

Spatial Data Analysis

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Introduction

This course materials are designed for a 1-day workshop on spatial data analysis. Rüttenauer (2024) provides a handbook chapter accompanying these workshop materials.

In recent years, more and more spatial data has become available, providing the possibility to combine otherwise unrelated data, such as social, economic, and environmental data. This also opens up the possibility of analysing spatial patterns and processes (e.g., spillover effects or diffusion).

Many social science research questions are spatially dependent such as voting outcomes, housing prices, labour markets, protest behaviour, or migration decisions. Observing an event in one region or neighbourhood increases the likelihood that we observe similar processes in proximate areas. As Tobler's first law of geography puts it: "Everything is related to everything else, but near things are more related than distant things". This dependence can stem from spatial contagion, spatial spillovers, or common confounders. Therefore, basic assumptions of standard regression models are violated when analysing spatial data. However, more importantly, spatial processes are interesting for their own sake. Spatial regression models can detect spatial dependence and explicitly model spatial relations, identifying spatial clustering, spillovers or diffusion processes.

The main objective of the course is the theoretical understanding and practical application of spatial regression models. This course will first give an overview on how to perform common spatial operations using spatial information, such as aggregating spatial units, calculating distances, merging spatial data as well as visualizing them. The course will further focus on the analysis of geographic data and the application of spatial regression techniques to model and analyze spatial processes, and furthermore, the course addresses several methods for defining spatial relationships, detecting and diagnosing spatial dependence and autocorrelation. Finally, we will discuss various spatial regression techniques to model processes, clarify the assumptions of these models, and show how they differ in their applications and interpretations.

The field has developed very quickly over the past few years, and *R* now provides a rich set of packages for various spatial data operations. For a more in-depth introduction into spatial data analysis in *R*, have a look into the materials references below.

The material introduces the use of geographical information to connect and analyze different spatial data sources very briefly. This introduction is limited to the fundamentals of using geographical information in *R*. Stefan Jünger & Anne-Kathrin Stroppe have provided a

comprehensive GESIS workshop on geospatial techniques in R. The focus of this workshop will be on techniques for spatial data analysis, such as spatial regression models.

Schedule

Time	Session
09:00 – 09:30	Refresher on R for spatial data
09:30 – 11:00	Spatial Relationships (W) and Spatial Dependence
11:15 – 12:00	Practical exercise
Lunch break	
13:00 – 14:30	Spatial Regression Models (SLX, Error, lagged DV)
14:45 – 16:00	Interpreting Results: Spatial Impacts
16:15 – 18:00	Practical exercise

Some useful packages

By now, *R* provides a lot of functionalities for GIS applications and spatial econometrics, and further extensions. There are lots of packages providing a huge variety of spatial functionalities and methods (see e.g. R. Bivand, Millo, and Piras 2021). Important packages for fundamental spatial operations are:

- Spatial data workhorses: [sf](#) (Pebesma 2018) and [terra](#)
- Visualization: [mapview](#) (Appelhans et al. 2021) and [tmap](#) (Tennekes 2018)
- Spatial weights and other relations: [spdep](#) (R. S. Bivand and Rudel 2018)
- Spatial interpolation and kriging: [gstat](#) (Gräler, Pebesma, and Heuvelink 2016)
- Spatial regression models: [spatialreg](#) and [sphet](#) (R. Bivand and Piras 2015)
- The packages have constantly developed over the past years, and older packages such as [rgdal](#), [rgeos](#), and [sp](#) are currently retiring ([Blog post](#))

Further Readings

- Great up-to-date introduction to spatial R: Lovelace, Nowosad, and Muenchow (2019), [updated version available online](#)
- Great open-science book on [Spatial Data Science](#) Pebesma and Bivand (2023)

- Comprehensive introduction to spatial econometrics: LeSage and Pace (2009)
- Relative intuitive introduction to spatial econometrics: Ward and Gleditsch (2008)
- Article-length introductions to spatial econometrics: Elhorst (2012), Halleck Vega and Elhorst (2015), LeSage (2014), Rüttenauer (2024), and Rüttenauer (2022)

Course materials

- I highly recommend the great Introduction to Geospatial Techniques for Social Scientists in R including, see [Stefan Jünger & Anne-Kathrin Stroppe's GESIS workshop materials](#). Nice materials on GIS, spatial operations and spatial data visualisation!
- For those looking for a more in-depth introduction, I highly recommend Roger Bivand's course on Spatial Data Analysis: [Youtube recordings](#), [Course Materials](#)
- I've learned most of what I know about spatial econometrics from [Scott J. Cook](#) and his workshop on Spatial Econometrics at the [Essex Summer school](#).

1 Refresher

Required packages

```
pkgs <- c("sf", "gstat", "mapview", "nngeo", "rnaturalearth", "dplyr",
         "nomisr", "osmdata", "tidyR", "texreg")
lapply(pkgs, require, character.only = TRUE)
```

Session info

```
sessionInfo()

R version 4.3.1 (2023-06-16 ucrt)
Platform: x86_64-w64-mingw32/x64 (64-bit)
Running under: Windows 10 x64 (build 19044)

Matrix products: default

locale:
[1] LC_COLLATE=English_United Kingdom.utf8
[2] LC_CTYPE=English_United Kingdom.utf8
[3] LC_MONETARY=English_United Kingdom.utf8
[4] LC_NUMERIC=C
[5] LC_TIME=English_United Kingdom.utf8

time zone: Europe/Berlin
tzcode source: internal

attached base packages:
[1] stats      graphics   grDevices utils      datasets  methods   base

other attached packages:
[1] texreg_1.38.6     tidyR_1.3.0      osmdata_0.2.3
```

```

[4] nomisr_0.4.7      dplyr_1.1.2      rnaturalearth_0.3.3
[7] nngeo_0.4.7       mapview_2.11.0    gstat_2.1-1
[10] sf_1.0-13

loaded via a namespace (and not attached):
 [1] xfun_0.39          raster_3.6-23     htmlwidgets_1.6.2  lattice_0.21-8
 [5] vctrs_0.6.3        tools_4.3.1       crosstalk_1.2.0   generics_0.1.3
 [9] stats4_4.3.1       tibble_3.2.1      proxy_0.4-27     spacetim_1.3-0
[13] fansi_1.0.4        xts_0.13.1       pkgconfig_2.0.3   KernSmooth_2.23-
21
[17] satellite_1.0.4    data.table_1.14.8 leaflet_2.1.2    webshot_0.5.5
[21] lifecycle_1.0.3    compiler_4.3.1   FNN_1.1.3.2     rsdmx_0.6-2
[25] munsell_0.5.0      terra_1.7-39     codetools_0.2-19 snakecase_0.11.0
[29] htmltools_0.5.5    class_7.3-22     pillar_1.9.0    classInt_0.4-9
[33] tidyselect_1.2.0    digest_0.6.32    purrr_1.0.1     fastmap_1.1.1
[37] grid_4.3.1         colorspace_2.1-0  cli_3.6.1      magrittr_2.0.3
[41] base64enc_0.1-3    XML_3.99-0.14   utf8_1.2.3     leafem_0.2.0
[45] e1071_1.7-13       scales_1.2.1     sp_1.6-1       rmarkdown_2.23
[49] httr_1.4.6          zoo_1.8-12      png_0.1-8     evaluate_0.21
[53] knitr_1.43         rlang_1.1.1     Rcpp_1.0.10    glue_1.6.2
[57] DBI_1.1.3          rstudioapi_0.14 jsonlite_1.8.5 R6_2.5.1
[61] plyr_1.8.8          intervals_0.15.4 units_0.8-2

```

1.1 Packages

Please make sure that you have installed the following packages:

```

pks <- c("dplyr",
"gstat",
"mapview",
"nngeo",
"nomisr",
"osmdata",
"rnaturalearth",
"sf",
"spatialreg",
"spdep",
"texreg",
"tidyR",
"tmap",
"viridisLite")

```

The most important package is [sf: Simple Features for R](#). users are strongly encouraged to install the sf binary packages from CRAN. If that does not work, please have a look at the [installation instructions](#). It requires software packages GEOS, GDAL and PROJ.

1.2 Coordinates

In general, spatial data is structured like conventional/tidy data (e.g. data.frames, matrices), but has one additional dimension: every observation is linked to some sort of geo-spatial information. Most common types of spatial information are:

- Points (one coordinate pair)
- Lines (two coordinate pairs)
- Polygons (at least three coordinate pairs)
- Regular grids (one coordinate pair for centroid + raster / grid size)

1.2.1 Coordinate reference system (CRS)

In its raw form, a pair of coordinates consists of two numerical values. For instance, the pair `c(51.752595, -1.262801)` describes the location of Nuffield College in Oxford (one point). The first number represents the latitude (north-south direction), the second number is the longitude (west-east direction), both are in decimal degrees.

However, we need to specify a reference point for latitudes and longitudes (in the Figure above: equator and Greenwich). For instance, the pair of coordinates above comes from Google Maps which returns GPS coordinates in ‘WGS 84’ ([EPSG:4326](#)).

```
# Coordinate pairs of two locations
coords1 <- c(51.752595, -1.262801)
coords2 <- c(51.753237, -1.253904)
coords <- rbind(coords1, coords2)

# Conventional data frame
nuffield.df <- data.frame(name = c("Nuffield College", "Radcliffe Camera"),
                           address = c("New Road", "Radcliffe Sq"),
                           lat = coords[,1], lon = coords[,2])

head(nuffield.df)
```

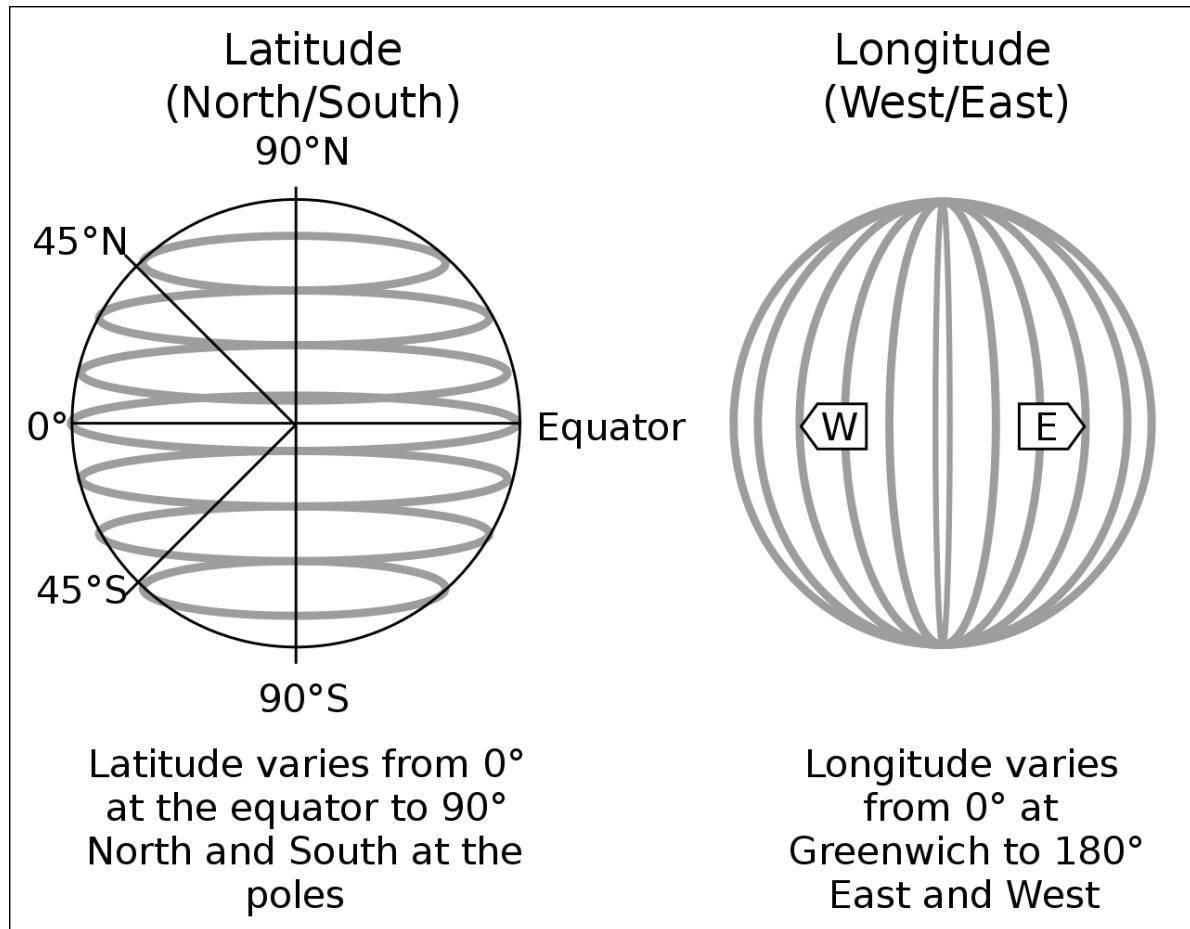


Figure 1.1: Figure: Latitude and longitude, Source: [Wikipedia](#)

```

      name      address      lat      lon
coords1 Nuffield College    New Road 51.75259 -1.262801
coords2 Radcliffe Camera Radcliffe Sq 51.75324 -1.253904

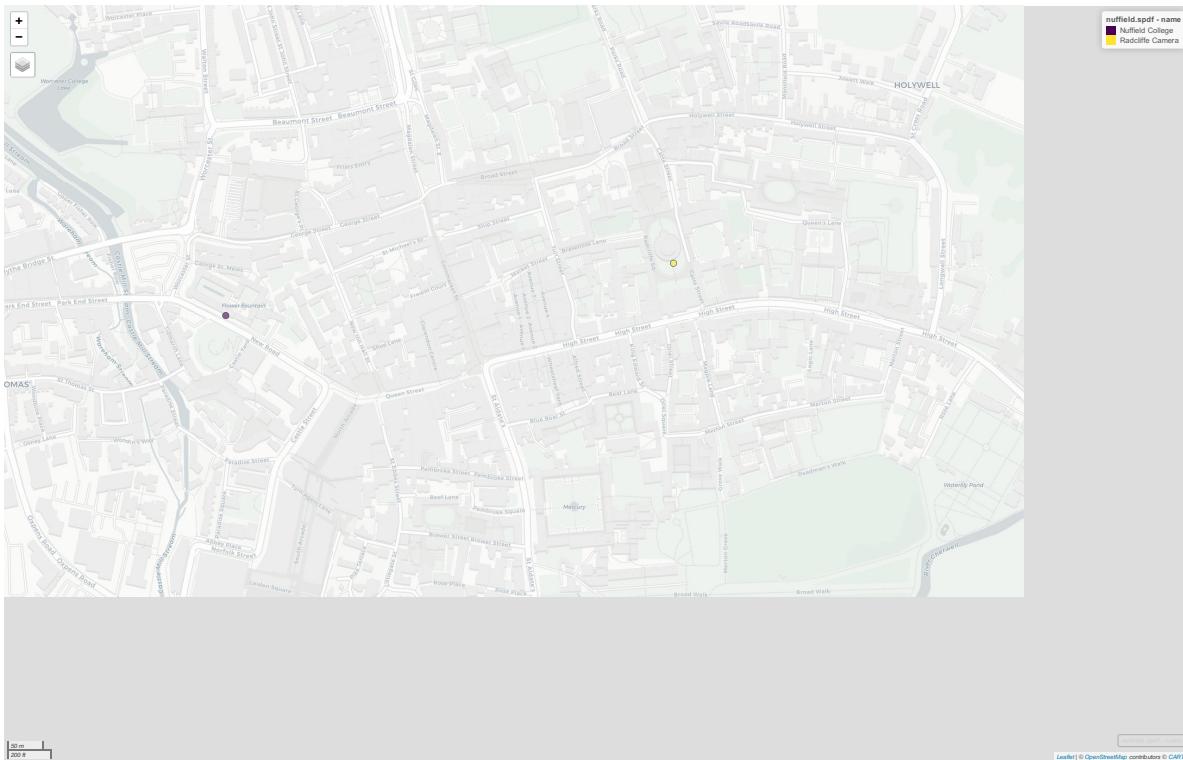
```

```

# Combine to spatial data frame
nuffield.spdf <- st_as_sf(nuffield.df,
                           coords = c("lon", "lat"), # Order is important
                           crs = 4326) # EPSG number of CRS

# Map
mapview(nuffield.spdf, zcol = "name")

```



1.2.2 Projected CRS

However, different data providers use different CRS. For instance, spatial data in the UK usually uses ‘OSGB 1936 / British National Grid’ ([EPSG:27700](#)). Here, coordinates are in meters, and projected onto a planar 2D space.

There are a lot of different CRS projections, and different national statistics offices provide data in different projections. Data providers usually specify which reference system they use.

This is important as using the correct reference system and projection is crucial for plotting and manipulating spatial data.

If you do not know the correct CRS, try starting with a standards CRS like [EPSG:4326](#) if you have decimal degree like coordinates. If it looks like projected coordinates, try searching for the country or region in CRS libraries like <https://epsg.io/>. However, you must check if the projected coordinates match their real location, e.g. using `mapview()`.

1.2.3 Why different projections?

By now, (most) people agree that [the earth is not flat](#). So, to plot data on a 2D planar surface and to perform certain operations on a planar world, we need to make some re-projections. Depending on where we are, different re-projections of our data (globe in this case) might work better than others.

```
world <- ne_countries(scale = "medium", returnclass = "sf")
class(world)

[1] "sf"           "data.frame"

st_crs(world)

Coordinate Reference System:
User input: +proj=longlat +datum=WGS84 +no_defs +ellps=WGS84 +towgs84=0,0,0
wkt:
BOUNDCRS[
  SOURCECRS[
    GEOGCRS["unknown",
      DATUM["World Geodetic System 1984",
        ELLIPSOID["WGS 84",6378137,298.257223563,
          LENGTHUNIT["metre",1]],
        ID["EPSG",6326]],
    PRIMEM["Greenwich",0,
      ANGLEUNIT["degree",0.0174532925199433],
      ID["EPSG",8901]],
    CS[ellipsoidal,2,
      AXIS["longitude",east,
        ORDER[1],
        ANGLEUNIT["degree",0.0174532925199433,
        ID["EPSG",9122]]]]]
```

```

        AXIS["latitude",north,
          ORDER[2],
          ANGLEUNIT["degree",0.0174532925199433,
            ID["EPSG",9122]]]],
TARGETCRS[
  GEOGCRS["WGS 84",
    DATUM["World Geodetic System 1984",
      ELLIPSOID["WGS 84",6378137,298.257223563,
        LENGTHUNIT["metre",1]]],
    PRIMEM["Greenwich",0,
      ANGLEUNIT["degree",0.0174532925199433]],
    CS[ellipsoidal,2],
      AXIS["latitude",north,
        ORDER[1],
        ANGLEUNIT["degree",0.0174532925199433]],
      AXIS["longitude",east,
        ORDER[2],
        ANGLEUNIT["degree",0.0174532925199433]],
    ID["EPSG",4326]]],
ABRIDGEDTRANSFORMATION["Transformation from unknown to WGS84",
  METHOD["Geocentric translations (geog2D domain)",
    ID["EPSG",9603]],
  PARAMETER["X-axis translation",0,
    ID["EPSG",8605]],
  PARAMETER["Y-axis translation",0,
    ID["EPSG",8606]],
  PARAMETER["Z-axis translation",0,
    ID["EPSG",8607]]]

```

```

# Extract a country and plot in current CRS (WGS84)
ger.spdf <- world[world$name == "Germany", ]
plot(st_geometry(ger.spdf))

```



```
# Now, let's transform Germany into a CRS optimized for Iceland
ger_rep.spdf <- st_transform(ger.spdf, crs = 5325)
plot(st_geometry(ger_rep.spdf))
```



Depending on the angle, a 2D projection of the earth looks different. It is important to choose a suitable projection for the available spatial data. For more information on CRS and re-projection, see e.g. Lovelace, Nowosad, and Muenchow (2019) or [Stefan Jünger & Anne-Kathrin Stroppe's GESIS workshop materials](#).

1.3 Importing some real world data

`sf` imports many of the most common spatial data files, like geojson, gpkg, or shp.

1.3.1 London shapefile (polygon)

Let's get some administrative boundaries for London from the [London Datastore](#). We use the `sf` package and its function `st_read()` to import the data.

```
# Create subdir (all data will be stored in "_data")
dn <- "_data"
ifelse(dir.exists(dn), "Exists", dir.create(dn))
```

```
[1] "Exists"
```

```

# Download zip file and unzip
tmpf <- tempfile()
boundary.link <- "https://data.london.gov.uk/download/statistical-gis-boundary-files-londo
download.file(boundary.link, tmpf)
unzip(zipfile = tmpf, exdir = paste0(dn))
unlink(tmpf)

# This is a shapefile
# We only need the MSOA layer for now
msoa.spdf <- st_read(dsn = paste0(dn, "/statistical-gis-boundaries-london/ESRI"),
                      layer = "MSOA_2011_London_gen_MHW" # Note: no file ending
)

```

Reading layer `MSOA_2011_London_gen_MHW' from data source
`C:\work\Lehre\Geodata_Spatial_Regression_short_data\statistical-gis-boundaries-london\ESRI'
using driver `ESRI Shapefile'
Simple feature collection with 983 features and 12 fields
Geometry type: MULTIPOLYGON
Dimension: XY
Bounding box: xmin: 503574.2 ymin: 155850.8 xmax: 561956.7 ymax: 200933.6
Projected CRS: OSGB36 / British National Grid

The object `msoa.spdf` is our spatial data.frame. It looks essentially like a conventional data.frame, but has some additional attributes and geo-graphical information stored with it. Most importantly, notice the column `geometry`, which contains a list of polygons. In most cases, we have one polygon for each line / observation.

```

head(msoa.spdf)

Simple feature collection with 6 features and 12 fields
Geometry type: MULTIPOLYGON
Dimension: XY
Bounding box: xmin: 530966.7 ymin: 180510.7 xmax: 551943.8 ymax: 191139
Projected CRS: OSGB36 / British National Grid
  MSOA11CD          MSOA11NM    LAD11CD          LAD11NM    RGN11CD
 1 E02000001      City of London 001 E09000001      City of London E12000007
 2 E02000002 Barking and Dagenham 001 E09000002 Barking and Dagenham E12000007
 3 E02000003 Barking and Dagenham 002 E09000002 Barking and Dagenham E12000007
 4 E02000004 Barking and Dagenham 003 E09000002 Barking and Dagenham E12000007
 5 E02000005 Barking and Dagenham 004 E09000002 Barking and Dagenham E12000007

```

```

6 E02000007 Barking and Dagenham 006 E09000002 Barking and Dagenham E12000007
  RGN11NM USUALRES HHOLDRES COMESTRES POPDEN HHOLDS AVHHOLDSZ
  1 London    7375     7187      188   25.5   4385     1.6
  2 London    6775     6724       51   31.3   2713     2.5
  3 London   10045    10033      12   46.9   3834     2.6
  4 London    6182     5937      245   24.8   2318     2.6
  5 London    8562     8562       0   72.1   3183     2.7
  6 London    8791     8672      119   50.6   3441     2.5
                                geometry
  1 MULTIPOLYGON (((531667.6 18...
  2 MULTIPOLYGON (((548881.6 19...
  3 MULTIPOLYGON (((549102.4 18...
  4 MULTIPOLYGON (((551550 1873...
  5 MULTIPOLYGON (((549099.6 18...
  6 MULTIPOLYGON (((549819.9 18...

```

Shapefiles are still among the most common formats to store and transmit spatial data, despite them being inefficient (file size and file number).

However, `sf` reads everything spatial, such as `geo.json`, which usually is more efficient, but less common (but we're getting there).

```

# Download file
ulez.link <- "https://data.london.gov.uk/download/ultra_low_emissions_zone/936d71d8-c5fc-4
download.file(ulez.link, paste0(dn, "/ulez.json"))

# Read geo.json
st_layers(paste0(dn, "/ulez.json"))

Driver: GeoJSON
Available layers:
  layer_name geometry_type features fields
  1 CentralUltraLowEmissionZone Multi Polygon      1      4
    crs_name
  1 OSGB36 / British National Grid

ulez.spdf <- st_read(dsn = paste0(dn, "/ulez.json")) # here dsn is simply the file

Reading layer `CentralUltraLowEmissionZone' from data source
`C:\work\Lehre\Geodata_Spatial_Regression_short\_data\ulez.json'
using driver `GeoJSON'

```

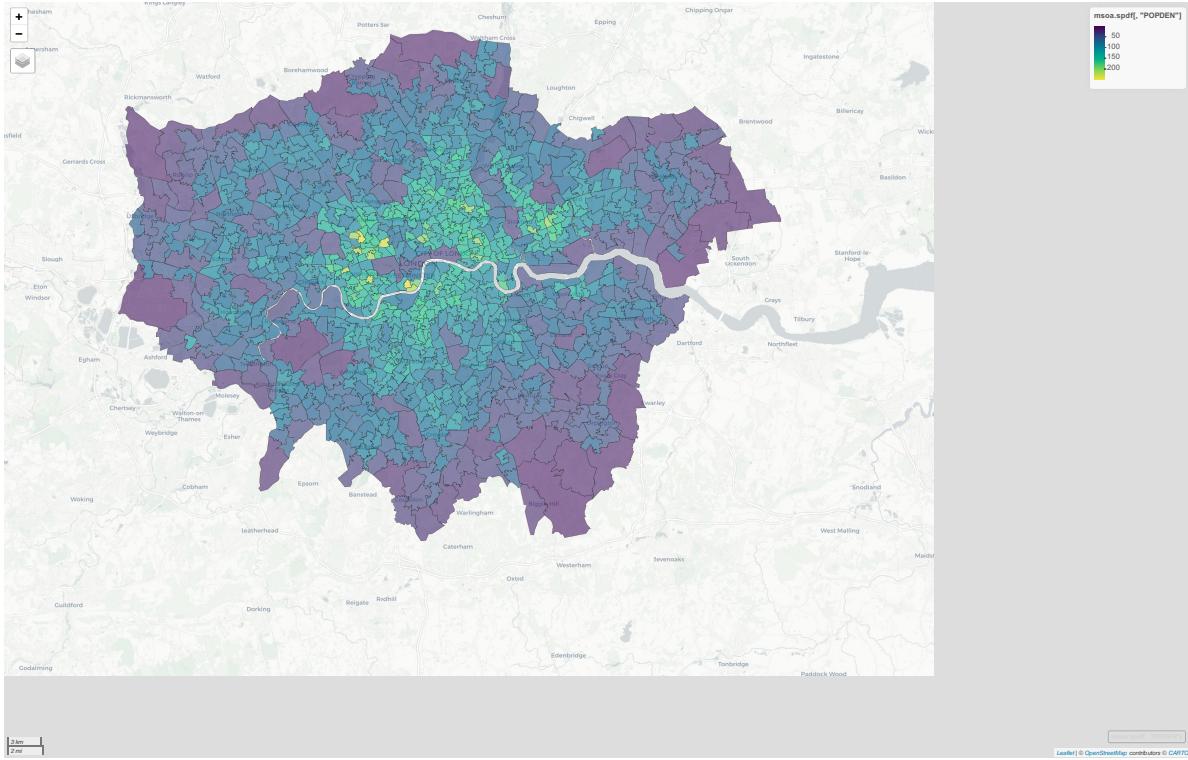
```
Simple feature collection with 1 feature and 4 fields
Geometry type: MULTIPOLYGON
Dimension:      XY
Bounding box:  xmin: 527271.5 ymin: 178041.5 xmax: 533866.3 ymax: 183133.4
Projected CRS: OSGB36 / British National Grid
```

```
head(ulez.spdf)
```

```
Simple feature collection with 1 feature and 4 fields
Geometry type: MULTIPOLYGON
Dimension:      XY
Bounding box:  xmin: 527271.5 ymin: 178041.5 xmax: 533866.3 ymax: 183133.4
Projected CRS: OSGB36 / British National Grid
  fid OBJECTID BOUNDARY Shape_Area           geometry
1   1         1 CSS Area    21.37557 MULTIPOLYGON (((531562.7 18...
```

Again, this looks like a conventional `data.frame` but has the additional column `geometry` containing the coordinates of each observation. `st_geometry()` returns only the geographic object and `st_drop_geometry()` only the `data.frame` without the coordinates. We can plot the object using `mapview()`.

```
mapview(msoa.spdf[, "POPDEN"])
```



1.3.2 Census API (admin units)

Now that we have some boundaries and shapes of spatial units in London, we can start looking for different data sources to populate the geometries.

A good source for demographic data is for instance the 2011 census. Below we use the nomis API to retrieve population data for London, See the [Vignette](#) for more information (Guest users are limited to 25,000 rows per query). Below is a wrapper to avoid some errors with sex and urban-rural cross-tabulation in some of the data.

```
### For larger request, register and set key
# Sys.setenv(NOMIS_API_KEY = "XXX")
# nomis_api_key(check_env = TRUE)

x <- nomis_data_info()

# Get London ids
london_ids <- msoa.spdf$MSOA11CD

### Get key statistics ids
```

```

# select requires tables (https://www.nomisweb.co.uk/sources/census\_2011\_ks)
# Let's get KS201EW (ethnic group), KS205EW (passport held), and KS402EW (housing tenure)

# Get internal ids
stats <- c("KS201EW", "KS402EW", "KS205EW")
oo <- which(grepl(paste(stats, collapse = "|"), x$name.value))
ksids <- x$id[oo]
ksids # This are the internal ids

[1] "NM_608_1" "NM_612_1" "NM_619_1"

#### look at meta information
q <- nomis_overview(ksids[1])
head(q)

# A tibble: 6 x 2
  name      value
  <chr>    <list>
1 analyses <named list [1]>
2 analysisname <chr [1]>
3 analysisnumber <int [1]>
4 contact <named list [4]>
5 contenttypes <named list [1]>
6 coverage <chr [1]>

a <- nomis_get_metadata(id = ksids[1], concept = "GEOGRAPHY", type = "type")
a # TYPE297 is MSOA level

# A tibble: 24 x 3
  id      label.en      description.en
  <chr>   <chr>        <chr>
1 TYPE265 NHS area teams  NHS area teams
2 TYPE266 clinical commissioning groups clinical commissi-
3 TYPE267 built-up areas including subdivisions built-up areas in-
4 TYPE269 built-up areas          built-up areas
5 TYPE273 national assembly for wales electoral regions 2010 national assembly-
6 TYPE274 postcode areas           postcode areas
7 TYPE275 postcode districts      postcode districts
8 TYPE276 postcode sectors       postcode sectors

```

```

9 TYPE277 national assembly for wales constituencies 2010      national assembly~
10 TYPE279 parishes 2011                                         parishes 2011
# i 14 more rows

b <- nomis_get_metadata(id = ksids[1], concept = "MEASURES", type = "TYPE297")
b # 20100 is the measure of absolute numbers

# A tibble: 2 x 3
  id    label.en description.en
  <chr> <chr>     <chr>
1 20100 value    value
2 20301 percent  percent

#### Query data in loop over the required statistics
for(i in ksids){

  # Determin if data is divided by sex or urban-rural
  nd <- nomis_get_metadata(id = i)
  if("RURAL_URBAN" %in% nd$conceptref){
    UR <- TRUE
  }else{
    UR <- FALSE
  }
  if("C_SEX" %in% nd$conceptref){
    SEX <- TRUE
  }else{
    SEX <- FALSE
  }

  # make data request
  if(UR == TRUE){
    if(SEX == TRUE){
      tmp_en <- nomis_get_data(id = i, time = "2011",
                                geography = london_ids, # replace with "TYPE297" for all MS
                                measures = 20100, RURAL_URBAN = 0, C_SEX = 0)
    }else{
      tmp_en <- nomis_get_data(id = i, time = "2011",
                                geography = london_ids, # replace with "TYPE297" for all MS
                                measures = 20100, RURAL_URBAN = 0)
    }
  }else{
}
}

```

```

if(SEX == TRUE){
  tmp_en <- nomis_get_data(id = i, time = "2011",
                            geography = london_ids, # replace with "TYPE297" for all MSAs
                            measures = 20100, C_SEX = 0)
} else{
  tmp_en <- nomis_get_data(id = i, time = "2011",
                            geography = london_ids, # replace with "TYPE297" for all MSAs
                            measures = 20100)
}

# Append (in case of different regions)
ks_tmp <- tmp_en

# Make lower case names
names(ks_tmp) <- tolower(names(ks_tmp))
names(ks_tmp)[names(ks_tmp) == "geography_code"] <- "msoa11"
names(ks_tmp)[names(ks_tmp) == "geography_name"] <- "name"

# replace weird cell codes
onlynum <- which(grepl("^[[:digit:]]+$", ks_tmp$cell_code))
if(length(onlynum) != 0){
  code <- substr(ks_tmp$cell_code[-onlynum] [1], 1, 7)
  if(is.na(code)){
    code <- i
  }
  ks_tmp$cell_code[onlynum] <- paste0(code, "_", ks_tmp$cell_code[onlynum])
}

# save codebook
ks_cb <- unique(ks_tmp[, c("date", "cell_type", "cell", "cell_code", "cell_name")])

### Reshape
ks_res <- tidyr::pivot_wider(ks_tmp, id_cols = c("msoa11", "name"),
                             names_from = "cell_code",
                             values_from = "obs_value")

### Merge
if(i == ksids[1]){
  census_keystat.df <- ks_res
}

```

```

    census_keystat_cb.df <- ks_cb
}else{
  census_keystat.df <- merge(census_keystat.df, ks_res, by = c("msoa11", "name"), all =
  census_keystat_cb.df <- rbind(census_keystat_cb.df, ks_cb)
}

# Descriptions are saved in the codebook
head(census_keystat_cb.df)

# A tibble: 6 x 5
  date cell_type      cell cell_code   cell_name
  <dbl> <chr>        <dbl> <chr>       <chr>
1 2011 Ethnic Group     0 KS201EW0001 All usual residents
2 2011 Ethnic Group    100 KS201EW_100 White
3 2011 Ethnic Group      1 KS201EW0002 White: English/Welsh/Scottish/Northern I-
4 2011 Ethnic Group      2 KS201EW0003 White: Irish
5 2011 Ethnic Group      3 KS201EW0004 White: Gypsy or Irish Traveller
6 2011 Ethnic Group      4 KS201EW0005 White: Other White

save(census_keystat_cb.df, file = "_data/Census_codebook.RData")

```

Now, we have one file containing the geometries of MSOAs and one file with the census information on ethnic groups. Obviously, we can easily merge them together using the MSOA identifiers.

```
msoa.spdf <- merge(msoa.spdf, census_keystat.df,
                     by.x = "MSOA11CD", by.y = "msoa11", all.x = TRUE)
```

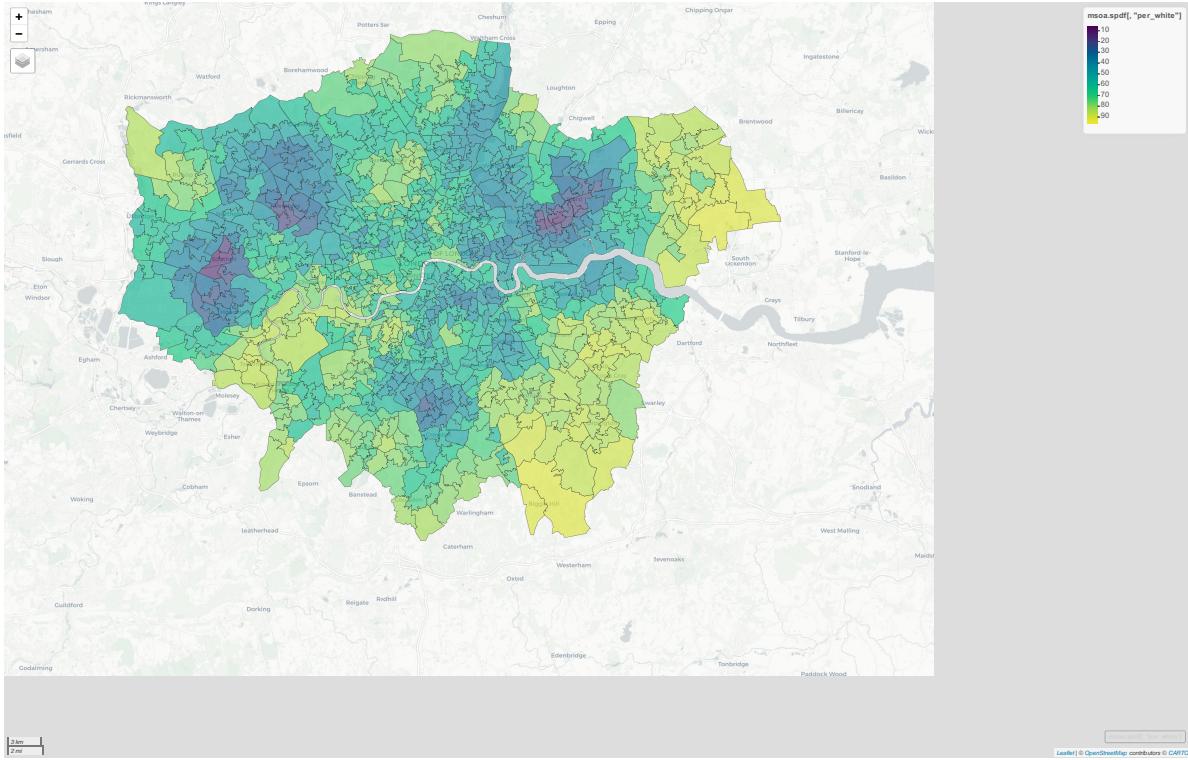
And we can, for instance, plot the spatial distribution of ethnic groups.

```

msoa.spdf$per_white <- msoa.spdf$KS201EW_100 / msoa.spdf$KS201EW0001 * 100
msoa.spdf$per_mixed <- msoa.spdf$KS201EW_200 / msoa.spdf$KS201EW0001 * 100
msoa.spdf$per_asian <- msoa.spdf$KS201EW_300 / msoa.spdf$KS201EW0001 * 100
msoa.spdf$per_black <- msoa.spdf$KS201EW_400 / msoa.spdf$KS201EW0001 * 100
msoa.spdf$per_other <- msoa.spdf$KS201EW_500 / msoa.spdf$KS201EW0001 * 100

mapview(msoa.spdf[, "per_white"])

```



If you're interested in more data sources, see for instance [APIs for social scientists: A collaborative review](#) by Paul C. Bauer, Camille Landesvatter, Lion Behrens. It's a collection of several APIs for social sciences.

