\bar{\phi}}\right)^{\frac{1}{\varphi}} & \text{if } n_{ij} \in [0, \bar{n}_{ij}] \\ \in \left[\left(\frac{n_{ij}}{n_j}\right)^{\frac{1}{\eta}} \left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} \left(\frac{N}{\bar{\phi}}\right)^{\frac{1}{\varphi}}, \infty\right) & \text{if } n_{ij} = \bar{n}_{ij} \end{cases} \quad (8)$$

where \mathbf{S} denotes aggregate consumption and aggregate labor disutility, which are taken as given by the firm.

Firms. Within each local labor market, firms seek to maximize profit taking into account the former labor supply schedule and competitor's best responses. In particular, they choose the amount of labor to hire n_{ij} :

$$\max \pi_{ij} = \bar{Z} z_{ij} n_{ij}^\alpha - w_{ij} n_{ij}, \quad 0 < \alpha < 1 \quad (9)$$

subject to:

$$w_{ij} \geq \underline{w}, n_{ij} \leq \bar{n}_{ij}, w_{ij} = w(n_{ij}, \bar{n}_{ij}, n_j(n_{ij}, n_{-ij}), \mathbf{S}) \quad (10)$$

where z_{ij} is the productivity of firm i in market j and \bar{Z} is an aggregate productivity shifter chosen to match properties of wage levels in the economy.

3.2 Minimum wages with heterogeneous firms

Minimum wages affect firms in different ways, depending on the wage level they were paying before the introduction of the minimum wage and their marginal cost curve.

1. Unconstrained firms. Firms where the minimum wage is not binding and labor is on its labor supply curve.
2. Constrained firms, where minimum wage is binding. Here there are two sub-cases.
 - Firms where the minimum wage reduces marginal cost and, hence, expands employment.
 - Firms where minimum wage increases marginal cost and shrink employment.

Unconstrained firms. When the minimum wage is not binding from the point of view of the firm, wages and employment remain identical to the situation with no minimum wage. In this case, wages, markdowns and market shares can be written as:

$$w_{ij} = \mu_{ij} mrpl_{ij}, \mu_{ij} = \frac{\varepsilon_{ij}}{\varepsilon_{ij} + 1}, \varepsilon_{ij} = \left[(1 - s_{ij}) \frac{1}{\eta} + s_{ij} \frac{1}{\theta} \right]^{-1}, s_{ij} = \frac{w_{ij} n_{ij}}{\sum_i w_{ij} n_{ij}} \quad (11)$$

Constrained firms. If however, the minimum wage is binding but decreases the marginal cost of the firm, employment is expanded. But if the minimum wage is too high, marginal cost increase and employment is reduced. In both cases, equations in (11) do not hold and make aggregation intractable. The reason is employment is not uniquely determined by wages and labor shares do not summarize all the information needed to optimize employment subject to strategic interactions.

3.3 Shadow wages economy

A solution of the previous aggregation problem is presented in [Berger et al. \(2025\)](#). The advantage of their framework is that the shadow wages summarize the constraint that the firm and their competitors are facing. Hence, it allows to write down the market equilibrium in terms of allocative prices, that is, the shadow wage. The wage, markdown and wage-bill share can be expressed as:

$$\tilde{w}_{ij} := w_{ij} \chi_{ij} = \left(\frac{n_{ij}}{n_j} \right)^{\frac{1}{\eta}} \left(\frac{n_j}{N} \right)^{\frac{1}{\theta}} \frac{(\bar{\phi} N)^{\frac{1}{\varphi}}}{C}, \quad \tilde{\mu}_{ij} := \frac{\tilde{w}_{ij}}{\alpha z_{ij} n_{ij}^{\alpha-1}}, \quad \tilde{s}_{ij} := \frac{\tilde{w}_{ij} n_{ij}}{\sum_{i=1}^{M_j} \tilde{w}_{ij} n_{ij}} \quad (12)$$

3.4 Market and economy-wide aggregation

As pointed out before, the advantage of defining firms' outcomes regarding shadow wages is that the economy admits aggregation. The sectoral output, shadow wage, and aggregate labor supply can be written as:

$$y_j = \omega_j \tilde{z}_j n_j^\alpha, \quad \tilde{w}_j = \mu_j \alpha \tilde{z}_j n_j^{\alpha-1}, \quad n_j = \left(\frac{\tilde{w}_j}{\tilde{W}} \right)^\theta N \quad (13)$$

Where \tilde{z}_j denotes the (undistorted) sectoral productivity:

$$\tilde{z}_j := \left[\sum_{i \in j} \tilde{z}_{ij}^{\frac{1+\eta}{1+\eta(1-\alpha)}} \right]^{\frac{1+\eta(1-\alpha)}{1+\eta}} \quad (14)$$

μ_j the market- j aggregate markdown:

$$\mu_j := \left[\sum_{i \in j} \left(\frac{\tilde{z}_{ij}}{\tilde{z}_j} \right)^{\frac{1+\eta}{1+\eta(1-\alpha)}} \mu_{ij}^{\frac{1+\eta}{1+\eta(1-\alpha)}} \right]^{\frac{1+\eta(1-\alpha)}{1+\eta}} \quad (15)$$

Moreover, it is possible to express the distortions that variable markdowns generate at the local labor market level. In the spirit of [Hsieh and Klenow \(2009\)](#), I define misallocation as the inefficient allocation of resources across firms that arise when the marginal revenue product of labor is not equated across firms. Through the lens of this model, larger and more productive firms could attract more resources, but because they charge markdowns—paying wages below the marginal product of labor—fewer resources are allocated to them. Firms exerting labor market power depress sectoral productivity, creating what I define as the market- j level of misallocation, ω_j . Formally, ω_j can be interpreted as a weighted geometric dispersion of firm markdowns, where the weights are given by each firm's share of total productivity in the local labor market. That is, larger firms contribute more to misallocation. Hence, labor market power constitutes a source of local misallocation and productivity loss. The market- j misallocation is:

$$\omega_j := \sum_{i \in j} \left(\frac{\tilde{z}_{ij}}{\tilde{z}_j} \right)^{\frac{1+\eta}{1+\eta(1-\alpha)}} \left(\frac{\mu_{ij}}{\mu_j} \right)^{\frac{\eta\alpha}{1+\eta(1-\alpha)}} \quad (16)$$

Analogously, the economy-wide aggregates can be written as:

$$Y = \Omega \tilde{Z} N^\alpha, \quad \tilde{W} = \mathcal{M} \alpha \tilde{Z} N^{\alpha-1}, \quad N = \bar{\varphi} \tilde{W}^\varphi \quad (17)$$

Where \tilde{Z} denotes the (undistorted) aggregate productivity:

$$\tilde{Z} := \left[\int \tilde{z}_j^{\frac{1+\theta}{1+\theta(1-\alpha)}} dj \right]^{\frac{1+\theta(1-\alpha)}{1+\theta}} \quad (18)$$

\mathcal{M} the aggregate markdown:

$$\mathcal{M} := \left[\int \left(\frac{\tilde{z}_j}{\tilde{Z}} \right)^{\frac{1+\theta}{1+\theta(1-\alpha)}} \mu_j^{\frac{1+\theta}{1+\theta(1-\alpha)}} dj \right]^{\frac{1+\theta(1-\alpha)}{1+\theta}} \quad (19)$$

At the aggregate level, misallocation captures the extent to which resources are not efficiently allocated across all firms in the economy. Aggregate misallocation arises when the marginal revenue product of labor is not equated across firms, leading to losses in aggregate productivity. In the context of this model, variable markdowns distort the allocation of labor by preventing more productive firms from expanding and absorbing more resources. When labor market power varies across firms and sectors, these micro-level distortions aggregate into an economy-wide loss of efficiency. Importantly, the contribution of a sector to aggregate misallocation depends not only on the degree of misallocation within the sector but also on the sector's relative size in the economy. Hence, aggregate misallocation reflects the combined effect of market-level misallocation weighted by each sector's share of total productivity. The aggregate level of misallocation, Ω , is defined as:

$$\Omega := \int \left(\frac{\tilde{z}_j}{\tilde{z}} \right)^{\frac{1+\theta}{1+\theta(1-\alpha)}} \left(\frac{\mu_j}{M} \right)^{\frac{\theta\alpha}{1+\theta(1-\alpha)}} \omega_j \quad (20)$$

3.5 Model mechanisms

A description of the model mechanisms can be found in [Berger et al. \(2025\)](#). Here I will present a short summary.

When a minimum wage is implemented, it affects a firm through two different ways, *i*) direct effect and *ii*) spillover. The direct effect corresponds to a situation where the minimum wage becomes binding from the point of view of the firm. These firms can be further classified into those for which the minimum wage is binding and the marginal cost of the firm decreases, and those firms for which the minimum wage is binding and the marginal cost of the firm increases.⁴ Besides direct effects, the model also exhibits spillover effects. Spillover effects are present because of strategic interactions. Firms that are not directly affected by the minimum wage can also increase wages because their smaller competitors are expanding employment and in consequence, increasing their market shares. Hence, non-affected firms may increase wages to prevent their market shares from dropping, expanding employment, and decreasing shadow markdowns as a consequence.

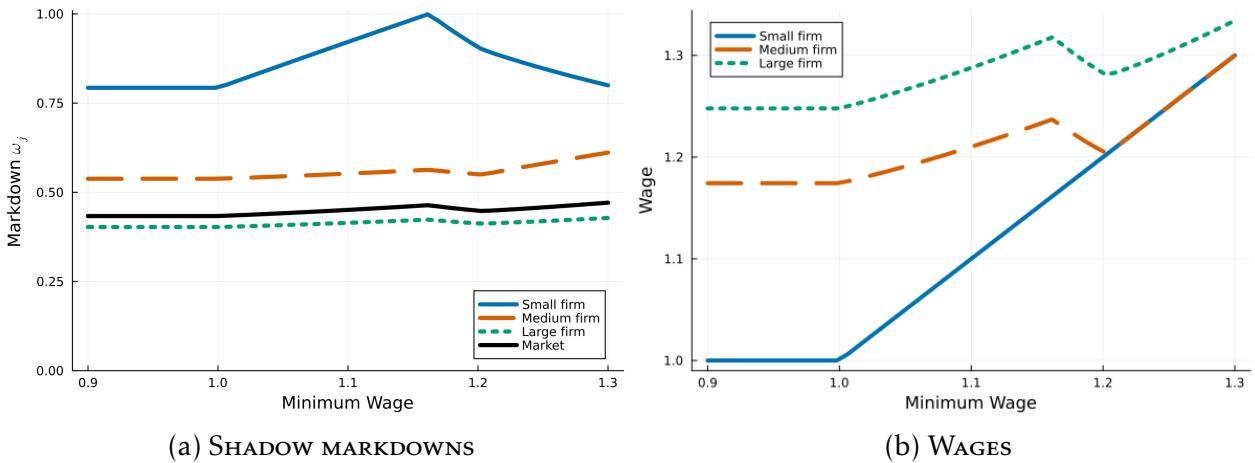
In figure (2) I show on panel *a*) the evolution of shadow markdowns for three different firms (small, medium, and large) when I continuously increase the minimum wage. The x-axis is the minimum wage. I normalize the initial minimum wage to 1 such that is equal to the wage of the smallest firm. When the minimum wage starts to increase, the smallest firm is the first to be affected and their shadow markdown starts to shrink. This

⁴The former case configures a situation where a binding minimum wage prevent efficiency losses from a firm exerting monopsony power [Robinson \(1969\)](#).

is the direct effect because the minimum wage is binding. This increase in the minimum wage can be also observed in panel *b*), where I plot the minimum wage against the wage of each firm. The spillover effect can be observed by noticing that the minimum wage also increases the wage of medium and big-size firms, even though the minimum wage is not binding from the point of view of these firms.

