Process Capability (Cp / Cpk / Pp / Ppk)
Global Training Material

Creator : Global Mechanics Process Manager
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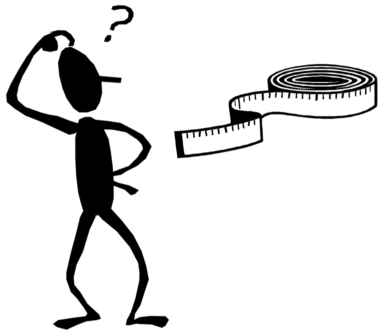
1.0	**21st Dec'01**	**Jim Christy & Søren Lundsfryd**	**Approved for Global Use**

NOTE – All comments and improvements should be addressed to the creator of this document.

Contents



Process Capability
- Evaluating Manufacturing Variation

Acknowledgements

- Benny Matthiassen (NMP CMT, Copenhagen, Denmark)
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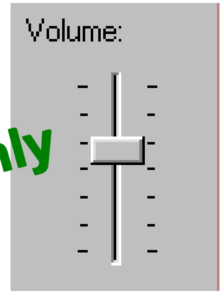
- Joni Laakso (NMP R&D, Salo, Finland)
- Jim Christy (NMP SRC, Southwood, UK)

Section 1

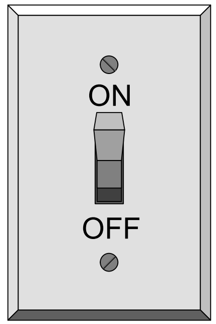
Variation, Tolerances and Dimensional Control

Two Types of Product Characteristics



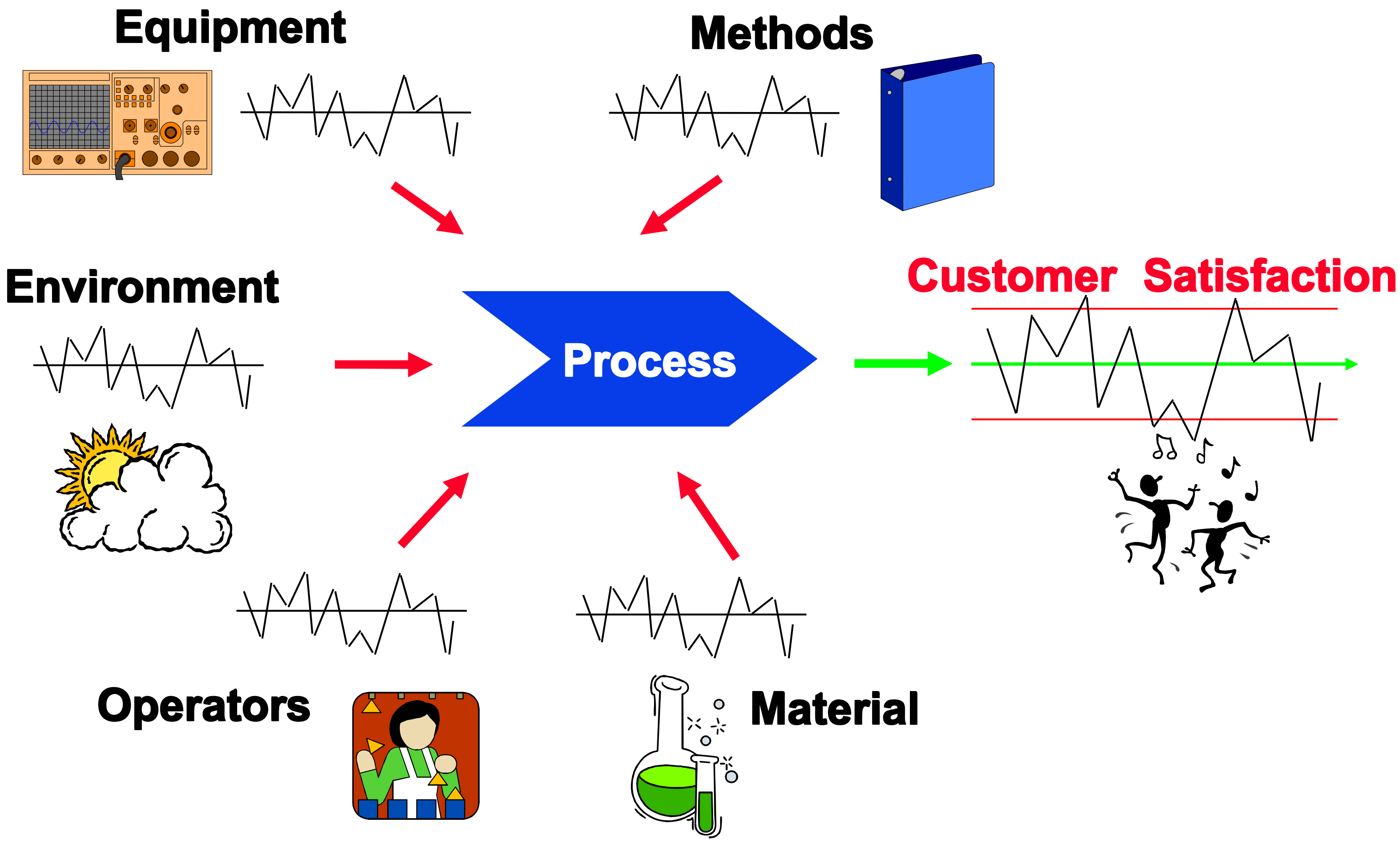
Variable: A characteristic measured in physical units, e.g. millimetres, volts, amps, decibel and seconds.

In this training we deal with variables only



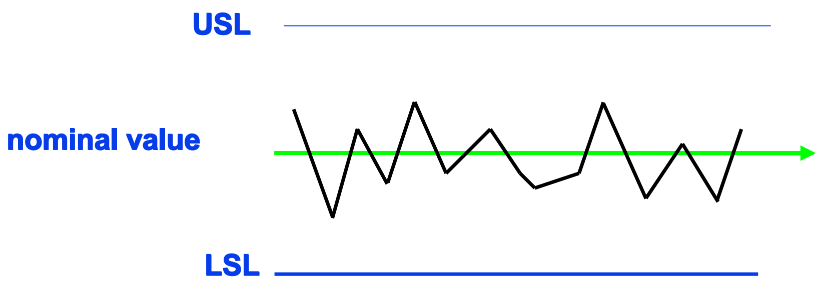
Attribute: A characteristic that by comparison to some standard is judged "good" or "bad", e.g. free from scratches (visual quality).

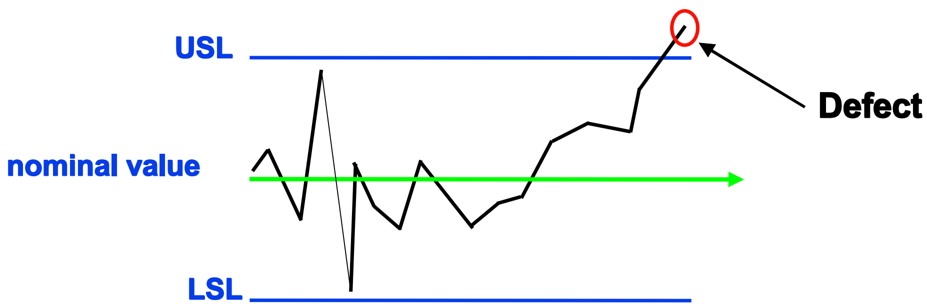
The Sources of Process/System Variation



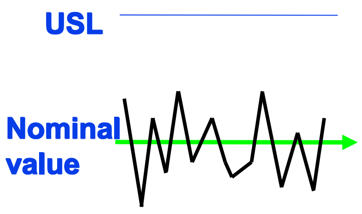
Two Types of Processes

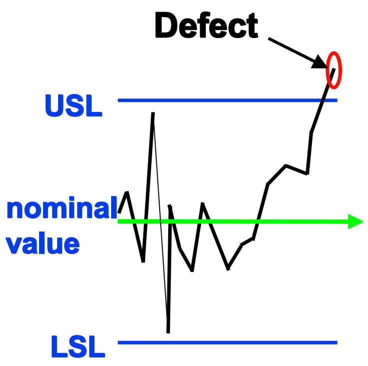
- All processes have:
  - Natural (random) variability => due to common causes
  - Unnatural variability => due to special causes
- Stable Process: A process in which variation in outcomes arises only from <u>common causes</u>
- Unstable Process: A process in which variation is a result of both <u>common and special causes</u>





The Two Causes of Variation





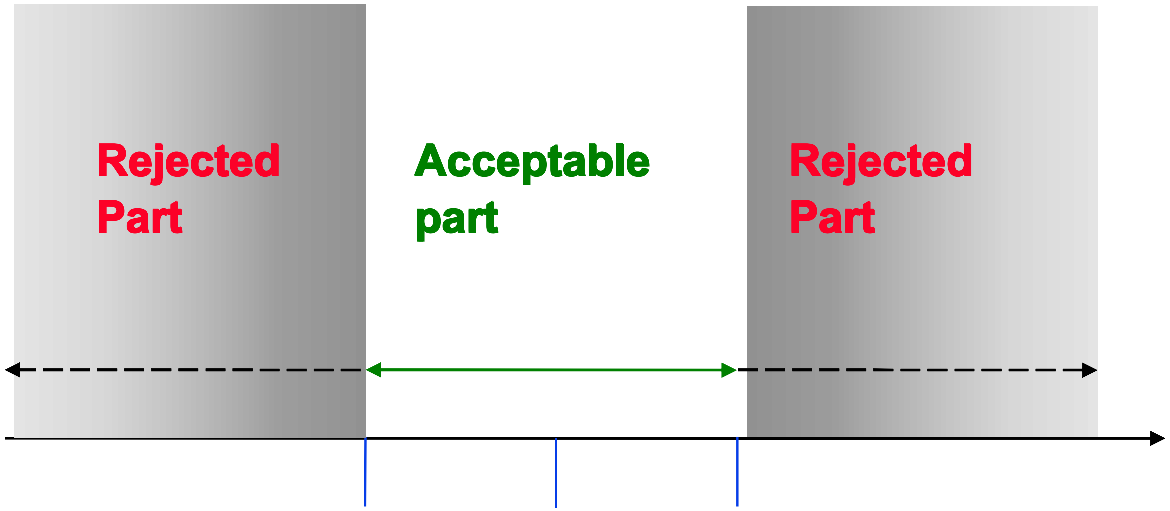
- Common Causes:
  - Causes that are implemented in the process due to the design of the process, and affect all outcomes of the process
  - Identifying these types of causes requires methods such as Design of Experiment (DOE), etc.

- Special Causes:
  - Causes that are not present in the process all the time and do not affect all outcomes, but arise because of specific circumstances
  - Special causes can be identified using Statistical Process Control (SPC)

Shewhart (1931)

Tolerances

A tolerance is a allowed maximum variation of a dimension.



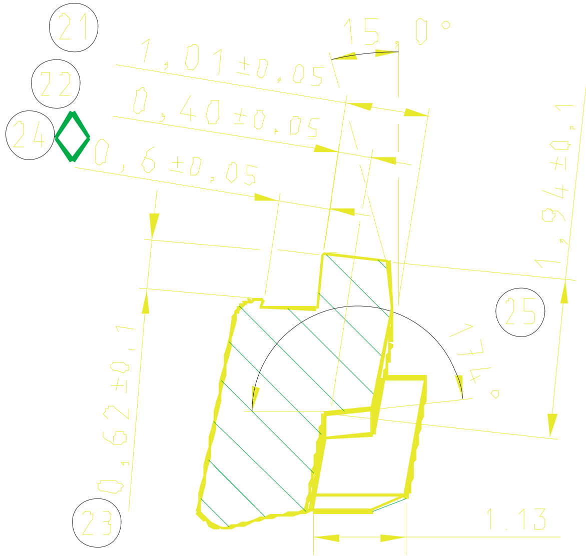
LSL (lower specification limit) 10,7

Nominal 10,8±0,1

USL (upper specification limit) 10,9

Measurement Report

In most cases we measure only one part per cavity for measurement report



	A	B	C	D	E	F	G	H	I	
36										
37								Dim out of Limit		
38								14	100,00%	
40	Dim. Number	Grid Reference	Measured Part	Nom. Dim.	Tolerance (FMC)	USL	LSL	Diff.	Result	Measured
41	21	A7		1,01	f	0,05	0,05	-1,01	NOT OK	
42	22	A7		0,40	f	0,05	0,05	-0,40	NOT OK	
43	23	B7		0,60	m	0,10	0,10	-0,60	NOT OK	
44	240	A7		0,60	f	0,05	0,05	-0,60	NOT OK	
45	240	A7		0,60	f	0,05	0,05	-0,60	NOT OK	
46	240	A7		0,60	f	0,05	0,05	-0,60	NOT OK	
47	240	A7		0,60	f	0,05	0,05	-0,60	NOT OK	
48	240	A7		0,60	f	0,05	0,05	-0,60	NOT OK	
49	240	A7		0,60	f	0,05	0,05	-0,60	NOT OK	
50	240	A7		0,60	f	0,05	0,05	-0,60	NOT OK	
51	240	A7		0,60	f	0,05	0,05	-0,60	NOT OK	
52	240	A7		0,60	f	0,05	0,05	-0,60	NOT OK	
53	240	A7		0,60	f	0,05	0,05	-0,60	NOT OK	
54	25	A8		1,94	m	0,10	0,10	-1,94	NOT OK	
55										