1.3.3 Gridded data

So far, we have queried data on administrative units. However, often data comes on other spatial scales. For instance, we might be interested in the amount of air pollution, which is provided on a regular grid across the UK from [Defra](#).

```
# Download
pol.link <- "https://uk-air.defra.gov.uk/datastore/pcm/mapno22011.csv"
download.file(pol.link, paste0(dn, "/mapno22011.csv"))
pol.df <- read.csv(paste0(dn, "/mapno22011.csv"), skip = 5, header = T, sep = ",",
                    stringsAsFactors = F, na.strings = "MISSING")

head(pol.df)
```

ukgridcode	x	y	no22011
------------	---	---	---------

```

1      54291 460500 1221500      NA
2      54292 461500 1221500      NA
3      54294 463500 1221500      NA
4      54979 458500 1220500      NA
5      54980 459500 1220500      NA
6      54981 460500 1220500      NA

```

The data comes as point data with x and y as coordinates. We have to transform this into spatial data first. We first setup a spatial points object with `st_as_sf`. Subsequently, we transform the point coordinates into a regular grid. We use a buffer method `st_buffer` with “diameter”, and only one segment per quadrant (`nQuadSegs`). This gives us a 1x1km regular grid.

```

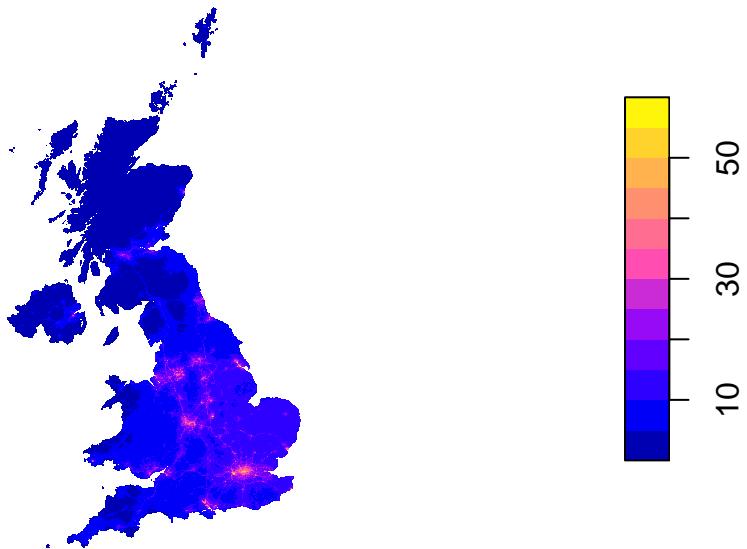
# Build spatial object
pol.spdf <- st_as_sf(pol.df, coords = c("x", "y"),
                      crs = 27700)

# we transform the point coordinates into a regular grid with "diameter" 500m
pol.spdf <- st_buffer(pol.spdf, dist = 500, nQuadSegs = 1,
                       endCapStyle = 'SQUARE')

# Plot NO2
plot(pol.spdf[, "no22011"], border = NA)

```

no22011



1.3.4 OpenStreetMap (points)

Another interesting data source is the OpenStreetMap API, which provides information about the geographical location of a series of different indicators. Robin Lovelace provides a nice introduction to the [osmdata API](#). Available features can be found on [OSM wiki](#).

First we create a bounding box of where we want to query data. `st_bbox()` can be used to get bounding boxes of an existing spatial object (needs CRS = 4326). An alternative would be to use `opq(bbox = 'greater london uk')`.

```
# bounding box of where we want to query data
q <- opq(bbox = st_bbox(st_transform(msoa.spdf, 4326)))
```

And we want to get data for all pubs and bars which are within this bounding box.

```
# First build the query of location of pubs in London
osmq <- add_osm_feature(q, key = "amenity", value = "pub")

# And then query the data
pubs.osm <- osmdata_sf(osmq)
```

Right now there are some results in polygons, some in points, and they overlap. Often, data from OSM needs some manual cleaning. Sometimes the same features are represented by different spatial objects (e.g. points + polygons).

```
# Make unique points / polygons
pubs.osm <- unique_osmdata(pubs.osm)

# Get points and polygons (there are barely any pubs as polygons, so we ignore them)
pubs.points <- pubs.osm$osm_points
pubs.polys <- pubs.osm$osm_multipolygons

# # Drop OSM file
# rm(pubs.osm); gc()

# Reduce to point object only
pubs.spdf <- pubs.points

# Reduce to a few variables
pubs.spdf <- pubs.spdf[, c("osm_id", "name", "addr:postcode", "diet:vegan")]
```

Again, we can inspect the results with `mapview`.

```
mapview(st_geometry(pubs.spdf))
```



Note that OSM is solely based on contribution by users, and the **quality of OSM data varies**. Usually data quality is better in larger cities, and better for more stable features (such as hospitals, train stations, highways) rather than pubs or restaurants which regularly appear and disappear. However, data from [London Datastore](#) would indicate more pubs than what we find with OSM.

1.3.5 Save

We will store the created data to use them again in the next session.

```
save(msoa.spdf, file = "_data/msoa_spatial.RData")
save(ulez.spdf, file = "_data/ulez_spatial.RData")
save(pol.spdf, file = "_data/pollution_spatial.RData")
save(pubs.spdf, file = "_data/pubs_spatial.RData")
```

1.4 Data Manipulation

Required packages

```
pkgs <- c("sf", "gstat", "mapview", "nngeo", "rnatural-earth", "dplyr",
         "nomisr", "osmdata", "tidyverse", "texreg", "downlit", "xml2")
lapply(pkgs, require, character.only = TRUE)
```

Having data with geo-spatial information allows to perform a variety of methods to manipulate and link different data sources. Commonly used methods include 1) subsetting, 2) point-in-polygon operations, 3) distance measures, 4) intersections or buffer methods.

The [online Vignettes of the sf package](#) provide a comprehensive overview of the multiple ways of spatial manipulations.

1.4.0.1 Check if data is on common projection

```
st_crs(msoa.spdf) == st_crs(pol.spdf)
```

```
[1] FALSE
```

```
st_crs(msoa.spdf) == st_crs(pubs.spdf)
```

```
[1] FALSE
```

```
st_crs(msoa.spdf) == st_crs(ulez.spdf)
```

```
[1] FALSE
```

The spatial data files are on different projections. Before we can do any spatial operations with them, we have to transform them into a common projection.

```
# MSOA in different crs --> transform
pol.spdf <- st_transform(pol.spdf, crs = st_crs(msoa.spdf))
pubs.spdf <- st_transform(pubs.spdf, crs = st_crs(msoa.spdf))
ulez.spdf <- st_transform(ulez.spdf, crs = st_crs(msoa.spdf))
```

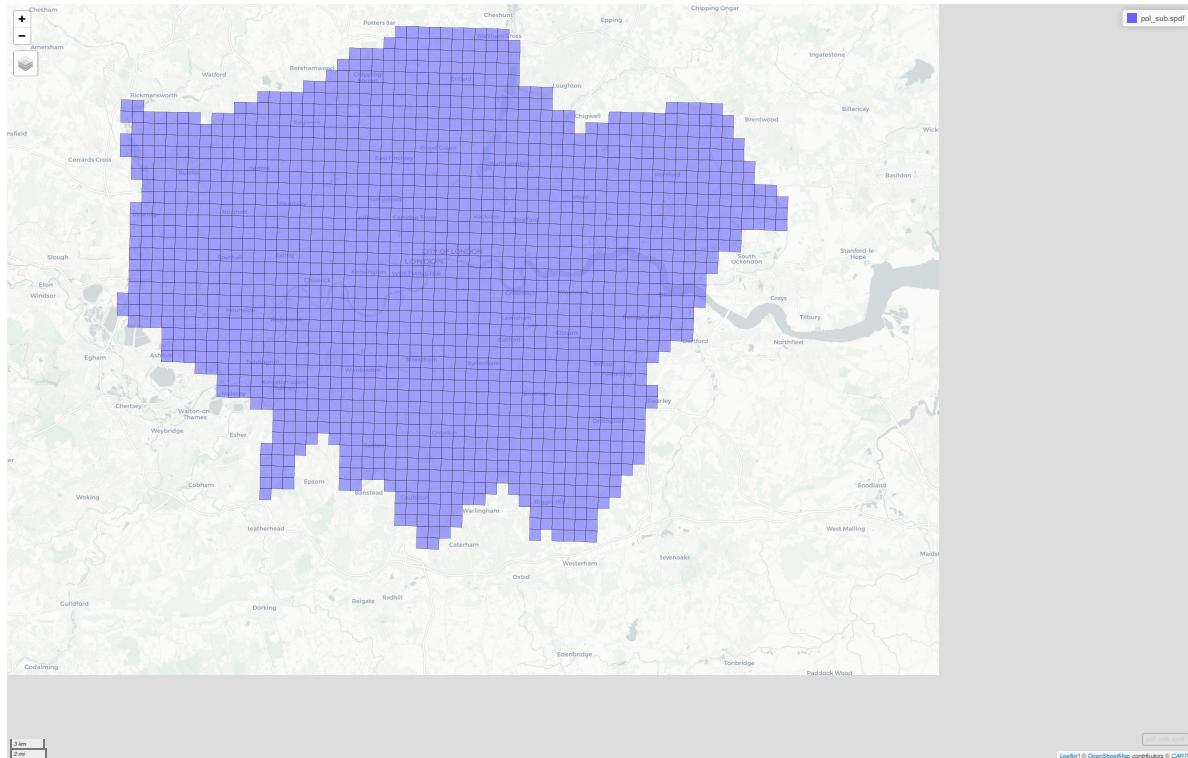
```
# Check if all geometries are valid, and make valid if needed
msoa.spdf <- st_make_valid(msoa.spdf)
```

The `st_make_valid()` can help if the spatial geometries have some problems such as holes or points that don't match exactly.

1.4.1 Subsetting

We can subset spatial data in a similar way as we subset conventional data.frames or matrices. For instance, below we simply reduce the pollution grid across the UK to observations in London only.

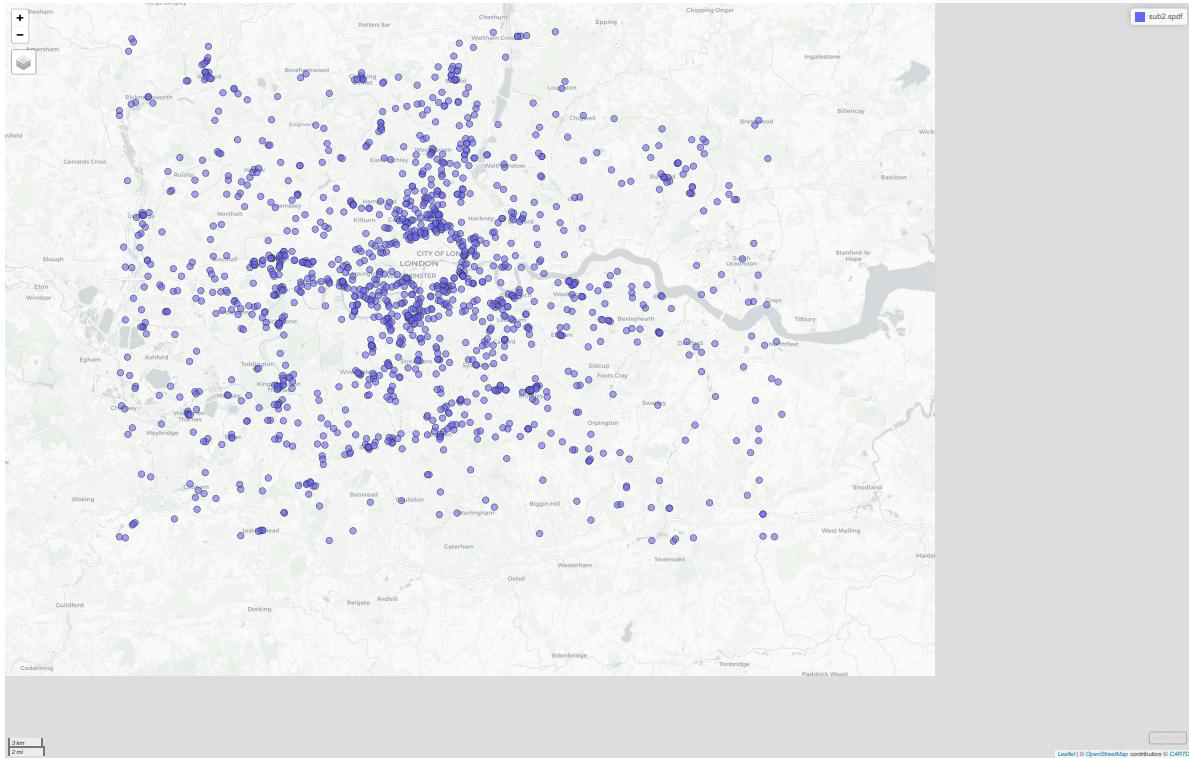
```
# Subset to pollution estimates in London
pol_sub.spdf <- pol.spdf[msoa.spdf, ] # or:
pol_sub.spdf <- st_filter(pol.spdf, msoa.spdf)
mapview(pol_sub.spdf)
```



Or we can reverse the above and exclude all intersecting units by specifying `st_disjoint` as

alternative spatial operation using the `op = option` (note the empty space for column selection). `st_filter()` with the `.predicate` option does the same job. See the [sf Vignette](#) for more operations.

```
# Subset pubs to pubs not in the ulez area
sub2.spdf <- pubs.spdf[ulez.spdf, , op = st_disjoint] # or:
sub2.spdf <- st_filter(pubs.spdf, ulez.spdf, .predicate = st_disjoint)
mapview(sub2.spdf)
```



We can easily create indicators of whether an MSOA is within ulez or not.

```
msoa.spdf$ulez <- 0

# intersecting lsoas
within <- msoa.spdf[ulez.spdf,]

# use their ids to create binary indicator
msoa.spdf$ulez[which(msoa.spdf$MSOA11CD %in% within$MSOA11CD)] <- 1
table(msoa.spdf$ulez)
```

```
0    1  
955  28
```

1.4.2 Point in polygon

We are interested in the number of pubs in each MSOA. So, we count the number of points in each polygon.

```
# Assign MSOA to each point  
pubs_msoa.join <- st_join(pubs.spdf, msoa.spdf, join = st_within)  
  
# Count N by MSOA code (drop geometry to speed up)  
pubs_msoa.join <- dplyr::count(st_drop_geometry(pubs_msoa.join),  
                                MSOA11CD = pubs_msoa.join$MSOA11CD,  
                                name = "pubs_count")  
sum(pubs_msoa.join$pubs_count)
```

```
[1] 1601
```

```
# Merge and replace NAs with zero (no matches, no pubs)  
msoa.spdf <- merge(msoa.spdf, pubs_msoa.join,  
                     by = "MSOA11CD", all.x = TRUE)  
msoa.spdf$pubs_count[is.na(msoa.spdf$pubs_count)] <- 0
```

1.4.3 Distance measures

We might be interested in the distance to the nearest pub. Here, we use the package `nngeo` to find k nearest neighbours with the respective distance.

```
# Use geometric centroid of each MSOA  
cent.sp <- st_centroid(msoa.spdf[, "MSOA11CD"])
```

```
Warning: st_centroid assumes attributes are constant over geometries
```

```
# Get K nearest neighbour with distance  
knb.dist <- st_nn(cent.sp,  
                   pubbs.spdf,
```

```
k = 1,                      # number of nearest neighbours
returnDist = TRUE, # we also want the distance
progress = FALSE)
```

projected points

```
msoa.spdf$dist_pubs <- unlist(knb.dist$dist)
summary(msoa.spdf$dist_pubs)
```

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
9.079	305.149	565.018	701.961	948.047	3735.478

1.4.4 Intersections + Buffers

We may also want the average pollution within 1 km radius around each MSOA centroid. Note that it is usually better to use a ego-centric method where you calculate the average within a distance rather than using the characteristic of the intersecting cells only (B. A. Lee et al. 2008; Mohai and Saha 2007).

Therefore, we first create a buffer with `st_buffer()` around each midpoint and subsequently use `st_intersection()` to calculate the overlap.

```
# Create buffer (1km radius)
cent.buf <- st_buffer(cent.sp,
                      dist = 1000) # dist in meters
mapview(cent.buf)
```



```
# Add area of each buffer (in this constant)
cent.buf$area <- as.numeric(st_area(cent.buf))

# Calculate intersection of pollution grid and buffer
int.df <- st_intersection(cent.buf, pol.spdf)
```

Warning: attribute variables are assumed to be spatially constant throughout all geometries

```
int.df$int_area <- as.numeric(st_area(int.df)) # area of intersection

# Area of intersection as share of buffer
int.df$area_per <- int.df$int_area / int.df$area
```

And we use the percent overlap areas as the weights to calculate a weighted mean.

```
# Aggregate as weighted mean
int.df <- st_drop_geometry(int.df)
```

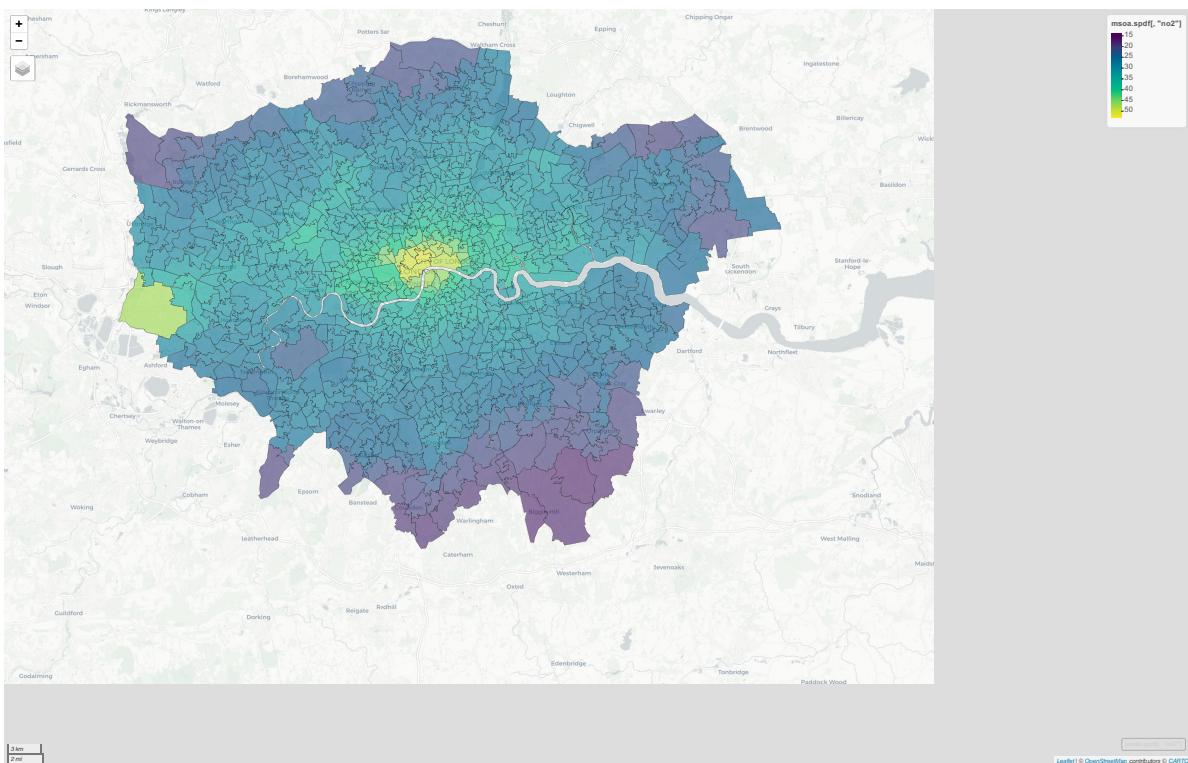
```

int.df$no2_weighted <- int.df$no22011 * int.df$area_per
int.df <- aggregate(list(no2 = int.df[, "no2_weighted"]),
by = list(MSOA11CD = int.df$MSOA11CD),
sum)

# Merge back to spatial data.frame
msoa.spdf <- merge(msoa.spdf, int.df, by = "MSOA11CD", all.x = TRUE)

mapview(msoa.spdf[, "no2"])

```



Note: for buffer related methods, it often makes sense to use population weighted centroids instead of geographic centroids (see [here](#) for MSOA population weighted centroids). However, often this information is not available.

1.4.5 and more

There are more spatial operation possible using sf. Have a look at the [sf Cheatsheet](#).

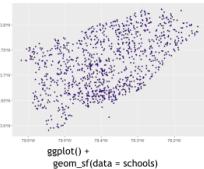
Spatial manipulation with sf: : CHEAT SHEET

The sf package provides a set of tools for working with geospatial vectors, i.e. points, lines, polygons, etc.



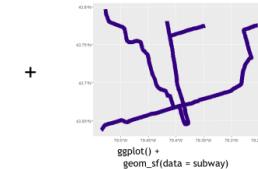
Geometric confirmation

- `st_contains(x, y, ...)` Identifies if y is within x (i.e. point within polygon)
- `st_covered_by(x, y, ...)` Identifies if x is completely within y (i.e. polygon completely within polygon)
- `st_covers(x, y, ...)` Identifies if any point from x is outside of y (i.e. polygon outside polygon)
- `st_crosses(x, y, ...)` Identifies if any geometry of x have commonalities with y
- `st_disjoint(x, y, ...)` Identifies when geometries from x do not share space with y
- `st_equals(x, y, ...)` Identifies if x and y share the same geometry
- `st_intersects(x, y, ...)` Identifies if x and y geometry share any space
- `st_overlaps(x, y, ...)` Identifies if geometries of x and y share space, are of the same dimension, but are not completely contained by each other
- `st_touches(x, y, ...)` Identifies if geometries of x and y share a common point but their interiors do not intersect
- `st_within(x, y, ...)` Identifies if x is in a specified distance to y



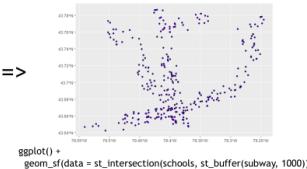
Geometric operations

- `st_boundary(x)` Creates a polygon that encompasses the full extent of the geometry
- `st_buffer(x, dist, nQuadSegs)` Creates a polygon covering all points of the geometry within a given distance
- `st_centroid(x, ..., of_largest_polygon)` Creates a point at the geometric centre of the geometry
- `st_convex_hull(x)` Creates geometry that represents the minimum convex geometry of x
- `st_line_merge(x)` Creates linestring geometry from sewing multi linestring geometry together
- `st_node(x)` Creates nodes on overlapping geometry where nodes do not exist
- `st_point_on_surface(x)` Creates a point that is guaranteed to fall on the surface of the geometry
- `st_polyonize(x)` Creates polygon geometry from linestring geometry
- `st_segmentize(x, dfMaxLength, ...)` Creates linestring geometry from x based on a specified length
- `st_simplify(x, preserveTopology, dTolerance)` Creates a simplified version of the geometry based on a specified tolerance



Geometry creation

- `st_triangulate(x, dTolerance, bOnlyEdges)` Creates polygon geometry as triangles from point geometry
- `st_voronoi(x, envelope, dTolerance, bOnlyEdges)` Creates polygon geometry covering the envelope of x, with x at the centre of the geometry
- `st_point(x, c(numeric vector), dim = "XYZ")` Creating point geometry from numeric values
- `st_multipoint(x = matrix(numeric values in rows), dim = "XYZ")` Creating multi point geometry from numeric values
- `st_linestring(x = matrix(numeric values in rows), dim = "XYZ")` Creating linestring geometry from numeric values
- `st_multilinestring(x = list(numeric matrices in rows), dim = "XYZ")` Creating multi linestring geometry from numeric values
- `st_polygon(x = list(numeric matrices in rows), dim = "XYZ")` Creating polygon geometry from numeric values
- `st_multipolygon(x = list(numeric matrices in rows), dim = "XYZ")` Creating multi polygon geometry from numeric values



This cheatsheet presents the sf package [Edzer Pebesma 2018] in version 0.6.3. See <https://github.com/r-spatial/sf> for more details.

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<https://creativecommons.org/licenses/by/4.0/>

1.5 Data visualisation

For mapping

```
pkgs <- c("tmap", "tmaptools", "viridisLite",
         "ggplot2", "ggthemes", "rmapshaper")
lapply(pkgs, require, character.only = TRUE)
```

A large advantage of spatial data is that different data sources can be connected and combined. Another nice advantage is: you can create very nice maps. And it's quite easy to do! [Stefan Jünger & Anne-Kathrin Stroppe](#) provide more comprehensive materials on mapping in their GESIS workshop on geospatial techniques in R.

Many packages and functions can be used to plot maps of spatial data. For instance, ggplot as a function to plot spatial data using `geom_sf()`. I am personally a fan of tmap, which makes

many steps easier (but sometimes is less flexible).

A great tool for choosing colour is for instance [Colorbrewer](#). `viridisLite` provides another great resource to chose colours.

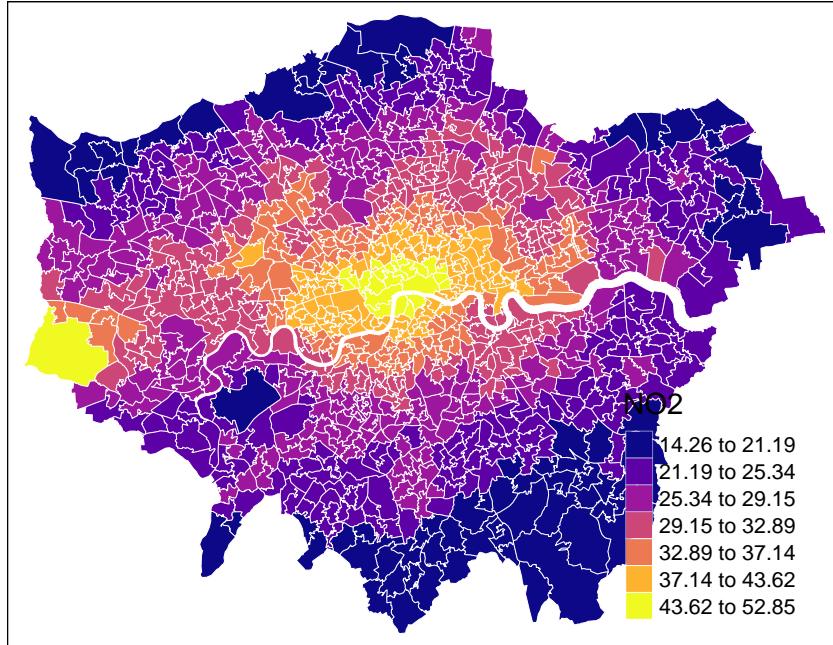
1.5.1 Tmaps

For instance, lets plot the NO2 estimates using `tmap + tm_fill()` (there are lots of alternatives like `tm_shape`, `tm_points()`, `tm_dots()`).

```
# Define colours
cols <- viridis(n = 7, direction = 1, option = "C")

mp1 <- tm_shape(msoa.spdf) +
  tm_fill(col = "no2",
          style = "fisher", # algorithm to def cut points
          n = 7, # Number of requested cut points
          palette = cols, # colours
          alpha = 1, # transparency
          title = "NO2",
          legend.hist = FALSE # histogram next to map?
    ) +
  tm_borders(col = "white", lwd = 0.5, alpha = 0.5)

mp1
```

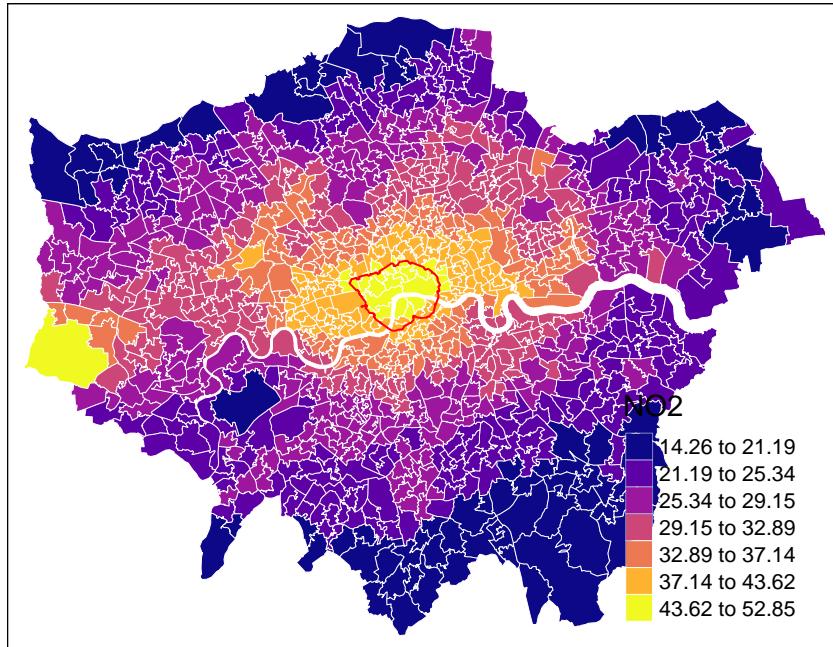


Tmap allows to easily combine different objects by defining a new object via `tm_shape()`.

```
# Define colours
cols <- viridis(n = 7, direction = 1, option = "C")

mp1 <- tm_shape(msoa.spdf) +
  tm_fill(col = "no2",
          style = "fisher", # algorithm to def cut points
          n = 7, # Number of requested cut points
          palette = cols, # colours
          alpha = 1, # transparency
          title = "NO2",
          legend.hist = FALSE # histogram next to map?
  ) +
  tm_borders(col = "white", lwd = 0.5, alpha = 0.5) +
  tm_shape(ulez.spdf) +
  tm_borders(col = "red", lwd = 1, alpha = 1)

mp1
```



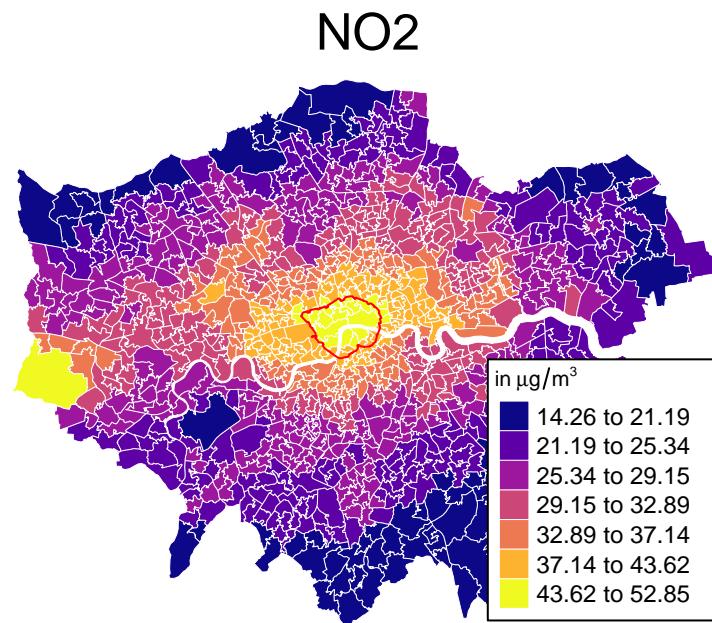
And it is easy to change the layout.

```
# Define colours
cols <- viridis(n = 7, direction = 1, option = "C")

mp1 <- tm_shape(msoa.spdf) +
  tm_fill(col = "no2",
    style = "fisher", # algorithm to def cut points
    n = 7, # Number of requested cut points
    palette = cols, # colours
    alpha = 1, # transparency
    title = expression('in'~mu*'g'/m^{3}),
    legend.hist = FALSE # histogram next to map?
  ) +
  tm_borders(col = "white", lwd = 0.5, alpha = 0.5) +
  tm_shape(ulez.spdf) +
  tm_borders(col = "red", lwd = 1, alpha = 1) +
  tm_layout(frame = FALSE,
    legend.frame = TRUE, legend.bg.color = TRUE,
    legend.position = c("right", "bottom"),
    legend.outside = FALSE,
    main.title = "NO2",
```

```
    main.title.position = "center",
    main.title.size = 1.6,
    legend.title.size = 0.8,
    legend.text.size = 0.8)
```

```
mp1
```

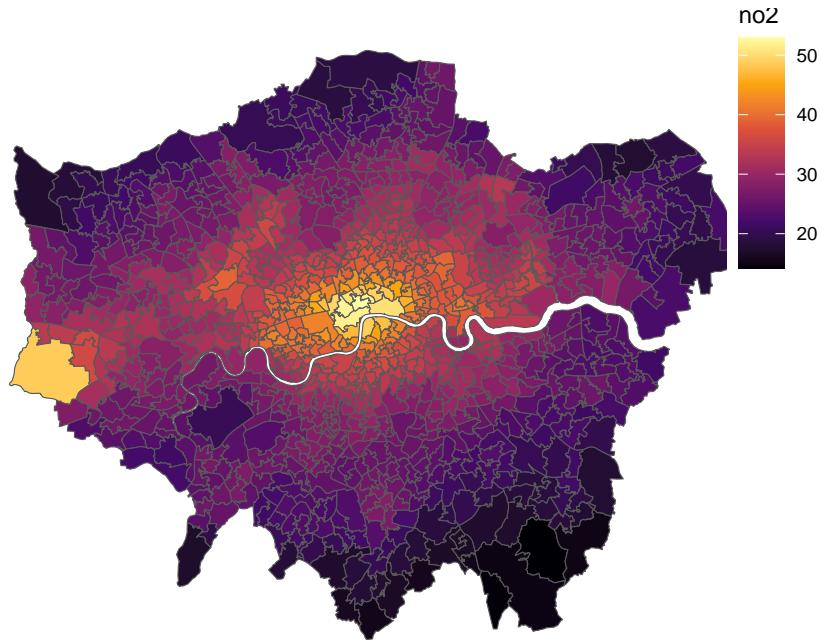


1.5.2 ggplot

For those of you have rather stick with the basic ggplot package, we can also use ggplot for spatial maps.

```
gp <- ggplot(msoa.spdf)+
  geom_sf(aes(fill = no2))+
  scale_fill_viridis_c(option = "B")+
  coord_sf(datum = NA)+
  theme_map()+
  theme(legend.position = c(.9, .6))

gp
```



```
# Get some larger scale boundaries
borough.spdf <- st_read(dsn = paste0("_data", "/statistical-gis-boundaries-london/ESRI"),
                         layer = "London_Borough_Excluding_MHW" # Note: no file ending
                         )

Reading layer `London_Borough_Excluding_MHW` from data source
`C:\work\Lehre\Geodata_Spatial_Regression_short\_data\statistical-gis-
boundaries-london\ESRI'
using driver `ESRI Shapefile'
Simple feature collection with 33 features and 7 fields
Geometry type: MULTIPOLYGON
Dimension:      XY
Bounding box:  xmin: 503568.2 ymin: 155850.8 xmax: 561957.5 ymax: 200933.9
Projected CRS: OSGB36 / British National Grid

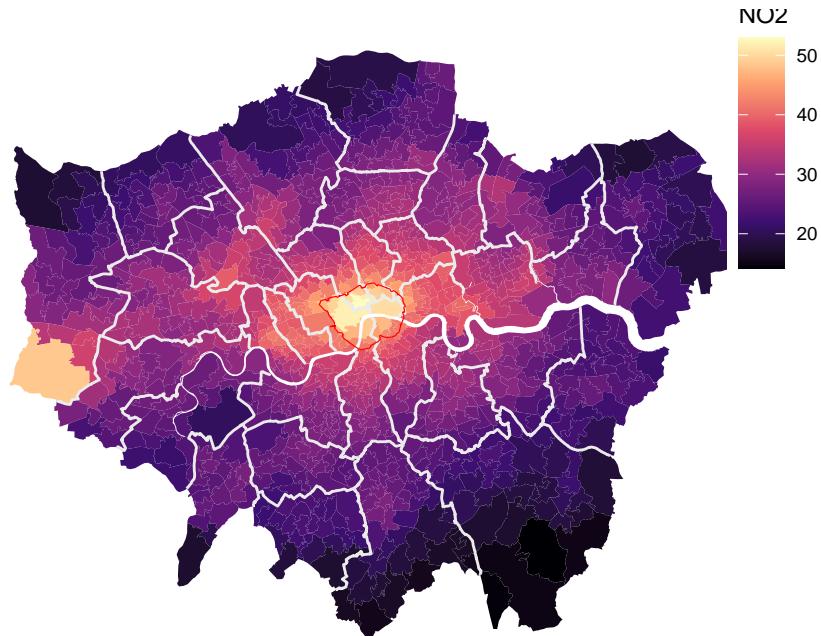
# transform to only inner lines
borough_inner <- ms_innerlines(borough.spdf)

# Plot with inner lines
gp <- ggplot(msoa.spdf)+
  geom_sf(aes(fill = no2), color = NA)+
```

```

scale_fill_viridis_c(option = "A")+
geom_sf(data = borough_inner, color = "gray92")+
geom_sf(data = ulez.spdf, color = "red", fill = NA)+
coord_sf(datum = NA)+
theme_map()+
labs(fill = "NO2")+
theme(legend.position = c(.9, .6))
gp

```



1.6 Exercise

- 1) What is the difference between a spatial “sf” object and a conventional “data.frame”? What’s the purpose of the function `st_drop_geometry()`?