When the shadow markdown of the small firm reaches 1, the minimum wage becomes optimal from the point of view of this firm, causing the marginal cost to equal the marginal revenue product of labor. However, if the minimum wage is still increasing, the shadow markdown of the small firm goes below 1, encoding sequentially higher allocative distortions. At that point, the spillover pressure over the medium and large firms ceases, and wages for these firms drop. With an even higher minimum wage, the medium size firm starts to be directly affected by the minimum wage, creating spillovers to the large firm

Figure 2: MARKDOWNS AND SPILLOVERS

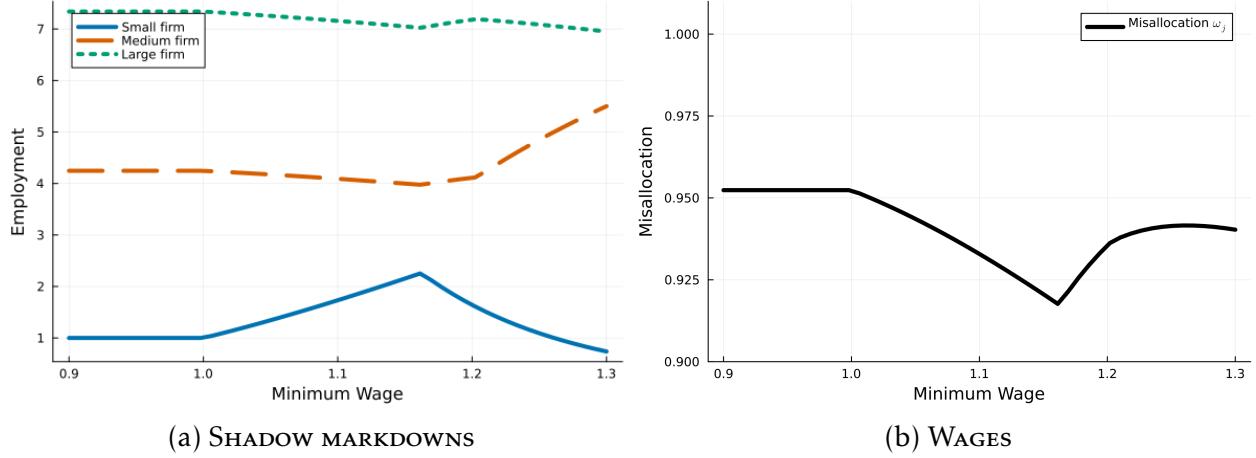


Notes: These figures show the market equilibrium effect of continuously increasing the minimum wage. The productivities of the firms are 6.0, 4.0, and 2.0 for big, medium, and small size firms. The minimum wage is expressed in terms of the small-size firm before the minimum wage becomes binding. The figure on the left shows the shadow markdown for each firm at different values of the minimum wage. The figure on the right shows the wage paid by each firm for different minimum wages.

In terms of employment, I show in figure (3) that once the minimum wage becomes binding from the point of view of the small firm, their employment starts to increase. This happens because the firm is in region II, where the minimum wage decreases its marginal costs and allows the firm to expand. This employment expansion creates a contraction in the employment level of medium and large firms because the small firms become relatively bigger. However, once the small firm starts to shrink, the employment level of the medium-sized firm expands. This is reinforced later, when the minimum wage becomes

binding for the medium size firm. In terms of misallocation, however, I show that even when there is employment expansion in the small firm, the minimum wage policy worsens misallocation. This happens because regardless of the improvement in the shadow markdown of the small firm, the *dispersion* in shadow markdown is increasing, lowering market level TFP.

Figure 3: EMPLOYMENT AND MISALLOCATION



Notes: These figures show the market equilibrium effect of continuously increasing the minimum wage. The productivities of the firms are 6.0, 4.0, and 2.0 for big, medium, and small size firms. The minimum wage is expressed in terms of the small-size firm before the minimum wage becomes binding. The figure on the left shows the employment level for each firm at different values of the minimum wage. The figure on the right shows the misallocation level for different minimum wages.

4 Calibration

4.1 Calibrated parameters

There are seven parameters to be calibrated plus the distribution of productivity: Two labor supply elasticities, the within firm and across sector (η, θ), the Frisch elasticity (φ), the decreasing return to scale parameter (α), the total number of markets (J), the labor disutility shifter ($\bar{\varphi}$) and the aggregate productivity shifter (\bar{Z})

Within labor supply elasticity. The across-firm elasticity of labor supply can be directly estimated from the firm-specific labor supply equation. Recall that the unconstrained wage equation presented in (8) is:

$$\log w_{ij} = \underbrace{\frac{1}{\eta} \log n_{ij} + \left(\frac{1}{\theta} - \frac{1}{\eta} \right) \log n_j + \frac{1}{\varphi} \log \frac{N}{\bar{\varphi}} - \frac{1}{\theta} \log N}_{\text{Common to all firms within a market } j} \quad (21)$$

The reduced-form expression for the previous equation can be written as:

$$\log w_{ijt} = \frac{1}{\eta} \log n_{ijt} + \delta_{jt} + e_{ijt} \quad (22)$$

I estimate the previous equation using the pre-2005 period, when no firm faced binding wage constraints; hence, wages were allocative for all firms. Table 2 reports the estimates for two different samples. The first two columns present the elasticity η considering all firms in the economy, with and without year fixed effects. An across-firm elasticity of 3.8 implies that workers at small firms take home approximately 79% of their marginal product of labor. However, given evidence of higher labor supply elasticities among workers more likely to be affected by the minimum wage (Delgado-Prieto, 2024), I re-estimate the regression restricting the sample to firms paying below the 2005 minimum wage (inflation-adjusted). The OLS estimation delivers a across firms labor supply elasticity of 7.9, which, through the lens of the model, implies that workers in atomistic firms receive about 90% of their marginal revenue product. This is the preferred estimate.

Table 2: WITHIN-FIRM LABOR SUPPLY ELASTICITY

	All firms		Low-wage firms	
	(1)	(2)	(3)	(4)
$\log n_{ij}$	0.264*** (0.003)	0.264*** (0.003)	0.127*** (0.003)	0.127*** (0.003)
Implied elasticity η	3.8	3.8	7.9	7.9
R^2	0.39	0.40	0.20	0.25
Observations	46,714	46,714	17,451	17,451
LLM FE	Yes	Yes	Yes	Yes
LLM x Year FE	No	Yes	No	Yes

Notes: This table reports estimates of the within-firm labor supply elasticity. Columns (1) and (2) present results for all firms, while columns (3) and (4) focus on low-wage firms. The dependent variable is the log of individual employment n_{ij} , and standard errors are clustered at the local labor market level (in parentheses). The implied elasticity η is computed from the estimated coefficient as described in the text. Columns (2) and (4) include local labor market (LLM) by year fixed effects, whereas columns (1) and (3) include only LLM fixed effects.

Decreasing return to scale. The decreasing return to scale parameter is pinned down by the aggregate labor share (0.62). Notice that through the lens of this model, the labor share is bounded above and equal to $\alpha\eta/(\eta+1)$, a case of monopsonistic competition where atomistic firms charge the same markdown. In the oligopsonistic economy, however, the labor share takes a more general form

$$LS = \frac{WN}{Y} = \alpha \left[(1 - HHI) \frac{\eta + 1}{\eta} + HHI \frac{\theta + 1}{\theta} \right]^{-1} \quad (23)$$

where $HHI = \int_0^1 s_j hhi_j dj$ and $hhi_j = \sum s_{ij}^2$, are the economy-wide and sector herfindahl-hirschman indexes, respectively. The labor share is taken from Penn World Tables, version 9.1 (PWT) and normalized after taking into account capital income share. Capital income is the product of the rental rate of capital times the stock of capital. I assume a risk-free interest rate of 4% and take the depreciation rate from PWT (3%).⁵

The total number of local labor markets is taken directly from the data and is defined as the intersection of municipalities (departamentos) and ISIC 3-digit industries, which configures 1,143 local labor markets. The Frisch elasticity is borrowed from Berger *et al.* (2025) and the across-market elasticity from Felix (2021).

Productivity. The Uruguayan labor market is populated by very few firms. The final sample is composed by 11,269 firms distributed across 1,143 local labor markets. The usual calibration of oligopsonistic or oligopolistic models requires a large number of total firms such that aggregates are stable and do not depend on the draw of productivity.⁶. However, small economies have necessarily less firms than bigger economies, which poses a challenge to estimate a model of imperfect competition in these economies. Besides, less developed economies have more markets with only one firm, which challenge further the calibration because more draws are needed to match the skewness of the distribution of firms across local labor markets⁷.

Instead of relying on a particular distribution, I invert the model for the pre-minimum wage period.⁸ For doing this, I proceed in two steps. First, I solve for the *relative produc-*

⁵Notice that the low depreciation rate is since the stock of capital in PWT is the summation of building, machines, and other types of capital, where structures having a lower depreciation rate.

⁶For instance, Berger *et al.* (2022) solve their model using 500,000 firms and Atkeson and Burstein (2008) with 400,000 firms

⁷In the US 15% of local labor markets have 1 firm (Berger *et al.*, 2025), in Portugal 29% (Delgado-Prieto, 2024), in Peru 39% (Amadio *et al.*) and Uruguay 49%.

⁸A similar approach to recover the productivity of Korean firms is employed in Choi *et al.* (2024).

tivities within a local labor market that rationalize the observed market shares. In a second step, I scale up the sectoral productivities to match the local labor market wage-bill share.

The relative productivities within a local labor market can be recovered as a system of non-linear equations. In particular, for each market j the market shares s_{ij} satisfy

$$s_{ij} = \frac{\left[\mu(s_{ij}) z_{ij} \right]^{\frac{\eta+1}{1+\eta(1-\alpha)}}}{\sum_{i \in j} \left[\mu(s_{ij}) z_{ij} \right]^{\frac{\eta+1}{1+\eta(1-\alpha)}}} \quad (24)$$

$$\varepsilon_{ij} = \left[(1 - s_{ij}) \frac{1}{\eta} + s_{ij} \frac{1}{\theta} \right]^{-1} \quad (25)$$

$$\mu(s_{ij}) = \frac{\varepsilon(s_{ij})}{\varepsilon(s_{ij}) + 1} \quad (26)$$

Hence, $\{z_{ij}, \mu_{ij}, \varepsilon_{ij}\}$ can be recovered by only observing equilibrium market shares and the solution display relative productivities.

In a second step, I match the wage bill share of each local labor market with its counterpart in the data. For doing this, I do not need to solve for the steady-state general equilibrium model, because sectoral aggregates are also independent of aggregates. In particular, the wage-bill share of local labor market j can be expressed as⁹

$$s_j := \frac{w_j n_j}{\int_0^1 w_j n_j dj} = \frac{\left[\mu_j z_j \right]^{\frac{\theta+1}{1+\theta(1-\alpha)}}}{\int_0^1 \left[\mu_j z_j \right]^{\frac{\theta+1}{1+\theta(1-\alpha)}} dj} \quad (27)$$

where z_j and μ_j are given by equations (14) and (15), respectively. Hence, with information on sector wage-bill data, the sectoral productivities can be solved for.¹⁰ In subsection (4.2) I show that the productivity inversion closely match empirical estimates of the (log) productivity moments.

Finally, the labor disutility shifter is chosen to match the average firm size (21.2 employees) while the aggregate productivity shifter is determined to match the share of workers binding at the 2005 minimum wage (15%).¹¹ Table (3) summarizes the set of parameters and their values.

⁹See appendix B.1.

¹⁰An overview of the algorithm can be find in appendix B.2.