Example of Capability Analysis Data

- For some critical dimensions we need to measure more than 1 part
- For capability data we usually measure 5 pcs 2 times/hour=100 pcs (but sampling plan needs to be made on the basis of production quantity, run duration and cycle time)

1st Subgroup	2nd Subgroup	3rd Subgroup	4th Subgroup
118.53	118.52	118.54	118.56
118.54	118.54	118.52	118.55
118.51	118.51	118.50	118.55
118.53	118.51	118.52	118.55
118.51	118.54	118.54	118.55
5th Subgroup	**6th Subgroup**	**7th Subgroup**	**8th Subgroup**
118.55	118.54	118.57	118.60
118.54	118.56	118.56	118.57
118.55	118.55	118.57	118.55
118.54	118.54	118.55	118.56
118.56	118.53	118.54	118.55
9th Subgroup	**10th Subgroup**	**11th Subgroup**	**12th Subgroup**
118.60	118.61	118.58	118.60
118.59	118.60	118.60	118.63
118.58	118.61	118.61	118.63
118.60	118.59	118.60	118.61
118.59	118.59	118.59	118.64

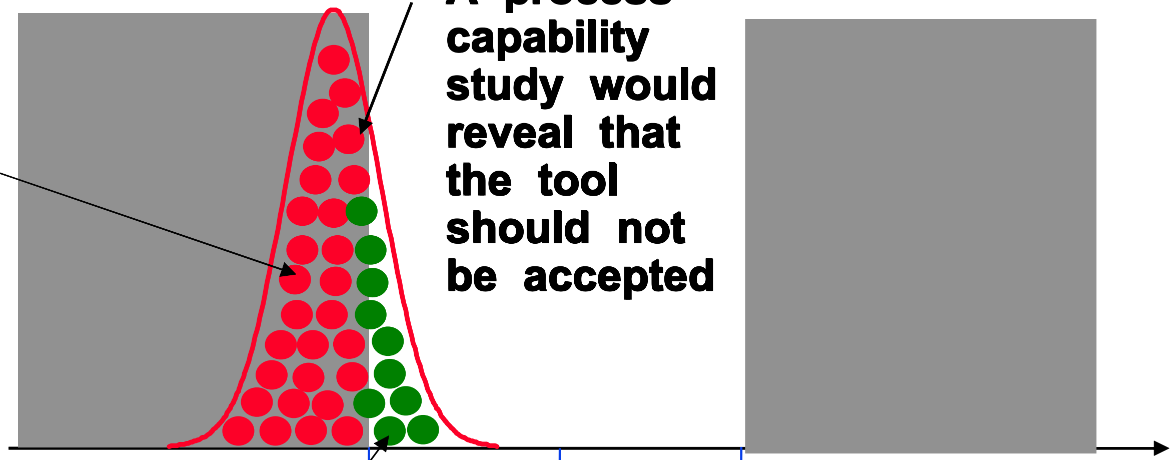
Process Capability - What is it?

- Process Capability is a measure of the **inherent** capability of a manufacturing process to be able to consistently produce components that meet the required design specifications
- Process Capability is designated by $\mathbf{C_p}$ and $\mathbf{C_{pk}}$

- Process Performance is a measure of the performance of a process to be able to ***consistently*** produce components that meet the required design specifications. Process Performance includes special causes of variation not present in Process Capability
- Process Performance is designated $\mathbf{P_p}$ and $\mathbf{P_{pk}}$

Why Make Process Capability Studies

These parts are out of spec and could be approved if only one good part was measured



A process capability study would reveal that the tool should not be accepted

LSL (lower specification limit) 10,7

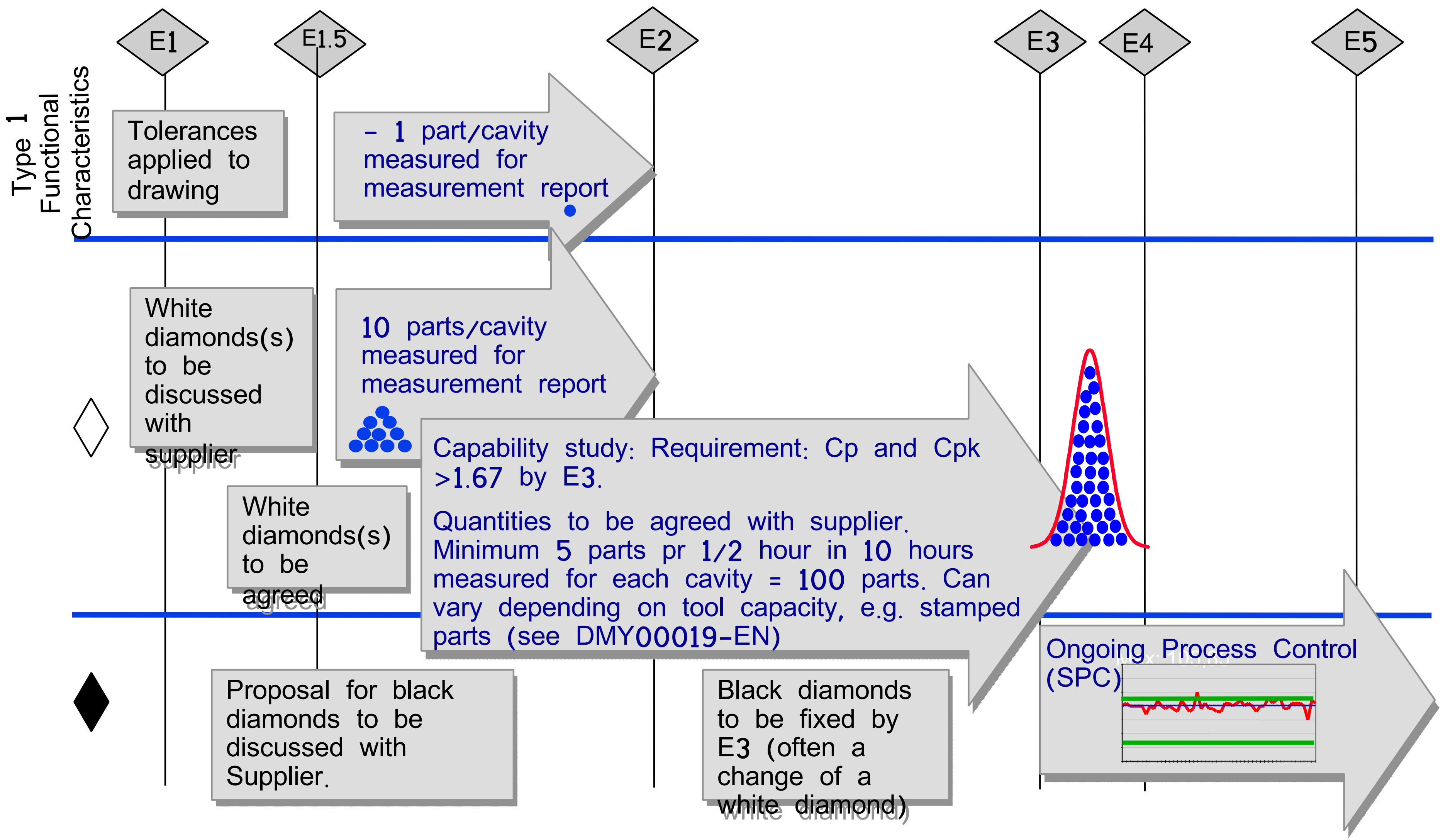
Nominal 10,8±0,1

USL (upper specification limit) 10,9

This part is within spec. The tool would be approved if only this part was measured

When a dimension needs to be kept properly within spec, we must study the process capability
.... but still this is no guarantee for the actual performance of the process as it is only an initial capability study

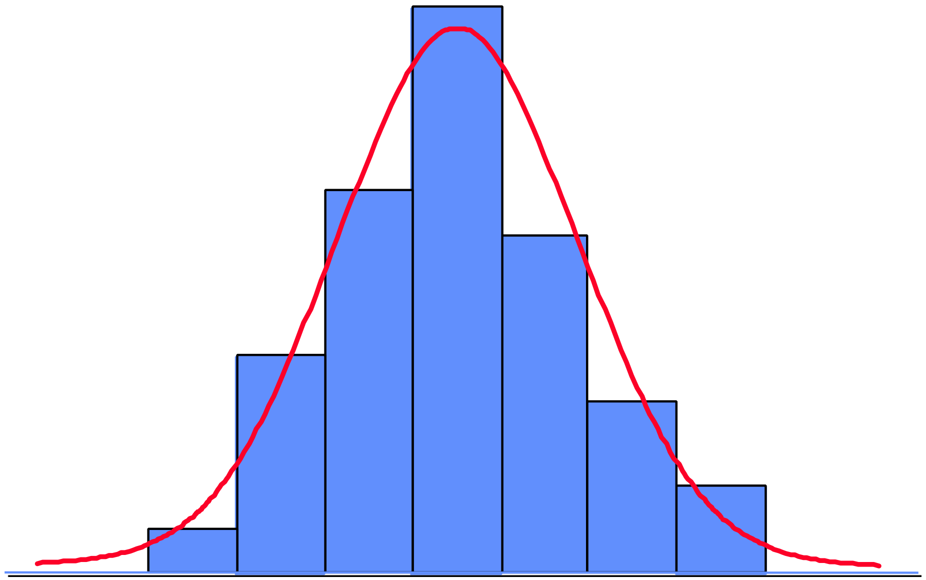
The Nokia Process Verification Process



Section 2.

Population, Sample and Normal Distribution

The Bell Shaped (Normal) Distribution

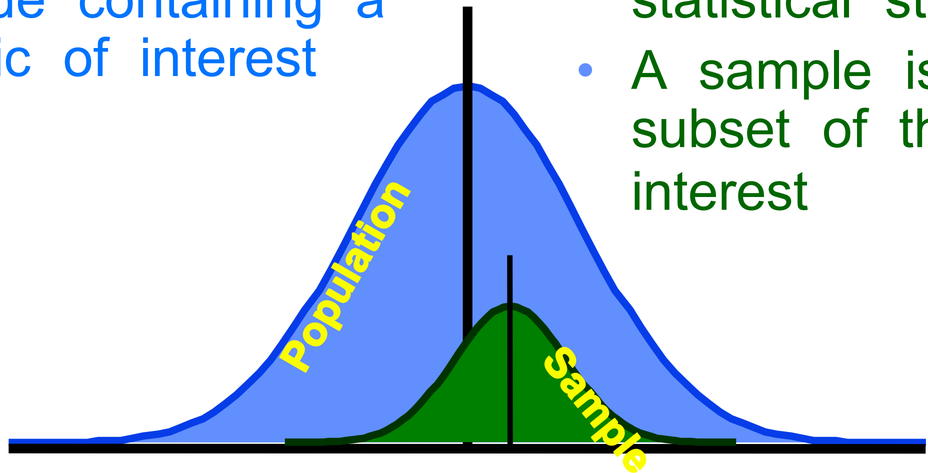


- Symmetrical shape with a peak in the middle of the range of the data.
- Indicates that the input variables (X's) to the process are randomly influenced.