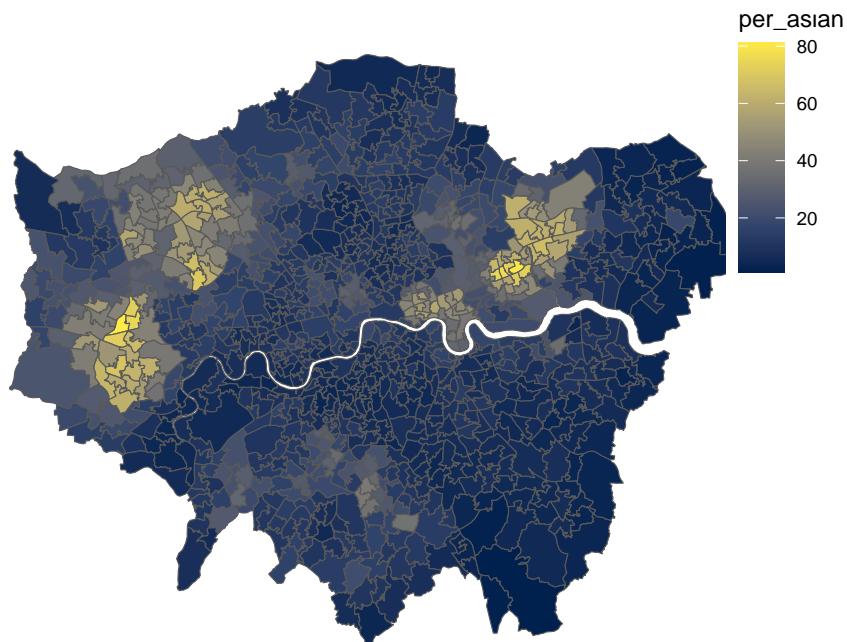
It’s the same. A spatial “sf” object just has an additional column containing the spatial coordinates.

- 2) Using `msoa.spdf`, please create a spatial data frame that contains only the MSOA areas that are within the ulez zone.

```
sub4.spdf <- msoa.spdf[ulez.spdf, ]
```

- 3) Please create a map for London (or only the msoa-ulez subset) which shows the share of Asian residents (or any other ethnic group).

```
gp <- ggplot(msoa.spdf)+  
  geom_sf(aes(fill = per_asian))+  
  scale_fill_viridis_c(option = "E") +  
  coord_sf(datum = NA) +  
  theme_map() +  
  theme(legend.position = c(.9, .6))  
gp
```



- 4) Please calculate the distance of each MSOA to the London city centre

- use google maps to get lon and lat,
- use `st_as_sf()` to create the spatial point
- use `st_distance()` to calculate the distance

```
#### Distance to city center  
# Define centre  
centre <- st_as_sf(data.frame(lon = -0.128120855701165,  
                             lat = 51.50725909644806),  
                     coords = c("lon", "lat"),
```

```

            crs = 4326)
# Reproject
centre <- st_transform(centre, crs = st_crs(msoa.spdf))
# Calculate distance
msoa.spdf$dist_centre <- as.numeric(st_distance(msoa.spdf, centre)) / 1000
# hist(msoa.spdf$dist_centre)

5) Can you create a plot with the distance to the city centre and pub counts next to each other?

# Define colours
cols <- viridis(n = 10, direction = 1, option = "B")
cols2 <- viridis(n = 10, direction = 1, option = "E")

mp1 <- tm_shape(msoa.spdf) +
  tm_fill(col = "dist_centre",
          style = "fisher", # algorithm to def cut points
          n = 10, # Number of requested cut points
          palette = cols, # colours
          alpha = 1, # transparency
          title = "Distance",
          legend.hist = FALSE # histogram next to map?
  ) +
  tm_borders(col = "white", lwd = 0.5, alpha = 0.5) +
  tm_layout(frame = FALSE,
            legend.frame = TRUE, legend.bg.color = TRUE,
            legend.position = c("right", "bottom"),
            legend.outside = FALSE,
            main.title = "Dist centre",
            main.title.position = "center",
            main.title.size = 1.6,
            legend.title.size = 0.8,
            legend.text.size = 0.8)

mp2 <- tm_shape(msoa.spdf) +
  tm_fill(col = "dist_centre",
          style = "quantile", # algorithm to def cut points
          n = 10, # Number of requested cut points
          palette = cols, # colours

```

```

alpha = 1, # transparency
title = "Distance",
legend.hist = FALSE # histogram next to map?
) +
tm_borders(col = "white", lwd = 0.5, alpha = 0.5) +
tm_layout(frame = FALSE,
          legend.frame = TRUE, legend.bg.color = TRUE,
          legend.position = c("right", "bottom"),
          legend.outside = FALSE,
          main.title = "Dist centre",
          main.title.position = "center",
          main.title.size = 1.6,
          legend.title.size = 0.8,
          legend.text.size = 0.8)

mp3 <- tm_shape(msoa.spdf) +
  tm_fill(col = "pubs_count",
          style = "fisher", # algorithm to def cut points
          n = 10, # Number of requested cut points
          palette = cols, # colours
          alpha = 1, # transparency
          title = "Count",
          legend.hist = FALSE # histogram next to map?
  ) +
  tm_borders(col = "white", lwd = 0.5, alpha = 0.5) +
  tm_layout(frame = FALSE,
            legend.frame = TRUE, legend.bg.color = TRUE,
            legend.position = c("right", "bottom"),
            legend.outside = FALSE,
            main.title = "Pubs",
            main.title.position = "center",
            main.title.size = 1.6,
            legend.title.size = 0.8,
            legend.text.size = 0.8)

mp4 <- tm_shape(msoa.spdf) +
  tm_fill(col = "pubs_count",
          style = "quantile", # algorithm to def cut points
          n = 10, # Number of requested cut points

```

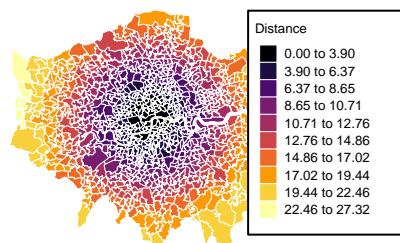
```

palette = cols, # colours
alpha = 1, # transparency
title = "Count",
legend.hist = FALSE # histogram next to map?
) +
tm_borders(col = "white", lwd = 0.5, alpha = 0.5) +
tm_layout(frame = FALSE,
          legend.frame = TRUE, legend.bg.color = TRUE,
          legend.position = c("right", "bottom"),
          legend.outside = FALSE,
          main.title = "Pubs",
          main.title.position = "center",
          main.title.size = 1.6,
          legend.title.size = 0.8,
          legend.text.size = 0.8)

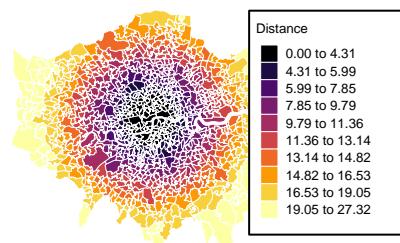
```

```
tmap_arrange(mp1, mp2, mp3, mp4, ncol = 2, nrow = 2)
```

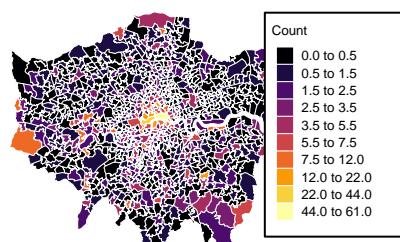
Dist centre



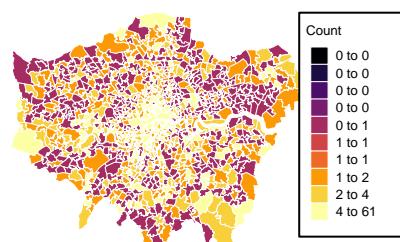
Dist centre



Pubs

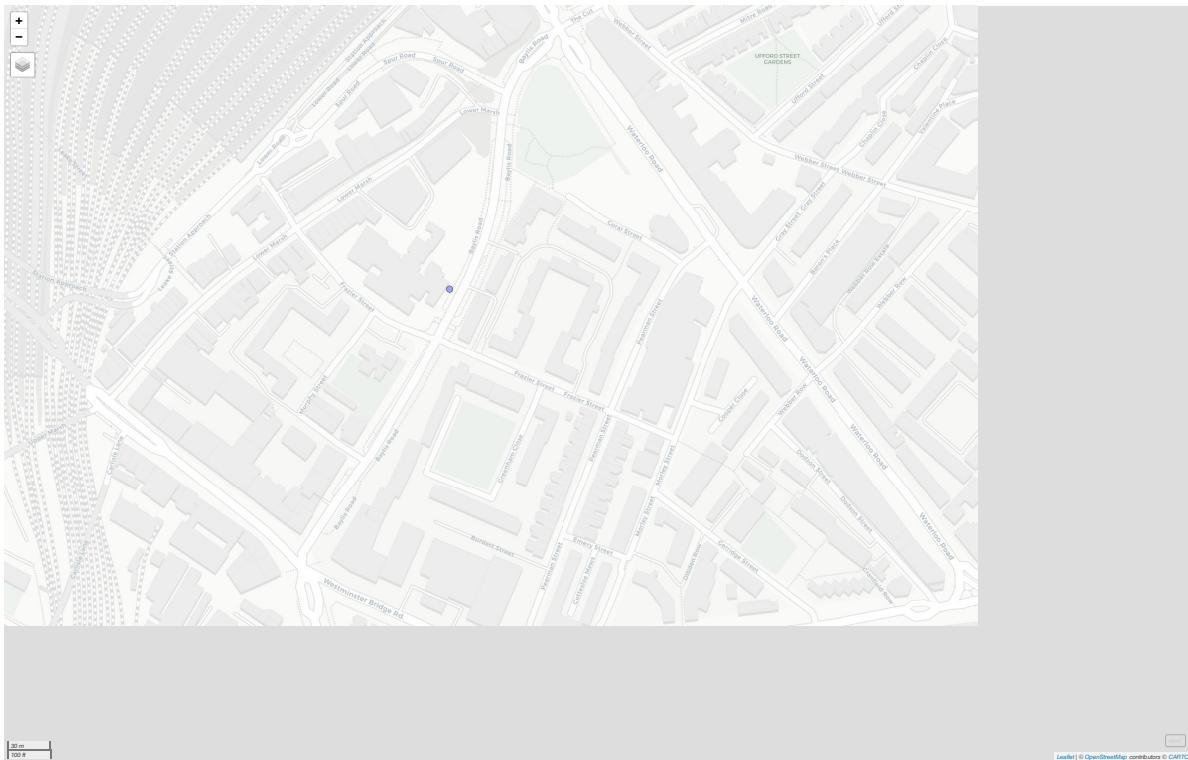


Pubs



1.6.1 Geographic center

```
# Make one single shape  
london <- st_union(msoa.spdf)  
  
# Calculate center  
cent <- st_centroid(london)  
  
mapview(cent)
```



2 Spatial Relationships

Required packages

```
pkgs <- c("sf", "mapview", "spdep", "spatialreg", "tmap", "viridisLite") # note: load spde
lapply(pkgs, require, character.only = TRUE)
```

Session info

```
sessionInfo()

R version 4.3.1 (2023-06-16 ucrt)
Platform: x86_64-w64-mingw32/x64 (64-bit)
Running under: Windows 10 x64 (build 19044)

Matrix products: default

locale:
[1] LC_COLLATE=English_United Kingdom.utf8
[2] LC_CTYPE=English_United Kingdom.utf8
[3] LC_MONETARY=English_United Kingdom.utf8
[4] LC_NUMERIC=C
[5] LC_TIME=English_United Kingdom.utf8

time zone: Europe/Berlin
tzcode source: internal

attached base packages:
[1] stats      graphics   grDevices  utils      datasets   methods    base

other attached packages:
[1] viridisLite_0.4.2 tmap_3.3-3       spatialreg_1.2-9  Matrix_1.5-4.1
[5] spdep_1.2-8     spData_2.3.0      mapview_2.11.0   sf_1.0-13
```

```

loaded via a namespace (and not attached):
[1] xfun_0.39           raster_3.6-23      htmlwidgets_1.6.2 lattice_0.21-8
[5] vctrs_0.6.3         tools_4.3.1       crosstalk_1.2.0   LearnBayes_2.15.1
[9] generics_0.1.3      parallel_4.3.1    sandwich_3.0-2   stats4_4.3.1
[13] tibble_3.2.1        proxy_0.4-27     fansi_1.0.4      pkgconfig_2.0.3
[17] KernSmooth_2.23-21 satellite_1.0.4 RColorBrewer_1.1-3 leaflet_2.1.2
[21] webshot_0.5.5       lifecycle_1.0.3  compiler_4.3.1   deldir_1.0-9
[25] munsell_0.5.0       terra_1.7-39     leafsync_0.1.0   codetools_0.2-
19
[29] stars_0.6-1         htmltools_0.5.5  class_7.3-22    pillar_1.9.0
[33] MASS_7.3-60          classInt_0.4-9   lwgeom_0.2-13   wk_0.7.3
[37] abind_1.4-5          boot_1.3-28.1   multcomp_1.4-25 nlme_3.1-162
[41] tidyselect_1.2.0     digest_0.6.32   mvtnorm_1.2-2   dplyr_1.1.2
[45] splines_4.3.1        fastmap_1.1.1   grid_4.3.1      colorspace_2.1-
0
[49] expm_0.999-7         cli_3.6.1       magrittr_2.0.3  base64enc_0.1-3
[53] dichromat_2.0-0.1    XML_3.99-0.14  survival_3.5-5 utf8_1.2.3
[57] TH.data_1.1-2        leafem_0.2.0    e1071_1.7-13   scales_1.2.1
[61] sp_1.6-1              rmarkdown_2.23  zoo_1.8-12    png_0.1-8
[65] coda_0.19-4          evaluate_0.21  knitr_1.43    tmaptools_3.1-1
[69] s2_1.1.4              rlang_1.1.1    Rcpp_1.0.10   glue_1.6.2
[73] DBI_1.1.3             rstudioapi_0.14 jsonlite_1.8.5 R6_2.5.1
[77] units_0.8-2

```

Reload data from previous session

```
load("_data/msoa2_spatial.RData")
```

2.1 Spatial interdependence

We can not only use coordinates and geo-spatial information to connect different data sources, we can also explicitly model spatial (inter)dependence in the analysis of our data. In many instance, accounting for spatial dependence might even be necessary to avoid biased point estimates and standard errors, as observations are often not independent and identically distributed.

Tobler's first law of geography has been used extensively (11,584 citation in 2023-06) to describe spatial dependence: 'Everything is related to everything else, but near things are more related than distant things' (Tobler 1970).

Note

Tobler's first law is a bit of story

And it has been labeled as an excuse to not think too much about the reasons for spatial dependence or auto-correlation. For instance, measurement error, omitted variables, or inappropriate levels of aggregation are among reasons for auto-correlation (Pebesma and Bivand 2023).

We will come back to the reasons of spatial dependence. However, for now, we are interested in some tools to detect and analyse spatial relations.

To analyse spatial relations, we first need to define some sort of connectivity between units (e.g. similar to network analysis). There are some obvious candidates that can be used to define these relations here: adjacency and proximity.

2.2 W: Connectivity between units

The connectivity between units is usually represented in a matrix \mathbf{W} . There is an ongoing debate about the importance of spatial weights for spatial econometrics and about the right way to specify weights matrices (LeSage and Pace 2014; Neumayer and Plümper 2016). The following graph shows some possible options in how to define connectivity between units.

In spatial econometrics, the spatial connectivity (as shown above) is usually represented by a spatial weights matrix \mathbf{W} :

\$\$

```
\boldsymbol{\mathbf{W}} = \begin{bmatrix} w_{11} & w_{12} & \dots & w_{1n} \\ w_{21} & w_{22} & \dots & w_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ w_{n1} & w_{n2} & \dots & w_{nn} \end{bmatrix}
```

\$\$ The spatial weights matrix \mathbf{W} is an $N \times N$ dimensional matrix with elements w_{ij} specifying the relation or connectivity between each pair of units i and j .

Note: The diagonal elements $w_{i,i} = w_{1,1}, w_{2,2}, \dots, w_{n,n}$ of \mathbf{W} are always zero. No unit is a neighbour of itself. This is not true for spatial multiplier matrices (as we will see later).

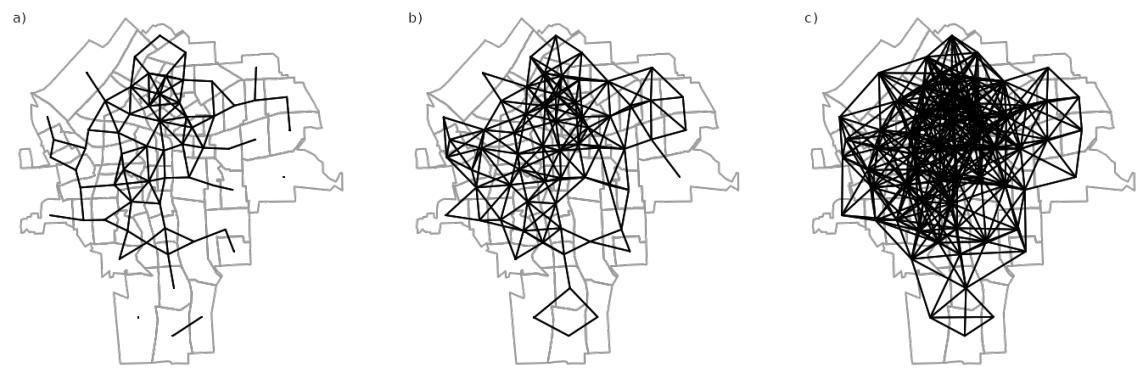


Figure 2.1: Figure: Different measures of connectivity, Source: R. S. Bivand and Rudel (2018)

2.2.1 Contiguity weights

A very common type of spatial weights. Binary specification, taking the value 1 for neighbouring units (queens: sharing a common edge; rook: sharing a common border), and 0 otherwise.

Contiguity weights $w_{i,j}$, where

$$w_{i,j} = \begin{cases} 1 & \text{if } i \text{ and } j \text{ neighbours} \\ 0 & \text{otherwise} \end{cases}$$

A contiguity weights matrix with three units, where unit 1 and unit 3 are neighbours, while unit 2 has no neighbours would look like this:

$$\mathbf{W} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

- Sparse matrices
- Problem of ‘island’: units without neighbours (if I calculate an average of their neighbours, would that be zero, or NA, or a mean?)

Lets create a contiguity weights matrix (Queens neighbours) for the London MSOAs: we create a neighbours list (`nb`) using `poly2nb()`, which is an efficient way of storing \mathbf{W} . A `snap` of 1 meter accounts for potential lacks of accuracy between lines and points.

```
# Contiguity (Queens) neighbours weights
queens.nb <- poly2nb(msoa.spdf,
                      queen = TRUE, # a single shared boundary point meets the contiguity condition
                      snap = 1) # we consider points in 1m distance as 'touching'
summary(queens.nb)
```

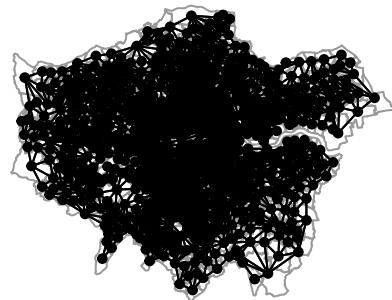
Neighbour list object:
Number of regions: 983
Number of nonzero links: 5648
Percentage nonzero weights: 0.5845042
Average number of links: 5.745677
Link number distribution:

2	3	4	5	6	7	8	9	10	11	12	13
9	39	130	264	273	169	66	19	5	6	2	1

9 least connected regions:

```
160 270 475 490 597 729 755 778 861 with 2 links  
1 most connected region:  
946 with 13 links
```

```
# Lets plot that  
plot(st_geometry(msoa.spdf), border = "grey60")  
plot(queens.nb, st_centroid(st_geometry(msoa.spdf)),  
     add = TRUE, pch = 19, cex = 0.6)
```



```
# We can also transform this into a matrix W  
W <- nb2mat(queens.nb, style = "B")  
print(W[1:10, 1:10])
```

```
[,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]  
1    0    0    0    0    0    0    0    0    0    0  
2    0    0    1    0    0    0    0    0    0    0  
3    0    1    0    0    1    0    0    0    0    0  
4    0    0    0    0    0    1    0    0    0    1  
5    0    0    1    0    0    1    1    0    0    0  
6    0    0    0    1    1    0    1    0    1    1  
7    0    0    0    0    1    1    0    1    1    0
```

8	0	0	0	0	0	0	1	0	0	0
9	0	0	0	0	0	1	1	0	0	1
10	0	0	0	1	0	1	0	0	1	0

💡 Question

Among those first 10 units that you see above, which are the neighbours of unit number 6?

Why is the diagonal of this matrix all zero?

Overall, the matrix W has dimensions $N \times N$, a row and a column for each observation. The value in a cell shows how units i (row number) and j (column number) are related to each other.

```
dim(W)
```

```
[1] 983 983
```

The row and column sums indicate the number of neighbours of each observation.

```
rowSums(W) [1:10]
```

```
1 2 3 4 5 6 7 8 9 10
11 6 7 5 5 6 6 6 6 5
```

```
colSums(W) [1:10]
```

```
[1] 11 6 7 5 5 6 6 6 6 5
```

Adjacency or graph-based neighbour's weights matrices are usually symmetric. If unit 1 is a neighbour of unit 55, then unit 55 is also a neighbour of unit 1.

💡 Higher Order Neighbours

Your neighbours have neighbours too, and they are called higher (second) order neighbours.

The neighbours of your neighbour's neighbours are third order neighbours.

You can use `nblag()` to calculate higher order neighbour relations.

2.2.2 Distance based weights

Another common type uses the distance d_{ij} between each unit i and j .

- Inverse distance weights $w_{i,j} = \frac{1}{d_{ij}^\alpha}$, where α define the strength of the spatial decay.

$$\mathbf{W} = \begin{bmatrix} 0 & \frac{1}{d_{ij}^\alpha} & \frac{1}{d_{ij}^\alpha} \\ \frac{1}{d_{ij}^\alpha} & 0 & \frac{1}{d_{ij}^\alpha} \\ \frac{1}{d_{ij}^\alpha} & \frac{1}{d_{ij}^\alpha} & 0 \end{bmatrix}$$

- Dense matrices
- Specifying thresholds may be useful (to get rid of very small non-zero weights)

For now, we will just specify a neighbours list with a distance threshold of 3km using `dneigh()`. An alternative would be k nearest neighbours using `kneigh()`. We will do the inverse weighting later.

```
# Create centroids
coords <- st_geometry(st_centroid(msoa.spdf))
```

Warning: `st_centroid` assumes attributes are constant over geometries

```
# Neighbours within 3km distance
dist_3.nb <- dneigh(coords, d1 = 0, d2 = 3000)
summary(dist_3.nb)
```

Neighbour list object:

Number of regions: 983

Number of nonzero links: 22086

Percentage nonzero weights: 2.285652

Average number of links: 22.46796

Link number distribution:

```
1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26
4  3  7 13 11 14 14 17 26 22 26 30 33 34 46 34 59 43 38 30 25 19 22 15 21 14
27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47
23 17 17 23 28 19 26 24 29 24 27 25 22 18  8 10 12  5  3  2  1
```

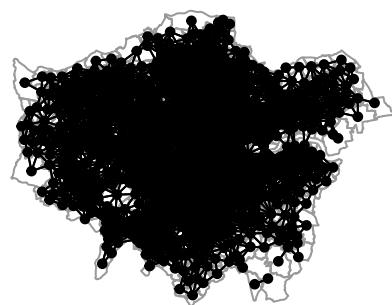
4 least connected regions:

158 160 463 959 with 1 link

1 most connected region:

545 with 47 links

```
# Lets plot that
plot(st_geometry(msoa.spdf), border = "grey60")
plot(dist_3.nb, coords,
     add = TRUE, pch = 19, cex = 0.6)
```



And you can see that the matrix is not so sparse anymore:

```
W2 <- nb2mat(dist_3.nb, style = "B")
W2[1:10, 1:10]
```

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[,7]	[,8]	[,9]	[,10]
1	0	0	0	0	0	0	0	0	0	0
2	0	0	1	0	1	0	0	0	0	0
3	0	1	0	0	1	1	1	0	0	0
4	0	0	0	0	1	1	1	0	1	1
5	0	1	1	1	0	1	1	1	1	1
6	0	0	1	1	1	0	1	1	1	1
7	0	0	1	1	1	1	0	1	1	1
8	0	0	0	0	1	1	1	0	1	0
9	0	0	0	1	1	1	1	1	0	1
10	0	0	0	1	1	1	1	0	1	0

2.3 Normalization of W

Normalizing ensures that the parameter space of the spatial multiplier is restricted to $-1 < \rho > 1$, and the multiplier matrix is non-singular (don't worry, more on this later).

The main message: Normalizing your weights matrix is always a good idea. Otherwise, the spatial parameters might blow up – if you can estimate the model at all. It also ensure easy interpretation of spillover effects.

Again, how to normalize a weights matrix is subject of debate (LeSage and Pace 2014; Neumayer and Plümper 2016).

2.3.1 Row-normalization

Row-normalization divides each non-zero weight by the sum of all weights of unit i , which is the sum of the row.

$$\frac{w_{ij}}{\sum_j^n w_{ij}}$$

- With contiguity weights, spatially lagged variables contain mean of this variable among the neighbours of i
- Proportions between units such as distances get lost (can be bad!)
- Can induce asymmetries: $w_{ij} \neq w_{ji}$

For instance, we can use row-normalization for the Queens neighbours created above, and create a neighbours list with spatial weights.

```
queens.lw <- nb2listw(queens.nb,
                        style = "W") # W ist row-normalization
summary(queens.lw)
```

Characteristics of weights list object:

Neighbour list object:

Number of regions: 983

Number of nonzero links: 5648

Percentage nonzero weights: 0.5845042

Average number of links: 5.745677

Link number distribution:

```
2   3   4   5   6   7   8   9   10  11  12  13
```

```

9 39 130 264 273 169 66 19 5 6 2 1
9 least connected regions:
160 270 475 490 597 729 755 778 861 with 2 links
1 most connected region:
946 with 13 links

```

```

Weights style: W
Weights constants summary:
  n      nn   S0      S1      S2
W 983 966289 983 355.1333 4017.47

```

To see what happened, let's look at our example in matrix format again.

```

# transform into matrix with row-normalization
W_norm <- nb2mat(queens.nb, style = "W")
print(W_norm[1:10, 1:10])

```

```

[,1]      [,2]      [,3]      [,4]      [,5]      [,6]      [,7]      [,8]
1 0 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000
2 0 0.0000000 0.1666667 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000
3 0 0.1428571 0.0000000 0.0000000 0.1428571 0.0000000 0.0000000 0.0000000
4 0 0.0000000 0.0000000 0.0000000 0.0000000 0.2000000 0.0000000 0.0000000
5 0 0.0000000 0.2000000 0.0000000 0.0000000 0.2000000 0.2000000 0.0000000
6 0 0.0000000 0.0000000 0.1666667 0.1666667 0.0000000 0.1666667 0.0000000
7 0 0.0000000 0.0000000 0.0000000 0.1666667 0.1666667 0.0000000 0.1666667
8 0 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.1666667 0.0000000
9 0 0.0000000 0.0000000 0.0000000 0.0000000 0.1666667 0.1666667 0.0000000
10 0 0.0000000 0.0000000 0.2000000 0.0000000 0.2000000 0.0000000 0.0000000
     [,9]      [,10]
1 0.0000000 0.0000000
2 0.0000000 0.0000000
3 0.0000000 0.0000000
4 0.0000000 0.2000000
5 0.0000000 0.0000000
6 0.1666667 0.1666667
7 0.1666667 0.0000000
8 0.0000000 0.0000000
9 0.0000000 0.1666667
10 0.2000000 0.0000000

```

Question

Overall, how many neighbours does unit 9 have (including all columns)? How do you know?

```
rowSums(W)[9]
```

We can also use the nb object to see which ones the neighbours are. Here, for instance, neighbours of unit 6:

```
queens.nb[6]
```

```
[[1]]  
[1] 4 5 7 9 10 462
```

This fits to what we see in the matrix above.

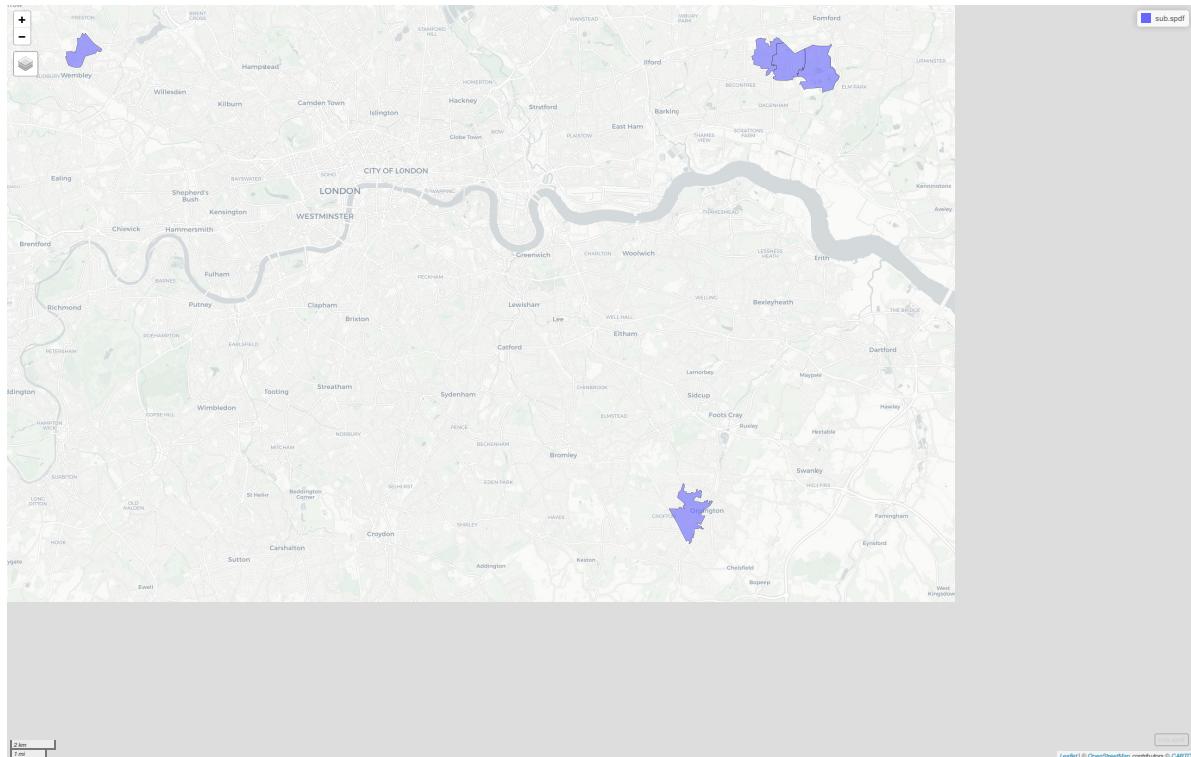
Warning

Note that row-normalization has some undesirable properties when we use some non-contiguous based neighbour relations, such as distance based neighbours.

The problem: It obscures the proportion due to dividing by a row-specific value.