¹¹Notice that the average firm size (e) can be expressed as $e = \frac{\int \sum n_{ij} dj}{\int M_j dj} = \frac{\bar{\varphi} \tilde{W}^\varphi}{\int M_j dj}$

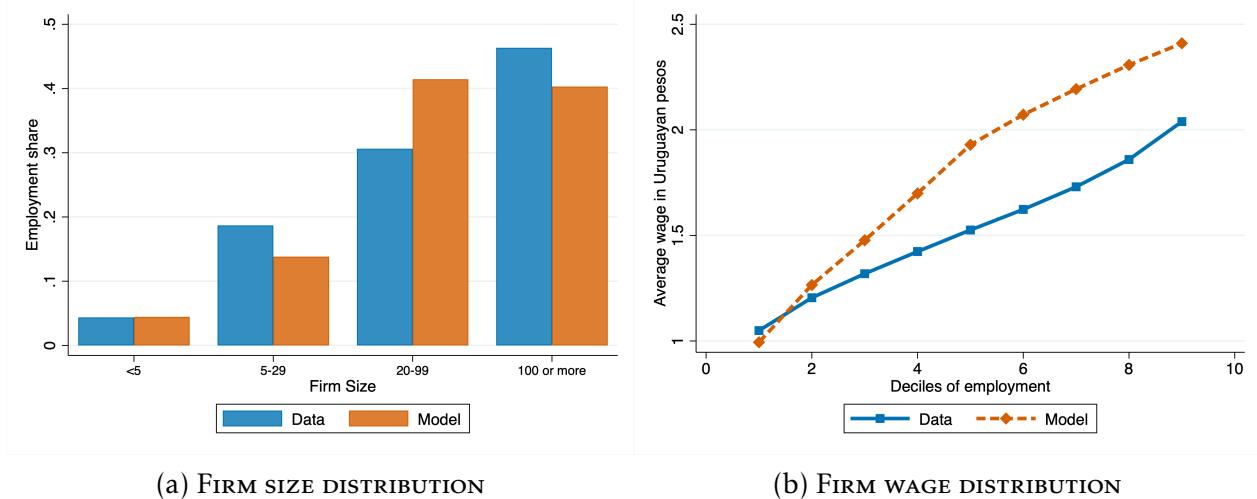
Table 3: CALIBRATED PARAMETERS

Concept	Parameters	Value	Source/Target	Data	Model
Externally calibrated					
Frisch elasticity	φ	0.62	Berger <i>et al.</i> (2025)		
Across-market elast	θ	0.80	Felix (2021)		
Within-market elast	η	7.9	Estimated		
Internally calibrated					
DRS parameter	α	0.88	Labor share	0.62	0.62
Relative productivities	z_{ij}		Weighted HHI	0.26	0.26
Labor disutility shifter	$\bar{\varphi}$	50.7	Average firm size	21.2	21.2
Productivity shifter	\bar{Z}	27.8	Binding at minimum wage	15%	15%

Untargeted moments. Even though the model calibration is not designed to target the firm size distribution or the average wages by employment deciles, the next figure examines how well the calibrated model aligns with these untargeted moments.

For the firm size distribution, the model reproduces the employment share for small firms, and it captures the increasing employment share with firm size. However, it under-predicts the data for the largest firm size (100+). For the wage distribution, the model aligns with the first two deciles of the log wage distribution but diverges from the data in higher deciles. These patterns indicate that refining the calibration of wage differentials across firm sizes may improve the fit.

Figure 4: UNTARGETED MOMENTS



Notes: These figures compare model predictions with the data. Panel (a) shows the firm size distribution, with data represented by blue bars and the model by red bars. Panel (b) shows the firm-level wage distribution by employment decile, with data as a solid blue line and the model as a dashed red line.

4.2 Model validation

I compare the model's predictions with empirical evidence on the dispersion of productivity and the existence of labor market power. The model generates a standard deviation of log productivity close to empirical estimates. However, it predicts an average wage markdown above empirical values, suggesting that the calibration of labor supply elasticities requires further scrutiny.

Standard deviation of log TFP. As described above, by inverting the model, I can calculate the entire distribution of productivity. The standard deviation of log productivity is 1.09. This is similar to estimates obtained by related papers. For example, [Busso et al. \(2013\)](#) report that the standard deviation of the log of quantity-based TFP during 1997–2005 was between 1.0 and 1.2. Also, [Casacuberta and Gandelman \(2009\)](#) estimate a value of 1.16. Since in the model all firms charge the same price, quantity- and revenue-based measures of productivity coincide. Furthermore, the model inversion yields an interquartile range of log TFPQ value of 1.49. [Casacuberta and Gandelman \(2009\)](#) reports an interquartile range of log TFPQ equals to 1.57

Table 4: PRODUCTIVITY. MODEL VS EMPIRICAL ESTIMATES

	Model inversion	CG09 estimates	Abs. log difference
Standard deviation	1.09	1.16	0.03
p50-p10	1.85	1.71	0.03
p75-p25	1.49	1.57	0.02
p90-p10	2.82	2.91	0.01

Notes: This table shows a comparison of the moments of the log of productivity between the model inversion and the empirical estimates from [Casacuberta and Gandelman \(2009\)](#) (CG09). The absolute log difference reports the absolute value of the difference between the model and the empirical moment.

Wage markdowns. I compare the model's predictions for wage markdowns with empirical estimates. The model predicts an average wage markdown of 0.74. For the Uruguayan context, [Casacuberta and Gandelman \(2023\)](#) estimate that the arithmetic average of the labor markdown prior to the minimum wage reform was close to 0.5.¹²

¹²They report markdowns as the ratio between the marginal revenue product of labor and wages; in my context, a value of 2 corresponds to a markdown of 0.5.

Table 5: AVERAGE WAGE MARKDOWN. MODEL VS EMPIRICAL ESTIMATES

	Model	CG23 estimates
Average markdown	0.74	0.50

Notes: This table shows a comparison between the average markdown implied by the model and the average markdown from the empirical estimates presented in [Casacuberta and Gandelman \(2023\)](#) (CG23).

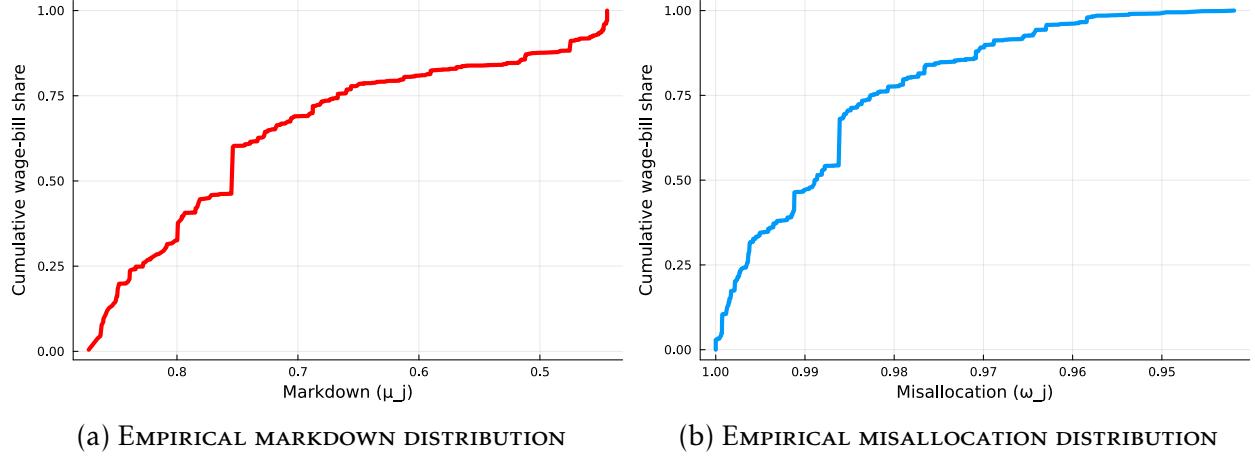
5 Results

The calibrated model yields the distribution of wage markdowns and misallocation across local labor markets. Figure (5) displays these distributions, weighting each local labor market by its share of the national wage bill. This weighting allows to assess how prevalent these distortions are at the economy-wide level.

Panel (a) shows the distribution of wage markdowns. The x-axis reports markdown levels, with higher values (toward the left) corresponding to weaker firm market power over workers, and lower values (toward the right) indicating stronger market power. For instance, workers in the median local labor market take home only 75% of their marginal product. However, there is substantial dispersion: while the most competitive quintile of local labor markets have markdowns of at most 85%, the least competitive quintile have markdowns of at least 67%..

Panel (b) presents the distribution of misallocation arising from variable markdowns within local labor markets. If all firms within a given market charged the same markdown, there would be no misallocation losses because there would be no dispersion in these wedges. Put differently, even if all firms charge markdowns that deviate from perfect competition, as long as small and large firms within a local labor market charge similar markdowns, labor market power would not distort resource allocation. However, this is not the case for Uruguay: while the most efficient quintile of local labor markets distort resource allocation by 0.4%, the least efficient do so by 1.6%.

Figure 5: EMPIRICAL DISTRIBUTION OF FIRMS MARKET POWER



Notes: The figure depicts the empirical cumulative distribution of wage markdowns and of misallocation stemming from firms' labor market power across local labor markets. Panel (a) displays the wage-bill-weighted distribution of markdowns, where lower values on the x-axis correspond to greater firms' labor market power. Panel (b) presents the wage-bill-weighted cumulative distribution of misallocation, where lower values on the x-axis reflect stronger misallocation due to dispersion in markdowns.

5.1 Welfare

To quantify the welfare implications, I measure the equivalent variation as the percentage increase in consumption that would leave households indifferent between the observed economy and a counterfactual economy without labor market power. Formally, the welfare loss is defined as:

$$U((1 + \lambda)C, N) = U(C^*, N^*), \quad (28)$$

In this calculation, I assume that firms do not internalize the effect of their own labor market power, their competitors' responses, or the shape of their labor supply curve. This approach highlights the aggregate cost of labor market distortions stemming from oligopsonistic behavior and provides a benchmark for evaluating the net impact of minimum wage policy on social welfare.

I calculate welfare losses, and output, aggregate markdown, average firm size and the share of binding workers at the minimum wage for three market structures. These are shown in Table 6. The first column shows the model's implications under the benchmark model, while the second shows the aggregates under monopsonistic competition. Finally, the third column displays the results for an economy with no market power.

Output under oligopsony is normalized to one, while monopsony produces 17 per-

cent more and the efficient allocation 25 percent more. This ranking is consistent with the aggregate markdown: wages are suppressed most strongly under oligopsonistic competition (0.683), followed by monopsonistic competition (0.89), with no markdown in the efficient benchmark. Similarly, the labor share of income is only 0.62 in the oligopsonistic case, compared to 0.78 under monopsony and 0.88 in the efficient outcome.

Firm-level outcomes also differ systematically across regimes. Average firm size is smallest under oligopsonistic competition (21.16), rising to 25.11 under monopsonistic competition and 26.89 in the efficient allocation. The incidence of workers bound by the minimum wage follows a similar pattern: 15 percent of workers are constrained in the oligopsonistic allocation, compared to 9 percent under monopsonistic and 8 percent in the efficient case. These results suggest that oligopsonistic competition generates both more fragmented firm size distributions and a higher prevalence of wage constraints, amplifying distortions in wage setting.

The welfare consequences of these distortions are substantial. Relative to the efficient benchmark, the welfare loss reaches 6.6 percent under oligopsonistic competition and 6.2 percent under monopsonistic competition. Although the magnitude of these losses is similar, the fact that oligopsonistic competition generates slightly higher welfare costs despite lower measured concentration (HHI of 0.26 versus 0.49) is notable. This divergence underscores that standard concentration metrics are insufficient to capture the degree of labor market power when firms differ in productivity and compete in hiring. Oligopsonistic competition can therefore be more damaging than monopsonistic competition in welfare terms, even though it appears less concentrated by conventional measures.