Population versus Sample

Population
- An entire group of objects that have been made or will be made containing a characteristic of interest

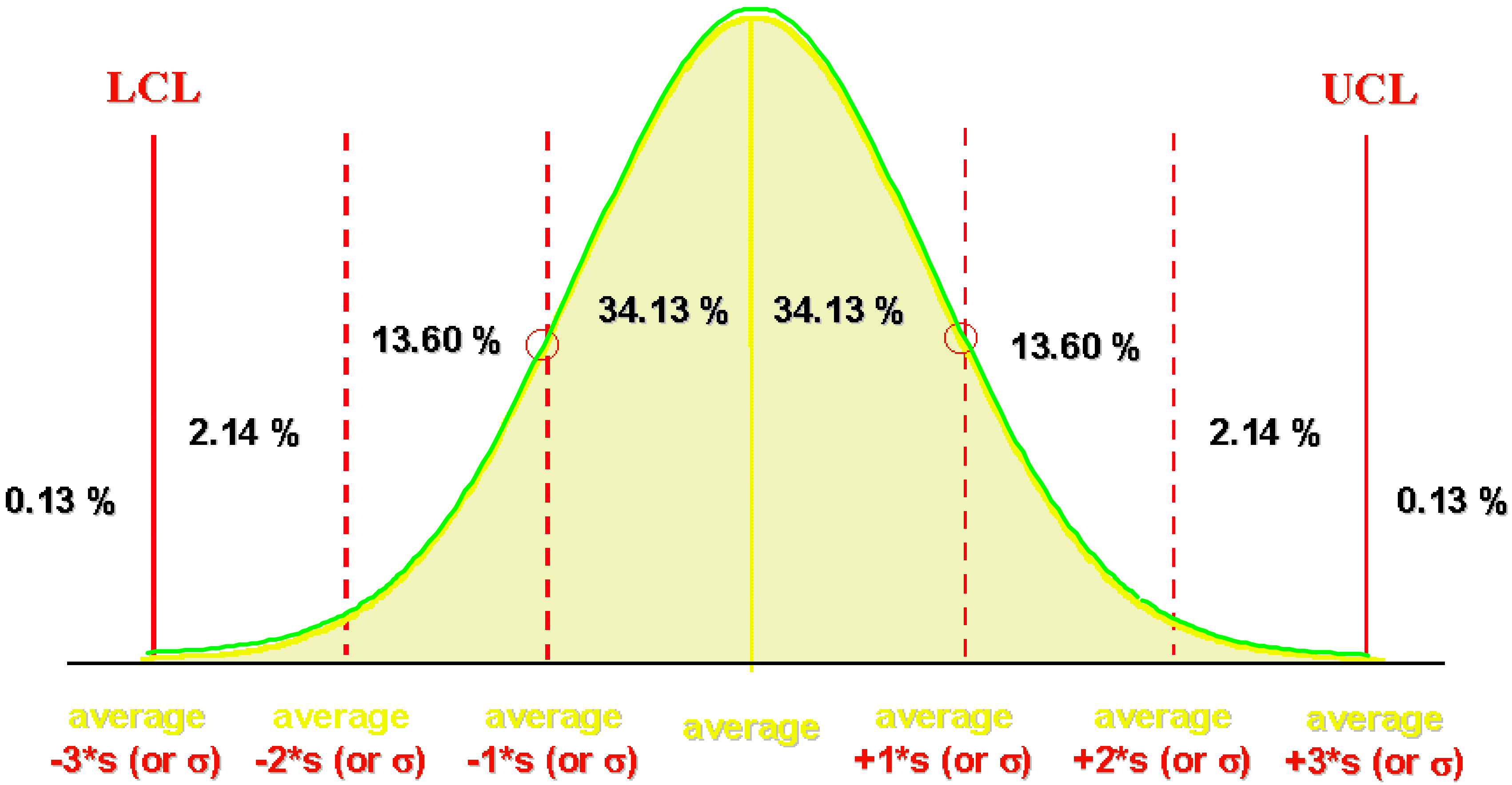
Sample
- The group of objects actually measured in a statistical study
- A sample is usually a subset of the population of interest



"Population Parameters"
μ = Population mean
σ = Population standard deviation

"Sample Statistics"
$\bar{x}$ = Sample mean
s = Sample standard deviation

The Normal Distribution



What Measurements Can Be Used to Describe a Process or System ?

- μ (mü), a measure of central tendency, is the *mean or average* of all values in the population. When only a sample of the population is being described, mean is more properly denoted as $\bar{x}$ (x–bar) :

$$\bar{x} = \frac{x_1 + x_2 + \ldots + x_N}{N}$$

- mean (average) or $\bar{x}$ describes the location of the distribution

Example: $x_1 = 5$ $x_2 = 7$ $x_3 = 4$ $x_4 = 2$ $x_5 = 6$

$$\bar{x} = \frac{5 + 7 + 4 + 2 + 6}{5} = \frac{24}{5} = 4.8$$

What Measurements Can Be Used to Describe Process variation?

- The most simple measure of variability is the *range*. The range of a sample is defined by as the difference between the largest and the smallest observation from samples in a sub-group, e.g. 5 consecutive parts from the manufacturing process.

$$R = \max(x_1, x_2, \ldots, x_N) - \min(x_1, x_2, \ldots, x_N)$$

Example: $x_1 = 5$ $x_2 = 7$ $x_3 = 4$ $x_4 = 2$ $x_5 = 6$

$$R = \max(5,7,4,2,6) - \min(5,7,4,2,6) = 7 - 2 = 5$$

What Measurements Can Be Used to Describe Process variation?

- s_{ST} - often notated as σ or sigma, is another measure of dispersion or variability and stands for "short-term standard deviation", which measures the variability of a process or system using "rational" sub-grouping.

$$s_{ST} = \left(\sum_{j=1}^{N} R_j / N \right) \Big/ d_2^* = \overline{R} / d_2^*$$

where $R_j = X_j^{\max} - X_j^{\min}$

is the *range* of subgroup *j*, *N* the number of subgroups, and d_2* depends on the number N of subgroups and the size *n* of a subgroup (see next slide)

d_2^* values for S_{ST}

N \ n →	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	1.414	1.906	2.237	2.474	2.669	2.827	2.961	3.076	3.178	3.268	3.348	3.423	3.490	3.552	3.610	3.663	3.713	3.760	3.805
2	1.276	1.806	2.149	2.405	2.603	2.767	2.905	3.024	3.129	3.221	3.304	3.380	3.449	3.513	3.571	3.626	3.677	3.725	3.770
3	1.227	1.767	2.120	2.379	2.580	2.746	2.886	3.006	3.112	3.205	3.289	3.365	3.435	3.499	3.558	3.613	3.665	3.713	3.759
4	1.206	1.749	2.105	2.365	2.569	2.736	2.876	2.997	3.104	3.197	3.281	3.358	3.428	3.492	3.552	3.607	3.659	3.707	3.753
5	1.189	1.738	2.096	2.353	2.562	2.729	2.870	2.992	3.098	3.192	3.276	3.354	3.424	3.488	3.548	3.603	3.655	3.704	3.749
6	1.179	1.731	2.090	2.353	2.557														
7	1.172	1.726	2.086	2.349	2.554														
8	1.167	1.722	2.082	2.346	2.552														
9	1.163	1.718	2.080	2.344	2.550														
10	1.159	1.716	2.078	2.342	2.548														
11	1.157	1.714	2.076	2.339	2.547														
12	1.154	1.712	2.075	2.339	2.546														
13	1.152	1.711	2.073	2.338	2.545														
14	1.151	1.709	2.072	2.337	2.544														
15	1.149	1.708	2.071	2.337	2.543														
16	1.148	1.707	2.071	2.336	2.543														
17	1.147	1.707	2.070	2.335	2.542														
18	1.145	1.706	2.069	2.335	2.542														
19	1.145	1.705	2.069	2.334	2.541														
20	1.144	1.705	2.068	2.334	2.541														
21	1.143	1.704	2.068	2.333	2.541														
22	1.143	1.704	2.068	2.333	2.540														
23	1.142	1.703	2.067	2.333	2.540														
24	1.141	1.703	2.067	2.333	2.540														
25	1.141	1.702	2.066	2.332	2.540														
∞	*1.128*	*1.693*	*2.059*	*2.326*	*2.534*	*2.704*	*2.847*	*2.970*	*3.078*	*3.173*	*3.258*	*3.336*	*3.407*	*3.472*	*3.532*	*3.588*	*3.64*	*3.689*	*3.735*

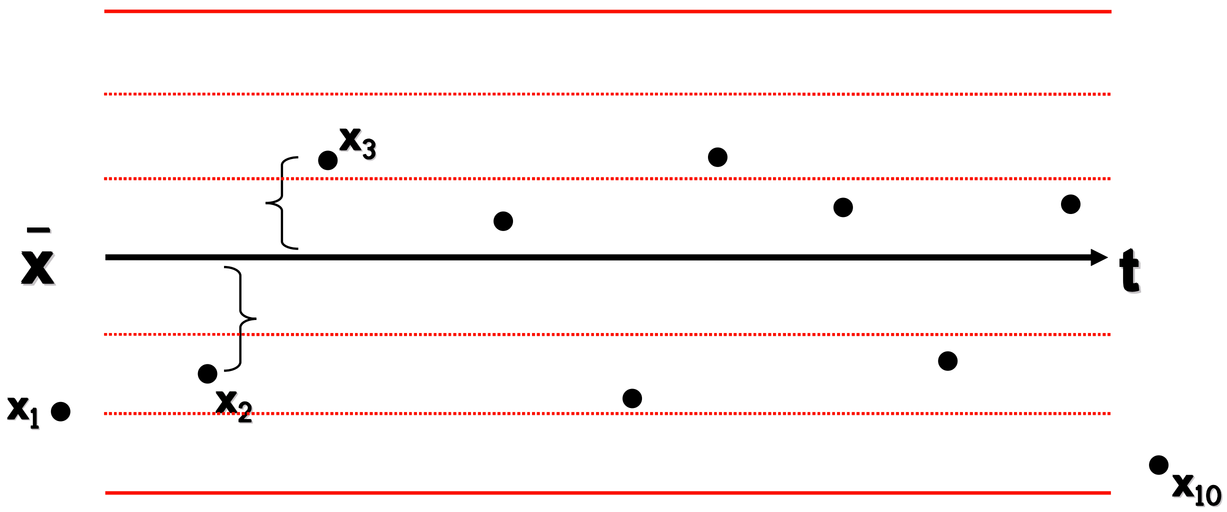
d_2^* (rows 1–25)

d_2 (row ∞)

Typical: N=20 & n=5

Where: N = no. of sub-groups, n = no. of samples in each sub-g

What Measurements Can Be Used to Describe Process variation?



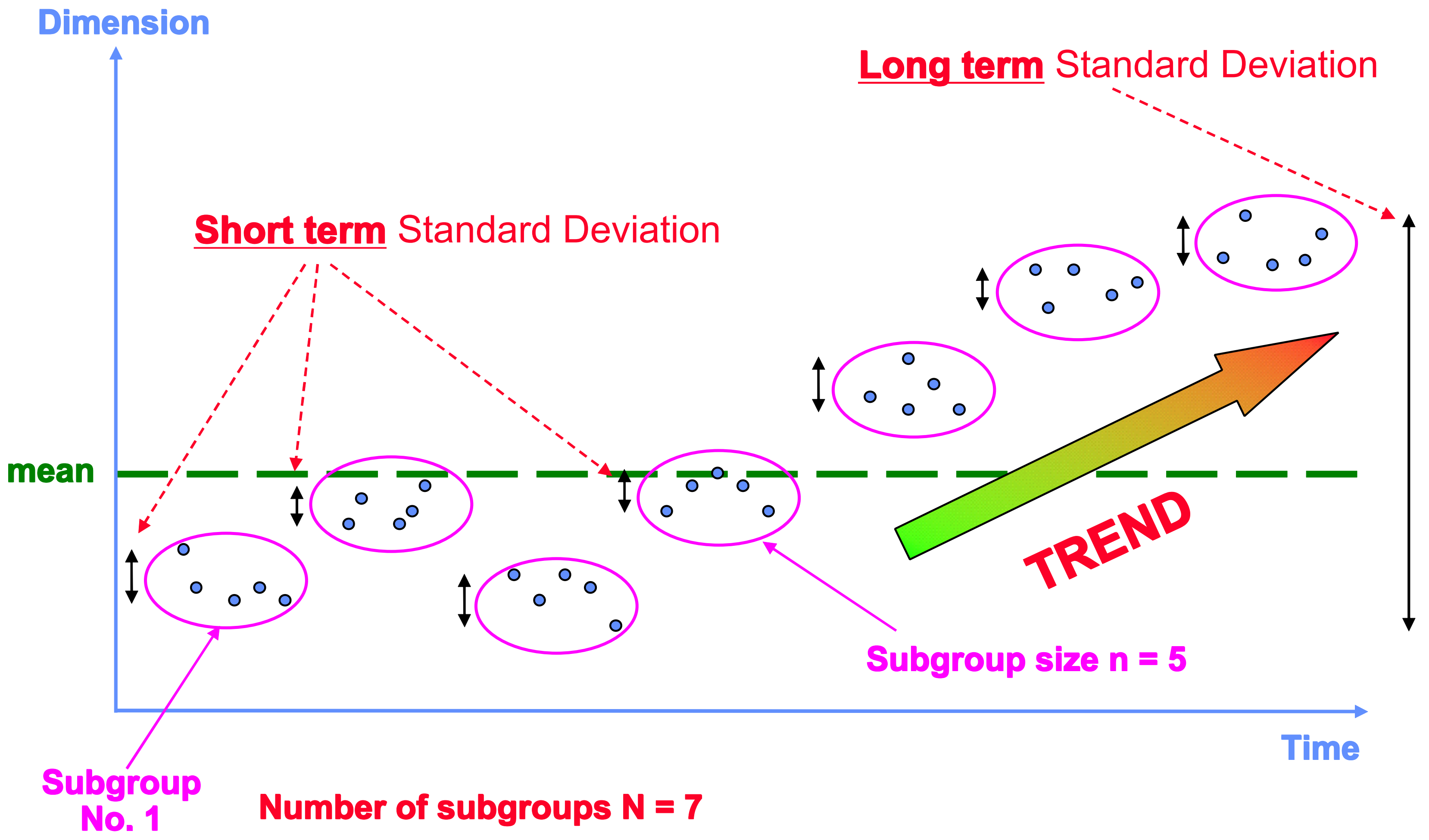
$$s_{LT} = \sqrt{s_{LT}^{\;2}} = \sqrt{\frac{(x_1 - \overline{x})^2 + (x_2 - \overline{x})^2 + \ldots + (x_N - \overline{x})^2}{N-1}}$$

Example:

$$s_{LT} = \sqrt{\frac{(5-4.8)^2 + (7-4.8)^2 + (4-4.8)^2 + (2-4.8)^2 + (6-4.8)^2}{5-1}}$$

$$= \sqrt{3.7} = 1.92$$

The Difference Between S_{ST} and s_{LT}!!