Let's construct a hypothetical example

```
# Subset of 5 units  
sub.spdf <- msoa.spdf[c(4, 5, 6, 102, 150), ]  
mapview(sub.spdf)
```



We construct the **inverse-distance weighted 2 nearest neighbors**.

```
# 2 closest neighbours
sub.coords <- st_geometry(st_centroid(sub.spdf))
```

Warning: `st_centroid` assumes attributes are constant over geometries

```
knn.nb <- knearneigh(sub.coords,
                      k = 2) # number of nearest neighbours
```

Warning in `knearneigh(sub.coords, k = 2)`: k greater than one-third of the number of data points

```
knn.nb <- knn2nb(knn.nb)
summary(knn.nb)
```

```

Neighbour list object:
Number of regions: 5
Number of nonzero links: 10
Percentage nonzero weights: 40
Average number of links: 2
Non-symmetric neighbours list
Link number distribution:

2
5
5 least connected regions:
1 2 3 4 5 with 2 links
5 most connected regions:
1 2 3 4 5 with 2 links

# listw with inverse-distance based weights
sub.lw <- nb2listwdist(knn.nb,
                        x = sub.coords, # needed for idw
                        type = "idw", # inverse distance weighting
                        alpha = 1, # the decay parameter for distance weighting
                        style = "raw") # without normalization

W_sub <- listw2mat(sub.lw)
formatC(W_sub, format = "f", digits = 6)

[,1]      [,2]      [,3]      [,4]      [,5]
1 "0.000000" "0.000414" "0.000723" "0.000000" "0.000000"
2 "0.000414" "0.000000" "0.000962" "0.000000" "0.000000"
3 "0.000723" "0.000962" "0.000000" "0.000000" "0.000000"
4 "0.000000" "0.000033" "0.000032" "0.000000" "0.000000"
5 "0.000049" "0.000000" "0.000049" "0.000000" "0.000000"

```

As you can see, units 1, 2, 3 have relatively proximate neighbours (.e.g inverse distance 0.000962: 3 zeros). Units 4 and 5, in contrast, have only very distant neighbours (e.g. inverse distance 0.000049: 4 zeros).

Now, see what happens when we use row-normalization.

```

sub.lw <- nb2listwdist(knn.nb,
                        x = sub.coords, # needed for idw
                        type = "idw", # inverse distance weighting
                        alpha = 1, # the decay parameter for distance weighting

```

```

style = "W") # for row normalization
W_sub <- listw2mat(sub.lw)
formatC(W_sub, format = "f", digits = 6)

 [,1]      [,2]      [,3]      [,4]      [,5]
1 "0.000000" "0.364083" "0.635917" "0.000000" "0.000000"
2 "0.300879" "0.000000" "0.699121" "0.000000" "0.000000"
3 "0.429123" "0.570877" "0.000000" "0.000000" "0.000000"
4 "0.000000" "0.507955" "0.492045" "0.000000" "0.000000"
5 "0.499360" "0.000000" "0.500640" "0.000000" "0.000000"

```

All rows sum up to 1, but the strength of the relation is now similar for the distant units 4 and 5, and the proximate units 1, 2, 3.

2.3.2 Maximum eigenvalues normalization

Maximum eigenvalues normalization divides each non-zero weight by the overall maximum eigenvalue λ_{max} . Each element of \mathbf{W} is divided by the same scalar parameter, which preserves the relations.

$$\frac{\mathbf{W}}{\lambda_{max}}$$

- Interpretation may become more complicated
- Keeps proportions of connectivity strengths across units (relevant esp. for distance based \mathbf{W})

We use eigenvalue normalization for the inverse distance neighbours. We use `nb2listwdist()` to create weight inverse distance based weights and normalize in one step.

```

coords <- st_geometry(st_centroid(msoa.spdf))

Warning: st_centroid assumes attributes are constant over geometries

idw.lw <- nb2listwdist(dist_3.nb,
                        x = coords, # needed for idw
                        type = "idw", # inverse distance weighting
                        alpha = 1, # the decay parameter for distance weighting
                        style = "minmax") # for eigenvalue normalization

```

```
summary(idw.lw)
```

Characteristics of weights list object:

Neighbour list object:

Number of regions: 983

Number of nonzero links: 22086

Percentage nonzero weights: 2.285652

Average number of links: 22.46796

Link number distribution:

```
 1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26  
 4  3  7 13 11 14 14 17 26 22 26 30 33 34 46 34 59 43 38 30 25 19 22 15 21 14  
27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47  
23 17 17 23 28 19 26 24 29 24 27 25 22 18  8 10 12  5  3  2  1
```

4 least connected regions:

158 160 463 959 with 1 link

1 most connected region:

545 with 47 links

Weights style: minmax

Weights constants summary:

n	nn	S0	S1	S2
minmax	983	966289	463.6269	23.92505
				1117.636

Examples from above: See how this keeps the proportions in our example. Instead of transforming values to sum up to 1 in each row, we now have much smaller values for 4 and 5 then we have for the proximate units 1, 2, 3.

```
sub.lw <- nb2listwdist(knn.nb,  
                         x = sub.coords, # needed for idw  
                         type = "idw", # inverse distance weighting  
                         alpha = 1, # the decay parameter for distance weighting  
                         style = "minmax") # for eigenvalue normalization
```

```
W_sub <- listw2mat(sub.lw)  
formatC(W_sub, format = "f", digits = 6)
```

```
[,1]      [,2]      [,3]      [,4]      [,5]  
1 "0.000000" "0.245687" "0.429123" "0.000000" "0.000000"  
2 "0.245687" "0.000000" "0.570877" "0.000000" "0.000000"  
3 "0.429123" "0.570877" "0.000000" "0.000000" "0.000000"
```

```

4 "0.000000" "0.019663" "0.019047" "0.000000" "0.000000"
5 "0.029099" "0.000000" "0.029174" "0.000000" "0.000000"

```

2.4 Islands / missings

In practice, we often have a problem with islands. If we use contiguity based or distance based neighbour definitions, some units may end up with empty neighbours sets: they just do not touch any other unit and do not have a neighbour within a specific distance. This however creates a problem: what is the value in the neighbouring units?

The `zero.policy` option in `spdep` allows to proceed with empty neighbours sets. However, many further functions may run into problems and return errors. It often makes sense to either drop islands, to choose weights which always have neighbours (e.g. k nearest), or impute empty neighbours sets by using the nearest neighbours.

2.5 Global Autocorrelation

If spatially close observations are more likely to exhibit similar values, we cannot handle observations as if they were independent.

$$E(\varepsilon_i \varepsilon_j) \neq E(\varepsilon_i)E(\varepsilon_j) = 0$$

This violates a basic assumption of the conventional OLS model. We will talk more about whether that is good or bad (any guess?).

2.5.1 Visualization

There is one very easy and intuitive way of detecting spatial autocorrelation: Just look at the map. We do so by using `tmap` for plotting the share of home owners.

```

mp1 <- tm_shape(msoa.spdf) +
  tm_fill(col = "per_owner",
          #style = "cont",
          style = "fisher", n = 8,
          title = "Median",
          palette = viridis(n = 8, direction = -1, option = "C"),
          legend.hist = TRUE) +
  tm_borders(col = "black", lwd = 1) +
  tm_layout(legend.frame = TRUE, legend.bg.color = TRUE,
            ...

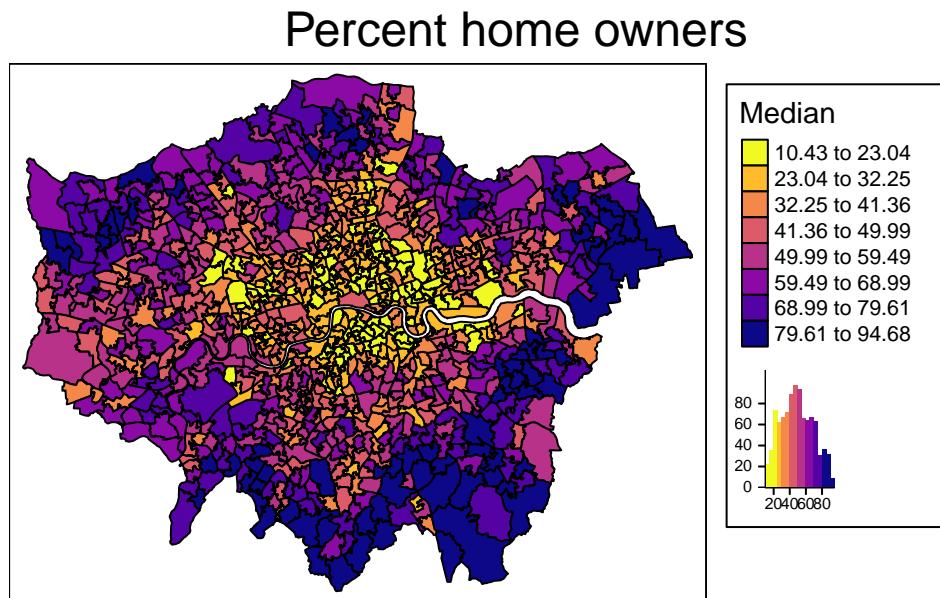
```

```

#legend.position = c("right", "bottom"),
legend.outside = TRUE,
main.title = "Percent home owners",
main.title.position = "center",
title.snap.to.legend = TRUE)

mp1

```

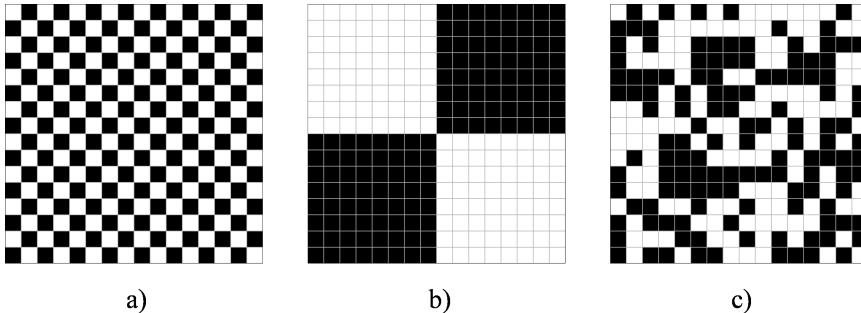


We definitely see some clusters with spatial units having a low share of home owner (e.g. in the city center), and other clusters where home ownership is high (e.g. suburbs in the south and east, such as Bromley or Havering).

However, this is (to some degree) dependent on how we define cutoffs and coloring of the map: the Modifiable Areal Unit Problem (Wong 2009).

💡 Question

Which of the following three checkerboards has no (or the lowest) autocorrelation?



Would your answer be the same if we would aggregate the data to four larger areas / districts using the average within each of the four districts?

2.5.2 Moran's I

The most common and well known statistic for spatial dependence or autocorrelation is Moran's I, which goes back to Moran (1950) and Cliff and Ord (1972). For more extensive materials on Moran's I see for instance Kelejian and Piras (2017), Chapter 11.

To calculate Moran's I, we first define a neighbours weights matrix W .

Global Moran's I test statistic:

$$I = \frac{N \sum_i \sum_j w_{ij} (y_i - \bar{y})(y_j - \bar{y})}{S_0 \sum_i (y_i - \bar{y})^2}, \text{ where } S_0 = \sum_{i=1}^N \sum_{j=1}^N w_{ij}$$

It is often written with deviations z

\$\$

```
\bm{I} = \frac{N}{S_0} \frac{\sum_i \sum_j w_{ij} (z_i)(z_j)}{\sum_i (z_i)^2}, \text{ where } S_0 = \sum_{i=1}^N \sum_{j=1}^N w_{ij}
```

\$\$

Note that in the case of row-standardized weights, $S_0 = N$. The I can be interpreted as: *Relation of the deviation from the mean value between unit i and neighbours of unit i* . Basically, this measures correlation between neighbouring values.

- Negative values: negative autocorrelation
- Around zero: no autocorrelation

- Positive values: positive autocorrelation

To calculate Moran's I, we first need to define the relationship between units. As in the previous example, we define contiguity weights and distance-based weights.

```
# Contiguity (Queens) neighbours weights
queens.nb <- poly2nb(msoa.spdf,
                      queen = TRUE,
                      snap = 1) # we consider points in 1m distance as 'touching'
queens.lw <- nb2listw(queens.nb,
                      style = "W")

# Neighbours within 3km distance
coords <- st_geometry(st_centroid(msoa.spdf))
```

Warning: `st_centroid` assumes attributes are constant over geometries

```
dist_3.nb <- dnearneigh(coords,
                           d1 = 0, d2 = 3000)
idw.lw <- nb2listwdist(dist_3.nb,
                         x = coords, # needed for idw
                         type = "idw", # inverse distance weighting
                         alpha = 1, # the decay parameter for distance weighting
                         style = "minmax") # for eigenvalue normalization
```

Subsequently, we can calculate the average correlation between neighbouring units.

For contiguity weights, we get:

```
# Global Morans I test of housing values based on contiguity weights
moran.test(msoa.spdf$per_owner, listw = queens.lw, alternative = "two.sided")
```

```
Moran I test under randomisation

data: msoa.spdf$per_owner
weights: queens.lw

Moran I statistic standard deviate = 38.161, p-value < 2.2e-16
alternative hypothesis: two.sided
sample estimates:
```

Moran I statistic	Expectation	Variance
0.728706855	-0.001018330	0.000365663

And for inverse distance weighting, we get:

```
# Global Morans I test of housing values based on idw
moran.test(msoa.spdf$per_owner, listw = idw.lw, alternative = "two.sided")
```

```
Moran I test under randomisation

data: msoa.spdf$per_owner
weights: idw.lw

Moran I statistic standard deviate = 65.853, p-value < 2.2e-16
alternative hypothesis: two.sided
sample estimates:
Moran I statistic      Expectation      Variance
0.6838957350     -0.0010183299     0.0001081719
```

Interpretation: In both cases, we have very strong autocorrelation between neighbouring/closer units (~.7). It barely matters which of the weights matrices we use. This autocorrelation is highly significant. we can thus reject the Null that units are independent of each other (at least at this spatial level and for the share of home owners).

2.5.3 Residual-based Moran's I

We can also use the same Moran's I test to inspect spatial autocorrelation in residuals from an estimated linear model.

Let's start with an intercept only model.

```
lm0 <- lm(per_owner ~ 1, msoa.spdf)
lm.morantest(lm0, listw = queens.lw, alternative = "two.sided")
```

```
Global Moran I for regression residuals

data:
model: lm(formula = per_owner ~ 1, data = msoa.spdf)
```

```

weights: queens.lw

Moran I statistic standard deviate = 38.177, p-value < 2.2e-16
alternative hypothesis: two.sided
sample estimates:
Observed Moran I      Expectation      Variance
0.7287068548     -0.0010183299    0.0003653613

```

This is exactly what we have received in the general case of Moran's I.

Now, lets add some predictors. For instance, the distance to the city centre, and the population density may be strongly related to the home ownership rates and explain parts of the spatial dependence.

```

### Distance to city center
# Define centre
centre <- st_as_sf(data.frame(lon = -0.128120855701165,
                                lat = 51.50725909644806),
                      coords = c("lon", "lat"),
                      crs = 4326)
# Reproject
centre <- st_transform(centre, crs = st_crs(msoa.spdf))
# Calculate distance
msoa.spdf$dist_centre <- as.numeric(st_distance(msoa.spdf, centre)) / 1000
# hist(msoa.spdf$dist_centre)

### Run model with predictors
lm1 <- lm(per_owner ~ dist_centre + POPDEN, msoa.spdf)
lm.morantest(lm1, listw = queens.lw, alternative = "two.sided")

```

Global Moran I for regression residuals

```

data:
model: lm(formula = per_owner ~ dist_centre + POPDEN, data = msoa.spdf)
weights: queens.lw

Moran I statistic standard deviate = 22.674, p-value < 2.2e-16
alternative hypothesis: two.sided
sample estimates:
Observed Moran I      Expectation      Variance
0.4298146060     -0.0024065617    0.0003633607

```

There is still considerable auto-correlation in the residuals. However, we have reduce it by a substantial amount with two very simple control variables.

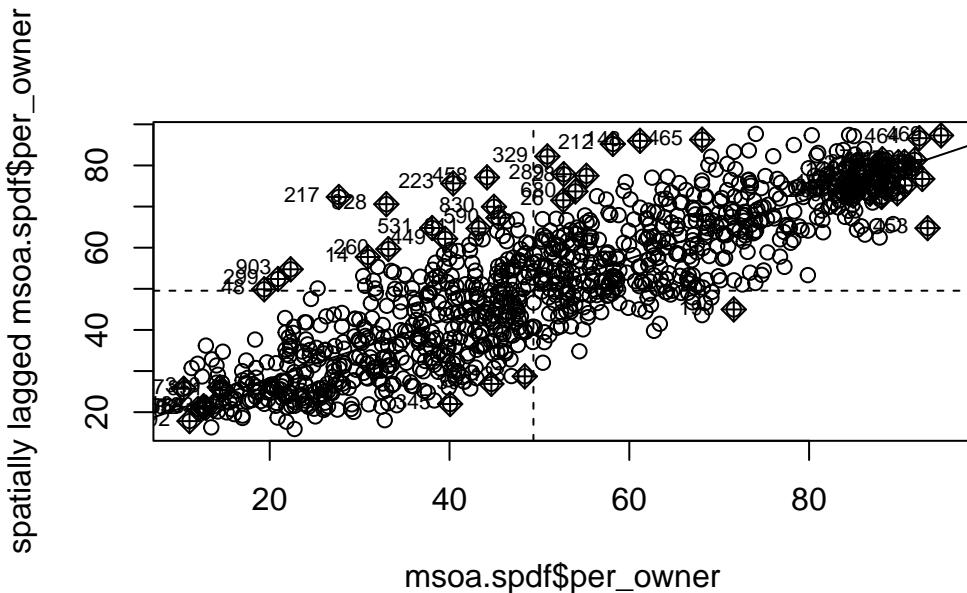
2.6 Local Autocorrelation

The Global Moran's I statistic above summarizes the spatial pattern by a single value. Although this is helpful to get a feeling of the strength of the general spatial association, it is often more helpful to inspect the spatial pattern in more detail.

The most prominent measure is the Local Indicators of Spatial Association (LISA) (Anselin 1995). LISA measures assess the importance and significance of a statistic at different spatial locations. For more information see for instance the [GeoData Materials](#) by Luc Anselin.

For instance, we can use the Moran Plot to identify how single (pairs of) units contribute to the overall dependence.

```
mp <- moran.plot(msoa.spdf$per_owner, queens.lw)
```

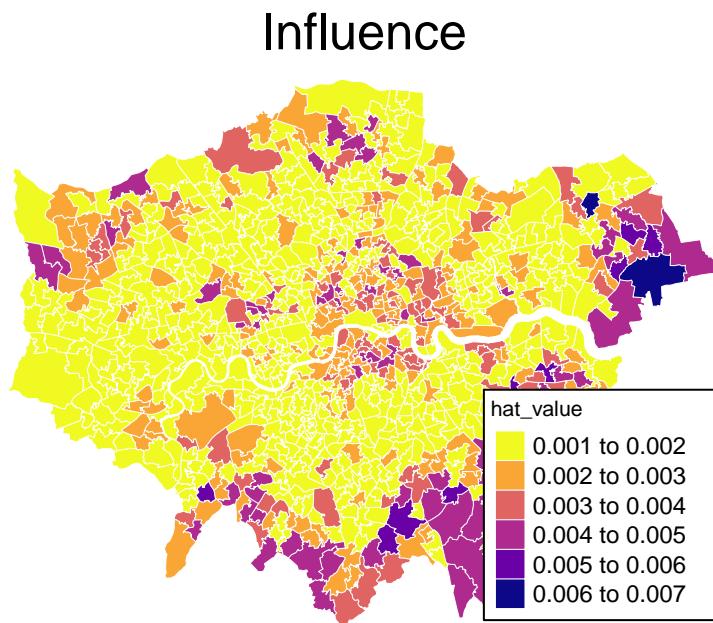


In the lower left corner, we see units with a low-low share of home ownership: focal and neighbouring units have a low share of home owners. In the top right corner, by contrast, we see high-high units.

And we can plot influence values on the Overall Moran statistic.

```
msoa.spdf$hat_value <- mp$hat
mp1 <- tm_shape(msoa.spdf) +
  tm_fill(col = "hat_value",
    palette = viridis(n = 10, direction = -1, option = "C"),
    ) +
  tm_borders(col = "white", lwd = 0.5, alpha = 0.5) +
  tm_layout(frame = FALSE,
    legend.frame = TRUE, legend.bg.color = TRUE,
    legend.position = c("right", "bottom"),
    legend.outside = FALSE,
    main.title = "Influence",
    main.title.position = "center",
    main.title.size = 1.6,
    legend.title.size = 0.8,
    legend.text.size = 0.8)

mp1
```



2.7 Local Moran's I

Local Moran's I is a local version of the overall Moran's I to identify local clusters and local spatial outliers (Anselin 1995). The Local Moran's I is just a local version which is calculated for each location:

\$\$

$$\text{bm I}_i = \frac{\sum_j w_{ij} z_j}{\sum_i (z_i)^2 / (n-1)}, \text{ where }$$

\$\$ We use the unfction `localmoran()` to calculate the local test statistic .

```
loci <- localmoran(msoa.spdf$per_owner, listw = queens.lw)
head(loci)
```

	I _i	E.I _i	Var.I _i	Z.I _i	Pr(z != E(I _i))
1	0.42322928	-1.285364e-04	0.011367934	3.9706976	7.166249e-05
2	-0.12775982	-2.229957e-05	0.003634711	-2.1187688	3.411001e-02
3	0.38111534	-6.569549e-04	0.091630752	1.2611995	2.072370e-01
4	1.02874685	-1.428679e-03	0.279333375	1.9491704	5.127507e-02
5	0.08553291	-2.108521e-04	0.041275789	0.4220412	6.729949e-01
6	-0.24014505	-2.228818e-04	0.036321252	-1.2588964	2.080678e-01

It also has an attribute with the Moran plot quadrant of each observation.

```
head(attr(loci, "quadr"))
```

	mean	median	pysal
1	Low-Low	Low-Low	Low-Low
2	Low-High	Low-High	Low-High
3	High-High	High-High	High-High
4	High-High	High-High	High-High
5	High-High	High-High	High-High
6	Low-High	Low-High	Low-High

This returns a data.frame with local moran statisic, the expectation of local moran statistic, its variance, and a p value for the statistical significance of each unit. Note that we obviously have a problem of multiple comparisons here and thus may want to correct the significance level, e.g. by Bonferroni adjustment (R. Bivand and Wong 2018).

```

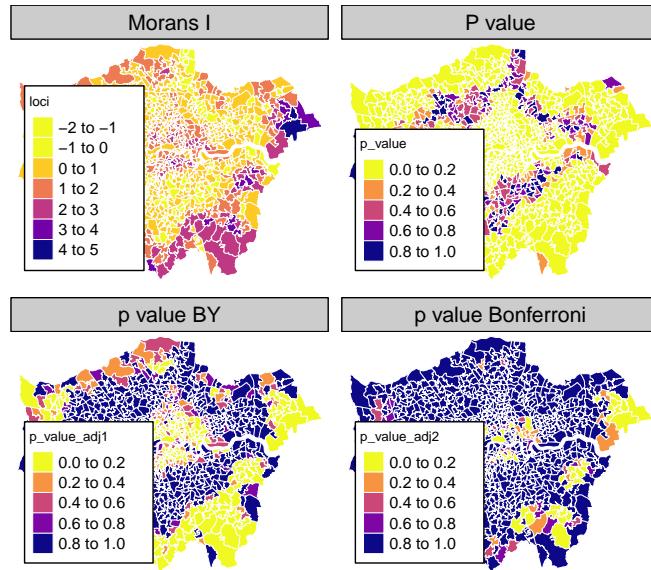
loci.df <- data.frame(loci)
names(loci.df) <- gsub("\\.", "", names(loci.df))
msoa.spdf$loci <- loci.df$II
msoa.spdf$p_value <- loci.df$PrzEIi
msoa.spdf$p_value_adj1 <- p.adjust(loci.df$PrzEIi, "BY")
msoa.spdf$p_value_adj2 <- p.adjust(loci.df$PrzEIi, "bonferroni")

mp1 <- tm_shape(msoa.spdf) +
  tm_fill(col = c("loci", "p_value", "p_value_adj1", "p_value_adj2"),
          palette = viridis(n = 10, direction = -1, option = "C"),
          ) +
  tm_borders(col = "white", lwd = 0.5, alpha = 0.5) +
  tm_layout(frame = FALSE,
            legend.frame = TRUE, legend.bg.color = TRUE,
            legend.position = c("left", "bottom"),
            legend.outside = FALSE,
            main.title = "Local Morans I",
            main.title.position = "center",
            main.title.size = 1.6,
            legend.title.size = 0.8,
            legend.text.size = 0.8,
            panel.labels = c("Morans I",
                            "P value",
                            "p value BY",
                            "p value Bonferroni"))

```

mp1

Local Morans I



Something you can often see are so called LISA hotspot maps. They are based on the same idea as the moran plot, and show cluster of high-high and low-low values. We can use the hotspot function to identify the clusters, with a cutoff for singificance and the adjustment for multiple testing.

```
# Calculate clusters
msoa.spdf$lisa_cluster <- hotspot(loci,
                                      "Pr(z != E(Ii))",
                                      cutoff = 0.05,
                                      quadrant.type = "mean",
                                      p.adjust = "BY")

# Map
mp1 <- tm_shape(msoa.spdf) +
  tm_fill(col = c("lisa_cluster"),
          palette = viridis(n = 3, direction = -1, option = "D"),
          colorNA = "white") +
  tm_borders(col = "grey70", lwd = 0.5, alpha = 0.5) +
  tm_layout(frame = FALSE,
            legend.frame = TRUE, legend.bg.color = TRUE,
            legend.position = c("left", "bottom"),
            legend.outside = FALSE,
```

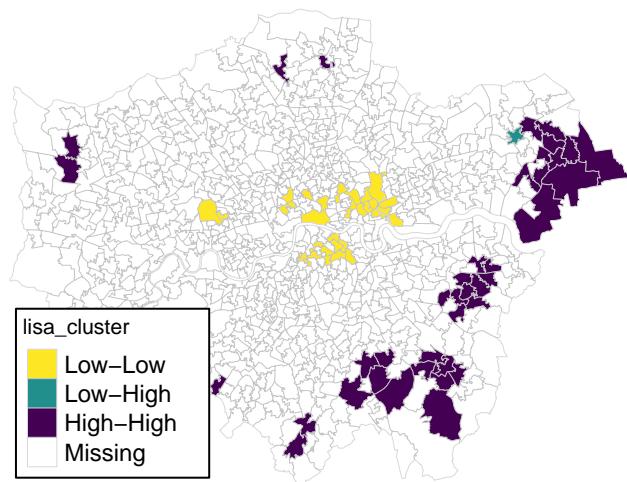
```

main.title = "Home Ownership \n LISA Clusters p(BY) < 0.05",
main.title.position = "center",
main.title.size = 1.6,
legend.title.size = 0.8,
legend.text.size = 0.8,)

mp1

```

Home Ownership LISA Clusters $p(\text{BY}) < 0.05$



Note that it is not suggested to interpret those cluster as significant in the strict statistical sense. Pebesma and Bivand (2023) suggest to speak of *interesting clusters*. After all, this is an explorative approach. Nevertheless, it can help to identify spatial patterns and clusters.

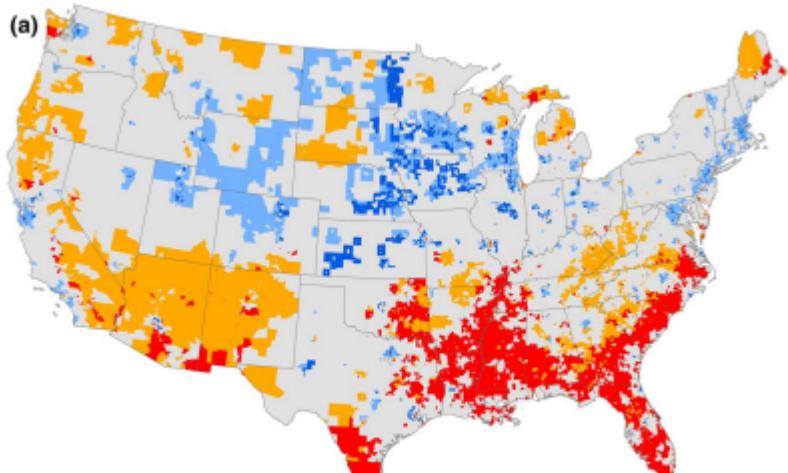
There are more ways of calculating these hotspot maps and more choices on the cutoffs and calculation of the statistical significance. For more materials see [Chapter 15](#) of Pebesma and Bivand (2023).

2.8 Example

Tate.2021

This study explores the geography of flood exposure and social vulnerability in the conterminous United States based on spatial analysis of fluvial and pluvial flood extent, land cover, and social vulnerability.

Mobile homes and racial minorities are most overrepresented in hotspots compared to elsewhere. The results identify priority locations where interventions can mitigate both physical and social aspects of flood vulnerability.



Flood Exposure and Surrounding Social Vulnerability

- High-High
- Low-High
- Not Statistically Significant
- High-Low
- Low-Low

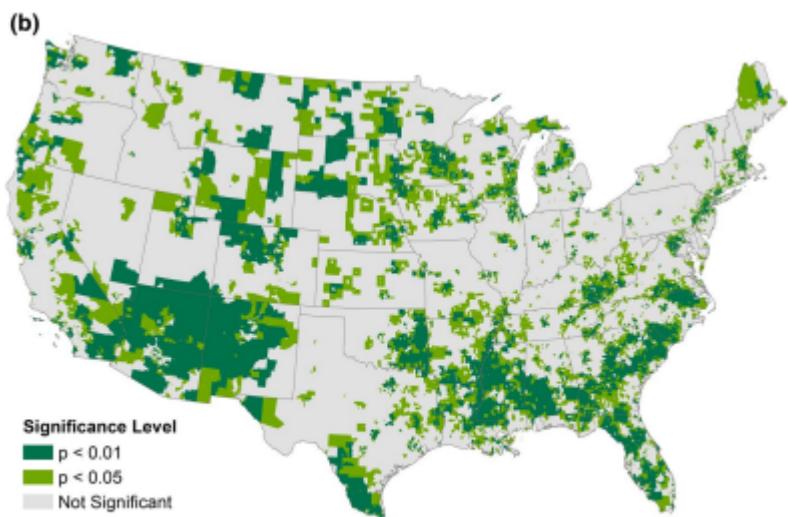


Fig. 4 Bivariate LISA of 100-year flood exposure and surrounding social vulnerability. **a** Cluster map and **b** Cluster significance

3 Exercise I

Required packages

```
pkgs <- c("sf", "mapview", "spdep", "spatialreg", "tmap", "viridisLite") # note: load spde  
lapply(pkgs, require, character.only = TRUE)
```

Session info

```
sessionInfo()  
  
R version 4.3.1 (2023-06-16 ucrt)  
Platform: x86_64-w64-mingw32/x64 (64-bit)  
Running under: Windows 10 x64 (build 19044)  
  
Matrix products: default  
  
locale:  
[1] LC_COLLATE=English_United Kingdom.utf8  
[2] LC_CTYPE=English_United Kingdom.utf8  
[3] LC_MONETARY=English_United Kingdom.utf8  
[4] LC_NUMERIC=C  
[5] LC_TIME=English_United Kingdom.utf8  
  
time zone: Europe/Berlin  
tzcode source: internal  
  
attached base packages:  
[1] stats      graphics   grDevices  utils      datasets  methods    base  
  
other attached packages:  
[1] viridisLite_0.4.2 tmap_3.3-3       spatialreg_1.2-9  Matrix_1.5-4.1  
[5] spdep_1.2-8     spData_2.3.0      mapview_2.11.0    sf_1.0-13
```

```

loaded via a namespace (and not attached):
[1] xfun_0.39           raster_3.6-23      htmlwidgets_1.6.2  lattice_0.21-8
[5] vctrs_0.6.3         tools_4.3.1       crosstalk_1.2.0   LearnBayes_2.15.1
[9] generics_0.1.3      parallel_4.3.1    sandwich_3.0-2   stats4_4.3.1
[13] tibble_3.2.1        proxy_0.4-27     fansi_1.0.4      pkgconfig_2.0.3
[17] KernSmooth_2.23-21 satellite_1.0.4 RColorBrewer_1.1-3 leaflet_2.1.2
[21] webshot_0.5.5       lifecycle_1.0.3  compiler_4.3.1   deldir_1.0-9
[25] munsell_0.5.0       terra_1.7-39    leafsync_0.1.0   codetools_0.2-
19
[29] stars_0.6-1         htmltools_0.5.5  class_7.3-22    pillar_1.9.0
[33] MASS_7.3-60          classInt_0.4-9   lwgeom_0.2-13   wk_0.7.3
[37] abind_1.4-5          boot_1.3-28.1   multcomp_1.4-25 nlme_3.1-162
[41] tidyselect_1.2.0     digest_0.6.32   mvtnorm_1.2-2   dplyr_1.1.2
[45] splines_4.3.1        fastmap_1.1.1   grid_4.3.1     colorspace_2.1-
0
[49] expm_0.999-7         cli_3.6.1       magrittr_2.0.3  base64enc_0.1-3
[53] dichromat_2.0-0.1    XML_3.99-0.14  survival_3.5-5  utf8_1.2.3
[57] TH.data_1.1-2        leafem_0.2.0    e1071_1.7-13   scales_1.2.1
[61] sp_1.6-1              rmarkdown_2.23  zoo_1.8-12    png_0.1-8
[65] coda_0.19-4          evaluate_0.21  knitr_1.43    tmapproj_3.1-1
[69] s2_1.1.4              rlang_1.1.1    Rcpp_1.0.10   glue_1.6.2
[73] DBI_1.1.3             rstudioapi_0.14 jsonlite_1.8.5 R6_2.5.1
[77] units_0.8-2

```

Reload data from previous session

```
load("_data/msoa2_spatial.RData")
```

3.1 General Exercises

1. Please calculate a neighbours weights matrix of the nearest 10 neighbours (see `spdep::knearneigh()`), and create a listw object using row normalization.