Table 6: AGGREGATES IN THREE MARKET SCENARIOS

	Oligopsonistic (1)	Monopsonistic (2)	Efficient (3)
Output	1.00	1.17	1.25
Aggregate markdown	0.683	0.89	1.00
Average firm size	21.16	25.11	26.89
Share of binding workers at MW	0.15	0.09	0.08
Labor share	0.62	0.78	0.88
HHI	0.26	0.49	0.49
Welfare loss %	6.6	6.2	

Notes: The table reports key outcomes under three alternative market scenarios, keeping the same model parameters without re-calibration. In the *oligopsonistic* scenario, firms recognize their impact on competitors' labor demand and behave strategically. In the *monopsonistic* scenario, firms consider only their own labor supply curve and behave as if they were atomistic. The *efficient* scenario assumes that firms do not internalize any distortions and labor allocation is socially optimal. Columns report output, aggregate markdown, average firm size, the share of workers affected by the minimum wage, labor share, the Herfindahl-Hirschman Index (HHI), and the percentage welfare loss relative to the efficient allocation.

Comparison with BHM. In the case of US, Berger *et al.* (2025) estimate that an economy with no labor market power delivers a welfare gain of 6.3%. In my case, this welfare gain is similar (6.6%) but for different reasons. While aggregate markdown is 0.72 for the US, is even wider for Uruguay and equal to 0.68. However, misallocation is higher in the US, depressing aggregate TFP by 4%¹³ and in Uruguay by 3%.

The lower degree of misallocation in the Uruguayan economy can be traced to the smaller dispersion in wage markdowns, which stems from relatively similar labor supply elasticities across firms and between sectors. For example, an atomistic Uruguayan firm within a local labor market that wishes to expand employment by 1% poaching from its competitors must raise wages by 0.13%, compared to only 0.09% in the United States.¹⁴ This difference reduces the extent of market power that atomistic firms can exercise in Uruguay. Similarly, a monopsony firm within a local labor market in Uruguay must increase wages by 1.25% to expand employment by 1% poaching from other local labor markets, whereas in the United States the required increase is 2.38%.¹⁵ Taken together, these elasticities compress wage markdowns in Uruguay to between 0.44 (for monopsony firms) and 0.88 (for atomistic firms). In contrast, in the U.S. labor market, markdowns

¹³It can be shown that $\Omega = \gamma\alpha(\mathcal{M}/LS)$, where LS is the labor share. Using the parameters of their paper, this yields $\Omega=0.96$

¹⁴From equation (21), $d\log w_{ij}/d\log n_{ij} = 1/\eta \approx 0.13$. BHM estimate $\eta_{US} = 10.85$, hence $1/\eta_{US} \approx 0.09$

¹⁵Analogously, taking logs on the inverse labor supply curve face by a monopsony yields to: $\log w_j = 1/\theta \log n_j + 1/\varphi \log N/\hat{\varphi} - 1/\theta \log N$, hence $d\log w_j/d\log n_j = 1/\theta \approx 1.25$. BHM estimate $\theta_{US} = 0.42$, therefore $1/\theta_{US} \approx 2.38$

range more widely, from 0.22 to 0.91 of the marginal revenue product of labor.

Another possible explanation for the lower dispersion in markdowns is reduced dispersion in idiosyncratic productivity. However, this does not appear to be the case for Uruguay. In the United States, the BLS's *Dispersion Statistics on Productivity (DiSP)* reports a mean within-industry standard deviation of log productivity across 4-digit NAICS industries of 0.49 in 2019. By comparison, productivity inversion for Uruguayan firms yield a very similar figure of 0.48. In sum, although labor market power is more pervasive in Uruguay, it does not translate into greater misallocation because the dispersion in wage markdowns remains comparatively limited.

5.2 Implementing a national minimum wage.

In 2005, the Uruguayan government raised the national minimum wage by 86% in real terms. This policy substantially expanded its coverage: whereas only about 3% of the workforce was initially bound by the minimum wage, the reform made it binding for roughly 15% of all workers. I use the model to evaluate the aggregate and sectoral consequences of this policy, as well as its implications for the distribution of shadow markdowns across firms.

Table (7) summarizes the aggregate equilibrium effects of minimum wage policies on labor market outcomes. The analysis compares two central measures—the misallocation index Ω and the aggregate markdown M —across three scenarios: the baseline economy, the 2005 minimum wage increase, and the 2009 policy adjustment. Both indicators capture different dimensions of efficiency: Ω measures the extent of productive misallocation arising from distortions in labor allocation across firms, while M reflects the gap between the marginal product of labor and the wage, capturing the degree of wage suppression due to firms' labor market power.

The results reveal that minimum wage increases lead to modest changes in both measures. In the baseline economy, the misallocation index Ω equals 0.970, indicating that allocative efficiency is high but not perfect. After the 2005 reform, Ω declines slightly to 0.968, implying a marginal deterioration of efficiency equivalent to 0.2%. The subsequent 2009 adjustment produces a further decline to 0.967. Although these changes are quantitatively small, they indicate that binding minimum wages can exacerbate misallocation by constraining the reallocation of workers across heterogeneous firms.

The markdown measure M displays a different pattern. Starting from a baseline value of 0.683, it rises to 0.684 under both the 2005 and 2009 regimes. This small increase of 0.15% suggests that the minimum wage modestly amplifies the wedge between productivity and wages, as constrained firms are less able to adjust employment flexibly

when facing binding wage floors. The direction of the effect is consistent with the theoretical prediction that stronger wage-setting institutions reduce firms' ability to extract rents, yet the magnitude remains economically limited.

Taken together, these findings indicate that minimum wage policies introduce competing forces in the labor market. On the one hand, they marginally worsen allocative efficiency by restricting the reallocation of labor. On the other hand, they reduce firms' monopsonistic power by narrowing the scope for wage markdowns, although in practice this effect is quantitatively small. The limited aggregate impact is consistent with the relatively modest weight of minimum wage firms in the overall economy. Even though 15% of the workforce became subject to the policy after 2005, such firms account for a disproportionately smaller share of total wage payments, which dampens economy-wide effects.

Overall, the evidence suggests that while minimum wage reforms do alter equilibrium outcomes in the model, their aggregate consequences are modest in scale. The policies shift efficiency and distributional measures in the expected directions, but the magnitude of the changes—less than 0.3 percentage points across both indicators—underscores the limited aggregate importance of minimum wage interventions in an economy where most employment occurs above the statutory wage floor.

Table 7: MINIMUM WAGE AND AGGREGATES

Scenario	Misallocation Ω	Markdown \mathcal{M}
Baseline	0.970	0.683
2005's MW	0.968	0.684
2009's MW	0.967	0.684

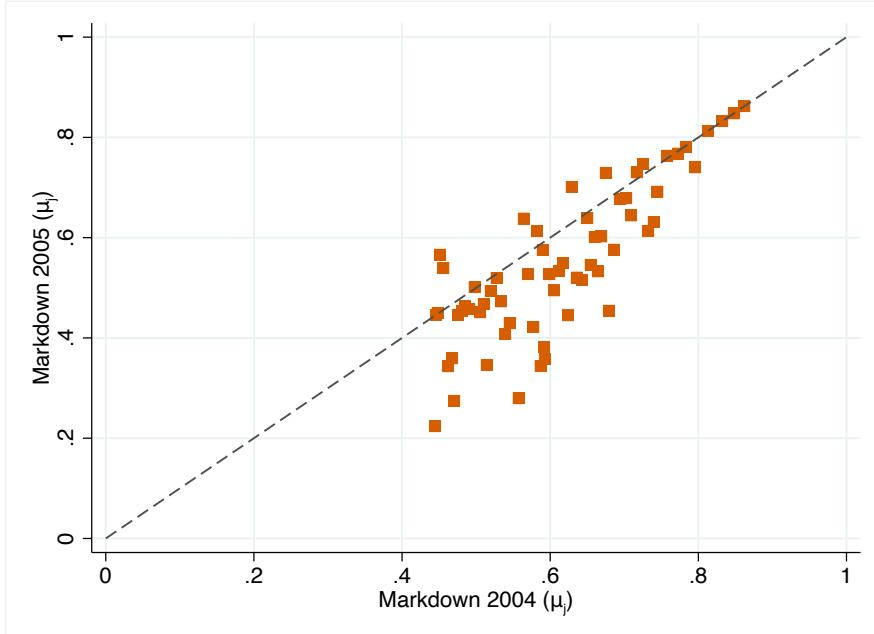
Notes: The table reports the effects of different minimum wage scenarios on aggregate labor market outcomes, keeping all other model parameters unchanged. The columns show overall misallocation (Ω) and the aggregate wage markdown (\mathcal{M}) under the baseline, the 2005 minimum wage, and the 2009 minimum wage. Differences across scenarios are small, indicating limited aggregate effects of the historical minimum wage changes in the model.

Sectoral responses. To examine the effects across local labor markets, figure 6 plots a binscatter of the baseline markdown, μ_j , in 2004 against the post-treatment markdown, $\tilde{\mu}_j$, in 2005. Each point represents the mean of μ_j and $\tilde{\mu}_j$ within 100 equal-frequency bins of the 2004 markdown distribution. The dashed 45-degree line indicates no change in markdowns across years. Observations above the line correspond to local labor markets where markdowns increased between 2004 and 2005, while points below the line indicate

reductions. The figure shows that, following the reintroduction of the minimum wage, local labor markets experienced a decline in their shadow markdowns, suggesting that the policy may have exacerbated resource misallocation across firms within local labor markets and increased overall inefficiencies.

The deterioration in shadow markdowns is not confined to oligopsonistic markets. Even in local labor markets that exhibit full monopsony, the 2005 minimum wage appears to have generated more distortions than improvements. Specifically, out of 447 monopsonistic local labor markets, only 92 experienced a decline in markdowns following the reform, while 304 saw their markdowns increase. This pattern highlights that the minimum wage level chosen in 2005 may have been excessively high, even in settings where theory predicts that binding minimum wages could improve efficiency by counteracting monopsonistic power.

Figure 6: MARKDOWN AND THE MINIMUM WAGE POLICY



Notes: The figure plots firm-level wage markdowns in 2004 (x-axis) against markdowns in 2005 (y-axis). Points below the 45-degree line indicate firms where the markdown decreased from 2004 to 2005, while points above the line indicate firms with an increased markdown. The figure illustrates the cross-year evolution of wage markups across firms.

Policy experiment: Employment response due to minimum wage. Finally, I evaluate employment changes, comparing model predictions with empirical estimates. I adapt [Harasztosi and Lindner \(2019\)](#), defining the treatment group as those firms that in year 2004 had an average wage below the 2005 minimum wage.