The difference between s_{ST} and s_{LT}

Long-term standard deviation :

$$s_{LT} = \sqrt{\frac{(x_1 - \overline{x})^2 + (x_2 - \overline{x})^2 + \ldots + (x_N - \overline{x})^2}{N-1}}$$

Short-term standard deviation :

$$s_{ST} = \left(\sum_{j=1}^{N} R_j / N \right) \Big/ d_2^* = \overline{R} / d_2^*$$

The difference between the standard deviations s_{LT} and s_{ST} gives an indication of how much better one can do when using appropriate production control, like **Statistical Process Control (SPC)**.

The difference between s_{ST} and s_{LT}

- The difference between s_{LT} and s_{ST} is **only** in the way that the standard deviation is calculated

- s_{LT} is always the same or larger than s_{ST}

- If s_{LT} equals s_{ST}, then the process control over the longer-term is the same as the short-term, and the process would **not** benefit from SPC

- If s_{LT} is larger than s_{ST}, then the process has lost control over the longer- term, and the process would benefit from SPC

- The reliability of s_{LT} is improved if the data is taken over a longer period of time. Alternatively s_{LT} can be calculated on several occasions separated by time and the results compared to see whether s_{LT} is stable

Exercise 1: Sample Distributions

1. **In Excel file** *"Data exercise 1.xls"* **you find 100 measurements being the result of a capability study. The specification for the dimension is 15,16±,01**

2. **How well does the sample population fit the specification, e.g. should we expect any parts outside spec?**

3. **Mention possible consequences of having a part outside spec .**

4. **Mention possible causes of variation for parts.**

5. **Calculate the sample mean and sample standard deviation for the 100 measurements. Use the average and stdev functions Excel.**

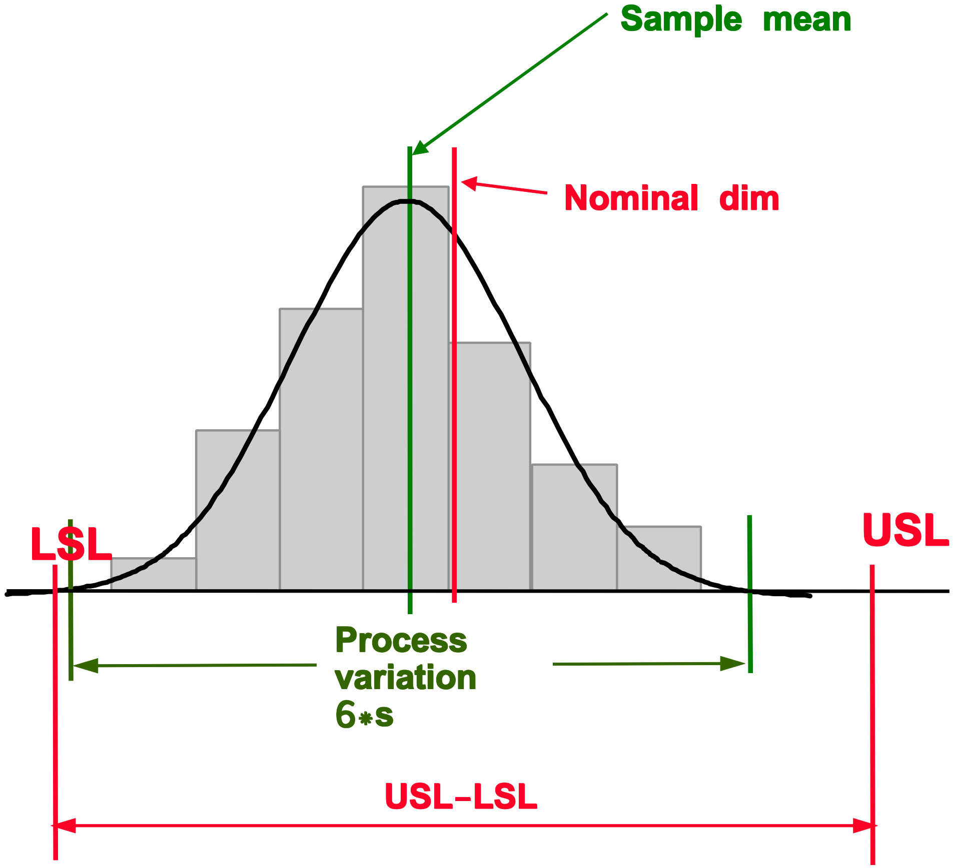
Section 3.

Cp and Cpk Concept

Defining Cp and Pp

$$C_p = \frac{USL-LSL}{6 * s_{st}}$$

$$P_p = \frac{USL-LSL}{6 * s_{lt}}$$

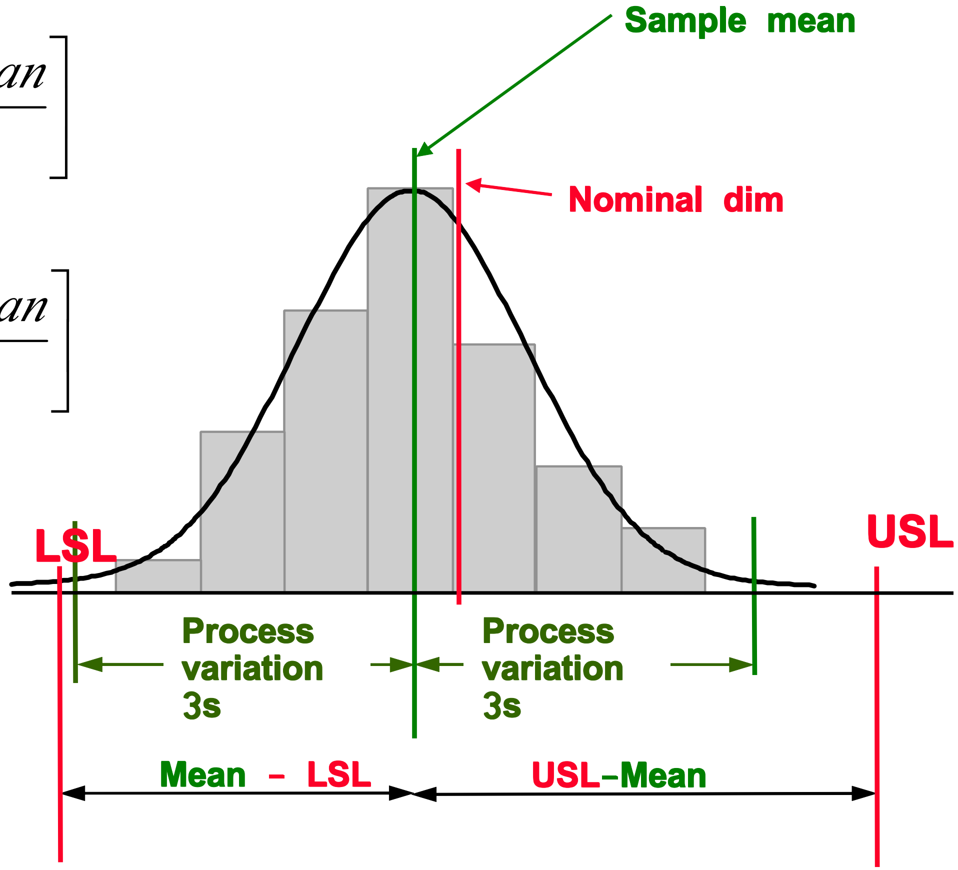


The tolerance area divided by the total process variation, irrespective of process centring.

Defining Cpk and Ppk

$$C_{pk} = \min\left[\frac{mean - LSL}{3 * s_{st}}, \quad \frac{USL - mean}{3 * s_{st}}\right]$$

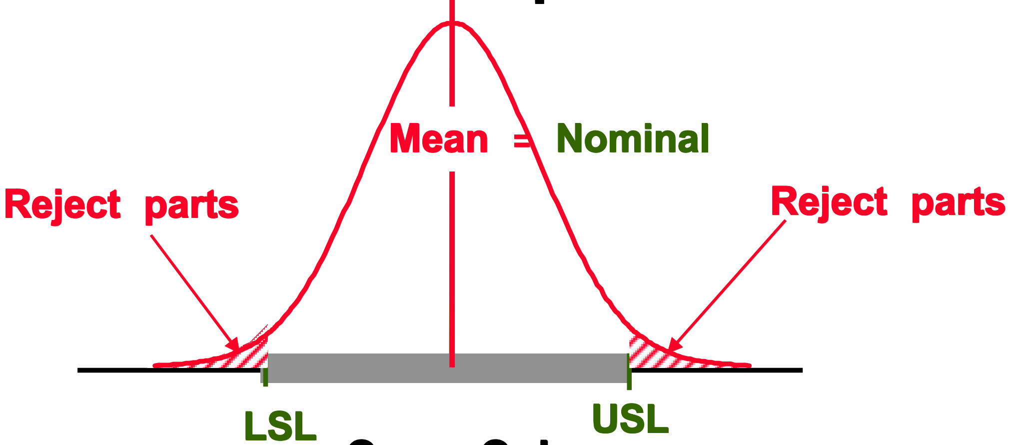
$$P_{pk} = \min\left[\frac{mean - LSL}{3 * s_{lt}}, \quad \frac{USL - mean}{3 * s_{lt}}\right]$$



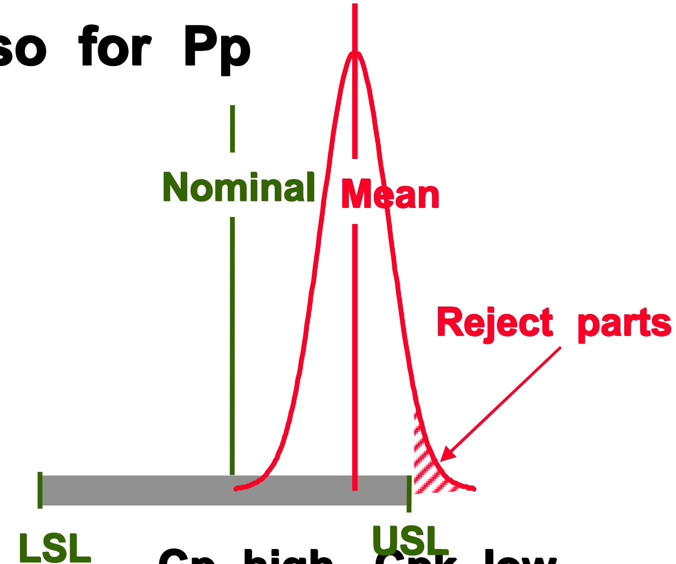
C_{pk} and P_{pk} Indexes account also for process centring.

What is the Difference Between Cp and Cpk?

- The Cp index only accounts for process variability
- The Cpk Index accounts for process variability and centering of the process mean to the design nominal
- Therefore, Cp ≥ Cpk
- NOTE: Same applies also for Pp and Ppk



Cp = Cpk
(both low)



Cp high, Cpk low
Process should be optimized!

What Do These Indexes Tell Us ??