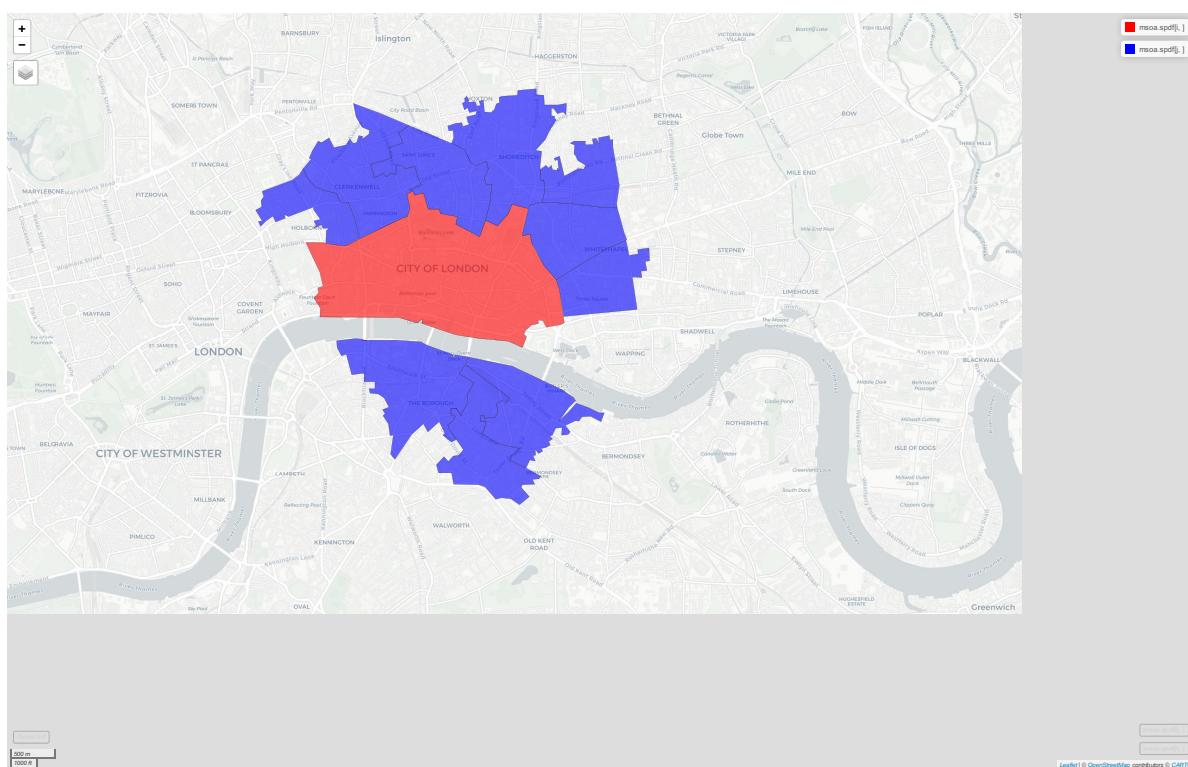
```
coords <- st_centroid(msoa.spdf)
```

Warning: `st_centroid` assumes attributes are constant over geometries

```
k10.nb <- knearneigh(coords, k = 10)
```

2. Can you create a map containing the City of London (MSOA11CD = "E02000001") and its ten nearest neighbours?

```
i <- which(msoa.spdf$MSOA11CD == "E02000001")  
  
# Extract neigbours  
j <- k10.nb$nn[i,]  
  
mapview(list(msoa.spdf[i,], msoa.spdf[j,]), col.regions = c("red", "blue"))
```



3. Chose another characteristics from the data (e.g. ethnic groups or house prices) and calculate global Moran's I for it.

```
# Gen nb object  
k10.nb <- knn2nb(k10.nb)  
  
# Gen listw object
```

```

k10.listw <- nb2listw(k10.nb, style = "W")

# Moran test
moran.test(msoa.spdf$per_white, listw = k10.listw)

Moran I test under randomisation

data: msoa.spdf$per_white
weights: k10.listw

Moran I statistic standard deviate = 55.733, p-value < 2.2e-16
alternative hypothesis: greater
sample estimates:
Moran I statistic      Expectation      Variance
0.7623842505     -0.00010183299    0.0001876235

```

4. Produce a LISA cluster map for the characteristic you have chosen.

```

loci2 <- localmoran(msoa.spdf$per_white, listw = k10.listw)

# Calculate clusters
msoa.spdf$lisa_cluster <- hotspot(loci2,
                                      "Pr(z != E(Ii))",
                                      cutoff = 0.05,
                                      quadrant.type = "mean",
                                      p.adjust = "BY")

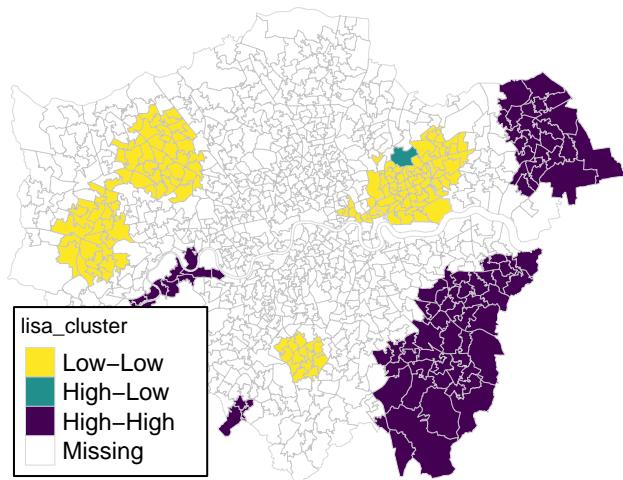
# Map
mp1 <- tm_shape(msoa.spdf) +
  tm_fill(col = c("lisa_cluster"),
          palette = viridis(n = 3, direction = -1, option = "D"),
          colorNA = "white") +
  tm_borders(col = "grey70", lwd = 0.5, alpha = 0.5) +
  tm_layout(frame = FALSE,
            legend.frame = TRUE, legend.bg.color = TRUE,
            legend.position = c("left", "bottom"),
            legend.outside = FALSE,
            main.title = "Percentage White \n LISA Clusters p(BY) < 0.05",
            main.title.position = "center",
            main.title.size = 1.6,

```

```
legend.title.size = 0.8,  
legend.text.size = 0.8,)
```

```
mp1
```

Percentage White LISA Clusters $p(BY) < 0.05$



3.2 Environmental inequality

How would you investigate the following descriptive research question: Are ethnic (and immigrant) minorities in London exposed to higher levels of pollution? Also consider the spatial structure. What's your dependent and what's your independent variable?

1) Define a neighbours weights object of your choice

You can choose any preferred neighbours-weights definition, such as for instance contiguity neighbours or all neighbourhoods within 2.5km.

```
coords <- st_centroid(msoa.spdf)
```

```
Warning: st_centroid assumes attributes are constant over geometries
```

```

# Neighbours within 3km distance
dist_15.nb <- dnearneigh(coords, d1 = 0, d2 = 2500)

summary(dist_15.nb)

```

Neighbour list object:
Number of regions: 983
Number of nonzero links: 15266
Percentage nonzero weights: 1.579859
Average number of links: 15.53001
4 regions with no links:
158 463 478 505
Link number distribution:

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	
4	5	9	23	19	26	36	31	53	39	61	63	59	48	42	35	24	31	28	30	27	26	25	19	38	29	
26	27	28	29	30	31	32	33	34																		
32	38	26	16	20	10	8	1	2																		
5	least connected regions:																									
160	469	474	597	959	with 1 link																					
2	most connected regions:																									
565	567	with 34 links																								

```

# There are some mpty one. Lets impute with the nearest neighbour
k2.nb <- knearneigh(coords, k = 1)

# Replace zero
nolink_ids <- which(card(dist_15.nb) == 0)
dist_15.nb[card(dist_15.nb) == 0] <- k2.nb$nn[nolink_ids, ]

summary(dist_15.nb)

```

Neighbour list object:
Number of regions: 983
Number of nonzero links: 15270
Percentage nonzero weights: 1.580273
Average number of links: 15.53408
Link number distribution:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
---	---	---	---	---	---	---	---	---	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----

```
9 9 23 19 26 36 31 53 39 61 63 59 48 42 35 24 31 28 30 27 26 25 19 38 29 32
27 28 29 30 31 32 33 34
38 26 16 20 10 8 1 2
9 least connected regions:
158 160 463 469 474 478 505 597 959 with 1 link
2 most connected regions:
565 567 with 34 links
```

```
# listw object with row-normalization
dist_15.lw <- nb2listw(dist_15.nb, style = "W")
```

2) Estimate the extent of spatial auto-correlation

The most common way would be to calculate Global Moran's I.

```
moran.test(msoa.spdf$no2, listw = dist_15.lw)
```

```
Moran I test under randomisation

data: msoa.spdf$no2
weights: dist_15.lw

Moran I statistic standard deviate = 65.197, p-value < 2.2e-16
alternative hypothesis: greater
sample estimates:
Moran I statistic      Expectation      Variance
0.891520698     -0.001018330     0.000187411
```

4 Spatial Regression Models

Required packages

```
pkgs <- c("sf", "mapview", "spdep", "spatialreg", "tmap", "viridisLite") # note: load spde
lapply(pkgs, require, character.only = TRUE)
```

Session info

```
sessionInfo()

R version 4.3.1 (2023-06-16 ucrt)
Platform: x86_64-w64-mingw32/x64 (64-bit)
Running under: Windows 10 x64 (build 19044)

Matrix products: default

locale:
[1] LC_COLLATE=English_United Kingdom.utf8
[2] LC_CTYPE=English_United Kingdom.utf8
[3] LC_MONETARY=English_United Kingdom.utf8
[4] LC_NUMERIC=C
[5] LC_TIME=English_United Kingdom.utf8

time zone: Europe/Berlin
tzcode source: internal

attached base packages:
[1] stats      graphics   grDevices  utils      datasets   methods    base

other attached packages:
[1] viridisLite_0.4.2 tmap_3.3-3       spatialreg_1.2-9  Matrix_1.5-4.1
[5] spdep_1.2-8     spData_2.3.0      mapview_2.11.0   sf_1.0-13
```

```

loaded via a namespace (and not attached):
[1] xfun_0.39           raster_3.6-23      htmlwidgets_1.6.2 lattice_0.21-8
[5] vctrs_0.6.3         tools_4.3.1       crosstalk_1.2.0   LearnBayes_2.15.1
[9] generics_0.1.3      parallel_4.3.1    sandwich_3.0-2   stats4_4.3.1
[13] tibble_3.2.1        proxy_0.4-27     fansi_1.0.4      pkgconfig_2.0.3
[17] KernSmooth_2.23-21 satellite_1.0.4 RColorBrewer_1.1-3 leaflet_2.1.2
[21] webshot_0.5.5       lifecycle_1.0.3  compiler_4.3.1   deldir_1.0-9
[25] munsell_0.5.0       terra_1.7-39    leafsync_0.1.0   codetools_0.2-
19
[29] stars_0.6-1         htmltools_0.5.5  class_7.3-22    pillar_1.9.0
[33] MASS_7.3-60          classInt_0.4-9   lwgeom_0.2-13   wk_0.7.3
[37] abind_1.4-5          boot_1.3-28.1   multcomp_1.4-25 nlme_3.1-162
[41] tidyselect_1.2.0     digest_0.6.32   mvtnorm_1.2-2   dplyr_1.1.2
[45] splines_4.3.1        fastmap_1.1.1   grid_4.3.1     colorspace_2.1-
0
[49] expm_0.999-7         cli_3.6.1       magrittr_2.0.3  base64enc_0.1-3
[53] dichromat_2.0-0.1   XML_3.99-0.14  survival_3.5-5 utf8_1.2.3
[57] TH.data_1.1-2        leafem_0.2.0    e1071_1.7-13   scales_1.2.1
[61] sp_1.6-1              rmarkdown_2.23  zoo_1.8-12    png_0.1-8
[65] coda_0.19-4          evaluate_0.21  knitr_1.43    tmaptools_3.1-1
[69] s2_1.1.4              rlang_1.1.1    Rcpp_1.0.10   glue_1.6.2
[73] DBI_1.1.3             rstudioapi_0.14 jsonlite_1.8.5 R6_2.5.1
[77] units_0.8-2

```

Reload data from previous session

```
load("_data/msoa2_spatial.RData")
```

There are various techniques to model spatial dependence and spatial processes (LeSage and Pace 2009). Here, we will just cover a few of the most common techniques / econometric models. One advantage of the most basic spatial model (SLX) is that this method can easily be incorporated in a variety of other methodologies, such as machine learning approaches.

For more in-depth materials see LeSage and Pace (2009) and Kelejian and Piras (2017). Franzese and Hays (2007), Halleck Vega and Elhorst (2015), LeSage (2014), Rüttenauer (2022), Rüttenauer (2024) and Wimpy, Whitten, and Williams (2021) provide article-length introductions.

4.1 Why do we need spatial regression models

4.1.1 Non-spatial OLS

Let us start with a linear model, where \mathbf{y} is the outcome or dependent variable ($N \times 1$), \mathbf{X} are various exogenous covariates ($N \times k$), and $(N \times 1)$ is the error term. We are usually interested in the coefficient vector ($k \times 1$) and its insecurity estimates.

$$\mathbf{y} = \mathbf{X} +$$

The work-horse for estimating in the social science is the OLS estimator (Wooldridge 2010).

$$\hat{\beta} = (\mathbf{X}^\top \mathbf{X})^{-1} \mathbf{X}^\top \mathbf{y}.$$

! OLS assumptions I

1. $E(\epsilon_i | \mathbf{X}_i) = 0$: for every value of X , the average / expectation of the error term equals zero – put differently: the error term is independent of X ,
2. the observations of the sample are independent and identically distributed (i.i.d),
3. the fourth moments of the variables \mathbf{X}_i and Y_i are positive and definite – put differently: extreme values / outliers are very very rare,
4. $\text{rank}(\mathbf{X}) = K$: the matrix \mathbf{X} has full rank – put differently: no perfect multicollinearity between the covariates,

! OLS assumptions II

5. $\text{Var}(\varepsilon|x) = \sigma^2$: the error terms ε are homoskedastic / have the same variance given any value of the explanatory variable,
6. $\varepsilon \sim \mathcal{N}(0, \sigma^2)$: the error terms ε are normally distributed (conditional on the explanatory variables X_i).

💡 Question

Which of the six assumptions above may be violated by spatial dependence?



4.1.2 Problem of ignoring spatial dependence

Does spatial dependence influence the results / coefficient estimates of non-spatial regression models, or in other words: is ignoring spatial dependence harmful?

I've heard different answers, ranging from "It only affects the standard errors" to "it always introduces bias". As so often, the true (or best?) answer is somewhere in the middle: *it depends* (Betz, Cook, and Hollenbach 2020; Cook, Hays, and Franzese 2020; Pace and LeSage 2010; Rüttenauer 2022).

The easiest way to think of it is analogous to the omit variable bias (Betz, Cook, and Hollenbach 2020; Cook, Hays, and Franzese 2020):

$$\text{plim } \hat{\beta}_{OLS} = \beta + \gamma \frac{\text{Cov}(\mathbf{x}, \mathbf{z})}{\text{Var}(\mathbf{x})},$$

where z is some omit variable, and γ is the conditional effect of \mathbf{z} on \mathbf{y} . Now imagine that the neighbouring values of the dependent variable $\mathbf{W}\mathbf{y}$ are autocorrelated to focal unit which we denote with $\rho > 0$, and that the covariance between the focal unit's exogenous covariates and $\mathbf{W}\mathbf{y}$ is not zero. Then we will have an omitted variable bias due to spatial dependence:

$$\text{plim } \hat{\beta}_{OLS} = \beta + \rho \frac{\text{Cov}(\mathbf{x}, \mathbf{W}\mathbf{y})}{\text{Var}(\mathbf{x})} \neq \beta,$$

For completeness, the entire bias is a bit more complicated (Pace and LeSage 2010; Rüttenauer 2022) and looks like:

$$plim \hat{\beta} = \frac{\sum_{ij} (\mathbf{M}(\delta) \mathbf{M}(\delta)^\top \circ \mathbf{M}(\rho))_{ij}}{\text{tr}(\mathbf{M}(\delta) \mathbf{M}(\delta)^\top)} \beta + \frac{\sum_{ij} (\mathbf{M}(\delta) \mathbf{M}(\delta)^\top \circ \mathbf{M}(\rho) \mathbf{W})_{ij}}{\text{tr}(\mathbf{M}(\delta) \mathbf{M}(\delta)^\top)} \theta,$$

where \circ denotes the Hadamard product, $\mathbf{M}(\delta) = (\mathbf{I}_N - \delta \mathbf{W})^{-1}$, and $\mathbf{M}(\rho) = (\mathbf{I}_N - \rho \mathbf{W})^{-1}$.

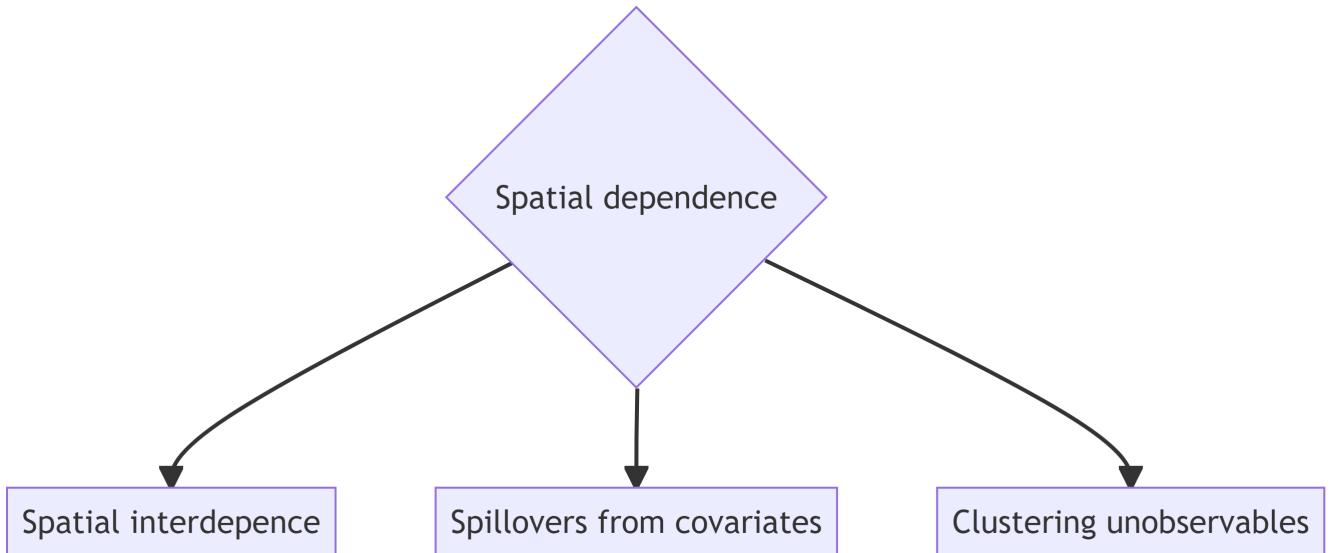
(Don't worry, no need to learn by hard!!)

Essentially, the non-spatial OLS estimator β_{OLS} is biased in the presence of either (Pace and LeSage 2010; Rüttenauer 2022):

- Spatial autocorrelation in the dependent variable ($\rho \neq 0$) and spatial autocorrelation in the covariate ($\delta \neq 0$). This bias increases with ρ , δ , and β .
- Local spatial spillover effects ($\theta \neq 0$) and spatial autocorrelation in the covariate ($\delta \neq 0$). This is analogous to the omitted variable bias resulting from the omission of \mathbf{Wx} . It increases with θ and δ , but additionally with ρ if $\theta \neq 0$ and $\delta \neq 0$.
- An omitted variable and $E(|\mathbf{x}|) \neq 0$. This non-spatial omitted variable bias γ is amplified by spatial dependence in the disturbances (λ) and spatial autocorrelation in the dependent variable (ρ), but also increases with positive values of δ if either $\rho \neq 0$ or $\lambda \neq 0$. Obviously, it also increases with γ .

4.2 Spatial Regression Models

Broadly, spatial dependence or clustering in some characteristic can be the result of three different processes:



Strictly speaking, there are some other possibilities too, such as measurement error or the wrong choice on the spatial level. For instance, imagine we have a city-specific characteristic (e.g. public spending) allocated to neighbourhood units. Obviously, this will introduce heavy autocorrelation on the neighbourhood level by construction.

There are three basic ways of incorporating spatial dependence, which then can be further combined. As before, the $N \times N$ spatial weights matrix \mathbf{W} defines the spatial relationship between units.

4.2.1 Spatial Error Model (SEM)

- Clustering on Unobservables

\$\$

```
\begin{split}
\{\mathbf{y}\} &= \alpha \{\mathbf{\iota}\} + \{\mathbf{X}\} \{\mathbf{\beta}\} + \{\mathbf{u}\}, \\
\{\mathbf{u}\} &= \lambda \{\mathbf{W}\} \{\mathbf{u}\} + \{\mathbf{\varepsilon}\}
\end{split}
```

\$\$

λ denotes the strength of the spatial correlation in the errors of the model: *your errors influence my errors.*

- > 0 : positive error dependence,
- < 0 : negative error dependence,
- $= 0$: traditional OLS model.

λ is defined in the range $[-1, +1]$.

4.2.2 Spatial Autoregressive Model (SAR)

- Interdependence

\$\$

$$\{\mathbf{y}\} = \alpha \{\mathbf{\iota}\} + \rho \{\mathbf{W}\} \{\mathbf{y}\} + \{\mathbf{X}\} \{\mathbf{\beta}\} + \{\mathbf{\varepsilon}\}$$

\$\$

ρ denotes the strength of the spatial correlation in the dependent variable (spatial autocorrelation): *your outcome influences my outcome.*

- > 0 : positive spatial dependence,
- < 0 : negative spatial dependence,
- $= 0$: traditional OLS model.

ρ is defined in the range $[-1, +1]$.

4.2.3 Spatially lagged X Model (SLX)

- Spillovers in Covariates

\$\$

$$\{\text{\\bm } y\}=\text{\\alpha}\{\text{\\bm } \iota\alpha\}+\{\text{\\bm } X\}\{\text{\\bm } \beta\}+\{\text{\\bm } W\}\{\text{\\bm } X\}\{\text{\\bm } \theta\}+ \{\text{\\bm } \varepsilon\}$$

\$\$

θ denotes the strength of the spatial spillover effects from covariate(s) on the dependent variable:
your covariates influence my outcome.

θ is basically like any other coefficient from a covariate. It is thus not bound to any range.

Moreover, there are models combining two sets of the above specifications.

4.2.4 Spatial Durbin Model (SDM)

- Interdependence
- Spillovers in Covariates

\$\$

$$\{\text{\\bm } y\}=\text{\\alpha}\{\text{\\bm } \iota\alpha\}+\text{\\rho}\{\text{\\bm } W\}\{\text{\\bm } y\}+\{\text{\\bm } X\}\{\text{\\bm } \beta\}+\{\text{\\bm } W\}\{\text{\\bm } X\}\{\text{\\bm } \theta\}$$

\$\$

4.2.5 Spatial Durbin Error Model (SDEM)

- Clustering on Unobservables
- Spillovers in Covariates

\$\$

```
\begin{split}
{\bm y}=&\alpha{\bm \iota}+{\bm X}{\bm \beta}+{\bm W}{\bm X}{\bm \theta}+{\bm u},\\
{\bm u}=&\lambda{\bm W}{\bm u}+{\bm \varepsilon}
\end{split}
```

\$\$

4.2.6 Combined Spatial Autocorrelation Model (SAC)

- Clustering on Unobservables
- Interdependence

\$\$

```
\begin{split}
{\bm y}=&\alpha{\bm \iota}+\rho{\bm W}{\bm y}+{\bm X}{\bm \beta}+{\bm u},\\
{\bm u}=&\lambda{\bm W}{\bm u}+{\bm \varepsilon}
\end{split}
```

\$\$

4.2.7 General Nesting Spatial Model (GNS)

- Clustering on Unobservables
- Interdependence
- Spillovers in Covariates

\$\$

```
\begin{split}
{\bm y}=&\alpha{\bm \iota}+\rho{\bm W}{\bm y}+{\bm X}{\bm \beta}+{\bm W}{\bm X}{\bm \theta}+{\bm u},\\
{\bm u}=&\lambda{\bm W}{\bm u}+{\bm \varepsilon}
\end{split}
```

\$\$

Manski's reflection problem

The General Nesting Spatial Model (GNS) is only weakly (or not?) identifiable (Gibbons and Overman 2012).

It's analogous to Manski's reflection problem on neighbourhood effects (Manski 1993): If people in the same group behave similar, this can be because a) imitating behaviour of the group, b) exogenous characteristics of the group influence the behaviour, and c) members of the same group are exposed to the same external circumstances. *We just cannot separate those in observational data.*

Note that all of these models assume different data generating processes (DGP) leading to the spatial pattern. Although there are specifications tests, it is generally not possible to let the data decide which one is the true underlying DGP (Cook, Hays, and Franzese 2020; Rüttenauer 2022). However, there might be theoretical reasons to guide the model specification (Cook, Hays, and Franzese 2020).

Just because SAR is probably the most commonly used model does not make it the best choice. In contrast, various studies (Halleck Vega and Elhorst 2015; Rüttenauer 2022; Wimpy, Whitten, and Williams 2021) highlight the advantages of the relative simple SLX model. Moreover, this specification can basically be incorporated in any other statistical method.

4.2.8 A note on missings

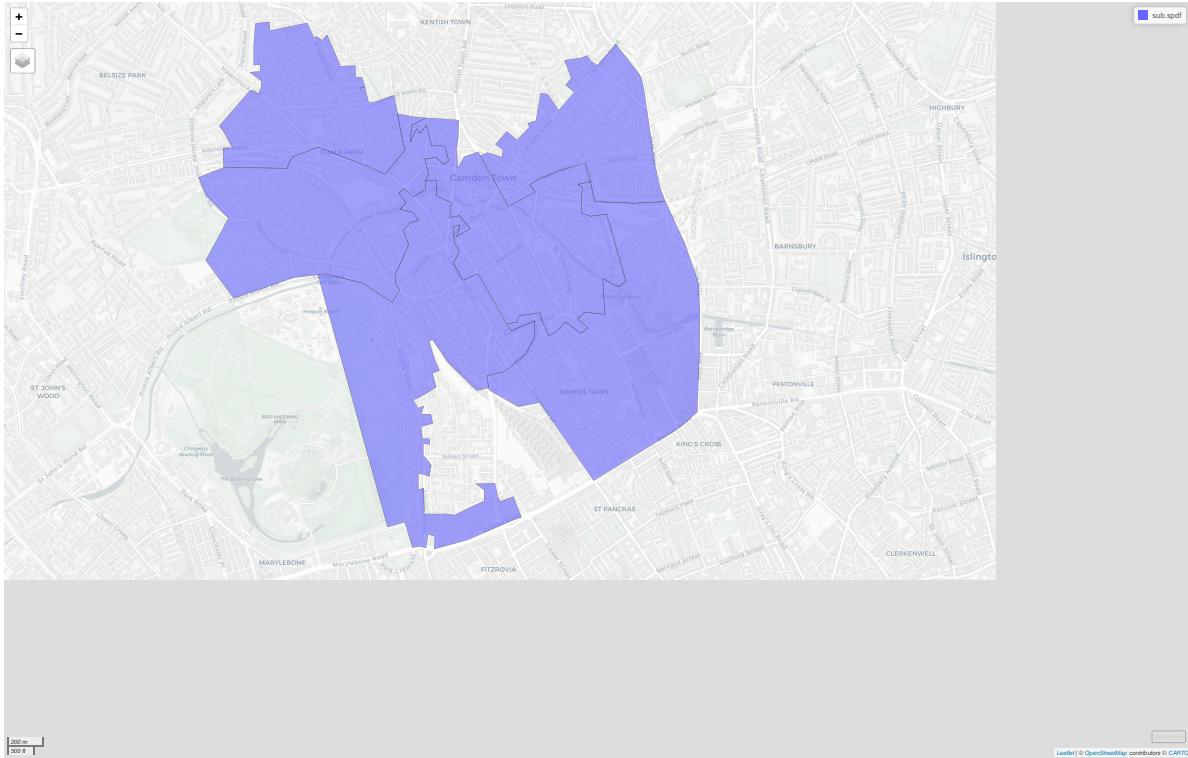
Missing values create a problem in spatial data analysis. For instance, in a local spillover model with an average of 10 neighbours, two initial missing values will lead to 20 missing values in the spatially lagged variable. For global spillover models, one initial missing will ‘flow’ through the neighbourhood system until the cutoff point (and create an excess amount of missings).

Depending on the data, units with missings can either be dropped and omitted from the initial weights creation, or we need to impute the data first, e.g. using interpolation or Kriging.

4.3 Mini Example

Let's try to make sense of this. We rely on a mini example using a few units in Camden

```
sub.spdf <- msoa.spdf[c(172, 175, 178, 179, 181, 182), ]  
mapview(sub.spdf)
```



We then construct queens neighbours, and have a look at the resulting non-normalized matrix \mathbf{W} .

```

queens.nb <- poly2nb(sub.spdf, queen = TRUE, snap = 1)
W <- nb2mat(queens.nb, style = "B")
W

[,1] [,2] [,3] [,4] [,5] [,6]
172    0    0    1    0    0    0
175    0    0    0    1    0    1
178    1    0    0    1    1    0
179    0    1    1    0    1    1
181    0    0    1    1    0    1
182    0    1    0    1    1    0

attr(,"call")
nb2mat(neighbours = queens.nb, style = "B")

```

We have selected 6 units. So, \mathbf{W} is a 6×6 matrix. we see that observation 1 has one neighbour: observation 3. Observation 2 has two neighbours: observation 4 and observation 6. The diagonal is zero: no unit is a neighbour of themselves.

No we row-normalize this matrix.

```
queens.lw <- nb2listw(queens.nb,
                      style = "W")
W_rn <- listw2mat(queens.lw)
W_rn
```

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]
172	0.0000000	0.0000000	1.0000000	0.0000000	0.0000000	0.0000000
175	0.0000000	0.0000000	0.0000000	0.5000000	0.0000000	0.5000000
178	0.3333333	0.0000000	0.0000000	0.3333333	0.3333333	0.0000000
179	0.0000000	0.2500000	0.2500000	0.0000000	0.2500000	0.2500000
181	0.0000000	0.0000000	0.3333333	0.3333333	0.0000000	0.3333333
182	0.0000000	0.3333333	0.0000000	0.3333333	0.3333333	0.0000000

No every single weight w_{ij} is divided by the total number of neighbours n_i of the focal unit. For observation 1, observation 3 is the only neighbour, thus a weight = 1. For observation two, both neighbours have a weight of 1/2. For observation 3 (with three neighbours) each neighbour got a weight of 1/3.

💡 Question

What happens if we multiply this matrix **W** with a $N \times 1$ vector **y** or **x**?

A short reminder on matrix multiplication.

$$\mathbf{W} * \mathbf{y} = \begin{bmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \\ w_{31} & w_{32} & w_{33} \end{bmatrix} * \begin{bmatrix} y_{11} \\ y_{21} \\ y_{31} \end{bmatrix} = \begin{bmatrix} w_{11}y_{11} + w_{12}y_{21} + w_{13}y_{31} \\ w_{21}y_{11} + w_{22}y_{21} + w_{23}y_{31} \\ w_{31}y_{11} + w_{32}y_{21} + w_{33}y_{31} \end{bmatrix}$$

Each line of $\mathbf{W} * \mathbf{y}$ just gives a weighted average of the other y -values y_j in the sample. In case of the row-normalization, each neighbour gets the same weight $\frac{1}{n_i}$. This is simply the mean of y_j of the neighbours in case of a row-normalized contiguity weights matrix.

Note that the *mean* interpretation is only valid with row-normalization. What would we get with inverse-distance based weights?

Let's look at this in our example

```
y <- sub.spdf$med_house_price
x <- sub.spdf$pubs_count

W_rn
```

```
[,1]      [,2]      [,3]      [,4]      [,5]      [,6]
172 0.0000000 0.0000000 1.0000000 0.0000000 0.0000000 0.0000000
175 0.0000000 0.0000000 0.0000000 0.5000000 0.0000000 0.5000000
178 0.3333333 0.0000000 0.0000000 0.3333333 0.3333333 0.0000000
179 0.0000000 0.2500000 0.2500000 0.0000000 0.2500000 0.2500000
181 0.0000000 0.0000000 0.3333333 0.3333333 0.0000000 0.3333333
182 0.0000000 0.3333333 0.0000000 0.3333333 0.3333333 0.0000000
```

```
y
```

```
[1] 376812.5 414625.0 713125.0 322750.0 495000.0 364000.0
```

```
x
```

```
[1] 1 3 3 1 9 7
```

```
W_rn_y <- W_rn %*% y
W_rn_x <- W_rn %*% x
W_rn_y
```

```
[,1]
172 713125.0
175 343375.0
178 398187.5
179 496687.5
181 466625.0
182 410791.7
```

```
W_rn_x
```

```
[,1]
172 3.000000
175 4.000000
178 3.666667
179 5.500000
181 3.666667
182 4.333333
```

Let's check if our interpretation is true

```

W_rn_y[1] == y[3]

[1] TRUE

W_rn_y[2] == mean(y[c(4, 6)])

[1] TRUE

W_rn_y[4] == mean(y[c(2, 3, 5, 6)])

```

4.4 Real Example

First, we need the a spatial weights matrix.

```

# Contiguity (Queens) neighbours weights
queens.nb <- poly2nb(msoa.spdf,
                      queen = TRUE,
                      snap = 1) # we consider points in 1m distance as 'touching'
queens.lw <- nb2listw(queens.nb,
                      style = "W")

```

We can estimate spatial models using `spatialreg`.

4.4.1 SAR

Lets estimate a spatial SAR model using the `lagsarlm()` with contiguity weights. We use median house value as depended variable, and include population density (POPDEN), the air pollution (no2), and the share of ethnic minorities (per_mixed, per_asian, per_black, per_other).

```

mod_1.sar <- lagsarlm(log(med_house_price) ~ log(no2) + log(POPDEN) +
                        per_mixed + per_asian + per_black + per_other,
                        data = msoa.spdf,
                        listw = queens.lw,

```

```
Durbin = FALSE) # we could here extend to SDM  
summary(mod_1.sar)
```

```
Call:lagsarlm(formula = log(med_house_price) ~ log(no2) + log(POPDEN) +  
per_mixed + per_asian + per_black + per_other, data = msoa.spdf,  
listw = queens.lw, Durbin = FALSE)
```

Residuals:

Min	1Q	Median	3Q	Max
-0.5281789	-0.1220524	-0.0099245	0.0992203	1.0936745

Type: lag

Coefficients: (asymptotic standard errors)

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	3.17383180	0.29041604	10.9286	< 2.2e-16
log(no2)	0.39705423	0.04452880	8.9168	< 2.2e-16
log(POPDEN)	-0.05583014	0.01242876	-4.4920	7.055e-06
per_mixed	0.01851577	0.00579832	3.1933	0.001407
per_asian	-0.00228346	0.00045876	-4.9775	6.442e-07
per_black	-0.01263650	0.00100282	-12.6009	< 2.2e-16
per_other	-0.00161419	0.00289082	-0.5584	0.576582

Rho: 0.66976, LR test value: 473.23, p-value: < 2.22e-16

Asymptotic standard error: 0.025311

z-value: 26.461, p-value: < 2.22e-16

Wald statistic: 700.19, p-value: < 2.22e-16

Log likelihood: 196.7203 for lag model

ML residual variance (sigma squared): 0.035402, (sigma: 0.18815)

Number of observations: 983

Number of parameters estimated: 9

AIC: -375.44, (AIC for lm: 95.786)

LM test for residual autocorrelation

test value: 8.609, p-value: 0.0033451

This looks pretty much like a conventional model output, with some additional information: a highly significant `mod_1.sar$rho` of 0.67 indicates strong positive spatial autocorrelation.

Remember that is the coefficient for the term $\mathbf{y} = \rho \mathbf{W} \mathbf{y}$ It is bound to be below 1 for positive autocorrelation.

In substantive terms, house prices in the focal unit positively influence house prices in neighbouring units, which again influences house prices among the neighbours of these neighbours, and so on (we'll get back to this).

⚠️ Warning

The coefficients of covariates in a SAR model are not marginal or partial effects, because of the spillovers and feedback loops in \mathbf{y} (see below)!

From the coefficient, we can only interpret the direction: there's a positive effect of air pollution and a negative effect of population sensitivity, and so on...

4.4.2 SEM

SEM models can be estimated using `errorsarlm()`.

```
mod_1.sem <- errorsarlm(log(med_house_price) ~ log(no2) + log(POPDEN) +
                           per_mixed + per_asian + per_black + per_other,
                           data = msoa.spdf,
                           listw = queens.lw,
                           Durbin = FALSE) # we could here extend to SDEM
summary(mod_1.sem)
```

```
Call:errorsarlm(formula = log(med_house_price) ~ log(no2) + log(POPDEN) +
per_mixed + per_asian + per_black + per_other, data = msoa.spdf,
listw = queens.lw, Durbin = FALSE)
```

Residuals:

Min	1Q	Median	3Q	Max
-0.581785	-0.105218	-0.012758	0.094430	0.913425

Type: error

Coefficients: (asymptotic standard errors)

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	12.92801104	0.35239139	36.6865	< 2.2e-16
log(no2)	0.15735296	0.10880727	1.4462	0.1481317
log(POPDEN)	-0.08316270	0.01254315	-6.6301	3.354e-11
per_mixed	-0.03377962	0.00811054	-4.1649	3.115e-05
per_asian	-0.00413115	0.00096849	-4.2656	1.994e-05
per_black	-0.01653816	0.00126741	-13.0488	< 2.2e-16
per_other	-0.01693012	0.00462999	-3.6566	0.0002556

```

Lambda: 0.88605, LR test value: 623.55, p-value: < 2.22e-16
Asymptotic standard error: 0.015803
      z-value: 56.068, p-value: < 2.22e-16
Wald statistic: 3143.6, p-value: < 2.22e-16

Log likelihood: 271.8839 for error model
ML residual variance (sigma squared): 0.026911, (sigma: 0.16405)
Number of observations: 983
Number of parameters estimated: 9
AIC: NA (not available for weighted model), (AIC for lm: 95.786)

```

In this case `mod_1.sem$lambda` gives us the spatial parameter. A highly significant lambda of 0.89 indicates that the errors are highly spatially correlated (e.g. due to correlated unobservables). Again, $\$ = 1$ would be the maximum.