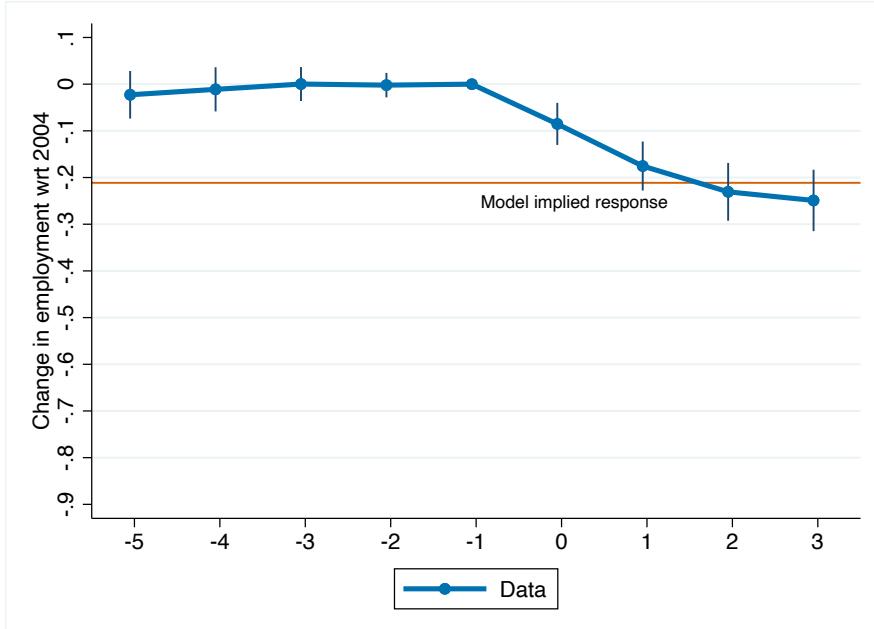
$$\begin{aligned} T_i &= 1 \quad \text{if } \bar{w}_{i2004} \leq MW_{2005} \\ T_i &= 0 \quad \text{if } \bar{w}_{i2004} > MW_{2005} \end{aligned}$$

I run the following equation

$$\frac{y_{it} - y_{i2004}}{y_{i2004}} = \alpha_{jt} + \beta_t T_i + \gamma_t X_{it} + \varepsilon_{it} \quad (29)$$

Where the LHS measures the change in employment with respect to the baseline year 2004, both at the intensive margin (layoff) and the extensive margin (shut down). The parameter of interest is β_t , and captures the cumulative change in the dependent variable for exposed firms with respect to non-exposed firms. Besides, the regression also controls for local labor market time fixed effect. Both regression (data and model based) are weighted by the employment at baseline year. The model predicts that in the new steady state, employment will decrease by 21% in directly affected firms with respect to non directly affected firms. In the data, this decline is achieved between the second and the third year.

Figure 7: EMPLOYMENT CHANGE DATA VS MODEL



Notes: This figure shows the employment response to the 2005 minimum wage in Uruguay. The treatment group consists of firms with average wages below the minimum in 2004. The left-hand side of the regression measures the change in employment relative to 2004, capturing both layoffs and firm closures. Both data and model-based regressions are weighted by baseline employment. The model predicts a 21% decline in employment in the new steady state, which is consistent with the observed decline in the second to third year after the reform.

6 Conclusion

This paper examined how labor market power shapes the effects of minimum wage policies in an economy where firms compete for workers under oligopsonistic conditions. Using matched employer–employee data from Uruguay, I found that the reintroduction of the minimum wage in 2005 led to sizable wage increases but modest aggregate employment losses, consistent with the presence of imperfect competition in labor markets. Firms initially paying below the new wage floor reduced employment, but this decline was smaller among firms with larger wage-bill shares within local labor markets, indicating that employers face different labor supply elasticities. The calibrated model reproduces these heterogeneous responses and shows that, before the reform, workers take home 67% of their marginal revenue product of labor and the existence of variable markdowns creates welfare loss of 6.6% relative to perfect competition. Once the minimum wage was implemented, markdowns and misallocation changed only slightly, suggesting that the policy had limited aggregate effects.

The estimated elasticities help explain these patterns. The within-market elasticity of labor supply implies that small firms face relatively elastic labor supply, limiting their wage-setting power. In contrast, the across-sector elasticity indicates that workers reallocate imperfectly between sectors, allowing sectoral labor market power to persist. Together, these elasticities generate heterogeneity in wage markdowns across firms while dampening reallocation effects across sectors. As a result, the minimum wage improved outcomes for some constrained, low-wage firms but did little to alter aggregate distortions driven by large employers with persistent market power. These findings highlight that the welfare and efficiency consequences of wage regulation depend critically on the structure of labor supply elasticities across firms and sectors that govern how competition and policy interact in imperfect labor markets.

Future work will focus on estimating labor supply elasticities more precisely to better match empirical moments such as the average wage markdown documented in the literature. Further extensions could incorporate the presence of informal firms that are not subject to minimum wage regulation, as well as the prevalence of self-employment as an alternative margin of adjustment. Another promising avenue is to test more directly for spillovers—both across firms within local labor markets and across workers within firms—and to explore their macroeconomic implications for wage dispersion, employment, and aggregate welfare.

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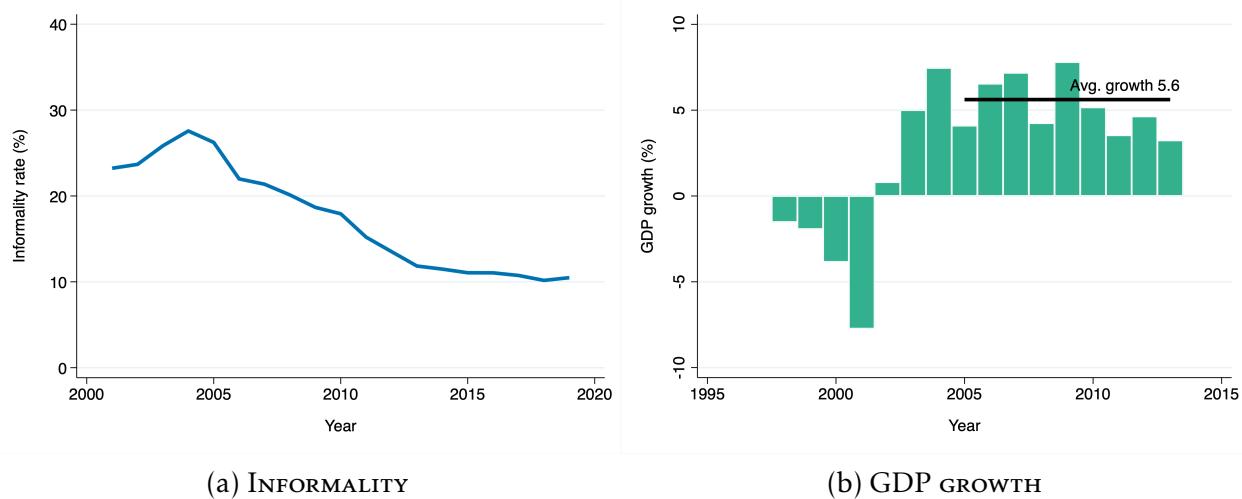
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A Appendix

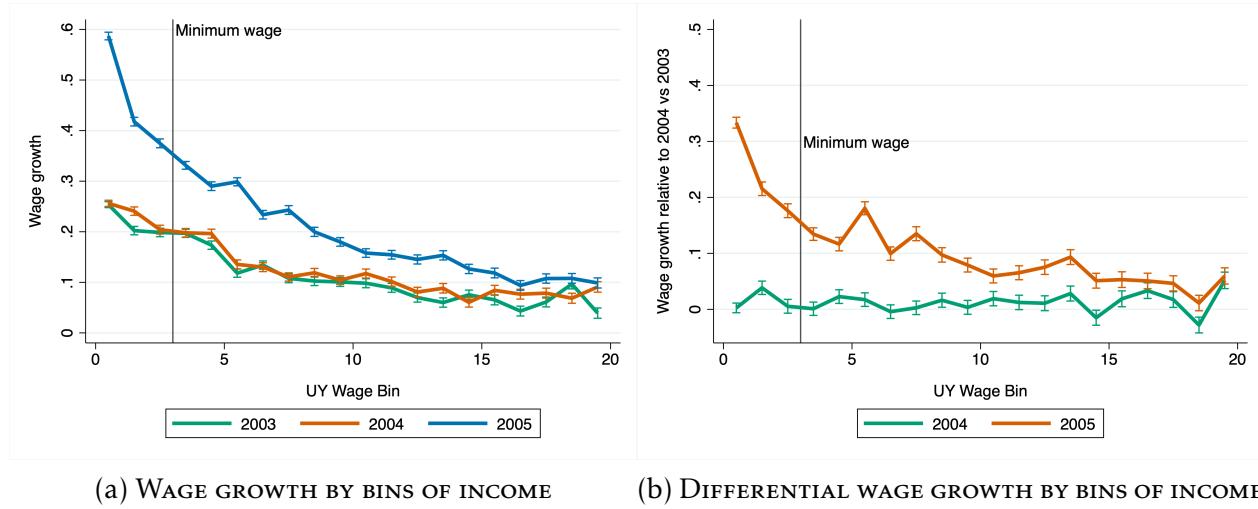
A.1 Tables and figures

Figure A1: THE MACRO ENVIRONMENT: INFORMALITY AND ECONOMIC GROWTH



Notes: These figures show the macroeconomic environment where the NMW and the CBA's policies were implemented. In panel a), is shown the drop in the informality rate. Informality rate is computed using Household Surveys from the National Institute of Statistics. Informality is defined as not having access to social security. In panel b), is shown the GDP growth for each year between 1998 to 2013. The calculations are based on National Accounts provided by the Central Bank of Uruguay.

Figure A2: WAGE GROWTH: GRAPHICAL EVIDENCE



Notes: tbc

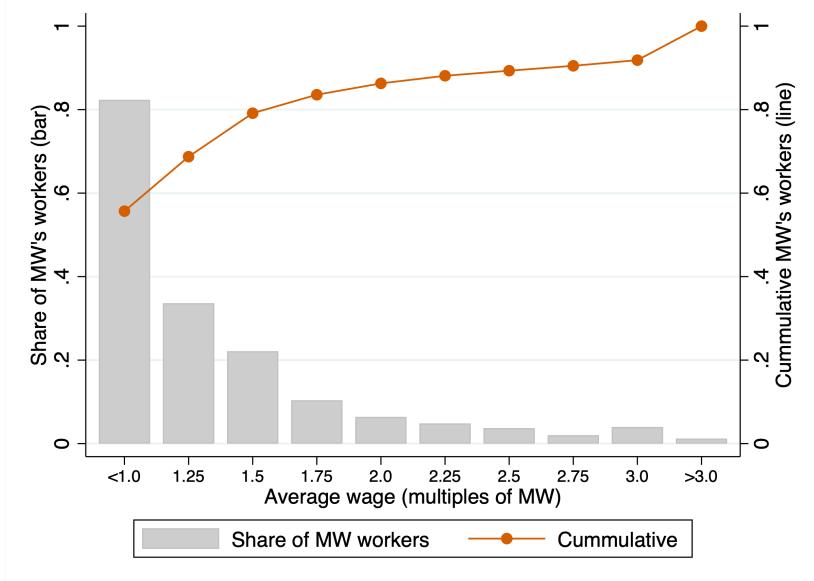


Figure A3: Caption

To motivate my model, I will present two sets of empirical analyses. The first one is based on incumbent workers and explores the bindingness of the minimum wage and the potential disemployment effects it creates. The second one is based on firms and explores the changes in employment levels concerning a pre-policy year and look for heterogeneity of the effect at relatively bigger firms. This data has widely used to study the effects of collective bargaining on labor income inequality, minimum wage and the distribution of firm wage premia, pension privatization, and

A.2 Incumbent workers

As widely documented in the literature, increases in the minimum wage can generate spillover effects on workers earning above the minimum. To capture this, I follow [Dustmann *et al.* \(2022\)](#) and define the treatment variable as a dosage measure. Specifically, I classify workers into three treatment groups based on their wage level in period $t - 1$

$$D_{it} = 1 \quad if \quad w_{it-1} \leq MW_{2005}$$

$$D_{it} = 2 \quad if \quad MW_{2005} < w_{it-1} \leq 1.7MW_{2005}$$

$$D_{it} = 3 \quad if \quad 1.7MW_{2005} < w_{it-1} \leq 2.7MW_{2005}$$

And the regression is therefore

$$\Delta y_{it} = \sum_{k=1}^3 \mathbb{1}(D_{it} = d_{tk}) \delta_{tk} + \gamma X_{i,t-1} + e_{it} \quad (\text{A.1})$$

Where Δy_{it} is the (log) change in wages for individual i at time t . The parameter δ_{tk} captures the wage growth for each treatment group at year $t \in \{2003, 2004, 2005\}$. However, it has no causal interpretation due to mean reversion and macroeconomic effects. Mean reversion is an important challenge in this setting because the wage growth for low-wage earners is expected to be higher than the ones for high-wage earners. To rule out this possibility, I re-write the previous equation but add a term that captures the wage growth in the baseline year for each income group. The equation reads

$$\Delta y_{it} = \sum_{k=1}^3 [\mathbb{1}(D_{it} = d_{tk}) \delta_{2003k} + \mathbb{1}(D_{it} = d_{tk}) \beta_{tk}] + \gamma X_{i,t-1} + e_{it} \quad (\text{A.2})$$

Where coefficient δ_{2003k} captures the baseline wage growth for each treatment group in year 2003. Henceforth, β_{kt} captures the differential wage growth with respect to the baseline for each year in $t \in \{2004, 2005\}$.