- Simple numerical values to describe the quality of the process **>> The higher the number the better**

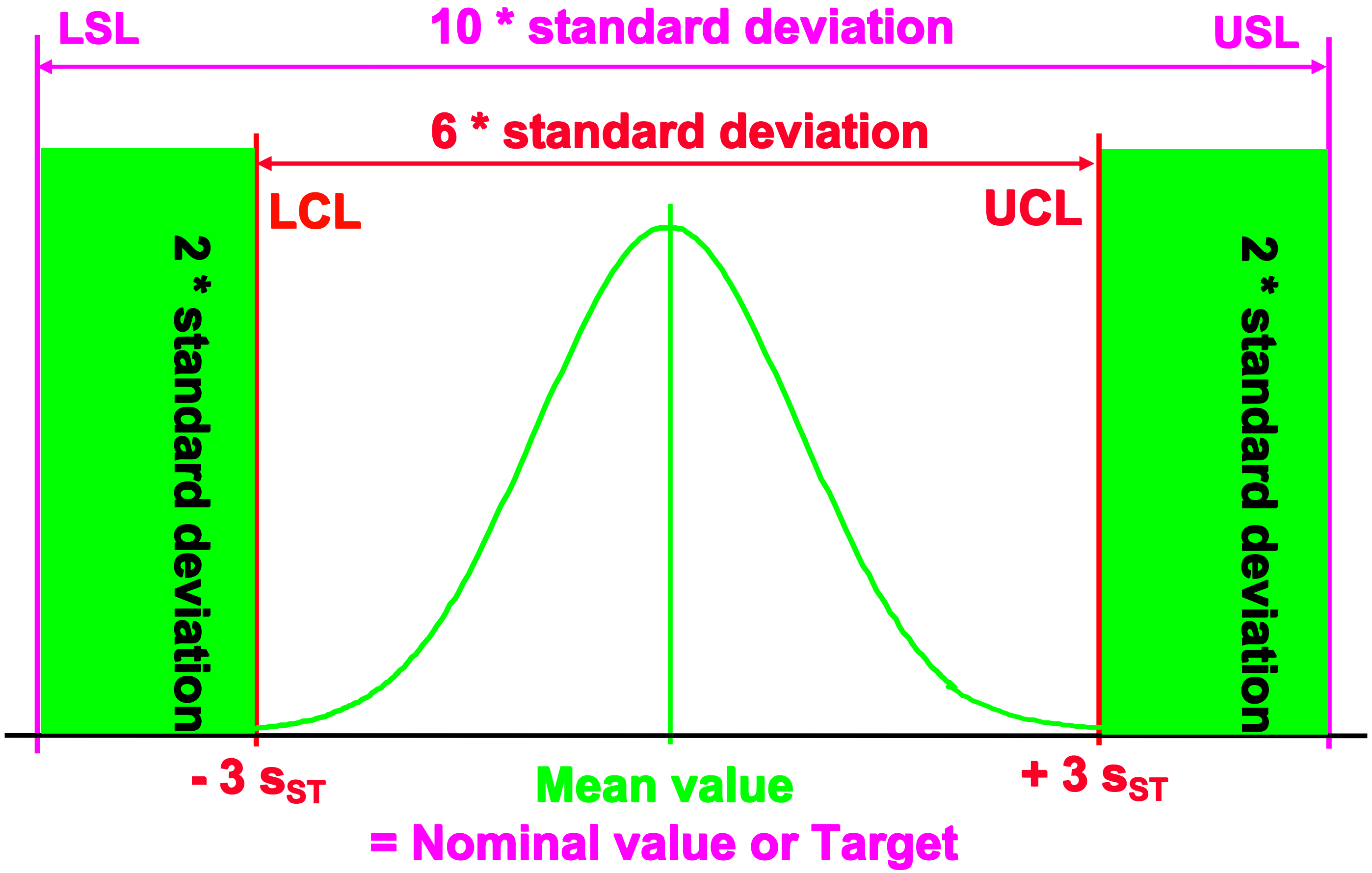
- **Requirement for C_p and C_{pk} is 1.67 min.**
- **Recommendation for P_p and P_{pk} is 1.33 min.**
  - This leaves us some space for the variation, i.e. a safety margin
  - Are we able to improve our process by using SPC?

- If index is low, following things should be given a thought:
  - Is the product design OK?
    - Are tolerance limits set correctly?
    - Too tight?
- Is the process capable of producing good quality products? Process variation? DOE required?

- Is the measuring system capable? (See Gage R&R)

C_{pk} - With a 2-sigma safety margin

- Requirement for C_p and C_{pk} is **1.67 min. 1.67 is a ratio of = 5/3 or 10/6.**



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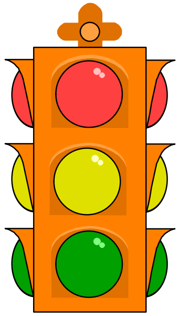
Acceptability of C_{pk} Index



- $C_{pk} < 1.67$ the process **NOT CAPABLE**

- $C_{pk} >= 1.67$ the process is **CAPABLE**

- $C_{pk} >= 2.0$ the process has reached **Six Sigma** level

What Do These Indexes Tell Us ??

- If $C_p = C_{pk}$,
- If $P_p = P_{pk}$,

… then process **perfectly centred**

- If $C_{pk} < C_p$,
- If $P_{pk} < P_p$,

… then process **not centred** (check process mean against design nominal)

- If $C_p = P_p$,
- If $C_{pk} = P_{pk}$,

… then process is not affected by special causes during the study run. SPC would **not be effective** in this case

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- If $P_p < C_p$,
- If $P_{pk} < C_{pk}$,

… then process is affected by special causes. Investigate X-bar/R-chart for out-of-control conditions. SPC **may be effective**

Cp and Cpk Indices and Defects

(both tails of the normal distribution)

Cpk /Ppk									Cp/Pp											
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.33	1.4	1.5	1.67	2.0	2.5	3.0	4.0	% / PPM
0.1	76.42	56.61	44.89	40.00	38.56	38.26	38.21	38.21	38.21	38.21	38.21	38.21	38.21	38.21	38.21	38.21	38.21	38.21	38.21	38.21	**%**
0.2		54.86	38.93	31.02	28.25	27.56	27.44	27.43	27.43	27.43	27.43	27.43	27.43	27.43	27.43	27.43	27.43	27.43	27.43	27.43	**%**
0.3			36.81	25.09	20.19	18.75	18.45	18.41	18.41	18.41	18.41	18.41	18.41	18.41	18.41	18.41	18.41	18.41	18.41	18.41	**%**
0.4				23.01	15.10	12.33	11.64	11.52	11.51	11.51	11.51	11.51	11.51	11.51	11.51	11.51	11.51	11.51	11.51	11.51	**%**
0.5					13.36	8.47	7.03	6.73	6.69	6.68	6.68	6.68	6.68	6.68	6.68	6.68	6.68	6.68	6.68	6.68	**%**
0.6						7.19	4.41	3.73	3.61	3.59	3.59	3.59	3.59	3.59	3.59	3.59	3.59	3.59	3.59	3.59	**%**
0.7							3.57	2.13	1.83	1.79	1.79	1.79	1.79	1.79	1.79	1.79	1.79	1.79	1.79	1.79	**%**
0.8								1.64	0.95	0.84	0.82	0.82	0.82	0.82	0.82	0.82	0.82	0.82	0.82	0.82	**%**
0.9									0.69	0.40	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	**%**
1.0										**2700**	1509	1363	1350	1350	1350	1350	1350	1350	1350	1350	**PPM**
1.1											967	532	485	484	483	483	483	483	483	483	**PPM**
1.2												318	165	160	159	159	159	159	159	159	**PPM**
1.33													**63**	38	33	33	32	32	32	32	**PPM**
1.4														27	14	13	13	13	13	13	**PPM**
1.5															7	3	3	3	3	3	**PPM**
1.67																**0.6**	0.3	0.3	0.3	0.3	**PPM**
2.0																	**0.0**	0.0	0.0	0.0	**PPM**
2.5																		0.0	0.0	0.0	**PPM**
3.0																			0.0	0.0	**PPM**
4.0																				0.0	**PPM**

Pp=Ppk=1,33
→ 63 ppm defects = 0,006%

Cp=Cpk=1,67
→ 0,6 ppm defects = 0,00006%

Note: Ppm reject rates calculated from Cp & Cpk are based on the short term variation which may not represent the long term reject rate

The Effects of Cpk and Cp on FFR

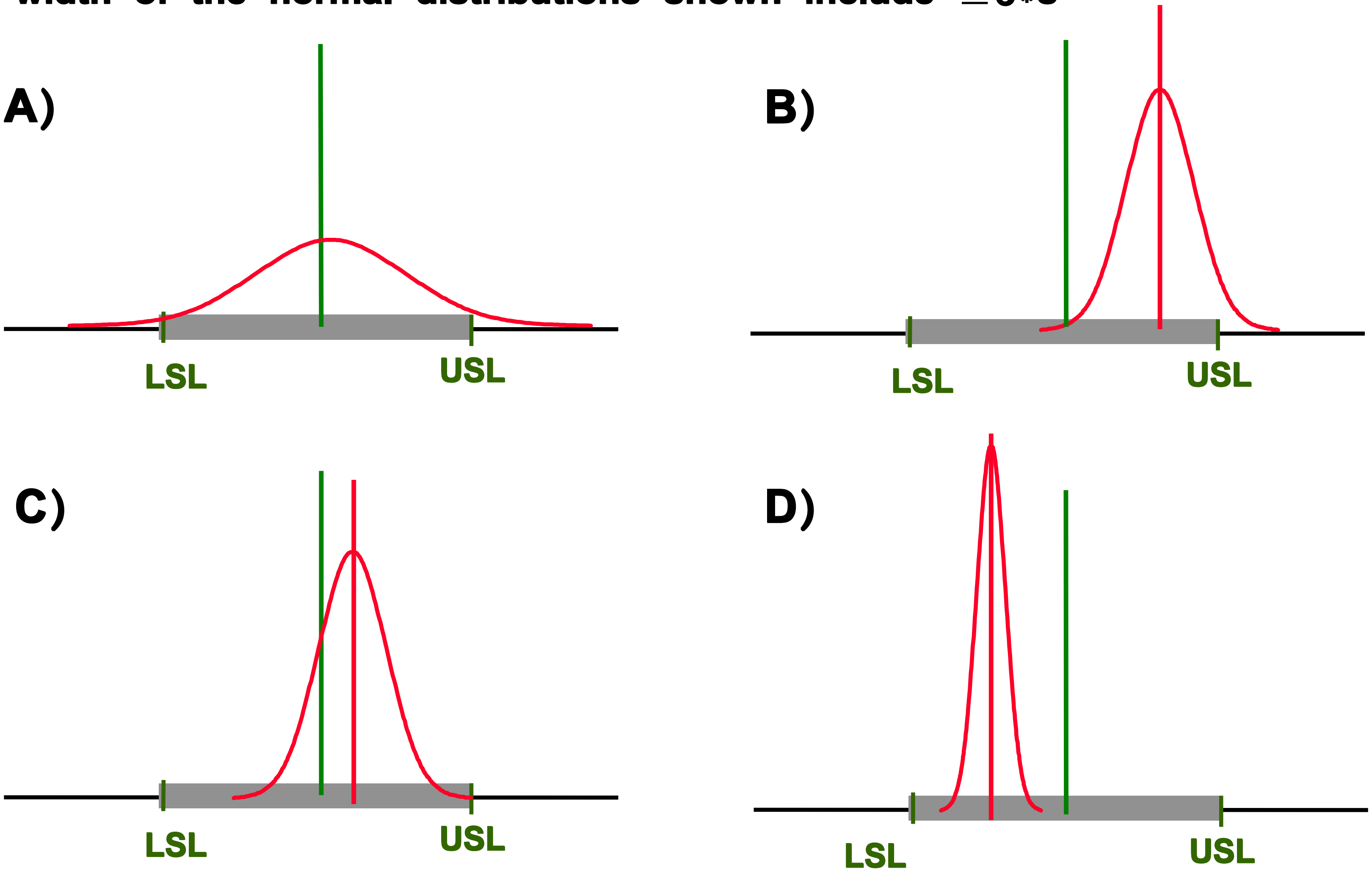
Cpk	Cp	Ppm defects	Total number of defects for 50,000,000 parts	Total number of defects if phone has 10 of these parts
0.8	1.33	8,200	410,000	4,100,000
1	1.33	1,350	67,500	675,000
1.33	1.33	63	3,150	31,500
1.33	1.67	33	1,650	16,500
1.50	1.50	7	350	3,500
1.67	1.67	1	30	300
2.00	2.00	0	0	1

Exercise 2: Cp and Cpk

- Calculate Cp and Cpk for the 100 measurements in the file "*Data exercise 1.xls*"
- Determine the approximate Cp and Cpk for the 4 sample populations on the following page
- Should actions be made to improve these processes. If yes, which?