In spatial error models, we can interpret the coefficients directly, as in a conventional linear model.

4.4.3 SLX

SLX models can either be estimated with `lmSLX()` directly, or by creating **WX** manually and plugging it into any available model-fitting function.

```

mod_1.slx <- lmSLX(log(med_house_price) ~ log(no2) + log(POPDEN) +
                     per_mixed + per_asian + per_black + per_other,
                     data = msoa.spdf,
                     listw = queens.lw,
                     Durbin = TRUE) # use a formula to lag only specific covariates
summary(mod_1.slx)

```

Call:
`lm(formula = formula(paste("y ~ ", paste(colnames(x)[-1], collapse = "+"))),`
 `data = as.data.frame(x), weights = weights)`

Residuals:

Min	1Q	Median	3Q	Max
-0.50809	-0.16605	-0.01817	0.13055	1.09039

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)							
(Intercept)	10.582440	0.153862	68.779	< 2e-16 ***							
log.no2.	-0.440727	0.181063	-2.434	0.01511 *							
log.POPDEN.	-0.076840	0.017345	-4.430	1.05e-05 ***							
per_mixed	-0.033042	0.011298	-2.925	0.00353 **							
per_asian	-0.002381	0.001474	-1.615	0.10655							
per_black	-0.016229	0.001801	-9.009	< 2e-16 ***							
per_other	-0.020391	0.006564	-3.107	0.00195 **							
lag.log.no2.	0.993602	0.199370	4.984	7.38e-07 ***							
lag.log.POPDEN.	0.113262	0.028752	3.939	8.76e-05 ***							
lag.per_mixed	0.126069	0.014294	8.820	< 2e-16 ***							
lag.per_asian	-0.003828	0.001661	-2.305	0.02140 *							
lag.per_black	-0.018054	0.002241	-8.056	2.30e-15 ***							
lag.per_other	0.048139	0.007971	6.039	2.20e-09 ***							

Signif. codes:	0	'***'	0.001	'**'	0.01	'*'	0.05	'.'	0.1	' '	1

Residual standard error: 0.2262 on 970 degrees of freedom
 Multiple R-squared: 0.653, Adjusted R-squared: 0.6487
 F-statistic: 152.1 on 12 and 970 DF, p-value: < 2.2e-16

In SLX models, we can simply interpret the coefficients of direct and indirect (spatially lagged) covariates.

For instance, lets look at population density:

Interpretation SLX

1. A high population density in the focal unit is related to lower house prices (a 1% increase in population density decreases house prices by -0.08%), but
2. A high population density in the neighbouring areas is related to higher house prices (while keeping population density in the focal unit constant). A 1% increase in the *average* population density *across the adjacent neighbourhoods* increases house prices *in the focal unit* by 0.11%)

Potential interpretation: areas with a low population density in central regions of the city (high pop density in surrounding neighbourhoods) have higher house prices. We could try testing this interpretation by including the distance to the city center as a control.

Also note how the air pollution coefficient has changed here, with a negative effect in the focal unit and positive one among the neighbouring units.

An alternative way of estimating the same model is lagging the covariates first.

```

# Loop through vars and create lagged variables
msoa.spdf$log_POPDEN <- log(msoa.spdf$POPDEN)
msoa.spdf$log_no2 <- log(msoa.spdf$no2)
msoa.spdf$log_med_house_price <- log(msoa.spdf$med_house_price)

vars <- c("log_med_house_price", "log_no2", "log_POPDEN",
         "per_mixed", "per_asian", "per_black", "per_other",
         "per_owner", "per_social", "pubs_count")
for(v in vars){
  msoa.spdf[, paste0("w.", v)] <- lag.listw(queens.lw,
                                              var = st_drop_geometry(msoa.spdf)[, v])
}

# Alternatively:
w_vars <- create_WX(st_drop_geometry(msoa.spdf[, vars]),
                      listw = queens.lw,
                      prefix = "w")

head(w_vars)

```

	w.log_med_house_price	w.log_no2	w.log_POPDEN	w.per_mixed	w.per_asian
1	12.98382	3.843750	4.662014	4.748368	23.899916
2	12.28730	3.098960	3.300901	3.978275	19.951593
3	12.21207	3.206338	4.009795	3.997487	20.793559
4	12.18176	3.169934	3.630360	2.759082	7.633439
5	12.11159	3.221203	3.993660	3.930061	12.791140
6	12.08393	3.217865	3.876070	3.419488	8.997514
	w.per_black	w.per_other	w.per_owner	w.per_social	w.pubs_count
1	7.879758	3.2080074	25.75738	33.85580	8.5454545
2	10.451828	1.6368986	66.42278	15.75042	0.6666667
3	12.965863	1.7526693	58.72637	21.38169	0.2857143
4	12.135478	0.6992118	66.52519	19.70500	0.2000000
5	16.108948	1.3817357	53.05539	29.44022	0.4000000
6	15.312652	0.9611710	59.49460	23.81126	0.1666667

And subsequently we use those new variables in a linear model.

```

mod_1.lm <- lm (log(med_house_price) ~ log(no2) + log(POPDEN) +
                  per_mixed + per_asian + per_black + per_other +
                  w.log_no2 + w.log_POPDEN +
                  w.per_mixed + w.per_asian + w.per_black + w.per_other,

```

```

data = msoa.spdf)
summary(mod_1.lm)

Call:
lm(formula = log(med_house_price) ~ log(no2) + log(POPDEN) +
    per_mixed + per_asian + per_black + per_other + w.log_no2 +
    w.log_POPDEN + w.per_mixed + w.per_asian + w.per_black +
    w.per_other, data = msoa.spdf)

Residuals:
    Min      1Q  Median      3Q     Max 
-0.50809 -0.16605 -0.01817  0.13055  1.09039 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 10.582440  0.153862  68.779 < 2e-16 ***
log(no2)    -0.440727  0.181063  -2.434  0.01511 *  
log(POPDEN) -0.076840  0.017345  -4.430 1.05e-05 *** 
per_mixed   -0.033042  0.011298  -2.925  0.00353 ** 
per_asian   -0.002381  0.001474  -1.615  0.10655  
per_black   -0.016229  0.001801  -9.009 < 2e-16 *** 
per_other   -0.020391  0.006564  -3.107  0.00195 ** 
w.log_no2    0.993602  0.199370  4.984  7.38e-07 *** 
w.log_POPDEN 0.113262  0.028752  3.939  8.76e-05 *** 
w.per_mixed  0.126069  0.014294  8.820 < 2e-16 *** 
w.per_asian  -0.003828  0.001661  -2.305  0.02140 *  
w.per_black  -0.018054  0.002241  -8.056  2.30e-15 *** 
w.per_other   0.048139  0.007971  6.039  2.20e-09 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.2262 on 970 degrees of freedom
Multiple R-squared:  0.653, Adjusted R-squared:  0.6487 
F-statistic: 152.1 on 12 and 970 DF,  p-value: < 2.2e-16

```

Looks pretty similar to `lmSLX()` results, and it should! A big advantage of the SLX specification is that we can use the lagged variables in basically all methods which take variables as inputs, such as non-linear models, matching algorithms, and machine learning tools.

Moreover, using the lagged variables gives a high degree of freedom. For instance, we could (not saying that it necessarily makes sense):

- Use different weights matrices for different variables
- Include higher order neighbours using `nblag()` (with an increasing number of orders we go towards a more global model, but we estimate a coefficient for each spillover, instead of estimating just one)
- Use machine learning techniques to determine the best fitting weights specification.

4.4.4 SDEM

SDEM models can be estimated using `errorsarlm()` with the additional option `Durbin = TRUE`.

```
mod_1.sdem <- errorsarlm(log(med_house_price) ~ log(no2) + log(POPDEN) +
                           per_mixed + per_asian + per_black + per_other,
                           data = msoa.spdf,
                           listw = queens.lw,
                           Durbin = TRUE) # we could here extend to SDEM
summary(mod_1.sdem)
```

```
Call:errorsarlm(formula = log(med_house_price) ~ log(no2) + log(POPDEN) +
per_mixed + per_asian + per_black + per_other, data = msoa.spdf,
listw = queens.lw, Durbin = TRUE)
```

Residuals:

Min	1Q	Median	3Q	Max
-0.617795	-0.106380	-0.014832	0.095826	0.927446

Type: error

Coefficients: (asymptotic standard errors)

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	10.4422703	0.3652148	28.5921	< 2.2e-16
log(no2)	-0.2057493	0.1264914	-1.6266	0.1038248
log(POPDEN)	-0.0769743	0.0132094	-5.8272	5.635e-09
per_mixed	-0.0222406	0.0079705	-2.7904	0.0052649
per_asian	-0.0037484	0.0010054	-3.7284	0.0001927
per_black	-0.0179751	0.0012383	-14.5161	< 2.2e-16
per_other	-0.0150218	0.0044895	-3.3460	0.0008199
lag.log(no2)	1.0004491	0.1739833	5.7503	8.911e-09
lag.log(POPDEN)	-0.0054241	0.0327802	-0.1655	0.8685763
lag.per_mixed	0.0669699	0.0169349	3.9545	7.668e-05

```

lag.per_asian   -0.0018566  0.0015957  -1.1635  0.2446368
lag.per_black   -0.0079949  0.0024833  -3.2195  0.0012842
lag.per_other    0.0273378  0.0087430   3.1268  0.0017671

Lambda: 0.76173, LR test value: 455.7, p-value: < 2.22e-16
Asymptotic standard error: 0.024949
z-value: 30.531, p-value: < 2.22e-16
Wald statistic: 932.15, p-value: < 2.22e-16

Log likelihood: 300.847 for error model
ML residual variance (sigma squared): 0.027504, (sigma: 0.16584)
Number of observations: 983
Number of parameters estimated: 15
AIC: NA (not available for weighted model), (AIC for lm: -117.99)

```

And this SDEM can be interpreted like a combination of SEM and SLX.

First, we still see highly significant auto-correlation in the error term. However, it's lower in magnitude now that we also include the **WX** terms.

Second, the coefficients tell a similar story as in the SLX (use the same interpretation), but some coefficient magnitudes have become smaller.

4.4.5 SDM

SDM models can be estimated using `lagsarlm()` with the additional option `Durbin = TRUE`.

```

mod_1.sdm <- lagsarlm(log(med_house_price) ~ log(no2) + log(POPDEN) +
                        per_mixed + per_asian + per_black + per_other,
                        data = msoa.spdf,
                        listw = queens.lw,
                        Durbin = TRUE) # we could here extend to SDM
summary(mod_1.sdm)

```

```

Call:lagsarlm(formula = log(med_house_price) ~ log(no2) + log(POPDEN) +
               per_mixed + per_asian + per_black + per_other, data = msoa.spdf,
               listw = queens.lw, Durbin = TRUE)

```

Residuals:

Min	1Q	Median	3Q	Max
-0.614314	-0.107947	-0.013509	0.092234	0.917398

```

Type: mixed
Coefficients: (asymptotic standard errors)
              Estimate Std. Error z value Pr(>|z|)
(Intercept)  2.7843426  0.2944721  9.4554 < 2.2e-16
log(no2)     -0.3112762  0.1308101 -2.3796 0.0173312
log(POPDEN)   -0.0802866  0.0125213 -6.4120 1.436e-10
per_mixed    -0.0368998  0.0081596 -4.5223 6.118e-06
per_asian     -0.0033726  0.0010636 -3.1711 0.0015189
per_black      -0.0159770  0.0013006 -12.2848 < 2.2e-16
per_other     -0.0209743  0.0047369 -4.4279 9.516e-06
lag.log(no2)   0.4880923  0.1456778  3.3505 0.0008067
lag.log(POPDEN) 0.0781188  0.0207600  3.7629 0.0001679
lag.per_mixed  0.0640880  0.0104646  6.1243 9.110e-10
lag.per_asian   0.0017665  0.0012101  1.4598 0.1443498
lag.per_black    0.0070487  0.0017938  3.9295 8.511e-05
lag.per_other   0.0284822  0.0057774  4.9299 8.226e-07

```

Rho: 0.73126, LR test value: 501.83, p-value: < 2.22e-16

Asymptotic standard error: 0.025889

z-value: 28.246, p-value: < 2.22e-16

Wald statistic: 797.86, p-value: < 2.22e-16

Log likelihood: 323.9111 for mixed model

ML residual variance (sigma squared): 0.026633, (sigma: 0.1632)

Number of observations: 983

Number of parameters estimated: 15

AIC: -617.82, (AIC for lm: -117.99)

LM test for residual autocorrelation

test value: 36.704, p-value: 1.3747e-09

And this SDM can be interpreted like a combination of SAR and SLX.

First, there's still substantial auto-correlation in \mathbf{y} , and this has become even stronger as compared to SAR.

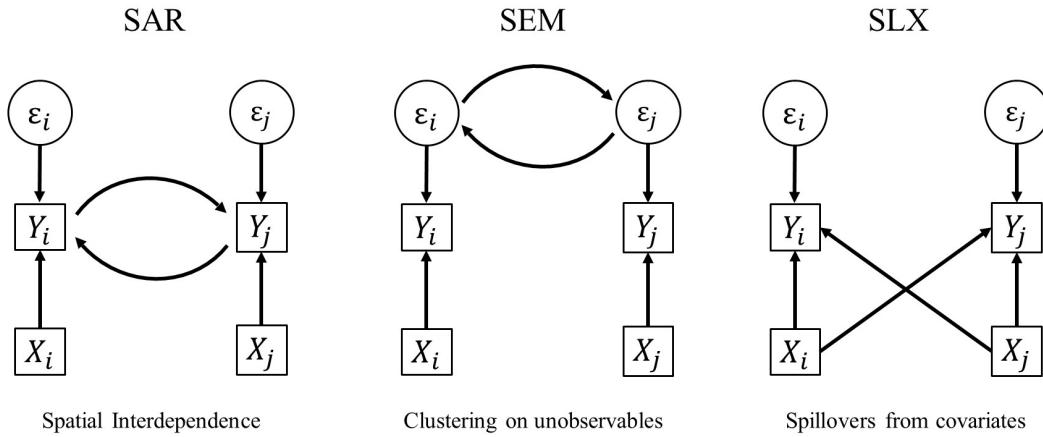
Second, we can interpret the direction of the effect, but we *cannot interpret the coefficient as marginal effects*.

4.5 Estimation

Note that most of the spatial model specifications can not be estimated by Least Squares (LS), as using (constrained) LS estimators for models containing a spatially lagged dependent variable or disturbance leads to inconsistent results (Anselin and Bera 1998; Franzese and Hays 2007). However, an extensive amount of econometric literature discusses different estimation methods based on (quasi-) maximum likelihood (Anselin 1988; L. Lee 2004; Ord 1975) or instrumental variable approaches using generalized methods of moments (Drukker, Egger, and Prucha 2013; Kelejian and Prucha 1998, 2010), in which the endogenous lagged variables can be instrumented by q higher order lags of the exogenous regressors ($\mathbf{X}, \mathbf{WX}, \mathbf{W}^2\mathbf{X}, \dots, \mathbf{W}^q\mathbf{X}$) (Kelejian and Prucha 1998).

4.5.1 Simultaneity bias

Remember what is happening when we estimate a spatial auto-regressive model.



Note the circular process here: My X influences my Y , which then influences your Y , which then influences my Y again. We write this as

\$\$

$$\{\bm{y}\} = \alpha\{\bm{\iota}\} + \rho\{\bm{W}\}\{\bm{y}\} + \{\bm{X}\}\{\bm{\beta}\} + \{\bm{\varepsilon}\}.$$

\$\$

If we ignore \mathbf{X} and write the pure auto-regressive term in its reduce form, we get:

$$\mathbf{y} = (\mathbf{I}_n - \rho\mathbf{W})^{-1} \varepsilon,$$

and the spatial lag term is

$$\mathbf{W}\mathbf{y} = \mathbf{W}(\mathbf{I}_n - \rho\mathbf{W})^{-1}\varepsilon.$$

The OLS estimator for the spatial lag term then is

$$\hat{\rho}_{OLS} = \left[\underbrace{(\mathbf{W}\mathbf{y})^\top}_{(1 \times n)} \underbrace{(\mathbf{W}\mathbf{y})}_{(n \times 1)} \right]^{-1} \underbrace{(\mathbf{W}\mathbf{y})^\top}_{(1 \times n)} \underbrace{\mathbf{y}}_{(n \times 1)}.$$

It can then be shown that the OLS estimators equals

$$\hat{\rho}_{OLS} = \rho + [(\mathbf{W}\mathbf{y})^\top (\mathbf{W}\mathbf{y})]^{-1} (\mathbf{W}\mathbf{y})^\top \varepsilon = \rho + \left(\sum_{i=1}^n \mathbf{y}_{Li}^2 \right)^{-1} \left(\sum_{i=1}^n \mathbf{y}_{Li} \varepsilon_i \right),$$

with \mathbf{y}_{Li} defined as the i th element of the spatial lag operator $\mathbf{W}\mathbf{y} = \mathbf{y}_L$. It can further be shown that the second part of the equation $\neq 0$, which demonstrates that OLS gives a biased estimate of ρ (Franzese and Hays 2007; Sarriás 2023).

⚠ Warning

Do not estimate spatial lags of the dependent variable in OLS. It will suffer from simultaneity bias.

4.5.2 Instrumental variable

A potential way of estimating spatial lag /SAR models is 2SLS (Kelejian and Prucha 1998).

We start with our standard model

$$\mathbf{y} = \alpha + \rho\mathbf{W}\mathbf{y} + \mathbf{X} + \varepsilon.$$

As we have seen above, there is a problem of simultaneity: the “covariate” $\mathbf{W}\mathbf{y}$ is endogenous. One way of dealing with this endogeneity problem is the Instrumental Variable approach.

So, the question is what are good instruments \mathbf{H} for $\mathbf{W}\mathbf{y}$? As we have specified the mode, we are sure that \mathbf{X} determines \mathbf{y} . Thus, it must be true that $\mathbf{W}\mathbf{X}$ and $\mathbf{W}^2\mathbf{X}, \dots, \mathbf{W}^l\mathbf{X}$ determines $\mathbf{W}\mathbf{y}$.

Note that \mathbf{W}^l denotes higher orders of \mathbf{W} . So \mathbf{W}^2 are the second order neighbours (neighbours of neighbours), and \mathbf{W}^3 are the third order neighbours (the neighbours of my neighbour’s neighbours), and so on...

We will discuss this in more detail later, but note for now that the reduced form of the SAR always contains a series of higher order neighbours.

$$(\mathbf{I}_N - \rho \mathbf{W})^{-1} \beta_k = (\mathbf{I}_N + \rho \mathbf{W} + \rho^2 \mathbf{W}^2 + \rho^3 \mathbf{W}^3 + \dots) \beta_k = (\mathbf{I}_N + \sum_{h=1}^{\infty} \rho^h \mathbf{W}^h) \beta_k.$$

Thus, Kelejian and Prucha (1998) suggested to use a set of lagged covariates as instruments for \mathbf{WY} :

$$\mathbf{H} = \mathbf{X}, \mathbf{WX}, \mathbf{W}^2\mathbf{X}, \dots, \mathbf{W}^l\mathbf{X},$$

where l is a pre-defined number for the higher order neighbours included. In practice, l is usually restricted to $l = 2$.

This has further been developed by, for instance, using a (truncated) power series as instruments (Kelejian, Prucha, and Yuzefovich 2004):

$$\mathbf{H} = \left[\mathbf{X}, \mathbf{W} \left(\sum_{l=1}^{\infty} \rho^l \mathbf{W}^l \right) \mathbf{X} \right].$$

We can estimate this using the pacakge **spatialreg** with the function **stscls()**,

```
mod_1.sls <- stscls(log(med_house_price) ~ log(no2) + log(POPDEN) +
                      per_mixed + per_asian + per_black + per_other,
                      data = msoa.spdf,
                      listw = queens.lw,
                      robust = TRUE, # heteroskedasticity robust SEs
                      W2X = TRUE) # Second order neighbours are included as instruments (else
summary(mod_1.sls)
```

```
Call:stscls(formula = log(med_house_price) ~ log(no2) + log(POPDEN) +
            per_mixed + per_asian + per_black + per_other, data = msoa.spdf,
            listw = queens.lw, robust = TRUE, W2X = TRUE)
```

Residuals:

Min	1Q	Median	3Q	Max
-0.5464924	-0.1238002	-0.0052299	0.0989150	1.0793093

Coefficients:

Estimate	HC0	std. Error	z value	Pr(> z)
----------	-----	------------	---------	----------

```

Rho          0.71004211   0.04678235 15.1776 < 2.2e-16
(Intercept) 2.73582523   0.50997823 5.3646 8.113e-08
log(no2)     0.37752751   0.04920257 7.6729 1.688e-14
log(POPDEN) -0.05710992   0.01684036 -3.3913 0.0006957
per_mixed    0.01634307   0.00588488 2.7771 0.0054842
per_asian    -0.00205426   0.00045905 -4.4750 7.640e-06
per_black    -0.01166456   0.00128557 -9.0734 < 2.2e-16
per_other    -0.00280423   0.00332302 -0.8439 0.3987377

```

Residual variance (sigma squared): 0.035213, (sigma: 0.18765)

4.6 Maximum Likelihood

Maximum Likelihood estimation of spatial models is the most common way of estimation. The procedure to estimate SAR models via ML is based on Ord (1975) and Anselin (1988). If you are interested in more details, please look at the more extensive [3-day workshop](#) or have a look into Sarrias (2023).

The package **spatialreg** Pebesma and Bivand (2023) provides a series of functions to calculate the ML estimator for all spatial models we have considered.

Table from Pebesma and Bivand (2023):

model	model name	maximum likelihood estimation function
SEM	spatial error	<code>errorsarlm(..., Durbin=FALSE)</code>
SEM	spatial error	<code>spautolm(..., family="SAR")</code>
SDEM	spatial Durbin error	<code>errorsarlm(..., Durbin=TRUE)</code>
SLM	spatial lag	<code>lagsarlm(..., Durbin=FALSE)</code>
SDM	spatial Durbin	<code>lagsarlm(..., Durbin=TRUE)</code>
SAC	spatial autoregressive combined	<code>sacsarlm(..., Durbin=FALSE)</code>
GNM	general nested	<code>sacsarlm(..., Durbin=TRUE)</code>

ML SAR

```

mod_1.sar <- lagsarlm(log(med_house_price) ~ log(no2) + log(POPDEN) +
                        per_mixed + per_asian + per_black + per_other,
                        data = msoa.spdf,
                        listw = queens.lw,
                        Durbin = FALSE) # we could here extend to SDM
summary(mod_1.sar)

```

```

Call:lagsarlm(formula = log(med_house_price) ~ log(no2) + log(POPDEN) +
  per_mixed + per_asian + per_black + per_other, data = msoa.spdf,
  listw = queens.lw, Durbin = FALSE)

Residuals:
    Min         1Q     Median        3Q        Max
-0.5281789 -0.1220524 -0.0099245  0.0992203  1.0936745

Type: lag
Coefficients: (asymptotic standard errors)
            Estimate Std. Error z value Pr(>|z|)
(Intercept) 3.17383180 0.29041604 10.9286 < 2.2e-16
log(no2)    0.39705423 0.04452880  8.9168 < 2.2e-16
log(POPDEN) -0.05583014 0.01242876 -4.4920 7.055e-06
per_mixed   0.01851577 0.00579832  3.1933 0.001407
per_asian   -0.00228346 0.00045876 -4.9775 6.442e-07
per_black   -0.01263650 0.00100282 -12.6009 < 2.2e-16
per_other   -0.00161419 0.00289082 -0.5584  0.576582

Rho: 0.66976, LR test value: 473.23, p-value: < 2.22e-16
Asymptotic standard error: 0.025311
z-value: 26.461, p-value: < 2.22e-16
Wald statistic: 700.19, p-value: < 2.22e-16

Log likelihood: 196.7203 for lag model
ML residual variance (sigma squared): 0.035402, (sigma: 0.18815)
Number of observations: 983
Number of parameters estimated: 9
AIC: -375.44, (AIC for lm: 95.786)
LM test for residual autocorrelation
test value: 8.609, p-value: 0.0033451

```

ML SEM

```

mod_1.sem <- errorsarlm(log(med_house_price) ~ log(no2) + log(POPDEN) +
  per_mixed + per_asian + per_black + per_other,
  data = msoa.spdf,
  listw = queens.lw,
  Durbin = FALSE) # we could here extend to SDEM
summary(mod_1.sem)

```

```

Call:errorsarlm(formula = log(med_house_price) ~ log(no2) + log(POPDEN) +
  per_mixed + per_asian + per_black + per_other, data = msoa.spdf,
  listw = queens.lw, Durbin = FALSE)

Residuals:
    Min         1Q     Median      3Q      Max
-0.581785 -0.105218 -0.012758  0.094430  0.913425

Type: error
Coefficients: (asymptotic standard errors)
            Estimate Std. Error z value Pr(>|z|)
(Intercept) 12.92801104  0.35239139 36.6865 < 2.2e-16
log(no2)     0.15735296  0.10880727  1.4462 0.1481317
log(POPDEN)  -0.08316270  0.01254315 -6.6301 3.354e-11
per_mixed    -0.03377962  0.00811054 -4.1649 3.115e-05
per_asian    -0.00413115  0.00096849 -4.2656 1.994e-05
per_black    -0.01653816  0.00126741 -13.0488 < 2.2e-16
per_other    -0.01693012  0.00462999 -3.6566 0.0002556

Lambda: 0.88605, LR test value: 623.55, p-value: < 2.22e-16
Asymptotic standard error: 0.015803
z-value: 56.068, p-value: < 2.22e-16
Wald statistic: 3143.6, p-value: < 2.22e-16

Log likelihood: 271.8839 for error model
ML residual variance (sigma squared): 0.026911, (sigma: 0.16405)
Number of observations: 983
Number of parameters estimated: 9
AIC: NA (not available for weighted model), (AIC for lm: 95.786)

```

5 Spatial Impacts

Required packages

```
pkgs <- c("sf", "mapview", "spdep", "spatialreg", "tmap", "viridisLite") # note: load spde
lapply(pkgs, require, character.only = TRUE)
```

Session info

```
sessionInfo()

R version 4.3.1 (2023-06-16 ucrt)
Platform: x86_64-w64-mingw32/x64 (64-bit)
Running under: Windows 10 x64 (build 19044)

Matrix products: default

locale:
[1] LC_COLLATE=English_United Kingdom.utf8
[2] LC_CTYPE=English_United Kingdom.utf8
[3] LC_MONETARY=English_United Kingdom.utf8
[4] LC_NUMERIC=C
[5] LC_TIME=English_United Kingdom.utf8

time zone: Europe/Berlin
tzcode source: internal

attached base packages:
[1] stats      graphics   grDevices  utils      datasets   methods    base

other attached packages:
[1] viridisLite_0.4.2 tmap_3.3-3       spatialreg_1.2-9  Matrix_1.5-4.1
[5] spdep_1.2-8     spData_2.3.0      mapview_2.11.0   sf_1.0-13
```

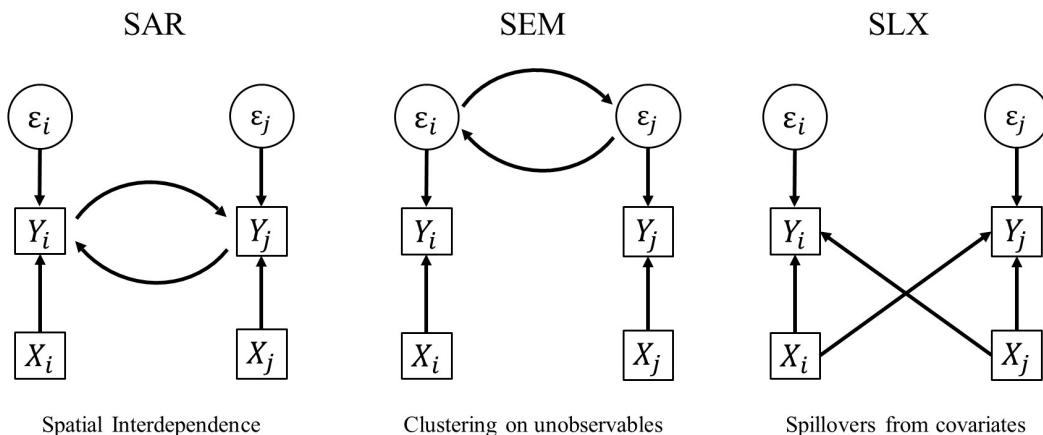
```

loaded via a namespace (and not attached):
[1] xfun_0.39           raster_3.6-23      htmlwidgets_1.6.2  lattice_0.21-8
[5] vctrs_0.6.3         tools_4.3.1       crosstalk_1.2.0   LearnBayes_2.15.1
[9] generics_0.1.3      parallel_4.3.1    sandwich_3.0-2   stats4_4.3.1
[13] tibble_3.2.1       proxy_0.4-27     fansi_1.0.4      pkgconfig_2.0.3
[17] KernSmooth_2.23-21 satellite_1.0.4 RColorBrewer_1.1-3 leaflet_2.1.2
[21] webshot_0.5.5      lifecycle_1.0.3   compiler_4.3.1   deldir_1.0-9
[25] munsell_0.5.0      terra_1.7-39     leafsync_0.1.0   codetools_0.2-
19
[29] stars_0.6-1        htmltools_0.5.5  class_7.3-22    pillar_1.9.0
[33] MASS_7.3-60         classInt_0.4-9   lwgeom_0.2-13   wk_0.7.3
[37] abind_1.4-5        boot_1.3-28.1   multcomp_1.4-25 nlme_3.1-162
[41] tidyselect_1.2.0    digest_0.6.32   mvtnorm_1.2-2   dplyr_1.1.2
[45] splines_4.3.1      fastmap_1.1.1   grid_4.3.1     colorspace_2.1-
0
[49] expm_0.999-7       cli_3.6.1       magrittr_2.0.3  base64enc_0.1-3
[53] dichromat_2.0-0.1  XML_3.99-0.14  survival_3.5-5  utf8_1.2.3
[57] TH.data_1.1-2      leafem_0.2.0    e1071_1.7-13   scales_1.2.1
[61] sp_1.6-1            rmarkdown_2.23  zoo_1.8-12    png_0.1-8
[65] coda_0.19-4        evaluate_0.21  knitr_1.43    tmaptools_3.1-1
[69] s2_1.1.4            rlang_1.1.1    Rcpp_1.0.10   glue_1.6.2
[73] DBI_1.1.3          rstudioapi_0.14 jsonlite_1.8.5 R6_2.5.1
[77] units_0.8-2

```

Reload data from previous session

```
load("_data/msoa2_spatial.RData")
```



5.1 Coefficient estimates ≠ ‘marginal’ effects

 Warning

Do not interpret coefficients as marginal effects in SAR, SAC, and SDM!!

At first glance, the specifications presented above seem relatively similar in the way of modelling spatial effects. **Yet, they differ in very important aspects.**

First, models with an endogenous spatial term (SAR, SAC, and SDM) assume a very different spatial dependence structure than models with only exogenous spatial terms as SLX and SDEM specifications. While the first three assume **global** spatial dependence, the second two assume **local** spatial dependence (Anselin 2003; Halleck Vega and Elhorst 2015; LeSage and Pace 2009).

Second, the interpretation of the coefficients differs greatly between models with and without endogenous effects. This becomes apparent when considering the reduced form of the equations above. Exemplary using the SAR model, the reduced form is given by:

$$\begin{aligned}\mathbf{y} - \rho \mathbf{W} \mathbf{y} &= \mathbf{X} +, \\ (\mathbf{I}_N - \rho \mathbf{W}) \mathbf{y} &= \mathbf{X} +, \\ \mathbf{y} &= (\mathbf{I}_N - \rho \mathbf{W})^{-1} (\mathbf{X} +),\end{aligned}$$

where \mathbf{I}_N is an $N \times N$ diagonal matrix (diagonal elements equal 1, 0 otherwise). This contains no spatially lagged dependent variable on the right-hand side.

If we want to interpret coefficient, we are usually in marginal or partial effects (the association between a unit change in X and Y). We obtain these effects by looking at the first derivative.

When taking the first derivative of the explanatory variable \mathbf{x}_k from the reduced form in (??) to interpret the partial effect of a unit change in variable \mathbf{x}_k on \mathbf{y} , we receive

$$\frac{\partial \mathbf{y}}{\partial \mathbf{x}_k} = \underbrace{(\mathbf{I}_N - \rho \mathbf{W})^{-1}}_{N \times N} \beta_k,$$

for each covariate $k = \{1, 2, \dots, K\}$. As can be seen, the partial derivative with respect to \mathbf{x}_k produces an $N \times N$ matrix, thereby representing the partial effect of each unit i onto the focal unit i itself and all other units $j = \{1, 2, \dots, i-1, i+1, \dots, N\}$.

Note that the diagonal elements of $(\mathbf{I}_N - \rho \mathbf{W})^{-1}$ are not zero anymore (as they are in \mathbf{W}). Look at the following minimal example:

$$\tilde{\mathbf{W}} = \begin{pmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix}, \text{ and normalized } \mathbf{W} = \begin{pmatrix} 0 & 0.5 & 0 & 0.5 & 0 \\ 0.33 & 0 & 0.33 & 0 & 0.33 \\ 0 & 0.5 & 0 & 0.5 & 0 \\ 0.33 & 0 & 0.33 & 0 & 0.33 \\ 0 & 0.5 & 0 & 0.5 & 0 \end{pmatrix}$$

and

$$\rho = 0.6,$$

then

$$\rho\mathbf{W} = \begin{pmatrix} 0 & 0.3 & 0 & 0.3 & 0 \\ 0.2 & 0 & 0.2 & 0 & 0.2 \\ 0 & 0.3 & 0 & 0.3 & 0 \\ 0.2 & 0 & 0.2 & 0 & 0.2 \\ 0 & 0.3 & 0 & 0.3 & 0 \end{pmatrix}.$$

If we want to get the total effect of X on Y we need to add the direct association within i and j and so on...

$$\begin{aligned} \mathbf{I}_N - \rho\mathbf{W} &= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0.3 & 0 & 0.3 & 0 \\ 0.2 & 0 & 0.2 & 0 & 0.2 \\ 0 & 0.3 & 0 & 0.3 & 0 \\ 0.2 & 0 & 0.2 & 0 & 0.2 \\ 0 & 0.3 & 0 & 0.3 & 0 \end{pmatrix} \\ &= \begin{pmatrix} 1 & -0.3 & 0 & -0.3 & 0 \\ -0.2 & 1 & -0.2 & 0 & -0.2 \\ 0 & 0.3 & 1 & 0.3 & 0 \\ -0.2 & 0 & -0.2 & 1 & -0.2 \\ 0 & -0.3 & 0 & -0.3 & 1 \end{pmatrix}. \end{aligned}$$

And finally we take the inverse of that

$$\begin{aligned}
 (\mathbf{I}_N - \rho \mathbf{W})^{-1} &= \begin{pmatrix} 1 & -0.3 & 0 & -0.3 & 0 \\ -0.2 & 1 & -0.2 & 0 & -0.2 \\ 0 & 0.3 & 1 & 0.3 & 0 \\ -0.2 & 0 & -0.2 & 1 & -0.2 \\ 0 & -0.3 & 0 & -0.3 & 1 \end{pmatrix}^{-1} \\
 &= \begin{pmatrix} \textcolor{red}{1.1875} & 0.46875 & 0.1875 & 0.46875 & 0.1875 \\ 0.3125 & \textcolor{red}{1.28125} & 0.3125 & 0.28125 & 0.3125 \\ 0.1875 & 0.46875 & \textcolor{red}{1.1875} & 0.46875 & 0.1875 \\ 0.3125 & 0.28125 & 0.3125 & \textcolor{red}{1.28125} & 0.3125 \\ 0.1875 & 0.46875 & 0.1875 & 0.46875 & \textcolor{red}{1.1875} \end{pmatrix}.
 \end{aligned}$$

As you can see, $(\mathbf{I}_N - \rho \mathbf{W})^{-1}$ has *diagonal elements* > 1 : these are feedback loops. My X influences my Y directly, but my Y then influences my neighbour's Y , which then influences my Y again (also also other neighbour's Y s). Thus the influence of my X on my Y includes a spatial multiplier.