Finally, wage growth could be affected by macroeconomic conditions, specially important in this setting due to high GDP growth. To rule out macroeconomic effects, I compare the differential wage growth of the third group with the differential wage growth of the first one. Where $\beta_{Maineffect}$ captures the causal effect of the NMW increase of 2005 on excess wage growth for incumbent binding workers with respect to incumbent non-affected workers. Also, is possible to estimate the spillover effect on non-binding workers. I express this as

$$\beta_{Maineffect} = \beta_1 - \beta_3, \quad \beta_{Spillover} = \beta_2 - \beta_3 \quad (\text{A.3})$$

Table (A7) shows that each treatment group has a differential baseline growth, as expected because of mean reversion. However, the highly exposed workers experienced a differential wage growth of 36% with respect to their baseline growth. Besides, the differential wage growth for each group is decreasing the further the income group is from the binding workers group. The last column indicates that the NMW increases wages by 29.5% to binding workers with respect to non-binding workers. Importantly, the placebo effect, i.e. the differential wage growth of 2004 with respect to baseline wage growth,

is not significant and close to 0. Finally, it is important to stress that there are sizeable spillovers given by the fact that low-exposure workers also experienced an increase in their wages compared to high-income workers, of about 8.6%.

Table A1: WAGE GROWTH - ALL WORKERS

Wage at $t - 1$	High-exposed	Low-exposed	Not exposed	Main effect	Spillover
	(1)	(2)	(3)	(1)-(3)	(2)-(3)
2005 vs 2004	0.363*** (0.014)	0.155*** (0.023)	0.068*** (0.015)	0.295*** (0.016)	0.086*** (0.014)
2004 vs 2003 (Placebo)	0.041*** (0.010)	0.020 (0.019)	0.022* (0.012)		
Baseline change	0.215*** (0.029)	0.148*** (0.031)	0.106*** (0.029)		
R^2	0.25	0.25	0.25		
N	370,633	370,633	370,633		

Notes: This table shows the change in wage growth with respect to a baseline change. The first and the second row shows the differential wage growth with respect to the baseline change. Column 1 shows the differential wage growth for high-exposed workers and column 2 shows the differential wage growth for low-exposed workers, while column 3 shows baseline change for non-exposed workers. The main effect is computed as the differential growth between high-exposed and not exposed workers while spillovers is calculated as the difference between low exposed wage growth and not exposed wage growth.

A first-order issue regarding minimum wage policies is to assess if the policy creates disemployment. However, I am not able to measure this margin because of data limitations; when a worker left the data it could be because she became unemployed or because she went to informality. Given this limitation, I can only measure if the worker is still employed in the formal sector or not. The LHS variable takes the value of -1 if the worker is no longer employed in the formal sector, and 0 otherwise. I do not find employment effects or differential effects across sectors.

As widely documented in the literature, increases in the minimum wage can generate spillover effects on workers earning above the minimum. To capture this, I follow [Dustmann et al. \(2022\)](#) and define the treatment variable as a dosage measure. Specifically, I classify workers into three treatment groups based on their wage level in period $t - 1$

$$\begin{aligned} D_{it} &= 1 \quad \text{if } w_{it-1} \leq MW_{2005} \\ D_{it} &= 2 \quad \text{if } MW_{2005} < w_{it-1} \leq 1.7MW_{2005} \\ D_{it} &= 3 \quad \text{if } 1.7MW_{2005} < w_{it-1} \leq 2.7MW_{2005} \end{aligned}$$

And the regression is therefore

$$\Delta y_{it} = \sum_{k=1}^3 [\mathbb{1}(D_{it} = d_{tk})\delta_{2003k} + \mathbb{1}(D_{it} = d_{tk})\beta_{tk}] + \gamma X_{i,t-1} + e_{it} \quad (\text{A.4})$$

Where coefficient δ_{2003k} captures the baseline wage growth for each treatment group in year 2003. Henceforth, β_{kt} captures the differential wage growth with respect to the baseline for each year in $t \in \{2004, 2005\}$, controlling for mean reversion.

Finally, wage growth could be affected by macroeconomic conditions, specially important in this setting due to high GDP growth. To rule out macroeconomic effects, I compare the differential wage growth of the third group with the differential wage growth of the first one. Where $\beta_{Maineffect}$ captures the causal effect of the NMW increase of 2005 on excess wage growth for incumbent binding workers with respect to incumbent non-affected workers. Also, is possible to estimate the spillover effect on non-binding workers. I express this as

$$\beta_{Maineffect} = \beta_1 - \beta_3, \quad \beta_{Spillover} = \beta_2 - \beta_3 \quad (\text{A.5})$$

Table (A7) shows that each treatment group has a differential baseline growth, as expected because of mean reversion. However, the highly exposed workers experienced a differential wage growth of 36% with respect to their baseline growth. Besides, the differential wage growth for each group is decreasing the further the income group is from the binding workers group. The last column indicates that the NMW increases wages by 29.5% to binding workers with respect to non-binding workers. Importantly, the placebo effect, i.e. the differential wage growth of 2004 with respect to baseline wage growth, is not significant and close to 0. Finally, it is important to stress that there are sizeable spillovers given by the fact that low-exposure workers also experienced an increase in their wages compared to high-income workers, of about 8.6%.

Table A2: WAGE GROWTH - ALL WORKERS

Wage at $t - 1$	High-exposed	Low-exposed	Not exposed	Main effect	Spillover
	(1)	(2)	(3)	(1)-(3)	(2)-(3)
2005 vs 2004	0.363*** (0.014)	0.155*** (0.023)	0.068*** (0.015)	0.295*** (0.016)	0.086*** (0.014)
2004 vs 2003 (Placebo)	0.041*** (0.010)	0.020 (0.019)	0.022* (0.012)		
Baseline change	0.215*** (0.029)	0.148*** (0.031)	0.106*** (0.029)		
R^2	0.25	0.25	0.25		
N	370,633	370,633	370,633		

Notes: This table shows the change in wage growth with respect to a baseline change. The first and the second row shows the differential wage growth with respect to the baseline change. Column 1 shows the differential wage growth for high-exposed workers and column 2 shows the differential wage growth for low-exposed workers, while column 3 shows baseline change for non-exposed workers. The main effect is computed as the differential growth between high-exposed and not exposed workers while spillovers is calculated as the difference between low exposed wage growth and not exposed wage growth.

Table A3: EMPLOYMENT EFFECT - ALL WORKERS

Wage at $t - 1$	High-exposed	Low-exposed	Not exposed	Main effect	Spillover
	(1)	(2)	(3)	(1)-(3)	(2)-(3)
2005 vs 2004	0.020*** (0.002)	0.012*** (0.001)	0.006*** (0.001)	0.014*** (0.002)	0.006*** (0.002)
2004 vs 2003 (Placebo)	0.019*** (0.002)	0.006*** (0.001)	0.005*** (0.001)		
Baseline change	-0.094*** (0.011)	-0.088*** (0.011)	-0.085*** (0.011)		
R^2	0.07	0.07	0.07		
N	393,585	393,585	393,585		

Notes: This table shows the change in employment growth with respect to a baseline change. The first and the second row shows the differential employment change with respect to the baseline change. Column 1 shows the differential employment change for high-exposed workers and column 2 shows the differential employment change for low-exposed workers, while column 3 shows baseline change for non-exposed workers. The main effect is computed as the differential employment between high-exposed and not exposed workers while spillovers is calculated as the difference between low exposed employment change and not exposed employment change.

Reallocation. In this section, I will present results on the reallocation of workers in two dimensions. The first one is to evaluate to what extent the minimum wage policy created reallocation of workers at all. This is what I call *reallocation rate*. The second one is in terms of the nature of reallocation, that is, conditional on switching to a different firm,

I explore if the worker moved to a bigger firm or not. This is what I call the *reallocation pattern*. I also look for differential effects depending on the sector of activity.

Reallocation rate. The LHS variable takes the value of -1 if the worker is reallocated to a different firm, and 0 otherwise. I found that the policy created reallocation to workers highly exposed to the minimum wage. I also find that there are spillovers. When analyzing by sector, I found that the reallocation rate is higher for the tradable sector.

Table A4: REALLOCATION RATE - ALL WORKERS

Wage at $t - 1$	High-exposed	Low-exposed	Not exposed	Main effect	Spillover
	(1)	(2)	(3)	(1)-(3)	(2)-(3)
2005 vs 2004	-0.074*** (0.008)	-0.057*** (0.009)	-0.030*** (0.006)	-0.045*** (0.010)	-0.027*** (0.007)
2004 vs 2003 (Placebo)	-0.041*** (0.007)	-0.031*** (0.006)	-0.010** (0.005)		
Baseline change	-0.123*** (0.025)	-0.118*** (0.025)	-0.109*** (0.026)		
R^2	0.20	0.20	0.20		
N	375,661	375,661	375,661		

Reallocation pattern. The LHS variable takes the value of -1 if the worker reallocated to a bigger firm, 0 otherwise. I found that the policy created upward mobility. However, I find that this mobility is explained by the non-tradable sector.

In conclusion, even though workers in the tradable sector are more likely to switch to a new firm, they are not more likely to move to a bigger firm.

Table A5: REALLOCATION PATTERN - ALL WORKERS

Wage at $t - 1$	High-exposed	Low-exposed	Not exposed	Main effect	Spillover
	(1)	(2)	(3)	(1)-(3)	(2)-(3)
2005 vs 2004	-0.047** (0.021)	0.039** (0.019)	0.043 (0.029)	-0.089*** (0.034)	-0.003 (0.029)
2004 vs 2003 (Placebo)	-0.028 (0.020)	0.026 (0.018)	-0.021 (0.032)		
Baseline change	-0.502*** (0.081)	-0.511*** (0.080)	-0.477*** (0.087)		
R^2	0.54	0.54	0.54		
N	41,893	41,893	41,893		

Once I classify the industries in tradable and non-tradable, the effect on wage growth is higher for tradable sector than from non-tradable sector. Both for the main effect as for the spillover effect.