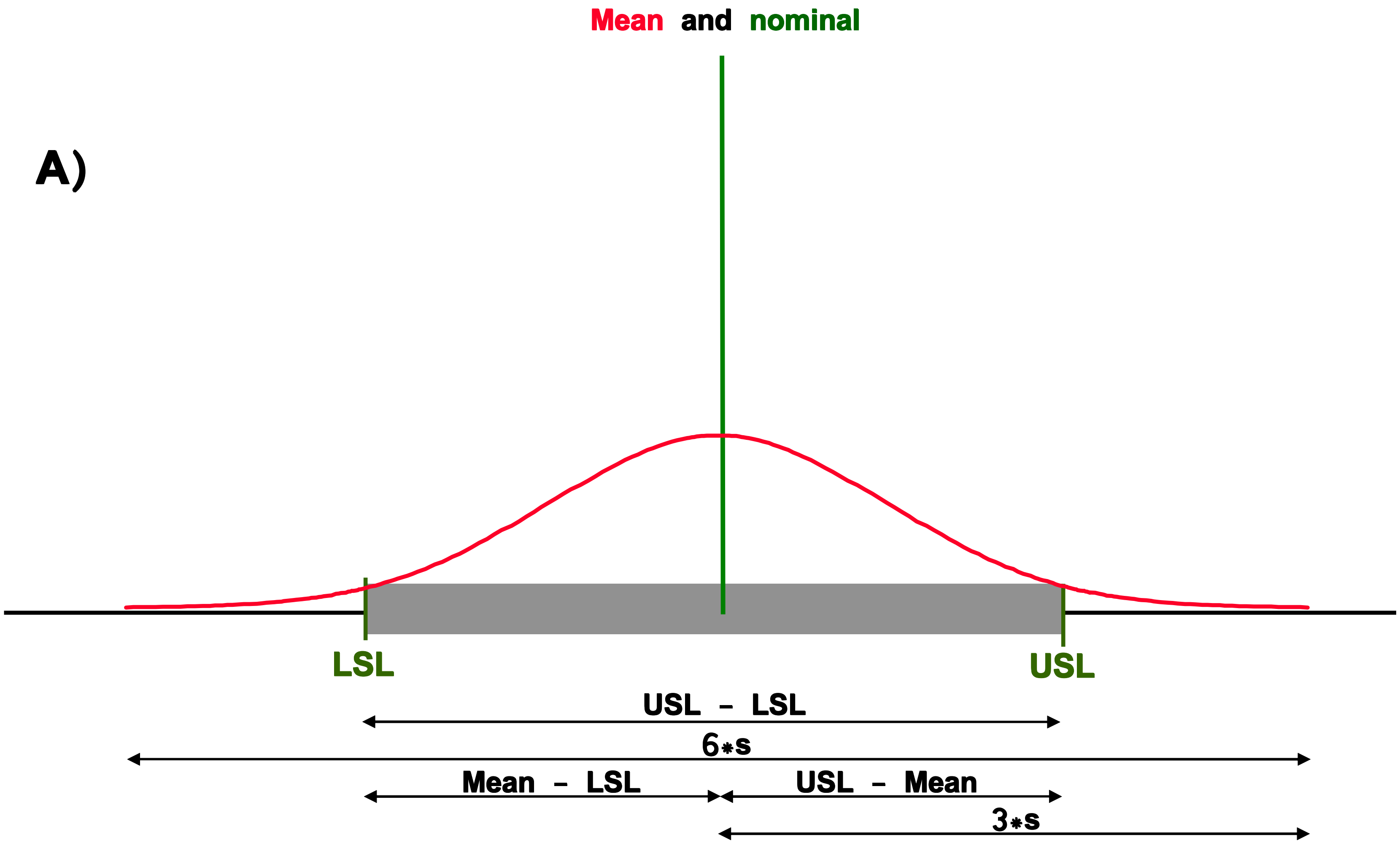
Estimate Cp and Cpk?

The width of the normal distributions shown include $\pm 3 * s$



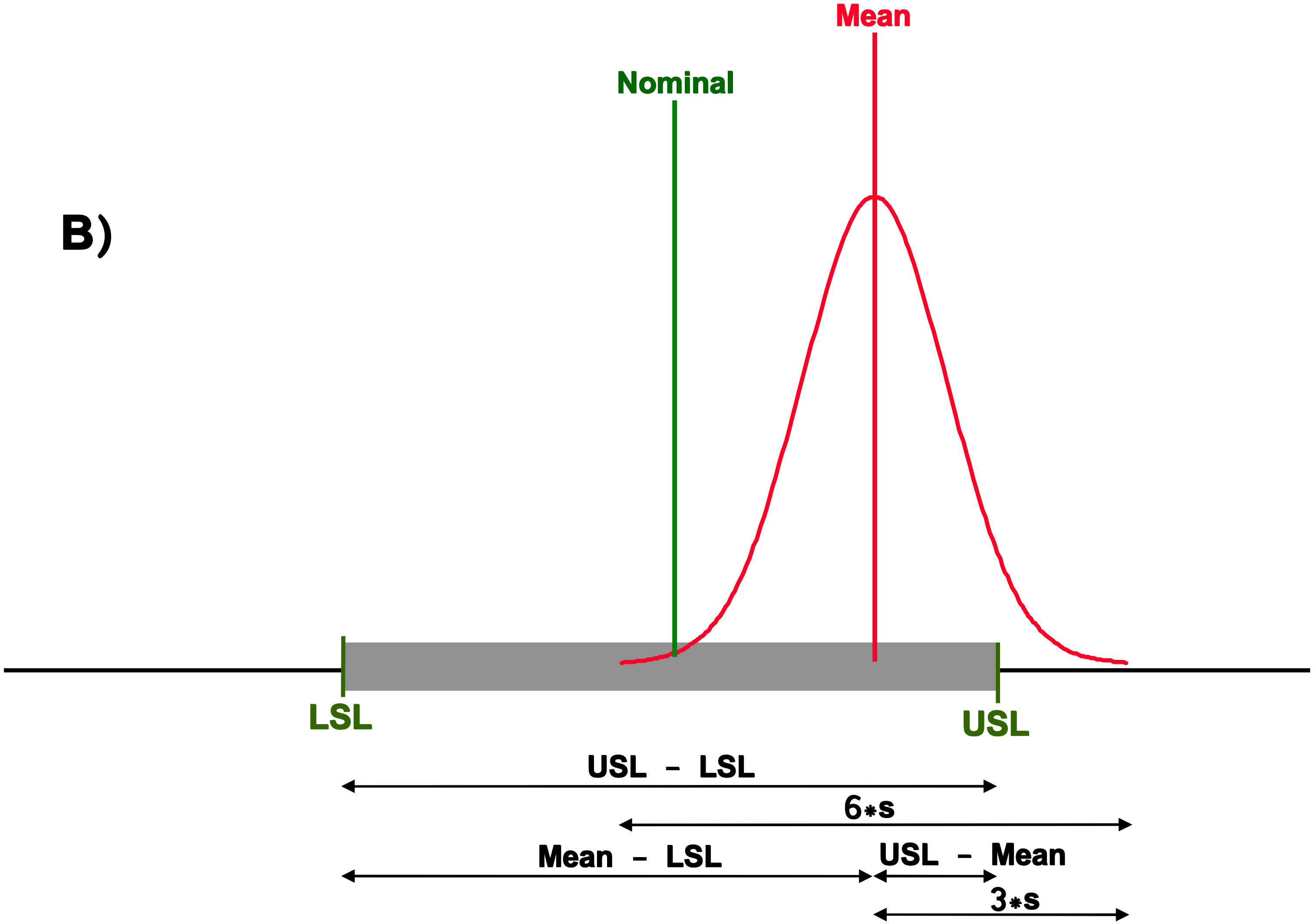
Estimate Cp and Cpk? - A)

A)

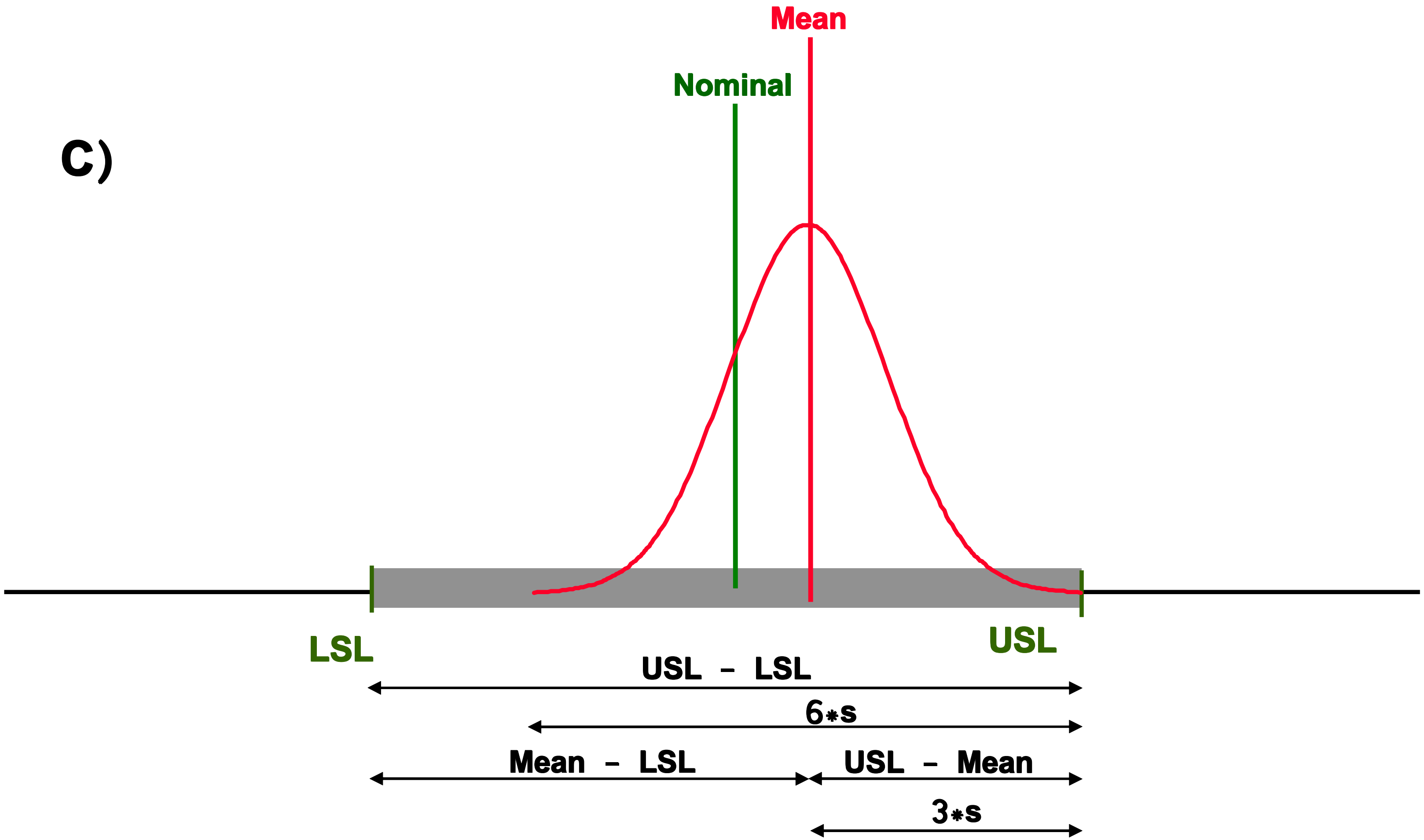


Estimate Cp and Cpk? - B)

B)

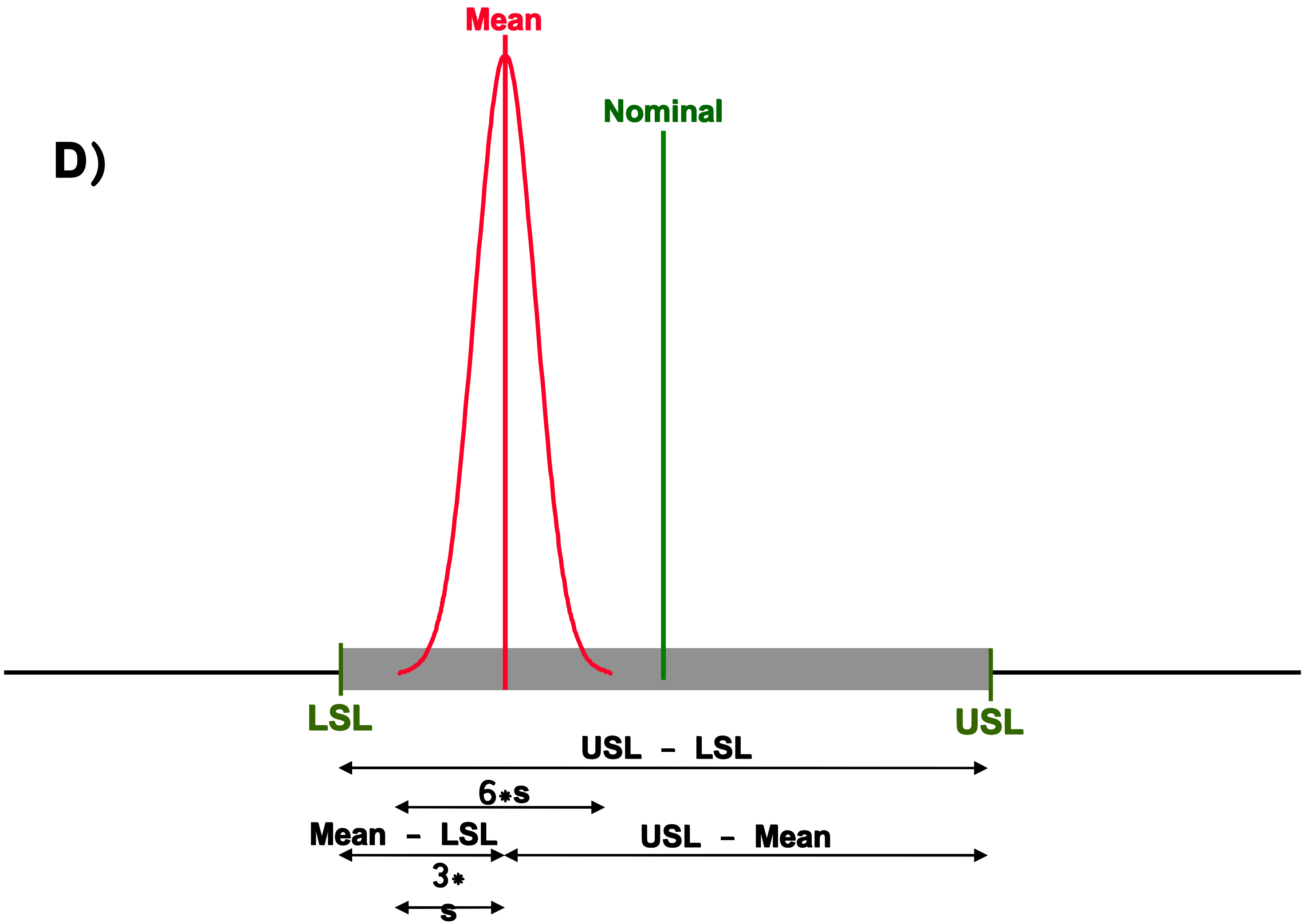


Estimate Cp and Cpk? - C)