Check yourself:

```

I = diag(5)
rho = 0.6
W = matrix(c(0 , 0.5 , 0 , 0.5 , 0,
             1/3 , 0 , 1/3 , 0 , 1/3,
             0 , 0.5 , 0 , 0.5 , 0,
             1/3 , 0 , 1/3 , 0 , 1/3,
             0 , 0.5 , 0 , 0.5 , 0), ncol = 5, byrow = TRUE)

(IrW = I - rho*W)

[,1] [,2] [,3] [,4] [,5]
[1,] 1.0 -0.3 0.0 -0.3 0.0
[2,] -0.2 1.0 -0.2 0.0 -0.2
[3,] 0.0 -0.3 1.0 -0.3 0.0
[4,] -0.2 0.0 -0.2 1.0 -0.2
[5,] 0.0 -0.3 0.0 -0.3 1.0

# (I - rho*W)^-1
(M = solve(IrW))

```

```
[,1]      [,2]      [,3]      [,4]      [,5]
[1,] 1.1875 0.46875 0.1875 0.46875 0.1875
[2,] 0.3125 1.28125 0.3125 0.28125 0.3125
[3,] 0.1875 0.46875 1.1875 0.46875 0.1875
[4,] 0.3125 0.28125 0.3125 1.28125 0.3125
[5,] 0.1875 0.46875 0.1875 0.46875 1.1875
```

The diagonal elements of M indicate how each unit i influences itself (change of x_i on change of y_i), and each off-diagonal elements in column j represents the effect of j on each other unit i (change of x_j on change of y_i).

$$\begin{pmatrix} 1.1875 & \textcolor{red}{0.46875} & 0.1875 & 0.46875 & 0.1875 \\ 0.3125 & 1.28125 & 0.3125 & 0.28125 & 0.3125 \\ 0.1875 & 0.46875 & 1.1875 & 0.46875 & 0.1875 \\ 0.3125 & 0.28125 & 0.3125 & 1.28125 & 0.3125 \\ 0.1875 & 0.46875 & \textcolor{blue}{0.1875} & 0.46875 & 1.1875 \end{pmatrix}.$$

For instance, $\textcolor{red}{W_{12}}$ indicates that unit 2 has an influence of 0.46875 on unit 1. On the other hand, $\textcolor{blue}{W_{53}}$ indicates that unit 3 has an influence of magnitude 0.1875 on unit 5.

💡 Question

Why does unit 3 have any effect on unit 5? According to \mathbf{W} those two units are no neighbours $w_{53} = 0$!

5.2 Global and local spillovers

The kind of indirect spillover effects in SAR, SAC, and SDM models differs from the kind of indirect spillover effects in SLX and SDEM models: while the first three specifications represent **global spillover effects**, the latter three represent **local spillover effects** (Anselin 2003; LeSage and Pace 2009; LeSage 2014).

5.2.1 Local spillovers

In case of SLX and SDEM the spatial spillover effects can be interpreted as the effect of a one unit change of \mathbf{x}_k in the spatially weighted neighbouring observations on the dependent variable of the focal unit: the weighted average among neighbours; when using a row-normalised contiguity weights matrix, \mathbf{Wx}_k is the mean value of \mathbf{x}_k in the neighbouring units.

Assume we have $k = 2$ covariates, then

$$\begin{aligned} \underbrace{\mathbf{W}}_{N \times N} \underbrace{\mathbf{X}}_{N \times 2} &= \begin{pmatrix} 0 & 0.5 & 0 & 0.5 & 0 \\ 0.33 & 0 & 0.33 & 0 & 0.33 \\ 0 & 0.5 & 0 & 0.5 & 0 \\ 0.33 & 0 & 0.33 & 0 & 0.33 \\ 0 & 0.5 & 0 & 0.5 & 0 \end{pmatrix} \begin{pmatrix} 3 & 100 \\ 4 & 140 \\ 1 & 200 \\ 7 & 70 \\ 5 & 250 \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} \\ &= \begin{pmatrix} 6 & 105 \\ 3 & 190 \\ 6 & 105 \\ 3 & 190 \\ 6 & 105 \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} \end{aligned}$$

```
X <- cbind(x1 = c(3,4,1,8,5),
             x2 = c(100,140,200,70,270))
WX <- W %*% X
```

```
x1  x2
[1,] 6 105
[2,] 3 190
[3,] 6 105
[4,] 3 190
[5,] 6 105
```

Thus, only direct neighbours – as defined in \mathbf{W} – contribute to those local spillover effects. The $\hat{\theta}$ coefficients only estimate how my direct neighbour's \mathbf{X} values influence my own outcome \mathbf{y} .

There are no higher order neighbours involved (as long as we do not model them), nor are there any feedback loops due to interdependence.

5.2.2 Global spillovers

In contrast, spillover effects in SAR, SAC, and SDM models do not only include direct neighbours but also neighbours of neighbours (second order neighbours) and further higher-order neighbours. This can be seen by rewriting the inverse $(\mathbf{I}_N - \rho \mathbf{W})^{-1}$ as power series: A power series of $\sum_{k=0}^{\infty} \mathbf{W}^k$ converges to $(\mathbf{I} - \mathbf{W})^{-1}$ if the maximum absolute eigenvalue of $\mathbf{W} < 1$, which is ensured by standardizing \mathbf{W} .}

$$(\mathbf{I}_N - \rho \mathbf{W})^{-1} \beta_k = (\mathbf{I}_N + \rho \mathbf{W} + \rho^2 \mathbf{W}^2 + \rho^3 \mathbf{W}^3 + \dots) \beta_k = (\mathbf{I}_N + \sum_{h=1}^{\infty} \rho^h \mathbf{W}^h) \beta_k,$$

where the identity matrix represents the direct effects and the sum represents the first and higher order indirect effects and the above mentioned feedback loops. This implies that a change in one unit i does not only affect the direct neighbours but passes through the whole system towards higher-order neighbours, where the impact declines with distance within the neighbouring system. Global indirect impacts thus are ‘multiplied’ by influencing direct neighbours as specified in \mathbf{W} and indirect neighbours not connected according to \mathbf{W} , with additional feedback loops between those neighbours.

$$\left(\underbrace{\mathbf{I}_N}_{N \times N} - \underbrace{\rho \mathbf{W}}_{\hat{=} 0.6 N \times N} \right)^{-1} \beta_k = \begin{pmatrix} 1.1875 & 0.46875 & 0.1875 & 0.46875 & 0.1875 \\ 0.3125 & 1.28125 & 0.3125 & 0.28125 & 0.3125 \\ 0.1875 & 0.46875 & 1.1875 & 0.46875 & 0.1875 \\ 0.3125 & 0.28125 & 0.3125 & 1.28125 & 0.3125 \\ 0.1875 & 0.46875 & 0.1875 & 0.46875 & 1.1875 \end{pmatrix} (\beta_1 + \beta_2)$$

All diagonal elements of $\text{diag}(\mathbf{W}) = w_{ii} = 0$. However, diagonal elements of higher order neighbours are not zero $\text{diag}(\mathbf{W}^2) = \text{diag}(\mathbf{WW}) \neq 0$.

Intuitively, $\rho \mathbf{W}$ only represents the effects between direct neighbours (and the focal unit is not a neighbour of the focal unit itself), whereas $\rho^2 \mathbf{W}^2$ contains the effects of second order neighbours, where the focal unit is a second order neighbour of the focal unit itself. Thus, $(\mathbf{I}_N - \rho \mathbf{W})^{-1} \beta_k$ includes feedback effects from $\rho^2 \mathbf{W}^2$ on (they are part of the direct impacts according to the summary measures below). This is why the diagonal above ≥ 1 .

In consequence, local and global spillover effects represent two distinct kinds of spatial spillover effects (LeSage 2014). The interpretation of local spillover effects is straightforward: it represents the effect of all neighbours as defined by \mathbf{W} (the average over all neighbours in case of a row-normalised weights matrix).

For instance, the environmental quality in the focal unit itself but also in neighbouring units could influence the attractiveness of a district and its house prices. In this example it seems reasonable to assume that we have local spillover effects: only the environmental quality in directly contiguous units (e.g. in walking distance) is relevant for estimating the house prices.

In contrast, interpreting global spillover effects can be a bit more difficult. Intuitively, the global spillover effects can be seen as a kind of diffusion process. For example, an exogenous event might increase the house prices in one district of a city, thus leading to an adaptation of house prices in neighbouring districts, which then leads to further adaptations in other units

(the neighbours of the neighbours), thereby globally diffusing the effect of the exogenous event due to the endogenous term.

Yet, those processes happen over time. In a cross-sectional framework, the global spillover effects are hard to interpret. Anselin (2003) proposes an interpretation as an equilibrium outcome, where the partial impact represents an estimate of how this long-run equilibrium would change due to a change in \mathbf{x}_k (LeSage 2014).

5.3 Summary impact measures

Note that the derivative in SAR, SAC, and SDM is a $N \times N$ matrix, returning individual effects of each unit on each other unit, differentiated in *direct, indirect, and total impacts*.

$$(\mathbf{I}_N - \rho\mathbf{W})^{-1}\beta = \begin{pmatrix} 1.1875 & 0.46875 & 0.1875 & 0.46875 & 0.1875 \\ 0.3125 & 1.28125 & 0.3125 & 0.28125 & 0.3125 \\ 0.1875 & 0.46875 & 1.1875 & 0.46875 & 0.1875 \\ 0.3125 & 0.28125 & 0.3125 & 1.28125 & 0.3125 \\ 0.1875 & 0.46875 & 0.1875 & 0.46875 & 1.1875 \end{pmatrix} \beta$$

However, the individual effects (how i influences j) mainly vary because of variation in \mathbf{W} .

⚠ Do not interpret these as “estimated” individual impacts

We estimate two scalar parameters in a SAR model: β for the direct coefficient and ρ for the auto-regressive parameter.

All variation in the effects matrix $(\mathbf{I}_N - \rho\mathbf{W})^{-1}$ comes from the relationship in \mathbf{W} which we have given a-priori!

Since reporting the individual partial effects is usually not of interest, LeSage and Pace (2009) proposed to average over these effect matrices. While the average diagonal elements of the effects matrix $(\mathbf{I}_N - \rho\mathbf{W})^{-1}$ represent the so called direct impacts of variable \mathbf{x}_k , the average column-sums of the off-diagonal elements represent the so called indirect impacts (or spatial spillover effects).

direct impacts refer to an average effect of a unit change in x_i on y_i , and the indirect (spillover) impacts indicate how a change in x_i , on average, influences all neighbouring units y_j .

Though previous literature (Halleck Vega and Elhorst 2015; LeSage and Pace 2009) has established the notation of direct and indirect impacts, it is important to note that also the direct impacts comprise a spatial ‘multiplier’ component if we specify an endogenous lagged

depended variable, as a change in \mathbf{x}_i influences \mathbf{y}_i , which influences \mathbf{y}_j , which in turn influences \mathbf{y}_i .

Usually, one should use summary measures to report effects in spatial models (LeSage and Pace 2009). Halleck Vega and Elhorst (2015) provide a nice summary of the impacts for each model:

Model	Direct Impacts	Indirect Impacts	type
OLS/SEM	β_k	—	—
SAR/SAC	Diagonal elements of $(\mathbf{I} - \rho\mathbf{W})^{-1}\beta_k$	Off-diagonal elements of $(\mathbf{I} - \rho\mathbf{W})^{-1}\beta_k$	global
SLX/SDEM	β_k	θ_k	local
SDM	Diagonal elements of $(\mathbf{I} - \rho\mathbf{W})^{-1} [\beta_k + \mathbf{W}\theta_k]$	Off-diagonal elements of $(\mathbf{I} - \rho\mathbf{W})^{-1} [\beta_k + \mathbf{W}\theta_k]$	global

$$(\mathbf{I}_N - \rho\mathbf{W})^{-1}\beta = \begin{pmatrix} 1.1875 & 0.46875 & 0.1875 & 0.46875 & 0.1875 \\ 0.3125 & 1.28125 & 0.3125 & 0.28125 & 0.3125 \\ 0.1875 & 0.46875 & 1.1875 & 0.46875 & 0.1875 \\ 0.3125 & 0.28125 & 0.3125 & 1.28125 & 0.3125 \\ 0.1875 & 0.46875 & 0.1875 & 0.46875 & 1.1875 \end{pmatrix} \beta$$

The different indirect effects / spatial effects mean conceptually different things:

- Global spillover effects: SAR, SAC, SDM
- Local spillover effects: SLX, SDEM

! Common ratio between direct and indirect impacts in SAR and SAC

Note that impacts in SAR only estimate one single spatial multiplier coefficient. Thus direct and indirect impacts are bound to a common ratio, say ϕ , across all covariates. if $\beta_1^{direct} = \phi\beta_1^{indirect}$, then $\beta_2^{direct} = \phi\beta_2^{indirect}$, $\beta_k^{direct} = \phi\beta_k^{indirect}$.

We can calculate these impacts using `impacts()` with simulated distributions, e.g. for the SAR model:

```
mod_1.sar.imp <- impacts(mod_1.sar, listw = queens.lw, R = 300)
summary(mod_1.sar.imp, zstats = TRUE, short = TRUE)
```

```

Impact measures (lag, exact):
      Direct    Indirect     Total
log(no2)   0.447853184  0.754466618  1.202319802
log(POPDEN) -0.062973027 -0.106086209 -0.169059236
per_mixed   0.020884672  0.035182931  0.056067603
per_asian   -0.002575602 -0.004338934 -0.006914536
per_black   -0.014253206 -0.024011369 -0.038264575
per_other   -0.001820705 -0.003067212 -0.004887917
=====
Simulation results ( variance matrix):
=====
Simulated standard errors
      Direct    Indirect     Total
log(no2)   0.0494175081  0.0971270096  0.134257535
log(POPDEN) 0.0145563259  0.0278341102  0.041558184
per_mixed   0.0063415668  0.0115411801  0.017649732
per_asian   0.0004991518  0.0008479782  0.001303205
per_black   0.0010099979  0.0022354462  0.002650574
per_other   0.0031091019  0.0053645307  0.008460770

Simulated z-values:
      Direct    Indirect     Total
log(no2)   9.0618695   7.8117374   8.9868007
log(POPDEN) -4.3161115  -3.8408895  -4.0842609
per_mixed   3.2820413   3.0711321   3.1874577
per_asian   -5.0924415  -5.0674192  -5.2478023
per_black   -14.1188639  -10.8059832  -14.4935469
per_other   -0.5849657  -0.5896361  -0.5888163

Simulated p-values:
      Direct    Indirect     Total
log(no2)   < 2.22e-16  5.5511e-15 < 2.22e-16
log(POPDEN) 1.5880e-05  0.00012259  4.4217e-05
per_mixed   0.0010306   0.00213249  0.0014353
per_asian   3.5348e-07  4.0325e-07  1.5392e-07
per_black   < 2.22e-16  < 2.22e-16 < 2.22e-16
per_other   0.5585708   0.55543464  0.5559845

# Alternative with traces (better for large W)
W <- as(queens.lw, "CsparseMatrix")
trMatc <- trW(W, type = "mult",

```

```

m = 30) # number of powers
mod_1.sar.imp2 <- impacts(mod_1.sar,
                           tr = trMatc, # trace instead of listw
                           R = 300,
                           Q = 30) # number of power series used for approximation
summary(mod_1.sar.imp2, zstats = TRUE, short = TRUE)

Impact measures (lag, trace):
      Direct    Indirect     Total
log(no2)   0.447853101  0.754459497  1.202312598
log(POPDEN) -0.062973015 -0.106085208 -0.169058223
per_mixed   0.020884668  0.035182599  0.056067267
per_asian   -0.002575601 -0.004338893 -0.006914494
per_black   -0.014253203 -0.024011142 -0.038264346
per_other   -0.001820704 -0.003067183 -0.004887888
=====

Simulation results ( variance matrix):
=====
Simulated standard errors
      Direct    Indirect     Total
log(no2)   0.0478560754  0.0946197332  0.129991489
log(POPDEN) 0.0137369704  0.0255446546  0.038410164
per_mixed   0.0064782366  0.0111640462  0.017421256
per_asian   0.0004784614  0.0008307828  0.001263728
per_black   0.0010661169  0.0022099065  0.002723152
per_other   0.0033933382  0.0059129401  0.009287672

Simulated z-values:
      Direct    Indirect     Total
log(no2)   9.3973681   8.0136104   9.2926609
log(POPDEN) -4.5523254  -4.1396603  -4.3811672
per_mixed   3.2607009   3.1906214   3.2571610
per_asian   -5.3918519  -5.2311851  -5.4804278
per_black   -13.3975484 -10.8922331 -14.0844783
per_other   -0.6910765  -0.6791833  -0.6848893

Simulated p-values:
      Direct    Indirect     Total
log(no2)   < 2.22e-16 1.1102e-15 < 2.22e-16
log(POPDEN) 5.3056e-06 3.4782e-05 1.1805e-05
per_mixed   0.0011114  0.0014197  0.0011253
per_asian   6.9735e-08 1.6843e-07 4.2430e-08

```

```

per_black < 2.22e-16 < 2.22e-16 < 2.22e-16
per_other  0.4895174 0.4970217 0.4934138

```

The indirect effects in SAR, SAC, and SDM refer to global spillover effects. This means a change of x in the focal units flows through the entire system of neighbours (direct neighbours, neighbours of neighbours, ...) influencing ‘their y ’. One can think of this as diffusion or a change in a long-term equilibrium.

If Log NO₂ increases by one unit, this increases the house price in the focal unit by 0.448 units. Overall, a one unit change in log NO₂ increases the house prices in the entire neighbourhood system (direct and higher order neighbours) by 0.754.

For SLX models, nothing is gained from computing the impacts, as they equal the coefficients. Again, it’s the effects of direct neighbours only.

```
print(impacts(mod_1.slx, listw = queens.lw))
```

```

Impact measures (SLX, glht):
      Direct    Indirect      Total
log(no2) -0.440727458 0.993602103 0.552874645
log(POPDEN) -0.076839828 0.113262218 0.036422390
per_mixed -0.033042221 0.126068686 0.093026466
per_asian -0.002380698 -0.003828126 -0.006208824
per_black -0.016229407 -0.018053503 -0.034282910
per_other -0.020391354 0.048139008 0.027747654

```

5.4 Examples

Boillat, Ceddia, and Bottazzi (2022)

The paper investigates the effects of protected areas and various land tenure regimes on deforestation and possible spillover effects in Bolivia, a global tropical deforestation hotspot.

Table 3
SDM Models (1–5) estimates of ADE, AIE and ATE expressed in terms of elasticity.

	SDM1			SDM2			SDM3		
	ADE	AIE	ATE	ADE	AIE	ATE	ADE	AIE	ATE
TRAVELTIME	-0.15** (0.06)	0.11 (0.19)	-0.04 (0.22)	-0.12* (0.06)	0.18 (0.20)	0.06 (0.22)	-0.01 (0.35)	0.40 (0.40)	0.40** (0.19)
SLOPE	-0.29*** (0.06)	-0.31*** (0.08)	-0.60*** (0.09)	-0.28*** (0.06)	-0.32*** (0.08)	-0.60*** (0.10)	-0.29 (0.26)	-0.32 (0.30)	-0.61*** (0.10)
POPDENS12	-0.004 (0.005)	0.09** (0.04)	0.09** (0.04)	0.00 (0.01)	0.09*** (0.04)	0.09** (0.03)	0.00 (0.03)	0.09 (0.09)	0.10*** (0.03)
WATERACCESS12	0.44*** (0.15)	1.59*** (0.35)	2.03*** (0.35)	0.41** (0.15)	1.44*** (0.35)	1.85*** (0.35)	0.44 (1.04)	1.19 (1.16)	1.63*** (0.32)
POVERTY12	-0.15* (0.08)	-1.08*** (0.20)	-1.22*** (0.22)	-0.13 (0.08)	-1.06*** (0.20)	-1.19*** (0.22)	-0.16 (0.67)	-0.78 (0.73)	-0.93*** (0.21)
PROTECTEDAREA	-0.13*** (0.03)	-0.21*** (0.05)	-0.34*** (0.07)	-0.12*** (0.03)	-0.20*** (0.05)	-0.32*** (0.07)	-0.12 (0.14)	-0.16 (0.14)	-0.28*** (0.06)
NOTITLELAND13	-0.11* (0.06)	0.85*** (0.18)	0.74*** (0.21)	0.17* (0.10)	0.95*** (0.36)	1.12*** (0.38)	0.21 (0.25)	0.25 (0.36)	0.47 (0.40)
INDIGENLAND13	-0.15*** (0.04)	0.22* (0.12)	0.06 (0.13)	-0.01 (0.05)	0.26 (0.19)	0.25 (0.20)	-0.01 (0.05)	-0.01 (0.18)	-0.01 (0.20)
COMMONLAND13	0.01 (0.03)	0.19** (0.09)	0.21** (0.10)	0.12*** (0.04)	0.23 (0.16)	0.35** (0.17)	0.12** (0.05)	0.04 (0.15)	0.15 (0.17)
STATELAND13	-0.20*** (0.06)	0.28* (0.15)	0.08 (0.17)	0.04 (0.09)	0.38 (0.28)	0.42 (0.30)	0.05 (0.13)	-0.12 (0.35)	-0.07 (0.34)
PRIVATELAND13				0.22*** (0.06)	0.11 (0.20)	0.33 (0.22)			
SMALLPROP13							0.24*** (0.03)	0.01 (0.06)	0.26*** (0.07)
MEDIUMPROP13							-0.06 (0.11)	-0.13 (0.16)	-0.19** (0.09)
BUSINESSLAND13							0.03 (0.03)	-0.02 (0.14)	0.01 (0.16)

Standard errors in parentheses.

*** p < 0.01, ** p < 0.05, * p < 0.1.

Protected areas – which in Bolivia are all based on co-management schemes - also protect forests in adjacent areas, showing an indirect protective spillover effect. Indigenous lands however only have direct forest protection effects.

Fischer et al. (2009)

The focus of this paper is on the role of human capital in explaining labor productivity variation among 198 European regions within a regression framework.

Table 2 Direct, indirect and total impact estimates (*t*-statistics in parentheses)

Variables	Spatial Durbin model		
	Mean	Mean	Mean
	direct impact	indirect impact	total impact
Initial labor productivity	0.6677 (27.5716)	0.0683 (1.8992)	0.7361 (26.3921)
Human capital	0.1317 (6.8644)	-0.1968 (-3.7637)	-0.0650 (-1.1847)

Note: *t*-statistics based on 10,000 sampled raw parameter estimates of the SDM

A ceteris paribus increase in the level of human capital is found to have a significant and positive direct impact. But this positive direct impact is offset by a significant and negative indirect (spillover) impact leading to a total impact that is not significantly different from zero.

The intuition here arises from the notion that it is relative regional advantages in human capital that matter most for labor productivity, so changing human capital across all regions should have little or no total impact on (average) labor productivity levels.

Rüttenauer (2018)

This study investigates the presence of environmental inequality in Germany - the connection between the presence of foreign-minority population and objectively measured industrial pollution.

Table 3

Community-fixed effects estimates. Dependent variable: Industrial air pollution.

	Overall	Urban	Rural	Diff
	M5	M6	M7	(M6-M7)
% Foreigners	0.062*** (0.009)	0.076*** (0.016)	0.035*** (0.007)	0.041* (0.017)
W [% Foreigners]	0.163*** (0.025)	0.391*** (0.070)	0.077*** (0.015)	0.315*** (0.071)
Population	-0.010 (0.007)	-0.011 (0.009)	-0.008 (0.006)	-0.002 (0.011)
W [Population]	-0.039 (0.023)	-0.124*** (0.030)	-0.023 (0.017)	-0.100** (0.034)
% 65 and older	-0.005 (0.004)	-0.001 (0.017)	-0.000 (0.003)	-0.000 (0.017)
W [% 65 and older]	-0.009 (0.007)	-0.101 (0.072)	0.004 (0.005)	-0.105 (0.072)
% Vacant housing	0.014** (0.005)	0.035 (0.027)	0.012** (0.004)	0.023 (0.027)
W [% Vacant housing]	0.021* (0.008)	0.087 (0.092)	0.020** (0.007)	0.068 (0.093)
Living space	-0.012** (0.004)	0.014 (0.022)	-0.008* (0.003)	0.022 (0.022)
W [Living space]	-0.036*** (0.007)	-0.161* (0.070)	-0.022*** (0.006)	-0.139* (0.070)
R ²	0.018	0.076	0.005	
Adj. R ²	-0.031	0.067	-0.050	
Num. obs.	93777	9061	84716	

*** $p < 0.001$, ** $p < 0.01$, * $p < 0.05$. Standardized coefficients. Cluster robust standard errors in parentheses. W is specified as contiguity neighbours weights matrix. The differences and standard errors in column 4 are obtained by an overall fixed-effects model with interaction terms of the urban dummy and all covariates.

Results reveal that the share of minorities within a census cell indeed positively correlates with the exposure to industrial pollution. Furthermore, spatial spillover effects are highly relevant: the characteristics of the neighbouring spatial units matter in predicting the amount of pollution. Especially within urban areas, clusters of high minority neighbourhoods are affected by high levels of environmental pollution.

5.5 Comparing and Selecting Models

As we have seen, a variety of spatial model specifications exist that can be used to account for the spatial structure of the data. Thus, selecting the correct model specification remains a crucial task in applied research.

One way of selecting the model specification is the application of empirical specification tests. In general, there are two different strategies: a specific-to-general or a general-to-specific approach (Florax, Folmer, and Rey 2003; Mur and Angulo 2009).

5.5.1 Specific-to-general

The specific-to-general approach is more common in spatial econometrics. This approach starts with the most basic non-spatial model and tests for possible misspecifications due to omitted

autocorrelation in the error term or the dependent variable.

Anselin et al. (1996) proposed to use Lagrange multiplier (LM) tests for the hypotheses $H_0: \lambda = 0$ and $H_0: \rho = 0$, which are robust against the alternative source of spatial dependence.

5.5.1.1 Lagrange Multiplier Test

We have earlier talked about methods to detect auto-correlation – visualisation and Moran's I. Both methods can tell us that there is spatial autocorrelation. However, both method do not provide any information on why there is autocorrelation. Possible reasons:

- Interdependence (ρ)
- Clustering on unobservables (λ)
- Spillovers in covariates ()

Lagrange Multiplier test (Anselin et al. 1996):

- (Robust) test for spatial lag dependence LM_ρ^*
- (Robust) test for spatial error dependence LM_λ^*

Robust test for lag dependence: $H_0: \rho = 0$

$$LM_\rho^* = G^{-1} \hat{\sigma}_\epsilon^2 \left(\frac{\hat{\mathbf{W}}^\top \hat{\mathbf{W}} \mathbf{y}}{\hat{\sigma}_\epsilon^2} - \frac{\hat{\mathbf{W}}^\top \hat{\mathbf{W}}}{\hat{\sigma}_\epsilon^2} \right)^2 \sim \chi^2$$

Robust test for error dependence: $H_0: \lambda = 0$

$$LM_\lambda^* = \frac{\left(\hat{\mathbf{W}}^\top \hat{\mathbf{W}} / \hat{\sigma}_\epsilon^2 - [T \hat{\sigma}_\epsilon^2 (G + T \hat{\sigma}_\epsilon^2)^{-1}]^\top \hat{\mathbf{W}} \mathbf{y} / \hat{\sigma}_\epsilon^2 \right)^2}{T \left[1 - \frac{\hat{\sigma}_\epsilon^2}{G + \hat{\sigma}_\epsilon^2} \right]} \sim \chi^2$$

with

$$\begin{aligned} G &= (\mathbf{W} \mathbf{X})^\top (\mathbf{I} - \mathbf{X} (\mathbf{X}^\top \mathbf{X})^{-1} \mathbf{X}^\top) (\mathbf{W} \mathbf{X}) \\ T &= \text{tr}[(\mathbf{W}^\top + \mathbf{W}) \mathbf{W}], \end{aligned}$$

where $\text{tr}(\mathbf{A})$ is the sum of the main diagonal of any square matrix \mathbf{A} .

5.5.1.2 Problem

The specific-to-general approach based on the robust LM test offers a good performance in distinguishing between SAR, SEM, and non-spatial OLS (Florax, Folmer, and Rey 2003).

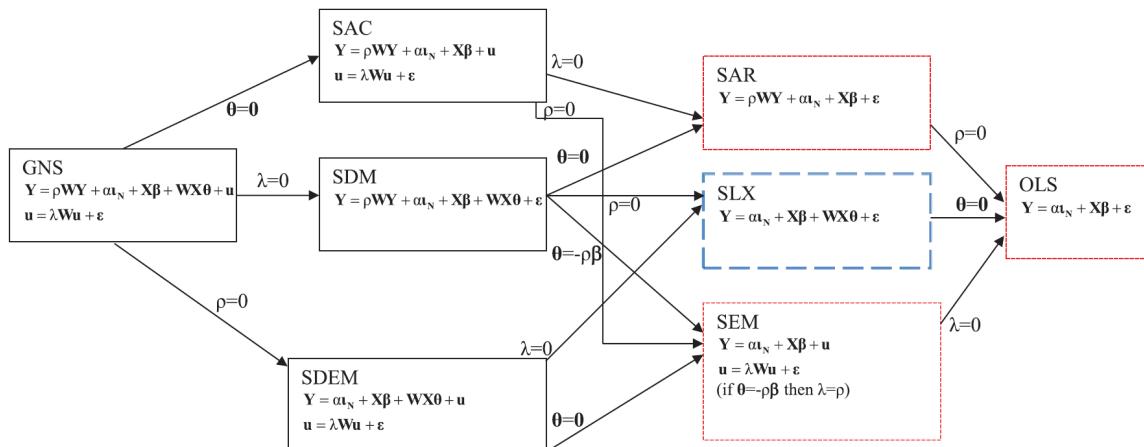
Still, in their original paper, Anselin et al. (1996) already note the declining power of the robust LM_λ test for spatial error dependence with increasing autocorrelation in the dependent variable (indicating some uncertainty under a SAC-like DGP).

Mur and Angulo (2009) demonstrate strong drawbacks of the specific-to-general approach under non-optimal conditions like heteroscedasticity or endogeneity.

Moreover, the test disregard the presence of spatial dependence from local spillover effects (θ is assumed to be zero), as resulting from an SLX-like process. Cook, Hays, and Franzese (2020), for instance, show theoretically that an SLX-like dependence structure leads to the rejection of both hypotheses $H_0: \lambda = 0$ and $H_0: \rho = 0$, though no autocorrelation is present (Elhorst and Halleck Vega 2017; Rüttenauer 2022).

5.5.2 General-to-specific approach

The general-to-specific approach depicts the opposite method of specification search. This approach starts with the most general model and stepwise imposes restrictions on the parameters of this general model.



Note: GNS = general nesting spatial model, SAC = spatial autoregressive combined model, SDM = spatial Durbin model, SDEM = spatial Durbin error model, SAR = spatial autoregressive model, SLX = spatial lag of X model, SEM = spatial error model, OLS = ordinary least squares model.

Figure 5.1: Halleck Vega and Elhorst (2015): Nesting of different Spatial Econometric Model Specifications

In theory, we would

- 1) start with a GNS specification and
- 2) subsequently restrict the model to simplified specifications based on the significance of parameters in the GNS.

The problem with this strategy is that the GNS is only weakly identified and, thus, is of little help in selecting the correct restrictions (Burridge, Elhorst, and Zigova 2016).

The most intuitive alternative would be to start with one of the two-source models SDM, SDEM, or SAC. This, however, bears the risk of imposing the wrong restriction in the first place (Cook, Hays, and Franzese 2020). Furthermore, Cook, Hays, and Franzese (2020) show that more complicated restrictions are necessary to derive all single-source models from SDEM or SAC specifications.

5.5.3 General advice?

LeSage and Pace (2009), LeSage (2014), Elhorst (2014) argue that there are strong analytical reasons to restrict the model specifications to a subset, as the SDM subsumes the SLX and SAR model, and the SDEM subsumes SLX and SEM.

It is easily observed that SDM reduces to SLX if $\rho = 0$ and to SAR if $= 0$, while the SDEM reduces to SLX if $\lambda = 0$ and to SEM if $= 0$. Less intuitively, (Anselin 1988) has also shown that the SDM subsumes the SEM. Therefore, we can express the reduced form and rearrange terms:

$$\begin{aligned} \mathbf{y} &= \mathbf{X} + (\mathbf{I}_N - \lambda \mathbf{W})^{-1} \\ (\mathbf{I}_N - \lambda \mathbf{W})\mathbf{y} &= (\mathbf{I}_N - \lambda \mathbf{W})\mathbf{X} + \\ (\mathbf{I}_N - \lambda \mathbf{W})\mathbf{y} &= \mathbf{X} - \lambda \mathbf{W}\mathbf{X} + \\ \mathbf{y} &= (\mathbf{I}_N - \lambda \mathbf{W})^{-1}(\mathbf{X} + \mathbf{W}\mathbf{X} +). \end{aligned}$$

Thus, the SEM constitutes a special case of an SDM with the relative simple restriction $= -\lambda$, meaning direct and indirect effects are constrained to a common factor (Anselin 1988, 2003).

The fact that SDM subsumes SAR, SLX, and SEM leads to the conclusion that applied research should only consider SDM and SDEM as model specifications (LeSage 2014). Especially in the case of a likely omitted variable bias, (LeSage and Pace 2009, ~68) argue in favour of using the SDM.

Nonetheless, others propose to use the SLX specification as point of departure (Gibbons and Overman 2012; Halleck Vega and Elhorst 2015). First, scholars have argued that SAC and SDM models are only weakly identified in practice (Gibbons and Overman 2012; Pinkse and Slade 2010). Second, the global spillover specification in SAR, SAC, and SDM often seems to be theoretically implausible.

And finally:

5.5.4 Design and Theory

Some argue that the best way of choosing the appropriate model specification is to exclude one or more sources of spatial dependence – autocorrelation in the dependent variable, autocorrelation in the disturbances, or spatial spillover effects of the covariates – by design Gibbons, Overman, and Patacchini (2015).

Natural experiments are probably the best way of making one or more sources of spatial dependence unlikely, thereby restricting the model alternatives to a subset of all available models. However, the opportunities to use natural experiments are restricted in social sciences, making it a favourable but often impractical way of model selection.

Cook, Hays, and Franzese (2020) and Rüttenauer (2022) argue that theoretical considerations should guide the model selection.

- 1) Rule out some sources of spatial dependence by theory, and thus restrict the specifications to a subset (*Where does the spatial dependence come from?*),
- 2) Theoretical mechanisms may guide the choice of either global or local spillover effects.

5.5.5 SLX is robust

Rüttenauer (2022) provides Monte Carlo experiments on how different spatial models perform under different scenarios of misspecification. SLX - despite its simplicity - does a very good job even if it is the wrong model for the specific situation. I personally advice for SLX (or SDEM) as the most useful specification if you do not have theoretical assumptions about spatial interdependence in the outcome.

Having a spatially lagged dependent variable comes with an extra layer of complexity. I suggest to only use these models if you have a-priori reasons to believe that it is the correct specification for your study.



Jeffrey Wooldridge
@jmwooldridge

...

In 2018 I was invited to give a talk at SOCHER in Chile, to give my opinions about using spatial methods for policy analysis. I like the idea of putting in spatial lags of policy variables to measure spillovers. Use fixed effects with panel data, compute fully robust ses.

[Tweet übersetzen](#)



1:30 vorm. · 10. März 2021

48 Retweets 7 Zitate 316 „Gefällt mir“-Angaben 69 Lesezeichen



Twittere deine Antwort!