Table A6: WAGE GROWTH - TRADABLE SECTOR

Wage at $t - 1$	High-exposed	Low-exposed	Not exposed	Main effect	Spillover
	(1)	(2)	(3)	(1)-(3)	(2)-(3)
2005 vs 2004	0.446*** (0.019)	0.233*** (0.012)	0.122*** (0.018)	0.324*** (0.026)	0.111*** (0.019)
2004 vs 2003 (Placebo)	0.077*** (0.017)	0.053*** (0.011)	0.052*** (0.017)		
Baseline change	0.203*** (0.030)	0.168*** (0.029)	0.139*** (0.032)		
R^2	0.31	0.31	0.31		
N	74,134	74,134	74,134		

Table A7: WAGE GROWTH - ALL WORKERS

Wage at $t - 1$	High-exposed	Low-exposed	Not exposed	Main effect	Spillover
	(1)	(2)	(3)	(1)-(3)	(2)-(3)
2005 vs 2004	0.344*** (0.015)	0.136*** (0.027)	0.054*** (0.017)	0.289*** (0.019)	0.082*** (0.015)
2004 vs 2003 (Placebo)	0.034*** (0.012)	0.013 (0.023)	0.015 (0.013)		
Baseline change	0.187*** (0.040)	0.112*** (0.041)	0.068* (0.041)		
R^2	0.24	0.24	0.24		
N	296,499	296,499	296,499		

Table A8: EMPLOYMENT EFFECT - TRADABLE SECTOR

Wage at $t - 1$	High-exposed	Low-exposed	Not exposed	Main effect	Spillover
	(1)	(2)	(3)	(1)-(3)	(2)-(3)
2005 vs 2004	0.026*** (0.005)	0.011*** (0.004)	0.013*** (0.003)	0.014** (0.006)	-0.002 (0.005)
2004 vs 2003 (Placebo)	0.023*** (0.006)	0.005 (0.004)	0.010*** (0.003)		
Baseline change	-0.044* (0.024)	-0.034 (0.024)	-0.041* (0.024)		
R^2	0.08	0.08	0.08		
N	80,811	80,811	80,811		

Table A9: EMPLOYMENT EFFECT - NON-TRADABLE SECTOR

Wage at $t - 1$	High-exposed	Low-exposed	Not exposed	Main effect	Spillover
	(1)	(2)	(3)	(1)-(3)	(2)-(3)
2005 vs 2004	0.019*** (0.002)	0.012*** (0.001)	0.005*** (0.001)	0.014*** (0.002)	0.007*** (0.002)
2004 vs 2003 (Placebo)	0.018*** (0.002)	0.006*** (0.001)	0.004*** (0.001)		
Baseline change	-0.099*** (0.013)	-0.094*** (0.013)	-0.089*** (0.013)		
R^2	0.07	0.07	0.07		
N	312,774	312,774	312,774		

Table A10: REALLOCATION PATTERN - TRADABLE SECTOR

Wage at $t - 1$	High-exposed	Low-exposed	Not exposed	Main effect	Spillover
	(1)	(2)	(3)	(1)-(3)	(2)-(3)
2005 vs 2004	-0.082*	0.055	-0.118*	0.035	0.172**
	(0.048)	(0.044)	(0.067)	(0.081)	(0.068)
2004 vs 2003 (Placebo)	-0.056	0.021	-0.136*		
	(0.045)	(0.045)	(0.069)		
Baseline change	-0.457***	-0.493***	-0.268*		
	(0.142)	(0.144)	(0.155)		
R^2	0.54	0.54	0.54		
N	9,013	9,013	9,013		

Table A11: REALLOCATION PATTERN - NON-TRADABLE

Wage at $t - 1$	High-exposed	Low-exposed	Not exposed	Main effect	Spillover
	(1)	(2)	(3)	(1)-(3)	(2)-(3)
2005 vs 2004	-0.036	0.035*	0.089***	-0.126***	-0.055**
	(0.023)	(0.020)	(0.023)	(0.030)	(0.024)
2004 vs 2003 (Placebo)	-0.021	0.025	0.016		
	(0.022)	(0.019)	(0.027)		
Baseline change	-0.618***	-0.620***	-0.641***		
	(0.097)	(0.096)	(0.097)		
R^2	0.54	0.54	0.54		
N	32,880	32,880	32,880		

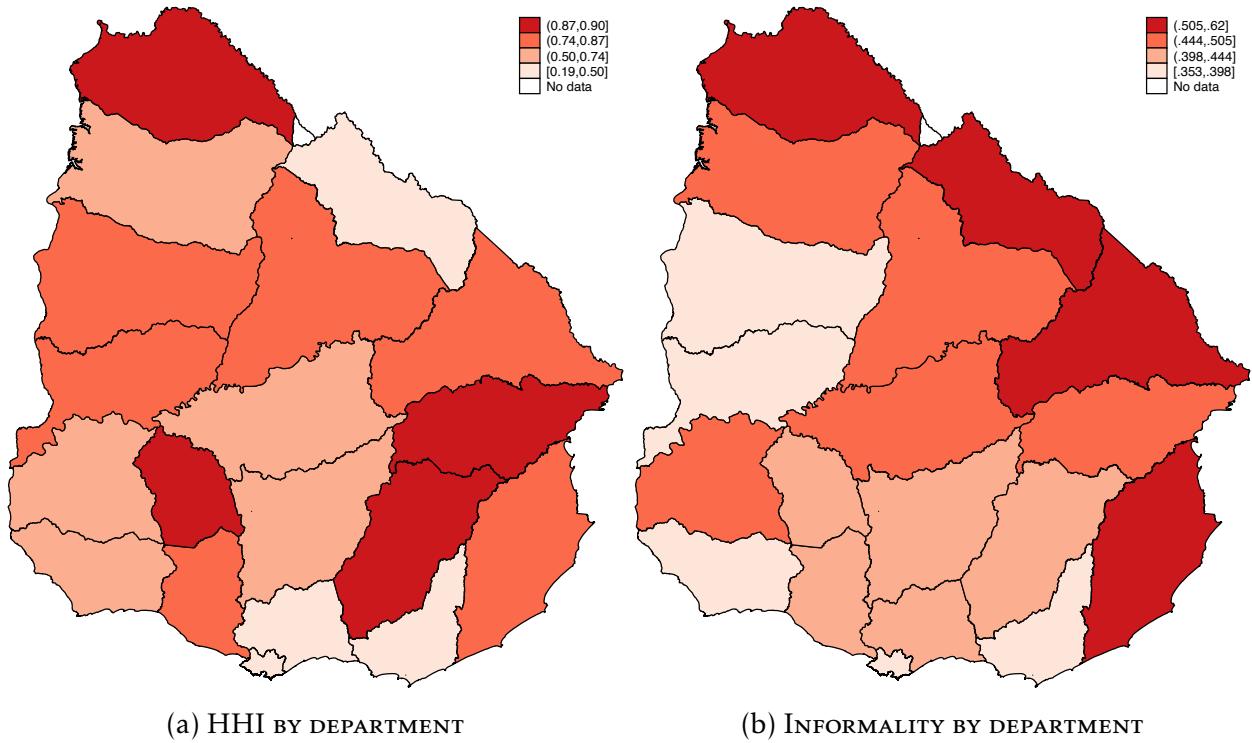
Table A12: REALLOCATION RATE - TRADABLE SECTOR

Wage at $t - 1$	High-exposed	Low-exposed	Not exposed	Main effect	Spillover
	(1)	(2)	(3)	(1)-(3)	(2)-(3)
2005 vs 2004	-0.083***	-0.080***	-0.014	-0.069***	-0.066***
	(0.017)	(0.014)	(0.014)	(0.021)	(0.018)
2004 vs 2003 (Placebo)	-0.042***	-0.026**	0.007		
	(0.016)	(0.012)	(0.014)		
Baseline change	-0.104***	-0.095***	-0.096**		
	(0.036)	(0.037)	(0.040)		
R^2	0.20	0.20	0.20		
N	75,125	75,125	75,125		

Table A13: REALLOCATION RATE - NON-TRADABLE

Wage at $t - 1$	High-exposed	Low-exposed	Not exposed	Main effect	Spillover
	(1)	(2)	(3)	(1)-(3)	(2)-(3)
2005 vs 2004	-0.072*** (0.009)	-0.050*** (0.010)	-0.034*** (0.007)	-0.038*** (0.012)	-0.016** (0.007)
2004 vs 2003 (Placebo)	-0.040*** (0.007)	-0.033*** (0.007)	-0.015*** (0.005)		
Baseline change	-0.133*** (0.030)	-0.130*** (0.029)	-0.118*** (0.030)		
R^2	0.20	0.20	0.20		
N	300,536	300,536	300,536		

Figure A4: CONCENTRATION AND INFORMALITY



Notes:

B Mathematical appendix

B.1 Local labor market wage bill share

The wage bill share of local labor market j can be expressed as

$$s_j := \frac{w_j n_j}{\int_0^1 w_j n_j dj} \quad (\text{A.6})$$

In what follow, I extend the results presented in [Berger et al. \(2022\)](#).

$$mrpl_j = \alpha z_j n_j^{\alpha-1} \quad (\text{A.7})$$

$$n_j = \left(\frac{w_j}{W}\right)^\theta \frac{N}{\bar{\varphi}} \quad (\text{A.8})$$

$$w_j = \mu_j mrpl_j \quad (\text{A.9})$$

Combining the pricing with the labor supply and replacing it on the labor demand, leads to

$$mrpl_j = \left[\alpha z_j \mu_j^{-\theta(1-\alpha)} X^{\alpha-1}\right]^{\frac{1}{1+\theta(1-\alpha)}} \quad (\text{A.10})$$

where X collects all variables that depends on aggregates, namely N and $\bar{\varphi}$. Then, plugging this expression into the pricing equation, leads to

$$w_j = \left(z_j \mu_j\right)^{\frac{1}{1+\theta(1-\alpha)}} \times g(X) \quad (\text{A.11})$$

Hence, the wage-bill of market j can be expressed as

$$w_j n_j = \left(z_j \mu_j\right)^{\frac{\theta+1}{1+\theta(1-\alpha)}} \times g(X) \quad (\text{A.12})$$

and finally, replacing this expression into the wage-bill share of market j yields to

$$s_j := \frac{w_j n_j}{\int_0^1 w_j n_j dj} = \frac{\left[\mu_j z_j\right]^{\frac{\theta+1}{1+\theta(1-\alpha)}}}{\int_0^1 \left[\mu_j z_j\right]^{\frac{\theta+1}{1+\theta(1-\alpha)}} dj} \quad (\text{A.13})$$

B.2 Algorithm

Algorithm 1: Solving Productivity Shifters from observed LLM Wage-Bill Shares

Input: Baseline productivities z_j^{baseline} , demand shifters μ_j^{baseline} , parameters (θ, α) , target wage-bill shares $\{\hat{s}_j\}_{j=1}^J$

Output: Productivity shifters $z^* = [1, z_2^*, \dots, z_J^*]$

Normalization: Set $z_1^{\text{shifter}} = 1$;

Step 1. Baseline model shares;

Compute wage-bill shares with all shifters equal to one:

$$s_j^{\text{baseline}} = \frac{(\mu_j^{\text{baseline}} z_j^{\text{baseline}})^{\frac{\theta+1}{1+\theta(1-\alpha)}}}{\sum_{k=1}^J (\mu_k^{\text{baseline}} z_k^{\text{baseline}})^{\frac{\theta+1}{1+\theta(1-\alpha)}}}.$$

Step 2. Initial guess;

Compute ratios $r_j = \hat{s}_j / s_j^{\text{baseline}}$;

Set $\tilde{z}_j = r_j^{\frac{1+\theta(1-\alpha)}{\theta+1}}$;

Normalize by \tilde{z}_1 : $z_j^{(0)} = \tilde{z}_j / \tilde{z}_1$, for $j = 2, \dots, J$;

Step 3. System of equations;

For candidate shifters $z = [1, z_2, \dots, z_J]$, compute model shares:

$$s_j^{\text{model}}(z) = \frac{(\mu_j^{\text{baseline}} z_j^{\text{baseline}} z_j)^{\frac{\theta+1}{1+\theta(1-\alpha)}}}{\sum_{k=1}^J (\mu_k^{\text{baseline}} z_k^{\text{baseline}} z_k)^{\frac{\theta+1}{1+\theta(1-\alpha)}}}.$$

Residuals: $F_j(z) = s_j^{\text{model}}(z) - \hat{s}_j$, for $j = 1, \dots, J-1$;

Step 4. Numerical solution Solve $F(z) = 0$ using a nonlinear solver (trust-region method) with initial guess $z^{(0)}$;

Stop when $|F_j(z)| < \varepsilon$ for all j ;

Step 5. Output;

Return $z^* = [1, z_2^*, \dots, z_J^*]$;

C Model with wedges and minimum wage

In the presence of wedges in the labor market, the profit maximization of the firm becomes

$$\pi_{ij} = z_{ij} n_{ij}^\alpha - \frac{1}{d_{ij}} w_{ij} n_{ij}, \quad d_{ij} \geq 0. \quad (\text{A.14})$$

In particular, a firm is subsidized when $d_{ij} > 1$ and taxed if $d_{ij} < 1$. As in Berger *et al.* (2025) we will have three cases.