Estimate Cp and Cpk? - D

D)



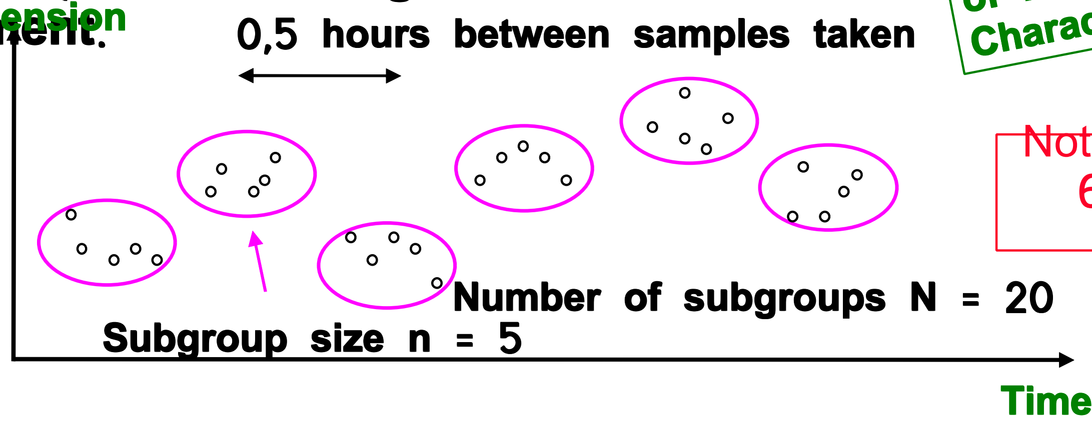
Section 4.

Use of the NMP Data Collection Spreadsheet

Example of how to Collect Data

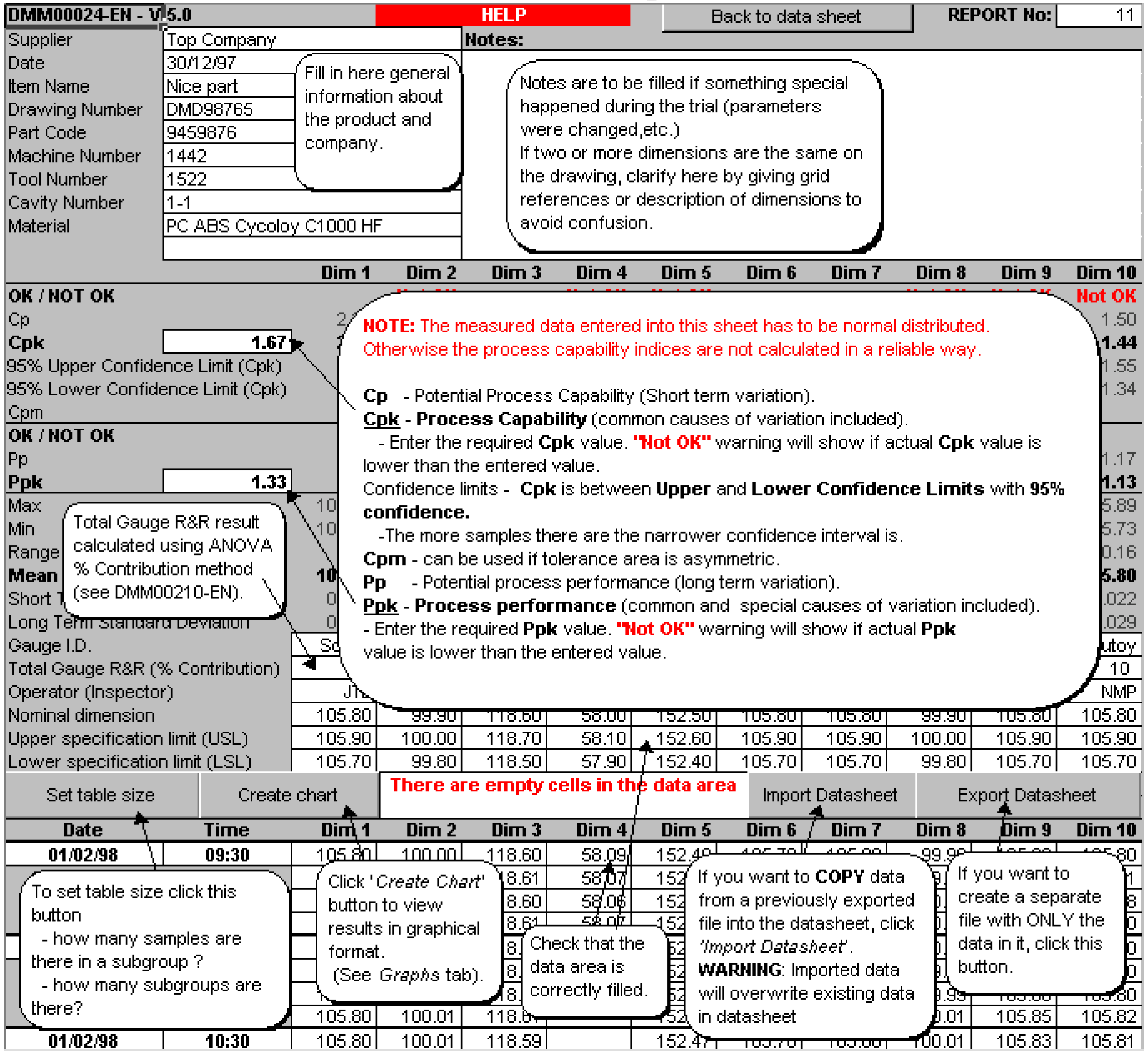
1. Run in and stabilise process
2. Note the main parameters for reference
3. When the process is stable run the tool for 10 hours
3. Take 5 parts out from each cavity every half hour and mark them with time, date and cavity. Total 20 sets of 5 parts from each cavity must be made, or according to agreement.
4. After the last sample lot note the main process parameters for reference
5. Measure and record the main functional characteristics (white diamonds)
6. Fill data into the NMP data collection spreadsheet
7. Analyse!

See DMY 00019-EN Classification and Marking of Functional Characteristics



Note: For clarity, only 6 subgroups are shown

Data Collection Sheet (DMM00024-EN-5.0)



Data Collection Sheet (DMM00024-EN-5.0)

		Dim 1	Dim 2
OK / NOT OK			**Not OK**
Cp		2.24	3.51
Cpk	**1.67**	**2.20**	**1.19**
95% Upper Confidence Limit (Cpk)		2.51	1.37
95% Lower Confidence Limit (Cpk)		1.88	1.01
Cpm			
OK / NOT OK			**Not OK**
Pp		1.93	1.62
Ppk	**1.33**	**1.90**	**0.55**
Max		105.850	100.010
Min		105.750	99.930
Range		0.100	0.080
Mean		**105.802**	**99.966**
Short Term Standard Deviation		0.015	0.009
Long Term Standard Deviation		0.017	0.021
Gauge I.D.		Scope	Scope
Total Gauge R&R (% Contribution)		15	11
Operator (Inspector)		JTL	JTL
Nominal dimension		105.80	99.90
Upper specification limit (USL)		105.90	100.00
Lower specification limit (LSL)		105.70	99.80

Set table size **Create chart**

Date	Time	Dim 1	Dim 2
13/06/01	**08:00**	105.800	100.000
Subgroup 1		105.810	100.000
		105.800	100.000
		105.800	100.010

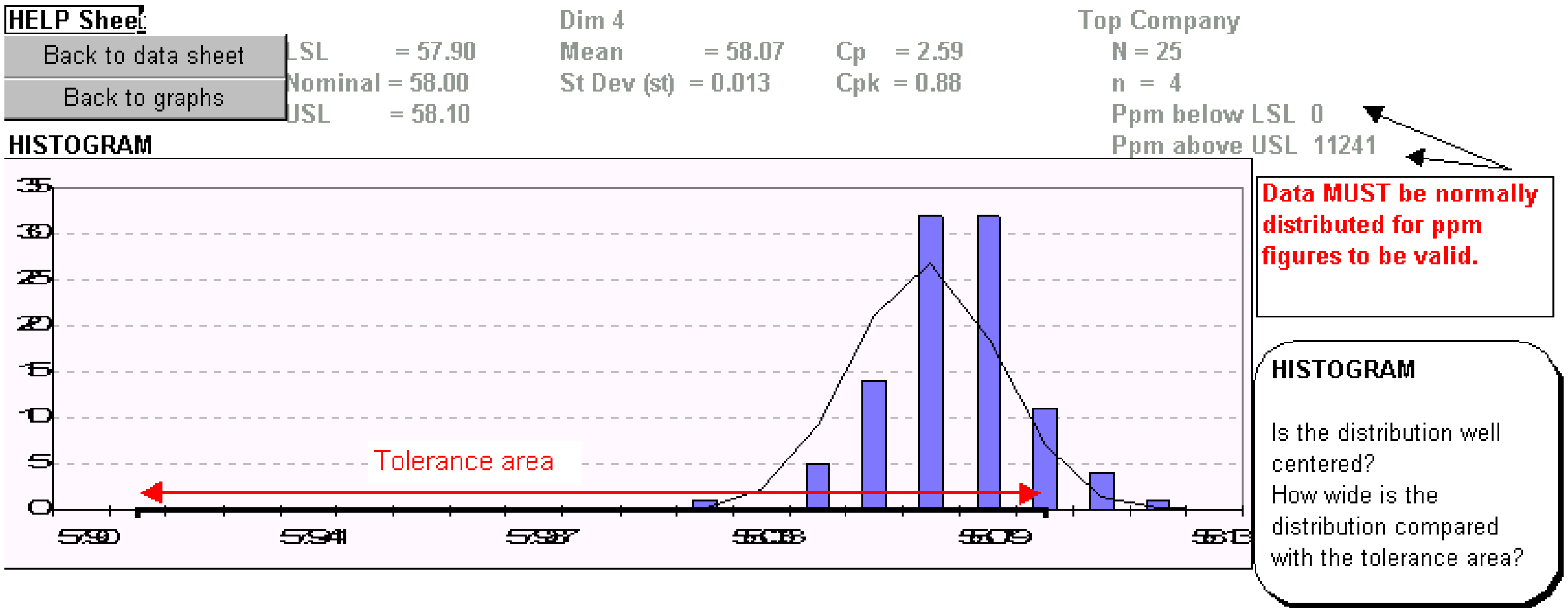
Graphical Presentation: Histogram

- What kind of distribution?