[Antworten](#)



Jeffrey Wooldridge @jmwooldridge · 10. März 2021

...

For the life of me, I couldn't figure out how putting in spatial lags of Y had any value. After preparing a course in July 2020, I was even more negative about this practice. It seems an unnecessary complication developed by theorists.

Figure 5.2: “I will use spatial lags of X, not spatial lags of Y”, J. Wooldridge on twitter

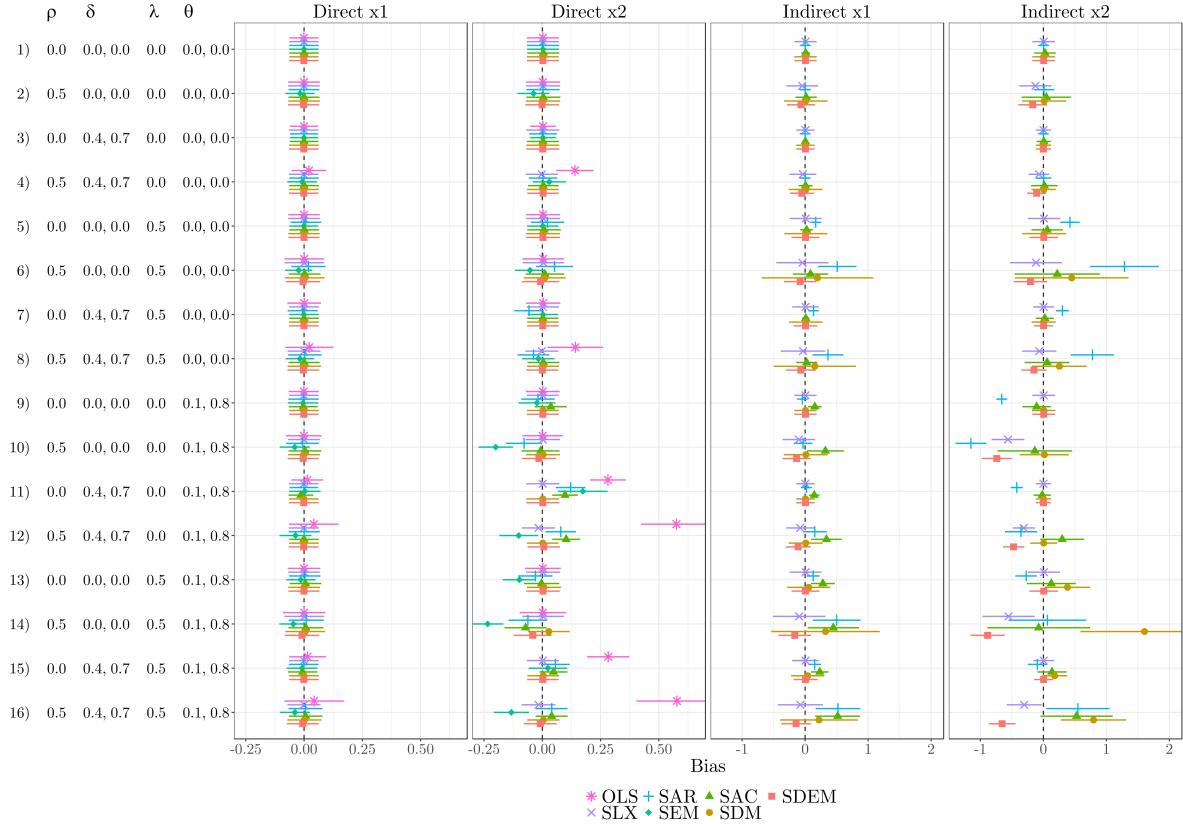


Figure 5.3: Bias of impacts and 95% confidence interval of empirical standard deviation without omv: $\rho = (0.2, 0.5)^\top$, $\delta = (0, 0)^\top$. ρ = autocorrelation in the dependent variable ($\mathbf{W}\mathbf{y}$); δ = autocorrelation in the covariates ($\mathbf{x}_k = f(\mathbf{W}\mathbf{x}_k)$); λ = autocorrelation in the disturbances ($\mathbf{W}\mathbf{u}$); θ = spatial spillover effects of covariates ($\mathbf{W}\mathbf{X}$); ρ = strength of omv.

6 Exercise II

Required packages

```
pkgs <- c("sf", "mapview", "spdep", "spatialreg", "tmap", "viridisLite") # note: load spde
lapply(pkgs, require, character.only = TRUE)
```

Session info

```
sessionInfo()

R version 4.3.1 (2023-06-16 ucrt)
Platform: x86_64-w64-mingw32/x64 (64-bit)
Running under: Windows 10 x64 (build 19044)

Matrix products: default

locale:
[1] LC_COLLATE=English_United Kingdom.utf8
[2] LC_CTYPE=English_United Kingdom.utf8
[3] LC_MONETARY=English_United Kingdom.utf8
[4] LC_NUMERIC=C
[5] LC_TIME=English_United Kingdom.utf8

time zone: Europe/Berlin
tzcode source: internal

attached base packages:
[1] stats      graphics   grDevices  utils      datasets   methods    base

other attached packages:
[1] viridisLite_0.4.2 tmap_3.3-3       spatialreg_1.2-9  Matrix_1.5-4.1
[5] spdep_1.2-8     spData_2.3.0      mapview_2.11.0   sf_1.0-13
```

```

loaded via a namespace (and not attached):
[1] xfun_0.39           raster_3.6-23      htmlwidgets_1.6.2 lattice_0.21-8
[5] vctrs_0.6.3         tools_4.3.1       crosstalk_1.2.0   LearnBayes_2.15.1
[9] generics_0.1.3      parallel_4.3.1    sandwich_3.0-2   stats4_4.3.1
[13] tibble_3.2.1        proxy_0.4-27     fansi_1.0.4      pkgconfig_2.0.3
[17] KernSmooth_2.23-21 satellite_1.0.4 RColorBrewer_1.1-3 leaflet_2.1.2
[21] webshot_0.5.5       lifecycle_1.0.3  compiler_4.3.1   deldir_1.0-9
[25] munsell_0.5.0       terra_1.7-39    leafsync_0.1.0   codetools_0.2-
19
[29] stars_0.6-1         htmltools_0.5.5  class_7.3-22    pillar_1.9.0
[33] MASS_7.3-60          classInt_0.4-9   lwgeom_0.2-13   wk_0.7.3
[37] abind_1.4-5          boot_1.3-28.1   multcomp_1.4-25 nlme_3.1-162
[41] tidyselect_1.2.0     digest_0.6.32   mvtnorm_1.2-2   dplyr_1.1.2
[45] splines_4.3.1        fastmap_1.1.1   grid_4.3.1      colorspace_2.1-
0
[49] expm_0.999-7         cli_3.6.1       magrittr_2.0.3  base64enc_0.1-3
[53] dichromat_2.0-0.1    XML_3.99-0.14  survival_3.5-5 utf8_1.2.3
[57] TH.data_1.1-2        leafem_0.2.0    e1071_1.7-13   scales_1.2.1
[61] sp_1.6-1              rmarkdown_2.23  zoo_1.8-12    png_0.1-8
[65] coda_0.19-4          evaluate_0.21  knitr_1.43    tmapproj_3.1-1
[69] s2_1.1.4              rlang_1.1.1    Rcpp_1.0.10   glue_1.6.2
[73] DBI_1.1.3             rstudioapi_0.14 jsonlite_1.8.5 R6_2.5.1
[77] units_0.8-2

```

Reload data from previous session

```
load("_data/msoa2_spatial.RData")
```

6.1 Environmental inequality

How would you investigate the following descriptive research question: Are ethnic (and immigrant) minorities in London exposed to higher levels of pollution? Also consider the spatial structure. What's your dependent and what's your independent variable?

1) Define a neighbours weights object of your choice

Assume a typical neighbourhood would be 2.5km in diameter

```
coords <- st_centroid(msoa.spdf)
```

```
Warning: st_centroid assumes attributes are constant over geometries
```

```
# Neighbours within 3km distance  
dist_15.nb <- dnearneigh(coords, d1 = 0, d2 = 2500)  
  
summary(dist_15.nb)
```

Neighbour list object:

Number of regions: 983

Number of nonzero links: 15266

Percentage nonzero weights: 1.579859

Average number of links: 15.53001

4 regions with no links:

158 463 478 505

Link number distribution:

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	
4	5	9	23	19	26	36	31	53	39	61	63	59	48	42	35	24	31	28	30	27	26	25	19	38	29	
26	27	28	29	30	31	32	33	34																		
32	38	26	16	20	10	8	1	2																		
5	least connected regions:																									
160	469	474	597	959	with 1 link																					
2	most connected regions:																									
565	567	with 34 links																								

```
# There are some mpty one. Lets impute with the nearest neighbour  
k2.nb <- knearneigh(coords, k = 1)
```

```
# Replace zero
```

```
nolink_ids <- which(card(dist_15.nb) == 0)  
dist_15.nb[card(dist_15.nb) == 0] <- k2.nb$nn[nolink_ids, ]
```

```
summary(dist_15.nb)
```

Neighbour list object:

Number of regions: 983

Number of nonzero links: 15270

```

Percentage nonzero weights: 1.580273
Average number of links: 15.53408
Link number distribution:

 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26
 9 9 23 19 26 36 31 53 39 61 63 59 48 42 35 24 31 28 30 27 26 25 19 38 29 32
27 28 29 30 31 32 33 34
38 26 16 20 10 8 1 2
9 least connected regions:
158 160 463 469 474 478 505 597 959 with 1 link
2 most connected regions:
565 567 with 34 links

# listw object with row-normalization
dist_15.lw <- nb2listw(dist_15.nb, style = "W")

```

2) Estimate the extent of spatial auto-correlation

```
moran.test(msoa.spdf$no2, listw = dist_15.lw)
```

```

Moran I test under randomisation

data: msoa.spdf$no2
weights: dist_15.lw

Moran I statistic standard deviate = 65.197, p-value < 2.2e-16
alternative hypothesis: greater
sample estimates:
Moran I statistic      Expectation      Variance
0.891520698     -0.001018330     0.000187411

```

3) Estimate a spatial SAR regression model

- a) Estimate a spatial autoregressive SAR model

```
mod_1.sar <- lagsarlm(log(no2) ~ per_mixed + per_asian + per_black + per_other
+ per_nonUK_EU + per_nonEU + log(POPDEN),
data = msoa.spdf,
```

```

listw = dist_15.lw,
Durbin = FALSE) # we could here extend to SDM
summary(mod_1.sar)

Call:lagsarlm(formula = log(no2) ~ per_mixed + per_asian + per_black +
   per_other + per_nonUK_EU + per_nonEU + log(POPDEN), data = msoa.spdf,
   listw = dist_15.lw, Durbin = FALSE)

Residuals:
    Min          1Q      Median          3Q         Max
-0.2140485 -0.0267085 -0.0021421  0.0238337  0.3505513

Type: lag
Coefficients: (asymptotic standard errors)
            Estimate Std. Error z value Pr(>|z|)
(Intercept) -1.7004e-02 1.8122e-02 -0.9383 0.348110
per_mixed    3.4376e-04 1.4758e-03  0.2329 0.815810
per_asian   -8.5205e-05 1.1494e-04 -0.7413 0.458507
per_black   -4.2754e-04 2.3468e-04 -1.8218 0.068484
per_other    1.9693e-03 7.4939e-04  2.6279 0.008591
per_nonUK_EU 8.9027e-04 3.9638e-04  2.2460 0.024703
per_nonEU   1.8460e-03 3.5159e-04  5.2506 1.516e-07
log(POPDEN)  1.8650e-02 2.7852e-03  6.6963 2.138e-11

Rho: 0.9684, LR test value: 2002.5, p-value: < 2.22e-16
Asymptotic standard error: 0.0063124
z-value: 153.41, p-value: < 2.22e-16
Wald statistic: 23535, p-value: < 2.22e-16

Log likelihood: 1562.401 for lag model
ML residual variance (sigma squared): 0.0020568, (sigma: 0.045352)
Number of observations: 983
Number of parameters estimated: 10
AIC: -3104.8, (AIC for lm: -1104.3)
LM test for residual autocorrelation
test value: 108.97, p-value: < 2.22e-16

```

- b) Have a look into the true multiplier matrix $(\mathbf{I}_N - \rho \mathbf{W})^{-1} \beta_k$

```

W <- listw2mat(dist_15.lw)
I <- diag(dim(W)[1])

rho <- unname(mod_1.sar$rho)

M <- solve(I - rho*W)

M[1:10, 1:10]

```

	1	2	3	4	5	6
[1,]	1.164650997	0.002433319	0.004089559	0.004034508	0.006545994	0.004882511
[2,]	0.010706605	1.407336301	0.643881932	0.370049927	0.464794934	0.385105188
[3,]	0.011246286	0.402426207	1.474021599	0.429011868	0.641526285	0.566175019
[4,]	0.008875918	0.185024963	0.343209495	1.684533322	0.614086824	0.631452476
[5,]	0.012000989	0.193664556	0.427684190	0.511739020	1.560840834	0.560656534
[6,]	0.010741524	0.192552594	0.452940016	0.631452476	0.672787841	1.571175245
[7,]	0.012779708	0.141953871	0.299247377	0.418234186	0.616895800	0.558170218
[8,]	0.014769006	0.125781189	0.253122442	0.295553039	0.500919513	0.357799398
[9,]	0.011708131	0.147549264	0.309080773	0.568442619	0.629156269	0.601077237
[10,]	0.009937859	0.152900148	0.306652041	0.727001926	0.553973310	0.679775059
	7	8	9	10		
[1,]	0.005808958	0.00872714	0.005854065	0.003613767		
[2,]	0.283907742	0.32703109	0.324608380	0.244640236		
[3,]	0.374059222	0.41132397	0.424986063	0.306652041		
[4,]	0.418234186	0.38421895	0.625286881	0.581601541		
[5,]	0.514079833	0.54266281	0.576726579	0.369315540		
[6,]	0.558170218	0.46513922	0.661184961	0.543820047		
[7,]	1.475511568	0.58520461	0.614170880	0.463886540		
[8,]	0.450157392	1.46638195	0.474994894	0.272339890		
[9,]	0.558337164	0.56135760	1.581077095	0.517983092		
[10,]	0.579858174	0.44255232	0.712226751	1.560083138		

c) Create an $N \times N$ effects matrix. What is the effect of unit 6 on unit 10?

```

# For beta 1

beta <- mod_1.sar$coefficients

effM <- beta[2] * M

effM[1:10, 1:10]

```

	1	2	3	4	5
[1,]	4.003610e-04	8.364789e-07	1.405829e-06	1.386904e-06	2.250254e-06
[2,]	3.680507e-06	4.837866e-04	2.213411e-04	1.272085e-04	1.597781e-04
[3,]	3.866028e-06	1.383382e-04	5.067103e-04	1.474773e-04	2.205314e-04
[4,]	3.051190e-06	6.360427e-05	1.179819e-04	5.790759e-04	2.110988e-04
[5,]	4.125465e-06	6.657422e-05	1.470209e-04	1.759156e-04	5.365554e-04
[6,]	3.692511e-06	6.619197e-05	1.557029e-04	2.170684e-04	2.312779e-04
[7,]	4.393158e-06	4.879813e-05	1.028694e-04	1.437724e-04	2.120644e-04
[8,]	5.077000e-06	4.323860e-05	8.701349e-05	1.015994e-04	1.721963e-04
[9,]	4.024792e-06	5.072160e-05	1.062497e-04	1.954081e-04	2.162790e-04
[10,]	3.416243e-06	5.256102e-05	1.054148e-04	2.499145e-04	1.904341e-04
	6	7	8	9	10
[1,]	1.678414e-06	1.996890e-06	3.000045e-06	2.012396e-06	1.242270e-06
[2,]	1.323839e-04	9.759625e-05	1.124204e-04	1.115875e-04	8.409764e-05
[3,]	1.946286e-04	1.285868e-04	1.413969e-04	1.460934e-04	1.054148e-04
[4,]	2.170684e-04	1.437724e-04	1.320793e-04	2.149489e-04	1.999316e-04
[5,]	1.927315e-04	1.767203e-04	1.865460e-04	1.982558e-04	1.269561e-04
[6,]	5.401079e-04	1.918768e-04	1.598965e-04	2.272892e-04	1.869438e-04
[7,]	1.918768e-04	5.072225e-04	2.011702e-04	2.111277e-04	1.594658e-04
[8,]	1.229973e-04	1.547463e-04	5.040841e-04	1.632845e-04	9.361968e-05
[9,]	2.066266e-04	1.919342e-04	1.929725e-04	5.435118e-04	1.780621e-04
[10,]	2.336798e-04	1.993323e-04	1.521320e-04	2.448354e-04	5.362949e-04

```
# "Effect" of unit 6 on unit 10
effM[10, 6]
```

```
6
0.0002336798
```

d) Estimate a spatial autoregressive SLX model

```
mod_1.slx <- lmSLX(log(no2) ~ per_mixed + per_asian + per_black + per_other
+ per_nonUK_EU + per_nonEU + log(POPDEN),
data = msoa.spdf,
listw = dist_15.lw,
Durbin = TRUE)
```

e) Calculate and interpret the summary impact measures for SAR and SLX.

```
mod_1.sar.imp <- impacts(mod_1.sar, listw = dist_15.lw, R = 300)
summary(mod_1.sar.imp)
```

Impact measures (lag, exact):

	Direct	Indirect	Total
per_mixed	0.0004939013	0.010385844	0.010879745
per_asian	-0.0001224192	-0.002574253	-0.002696672
per_black	-0.0006142789	-0.012917166	-0.013531445
per_other	0.0028294759	0.059498722	0.062328198
per_nonUK_EU	0.0012791011	0.026897166	0.028176267
per_nonEU	0.0026523198	0.055773451	0.058425770
log(POPDEN)	0.0267960076	0.563471199	0.590267206

=====

Simulation results (variance matrix):

Direct:

Iterations = 1:300
Thinning interval = 1
Number of chains = 1
Sample size per chain = 300

1. Empirical mean and standard deviation for each variable,
plus standard error of the mean:

	Mean	SD	Naive SE	Time-series SE
per_mixed	0.0005168	0.0021363	1.233e-04	1.233e-04
per_asian	-0.0001284	0.0001703	9.834e-06	9.834e-06
per_black	-0.0006044	0.0003390	1.957e-05	1.957e-05
per_other	0.0027862	0.0011443	6.607e-05	8.115e-05
per_nonUK_EU	0.0012862	0.0005662	3.269e-05	2.910e-05
per_nonEU	0.0026916	0.0005228	3.019e-05	3.019e-05
log(POPDEN)	0.0269022	0.0038502	2.223e-04	2.223e-04

2. Quantiles for each variable:

	2.5%	25%	50%	75%	97.5%
per_mixed	-3.418e-03	-0.0008815	0.0004794	2.083e-03	4.509e-03
per_asian	-4.828e-04	-0.0002322	-0.0001293	-2.203e-05	1.944e-04
per_black	-1.201e-03	-0.0008450	-0.0006120	-3.966e-04	4.135e-05
per_other	6.681e-04	0.0019826	0.0027552	3.581e-03	5.097e-03
per_nonUK_EU	9.799e-05	0.0009593	0.0012686	1.636e-03	2.565e-03
per_nonEU	1.672e-03	0.0023237	0.0026867	3.014e-03	3.738e-03
log(POPDEN)	1.983e-02	0.0242592	0.0269936	2.927e-02	3.477e-02

=====

Indirect:

```

Iterations = 1:300
Thinning interval = 1
Number of chains = 1
Sample size per chain = 300

```

1. Empirical mean and standard deviation for each variable,
plus standard error of the mean:

	Mean	SD	Naive SE	Time-series SE
per_mixed	0.010778	0.048364	0.0027923	0.0027923
per_asian	-0.002961	0.004097	0.0002366	0.0002366
per_black	-0.013377	0.008198	0.0004733	0.0004733
per_other	0.061068	0.029025	0.0016758	0.0016758
per_nonUK_EU	0.027942	0.014016	0.0008092	0.0009475
per_nonEU	0.059964	0.020931	0.0012084	0.0012084
log(POPDEN)	0.588307	0.130589	0.0075396	0.0066601

2. Quantiles for each variable:

	2.5%	25%	50%	75%	97.5%
per_mixed	-0.074574	-0.020006	0.009793	0.042135	0.1026677
per_asian	-0.011613	-0.005154	-0.002918	-0.000434	0.0041665
per_black	-0.030361	-0.018823	-0.012962	-0.007967	0.0009654
per_other	0.014982	0.041674	0.057886	0.077804	0.1289661
per_nonUK_EU	0.002507	0.020833	0.026776	0.036080	0.0566664
per_nonEU	0.031002	0.046488	0.055934	0.068925	0.1085468
log(POPDEN)	0.384109	0.497057	0.584798	0.656872	0.9202922

=====

Total:

```

Iterations = 1:300
Thinning interval = 1
Number of chains = 1
Sample size per chain = 300

```

1. Empirical mean and standard deviation for each variable,
plus standard error of the mean:

	Mean	SD	Naive SE	Time-series SE
per_mixed	0.01130	0.050442	0.0029122	0.0029122
per_asian	-0.00309	0.004259	0.0002459	0.0002459

per_black	-0.01398	0.008509	0.0004913	0.0004913
per_other	0.06385	0.030027	0.0017336	0.0017336
per_nonUK_EU	0.02923	0.014502	0.0008373	0.0009764
per_nonEU	0.06266	0.021319	0.0012309	0.0012309
log(POPDEN)	0.61521	0.132209	0.0076331	0.0073519

2. Quantiles for each variable:

	2.5%	25%	50%	75%	97.5%
per_mixed	-0.078305	-0.020867	0.010305	0.0436707	0.106898
per_asian	-0.012144	-0.005456	-0.003084	-0.0004561	0.004375
per_black	-0.031577	-0.019618	-0.013483	-0.0084129	0.001006
per_other	0.015752	0.043799	0.060472	0.0814409	0.133176
per_nonUK_EU	0.002605	0.021835	0.028154	0.0375371	0.059010
per_nonEU	0.032868	0.048889	0.058619	0.0720005	0.111944
log(POPDEN)	0.407313	0.527129	0.614030	0.6865889	0.947818

For SLX, you can just interpret the coefficients. Impacts will give you the same results.

4) Is SAR the right model choice or would you rather estimate a different model?

- f) How do results change once you specify a spatial Durbin model?
- g) Please calculate and interpret the impacts for a spatial Durbin model.

6.2 Life Expecatancy in Germany

Below, we read and transform some characteristics of the [INKAR](#) data on German counties.

```
load("_data/inkar2.Rdata")
```

Variables are

Variable	Description
“Kennziffer”	ID
“Raumeinheit”	Name
“Aggregat”	Level
“year”	Year
“poluation_density”	Population Density
“median_income”	Median Household income (only for 2020)

Variable	Description
“gdp_in1000EUR”	Gross Domestic Product in 1000 euros
“unemployment_rate”	Unemployment rate
“share_longterm_unemployed”	Share of longterm unemployed (among unemployed)
“share_working_industry”	Share of employees in undustrial sector
“share_foreigners”	Share of foreign nationals
“share_college”	Share of school-finishers with college degree
“recreational_space”	Recreational space per inhabitant
“car_density”	Density of cars
“life_expectancy”	Life expectancy

And we get the respective county shapes:

```
kreise.spdf <- st_read(dsn = "_data/vg5000_ebenen_1231",
                         layer = "VG5000_KRS")
```

```
Reading layer 'VG5000_KRS' from data source
`C:\work\Lehre\Geodata_Spatial_Regression_short\_data\vg5000_ebenen_1231'
using driver 'ESRI Shapefile'
Simple feature collection with 400 features and 24 fields
Geometry type: MULTIPOLYGON
Dimension:      XY
Bounding box:  xmin: 280353.1 ymin: 5235878 xmax: 921261.6 ymax: 6101302
Projected CRS: ETRS89 / UTM zone 32N
```

6.2.1 1) Merge data with the shape file (as with conventional data)

```
# Merge
inkar_2020.spdf <- merge(kreise.spdf, inkar.df[inkar.df$year == 2020, ],
                           by.x = "AGS", by.y = "Kennziffer")
```

6.2.2 2) Create a map of life-expectancy

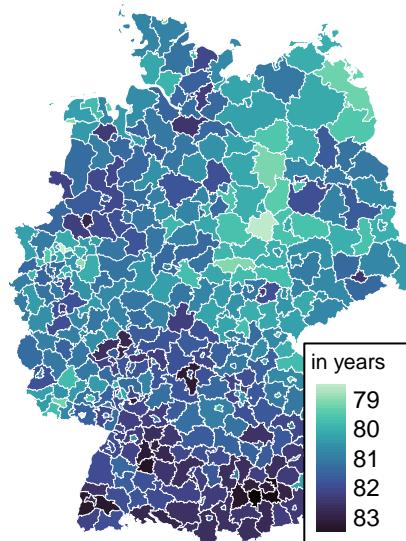
```
cols <- viridis(n = 100, direction = -1, option = "G")

mp1 <- tm_shape(inkar_2020.spdf) +
```

```
tm_fill(col = "life_expectancy",
        style = "cont", # algorithm to def cut points
        palette = cols, # colours
        stretch.palette = TRUE,
        title = "in years"
    ) +
tm_borders(col = "white", lwd = 0.5, alpha = 0.5) +
tm_layout(frame = FALSE,
          legend.frame = TRUE, legend.bg.color = TRUE,
          legend.position = c("right", "bottom"),
          legend.outside = FALSE,
          main.title = "Life expectancy",
          main.title.position = "center",
          main.title.size = 1.6,
          legend.title.size = 0.8,
          legend.text.size = 0.8)
```

mp1

Life expectancy



6.2.3 3) Choose some variables that could predict life expectancy. See for instance the following paper.

6.2.4 4) Generate a neighbours object (e.g. the 10 nearest neighbours).

```
# nb <- poly2nb(kreise.spdf, row.names = "ags", queen = TRUE)
knn <- knearneigh(st_centroid(kreise.spdf), k = 10)
```

Warning: st_centroid assumes attributes are constant over geometries

```
nb <- knn2nb(knn, row.names = kreise.spdf$ags)
listw <- nb2listw(nb, style = "W")
```

6.2.5 5) Estimate a cross-sectional spatial model for the year 2020 and calculate the impacts.

Use a spatial Durbin Error model

```
# Spec formula
fm <- life_expectancy ~ median_income + unemployment_rate + share_college + car_density

# Estimate error model with Durbin = TRUE
mod_1.durb <- errorsarlm(fm,
                           data = inkar_2020.spdf,
                           listw = listw,
                           Durbin = TRUE)

summary(mod_1.durb)
```

Call:errorsarlm(formula = fm, data = inkar_2020.spdf, listw = listw,
Durbin = TRUE)

Residuals:

Min	1Q	Median	3Q	Max
-1.343988	-0.349564	0.013309	0.333105	1.819014

Type: error

```

Coefficients: (asymptotic standard errors)
                Estimate Std. Error z value Pr(>|z|)
(Intercept)      8.4970e+01 1.4366e+00 59.1460 < 2.2e-16
median_income    5.4013e-04 8.2285e-05  6.5642 5.233e-11
unemployment_rate -3.8970e-01 2.0095e-02 -19.3923 < 2.2e-16
share_college    6.7806e-03 3.2502e-03   2.0862 0.036957
car_density      -3.2042e-03 4.9774e-04 -6.4376 1.214e-10
lag.median_income 4.9282e-04 1.8112e-04   2.7209 0.006510
lag.unemployment_rate -3.4685e-02 4.5454e-02 -0.7631 0.445415
lag.share_college -1.7066e-03 7.0324e-03 -0.2427 0.808256
lag.car_density   -5.2210e-03 1.7540e-03 -2.9766 0.002915

```

Lambda: 0.57895, LR test value: 48.146, p-value: 3.9563e-12

Asymptotic standard error: 0.069524

z-value: 8.3274, p-value: < 2.22e-16

Wald statistic: 69.345, p-value: < 2.22e-16

Log likelihood: -305.6855 for error model

ML residual variance (sigma squared): 0.26001, (sigma: 0.50991)

Number of observations: 400

Number of parameters estimated: 11

AIC: NA (not available for weighted model), (AIC for lm: 679.52)

```

# Calculate impacts (which is unnecessary in this case)
mod_1.durb.imp <- impacts(mod_1.durb, listw = listw, R = 300)
summary(mod_1.durb.imp, zstats = TRUE, short = TRUE)

```

Impact measures (SDEM, glht, n):

	Direct	Indirect	Total
median_income	0.0005401288	0.0004928219	0.001032951
unemployment_rate	-0.3896966870	-0.0346850573	-0.424381744
share_college	0.0067806074	-0.0017065824	0.005074025
car_density	-0.0032042421	-0.0052210252	-0.008425267

=====

Standard errors:

	Direct	Indirect	Total
median_income	8.228463e-05	0.0001811231	0.0001813201
unemployment_rate	2.009540e-02	0.0454539507	0.0455753453
share_college	3.250157e-03	0.0070323527	0.0069549479
car_density	4.977409e-04	0.0017540485	0.0018942462

Z-values:

	Direct	Indirect	Total
median_income	6.564152	2.7209218	5.6968356
unemployment_rate	-19.392336	-0.7630812	-9.3116518
share_college	2.086240	-0.2426759	0.7295561
car_density	-6.437570	-2.9765569	-4.4478205

p-values:

	Direct	Indirect	Total
median_income	5.233e-11	0.0065100	1.2205e-08
unemployment_rate	< 2.22e-16	0.4454149	< 2.22e-16
share_college	0.036957	0.8082565	0.46566
car_density	1.214e-10	0.0029151	8.6746e-06

6.2.6 6) Calculate the spatial lagged variables for your covariates (e.g. use `create_WX()`, which needs a non-spatial df as input) .

```
# Extract covariate names
covars <- attr(terms(fm), "term.labels")

w_vars <- create_WX(st_drop_geometry(inkar_2020.spdf)[, covars],
                     listw = listw,
                     prefix = "w")

inkar_2020.spdf <- cbind(inkar_2020.spdf, w_vars)
```

6.2.7 6) Can you run a spatial machine learning model? (for instance, using `randomForest`)?

```
library(randomForest)
```

Warning: package 'randomForest' was built under R version 4.3.2

`randomForest` 4.7-1.1

Type `rfNews()` to see new features/changes/bug fixes.

```

# Train
rf.mod <- randomForest(life_expectancy ~ median_income + unemployment_rate + share_college
                        + w.median_income + w.unemployment_rate + w.share_college + w.car_density,
                        data = st_drop_geometry(inkar_2020.spdf),
                        ntree = 1000,
                        importance = TRUE)

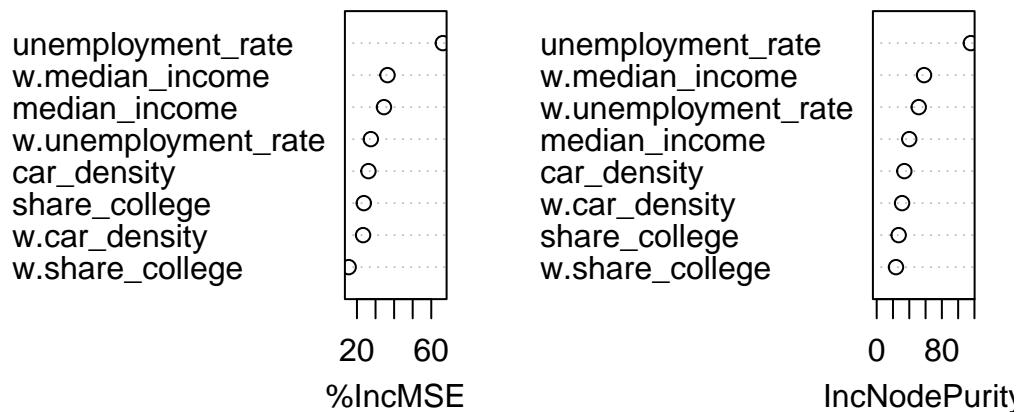
# Inspect the mechanics of the model
importance(rf.mod)

```

	%IncMSE	IncNodePurity
median_income	34.53254	39.96644
unemployment_rate	66.22250	115.57589
share_college	23.70963	26.98842
car_density	26.18910	33.92903
w.median_income	36.44083	58.15672
w.unemployment_rate	27.47648	51.63296
w.share_college	15.54756	23.69628
w.car_density	23.29674	31.15653

```
varImpPlot(rf.mod)
```

rf.mod



You could even go further and use higher order neighbours (e.g. `nblag(queens.nb, maxlag = 3)`) to check the importance of direct neighbours and the neighbours neighbours and so on ...

```
# Create higher order NB object
listw.lag <- nblag(nb, maxlag = 3)

# Create listwise of 1st, 2nd and 3rd order neighbours
listw.lw1 <- nb2listw(listw.lag[[1]], style = "W")
listw.lw2 <- nb2listw(listw.lag[[2]], style = "W")
listw.lw3 <- nb2listw(listw.lag[[3]], style = "W")

# Create lagged X
w_vars2 <- create_WX(st_drop_geometry(inkar_2020.spdf)[, covars],
                      listw = listw.lw2,
                      prefix = "w2")

w_vars3 <- create_WX(st_drop_geometry(inkar_2020.spdf)[, covars],
                      listw = listw.lw3,
                      prefix = "w3")

inkar_2020.spdf <- cbind(inkar_2020.spdf, w_vars2, w_vars3)

# Train
rf.mod <- randomForest(life_expectancy ~ median_income + unemployment_rate + share_college +
                         w.median_income + w.unemployment_rate + w.share_college + w.car_density +
                         w2.median_income + w2.unemployment_rate + w2.share_college + w2.car_density +
                         w3.median_income + w3.unemployment_rate + w3.share_college + w3.car_density,
                         data = st_drop_geometry(inkar_2020.spdf),
                         ntree = 1000,
                         importance = TRUE)

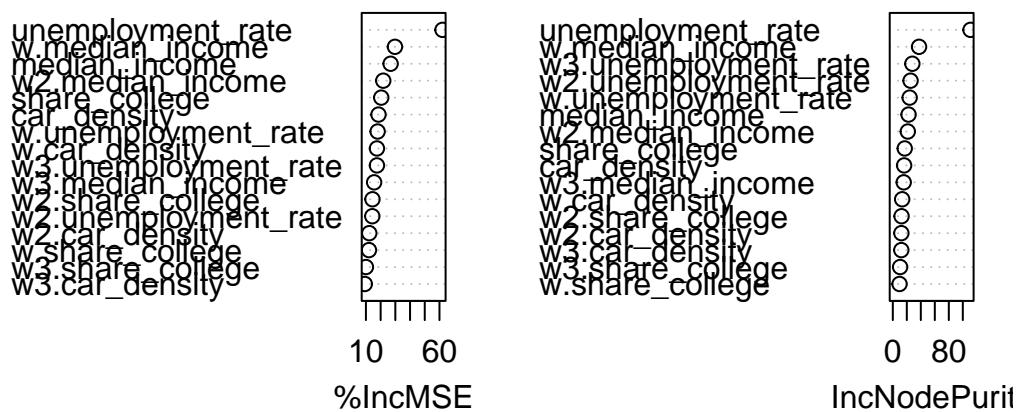
# Inspect the mechanics of the model
importance(rf.mod)
```

	%IncMSE	IncNodePurity
median_income	26.780407	22.49660
unemployment_rate	62.055773	111.00184
share_college	20.424443	17.00946
car_density	18.563498	16.13292
w.median_income	29.735352	37.49061
w.unemployment_rate	17.965260	24.08801

w.share_college	12.068157	9.62800
w.car_density	17.494244	13.36912
w2.median_income	21.788027	21.42375
w2.unemployment_rate	14.326078	25.35485
w2.share_college	14.447565	12.49382
w2.car_density	12.275740	12.23329
w3.median_income	15.581895	15.57669
w3.unemployment_rate	17.482982	27.89588
w3.share_college	10.004782	10.44678
w3.car_density	9.239005	12.14852

```
varImpPlot(rf.mod)
```

rf.mod



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