Region I:

$$w_{ij} = \frac{\varepsilon_{ij}}{d_{ij}} \mu_{ij} mrpl_{ij}, \quad \mu_{ij} = \frac{\varepsilon_{ij}}{\varepsilon_{ij} + 1}, \quad \varepsilon_{ij} = \left[(1 - s_{ij}) \frac{1}{\eta} + s_{ij} \frac{1}{\theta} \right]^{-1}, \quad s_{ij} = \frac{w_{ij} n_{ij}}{\sum w_{ij} n_{ij}} \quad (\text{A.15})$$

Region II:

$$\tilde{w}_{ij} = \frac{\bar{w}}{d_{ij}}, \quad \tilde{\mu}_{ij} = \frac{\bar{w}}{d_{ij} mrpl_{ij}} \quad (\text{A.16})$$

Region III:

$$\tilde{w}_{ij} = \frac{\bar{w}}{d_{ij}} = mrpl_{ij}. \quad (\text{A.17})$$

Hence, with firm specific distortions the minimum wage becomes *firm specific*. For instance, firms that were receiving a payroll subsidy $d_{ij} > 1$ will face a lower effective minimum wage while firms taxed will face a higher effective minimum wage. The firm-specific effective minimum wage is equal to $\tilde{w}_{ij} = \frac{\bar{w}}{d_{ij}}$.

To solve for the optimal rationing constraint, I intersect the $mrpl_{ij}$ with the firm-specific minimum wage, which gives

$$\bar{n}_{ij} = \left(\frac{d_{ij} \alpha z_{ij}}{\bar{w}} \right)^{\frac{1}{1-\alpha}} \quad (\text{A.18})$$

Hence –everything else equal– a firm that is subject to a payroll subsidy will have a higher optimal rationing constraint than a taxed firm will.

D Aggregation with wedges

Claim. In the presence of labor market wedges, the aggregates $\{y, w, n\}$ can be written as

$$y = \omega^{distorted} z n^\alpha \quad (\text{A.19})$$

$$w = \mu^{distorted} z \alpha n^{\alpha-1} \quad (\text{A.20})$$

where

$$\omega = \left[\int z_i^{\frac{1+\eta}{1+\eta(1-\alpha)}} d_i \right]^{\frac{1+\eta(1-\alpha)}{1+\eta}} \quad (\text{A.21})$$

$$\mu = \left[\int \left(\frac{z_i}{z} \right)^{\frac{1+\eta}{1+\eta(1-\alpha)}} \mu_i^{\frac{1+\eta}{1+\eta(1-\alpha)}} d_i^{\frac{1+\eta}{1+\eta(1-\alpha)}} d_i \right]^{\frac{1+\eta(1-\alpha)}{1+\eta}} \quad (\text{A.22})$$

Proof.

$$y_i = z_i n_i^\alpha \quad (\text{A.23})$$

$$n_i = \left(\frac{w_i}{w} \right)^\eta n \quad (\text{A.24})$$

$$w_i = \mu_i d_i mrpl_i \quad (\text{A.25})$$

$$mrpl_i = \alpha z_i n_i^{\alpha-1} \quad (\text{A.26})$$

$$y = \int y_i d_i \quad (\text{A.27})$$

$$w = \left[\int w_i^{1+\eta} d_i \right]^{\frac{1}{1+\eta}} \quad (\text{A.28})$$

$$mrpl_i = \left(\frac{z_i}{z} \right)^{\frac{1}{1+\eta(1-\alpha)}} \mu_i^{-\frac{\eta(\alpha-1)}{1+\eta(1-\alpha)}} d_i^{-\frac{\eta(\alpha-1)}{1+\eta(1-\alpha)}} \left(\alpha z n^{\alpha-1} \right)^{\frac{1}{1+\eta(1-\alpha)}} w^{\frac{\eta(1-\alpha)}{1+\eta(1-\alpha)}} \quad (\text{A.29})$$

$$w_i = \mu_i d_i mrpl_i \quad (\text{A.30})$$

$$w_i = \left(\frac{z_i}{z} \right)^{\frac{1}{1+\eta(1-\alpha)}} \mu_i^{\frac{1}{1+\eta(1-\alpha)}} d_i^{\frac{1}{1+\eta(1-\alpha)}} \left(\alpha z n^{\alpha-1} \right)^{\frac{1}{1+\eta(1-\alpha)}} w^{\frac{\eta(1-\alpha)}{1+\eta(1-\alpha)}} \quad (\text{A.31})$$

Now, using (A.23) we can write

$$w = \left[\int \left(\frac{z_i}{z} \right)^{\frac{1+\eta}{1+\eta(1-\alpha)}} \mu_i^{\frac{1+\eta}{1+\eta(1-\alpha)}} d_i^{\frac{1+\eta}{1+\eta(1-\alpha)}} di \right]^{\frac{1+\eta(1-\alpha)}{1+\eta}} \alpha z n^{\alpha-1} \quad (\text{A.32})$$

$$w = \mu z \alpha n^{\alpha-1} \quad (\text{A.33})$$

Turning to the proof of value added aggregation, we have.

$$y_i = z_i n_i^\alpha \quad (\text{A.34})$$

$$y_i = z_i \left(\left(\frac{w_i}{w} \right)^\eta n \right)^\alpha \quad (\text{A.35})$$

$$y_i = z_i \left(\left(\frac{\mu_i d_i m r p l_i}{w} \right)^\eta n \right)^\alpha \quad (\text{A.36})$$

$$y_i = z_i \mu_i^{\alpha\eta} d_i^{\alpha\eta} m r p l_i^{\alpha\eta} \left(\frac{1}{w} \right)^{\alpha\eta} n^\alpha \quad (\text{A.37})$$

$$y_i = z_i \mu_i^{\alpha\eta} d_i^{\alpha\eta} \left[\left(\frac{z_i}{z} \right)^{\frac{1}{1+\eta(1-\alpha)}} \mu_i^{-\frac{\eta(\alpha-1)}{1+\eta(1-\alpha)}} d_i^{-\frac{\eta(\alpha-1)}{1+\eta(1-\alpha)}} (\alpha z n^{\alpha-1})^{\frac{1}{1+\eta(1-\alpha)}} w^{\frac{\eta(1-\alpha)}{1+\eta(1-\alpha)}} \right]^{\alpha\eta} \left(\frac{1}{w} \right)^{\alpha\eta} n^\alpha \quad (\text{A.38})$$

$$y_i = z \left[\left(\frac{z_i}{z} \right)^{1+\frac{\alpha\eta}{1+\eta(1-\alpha)}} \mu_i^{\alpha\eta(1-\frac{\eta(\alpha-1)}{1+\eta(1-\alpha)})} d_i^{\alpha\eta(1-\frac{\eta(\alpha-1)}{1+\eta(1-\alpha)})} \left((\alpha z n^{\alpha-1}) w^{\eta(1-\alpha)} \right)^{\frac{\alpha\eta}{1+\eta(1-\alpha)}} \left(\frac{1}{w} \right)^{\alpha\eta} \right] n^\alpha \quad (\text{A.39})$$

$$y_i = z \left[\left(\frac{z_i}{z} \right)^{\frac{1+\eta}{1+\eta(1-\alpha)}} \mu_i^{\frac{\alpha\eta}{1+\eta(1-\alpha)}} d_i^{\frac{\alpha\eta}{1+\eta(1-\alpha)}} \left((\alpha z n^{\alpha-1}) w^{\eta(1-\alpha)} \right)^{\frac{\alpha\eta}{1+\eta(1-\alpha)}} \left(\frac{1}{w} \right)^{\alpha\eta} \right] n^\alpha \quad (\text{A.40})$$

$$y_i = z \left(\frac{z_i}{z} \right)^{\frac{1+\eta}{1+\eta(1-\alpha)}} \mu_i^{\frac{\alpha\eta}{1+\eta(1-\alpha)}} d_i^{\frac{\alpha\eta}{1+\eta(1-\alpha)}} \left(\left(\frac{w}{\mu d} \right) w^{\eta(1-\alpha)} \right)^{\frac{\alpha\eta}{1+\eta(1-\alpha)}} \left(\frac{1}{w} \right)^{\alpha\eta} n^\alpha \quad (\text{A.41})$$

$$y_i = z \left[\left(\frac{z_i}{z} \right)^{\frac{1+\eta}{1+\eta(1-\alpha)}} \left(\frac{\mu_i}{\mu} \right)^{\frac{\alpha\eta}{1+\eta(1-\alpha)}} \left(\frac{d_i}{d} \right)^{\frac{\alpha\eta}{1+\eta(1-\alpha)}} \right] n^\alpha \quad (\text{A.42})$$

$$y = \int y_i d_i \quad (\text{A.43})$$

$$y = \left[\int \left(\frac{z_i}{z} \right)^{\frac{1+\eta}{1+\eta(1-\alpha)}} \left(\frac{\mu_i}{\mu} \right)^{\frac{\alpha\eta}{1+\eta(1-\alpha)}} \left(\frac{d_i}{d} \right)^{\frac{\alpha\eta}{1+\eta(1-\alpha)}} di \right] z n^\alpha \quad (\text{A.44})$$

$$y = \left[\int \left(\frac{z_i}{z} \right)^{\frac{1+\eta}{1+\eta(1-\alpha)}} \left(\frac{\mu_i}{\mu} \right)^{\frac{\alpha\eta}{1+\eta(1-\alpha)}} \right] \frac{\int \left(\frac{z_i}{z} \right)^{\frac{1+\eta}{1+\eta(1-\alpha)}} \left(\frac{\mu_i}{\mu} \right)^{\frac{\alpha\eta}{1+\eta(1-\alpha)}} \left(\frac{d_i}{d} \right)^{\frac{\alpha\eta}{1+\eta(1-\alpha)}} di}{\int \left(\frac{z_i}{z} \right)^{\frac{1+\eta}{1+\eta(1-\alpha)}} \left(\frac{\mu_i}{\mu} \right)^{\frac{\alpha\eta}{1+\eta(1-\alpha)}} di} z n^\alpha \quad (\text{A.45})$$

$$y = \omega \phi z n^\alpha \quad (\text{A.46})$$

Claim. In the presence of labor market wedges, the aggregates $\{Y, W, N\}$ can be written as

$$Y = \Omega \Phi Z N^\alpha \quad (\text{A.47})$$

$$W = M D Z \alpha N^{\alpha-1} \quad (\text{A.48})$$

Claim. In the presence of labor market wedges, the market shares can be written as

$$s_{ij} = \frac{\left[\mu(s_{ij}) z_{ij} d_{ij} \right]^{\frac{\eta+1}{1+\eta(1-\alpha)}}}{\sum_{i \in j} \left[\mu(s_{ij}) z_{ij} d_{ij} \right]^{\frac{\eta+1}{1+\eta(1-\alpha)}}} \quad (\text{A.49})$$