  Location versus tolerance area

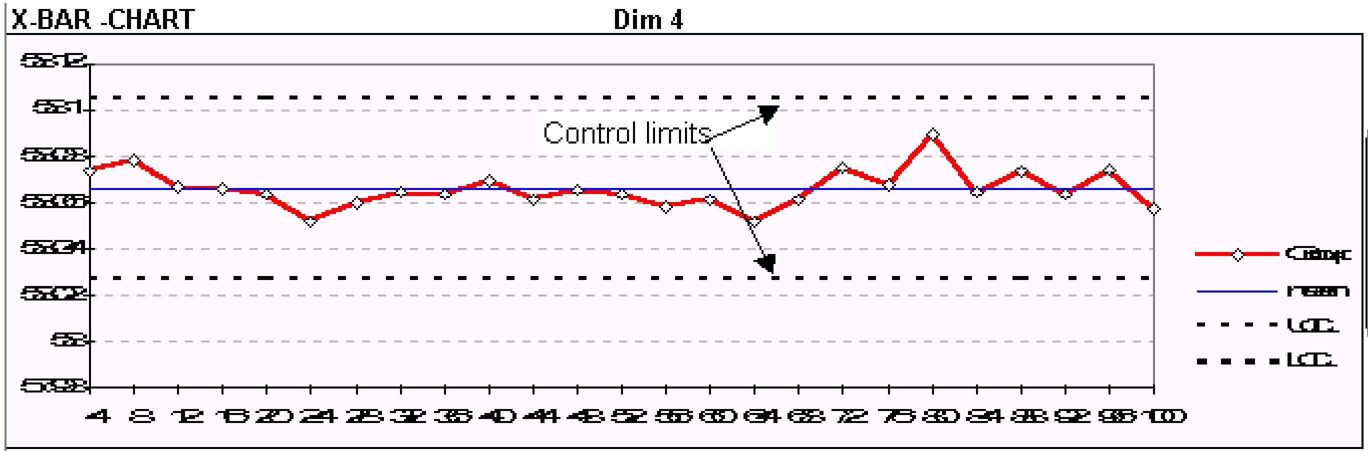
  Width (deviation)

- Example : C_p 2.59 P_p 1.86
- C_{pk} 0.88 P_{pk} 0.63



Graphical Presentation: X-bar and R-Chart

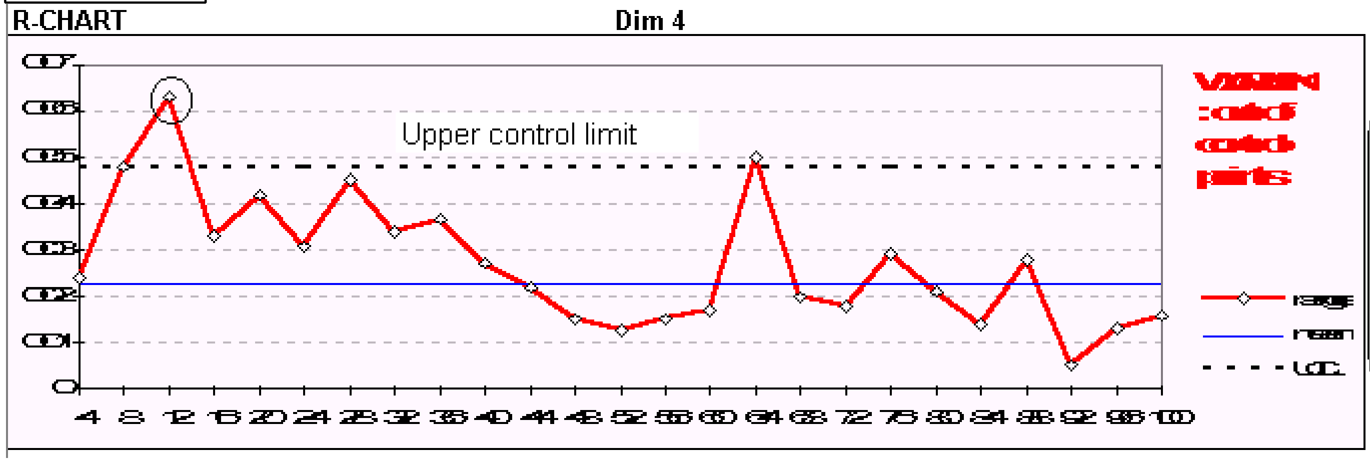
X-Bar Chart



X-BAR CHART -
Averages of the subgroups over time.

Check for trends, unexplained changes, out of control points, etc.

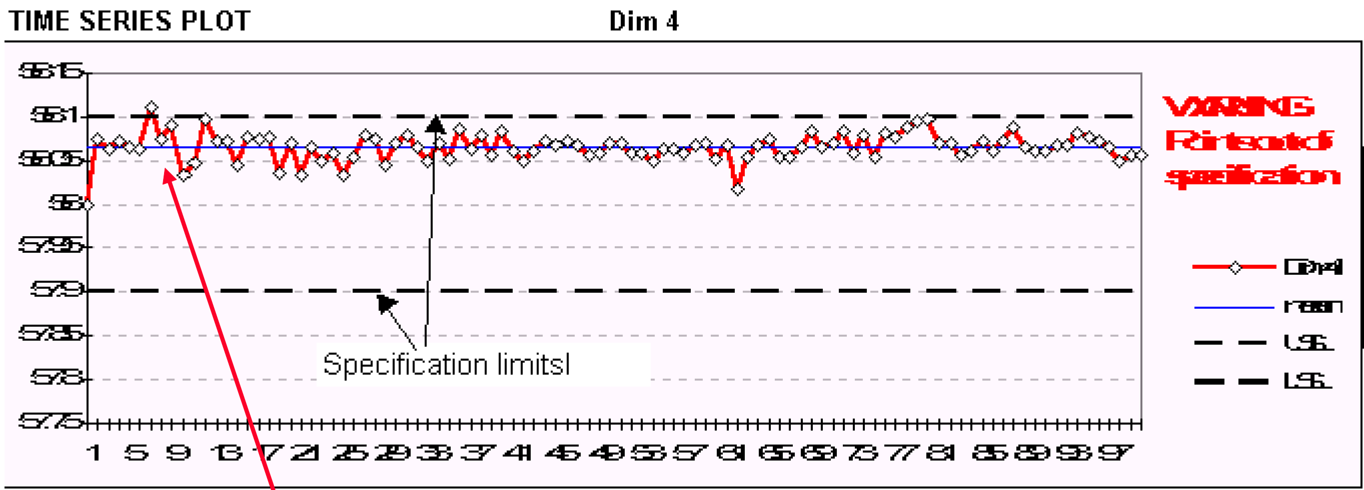
R-Chart



R CHART -
Ranges within the subgroups over time

Check for unexplained changes, out of control points, etc.
The lower the better.

Graphical Presentation - Time Series Plot



Time series plot - Each measurement plotted over time

Check for trends, sudden changes, out of specification points, etc.

Something happened here !!!

Exercise 3: Cpk Data Spreadsheet

- **Open spreadsheet "*Data exercice 3.xls*". Dim 13 is identical to the data from the previous exercises.**
- **Verify the results from the previous exercises for dimension 13.**
- **Analyse the remaining data sets an comment the process, should any actions be made? Remember to create and look at the charts.**

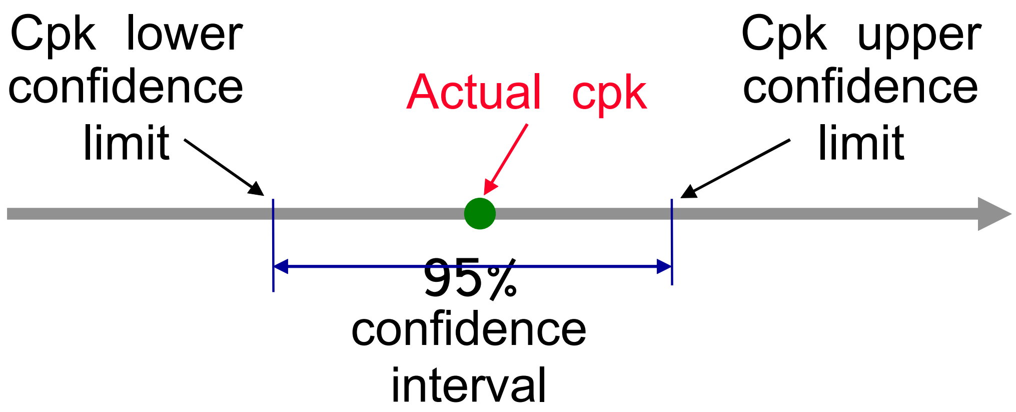
Section 5.

Confidence of Cpk

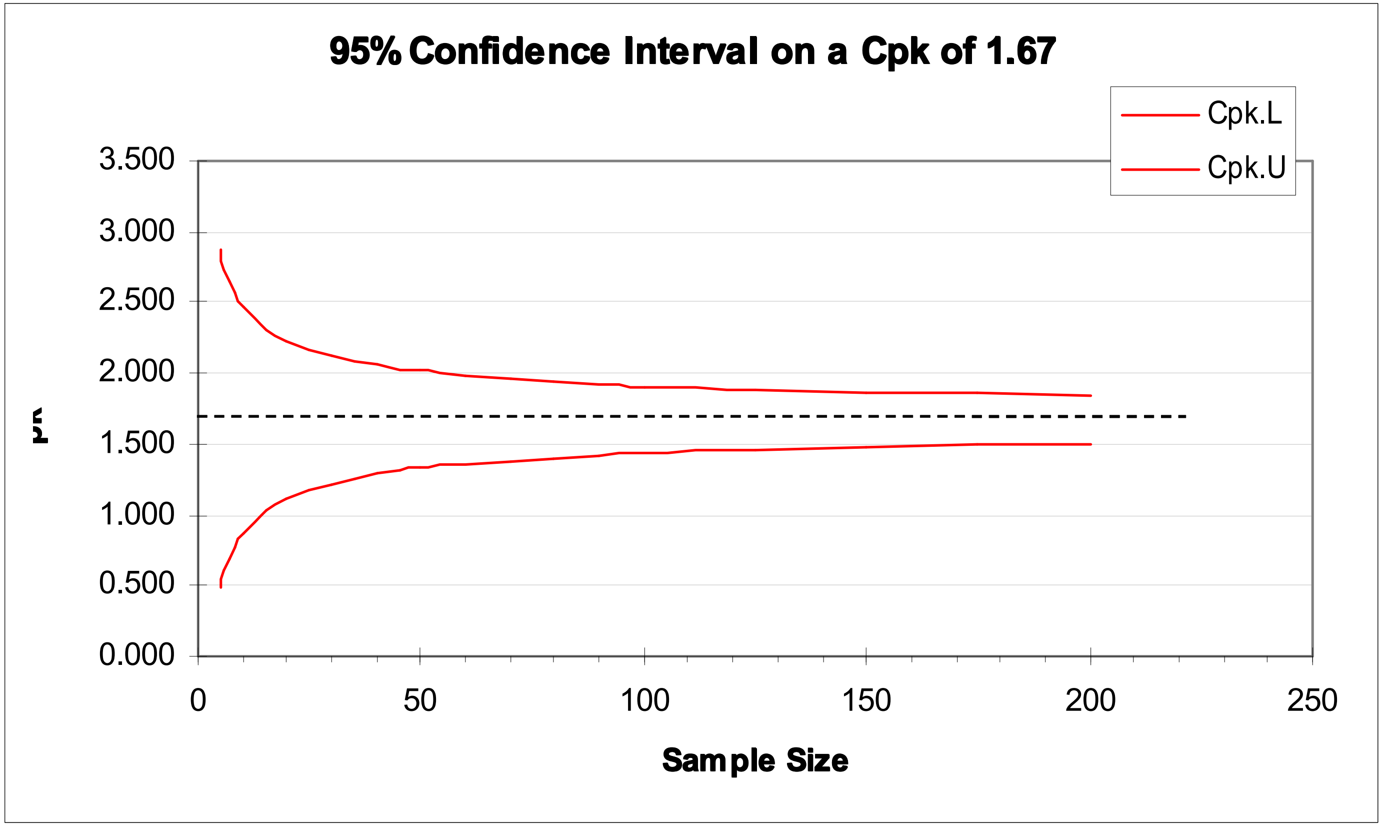
Confidence of Cpk

- Cpk values are not definite numbers as they are based on relatively small samples of a population.

- The 95% confidence interval determines the interval which includes the true Cpk value with a probability of 95%, i.e. "there is a probability of 5% that Cpk is either lower or higher" than this confidence interval.



Confidence of Cpk



Small sample sizes gives wide confidence intervals

  T0001801.PPT/ 21-Dec-2001 / Jim Christy

Cpk Confidence Limits with a sample size of 100 and a nominal Cpk of 1.67

Calculated Cpk

1.67

Change the values on this row

95% Confidence by Sample Size				95% Confidence, Predicted Tail Area			
Cpk	N	Cpk.L	Cpk.U	Lower Limit	Nominal	Upper Limit	
1.67	5	0.476	2.864	0	0	76448	ppm
1.67	6	0.601	2.739	0	0	35663	ppm
1.67	7	0.693	2.647	0	0	18754	ppm
1.67	8	0.765	2.575	0	0	10846	ppm
1.67	9	0.823	2.517	0	0	6761	ppm
1.67	10	0.871	2.469	0	0	4475	ppm
1.67	15	1.029	2.311	0	0	1013	ppm
1.67	20	1.119	2.221	0	0	393	ppm
1.67	25	1.180	2.160	0	0	200	ppm
1.67	30	1.224	2.116	0	0	120	ppm
1.67	35	1.258	2.082	0	0	80	ppm
1.67	40	1.285	2.055	0	0	58	ppm
1.67	45	1.308	2.032	0	0	44	ppm
1.67	50	1.327	2.013	0	0	34	ppm
1.67	60	1.357	1.983	0	0	23	ppm
1.67	70	1.381	1.959	0	0	17	ppm
1.67	80	1.400	1.940	0	0	13	ppm
1.67	90	1.415	1.925	0	0	11	ppm
1.67	100	1.428	1.912	0	0	9	ppm